UNIVERSIDAD SAN FRANCISCO DE QUITO USFQ

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Theoretical analysis of the Washboard Effect in Unpaved Roads using numerical modelling

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Theoretical analysis of the Washboard Effect in Unpaved Roads using numerical modelling

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ABSTRACT

This study develops and optimizes a numerical model to simulate the washboard phenomenon on unpaved roads, using granular transport principles and theories from wind-blown sand ripple formation. The model identifies a grain transport mechanism driving ripple formation and emphasizes the significance of critical velocity and initial surface, in initiating undulations. Optimization through Particle Swarm Optimization (PSO) successfully aligns the numerical model with experimental data, particularly in the frequency and amplitude of undulations. The study highlights the need for further validation of saltation length calculations and suggests improvements through image processing to refine the model. The results have practical applications in improving road maintenance, enhancing road safety, and reducing maintenance costs, particularly in regions with unimproved roads. Future research could explore variations in road conditions, vehicle speeds, and grain properties to further refine the model.

Key words: Washboard phenomenon, numerical model, granular transport, ripple formation, particle swarm optimization (PSO), unpaved roads, road maintenance, shear stress.

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INTRODUCTION

The washboard phenomenon refers to the formation of periodic undulations that commonly appear on unpaved roads due to the repetitive passage of vehicles. These undulating patterns, visible in Figure 1, can cause discomfort for drivers and safety risks by reducing tire-road contact, compromising vehicle stability. While the presence of washboard patterns has been widely studied, its exact mechanism of formation remains a subject of ongoing investigation.

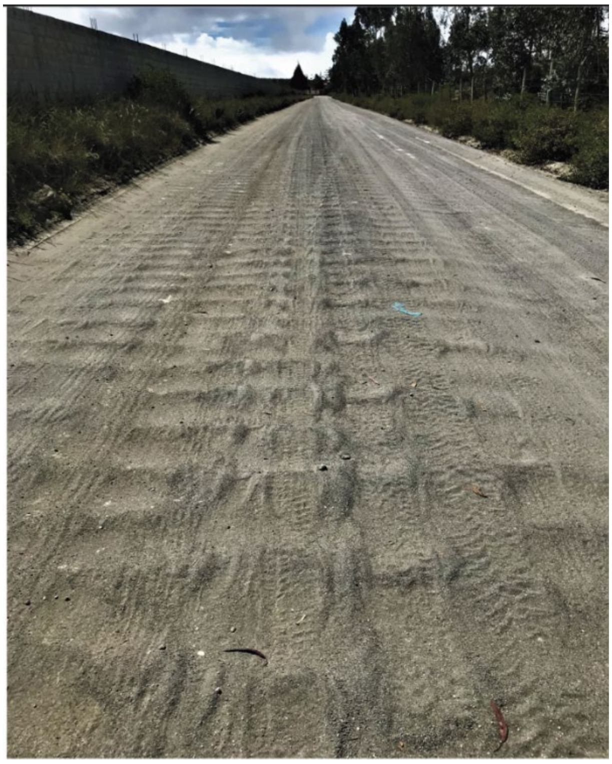

Figure 1 – Washboard Effect on unpaved roads

The phenomenon is generally considered to generate after repetitive passing of vehicles, where the road surface gradually deteriorates with each successive vehicle pass. Early studies began to examine this behavior in the context of granular materials. Identifying a connection between vehicle-induced forces and the development of these undulations. Notably, Relton (1938) was among the first to propose a theoretical model. Highlighting that the vehicle's motion pushes material ahead of the wheel, leading to the formation of ripples. Subsequent work, such as that by K. B. Mather (1963) and G. Riley & R. B. Furry (1973) expanded on these concepts, using experimental and theoretical models to investigate the influence of vehicle speed, weight, and tire properties on wave formation.

Despite all the research, a clear understanding of the fundamental mechanisms behind washboard formation is still lacking. While many studies have linked the formation of ripples to factors such as vehicle speed, weight, and tire inflation, the physics in transport of granular material has not been thoroughly explored in the context of vehicle-induced waves.

This thesis seeks to address the existing gaps in understanding washboard formation by proposing a new approach based in the physics of ripple patterns formed by wind-blown sand. The theoretical model developed by Nishimori & Ouchi (1993) for sand ripple formation will be adapted to simulate the formation of washboard patterns induced by vehicle passage. The aim of this research is to demonstrate that washboard formation can be understood as a granular transport phenomenon, where the wheel acts similarly to wind shear, driving the movement and redistribution of soil particles.

