UNIVERSIDAD SAN FRANCISCO DE QUITO USFQ

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Logrolling and strategic preferences for high-transaction-cost legislative dynamics: A quid pro quo level-k approach.

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RESUMEN

El presente artículo presenta una propuesta para agrupar conceptos de distintas ramas del estudio de la economía, como el logrolling y la teoría de niveles de razonamiento-k, con la finalidad de argumentar cómo podrían variar las preferencias de los legisladores en un intercambio de votos mutuamente beneficioso. Se explora qué formas podrían tener las funciones de utilidad bajo contextos de costos de transacción como la distancia ideológica y la demora en el procesamiento de una propuesta de ley, precisando identificar cuáles serían las nociones con las que se podría argumentar que un nivel k superior podría hacer que los legisladores tienden a votar en favor de una coalición. Se incorpora, del mismo modo, una lógica de logroll a través de la probabilidad condicional que una misma coalición se forma por dos periodos consecutivos. Conjuntamente, se hace un recorrido histórico del estudio del logrolling en la economía política public choice y las dinámicas legislativas, considerando aspectos psicosociales de la ciencia comportamental.

Palabras clave: Logrolling,, Intercambio, Votación, Legislador, Coalición, Niveles-k, Razonamiento, Propuesta, Legislativo.

ABSTRACT

The present article introduces a proposal to consolidate concepts from different branches of economics studies, such as logrolling and the theory of levels of k-reasoning, aiming to argue how legislators' preferences might vary in a mutually beneficial exchange of votes. It explores the possible forms that utility functions could take under transaction cost contexts such as ideological distance and delay in the processing of a bill proposal. It points out the arguments suggesting that a higher k-level could lead legislators to tend toward voting in favor of a coalition. Additionally, it incorporates a logrolling logic by means of the conditional probability that the same coalition forms over two consecutive periods. Simultaneously, the article provides a historical overview of logrolling studies in public choice political economy and legislative dynamics considering psychosocial and behavioral social-choice aspects.

Key words: Logrolling, Vote-trading, Legislator, Coalition, K-levels, Reasoning, Proposal, Legislative.

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Introduction

When studying economic transactions in the legislative procedure, the currency displayed can be understood as the legislators votes on a specific issue. The procedure, as it is known, is not the same for every country or congress. But it can be identified a mechanism to classify and analyze the voting process in this sphere using economic concepts to reach a major understanding on the nature and the motives of an individual vote where a mutually beneficial environment can be reached. The intention of this paper is to propose a more rigorous and detailed theoretical approach to comprehend these systems in the legislative scenario where cooperation is the main goal of the individuals whose votes are displayed.

The core is to describe a situation where the legislative logrolling reflects a level-k adaptation in the formation of coalitions or voting strategies that could be studied within a rule of majority voting. The importance of these conjectures for a determined context can vary within the electoral rules that a congress reinforces or the institutional arrangements of the country´s constitution, but this analysis seeks to describe fundamental relationships between the votes of the legislators, the incentives that each of them have to vote in a certain way and the transaction costs that they can confront. To determine its mechanism process, the public choice concept of logrolling is used to study the preferences of the legislators and mingle their decisions.

Logrolling is described as a situation where one legislator votes in favor of a specific project or policy in exchange of a reciprocatively action in the future. This process is described as procedures that generate improving outcomes from “nonPareto optimal alternative towards a Pareto optimal one” (Cheung 2012, p.62). The happening of this type of vote trading is characterized by legislators that tacitly understands each other’s interests and relies in the on subtle signals and reciprocal understandings among lawmakers. Informal networks and implicit agreements are part of the legislative settings for individual preferences.

Linking legislators' preferences so that they are recursive may involve an additional step in

the aim of voting in the same way on the bill under discussion. However, this is only one type of transaction cost that relates to any kind of disagreements between legislators. Other transaction costs (TC) that can shape these preferences may include the duration of the legislative process, the non-formation of coalitions in the congress and the partisan and ideological preferences a legislator may have, especially if they are part of a representative caucus of a political party. Concepts from strategic game theory for vote exchange situations are utilized with an adaptation of "level-k reasoning models" to establish qualitive magnitudes. Above all, this modelling tries to emphasize the allocation process of resources and favors, creating dynamics within political systems that are important to study if understanding human behavior is an academic grail.

Literature Review

The study of legislative decision-making spans numerous fields within social choice theory. Formal investigatory approaches have sought to focus on contexts within Anglo-Saxon countries, evaluating congressional behavior and seeking to determine preferences regarding voting for or against proposed legislation. The stated objective of studying variations in legislative voting aims to explore additional causes or impacts of factors external to the voting process.

