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Design of a Hybrid Acoustic-Electronic Musical Instrument

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HOJA DE APROBACIÓN DE TESIS

Design of a Hybrid Acoustic-Electronic Musical Instrument

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DEDICATORIA

Este trabajo se lo dedico a mis padres y hermanos quienes me brindaron su apoyo incondicionalmente. Sin ustedes, su apoyo y su incesante insistencia, esto no habría sido posible.

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RESUMEN

El presente proyecto discute el diseño de un instrumento musical híbrido que combina producción de sonido mediante métodos acústicos y electrónicos. Se discute la evolución del proyecto desde la definición de la idea como tal para después determinar los requerimientos y proceder a realizar el diseño. Después de realizado el diseño, se realizan varios modelos para verificación del comportamiento y correcto diseño del instrumento. Estos análisis incluyen métodos de dinámica multicuerpo y utilización del método de elementos finitos.

ABSTRACT

The present project discusses the design of a hybrid musical instrument that combines sound production through acoustic and electronic methods. The evolution of the project is discussed from the definition of the idea for the instrument to later determination of the design requirements and then proceed to design. After the design of the instrument is done, various models are created for instrument's behavior verification as well as evaluation of the created design. These analyses include multibody dynamics and Finite element Methods utilization.

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1. Introduction

This Chapter briefly introduces the reader into musical instrument design, the main topic of this present work. It discusses previous work that has been done regarding this topic, provides a methodology to be followed for the hybrid instrument design and the objectives to be achieved with this work.

1.1.Previous Work

Hybrid musical instrument design is a relatively new topic; still, this type of musical instrument design has become very popular worldwide. Lots of instruments with very different and innovative features are created daily; instruments that combine different forms of sound production to provide the user with a totally new experience for sound production. It is important to note that most of the work done has been for creative purposes rather than for study ones.

Instruments combining different mechanisms and sound sources, being these acoustic or electronic, show very innovative characteristics with respect to single source instruments. The combination sounds coming from two or more different sources produces new timbre features, creating very different sound characteristics. (Clarke, 2012) The possibilities for sound combination has shown that creating hybrid instruments could revolutionize music. Therefore, with the musical revolution idea in mind, a study was conducted regarding the relationship between electronic and electronic acoustic instruments have when it comes to sound production and timbre features. (Emmerson, 1998) It was found that people seem to like the sounds produced by electronic and electronic acoustic instruments due to music richness that this combination provides.

Hybrid musical instruments seem to have very interesting properties and people appear to like them. Still, mostly empirical work has been done in the topic which means very little theoretical insight can be obtained from it.

1.2. Literature Review

In modern times, sound production and instrument design has become a very relevant topic. Everywhere around the world, people create and combine all sorts of sounds produced through different sources to create new devices that would be able to revolutionize the music and sound production industry (Oppenheimer, 2006). As being said before, music instruments with more than one sound source can produce sounds with different properties like timbre and sound diffusion. Thus, hybrid musical instrument design needs to be evaluated through some parameters.

The first parameter to be discussed is the timbre. Hybrid instruments produce sounds with different timbre, and thus enriches sound production (Moore, et al., 2013). Studies conducted through the Computer Music Journal (n.d.) regarding sound perception revealed that more complex timbre features, if at pleasant amplitudes and pitches, tend to be more pleasurable to hear than sounds with simpler timbres. Instruments with more complex features are more popular. It is important to also note that timbre features are qualitative and quite subjective. Every person has a different perception of the sound and this makes harder to

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quantify how pleasurable a sound can be. "Timbre seems to be a very important feature for sound qualification, but its subjective properties make it hard to measure." (Cazau, 2014).

Another parameter worth examining for this project is the listener's perception regarding the sound diffusion in the place of performance. It is well known that every person perceives sound in very different ways. People listening to the same sound can easily disagree in some of the features of the sound itself. Additionally, the environment that surrounds us can substantially affect the sound from its production to its perception (Lapp, 2012). The most common effect regarding this perception is reverb, which is produced when the sound goes through reflection by bouncing of objects and sums up to the original sound giving a sense that the sound echoes or repeats several times. This and other perception features can alter the way people perceive the produced sound.

Materials used are another parameter that can affect sound production. Each material has its own characteristic properties which are directly related to sound production and amplification. Stiffer materials usually are very good transmitters while softer ones are better for transforming mechanical vibration into sound waves (Fletcher, 1998); metal has better amplification qualities and wood produces richer timbres (Lapp, n.d.). Materials are one of the most important factors for instrument design and a correct selection of what materials should be used is necessary to achieve a better design.

Hybrid musical instruments are the future of music since they can be designed to have very useful and interesting properties. Nevertheless, there are many parameters to consider in the design of the instrument, both qualitative and quantitative in order to achieve the best possible design.

1.3. Project Justification

This project will provide good insight regarding the necessities and challenges that the design of a hybrid instrument can have. It will also be a base for future research projects in this specific topic facilitating the future construction of the designed instrument for quantitative tests and experimentation.

1.4. Methodology

This project will be made in a step by step manner, first defining the design characteristics to be achieved to later design the different components of the instrument using CAD aid. Finally after designing each of the components, models to verify the correct design and behavior will be created. This will all be documented and discussed in the next chapters.

1.5.Objectives

The main objective to be achieved in the present thesis is to design a fully functional hybrid instrument that combines acoustic with electronic sound production. Additionally, some secondary objectives for the project to accomplish are desired:

- Create a visual 3D model of the design using CAD software
- Verify the correct design and behavior through different analysis
- Create design drawings of the designed components to facilitate future construction of the designed instrument.

2. Theoretical Framework

This chapter briefly discusses all the theoretical topics considered to be important for the making and understanding of the present thesis. It includes vibrations, sound wave analysis, electronic oscillating systems and devices and some background with respect to finite element methods and multibody dynamics.

2.1. Vibrations

Vibration is a physical phenomenon that refers to the oscillatory movement of an object out of its equilibrium position [38]. There are various types of vibrating systems: mechanical, acoustical, electrical, etc. They can all be modeled in the same way and are usually represented by a stiffness factor, an inertial factor and a dissipating factor. Vibrating systems are the basics of sound generation so their understanding is necessary for spring mass systems, free and forced vibrations, and sound radiation systems.

2.1.1. Spring-mass Systems

The simplest model used to analyze a vibrating system is a spring-mass system. It consists of a mass m attached to a spring with stiffness k, which is also attached to a rigid wall as seen in Figure 2.1. When the mass is either set to a position x different from equilibrium or with an initial velocity, the system will oscillate around the equilibrium position producing a harmonic motion.



Figure 2.1.: Spring-mass System [38]

To analyze the system, Newton's Laws for dynamics are used. From Figure 2.1, A Free body Diagram (FBD) like the one in Figure 2.2 can be drawn to solve the dynamics. From it, Equation (2.1) is obtained.



Figure 2.2: Free body Diagram of the system [38]

$$-kx = ma$$
 (2.1)

Rearranging the equation, and setting $a = \frac{d^2x}{dt^2} = \ddot{x}$, Eq. (2.2) is obtained.

$$kx + m\ddot{x} = 0 \quad (2.2)$$

Equation (2.2) is a second order differential equation. The answer for this equation (2.2) is well-known and describes harmonic motion as shown in equation (2.4) where A is the

amplitude of the harmonic motion and φ is the initial phase of the motion. This solution is only valid if $\omega_0 = \sqrt{k/m}$. Using this constant, Equation (2.2) can be rearranged into Equation (2.3).

$$\ddot{x} + \omega_0^2 x = 0 \quad (2.3)$$
$$x(t) = A\cos(\omega_0 t + \varphi) \quad (2.4)$$

The values for amplitude A and phase φ are obtained by replacing the initial conditions of the system which are initial displacement (x_0) and velocity (v_0) as shown in Equation (2.5).

$$A = \sqrt{x_0^2 + \left(\frac{v_o}{\omega_0}\right)^2}$$
(2.5)

$$\varphi = \tan^{-1}\left(\frac{-\nu_0}{\omega_0 x_0}\right)$$

The mass of the system represents the inertial component which is the one in charge of storing the kinetic energy of the system. This representation is analogous to other vibrating systems, for example the inertia in rotating systems and capacitors in electrical systems.

The spring of the system represents the stiffness component which is the one in charge of storing the potential energy of the system. This representation is analogous to other vibrating systems, for example the inductance in electrical systems. Analyzing vibrations from an energy point of view, the oscillations are caused by the transmission and transformation of energy between the stiffness component of the system and the inertial component.

2.1.2. Damped vibrations

A spring-mass system is a simple representation of a vibrating system, however, real life systems are more complicated and contain an additional term which comes from the energy dissipation Forces that act inside the system like for example friction. In the mechanical representation of these forces, dampers are used. Viscous damping, which generates a force proportional to the velocity of the system is usually used to model this systems. This extra term needs to be added to equation (2.2) as shown in equation (2.6) to obtain a damped vibration problem. In Figure 2.3, a diagram of the spring-mass model with damping can be appreciated.

$$kx + b\dot{x} + m\ddot{x} = 0 \quad (2.6)$$



Figure 2.3.: Spring-mass-damper system [46]

Rearranging the terms and defining the constant $\xi = b/2\sqrt{km}$, which is known as the damping factor, equation (2.7) is obtained.

$$\ddot{x} + 2\xi\omega_0 \dot{x} + \omega_0^2 x = 0 \quad (2.7)$$

Equation (2.7) is a second order differential equation where the harmonic solution $x = Ae^{\omega t}$ is assumed. Substitution in equation (2.7) gives three possible types of solutions

depending on the values of mass, stiffness and damping factors. Figure 2.4 shows the behavior of the system under these three conditions for the same initial displacement and velocity values.

The first type of solution, which is known as critically damped appears when $\xi = 1$ or $b = 2\sqrt{km}$. Then the solution to equation (2.7) that is obtained is

$$x(t) = A(1 + \xi \omega_0 t) e^{-\xi \omega_0 t} \quad (2.8)$$

Equation (2.8) shows an exponential decay with respect to the equilibrium position. It is called critical damping since it is the fastest restitution to the equilibrium position.

The second type is called the over damped solution, and is obtained when $\xi > 1$ or $c > 2\sqrt{km}$. Here, the harmonic solution assumed yields two real solutions that, through linear combination, give equation (2.9)

$$x(t) = e^{-\xi\omega_0 t} \left(A e^{\sqrt{(\xi\omega_0)^2 - \omega_0^2} t} + B e^{-\sqrt{(\xi\omega_0)^2 - \omega_0^2} t} \right) \quad (2.9)$$

This solution shows a system that will slowly return to the equilibrium position with an exponential decay that is proportional to the damping factor.

Finally, the third and most interesting type of answer, since this one gives an oscillation, is the under-damped solution which occurs when $\xi < 1$ or $c < 2\sqrt{km}$. When this happens, solution for equation (2.7) gives equation (2.10)

$$x(t) = Ae^{-\xi\omega_0 t}\cos(\omega_d t + \varphi) \quad (2.10)$$

Where ω_d is the damped natural frequency of the system, and it is smaller than the natural frequency without damping.

Using initial conditions and replacing them in equation (2.10) gives equation (2.11) and (2.12) which shows the values of A and φ in terms of initial displacement (x_0) and velocity (v_0).

$$A = \sqrt{(\omega_d x_0)^2 + (v_0 + \xi \omega_n x_0)^2} \quad (2.11)$$
$$\varphi = tan^{-1} \left(-\frac{v_0 + \xi \omega_n x_0}{\omega_d x_0} \right) \quad (2.12)$$

This solution shows a system that vibrates at the damped frequency with an initial amplitude and phase that depend on the initial conditions. Its amplitude decreases exponentially depending on the damping factor. After a period of time, the motion will cease completely since all the energy is dissipated by the damper.



Figure 2.4.: Motion of a damped system with different damping values.

As it can easily be appreciated, when the damping is increased, the oscillation decays faster and tends to behave more like the critical damping. Also, if damping is increased to a value higher than critical damping, the system takes more time to return to equilibrium position.

2.1.3. Forced Oscillations

When a simple vibrating system is driven by an external force f (t) as shown in Figure 2.3, the equation of motion becomes

$$kx + c\dot{x} + m\ddot{x} = \ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = f(t) \quad (2.13)$$

The driving force may be constant or have time dependence; it could be harmonic, impulsive or a random function of time. For the case of a harmonic driving force of the form $f(t) = Fcos\omega t$, the solution to equation (2.13) consists of two parts: a transient term that depends on the initial conditions, and a steady-state response that depends only on the force amplitude F and its driving frequency ω .