To comprehensively address the washboard phenomenon, this thesis is structured to combine theoretical modeling, experimental validation, and optimization techniques. The initial section presents a detailed description of the adapted granular transport model used to simulate washboard formation. The subsequent Methodology validates this model using experimental results from Ibagón et al. (2023) and a parametric analysis, to explore the system's dynamics. This leads to the Optimization phase, where Particle Swarm Optimization (PSO) is applied to refine the model’s accuracy. Finally, the Results and Discussion analyze the outcomes of the optimized model, while the Conclusions summarize key findings, and propose ideas for future work.

THEORICAL CONSIDERATIONS

According to the previous section, some studies imply particle movement due to wheel displacing material. However, none explicitly frames the mechanism as a transport phenomenon driven by the wheel. Therefore, this research aims to frame washboard formation as a grain transport phenomenon, directly applying theories from ripple formation by wind-blown sand to vehicle-induced undulations. This conceptual framework is not explicitly addressed in prior research, presenting a new perspective on the understanding of the mechanism behind washboard formation.

The proposed theorical model was based on the findings of Nishimori & Ouchi (1993). They developed a discrete model, describing spontaneously formation of ripple patterns. These dynamics of sand grains when released into the air due to wind forces are based on the fundamental processes reported by Bagnold (1974). Highlighting three elementary processes which are: suspension, saltation and surface creep. These processes describe the fundamental physics behind transportation of sand grains and are the basis for the development of the equations described in this model. A simplified version of the model and the interactions is presented in figure 2.

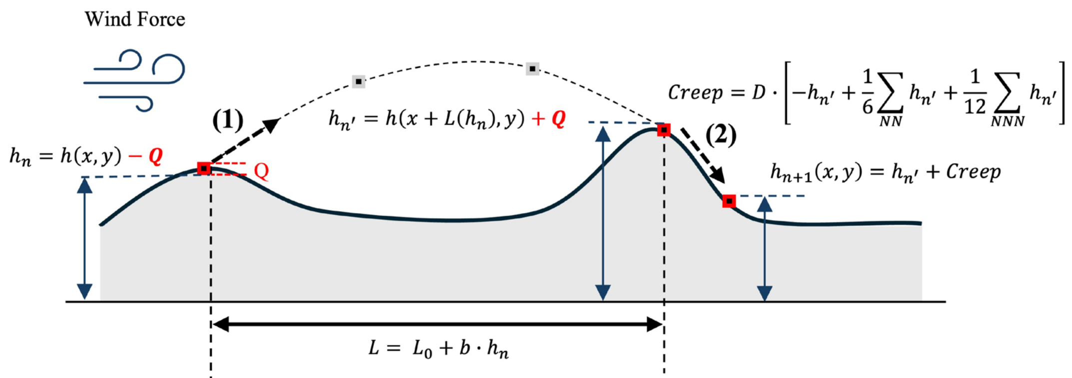

Figure 2 – Theorical Model of Formation of Ripple Patterns by wind-blown sand

As for the elementary processes described, only two were considered, saltation and surface creep. Since, suspension was referred to the process of flying grains that got into the wind and left the system.

First, we have the dynamics of the saltation process, which is described in step **(1)** from figure 2. The saltation step involves the transportation of a height of grains defined as $\boldsymbol{Q}$ from one position into a new position located at a distance $\boldsymbol{L}$ from the original position after one step. Specifically expressed as:

$$\boldsymbol{h_n = h(x, y) - Q}$$

$$\boldsymbol{h_{n'} = h(x + L(h_n), y) + Q}$$

Where $\boldsymbol{L(h_n)}$ is referred to the length of saltation and it is obtained as:

$$\boldsymbol{L = L_o + b \cdot h_n}$$

Where, $\boldsymbol{L_o}$ represents a control parameter proportional to the wind force induced as shear stress at the sand surface. And $\boldsymbol{b}$ represents a coefficient proportional to the average velocity a grain experiences in flight.