For instance, Chappell Jr (1981) found no evidence indicating that congressional votes have any dependency on private financial interests. In "*Conflict of interest and congressional voting: A note*", he discusses how conflicts of interest among legislators hold significant weight in their voting behavior through a logit econometric model. However, factors such as ideology or the urban area of the legislator could influence their decision-making. Similarly, Kau & Robin (1979) found alike results regarding individual interest and voter ideology using a linear probability regression model. Moreover, Laband (1988) highlights transaction costs in the competitive design of the legislature through ordinary least squares regression, he finds that the duration of legislative sessions could be affected by legislative majorities and the level of party domination in the process. This aspect is

important in considering the contribution sought by this manuscript, as transaction costs might alter legislators' decisions and preferences in contexts of uncertainty or delay in the passage of laws. Wood & Bohte (2004) also consider political transaction costs in administrative design, concluding that temporal delays in coalition formation could manipulate the formation of federal administrative agencies during 1879-1988 in the United States. This serves as a complementary study to the theory developed for legislative elections up to that period.

For societies under democratic regimes, considering voting rules that conjecture legislative decisions is crucial for understanding the electoral outcomes of legislators and their impact on a bill. Sean Ingham developed a game theory model in 2016 in his paper "Social Choice and Popular Control" to illustrate the importance of legislative majorities in achieving political objectives through law passage. Ensuring that legislators' preferences are expressed in each vote requires voting schemes capable of reflecting honest expression of individual preferences. Gibbard (1973) provided arguments for validating such schemes as dominant strategies compared to their alternatives, particularly in non-trivial situations. The starting point for describing a mutually beneficial situation in a country's legislative voting should focus on internalizing negative externalities that may render a logrolling situation mutually beneficial. Bernholz (2012) conceives these negative effects in cyclic social preference formations as problematic for a Pareto optimal point unless legislators can engage in logrolling in each vote. Though, Riker (1973) and Uslaner (1975) have previously analyzed the paradox of vote trading, highlighting the conditions and possibility of cyclic majorities. Despite this, Joe Oppenheimer argues in "*Outcomes of Logrolling in the Bargaining Set and Democratic Theory: Some Conjectures*" that logrolling mechanisms always entail a mutually beneficial exchange of votes. Clifford, Carrubba, and Volden (2000) contribute to the literature by indicating an electoral need for districts representing these legislators, as politicians would prefer universal coalitions *ex ante* to ensure minimal cooperation. They also find that additional members contingent on a coalition can increase costs and reduce benefits.

The "all-inclusive logroll" situation was pivotal in the study of legislative dynamics for political science and public-choice political economy. However, it wasn't until the Nobel laureate in economics, James J. Buchanan, along with his co-author from the Virginia school, Gordon Tullock, started describing logrolling as a pervasive political process applicable in specific contexts in their book "*The Calculus of Consent, Logical Foundations of Constitutional Democracy*." Additionally, Buchanan and Tullock analysis advanced the mathematical modeling of conditions and assumptions for conceiving voting models. McKelvey (1979) explores general conditions of intransitivity in votes within balanced institutional orders, implying that constitutional orders shape sustainable Pareto-optimal solutions. Tullock (1962) suggested that this stability also depends on the probability of forming majorities that include the largest number of participants. Eneloy (1986) and Groseclose & Snyder (1996) incorporated formal models to lend weight to this argument. The concept of risk aversion in legislators is also pivotal in coordinated voting since having risk-averse voters in these contexts helps support the hypothesis that logrolling can form.

Buchanan and Tullock posit that vote allocation after agreements represents the most desirable vote exchange or trade. Similarly, they theorize on efficiency properties to find an optimal outcome resolving indeterminacies under a majority voting rule. Each voter's decision indicates the direction of their preferences, but not the intensity; however, vote trading could resolve this final uncertainty. Arthur F. Bentley had previously analyzed logrolling dynamics, noting that if legislature members couldn't or wouldn't exchange votes, the activities related to that axis of power would cease to function. These arguments were part of his treatise "*The Process of Government*" almost a century ago.