To obtain the steady-state solution for Equation 2.14, it is expressed in complex form (The symbol ~ on top of a variable means it is complex):

$$k\tilde{x} + c\tilde{\dot{x}} + m\tilde{\ddot{x}} = Fe^{j\omega t} \quad (2.14)$$

Since it is a linear equation, the solution should be similar to the right-hand side of the equation which in this case is harmonic. Thus, we replace \tilde{x} by $\tilde{A}e^{j\omega t}$, derive and obtain Equation (2.15)

$$\tilde{A}e^{j\omega t}(-\omega^2 m + j\omega c + k) = Fe^{j\omega t} \quad (2.15)$$

Then the complex displacement is obtained:

$$\tilde{x} = \frac{Fe^{j\omega t}}{k - \omega^2 m + j\omega c} = \frac{Fe^{j\omega t}/m}{\omega_n^2 - \omega^2 + 2j\omega\xi\omega_n} \quad (2.16)$$

Where $\omega_n^2 = k/m$ and $\xi = c/2\sqrt{km}$. Taking the real part of Equation (2.16), a solution for the displacement is obtained as shown in Equation (2.17).

$$x(t) = C\cos(\omega t + \varphi) \quad (2.17)$$

Where the amplitude C and the phase angle are:

$$C = \frac{F/m}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\omega\xi\omega_n)^2}}$$

$$\varphi = tan^{-1} \left(\frac{2\omega \xi \omega_n}{(\omega_n^2 - \omega^2)} \right)$$

This shows that the system moves at the driving frequency with a certain phase depending in the stiffness, mass and damping parameters.

2.1.4. Resonance

When a harmonic oscillator is driven very close to its natural frequency, a phenomenon called resonance occurs. Physically, when resonance occurs, the system is excited at the same pace as it would naturally oscillate and thus it is capable to absorb a lot more energy from the exciting source. This results in an increment in the amplitude of the vibration [38].

If no damping exists, then theoretically the system would start increasing in amplitude infinitely. But in real systems, there is always some damping present, thus the amplitude at resonance reaches a maximum value several times bigger that the static amplitude, which is the maximum amplitude the system would reach when excited by the driving force (without resonance) and is defined as $x_s = F/k$ but of finite magnitude. The magnitude of the amplification depends on the value of the damping being higher when damping is small and decreasing as damping increases. Figure 2.5 shows the amplitude and phase behavior depending in the frequency for systems with different damping values. Amplitude is normalized using the static amplitude defined above, while frequency is normalized as a ratio of the driving frequency ($\omega = 2\pi f$) and the natural frequency ($\omega_0 = 2\pi f_0$).

(2.18)



Figure 2.5.: Amplitude and phase response versus Frequency for driven systems with different damping factors [29]

As it can be seen in Figure 2.5, when the driving frequency is smaller than the natural frequency of the system, the amplitude of the motion stays close to the value of the static displacement x_s increasing to its maximum value at resonance. And when $\omega > \omega_0$, the amplitude decays to zero as the frequency gets higher.

As for the phase, it is zeros when the driving frequency is zero and gradually increases to 90° as the driving frequency approaches the natural frequency. When $\omega = \omega_0$, phase is equal to 90°, and when $\omega > \omega_0$, the phase shifts and tends to 180° as the driving frequency increases. When phase reaches 180°, the system is considered to be out of phase.

The width of the resonance peak is determined by the damping factor ξ . Since the amplitude starts increasing when $\omega \approx \omega_0$, the denominator of the amplitude found in Equation (2.18) and approximate it to $2\omega_0\sqrt{[(\omega - \omega_n)^2 + (\xi\omega_n)^2]}$. The magnitude of the amplitude denominator, thus increases by a factor of $\sqrt{2}$ relative to its value at $\omega = \Box_0$, when $|\omega - \omega_0|$
$\omega_0 | \approx \xi \omega_n$. The response decreases by the same factor, which represents a 3 dB decline from the peak value at resonance. This is known as the half width of the peak and is equal to the damping coefficient ξ . This means that the 3 dB full-width $\Delta \omega$ of the curve is $2\xi \omega_n = c/m$.

2.2. Sound Waves

The sensation and perception of what is commonly called sound is produced by variations in the air pressure that are detected by our ear mechanism which basically vibrates with the changes in pressure transmitting this motion to a spiral cavity called the cochlea, which stimulates little hairs that send nervous stimuli to the brain producing the perception of what we called sound.

Since the human hearing can perceive sounds from around the 20 Hz to 20 kHz, this range has been defined as the sound spectrum. Although, music tries to stimulate the entire spectrum, it is mostly concentrated in the range of 100 Hz-3 kHz since the sensitivity of human hearing drops significantly below the 100 Hz and over the 10 kHz, and since most of the energy from human speech lies in this range and thus we are more sensitive to these frequencies.

Since sound waves travel through air, which does not have elastic resistance to shear, only longitudinal waves are transferred with the local motion of air being in the same direction as the propagation direction of the wave itself.

When sound waves are produced, they spread out in a spherical fashion (spherical waves) but it is also useful to analyze the waves in a plane (plane waves) which is acceptable

if the wave has travelled enough distance away from the source such that the wave fronts can be treated as planes normal to the direction of propagation. These waves are described below.

2.2.1. Plane Waves

When a wave passes through the air, it creates a displacement. Suppose this displacement is given by ξ , so that a rectangular volume of air as the one shown in Figure 2.6 moves from points ABCD to A'B'C'D'. Being S the area normal to x, the volume of the element becomes

$$V + dV = Sdx \left(1 + \frac{\partial \xi}{\partial x}\right)$$
 (2.19)



Figure 2.6.: Displaced air volume due to the passage of a sound wave [29]

And defining p_a as the total pressure of the air and K as the bulk modulus which represents the substance's resistance to uniform compression, we can get the relation shown in Equation (2.20) from the Bulk modulus definition.

$$dp_a = -K \frac{dV}{V} [Pa] \quad (2.20)$$

Where dp_{\Box} , the varying pressure, will be considered as the sound pressure and simply be defined as p. And noting from equation (2.19) that V = Sdx and $dV = sd\xi$, these can be replaced into Equation (2.20) to obtain Equation (2.21).

$$p = -K \frac{\partial \xi}{\partial x} \quad (2.21)$$

The motion from the element in Figure 2.6 should be described by Newton's second law where the pressure gradient force should equal mass times acceleration in the x direction

$$-S\left(\frac{\partial p}{\partial x}dx\right) = \rho S dx \frac{\partial^2 \xi}{\partial t^2}$$

Or

$$-\frac{\partial p}{\partial x} = \rho \frac{\partial^2 \xi}{\partial t^2} \quad (2.22)$$

Then, replacing Equation (2.21) in (2.22)

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{K}{\rho} \frac{\partial^2 \xi}{\partial x^2} \quad (2.23)$$

And, differentiating Equation (2.22) with respect to x and Eq. (2.21) twice with respect to t, we can obtain Equation (2.24)

$$\frac{\partial^2 p}{\partial t^2} = \frac{K}{\rho} \frac{\partial^2 p}{\partial x^2} \quad (2.24)$$

This represents, along with Eq. (2.24) the one dimensional wave equation expressed for acoustic displacement ξ and acoustic pressure p. These two equations are generalized for waves travelling through any fluid.

In the case of air, there can be two possibilities depending if the elastic behavior is isothermal (pV = constant = nRT where p stands for pressure, V for volume, n for number of molecules, and R is the universal gas constant) with T being an absolute temperature, or whether it is adiabatic ($PV^{\gamma} = consta \Box t$) where $\gamma = C_p/C_v = 1.4$ is the ratio of specific heat at constant pressure and volume respectively. In the case of acoustic waves, the process can be considered adiabatic. This happens because the lengths implied make air compress and expand in such a way that the maximum and minimum pressure, and thus the points of maximum and minimum temperature, be so far apart from one another that there is no appreciable heat transfer.

Taking logarithms on each side of the adiabatic case equation ($PV^{\gamma} = constant$) and differentiating it we can obtain

$$K = \gamma p_a$$
 (2.25)

so that equation (2.24) can be rewritten as

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} \quad (2.26)$$

where

$$c^2 = \frac{K}{\rho} = \frac{\gamma p_a}{\rho} \quad (2.27)$$

The magnitude c is interpreted as the speed of propagation of the wave. Since these waves are travelling through air, this factor c can also be defined as the speed of sound which is c = 343 m/s at room temperature.

This equation can be solved using d'Alambert's general solution

$$p = f_1(ct - x) + f_2(ct + x) \quad (2.28)$$

Where f_1 and f_2 represent a function travelling with speed c to the right and to the left respectively. The nature of these two waves is completely arbitrary so they can have any form and can be chosen so that their sum represents any desired initial displacement and velocity.

And assuming f_1 and f_2 as simple harmonic waves, we can rewrite them as a sum of sine and cosine terms which can be written in complex notation as

$$p = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)} \quad (2.29)$$

Where $k = \omega/c$, and A and B represent waves going to the right and left respectively. From Eq. (2.23) we can see that ξ must have a similar form. This happens because the equation is similar to Equation (2.26). We can see from equation (2.22) that p and ξ relate, and assuming a wave travelling to the right (A=1, B=0) and replacing these values inside equation (2.22) we obtain

$$jkp = jp\omega \frac{\partial \xi}{\partial t}$$
 (2.30)

Rewriting this equation, defining u as the acoustic fluid velocity $\partial \xi / \partial t$, and remembering that $k = \omega/c$, then

$$p = \rho c u$$
 (2.31)

Therefore the acoustic fluid velocity is in phase with the acoustic pressure, which makes it useful to define the term $z = \rho c$ as the acoustic impedance of the wave. This quantity is measured in Rayls which is equivalent to $Kgm^{-2}s^{-1}$.

2.2.2. Spherical waves

Spherical waves are, as it name states, a type of wave which originates in a center point and then propagates into every direction (spherically). To study this type of waves, Equation (2.32) which represents the wave Equation in a three dimensional space, is used. This wave equation can be transformed into the Hemholtz equation by assuming a time independence as shown in Eq. (2.33)

$$\frac{\partial^2 p}{\partial t^2} = c^2 \nabla^2 p \quad (2.32)$$
$$\nabla^2 p + k^2 p = 0 \quad (2.33)$$

This equation is linear and easy to treat in any coordinates. So transforming it into spherical coordinates using Figure 2.7 as a reference ($x = rsin\theta cos\phi$, $y = rsin\theta sin\phi$, $z = rcos\theta$) we get Equation (2.34).

$$\nabla^2 p = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial p}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial p}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 p}{\partial \phi^2} \quad (2.34)$$

It is assumed that the wave propagates uniformly around the sphere, thus p has no dependence of θ and Φ simplifying the wave to only radial dependence. The general solution of this equation can be written as a superposition of outgoing and incoming waves of the form

$$p = \left(\frac{A}{r}e^{-jkr} + \frac{B}{r}e^{-jkr}\right)e^{j\omega t} \quad (2.35)$$

To find the acoustic particle velocity v, we use the equivalent of Eq. (2.22)

$$-\nabla p = -\frac{\partial p}{\partial r} = \rho \frac{dv}{dt} \quad (2.36)$$

Equation (2.36) represent the relationship between pressure and fluid particle velocity, it only depends on the radius since uniform spread along the sphere was assumed. In the case of an outgoing wave (B=0)

$$v = \frac{A}{r\rho c} \left(1 + \frac{1}{jkr} \right) e^{-jkr} e^{j\omega t} \quad (2.37)$$

Also, we can define the spherical wave impedance which depends on the distance from the origin

$$z = \frac{p}{v} = \rho c \left(\frac{jkr}{1+jkr}\right) \quad (2.38)$$

Thus, spherical waves propagate uniformly to the medium and depend directly in the medium density and speed of sound in it.



Figure 2.7: Spherical Coordinates reference [38]

2.2.3. Sound Pressure Level and Intensity

Since there is a factor of about 10^6 between acoustic pressure and intolerable pressure, it is convenient to use a logarithmic scale for better visualization.

$$L_p = 20 \log_{10} \left(\frac{p}{p_0}\right) (2.39)$$

Where the reference pressure p_0 is taken to be 20 μPa , which is approximately the threshold of human hearing in the most sensitive range.

In many cases it is more useful physically to know the acoustic energy carried through a surface by sound waves. This quantity is called acoustic intensity and is measured in $[Wm^{-2}]$. This is also taken into logarithmic scale which is defined as the intensity level.

$$L_{I} = 10 \log_{10} \left(\frac{I}{I_{0}} \right) (2.40)$$

The factor in the Intensity level is 10 because intensity is proportional to p^2 . The reference intensity I_0 is defined to be $10^{-12}Wm^{-2}$. For calculating intensity, Equation (2.41) is used.

$$I = \rho c u = \frac{p^2}{\rho c} = p u (2.41)$$

2.3. Sound Radiation

Sound radiation is the effect through which mechanical vibrations transform into sound waves. This is the necessary process in order to have sound. Most of the time this effect is assumed as granted but since it involves some important physics regarding the sound generation of musical instruments, it is worth examination.