Next, the second process refers to surface creep, **(2)** in figure 2. Which takes into consideration the accommodation or relaxation of a local height by the presence of gravity. This relaxation is proportional to a rate of relaxation $\boldsymbol{D}$ and the sum of heights of the nearest and second nearest neighboring heights. Expressed as:

$$\boldsymbol{Creep = D \cdot \left[-h_{n'} + \frac{1}{6}\sum\nolimits_{NN} h_{n'} + \frac{1}{12}\sum\nolimits_{NNN} h_{n'}\right]}$$

Where $\sum_{NN} \boldsymbol{h_{n'}}$ refers to the nearest neighbors and $\sum_{NNN} \boldsymbol{h_{n'}}$ to the second nearest.

The above detailed processes represent the dynamics of a grain on unit time step, simplified in:

$$\boldsymbol{h_{n+1} = h_{n'} + Creep}$$

The Simulation process of this model will be developed in the next section of methodology.

METHODOLOGY

This research begins with the theoretical considerations outlined in the previous section, which provide the foundation for the model and simulation. The simulation process was performed based on the equations described earlier, starting with the generation of an initial random surface with specified dimensions. As proposed in the theory, a periodic boundary condition was applied in both the x and y directions of the surface, ensuring that the total number of grains remained constant. Grains exiting one side of the simulation re-entered from the opposite side, preserving the overall grain distribution. Once the basic interactions were established, different sets of control variables, such as $L_o, b, Q \text{ and } D$, were selected to replicate the expected ripple formation. This led to the initial set of parameters that allowed the evolution of the system to form ripple patterns, as shown in the figure below:

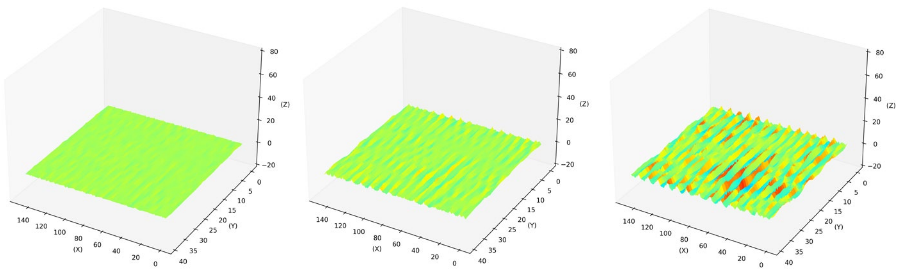

Figure 3 – Evolution of Theorical Model Implementation

After achieving a model capable of representing the evolution of ripple formation, the next step was to validate it using experimental results from Ibagón et al. (2023), obtained through a circular sandy path with an instrumental rotating wheel. To accomplish this, a parametric analysis was proposed using a one-variable-at-a-time approach, as the theoretical model involves multiple variables:

$$Ripple\ Formation = f(\,Initial\ Surface, L_o, b, Q, D\,)$$

Initially, the focus was placed on the initial surface. A comparison was made using a Y-axis cut through the ripple formation at the center of the surface after 80 simulation steps, since the experimental results by Ibagón et al. (2023) only reached 80 laps. The results were compared with three different initial surface conditions: the measured surface from the experimental validation at the 5th lap, a completely flat surface, and a random surface as shown in Figure 4.

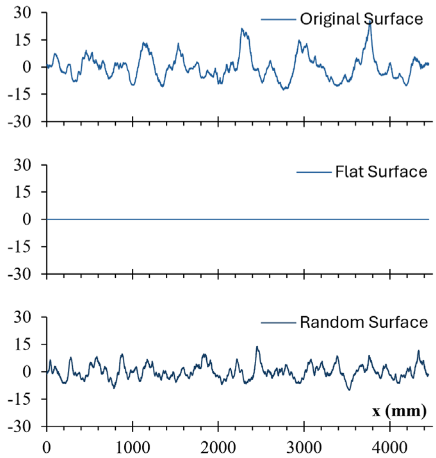

Figure 4 – Initial Surface Comparison

Once the effect of the initial surface was understood, the next step was to analyze the parameters involved in both the saltation and creep processes, starting with the rate of relaxation $\boldsymbol{D}$. It was found in figure 5 that changes in $\boldsymbol{D}$ had minimal effect on the formation of the washboard pattern.