On another front, Casella, and Palfrey (2019) sought to study the dynamics of vote exchange for multiple law proposals where two laws are voted on as a group for a finite number of proposals with separable legislator preferences. Most recent studies within the fields of experimental and behavioral economics incorporate the concept-solution of the 'level-k-reasoning model' in evaluating

the most sincere responses to a voting process. Ana Bassi (2015) showed that these responses did not always align with the best-response strategies, because it depends on the type of game conducted to reveal that plurality rule-voting encourages sophisticated, but not insincere voting, while the borda-count leads to the highest level of insincere voting. A laboratory experiment that followed a similar line of inquiry is "*Trading Votes for Votes: A Laboratory Study"* conducted by Alessandra Casella and Thomas Palfrey in 2020. They introduced a vote-trading model where voters were paired and exchanged votes whenever mutually beneficial. The dynamics ultimately converged to stable vote allocations and demonstrated that the collected data could hold predictive power for generating projections.

Logrolling in the economic theory

In the neoclassical conception of microeconomics, assumptions are studied, such as comparability, where any economic agent is capable of expressing preference or indifference between combinations of goods. Additionally, these bundles of goods would be ordered and preferred under transitivity, referring to the consistency among the mentioned preferences, ensuring that curves forming the graphical representations of voters' preferences do not intersect. Differentiability ensures that their utility functions can be differentiated to an order assigned by the model. For the analysis of a voting scheme converging to a coordinating equilibrium in the legislative bloc, studying the logrolling phenomenon from microeconomic theory is necessary to observe the dynamics of vote exchanges, as previously defined in existing literature. A logrolling situation occurs when there are two mutually exclusive legislative proposals and there exist legislator preferences in a transitive ordering, meaning one proposal is preferred over another. Let these be:

$$(X, Y) \wedge (Z, W)$$

And the preferences of legislators for each pair are separable, indicating that the way a legislator values a bill is not influenced by the existence of other proposals they would choose regardless of their

interest. Following this formulation, legislators can only choose one proposal from each pair. They would choose X over Y and Z over W, but when both pairs are considered together, the voter prefers the set of issues Y and W over the set of issues X and Z according to Stratman (1997). This means that:

$$X \succ Y$$

$$Z \succ W$$

$$Y\ e\ W \succ X\ e\ Z$$

This situation represents logrolling, where individual preferences in each pair of issues do not align when both pairs are considered together. Social preferences are also defined by the voting rule employed according to Bernholz (1994), and these characterizations may or may not hold, as there could be situations of indifference between proposals. In this case, the simple majority rule governs the voting. To provide a more concise idea about the implication of logrolling and the social gain or loss it carries, an example of a payoff matrix for the decision of three legislators (A, B, C) on two bills (X and Y) is illustrated.

	Legislator A	Legislator B	Legislator C
Issue X	6	-2	-2
Issue Y	-2	-2	6

Table 1: Voting scheme

Table 1 reflects the payoffs legislators would obtain if they supported the proposed bills. These payoffs represent the benefit or utility for the voter when a project passes or is approved by a simple majority. In a preliminary analysis, neither of the two proposals would pass because if we observe option X, legislators B and C would have no interest in voting for that option. The same occurs with Y when analyzing the strategies of A and B. Given this voting behavior, both proposals would have two votes against and one in favor, resulting in zero total utility for this society of voters, as explained by

Mauerberg, Strachman, and Reami (2013). However, if legislators A and C cooperate through logrolling, vote swapping would allow them to generate an alternative Pareto-optimal outcome. This would occur because A would agree to vote in favor of proposal Y, and voter C would do the same with X. In this case, both bills would be accepted, and the total social benefit would be 4, signifying an improvement. Summarizing this example with the economic definitions previously provided for the logrolling dynamics, legislators A and C would have a change in their strategic preferences to choose differently from what they would have chosen previously, as they would gain a benefit by doing so. It is necessary to note that this dynamic is not always so straightforward, as there might be transaction costs not internalized by legislators, such as pre-formation of coalitions, the impossibility for legislators to reach mutually beneficial agreements, and an imminent problem of a voting cycle.

	Legislator A	Legislator B	Legislator C	Total
Issue X	6	−2	−2	2
Issue Y	−2	−2	6	2
SB	4	−4	4	4

Table 2: Logrolling

	Legislator A	Legislator B	Legislator C	Total
Issue X	6	−4	−4	−2
Issue Y	−4	−4	6	−2
SB	2	−8	2	−4

Table 3: Logrolling for a negative sum-game

If the payments to legislators when voting for *X* or *Y* were different, let's say instead of the negative -2, we had a -4 for each legislator, the social benefit would still be negative. This is shown in Table 3, indicating that rather than finding a better outcome, the situation would worsen. Hence, logrolling would align with cardinal preferences, prioritizing characteristics based on their intensity rather than their specific position. What is originally stipulated in a simple majority voting, as studied in the literature of spill-over effects and theorized for these contexts by Gordon Tullock in his paper "*Problems of Majority Voting*." An additional crucial consideration for studying these dynamics is the potential for the formation of infinite behavioral voting cycles, which would be a clear signal of the intransitivity of preferences. According to Bernholz (1973), the individual transitivity of legislator preferences does not guarantee aggregated transitivity. To theorize about the voting paradox in logrolling contexts, many scholars focus on the inefficiencies in collective decision-making processes that arise when the aggregation of individual preferences does not reflect an option preferred by the majority of legislators. This might even result in a situation where social benefit consecutively remains negative despite some legislators accruing individual benefits (Riker & Brams 1973).