2.3.1. Simple Multipole Sources

The simplest possible source is a point source known as a monopole which can be modeled as a pulsating sphere as its radius tends to zero. Suppose these sphere has a small radius *a* and produces a pulsating flow with a frequency ω and an amplitude Q given by

$$Q = 4\pi a^2 v(a)$$
 (2.42)

Where the term v(a) corresponds to the radial velocity amplitude at the surface of the sphere. This amplitude Q is also called strength of the monopole. To determine the pressure

that this monopole generates, Eq. (2.42) is matched with Eq. (2.37) which represents the velocity at *a*; and assuming $ka \ll 1$ (the sphere is so small it tends to a point), we find a value for the constant A (given that it is an outgoing wave so that B=0) and replace it in the equation of pressure of the sphere (2.35) to get

$$p(r) = \frac{j\omega\rho}{4\pi r} Q e^{-jkr} \quad (2.43)$$

And its related power P is $\frac{1}{2}p^2/\rho c$ integrated over a spherical surface

$$P = \frac{\omega^2 \rho Q^2}{8\pi c} \quad (2.44)$$

Which, as it can easily be seen, has a strong dependence in frequency.

More complex radiators consist in joints and arrays of monopoles that irradiate together with certain strength (dipoles, quadrupoles, etc.) Its analysis is quite similar to the monopole with the difference that they have angle dependency since there is a distance between them which varies with the angle of the midpoint between these sources and the listener's position.

2.3.2. Radiation from Large Plates

It is important to get some insights of the radiation properties of large objects such as large plates. These plates are usually modeled as infinite, though in real situations these plates have boundaries that change the idealized situation described. Suppose we have an infinite plate upon which a plane wave of frequency ω is propagating. The speed of this wave depends on the thickness and elastic properties of the pate as well as the frequency of the wave itself as shown in Eq. (2.45)

$$v(f) = \frac{\omega}{k} = \sqrt{\frac{\omega h c_L}{\sqrt{12}}} \quad (2.45)$$

Where $c_L = \sqrt{E/\rho(1-v^2)}$ is the velocity of longitudinal waves in an infinite plate. If we assume that the infinite plate is lying in the plane z=0 and that the waves propagates in the x direction, then the displacement velocity of the plate surface can be represented as shown in Equation (2.46)

$$u(x,0) = u_p e^{-jk_p x} e^{j\omega t}$$
 (2.46)

Where the wave number on the surface is given by $k_p = \omega/v_p$ being v_p the speed of this wave as represented in Eq. (2.45).

In the air above the plate, there is no changes in any relevant physical quantities in the y direction, thus the air pressure and particle velocity only depend on x and z, and satisfy the wave equation in the form

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad (2.47)$$

Equation (2.47) has a solution on the form of Equation (2.48)

$$p(x,z) = \rho c u(x,z) = A e^{-j(k_x x + k_z z)}$$
 (2.48)

Where k_x and k_z are the components of the wave vector k in the respective direction. There is also a time factor implied which was not written for simplicity.

The particle velocity of the air must match the normal velocity of the plate in the surface of the plane, we must have $k_x = k_p$ and $A = u_p$. So substituting these values into equation (2.48) and the result of it into Eq. (2.49) we find that

$$k_{z} = \frac{\omega^{2}}{c^{2}} - k_{p}^{2} = \omega^{2} \left(\frac{1}{c^{2}} - \frac{1}{v_{p}^{2}} \right) \quad (2.49)$$

This result has very important implications; if the velocity of the plate v_p is less than the velocity of sound c, then k_z is imaginary and the acoustic disturbance in the z direction is attenuated. The whole motion of air then constrains to the immediate vicinity of the plate and there is no perceivable acoustic radiation. If, on the other hand, v_p is bigger than c, then an acoustic wave is irradiated in a direction making an angle θ with the plate surface. This angle is given by

$$tan\theta = \frac{k_z}{k_x} = \left(\frac{v_p^2}{c^2} - 1\right)^{1/2} \quad (2.50)$$

Since the surface velocity increases with frequency, a coincidence frequency or matching frequency can be determined. This matching frequency represents the frequency at which there can be sound radiation.

2.4. Frequency analysis

Since a response from a real vibrating system is composed of a combination of modes, at first sight it is very difficult to find the harmonics present inside the response. That is the reason why spectral analysis are performed using Fourier transformation to find the frequencies that compose the spectrum of the response [29].

These transformations mathematically convert the complete response in Fourier series which are infinite sums of sine and cosine waves with different frequencies and amplitudes. This allows creating a spectrum of the power or amplitude in function of the frequency range. These types of graphics show peaks at the frequencies present in the response since they are the ones that hold the most power and thus are responsible for the amplitude of the response. Figure 2.8 shows a typical spectral diagram, the peak is the frequency contained in the analyzed motion.



Figure 2.8.: Typical Spectral diagram showing a peak at the frequency contained in the

analyzed wave

As it can be seen, each peak has different amplitude depending on how much of the motion it is responsible for, it can also be seen that the highest peak corresponds to the fundamental frequency, and that there are other peaks at integer multiples of the fundamental frequency, these are the harmonics.

2.5.Pitch

Pitch is usually described as the auditory perception of a sound in which a person assigns musical tones based on the frequency of the sound. But more than a physical property is more of a psychoacoustic and perceptual one since it depends in the perception of the person listening to the sound.

Pitch is usually classified as high (also called sharp) or low (also called flat), and based in this the musical scale of tones have been created. But it is indispensable to remember that this is a very subjective property that can vary from a person to another even though it is based on a physical property like the frequency.

2.6. Reverberance

The reverb effect is a phenomenon that appears when the sound that was emitted by a specific source bounces back and reflects in walls or obstacles and sums up to the original sound creating a mild permanence of the sound even after the sound source has stopped producing sounds.

Since the reverb effect changes the sound perception, it is a very important property to take into account for places where music will be played. To quantify this sound property, the

reverberation time measure is used. This is defined as the period of time that it takes for the sound to decay 60 dB from its original amplitude when the sound source is turned off. This is widely used to measure and value the acoustics of different places.

2.7.Strings

Strings are considered a continuous system which can be described as a joint of various simple vibrating systems joined together by same elasticity springs. Strings can also be modeled as long cylinders with a certain linear density and transverse area. These systems can move transversely or in a torsional fashion, but since transverse vibrations in the strings are the most relevant vibrations since that is the one that is transmitted to create sound, it is the one which should be subject of examination.

Consider an infinitesimal part of a uniform string with linear density $\mu \left[\frac{Kg}{m}\right]$, stretched to a tension T [N] as the one shown in Figure 2.9. The net force of the restoring segments ds to its equilibrium position is the difference of the y components of T at the two ends of the segment

$$dF_{y} = (Tsin\theta)_{x+dx} - (Tsin\theta)_{x} \quad (2.51)$$



Figure 2.9.: Infinitesimal element of an ideal string [46]

Using a Taylor series expansion for $Tsin\theta$, keeping only the first order terms which are the most relevant ones and replacing it in Eq. (2.51) gives

$$dF_{y} = \left[(Tsin\theta)_{x} + \frac{\partial (Tsin\theta)}{\partial x} dx \right] - (Tsin\theta)_{x} = \frac{\partial (Tsin\theta)}{\partial x} dx \quad (2.52)$$

For small transversal displacement in y, sin θ can be replaced by $tan\theta$, which is also $\frac{\partial y}{\partial x}$

$$\frac{\partial \left(T\frac{\partial y}{\partial x}\right)}{\partial x}dx = T\frac{\partial^2 y}{\partial x^2}dx \quad (2.53)$$

Then applying Newton's second law of motions and remembering that the mass of the segment is μds gives

$$T\frac{\partial^2 y}{\partial x^2}dx = \mu ds\frac{\partial^2 y}{\partial t^2} \quad (2.54)$$

Since dy is very small, $ds \approx dx$. Also, we can define $c^2 = T/\mu$ to obtain Equation (2.54)

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \quad (2.55)$$

Equation (2.55) is the wave equation for transverse waves in vibrating strings. These equation has for solution the d'Alembert's general solution as shown in Eq. (2.28) and is solved in the same fashion as other waves motions as explained before.

Now assuming that the functions of the solution are harmonic, we can assume functions f_1 and f_2 to be composed of a sine and cosine term as shown in Eq. (2.56)

$$y(x,t) = A\cos(\omega t - kx) + B\sin(\omega t - kx) + C\cos(\omega t + kx) + D\sin(\omega t + kx)$$
(2.56)

Where $k = \omega/c = 2\pi/\lambda$ is known as the wave number. This number represents how the wave behaves spatially.

Since strings in musical instruments are usually fixed at both ends (x=0 and x=L) it is useful to solve the general equation using these boundary conditions. The first boundary condition y(0, t) = 0 requires that A = -C and b = -D in the solution in Eq. (2.63) to obtain Equation (2.57)

$$y = 2Asinkxcos\omega t - 2Bsinkxsin\omega t = 2[Acos\omega t - Bsin\omega t]sinkx$$
 (2.57)

Equation (2.57) is obtained by using trigonometric identities. Now using the second boundary condition: y(L, t) = 0 we see that it requires that sinkL = 0 or $\omega L/c = n\pi$. This restricts the frequency values to $\omega_n = n\pi c/L$, thus the string only vibrates under certain frequencies which are the normal modes of vibration. These are harmonic modes since each one is an integer value of the fundamental. The general solution then can be expressed as a sum of all the normal modes of the string

$$y(x,t) = \sum_{n=1}^{\infty} (A_n sin \omega_n t + B_n cos \omega_n t) sin k_n x \quad (2.58)$$

Equation (2.58) expresses the general behavior of the string motion, the constants A_n and B_n can be found using the initial conditions given depending on the specific problem. These terms can be determined by Fourier analysis which gives the values of these coefficients as functions of the initial conditions as shown in Eq. (2.59) and (2.60) respectively.

$$A_{n} = \frac{2}{\omega_{n}L} \int_{0}^{L} \dot{y}(x,0) \sin \frac{n\pi x}{L} dx \quad (2.59)$$
$$B_{n} = \frac{2}{L} \int_{0}^{L} y(x,0) \sin \frac{n\pi x}{L} dx \quad (2.60)$$

2.8. Resonance Boxes

A resonance box is a structure used to amplify the mechanical vibrations produced by another source such as a string in order to create a difference in the pressure of the air nearby to produce sound waves.

These boxes are usually used in various types of instruments, specially string ones since a string does not have enough area to produce a significant movement in the air and thus cannot create sound with enough power to be heard unless it is played very close to the person. These boxes are designed to have some of its normal modes of vibration close enough to the vibration frequency of the string in order to get the maximum amplitude possible. That is why some frequencies are higher amplified while others can be damped out.

In order to obtain the normal modes of oscillation of these boxes, finite element methods are used or, if possible, an experimental analysis in order to find these frequencies and modes.

2.9. Musical Instruments

Musical instruments are complex vibration systems which involve different kinds of vibrations created by various excitement sources in order to transform them into sound waves that can be heard.

They can produce various sound frequencies that give a wide spectrum range of tones available to create music.

Acoustic Instruments

Acoustic instruments are defined as all types of instruments that create sound through mechanical vibrations that transform into sound waves. The excitation mechanisms inside these categories are subcategorized into strings, wind and percussion instruments [29].

String Instruments

String instruments are acoustical instruments that produce sound through plucking, bowing or striking strings which vibrate and creating sound with an amplification of this vibration through a resonance box.

Wind Instruments

Wind instruments are instruments that produce sound by blowing or lip vibration that is amplified inside the instrument creating the characteristic sounds we can hear. These instruments generally produce their sound automatically, not without transmission of the vibrations.

Percussion Instruments

Percussion instruments are the oldest type of instruments in human history. They produce sound by hitting a membrane placed over a hole or a chamber or air, the membrane vibrates creating the sound waves with air expansion and compression inside the chamber.

Electronic Instruments

Electronic instruments are a new type of musical instrument which appeared around the 18th century, but it was not until the beginnings of the 20th century that they started to take an active part in musical composition and creation [15].

Electronic instruments are special types of musical instrument that, instead of creating sound waves using mechanical vibrations, they create the using electrical signals that could be

produced by analog or digital oscillators that go to speakers which transform them into sound waves that can be heard.

2.10. Sound Synthesizers

Sound synthesis is a method used in electronic music production where various waves with different frequencies are produced to electronically produce signals that would create sound. Thus, synthesizers are the instruments used to produce the sound synthesis.

2.10.1. Oscillators

An oscillator is an electronic circuit that produces a periodic signal of a certain amplitude and frequency in the form of a voltage or current time-dependent function. These voltage or current signals produced are used for different purposes such as changing direct current into alternate current which is the one that comes from the electrical power sources, producing radio signals or audio signals, etc. When talking about sound and music these signals electronically resemble a sound wave and could be converted into physical sound through a device such as a speaker.

Barkhausen Conditions

Barkhausen conditions are restriction for frequency dependant circuits using feedback. They are used to obtain minimum gain and oscillation frequency. The conditions are given by Equations (2.61) and (2.62).

$$|\beta(j\omega)A| = 1 \quad (2.61)$$

$$\angle \beta(j\omega)A = 0 \quad (2.62)$$

Where A is the gain of the amplifier's circuit and $\beta(j\omega)$ is the transfer function of the feedback path.