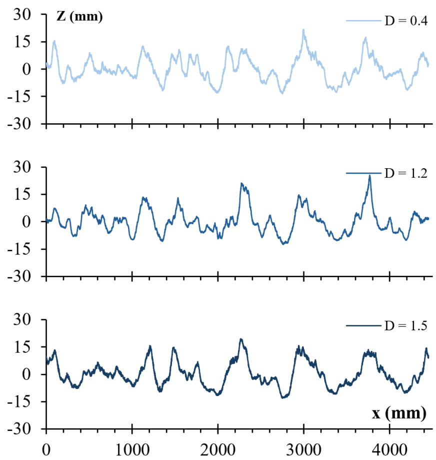

Figure 5 – Parameter D variation

Next, the variability of L_o was analyzed. Considering that the key parameter for washboard formation is the force induced by the vehicle's speed, which is equivalent to the wind velocity in the sand ripple model. Therefor, due to L_o being proportional to the shear force generated by the wheel or wind, the results indicated that this parameter plays a significant role in the formation of washboard patterns as shown in figure 6.

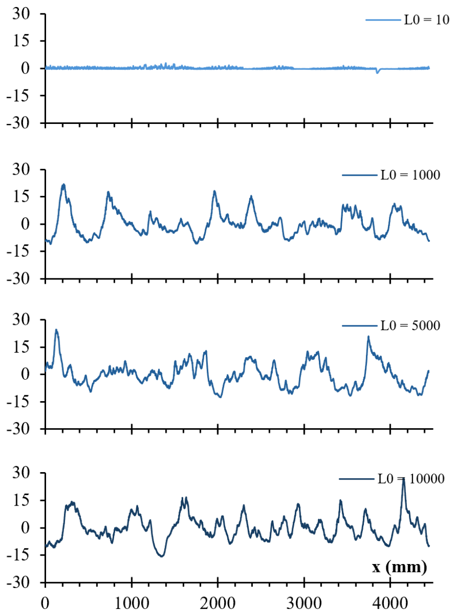

Figure 6 – L0 Variation in Numerical Model

Finally, the influence of the coefficient b, which affects the saltation length, was examined. The analysis highlighted the importance of b in the calculation of the saltation distance and, consequently, in the formation of washboard undulations.

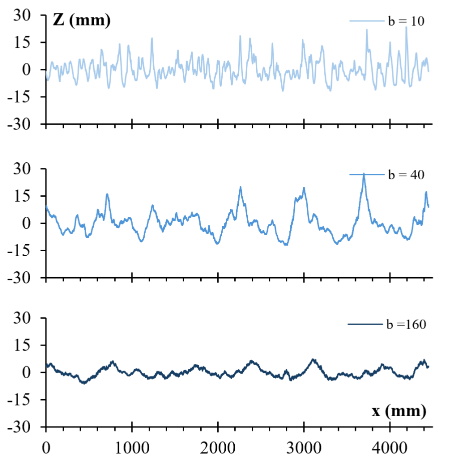

Figure 7 - L0 Variation in Numerical Model

Having established a reliable range of parameters from the parametric analysis, the next phase involved a more comprehensive experimental validation. Manual tuning of the model proved inefficient and prone to error due to the complexity and number of parameters involved. Therefore, optimization algorithms were used to systematically identify the best-fit parameters.

To demonstrate the feasibility of applying a model based on the transportation of grains by wind, driven by wheel-induced forces, Particle Swarm Optimization (PSO) was employed. PSO, a robust optimization algorithm inspired by the social behavior of swarms, was well-suited for exploring the multi-dimensional, non-linear parameter space of the model (Shi & Eberhart, 1998). The goal of PSO was to minimize the error between the experimental profile and the numerical model by adjusting key simulation parameters.

The PySwarms Python library (Lester James V., 2017) was used to implement the PSO algorithm, facilitating the configuration and execution of the optimization process. This library allowed for efficient management of the particle swarm, objective function evaluation, and iteration through optimization cycles. It also provided flexibility in customizing hyperparameters, such as the number of particles and the influence of individual and global best solutions.

In this optimization process, the objective function was defined using Mean Squared Error (MSE), ensuring that the numerical model's output closely matched the experimental data. The parameters optimized by the PSO algorithm were validated for physical relevance, confirming the model's accuracy and reliability.

Additionally, to enhance the comparison between the numerical and experimental results, Fast Fourier Transform (FFT) was used to analyze the frequency and magnitude of the undulations. The FFT converts the profiles from the time domain to the frequency domain, revealing periodic components. The objective function minimized the difference between the experimental and numerical FFTs, focusing on aligning key characteristics like frequency and amplitude, rather than requiring a perfect match. This approach proved effective for validating the numerical model's accuracy.