Level-k-reasoning approach

Part of the paper's objectives, in addition to tracing the historical evolution in the study of logrolling, is to provide an approach from different branches of economic study, such as public-choice political economy and behavioral economics, to this phenomenon within a context of high transaction costs that can be modeled to coordinate political decisions. Therefore, a relatively modern conception is employed to reach equilibriums in social choice, known as the "level-k-reasoning" theory of cognitive hierarchy, wherein participants in these games consider their peers' strategies as more or less akin to their own. The level-k-reasoning framework offers a means to achieve agreements beneficial to their individual or collective interests.

The limited rationality of legislators in making strategic decisions is contingent upon the political dynamics within legislative contexts. Incumbents have incentives to find solutions to pass proposed legislation. This involves coordinating actions in advance to achieve beneficial agreements within a potential coalition. In legislative decision-making, akin to various other aspects of social choice theory, decision-makers do not possess infinite reasoning capacity to rationally choose their best option by anticipating the actions of others. Legislators have a limited level of understanding regarding their peers' actions, which can influence their final vote.

The level-k reasoning theory proposes distinct levels for players to think that they have a "strategic reasoning with one more step than the others" (Zhang, 2021, p.5). It is theorized that at level 0, participants in the voting process will make decisions solely based on their personal preferences without considering the actions of other legislators. These players might make decisions randomly or according to a predetermined pattern, disregarding the strategies of other voters. Conversely, level 1 players assume that level 0 players act randomly and try to predict their actions, shaping their decisions based on what they know so far. However, these players do not take into account that the decisions of other players (i.e., players of different levels) are also conditioned by their own. Level 2 players assume that some players might be level 1 and seek to predict their moves, while level 3 players do the same

with level 2 players. This continues to escalate levels. Players presume each lower level possesses some degree of rationality or relatively predictable behavior. This behavior is part of decision-making models where some agents view others as less sophisticated agents than themselves, as indicated by Clippel et al. (2016), and can even result in better equilibriums than the well-known Nash Equilibrium.

Applied to legislative bargaining, the concept aims to incorporate the idea that higher-level decision-makers would attempt to predict the actions of lower-level legislators and adjust their strategies accordingly, generating variations in their preferences within contexts with limited information. This process would continue until legislators are completely certain of their peers' decisions, as further prediction becomes unnecessary. In a logrolling context, it would be possible to model a scenario with multiple legislators, each with different levels of knowledge and preferences regarding the outcome of legislative proposals. Furthermore, additional factors influencing preference formation could be implemented due to the costs and benefits that legislators may obtain through the outcome of legislative approval. Indeed, Bassi & Williams (2014) propose in their paper "*Examining Monotonicity and Saliency Using Level-k Reasoning in a Voting Game*" the inclusion of financial incentives for complex decision situations through a laboratory experiment of fifteen sessions to observe students' focal decisions regarding the transitive property of monetary gain. They use a prior model of behavioral decisions based on Nagel (1995), who highlights theoretical solution concepts in a study on the importance of levels of behavioral reasoning. Zhang (2021) also adapts the general level-k model to study student decisions regarding school-choice and introduces a *naive belief setting*, where students at a sophisticated level of reasoning report their true preferences.

When players face transaction costs, such as in the proposed core, they can rely on these hierarchical levels of cognitive reasoning to find solutions and agreements. In the case of TC existing due to ideological distortions, legislators could be represented by levels reflecting their ideological affinity on a scale where higher levels indicate greater ideological affinity. A legislator with a lower level might be more willing to form coalitions with those who share similar ideologies, while a

legislator with a higher level might be more willing to negotiate with those of lower ideological levels if sufficient benefits are obtained. In instances of difficulty in generating coalitions, a higher-level legislator might be willing to associate with legislators of different levels if the sum of their combined levels exceeds a certain threshold, reflecting coalition formation despite differences. Similarly, in contexts where TC is attributed to waiting time in the processing of a bill, a higher-level legislator might be willing to reduce their demand on a specific issue if the resolution time is excessively prolonged, while a lower-level legislator might not be as inclined to quickly compromise on their positions. Such adaptations could be explored through mathematical modeling or the incorporation of specific parameters to provide a more assertive exploration in attempting to theorize these dynamics.