2.10.2. Operational Amplifiers

Operational amplifiers are components created using transistors. They are used to amplify an input voltage by a factor commonly known as gain. These components ideally have infinite input impedance and zero output impedance and also can amplify the input voltage infinitely. In reality, this components get can only amplify the input signal to a maximum of the supply voltage that is being fed to them. To analyze this type of component's influence in a circuit the ideal Op amp conditions which are shown in Equation (2.63) are used. Figure 2.10 shows a schematic of an operational amplifier.

Figure 2.10: Schematic of an operational Amplifier

These components have several uses in circuits such as inverters, sum inventers, buffers, comparators, and amplifiers in general. Many types of circuits can be accomplished with the use of these components. Some of the most commonly used sets of these components are discussed next to show their function.

Inverters

When amplifiers are used as inverters, as shown in Figure 2.11, a feedback loop is connected between the output and the negative input, also known as non-inverting connection. This setting generates an output which is equal to the inverse of the input multiplied by a gain factor given by the resistance configuration as shown in Equation (2.64).

$$V_0 = -\left(\frac{R_2}{R_1}\right)V_i \quad (2.64)$$



Figure 2.11.: Inverter Circuit using an Op Amp

Sum-Inverters

This configuration is just an extension of the inverter one. It is shown in Figure 2.12. It acts in exactly the same way; the only difference is that the output is the inverse of a sum of all the inputs provided multiplied by a gain value. This gain value is again given by the resistance configuration existing in the model as shown in Equation (2.65).

$$V_0 = -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \dots + \frac{V_n}{R_n} \right) \quad (2.65)$$



Figure 2.12.: Sum Inverter Circuit with Op Amp

Buffers

A buffer configuration, also known as voltage follower, of the operational amplifier refers simply to making a feedback loop between the output and the non-inverting connection as shown in Figure 2.13. There are no components such as resistors or capacitors of any kind in this setting. This configuration simply avoids loading of the output by maintaining the signal as a voltage one because of the high impedance it provides. It is commonly used between connections of different circuits to assure a proper response.



Figure 2.13.: Buffer configuration of an Op Amp

Comparators

An operational amplifier used as a comparator, as the one shown in Figure 2.10., consists of taking advantage of the saturation feature of the component. This is done by connecting the signals to be compared as inputs into the op amp, one at the non-inverting connection (positive connection), and the other one at the inverting one (negative connection). Because a feedback loop is not used in this configuration, the output of the amplifier will be equal to the supply voltage (the maximum voltage that the amplifier can give); and, the sign of the output will depend on which of the two input signals is higher. If the one at the noninverting connection is higher, the output signal will be positive. On the contrary, if the signal at the inverting connection is higher, the output will be negative.

2.10.3. Analog Synthesis

The analog synthesis process consists in producing the oscillatory periodic signal through an electronic circuit created with analog parts such as capacitors, inductors, resistors, etc. This is achieved by creating a circuit that, when provided a certain input, would give the required signal output.

2.11. Signal Processing

Signal processing is a process which involves the storing, processing and transferring the information contained in different sorts of physical, symbolic or abstract formats through signals, which are typically electromagnetic or digital. This process encompasses fundamental theory, algorithms, and applications of a broad profile of things around the world. It is widely used nowadays.

2.11.1. Filtering

Filtering is a process in which a signal is decomposed into a sum of its components and the relevant ones are chosen and recomposed back so that the resulting signal only possesses the information we need. This is broadly used in data analysis and musical equalizing since the original signal usually contains noise or information that could distort the results. In electronics, this is commonly done with the use of filter circuits which take an input signal and, as it name states, filter it letting some of the frequencies get through while attenuating others. There are four basic types of filters: low pass filters, high pass filters, bandpass filters, and band stop filters. The first three mentioned ones are relevant to this thesis, and thus will be discussed.

Low Pass Filter

A low pass filter lets a range of frequencies lower than a specific one pass, while attenuates all the higher ones. This specific frequency is commonly known as the cutoff frequency, in which the attenuation gets a decay value of 3dB per decade. A common frequency response for a low pass filter is shown in Figure 2.14.



Figure 2.14.: Frequency Response of a low pass filter

Another important characteristic in this, and all types of filters is the resonance value. This value refers to the amplification factor of the signal when it approaches the cutoff frequency. It is dependent on the resistance used in the configuration of the filter. This resonance is equivalent to the mechanical one that appears in vibrating systems.

High Pass Filter

A high pass filter, on the other hand attenuates all the frequencies lower than the cutoff frequency and lets all the higher ones pass. The cutoff frequency is also the frequency in which the decay reaches 3dB per decade. Also, this type of filter can show a resonant behavior near the cutoff frequency depending on the resistance. The frequency response of a high pass filter can be appreciated in Figure 2.15.



Figure 2.15.: Frequency Response of a high pass filter

Bandpass Filter

A band pass filter is a combination of a high pass and a low pass filters. This type of filter lets pass only a range of frequencies, commonly known as a band, and attenuates all the other ones. A common frequency response for this filter is shown in Figure 2.16.



Figure 2.16.: Frequency response of a bandpass filter

This type of filter has five important characteristics: the central frequency f_c , low cutoff frequency f_L , high cutoff frequency f_H , resonance value, and quality Factor Q. The central frequency refers to the resonant frequency of the filter, or the frequency at which the amplitude is highest. This characteristic defines the location of the band in the frequency domain. Low and high cutoff frequencies are the frequencies at which the decay reaches 3dB per decade. These two frequencies define the width of the band of frequencies the filter lets pass. The resonant value, as in other filters is the amplification factor at the resonant frequency. Finally, the quality Factor Q, is a value that makes reference to the width of the band. It is the ratio of the central frequency to the cutoff frequency as shown in Equation (2.66). A higher quality factor implies a thinner band, while a lower quality factor implies a wider band.

$$Q = \frac{f_c}{\Delta f} = \frac{f_c}{f_H - f_L} \quad (2.66)$$

2.11.2. Altering

Altering a signal refers to any process that changes the original signal into something else. This could be made through adding, subtracting, or multiplying the original signal with other signals to produce a new one with different characteristics.

This process in broadly used in the electronic music composition since adding different signals with other frequencies and amplitudes can produce a new wave with more acoustically rich properties.

2.12. Finite Element methods

The method of finite elements is a process in which a solid body is modeled as a discrete set of elements which connect with each other through points called nodes. This method is used to model different systems numerically to obtain a more precise, close to reality model. It can be used to model different processes such as statics, dynamics, heat transfer and much more.

2.13. Multibody Dynamics

Multibody dynamics is a type of modeling in which a complex system is represented as a set of different rigid bodies connected between each other by joints with different degrees of freedom from one another. The idea of this is to simplify the complex behavior by analyzing how each body affects one another to get a more close to reality idea of how the system works.

3. Instrument Definition

This Chapter describes the process that is followed to define the diverse properties and characteristics that the prototype instrument would have in order to proceed with the design process described in the following Chapter. The characteristics discussed in this Chapter include: the type of acoustic sound generation for the instrument, electronic features for synthesized sound generation, material needs, sound requirements and other general design requirements.

3.1. Definition of the acoustic part

In this section, the acoustic sound generating type is defined. The possible acoustic sources are discussed; their advantages and disadvantages are analyzed and one of these sources is chosen to be used in the present project

3.1.1. Types of Instruments

The three main types of acoustic instruments are string instruments, wind instruments and percussion instruments. Each one of these produces acoustic sounds through vibration of strings, air blown through a cavity or membranes over an air enclosure.

String instruments, as it name states, produce sound through vibration of strings. These instruments produce different sounds by manipulating the string features such as length, diameter and tension. Also, they usually need some sort of amplification board or box since they do not have enough area to move a considerable mass of air to produce audible sound by

themselves. They are very popular due to their broad sound production range and sound timbre. [29]

Wind instruments produce acoustic sound through vibrations caused by air passing through a cavity. They are generally made of wood or brass and come in many sizes and forms. These instruments need the user to blow inside of them to produce air motion through them. When air is driven through the instrument's inner cavity, it bounces on and off the walls making the instrument vibrate and create the different tones depending on the length the air travels and the open escape valves or holes in the instrument's body itself. They have very specific timbre features, but reduced pitch range. [38]

Percussion instruments create sounds through a clamped membrane over a cavity with air. Vibration of the membrane starts when it is excited through an impact or pulse and starts moving back and forth creating motion in the air beneath it and produces sound. These sounds are very specific and have low sustain features, which means that they do not last long. Different sounds can be created depending on the portion of the membrane hit. They are very simple, but still very popular and quite used for rhythmical music. [38]

3.1.2. Selection criteria, analysis and final selection

After getting a little insight in the different types of acoustic instruments, an analysis of these sound sources is made. A SWOT analysis, is created to get a better idea of what characteristics each type could provide. Based on this analysis, a sound source is chosen for design of the present thesis.

A Strength, Weakness, Opportunity and Threat (SWOT) Analysis is a commonly used tool to study the advantages and disadvantages, making a more objective decision. By this analysis, the pros and cons of each one of the acoustic sound sources are determined.

The SWOT analysis is done by listing in a graph each of the attributes of the SWOT. Strength attributes are the advantages that the topic has, weaknesses refer to the disadvantages. Opportunity refers to the characteristics that the topic discussed has which makes it better compared to other options; and threats refer to characteristics that would make it less desirable compared to other. The SWOT analysis done for each of the instrument types are shown in Figures 3.1 to 3.3.



Figure 3.1.: SWOT Analysis for the String Instrument's type

As it can be seen from Figure 3.1, string instruments have great advantages like broad pitch range, versatile design properties for frequency definition, and timbre features. It also

shows that these types of instruments can be created using several different string excitation methods which provides more choices for design. On the other hand, a larger need for components and materials are needed which can be an impediment for future construction due to material availability and costs. Still, it seems as a great option for the proposed design.



Figure 3.2.: SWOT Analysis for the Wind Instrument's type

Taking a look at Figure 3.2 to examine the characteristic of wind instruments, it is noted that they possess desirable qualities such as timbre features and compact dimensions for transportation. Also, these types of instruments produce very adaptable sounds in a lot of musical styles. But they also have some disadvantages such as the complex shapes for sound production which can make an instrument of these type harder to construct, due to its compact design. Also, need for smaller components for the electronic sound source would be needed increasing costs. Nevertheless, it is an interesting option worth revising.



Figure 3.3.: SWOT Analysis for the Percussion Instrument's type

From Figure 3.3, it can be seen that choosing a percussion type of sound production could provide some interesting advantages such as rapidity, high sound amplitudes and adaptability to the user since percussion instruments are easy to play. They are also widely used in all sorts of music styles for rhythm purposes. On the other hand, there are also some disadvantages that should be taken into account. These instruments provide low sustain and their high amplitudes could be considered a problem if player in closed places. Also, these instruments have a reduced frequency range, which means they are not able to produce different sounds at will.

After discussing all the advantages and disadvantages of the different types of acoustic instruments, string sound production is chosen to be used for the present project. This is decided since features such as: wide frequency range of sounds, rapidity for playing and

versatility for design are desired; and string sound production is the type of acoustic sound generation that best adapts to these features.

3.2.Electronic Features Definition

After acoustic sound production is defined, electronic features are decided. The features necessary for the present project are: oscillators, envelope generators, filters, amplification circuits, equalization, and circuits to generate effects such as vibrato and Tremolo.

In order to produce electronic sounds, an electric signal is needed to be later processed and transformed into sound through a transducer [10]. First of all, the electric signal is produced using an oscillator. Then, this signal is modulated using an envelope signal created with the envelope generator circuit. The resulting signal is then passed through a filter to eliminate some frequencies and get a signal closer to the one that an acoustic source would produce. After this, the signal is passed through sound processing which adds Tremolo or vibrato effects if desired. Finally, this signal is amplified and sent to the speakers to produce sound.

With all the features set and decided, the electronic devices needed can be defined. For signal generation, electronic oscillators are chosen. These oscillators produce periodic signals where frequency can be set through resistance and capacitance values, and amplitude of the signal is defined through voltage supply.

An envelope generator is used to modulate a signal to give it a rise, sustain, and decay time [46]. It consists of a circuit that generates a signal that modulates the oscillators signal's

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amplitude through a voltage controlled amplification (VCA). And, for signal filtering, simple low-pass, high-pass and band-pass filter circuits will be used [32].

For the signal amplification before output, a frequency-independent amplifier circuit is chosen [32]. This would allow any signal to be amplified leaving spectral characteristics unchanged providing a volume control when connected to speakers.

A six band equalizer circuit is chosen for equalization [7]. This consists of six coupled band-pass filters with adjustable amplitude outputs for the different signals generated. The existence of equalization will provide better sound manipulation for the user.