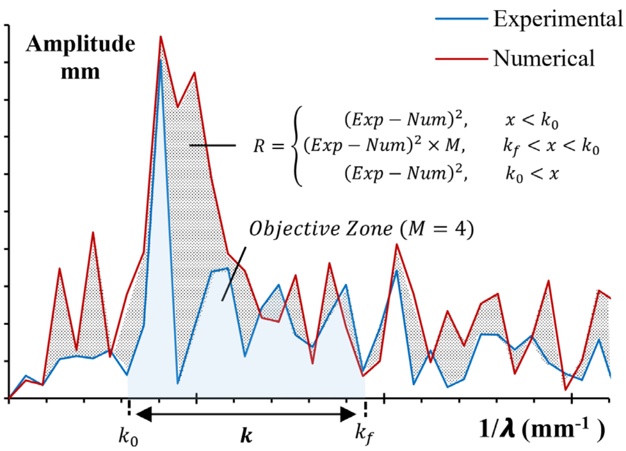

Figure 8 – Objective function definition for PSO

In Figure 8, the objective function is clearly illustrated, with a focus on the defined objective zone that was highly emphasized during the parameter search. This zone specifically targeted the most relevant section of the experimental FFT. For the optimization process, the only parameters adjusted were L_0 *and* b, while $D = 1.2$ *and* $Q = 0.2$ were kept constant throughout each iteration. The minimal effect of changing D, coupled with the fact that Q (in mm) represents the known value for sand grain heights, justified the decision to fix these parameters. Additionally, the initial surface was defined using the 5th passage from the experimental data as the input, providing a reliable starting point for the simulation.

This approach ensured that the optimization focused on the most critical parameters influencing the formation of the washboard effect, ultimately improving the model's ability to replicate experimental results.

RESULTS AND DISCUSSION

The optimization process achieved significant success in refining the numerical model, allowing it to closely match the experimental results. Using the experimental results from the velocity of 2.08 m/s, the optimized parameters led to a strong alignment of the simulated washboard profile with the experimental data. The accuracy of the optimization was confirmed through the Fast Fourier Transform (FFT) analysis, which showed that the frequency and amplitude of the undulations in the numerical model closely mirrored those in the experimental results. This validation through FFT allowed for a precise comparison of the periodic components between the two profiles, ensuring that the optimized model captured the key characteristics of the washboard phenomenon as shown in figure 9.

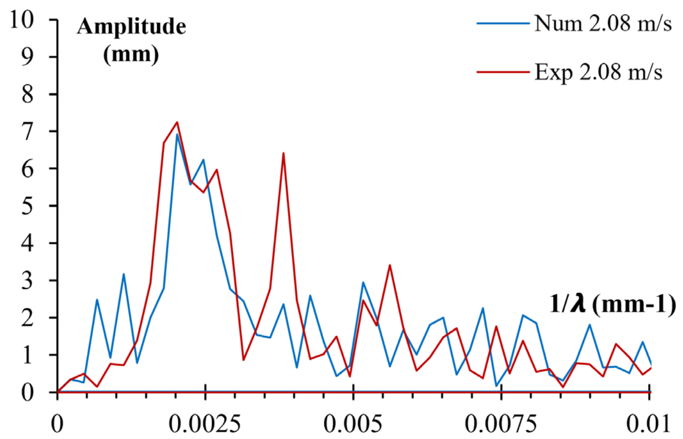

Figure 9 – Experimental and Numerical FFT comparison

Moreover, the comparison of the experimental and numerical profiles, shown in the figure below, illustrates the effectiveness of the optimization process. Not only did the optimized model replicate the final representation of the 80th lap, but it also successfully approximated the evolution of the system across successive steps. This demonstrates that the numerical model, with its optimized parameters, can simulate the washboard phenomenon over time, from initial ripple formation to the stabilization of undulations. The optimized parameters were: $D = 1.2,\ Q = 0.2,\ b = 50.94\ and\ L_0 = 4983.85$