Modelling some conjectures

Introducing high transaction costs to account for the complexities and difficulties that legislators face in reaching agreements, the level-k-reasoning perspective theory could serve as an alternative for examining the understanding of logrolling. To illustrate a scenario wherein legislators exhibit ideological distances and experience changes in their preferences over the time delay for the processing or discussion of a proposal, an attempt is made to model conjectures for the utility functions of legislators, each possessing varying levels of strategic reasoning. These levels are defined on a scale from 0 to K, with K representing the highest level of reasoning. Legislators at level 0 would choose according to their personal preferences, those at level 1 would seek to predict the actions of those at level 0 and adjust their decisions accordingly, and so forth up to level K, as explained before.

Ideological Reasoning

Regarding the ideological distances among legislators, we can employ a utility function that combines ideological proximity and levels of rationality to determine each legislator's inclination towards fostering cooperation and voting similarly. Assuming a legislative framework with N

legislators, each possessing a level of ideological affinity on a scale from 0 to 1, where 1 denotes complete ideological alignment.

$$N = (L_1, L_2, L_3, \dots, L_n)$$

$$I = [0,1]$$

For each legislator, their level of ideological affinity, denoted as Ii, is considered. A legislator with a lower level of affinity Ii would be more inclined to enter into a voting agreement with legislators whose Ij affinities are closer to their own. It can be inferred that as voters are more ideologically aligned, their utility will increase. Legislators with higher levels of rationality might be willing to collaborate with other levels if they perceive sufficient benefits or have the capacity to predict the actions of lower levels, what we could label as ideological rationality. Then, describing the type of function that can be used to center the analysis:

$$e^{-\delta(I_i - I_j)}$$

The proximity or similarity between two values, in this case, the ideological closeness of legislators i and j, when raised to a negative power of the absolute value of their ideological affinity, might yield a result that exponentially decreases as the absolute value increases. The inclusion of a parameter δ enable to adjust the exponential decay rate of the function. With a larger δ, the function decays more rapidly when there is greater ideological proximity, as opposed to a gradual decay with a smaller parameter.

Now, to incorporate the levels of reasoning of the legislators:

$$e^{-\rho(K_i, K_j)\cdot(I_i - I_j)} \qquad (1)$$

$$\rho(K_i, K_j) = \frac{1}{K_i + K_j + r} \tag{2}$$

Where r is a constant that adjust the relative influence of the reasoning level in the ideological affinity perception. For example, the perception of how the other legislator would respond the logroll is adjusted by the other´s legislator k level. Additionally, as the levels of reasoning increase, the exponential decay rate decreases. This implies that legislators with higher levels of reasoning perceive ideological differences as less significant compared to those with lower levels. It is important to mention that this formulation does not conceives two legislators with the exact same ideology, since that could be difficult to find. This means that the difference between $I_i$and I_j cannot be 0.

Therefore:

$$U_i = I_i \cdot K_i \cdot \sum_{j \neq i} e^{-\rho(K_i, K_j) \cdot (I_i - I_j)}$$

$$U_i = I_i \cdot K_i \cdot \sum_{j \neq i} e^{-\frac{I_i - I_j}{K_i + K_j + r}} \tag{3}$$

The multiplication of the ideological affinity for legislator i with their level of rationality weighted by the exponential sum of ideological similarities between legislators and their respective levels of reasoning accounts for the utility derived from logrolling. The greater the ideological affinity and the level of reasoning of legislator i, the higher the utility for that legislator. Regarding the summation used, it considers all other legislators different from i, taking into account their ideological affinity, their level of reasoning, and their perception of conceding on an agreement.

Delay time under k-reasoning

In this scenario, legislators with higher level-k might adjust their demands or preferences on a legislative proposal if they perceive the resolution time as excessively prolonged. They could lean

towards the possibility of compromising on certain aspects and voting favorably to expedite the process. Conversely, those with lower-level k would be less likely to alter their stance as the process extends. Assuming, as in the previous context, a set of N legislators with varying levels of reasoning, an adaptive utility function is proposed to reflect how time affects preferences and the willingness to form alliances when voting.