Two effects are chosen for the prototype instrument: Vibrato and Tremolo [32]. Vibrato refers to a change in frequency of the signal, which will be obtained by adding a low frequency signal to the main oscillation. Tremolo, on the other hand, refers to an oscillatory change in amplitude of the signal. This will be done by taking a low frequency signal and using it as control signal for the Voltage-controlled Amplifier.

Finally, a controlling interface is needed to let the user manipulate the signal outputs and produce the sounds at will. Since features such as wide frequency range and the ability to play various sounds at once are desired, a keyboard is decided as control mechanism for the system. This mechanism would let the user send various signals at once letting the performer to play rapidly and mix different sounds.

3.3.Signal Flow

With both acoustic and electric properties defined it is necessary to define how they will interact. Both acoustic and electronic sounds should combine and interact with one

another to achieve hybrid sound properties. To do this, it is decided to place the speakers inside the resonance box of the instrument. Since the resonance box is the place in the instrument where acoustic sound is amplified, placing the speakers there will provide good combination of sounds. This is because it is the place where both sources of sound connect.

The basic idea of how the sound will interact inside the box is illustrated in Figure 3.4 with a flow chart showing the path the sound will take from its source until it goes outside the instrument.



Figure 3.4.: Signal Flow Chart for the hybrid instrument

In the Acoustic part, a key from the keyboard will be pressed by the performer. This motion is transferred through the jack, which is a percussion excitation lever that moves a hammer that then hits the string and makes it vibrate. This vibration is then transmitted to the resonance box through the bridge, which is a piece of wood that defines the strings' length and connects them with the soundboard, where it is amplified. [35]

In the electronic part, a signal is produced in the synthesizer when a key from the electronic keyboard is pressed. This signal passes through the sound processing effects and filters. The signal is then amplified and sent to the speakers where it is transformed into sound in the resonance box. The sound then is blended with the acoustic one and the final combined sound is then heard by the listener.

3.4.Design Requirements

This section describes all the requirements to be set for the design of the hybrid instrument. These requirements include: structure, materials, and sound requirements referring to frequency ranges and loudness.

3.4.1. Structure

The structure for the hybrid instrument should be resistant and not too heavy for transportation. It should be able to resist the tensions that the strings would exert on it without excessive deformation. It also needs to be esthetic at sight. To meet these requirements, wood is chosen as the basic material for the general structure of the instrument. Additionally, an aluminum frame that would go inside the instrument to withstand the tension should be designed.

Regarding the size of the structure, it should be big enough to be able to adjust string length for low frequency notes, but not so large that it would be unpractical. The keyboard mechanisms as well as the other electronic functions should be near the performer for comfort and better playability. In order to achieve this, maximum dimensions for the instrument are set. The instrument's maximum size should be 1.30 m long, 1.30 m wide and 0.30 m tall. These dimensions were chosen assuming that the instrument would be played on a table or support with an average size person playing it comfortably by sitting on a chair in front of it.

3.4.2. Materials

Materials play a very important role in the design of the instrument. The selection of the right materials for each of the parts of the instrument is crucial for it to be able to create sounds properly and withstand the different forces applied through strings and excitation mechanisms. The three main materials used in the instrument are wood, aluminum, and steel.

Wood is the chosen material for almost all the structural pieces in the instrument (legs, resonance box, soundboard, percussion mechanism, etc.) It is strong and relatively light which makes it the ideal material for conforming most of the pieces.

Aluminum is used for the frame where all the strings will be tensed. This frame is very important since it must withstand the tension forces of all the strings. Aluminum is chosen because it is very light, can be casted through a mold, and it possesses good tensile properties. Steel is the material chosen for pieces such as screws, pins, and small parts. It is chosen

because this parts are commercially available in steel. Also, it is a very resistant material able to stand high stress without failing.

3.4.3. Sound Requirements

The instrument should be able to play a high range of musical notes and do so at a moderated loudness. This loudness should be high enough for a person to easily hear the performer playing if close enough to the instrument. To meet these requirements, it is decided

to use six octaves for the instrument which would give an approximate frequency range of 20Hz – 4000Hz. Also, the resonance box would need to provide enough amplification to be able to hear the music played from a moderate distance.

4. Design of the Instrument

This chapter describes the steps and analysis that were followed to design the hybrid musical instrument, as well as the different features and properties of the most important components. In the first part of the chapter, the design of the acoustic part including the excitation mechanism, strings, bridge, soundboard (principal component of the resonance box) and structure in general are discussed. After that, the electrical part of the design is analyzed including the different circuits to be used with its behavior, functionality and required components. Other parts were also designed for the instrument including pins, buttons and small supports, but since they do not play an active role in the sound production they are not discussed. Drawings of the final design of these and the active components discussed below can be found in Appendix C.

4.1.Design of the Acoustic Part

The acoustic part of the design involves all parts and mechanisms that contribute to the mechanical sound production, which in the case of the present project is via string excitation. In this part of the design, the general structure of the instrument will also be discussed since, even though they do not play an active role in the acoustic sound production, they are used to hold in position and as support for all the parts that produce sound.

4.1.1. General Structure

The first part of the instrument that will be discussed is the general structure and body. The body, also called the rim of the instrument, is a wooden structure build as the main support of the instrument. The function of the rim is to act as support box for the other mechanisms and components. Additionally, it acts as part of the resonance box where the acoustic and electronic sounds will combine, and it also provides esthetic looks to the instrument.

The structural rim designed for the project was designed using a French Zither model from the 1800s as shown in Figure 4.1. The original esthetic looks were replicated, but the dimensions were modified in order to obtain a body of about 84 cm long, 125 cm wide and 36 cm tall.



Figure 4.1 Zither used as base model for the rim [17]

Besides the main rim, other structures were designed and attached to it to place the electronic components. These pieces were made to hold the circuits and to provide the user with manipulation tools. They were placed near the keyboard at strategic positions for user comfort. Additionally some wooden legs were also used to support the entire instrument. All these components are shown in Fig. 4.2



Figure 4.2 General structure of the instrument before other components implementation

4.1.2. Percussion mechanism

The percussion mechanism is the excitation source of the acoustic part of the instrument. It consists of a set of levers that act as an interface between the user and the instrument, transferring the force that the performer inputs on the keys to the strings as a hammer blow. A schematic of the percussion mechanism used as reference is shown in Figure 4.3.



Figure 4.3 Schematic of the percussion mechanism used as reference for the instrument [24]

As can be seen from Figure 4.3, the mechanism is nothing but a complex set of levers that are used to transfer the force from the key to the string. First, the key is depressed by the user and it lifts a second lever called the whippen with the Capstan screw, which is a big headed screw connected to the key lever. When the whippen is raised, it lifts the jack and the repetition lever at the same time. The jack is raised until it touches a stop screw and rotates raising the hammer and shooting it to the string where the hammer hits it and begins the string vibration.

After the hammer hits the string (represented in the schematic form Fig. 4.3 as a hard point), it falls and stops at the back check. This position is known as check position. When the hammer is at check, the key is still fully pressed. This special position is used for double tap playing. If the user releases the key at half its original position, the repetition lever brings the jack back to the shooting position allowing the user to quickly play the note again only by hitting the key as shown is Figure 4.4. If the user releases the key completely, the mechanism goes back to its original position.



Figure 4.4 Percussion mechanism held at check position [35]

When the key is pressed, it also lifts a felt damper which is located over the string and used to stop vibration. This damper is raised by a small lever at the end of the key. With this mechanism, the damper lifts at the same time they key is pressed and falls when the key is released. The damping mechanism stops the vibrating string the moment the key is released to prevent it to keep vibrating and make sounds that can distort the melody being played. The final model design of the percussion mechanism is shown in Figure 4.5.



Figure 4.5 Final model of the percussion mechanism

As for the materials for these components, wood is chosen since it is strong and esthetic. Rubber or felt are good options for the stop spots. For the hammer head, felt or wool will be the best material since hammer heads made of these materials are available on the market. Also, felt or wool properties provide better tuning options for the heads.

4.1.3. Strings

The strings are the most important components in the acoustic design. Each string must provide a different note, which means that each string needs to be tuned at a specific fundamental frequency in order to produce this particular note.

Since the instrument is designed to have six octaves that go from C2 (65 Hz) to B7 (3951 Hz), with twelve notes per octave, it will require a total of 72 strings with different fundamental frequencies. From string vibration theory as the one previously discussed in Chapter 2, it was shown that the string's fundamental frequency depends on its length L [m], its linear density $\mu \left[\frac{Kg}{m} \right]$ and the tension T [N] that is applied on the string as shown in Equation (4.1).

$$f = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \quad [Hz] \quad (4.1)$$

Since all the strings will be made of music wire steel, it is more convenient to express equation (4.1) in the form of equation (4.2) remembering that the linear density is equivalent to the volumetric density ρ times the transversal area $A = \pi \frac{d^2}{4}$.

$$f = \frac{1}{Ld} \sqrt{\frac{T}{\pi\rho}} \quad [Hz] \quad (4.2)$$

Equation (4.2) shows that frequency is dependent of Tension, Length and diameter since the volumetric density is a constant for all the strings. Using this approach, the fundamental frequency of each string will be obtained by regulation of these parameters.

First of all, each of the frequencies of the string needs to be determined. This is done using music theory which states that the frequency of a note an octave higher is the double of the equivalent note in the previous octave as expressed in Equation (4.3) where n is the number of the octave.

$$f_n = 2f_{n-1}$$
 (4.3)

Given the fact that there are seven fundamental notes and five half tones in an octave giving a total of twelve frequencies per octave, the relationship between the frequencies of each note can be achieved. Using equation (4.3) as a base and making use of the 12-tone equal temperament scale [21] that says that the interval between two semitones is given by $2^{1/12}$, an equation to obtain the frequency values of each note can be obtained.

Replacing this value into equation (4.3) yields equation (4.4) with m being the number of musical notes (or half tones) up or down the frequency of the note used as reference.

$$f_m = 2^{\pm m/_{12}} f_{ref} \quad (4.4)$$

Using equation (4.4) and knowing that the frequency of A4, which is 440 Hz, is used as a reference value to calculate the other frequencies, all the fundamentals for the strings are calculated. These values are given in Table 4.1

Octave	С	C#	D	D#	E	F	F#	G	G#	А	A#	В
Number												
1	65.41	69.30	73.42	77.78	82.41	87.31	92.50	98	103.83	110	116.54	123.47
2	130.81	138.59	146.83	155.56	164.81	174.61	185	196	207.65	220	233.08	246.94
3	261.63	277.18	293.66	311.13	329.63	349.23	369.99	392	415.30	440	466.16	493.88
4	523.25	554.37	587.33	622.25	659.26	698.46	739.99	783.99	830.61	880	932.33	987.77
5	1046.50	1108.73	1174.66	1244.51	1318.51	1396.91	1479.98	1567.98	1661.22	1760	1864.66	1975.53
6	2093	2217.46	2349.32	2489.02	2637.02	2793.83	2959.96	3135.96	3322.44	3520	3729.31	3951.07

Table 4.1.: Fundamental frequency values [Hz] for each musical note

With the fundamental frequencies of the strings, the selection of the parameters of tension, length and diameter is the next step. But, before the selection of each of these parameters, it is necessary to discuss the limitations in the range of values for each one.

Tension will be the first parameter to be discussed. The material selected for the strings is steel ASTM – A228 commonly known as music wire or spring wire [1]. It is a high strength steel with excellent elastic and fatigue properties. It has a Young Modulus of 205 GPa and a tensile strength between 1585 and 2750 MPa depending on the diameter [ASTM] which means it is very strong and capable of withstand high tensions. With this properties in mind, the maximum capable tension one of these strings would be capable of withstand can be obtained using equation (4.5).

$$T = \sigma_y \frac{\pi d^2}{4} \quad (4.5)$$

After determining the maximum tension value, it is advisable to take only a fraction of this value and set it as maximum tension in order to provide a factor of safety for the strings and avoid problems such as string breaks and detuning. According to the ASTM, the recommended tension operating value is at about 60% of the maximum tension. With this being said, the tension range was set up to a maximum value of 750 N.

The diameter parameter mainly depends on the commercial standard values of the spring wire, which can be obtained in 24 different diameter values which are listed in Table 4.2.

Spring Wire	Diameter [mm]
12	0.7366
12 1/2	0.762
13	0.7874
13 1/2	0.8128
14	0.8382
14 1/2	0.8636
15	0.889
15 1/2	0.9144
16	0.9398
16 ½	0.9652
17	0.9906
17 1⁄2	1.016
18	1.0414
18 1⁄2	1.0668
19	1.0922
19 1⁄2	1.1176
20	1.143
20 1/2	1.1684
21	1.1938
21 1/2	1.2192
22	1.2446
23	1.2954
24	1.397
25	1.4986

Table 4.2.: Spring wire diameters available commercially [1]

Finally, the length parameter is only restricted by the general dimension of the musical instrument itself. Since the structure total length is 1.25 m, the length is chosen to go from 0 to 0.90 m. It is important to remember that the length taken into account in these calculations is the speaking length, which is commonly known as the effective longitude of the string that will vibrate to create sound. This parameter is adjusted with the bridge. In the design of the instrument, all the strings will be attached to a single structure (the frame) which will be discussed further in this chapter.