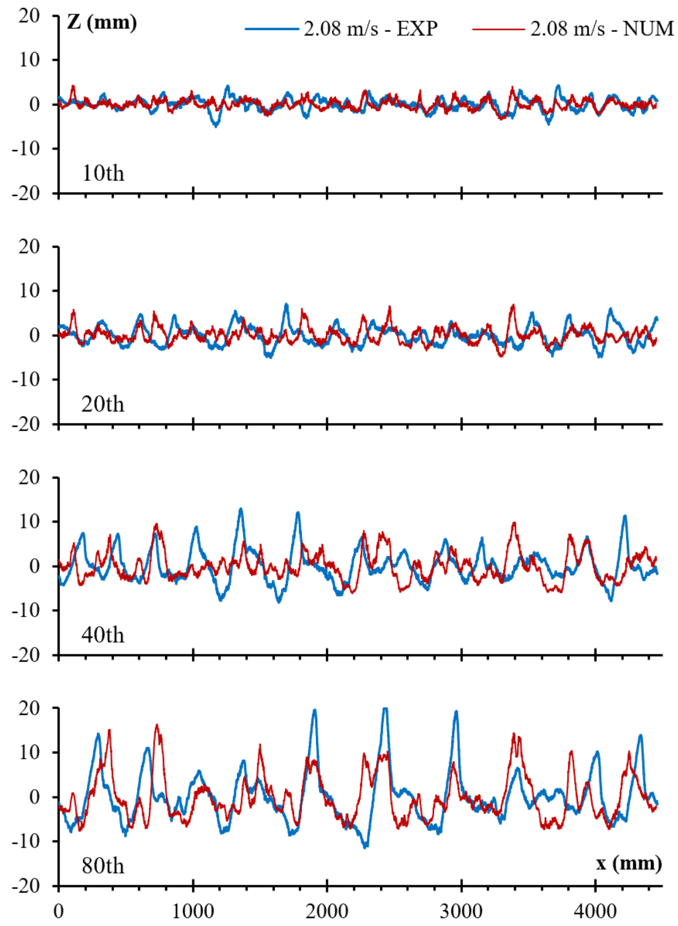

Figure 10 – Experimental and Numerical Profile Comparison

The optimized model's success in replicating both the final profile and the evolution of the washboard effect provides a reliable representation of the phenomenon. The model was able to generate ripple patterns consistent with the experimental observations, confirming that the underlying grain transport mechanisms were accurately captured.

In these results, the saltation equation that governs grain movement plays a crucial role in the washboard formation. Higher starting points of grains lead to longer saltation distances, which aligns with the physical understanding of the process. However, further refinement is needed in the model to account for the specific locations of grains, as the behavior of grains departing from peaks may differ from those departing from valleys. This is an important consideration that could influence the dynamics of ripple formation.

The parametric analysis also revealed that changes in the rate of relaxation D had minimal impact on the washboard pattern, suggesting that other factors, such as the wind or wheel-induced force (represented by L0), play a more significant role in the formation of washboard ripples. These insights suggest that the velocity-driven forces are key to generating the washboard effect, supporting the idea that the model can be refined to better represent the underlying dynamics of granular transport under vehicle-induced forces.

The optimized numerical model represents a successful approach to simulating the washboard phenomenon. The model, validated through experimental data and optimized using Particle Swarm Optimization (PSO), provides an effective framework for understanding granular transport dynamics and their role in the formation of undulations on unpaved roads.

CONCLUSIONS

This study successfully developed and optimized a numerical model to simulate the washboard phenomenon. Validating it against experimental data and providing a deeper understanding of the underlying mechanisms driving washboard phenomenon. The model clearly establishes a relationship between washboard formation and the transport of grains, allowing the use of the theoretical basis of the model, which can be applied to further research on washboard dynamics.

One important aspect that requires additional validation is the theoretical calculation of saltation length. While the current model assumes that saltation length depends only on the height from which grains are ejected, future studies should examine whether the location of the grain (e.g., peak vs. valley) influences its trajectory and the resulting ripple formation. Image processing techniques could offer valuable insights to capture the actual behavior of grains during saltation, improving the model's accuracy.

The model shows the importance of the critical velocity, represented L0, which was shown to play a crucial role in initiating washboard formation. The results validate the relevance of this critical value, highlighting that a specific threshold is necessary for ripple formation. Furthermore, L0 needs to be directly related to the force induced by the wheel, as the model suggests that wheel-induced shear stress is a key factor in generating washboard patterns.

The insights gained from this research with the model opens the door for further studies involving different densities, velocities, wheel masses, and other factors that experimental data can provide. This could lead to a broader understanding of washboard dynamics.

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