The adaptation concerning waiting time would involve a gradual adjustment of legislators' preferences and could be conceptualized through a logistic *sigmoid* function to explain how waiting time (T) and the level of reasoning (K_i) would influence a legislator's willingness to compromise in negotiation. In this regard, recognizing that the gained utility would measure the willingness to concede for a joint vote among legislators, it can be inferred that such willingness to agree might increase or decrease depending on the waiting time and the adaptability levels (i.e., reasoning) of legislators. The characterizations of these functions are that they have a domain of all real numbers, with return (response) value commonly monotonically increasing but could be decreasing as well. So, defining the utility function by:

$$U_i = \frac{1}{1 + e^{-a \cdot (K_i - b) \cdot T}} \qquad (4)$$

$$T \in [0,1]$$

Being a and b constants embedded in the function, a determines the slope of the logistic curve, signifying how rapidly the willingness to compromise changes over time and the level of rationality. On the other hand, constant b represents the inflection point where the function changes direction and reflects the level K of reasoning at which the waiting time begins to have a greater effect on the willingness to compromise.

It can be observed that as the waiting time T increases, the function changes its value. When T is small, more agreements can be reached, achieving stability over a broader range. However, as T

increases and the time cost becomes more present, legislators with higher levels of reasoning might opt for a coalition to facilitate logrolling. Legislators with lower levels might be less convinced of this, although the commitment to a future vote may also be crucial. The parameters expressed in the logistic function can be utilized to adjust the shape of the logistic curve (exponential) up to a point where the time and impact for the willingness to form agreements are determined.

It is noteworthy that legislators may have limited information about the preferences and actions of others, which could influence their ability to predict behaviors and form coalitions.

Coordination in the Political Process

After outlining potential forms that legislators' utility functions may take within a coalition-building process, an additional critical step is necessary regarding the situation of logrolling. It is sought, in addition to incorporating levels of reasoning based on voters' cognitive differences, to secure coalitions across two consecutive periods. Thus, if one legislator agrees to vote in alignment with their counterpart, for logrolling to occur, the other legislator must reciprocate in the second period. As described by Smith & Banks (1988), electoral behavior responds to an environment where voters' beliefs hold vital significance. In this scenario, legislators anticipate reciprocal voting in the second period within a strategic negotiation game, considering the incentives that legislators, who were favored in the first period, possess to continue and respond similarly in the second period. Without this, coalitions might form under certain contexts but would not culminate in logrolling. One could argue that as long as logrolling remains feasible, levels of reasoning will influence its formation and structure, including potential costs generated from a negative sum-game, as explained in the section of this paper that describes economic logrolling.

According to Stratman (1997), the costs of an already approved bill must be borne collectively by all participants in the voting process, irrespective of their individual decisions. Moreover, these costs are likely to be felt to a lesser extent among the “final” voters. In the subsequent description

presented, this can be understood as a reassessment of the incentives that legislators have to "respond" to logrolling. A certain degree of stability in logrolling, as Coleman (1967) and Tullock (1981) suggest, can be achieved if there is credibility among the agents involved. Enelow (1986) contributes to the debate by proposing a model where each legislator forecasts the future using a random variable with a linear mean across present proposal alternatives. Nevertheless, in a much simplistic view, to illustrate a logrolling scenario, consider a set of three legislators across two time periods:

$$N = \{L_1, L_2, L_3\}$$

$$P_t = \{t_1, t_2\}$$

The coalition C_{t1} is a subset of the legislator set N, consisting of two members. This implies that in the first period, a coalition comprised of two out of three legislators was established. Considering the possibility of the same coalition forming again in the second period (logrolling), this introduces levels of reasoning using the level-k-thinking framework, along with the conditional probability that the identical coalition will reassemble in the second period:

$$C_{t1} \subseteq N$$

$$|C_{t1}| = 2$$

Over a C_{t1} formation it is specified that $k = \{1,2\}$ and $K = 2$. In the equation above, k refers to the level of the legislator in question and K denotes the highest level of reasoning. The conditional probability would be described as:

$$P(C_{t2} = C_{t1}|C_{t1})$$

$$P_C^{(k)} = \frac{k}{K}$$

Where $P_C^{(k)}$ represents the probability based on the k-level of reasoning for the coalition to reoccur.

$$P(C_{t2} = C_{t1}|C_{t1}) = P_C^{(k)} \cdot P_{IgD}$$

$$P(C_{t2} = C_{t1}|C_{t1}) = P_C^{(k)} \cdot P_{td}$$

$$P_{IgD} = 1 - (I_{L_1} - I_{L_2})$$

$$P_{td} = e^{-\mu(T)}$$

P_{IgD} and P_{td} are the probabilities under TC contexts for ideological distance and time delay, respectively. Ideological distance is normalized within a range from 0 to 1. P_{td} is considered to determine the probability based on the delay (waiting) time on a proposal to see coalition formation, as T decreases—meaning less time required for the approval of a bill—the fraction increases, hence the probability of maintaining the same coalition also increase. T in this case is a proportional fraction between 0 and 1, where 0 is null time of waiting and 1 is a very long time of delay.