With all the parameters defined and considering their restrictions, a calculation of the parameter values to match the fundamental frequencies is made. Taking a look at the different factors that influence the string fundamental frequency, it is decided to rearrange equation (4.2) to get Tension as a function of the other parameters as shown in equation (4.6).

$$T = \rho \pi (fLd)^2 \quad (4.6)$$

For each musical note, the fundamental frequency is a constant and the diameter can be varied between the ranges of commercial values to get different curves. This leaves tension only as a function of length as shown in equation (4.7) where $K = \rho \pi (fd)^2$ is a constant value for every curve and note.

$$T = f(L) = KL^2 \quad (4.7)$$

With equation (4.7) as reference, a script in Matlab is created to introduce the values previously discussed and generate the necessary data. With these data, various curves of the values of tension as a function of length for different diameters are plotted for every fundamental frequency as the one shown in Figure 4.6. A straight line in the maximum value picked for tension of 750 N is also drawn to limit the possible values. Using these curves, values for the different parameters are chosen. In most of the curves, the intersection between the tension limit and the curve is used as a reference for choosing the values. This approach is not used in the lower frequency values because the curves do not cross this limit. Diameter and length are chosen so that length would decrease slowly and to minimize the amount of strings with different diameters. An exact tension is then calculated using the chosen values. All the plots made for the parameter selection are shown in Appendix A.1 and the script can be found in Appendix B.



Figure 4.6 Tension as a function of length for various diameters for C4 (262 Hz)

The values chosen for tension, length, and diameter are shown in Tables 4.3, 4.4, and 4.5 respectively. In each table, the values are listed as the note name in the columns and the octave in the rows. Also, a complete list of the values of frequency, diameter, tension and length for each string can be found in Appendix A.2.

Table 4.3.: Calculated values for the tension parameter T [N] for each musical tone using

Equation 4.6

Octave	С	C#	D	D#	E	F	F#	G	G#	А	A#	В
1	191.92	215.42	241.80	271.41	304.65	341.96	383.84	430.84	483.60	542.83	609.30	683.92
2	667.11	643.85	667.13	732.28	730.95	732.95	732.42	732.35	732.52	739.85	734.99	733.47
3	730.32	733.28	730.02	733.96	763.54	732.20	730.62	729.94	727.33	724.46	731.36	730.11
4	731.67	732.56	732.94	728.18	733.05	733.33	734.21	730.02	723.76	730.52	732.97	732.24
5	734.84	732.56	733.44	729.26	738.75	723.70	733.25	731.04	735.21	734.70	728.95	741.66
6	728.06	722.06	745.68	733.55	732.48	736.95	731.66	678.38	744.58	735.39	738.62	751.99

Octave	С	C#	D	D#	Ε	F	F#	G	G#	А	A#	В
1	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
2	0.9	0.9	0.9	0.89	0.875	0.845	0.815	0.805	0.778	0.756	0.729	0.705
3	0.664	0.628	0.607	0.59	0.568	0.525	0.495	0.467	0.44	0.426	0.404	0.381
4	0.36	0.34	0.321	0.302	0.286	0.27	0.255	0.24	0.232	0.22	0.208	0.202
5	0.191	0.18	0.17	0.16	0.152	0.142	0.139	0.131	0.124	0.117	0.11	0.108
6	0.101	0.098	0.094	0.088	0.084	0.083	0.079	0.0718	0.071	0.0666	0.063	0.06

Table 4.4.: Chosen values for the length parameter L [m] for each musical note

Table 4.5.: Chosen values for the diameter parameter d [mm] for each musical note

Octave	С	C#	D	D#	Е	F	F#	G	G#	А	A#	В
1	1.4986	1.4986	1.4986	1.4986	1.4986	1.4986	1.4986	1.4986	1.4986	1.4986	1.4986	1.4986
2	1.397	1.2954	1.2446	1.2446	1.1938	1.1684	1.143	1.0922	1.0668	1.0414	1.016	0.9906
3	0.9906	0.9906	0.9652	0.9398	0.9398	0.9398	0.9398	0.9398	0.9398	0.9144	0.9144	0.9144
4	0.9144	0.9144	0.9144	0.9144	0.9144	0.9144	0.9144	0.9144	0.889	0.889	0.889	0.8636
5	0.8636	0.8636	0.8636	0.8636	0.8636	0.8636	0.8382	0.8382	0.8382	0.8382	0.8382	0.8128
6	0.8128	0.7874	0.7874	0.7874	0.7874	0.7366	0.7366	0.7366	0.7366	0.7366	0.7366	0.7366

With all the values chosen, strings can be made and tuned to reach the fundamental frequency required. It is important to remember that these are referential values which can be slightly changed if needed. Also, it is necessary to note that in the design, each note in the first three octaves should have two strings per note while the next three octaves should have three per note. This is done to amplify the sound production as it is commonly done in pianos [35].

4.1.4. Bridge

The bridge, as the one shown in Figure 4.7 is the element that defines the speaking length of the strings, thus its design is very important. It will be made of hard wood in order to effectively transfer the energy from the strings to the soundboard (due to its rigidity) and to withstand the forces that the strings will apply.

The most important feature of the bridge is its curvature; it needs to have a curvy shape to lift the strings at the precise location so the calculated length is achieved. In order to determine this curvy shape, a graphic using the chosen lengths for the string as a function of the total bridge length was made as shown in figure 4.8.



Figure 4.7.: Example of a bridge of a piano [35]

A sixth order polynomial regression was made to obtain an average value of the positions as shown in Figure 4.8.



Figure 4.8.: Polynomial regression for bridge shape

The equation obtained from the regression is shown in Equation (4.8). Using this equation, new values for speaking length were determined in order for the lengths to match the shape chosen by the bridge. Also, using this new length values, the previously obtained tension values were recalculated. These new values are given in Appendix A.2.

$$y(x) = -36.06x^{6} + 143.96x^{5} - 231.38x^{4} + 188.43x^{3} - 78.48x^{2} + 13.425x$$
$$+ 0.133 \quad (4.8)$$

Finally, this data was used to design the bridge with the optimal shape as shown in Figure 4.9. It is important to note that the design of the bridge could be hard to construct since a theoretical curve was used, but since the tension parameters are tunable a simpler bridge design could be used and the fundamental frequencies of the strings can be achieved by tension tuning.



Figure 4.9.: Final design of the bridge

4.1.5. Soundboard

The soundboard is a large plank of wood with the same shape of the rim that fits perfectly inside. It is the part main part of the resonance box since it is responsible for the amplification of the sound due to its large superficial area capable of moving large masses of air [38].

The best material for constructing the soundboard is Sitka Spruce [20], a soft wood that grows in northern Canada and Alaska. It is widely known for its excellent acoustic properties.

The soundboard is driven by the vibration of the strings which are transferred to it through the bridge [22]. When a string is excited, its energy is transferred to the soundboard which vibrates at the same frequency moving the mass of air around it to produce sound. In other words, it transforms the mechanical energy from the strings into acoustic energy. A drawing of the soundboard designed is shown in Figure 4.10.



Figure 4.10.: Drawing of the soundboard

4.1.6. Frame

The frame is a structure placed inside the rim to secure the strings. Since there are 180 strings in the instrument, each with a tension up to 750 N, the total force that the frame needs to withstand is about 135 KN (14 tons), and thus it needs to be very strong.

The best material for this structure is aluminum, since it is cheap, light, and has good mechanical properties. The structure has to be as light as possible to make the instrument easy to travel and as portable as possible. For this reason, an optimization process, which will be discussed in chapter 5, is made in order to minimize the weight of the structure. The frame model created and the optimized frame model are shown in Figures 4.11 and 4.12 respectively.



Figure 4.11.: Base model of the Frame



Figure 4.12.: Final model of the Frame (after optimization)

4.2. Design of the Electric part

The electric part corresponds to all the circuits and components that produce electronic sounds and allow the user to manipulate them, as shown in Figure 4.13. In this part of the design process, the different electronic features of the instrument such as amplification, filtering and electronic sound synthesis by oscillating signals will be discussed [7]; showing how they work and what can be done with this part of the instrument. The components that will be discussed are the synthesizer and its components, the equalizer, and the amplifier.

Synthesizer Equalizer Amplifier Speakers

Figure 4.13.: Flow chart of electronic sound generation

4.2.1. Sound Synthesizer

The sound synthesizer is the part of the instrument in charge of producing the electronic sounds [32]. It is composed of several parts that transform voltage signals generated into oscillatory signals that will later produce the sound. It is composed of a keyboard circuit, 72 oscillator circuits (one for each note), a voltage controlled filter, a voltage controlled amplifier (VCA), an envelope generator, and a low frequency oscillator. All these circuits are further discussed next in this chapter. A flow diagram of the sound synthesizer is shown in Figure 4.14. An enlargement of the Flow Chart can also be found in Appendix A.3.



Figure 4.14.: Signal Flow chart of the sound Synthesizer

As can be seen from Figure 4.14, when the key is stroke a voltage control signal is sent to the envelope generator and to the corresponding oscillator to create the signal. The envelope generator then sends a control voltage that turns on the filters and the voltage controlledamplifier (VCA). The oscillator signal goes through the filters to alter its properties and get different sounds out of the different signals. After that, the oscillation signal goes into the VCA where it amplifies it using the envelope as reference and creating a sound that resembles being acoustically generated. This signal then goes out to the volume amplifiers (speakers) which are discussed further in this chapter. On the other side, if the low frequency oscillator is turned on, its signal can be added to the oscillator's signal for frequency modulation (vibrato) or used as a control signal in the VCA to create amplitude modulation (tremolo) [32]. All these features are discussed further in this chapter.

4.2.1.1. Keyboard

The keyboard circuit is the interface between the user and the electronic sound production. It consists of a set of switches, one for each key, connected to a resistor that goes to an oscillator. When the user presses one of the keys, the acoustic mechanism is excited and, at the same time, it sends a voltage control signal to the circuit. This signal turns on the oscillator corresponding to that key; which then produces the tone requested by the user. A schematic of the keyboard switch is presented in Figure 4.15. The enlarged schematic can be found in Appendix C.



Figure 4.15 Schematic of the keyboard circuit

The circuit shown in Figure 4.15 works in a very simple way, when a key is pressed the switch is activated and a control voltage can pass through the circuit and go directly as input of the oscillator which in turn creates the tone with the fundamental frequency corresponding to the key that was pressed. Between the switch and the oscillator, a buffer is set to avoid altering the output of the circuit. Also, a comparator is added so that the control signal is sent whenever a key is pressed. This is done to activate the other components.

4.2.1.2. Voltage-Controlled Filter

This part of the synthesizer is used to adjust and manipulate harmonics and overtones in the signals as well as reducing electrical noise produced by the circuits. This allows the instrument to change timbre features of the generated tones and, therefore creating different sounds. The voltage-controlled filter suggested was designed by Andre Lunkvist [32] and can be seen in Appendix C.2. In this design, a state variable filter is used which provides a high pass; a low pass and a band pass filter outputs, like the ones seen in Figure 4.16. The resonance and cutoff frequency values are given by resistance configurations.



Figure 4.16.: High pass, low pass, and bandpass behavior [7]

Voltage signal control is achieved by replacing the resistors that give the resonant frequency of the circuit and making use of the LM13700 components which, if set in the particular configuration seen in Figure 4.17 acts as a voltage controlled resistance [32]. When this is done, cutoff frequencies can be set using R_{tune} potentiometer. Additionally, the resonance and quality factors, which were defined in Chapter 2, can be adjusted using the tuning potentiometer R_{RES} in the feedback gain. The control voltage is obtained from the instrument's voltage supply and is set using a voltage divider. The filter is designed to cover a frequency range of 30Hz - 4000 Hz.



Figure 4.17.: LM13700 in voltage controlled resistance set [32]

With the voltage controlled filters, as mentioned before, a manipulation of the signal before the output can be obtained depending on what signal is passed through which filter. Therefore, switches are added for the user to be able to manually manipulate the signal by selecting what filter to use. These controls are located in the electronic panel of the instrument.

4.2.1.3. Voltage-Controlled Amplifier

The voltage-controlled amplifier is a circuit that amplifies the input signal coming from the oscillator using a voltage signal as reference, as it name states. This circuit is the one in charge of coupling the envelope generator and the sound producing signal to get a sound that resembles an acoustic generation. There are commercially available VCAs; therefore there is no need to design this circuit. A schematic of this circuit can be found in Appendix C.2.

4.2.1.4. Oscillators

The oscillator circuit is one of the most important parts of the electronic part design. It is the responsible for producing the electric signal with the frequency necessary to produce the

sound. For the present project, a Wien Bridge oscillator as the one shown in Figure 4.18 was used for this necessary component. This oscillator was chosen because is one of the most used in sound generation [32]. It is important to remember that each note will have an individual oscillator circuit.