Thus:

$$P(C_{t2} = C_{t1}|C_{t1}) = \frac{k \cdot (1 - I_{L_1} + I_{L_2})}{K} \quad (7)$$

$$P(C_{t2} = C_{t1}|C_{t1}) = \frac{e^{-\mu(T)} \cdot k}{K} \quad (8)$$

It is important to note that these probabilities are described to calculate a probabilistic value on C_{t2} for each of the exemplified TCs. The level-k reasoning plays a role in the incidence depending on the level of the legislators in the second period when returning the favor in the vote. It should be noted that the shapes of the functions may differ, and their values will depend on the legislative processes being attained at the time. It could depend on if we metric that in number of sessions for project approval or if it refers to days of delay. The level of reasoning will provide legislators with a modification in their preferences to vote together and achieve a logroll.

The k-levels are influential, on that account, in the utility gained by legislators in a first period

and also in the probability that those benefiting from the first coalition decide to cooperate in a second period, even if it means disrupting the transitivity of preferences regarding legislative proposals: reaching a logrolling logic.

Discussion

The attainment scenario of logrolling is not simplified to a probabilistic approach determined by a specific voting strategy. Rather, it encompasses the evolving conceptions and preferences of legislators in a context where coordination within a coalition mutually benefits them. The assumption of a positive sum-game for logrolling formation is shaped by the voting design and the type of voting rule employed. A discursive argument is presented to justify the form of utility functions, probabilities of a repeated coalition and the strategic reasoning behind why coalitions should form to conceive logrolling. The economic argument, when delving into distinctions in preferences and the incentives legislators have for decision-making, is extended far beyond of psychological determinants and can be attributed to several factors not encapsulated in a cognitive differentiation within legislators' minds. Consequently, additional elements need to be integrated with the level-k theory and its representation under TC to explore logrolling further.

To accelerate the argument that cognitive differences decisively impact stable and reciprocal voting strategies, it is essential to adequately define the environment where vote allocations converge to stable assignments, as described by Casella & Palfrey (2020) within strictly preferred coalitions when vote-trading exists. These allocations may be explained through attention biases, risk aversion, persuasive abilities, and more—all elements extensively studied in behavioral literature to estimate an economic effect not explained by classical utility maximization.

In a complex environment, axioms are indispensable for modeling the real world and deriving falsifiable interpretations. Wilson (1969) develops a model associating votes in *social states* exchange, aligning individual preferences regarding generalized legislative proposals. While experimentation

could explore additional elements to construct indices and probability distributions guiding agents' decisions, studying vote-trading in the presence of TCs, specifically ideological distances and law processing delay time, could provide insights into the implications of different cognitive reasoning levels '*k*' in handling each case. That is why it is necessary to note the attribution of reasoning levels to previsualize possible gains for each agent.

As the argument for the first case was constructed, the fact that a legislator possesses a higher reasoning level allows them to perceive net benefits surpassing distributed costs (Buchanan, 1962), serving as an impetus to vote in favor of an agreement. Clearly, the political context involves communication and some interests in legislative dynamics serve as reflected information or signaling for an explicit understanding of these benefits. This could be the case of a legislator abstaining from a project that doesn't favor them until a peer offers something more valuable. In this way, both would operate with a reasoning level guiding their decisions on whether to concede (despite differing ideologies) or form a coalition outside the legislative realm, focusing more on power distribution and favor return (Hortala-Vallve et. al, 2023).

An attention or persuasion bias could similarly influence vote structuring if a legislator with a higher level has the ability to influence the votes of lower-level legislators. Higher levels of reasoning might be associated with more sophisticated cognitive qualities enabling them to induce others into a situation more favorable to this player. Fortunately, in this modeling, what is more favorable for one player is also beneficial for the rest. However, in a context of temporal delay for the passage or rejection of a bill, the type of voting and the number of legislators in a congress become pivotal. McGann (2019) posits that an institutional aspect could be the communication lines among legislators and the how discipline is the party that they conform. Associating these qualities with varying levels-k of rationality could potentially lend credence to the hypothesis that a higher k might prompt a legislator to express their vote in favor of a coalition and dismantle the threshold incentive contract, described by Caplan (2006) as a minimal political benefit.