Figure 4.18.: Schematic of the Wien Bridge Oscillator [32]

This circuit's behavior can be determined using the ideal conditions of operational amplifiers, Barkhausen conditions and general circuit theory, previously discussed in Chapter 2.

First, analyzing the negative connection of the amplifier using the node rule, the gain in the connection can be obtained as seen in Equation (4.9)

$$G\left(\frac{V}{V}\right) = \frac{V_o}{V^+} = 1 + \frac{R_2}{R_1}$$
 (4.9)

Then, analyzing the positive connection a value of the amplifier's input voltage as a function of the output voltage can be obtained as seen in Equation (4.10)

$$V^{+} = \frac{V_o}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)} \quad (4.10)$$

Applying the Barkhausen Condition for oscillation in Equation (4.10), the oscillation frequency which is shown in Equation (4.11) is found.

$$f_0 = \frac{1}{2\pi RC}$$
 (4.11)

And doing the same in Equation (4.9), the condition for oscillation sustain can be obtained from Barkhausen conditions as shown in Equation (4.12)

$$A_{Vo}(j\omega) = \frac{1}{\beta(\omega_o)} = 3$$
(4.12)

$$1 + \frac{R_2}{R_1} = 3 \to R_2 = 2R_1$$

Thus, R_2 has to be at least twice as R_1 to sustain the oscillation. Nevertheless, a gain a little higher than 3 is recommended to ensure sustaining.

As seen from Equation (4.11), frequency of the oscillator is determined by resistance R and capacitance C values used, which lets the oscillator to be designed in a high range of frequencies.

The Wien Bridge circuit is used as base for two parts of the synthesizer: the low frequency oscillator and main oscillator circuitry for sound producing.

Low Frequency Oscillator

The low frequency oscillator can be seen in Figure 4.19. It provides the vibrato effect when coupled to the sound oscillators and the tremolo effect when connected to the voltage controlled amplifier. The resistors in the Wien Bridge (the ones that determine the frequency of oscillation) are set in series with a dual potentiometer in order to manipulate this oscillator's frequency manually. Dual potentiometers are needed since resistance magnitude needs to change in the same rate. Also, to avoid loading in the output some buffer unity-gain opamps as the ones discussed in Chapter 2 are used. The values seen in Figure 4.19 are set to achieve frequencies between 1 Hz and 33 Hz. These values can also be found in Appendix A.2.



Figure 4.19.: Low Frequency oscillator Schematic [32]

Sound Producing Oscillator

The main sound oscillator circuit as the one shown in Figure 4.20 is the one used in the design of the instrument. It is basically the same as the low frequency oscillator only with some modifications in the components. Resistance and capacitance magnitudes are set to achieve each fundamental frequency needed. A list of the values of the components for the oscillators can be found in Appendix A.2. The values used for the capacitance components can be obtained commercially while the resistance values need to be obtained by the use of dual potentiometers. This was done since precision for the frequencies is required and values

needed to precisely achieve are not available commercially. The Code used to find these values can be found in Appendix B.

For the oscillator's circuit, some buffer unity-gain amplifiers, as the ones used in the Low Frequency Oscillator (LFO) are set to avoid loading in the output; and a comparator amplifier, as the ones discussed in Chapter 2, is used after the sine oscillator to produce a square wave oscillator. A Switch in the end of the outputs could be placed to choose between square and sine outputs if multiple output connections are not wanted.

The values for R_1 and R_2 which provide the gains of the amplifiers are the same for all the oscillators. The chosen values are $R_1 = 10 K\Omega$ and $R_2 = 22 K\Omega$ leading to a gain of 3.2 which would be more than enough to sustain oscillation. But, as can be seen in Figures 4.19 and 4.20, a different set of elements is set to obtain the gain. This is done to stabilize the gain and reduce non linearity issues that could destabilize the oscillator's gain.

The circuitry used for R_2 value will be explained using Figure 4.20 notation. This part of the circuit consists of a resistance of 10k (R_1) in series with a parallel resistance setting consisting of a resistance of 24k (R_9) and a resistance of 12k (\Box_{10}). Along with R_9 , diodes D_1 and D_2 are placed in a parallel setting. These diodes are off when the gain is equal to 3.2. When the diodes are off, the upper resistor receives no current, which means no voltage drops in it. This causes the value for equivalent resistance R_t to be the sum of the 10k and 12k resistance obtaining a total of 22k and thus getting the wanted gain value.

When gain is lower than 3.2, diode D_2 turns on causing the 24k resistor to act and thus reducing the total resistance for the value of R_t to $R_t = R_{10k} ||R_{24k} = 18k\Omega$. This causes gain
in the circuit to reduce. This makes the input voltage lower that the output one generating more current flow and increasing the amplitude until gain is set back to 3.2.

In the opposite case, when gain is higher than 3.2, diode D_1 turns on causing total resistance to be $R_t = R_{10k} ||R_{24k} = 18k\Omega$. This causes gain in the circuit to reduce too. But, since current flow is set in the other direction, it reduces the amplitude of the output signal until gain is set again to 3.2



Figure 4.20.: Sound producing oscillator Schematic [32]

4.2.1.5. Envelope generator

The envelope generator is just a simple circuit that is activated at the same time as the oscillator. What this circuit does is to generate a function that would modulate the oscillator's

output. The idea of this function is to produce more realistic sounds with attack, decay, sustain, and release times, as shown in Figure 4.21. Attack is the time the signal takes to achieve its maximum amplitude. Decay is the time the signal takes to decrease from the maximum value to the sustain amplitude. Sustain is the time the signal is held; it has a constant amplitude that last while the key is kept pressed. Release is the time that the signal takes to return zero amplitude.



Figure 4.21.: Attack, decay, sustain, and release times

The envelope circuit used for the design of the synthesizer is shown in Figure 4.22.



Figure 4.22: Schematic of the envelope generator circuit [32]

This circuit is just an RC circuit in series with the voltage output measure at the capacitor. The circuit is connected to a gate circuit with two transistors. When the gate supplies voltage, transistor Q1 opens and lets the capacitor charge and when gate control voltage is stopped, Q2 opens allowing the capacitor to discharge.

Using Kirchhoff's laws to find the capacitor's charging behavior gives Equation (4.13), and using the same analysis for the capacitor's discharge behavior gives Equation (4.14).

$$V_{c}(t) = V_{in} \left(1 - e^{-\frac{t}{RC}} \right) = V_{in} \left(1 - e^{-\frac{t}{\tau}} \right) \quad (4.13)$$
$$V_{c}(t) = V_{in} e^{-\frac{t}{RC}} = V_{in} e^{-\frac{t}{\tau}} \quad (4.14)$$

This provides an attack time and release time proportional to the time constant $\tau = RC$. The function stays at maximum value while the key is pressed setting a variable sustain that mainly depends on the user. This behavior is modeled using a Script which can be found in Appendix B. The envelope function that is modeled can be seen in Figure 4.23.



Figure 4.23: Envelope function of the generator

The envelope generator's attack and release time can be regulated through variable resistance achieved by the use of potentiometers in the circuit.

4.2.2. Equalizer

The equalizer is a circuit that decomposes the input signal and passes it through a series of band-pass filters, in this case six, which let the pass or block certain frequencies reducing or amplifying its contribution to the original signal, summing the modified signal and passing it to the output. It is used as a filter for the input signal to manipulate it and modify it at will. The circuit designed for this component can be seen in Figure 4.24 and an enlarged

schematic in Appendix C. Additionally, a flow chart of how the signal travels through the circuit is shown in Figure 4.25



Figure 4.24.: Circuit designed for the six-band equalizer



Figure 4.25.: Flow Chart of the Equalizer

This circuit has three important parts which will be discussed in detail: the inverter, the band-pass filters and the sum inverter. Additionally, the circuit has buffer operational amplifiers which reduce the load effect between each phase of the circuit.

4.2.2.1. Inverter

The inverter, as it names states, inverts the input signal without any gain of phase shift. This is made through resistance tuning to achieve a unity gain from the amplifier. This circuit is used only to reduce the need of resistors since an inverter amplifier requires fewer components than a non-inverter. Figure 4.26 shows a diagram of the inverter amplifier.



Figure 4.26.: Inverter amplifier

In this circuit, the voltage output is characterized by equation (4.15) where Vin is the input voltage and R1 and R2 are the resistors used.

$$V_{out} = -\left(\frac{R_2}{R_1}\right) V_{in} \quad (4.15)$$

The resistors used in this circuit have the same value giving a unity gain and just inverting the sign.

4.2.2.2. Band-pass filters

A band-pass filter is a circuit that filters the input signal. It lets a band of certain frequencies pass, and attenuates the rest of the frequencies present in the input signal. The central frequency and the cut-off frequency, which are the frequency in the middle of the band and the frequency corresponding to 3dB attenuation respectively, are determined by the components present in the circuit. Figure 4.27 shows a schematic of the band-pass circuit used.



Figure 4.27 Schematic of the band-pass filter. [7]

Analyzing the behavior of the circuit using the operational amplifier ideal properties, shown in Chapter 2, the circuit's behavior can be found. First of all, using Kirchhoff's law for currents V_1 can be calculated using Equation (4.16).

$$V_1 = -\frac{V_s}{jc\omega R_2} \quad (4.16)$$

Now using Millman's theorem, an equivalent form for V_1 using the other side of the circuit is found as seen in Equation (4.17)

$$V_{1} = \frac{\frac{V_{e}}{R_{1}} + \frac{V_{s}}{1/jc\omega}}{\frac{1}{R_{1}} + 2jC\omega + \frac{1}{R_{3}}} \quad (4.17)$$

Now Equations (4.16) and (4.17) are compared, and V_s/V_e which is the gain function of the circuit, is found to obtain Equation (4.18)

$$\frac{V_s}{V_e} = G(j\omega) = -\frac{R_2 R_3 C j\omega}{R_1 + R_3 + 2R_1 R_3 C j\omega - R_1 R_2 R_3 C^2 \omega^2} \quad (4.18)$$

And rearranging the terms and replacing the value $s = j\omega$, the transfer function shown in Equation (4.19) is obtained.

$$G(s) = -\frac{\frac{R_2 R_3 C}{R_1 + R_3} s}{1 + \frac{2R_1 R_3 C}{R_1 + R_3} s + \frac{R_1 R_2 R_3 C^2}{R_1 + R_3} s^2} \quad (4.19)$$

Finally, from the generic transfer function of a band-pass filter shown in Equation (4.20) where ω_0 is the central angular frequency of the filter, Q is the quality factor and K is just a gain constant, the values of this parameters are obtained and shown in Equations (4.21), (4.22) and (4.23).

$$G(s) = \frac{K\frac{Q}{\omega_0}s}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1} \quad (4.20)$$
$$K = \frac{2R_3}{R_1 + R_3} \quad (4.21)$$
$$\omega_0 = \sqrt{\frac{R_1 + R_3}{R_1 R_2 R_3 C^2}} \quad (4.22)$$
$$Q = \frac{R_2 C\omega_0}{2} \quad (4.23)$$

Using this values the central frequency $f_0 = \frac{\omega_0}{2\pi}$ and the cutoff frequency $\Delta f = \frac{f_0}{Q}$ are found as shown in Equations (4.24) and (4.25)

$$f_{0} = \frac{1}{2\pi C} \sqrt{\frac{R_{1} + R_{3}}{R_{1}R_{2}R_{3}}} \quad (4.24)$$
$$\Delta f = \frac{1}{\pi R_{2}C} \quad (4.25)$$

Using Equations (4.24) and (4.25), the different band-pass filters are set to let a certain band of frequencies pass and fade away the others. The cutoff frequency is calculated using a constant Q = 2 which was chosen for all the filters. The central frequencies and quality factors for the different filters were chosen taking into account the ISO normative for filtering audio signals. The central frequencies chosen for design can be appreciated in Table 4.6.

Band Filter number	Central Frequency [Hz]	Quality Factor	Cutoff Frequency
			[Hz]
1	65	2	32.5
2	125	2	62.5
3	500	2	250
4	1000	2	500
5	8000	2	4000
6	16000	2	8000

Table 4.6: Central Frequencies chosen for the filters

Using these values and the equations previously found, setting the values for the capacitors to be 100 nF for the first four bands and C=10nF for the other two bands (this is done since there not enough independent equations to determine its value, the value of C for the last two bands is smaller in order to get a higher resistance value to avoid excessive power use due to current and overheating) and also setting $R_2 = 2R_1$ (for signal stability), the values for the resistors are calculated. It is important to note that the obtained values for the resistors and capacitors using the equation are not all available commercially; so the calculated values

are used as a reference and approximate commercial values were chosen [18]. With these values the central frequency, quality factor and cutoff frequency were recalculated. All these values are shown in Tables 4.7, 4.8 and 4.9.