The number of legislators and the type of legislative body are crucial in discerning whether it's a "grand-logroll" situation or even an implicit logrolling scenario, where communication among legislators is almost absent. The directions these prescriptions would influence a legislator's decision depend on how much their '*k*' level determines their willingness to concede when the processing of a law is delayed in congress. The institutional cultivation preceding this logrolling, as expressed by François (2003) in their writings on political entrepreneurship, constitutes the existence of stable institutions and communicative processes of preferences. Although, high *k* levels already comprehend this to a greater extent, encouraging them to advocate for cooperation.

Establishing relations between cognitive levels of reasoning and the social characteristics possessed by legislators opens an empirical research avenue to determine whether these elements are significant enough to guide a logrolling situation. The conditionality between two consecutive periods with consequent responses must be a reality for this to occur, hence, cooperative games can be proposed where the probability of a coalition forming is subject to the influence of psychosocial qualities, political philosophical conceptions, and ultimately, the main legislative survival. This paper considers elements such as attention and persuasion biases, institutional design, and the number of legislators to assess if a logrolling situation can be explained by strategic cognitive differences. However, the limitations of these theoretical intuitions and their mathematical modeling approaches are extensive and have been a subject of study in political science for decades. The configurations of legislative *modus operandi* can encompass myriad facets to explain strategic voting, thus the challenge lies in how well the level-k theory predicts vote exchanges.

Further developments and limitations

Within the realm of conceptual analysis and approaches to theoretical level-k reasoning, it is imperative to acknowledge the explanatory limitations inherent in this concept of legislative vote-trading. It is recognizable and evident that numerous factors beyond a cognitive difference among

legislators account for the occurrence of logrolling situations. In the political science literature, it is argued that market dynamics do not singularly shape the voting process and its outcomes. There exists a distinction between how a firm maximizes its profits and how legislative institutions operate, as explained by Weingast & Marshall (1988). Furthermore, the conceptual application concerning transaction costs constitutes a very specific sphere for inferring reasoning levels, as hypothetical scenarios involving exchanges of favors between parties or representative blocs highlight the presence of extra-legislative transactions. These could take the form of favors being bought and sold between political parties, as proposed by Koford (1982), focusing on personal qualities of legislators such as leadership within their blocs. Additionally, situations may arise where legislators are compelled to act in specific ways due to the socio-political environment of a particular country, especially in semi-presidential regimes to bicameralism influences executive and judicial decisions.

The interest groups represented by legislators also warrant consideration when explaining why a voting outcome unfolds in a particular manner. Power distribution issues and inherent incentives to advocate for self-interests present the primary counterarguments against employing cognitive differentiation reasoning to solidify cooperative strategies. Potential infinite cycles in majority voting schemes could also lead to a equalization of reasoning levels, but verifying this hypothesis would necessitate laboratory experiments. Another clarification, concerning transaction cost situations, is that delays in the processing of a law could be an endogenous phenomenon within each voting scheme. In such cases, it wouldn't serve as an explanation to resolve suboptimal equilibriums through reasoning but rather as an additional parameter within the same explanation of logrolling. Various functional forms could be attributed to reasons shaping a vote; however, only one of them is explored and linked within the literature—the legislator considering actions of others at a lower level. Exploring new functions that align with the schematization and assumptions of logrolling, which might not always fall within a positive sum-game, would thus be necessary.

These considerations form part of the constraints encountered when attempting to associate

reasoning levels and logrolling more directly, especially when there are ideological distances and delays in proposing laws. It would be essential in the future to engage in this type of analysis to empirically confirm or, conversely, refute whether this combination of concepts across various economic branches is suitable for explaining legislative decisions.

Conclusions

The voting strategies can be driven by legislators' desire to reach agreements. Previously, this was understood as a situation of logrolling, where vote-trading can be explained by the natural and cognitive incentives of voters, providing formal research opportunities for theoretical and empirical proposals regarding the formation of these coalitions. In this context, an approach stemming from level-k reasoning has allowed us to theorize on how different levels of reasoning influence voting decisions. The described scenario plays a role in the incidence depending on the legislators' level in the second period when returning the favor in voting because it will provide legislators with a modification in their preferences to vote together and achieve a logroll. It's worth mentioning that assumptions arguing that a fixed level of reasoning influences fostering cooperation when legislators have different ideologies or perceive time passing as a detrimental factor in cooperation are not intuitive and may connect with psychosocial qualities of legislators, such as persuasion capacity and the ability to lead legislative blocs.

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