Band Number	R1 [ohm]	R2 [ohm]	R3 [ohm]
1	48970.75	97941.50	6995.82
2	25464.80	50929.60	3637.83
3	6366.20	12732.40	909.46
4	3183.10	6366.20	454.73
5	3978.87	7957.75	568.41
6	1989.44	3978.87	284.21

Table 4.7: Calculated values for the components of the filters

Table 4.8: Selected commercial values for the components of the filters

Band Number	R1 [ohm]	R2 [ohm]	R3 [ohm]
1	57000	100000	6800
2	27000	57000	3900
3	6800	10000	1000
4	3900	6800	440
5	3900	7500	560
6	2200	3900	270

Table 4.9: Recalculated central frequencies cutoff frequencies and quality factors

Band Filter number	Central Frequency [Hz]	Quality Factor	Cutoff Frequency [Hz]
1	64.57	2.03	31.83
2	114.20	2.04	55.84
3	539.03	1.69	318.31
4	970.62	2.07	468.1
5	8304.82	1.96	4244.13
6	16434.01	2.01	8161.79

Using the commercial values of the resistors and the equations from the circuit behavior, a script is created to model its behavior and create a bode plot of each of the bands. These plots show how magnitude and phase of the output signal vary as a function of frequency. Bode plot for the first band is shown in Figure 4.28. Bode plots for the rest of the bands can be found in Appendix A.1 and the script created in Appendix B.



Figure 4.28 Bode plot of the first band of the equalizer

4.2.2.3. Sum-Inverter

The sum inverter, as is name states sums the different inputs and inverses the signal. The output signal then, is a result of the sum of all the input signals inversed. The sum-inverter circuit used is shown in Figure 4.29. This part of the circuit takes all the input signals from the filters and sums them into one output signal. It also inverts it to reverse the effect of the initial inverter.



Figure 4.29 Sum-Inverter amplifier's Circuit

The voltage output is determined by Equation (4.26)

$$V_{out} = R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots \frac{V_n}{R_n} \right) \quad (4.26)$$

In order to get unity gain from the inverter R_f should be chosen have the same value as the sum of the inverse of the input resistors. To do that, all the resistors in the circuit should have the same value.

4.2.3. Amplifiers

The amplifier is just a circuit that takes the signal input and, using an external source, magnifies its amplitude without changing properties such as phase or frequency. This is the last electronic component which will be used to modulate the volume of the produced electronically synthesized sounds. A Schematic of the circuit to be used can be found in Figure 4.30



Figure 4.30 Schematic of the amplifier circuit [7]

Using the same properties of operational amplifiers previously discussed in Chapter 2, the behavior of the amplifier can be found.

First of all, the potentiometer located at the initial part of the circuit acts as a voltage divider, so the voltage input to the amplifier V_{in} is given by equation (4.27) where V_s is the

voltage source, R_a is the acting resistance in the potentiometer and R_p is the total value of the potentiometer resistance.

$$V_s = \left(1 + \frac{R_a}{R_p - R_a}\right) V_{in} \quad (4.27)$$

Since, ideally no current enter the operational amplifier, the capacitors C1, C2 and C3 have no effect on the analysis. From the operational amplifier's ideal conditions, the voltages in both entries of the amplifier are the same and equal in magnitude to the voltage input after the potentiometer. Now, using Kirchhoff's current law, the output voltage can be determined as a function of the input voltage to the amplifier as seen in Equation (4.28)

$$V_{\Box ut} = \left(1 + \frac{R_1}{R_2}\right) V_{in} \quad (4.28)$$

And obtaining the value of V_{in} from Equation (4.27) and replacing it into Equation (4.28), Equation (4.29) is obtained

$$V_{out} = \frac{1}{\left(1 + \frac{R_a}{R_p - R_a}\right)} \left(1 + \frac{R_1}{R_2}\right) V_s \quad (4.29)$$

This equation describes the behavior of the amplifier to an input voltage. As it can be seen, the amplifier does not depend on frequency so it does not provide any filtering or phase shifting. It can also be seen that the amplifier's gain depends on the values of the resistors R1 and R2 while the first factor on the right side of the equation provides a linear gain depending on the resistance of the potentiometer. When the effective resistance is tending to zero, this factor goes to 1 providing the maximum gain (equivalent to maximum volume). On the other

side, when the effective resistance R_a tends to R_p , goes to zero nullifying the amplifier's gain (equivalent to zero volume).

Components	Value
R1	2200 [ohm]
R2	1000 [ohm]
Rp	10000 [ohm]
C1	2.2 [uF]
C2	470 [uF]
C3	2200 [uF]
C4	100 [nF]

Table 4.10: Values of the components of the amplifier

The components values used for the project can be found in Table 4.10 where the values were chosen to get a 10 dB gain and a 10 W power consumption. If bigger amplification wants to be achieved, the resistors that provide the gain should be changed to other values to match the desired gain. In Figures 4.31 and 4.32, the gain behavior of the circuit as a function of the potentiometer resistance can be seen. This was made by the creation of a script that models the amplifier's behavior which can be found in Appendix B.



Figure 4.31: Gain of the amplifier [V/V] as a function of potentiometer resistance



Figure 4.32: Gain of the amplifier [dB] as a function of potentiometer resistance

It is important to note that even though this behavior will approach reality, there are some system non-linearity and not ideal behaviors that can slightly change the model behavior. The capacitors are used to reduce noise and give a better signal stability and a closer approach to ideality.

4.3. Final assembled Design

After discussing the most important components in the instrument, a view of the instrument as a whole is needed. In order to do that Autodesk Inventor, a CAD software was used to design and assemble the different components previously discussed. The electronic components, with the exception of the keyboard were not modeled since they do not provide significant value to the CAD view. The assembled model created using the CAD software can be seen in Figure 4.33



Figure 4.33: Final assembled design of the hybrid instrument.

Each piece of the instrument was modeled in this software giving specifications of geometry and material to obtain the final assembled model. As it can be seen from Figure 4.33, the resonance box of the model resembles the shape of the Zither shown in Figure 4.1 as desired. On wider end of the resonance box, the acoustic keyboard and the electronic circuits

are held. On the top side of the keyboard, the electronic panel is shown with all controls needed for user manipulation of the sound. Additionally, an auxiliary input is set in order for the user to be able to use music from MP3 devices as additional input. A detailed view of the electronic panel is shown in Figure 4.34.



Figure 4.34.: Electronic Panel

On the back side of the instrument, as shown in Figure 4.35, two speakers are coupled to the resonance box in order for the electronic sound to get into the resonance box to combine with the acoustic sounds. The volume of the speakers can be controlled using the knob in the right panel in order to modulate the combination of sounds and to keeping the electronic sound from overcoming the acoustic one.



Figure 4.35.: Side view of the Final assembled instrument

The instrument is designed for hybrid playing and to allow the user to create different types of sounds by a friendly, easy to use interface. Any performer with previous experience in instruments such as the piano or the synthesizer could easily adapt and play this instrument. Also, it can also be played only as an acoustic instrument, which is a plus feature. The careful design of each of the components makes the instrument as a whole to be very innovative, diverse and easy to manipulate for the user.

5. Analyses and processes made for the Designed hybrid Instrument

This chapter describes all the processes and analyses done for the design of the hybrid instrument and for the verification of the design criteria. These analyses include string behavior and simulation, mechanism simulation, modal analysis, steady state analysis, frame optimization, and sound radiation analysis. The purpose of these analyses is to evaluate and verify that the hybrid instrument is designed correctly, accomplishing the objectives that were set for the present thesis.

5.1. Mechanism Analysis

After designing and dimensioning the different components of the percussion mechanism, a model of its behavior is made in order to estimate the initial velocity that would be produced when struck by the hammer. This value is then used as input for the string behavior analysis which will be discussed further in this chapter.

The mechanism simulation is made with multibody dynamics theory with the aid of a Matlab & Simulink Add-in for this type of procedures known as SimMechanics. SimMechanics works with block diagrams that can be analyzed using Simulink to dynamically simulate the model using dynamic or kinematic solving methods depending on the inputs provided.

The first thing to do to create the model is to import all the geometric, mass and inertia parameters of the designed components into Simulink to create the body blocks which represent the solid parts of the model. To do this, an Add-in was installed in Autodesk

Inventor. This Add-in allows the CAD software to export all these parameters through xml (extensible Markup Language) and STL (Stereo Lithography) file for creation of the block diagram in SimMechanics. This procedure can be visualized in Figure 5.1.



Figure 5.1.: Process of feature export from Inventor to Matlab as Block diagram model

After exporting the component's features into SimMechanics, the block diagram is created with correct application of the joint blocks to connect the bodies between them, the ground blocks to provide reference to the system, and the force blocks to introduce the forces and moments applied to the system. Additionally, some sub-models for part's contact are created and applied between the bodies for correct simulation. The block model can be found in Appendix A.

For the model simulation, the force profile shown in Figure 5.2, which is obtained from a study of the techniques of piano playing [24], was used as input and four variables requested as output. These four variables are: hammer angular rotation, hammer angular velocity, whippen angular displacement and key angular displacement. These variables are recorded setting the reference position (initial position of the components) as zero, so only variation from the initial position can be appreciated. When the model is run, a graphic aid like the one shown in Figure 5.3 appears to show how the model behaves and the required data is extracted and saved into Matlab workspace. The model was simulated for a single forte blow into the key, which means the key is depressed once (raise position), held for a small time (check position) and then released. The force is maintained for a small portion of time to show check position for the components.



Figure 5.2.: Force profile used for the model



Figure 5.3.: Graphic Aid of the model

The key angular displacement θ , whippen angular displacement α , and hammer angular displacement β , as shown in Figure 5.3 change the moment the force is applied to the key by the user. The variation of these three angles with respect to time is shown in Figure 5.4.



Figure 5.4.: Angular displacement for the key, whippen, and hammer

As it can be appreciated, the key angular displacement θ changes in a parabolic manner and then reaches a peak when gets to contact position until it stabilizes to a regular position. This peak appears due to the contact model which is set as a very stiff spring, therefore allowing an over displacement for the key before the force reaches equilibrium. Once the position stabilizes, it stays there as long as there is force being applied to the system and then drops back to the initial position when force in the key disappears. It can be observed that the position drops slower compared to the acceleration it suffers to get to its maximum position; this happens since force is slowly decreased as in can be seen from the force profile in Figure 5.2. Also, there are peaks in the position drop as well due to the contact model which creates a small bouncing when the key drops back since small damping features were applied to the contacts.

Whippen's angular displacement α , as shown in Figure 5.4, exhibits a similar behavior as the key, which is expected since it is actioned through the key movement. It has similar peaks when raised and dropped that are caused by the contact definitions with the key through the Capstan screw. Its rotational magnitude is different due to the lever gain and the radius of gyration which is smaller than the one from the key. Key displaces a maximum of 10 mm when pressed, but due to the lever effect it applies, it can raise the whippen up to 30 mm.

Hammer rotational displacement β is the last position value evaluated in order to verify the model. It is also shown in Figure 5.4. Hammer exhibits similar conditions as the ones from the key and the whippen with some differences. The peak obtained when the displacement starts changing due to the force is quite higher than the ones shown in the key or the whippen. This happens since the hammer is launched into the string, thus it raises a lot more before

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being stopped by the string (which in the case of this model is just a solid reference point). It also exhibits less bouncing peaks when it falls into check (first stable position) and release (initial position). This is probably due to energy absorption through the hard point that resembles the string, which was modeled as a hard contact with some damping (see Figure 4.3). Also, this motion could have being absorbed by the other component's damping contacts. There is some small bouncing when it falls into release, but it is smaller than the ones observed in the other components.

Finally, the hammer rotational velocity was obtained in order to be able to provide a value for initial velocity for the string in the contact point. The hammer rotational velocity for the simulation is shown in Figure 5.5. As it can be seen, the velocity spikes quite fast due to the acceleration applied when the key is depressed reaching a peak value of 48 [rad/s] when it strikes the string. The second the string is struck, a sudden shift in velocity is achieved and it becomes negative really fast until the hammer is stopped by the whippen in the check position. Then it grows negatively when the key is being released and the hammer goes down until it is stopped at the release position and velocity goes to zero. With the peak value of velocity and hammer rotation radius, an initial velocity for string excitation of 4.56 [m/s] can be estimated.



Figure 5.5.: Hammer Rotational velocity

5.2. String Behavior simulation

A string behavior analysis is done in order to verify the correct behavior of the strings and to use this model to get string excitation to the instrument for a steady state dynamic analysis that will be further discussed in this chapter. This simulation is made using theoretical approaches for the dynamic behavior of the strings previously discussed in Chapter 2 and Matlab to obtain the numerical data for the model.

Using equation (5.1) which describes the general behavior of a string clamped at both edges [46] and replacing some values, an expression for struck strings behavior can be obtained.

$$y(x,t) = \sum_{n=1}^{\infty} \left(A_n sin(\omega_n t) + B_n cos(\omega_n t) \right) sin\left(\frac{\omega_n}{c}x\right) \quad (5.1)$$

$$y(x, 0) = 0$$

 $\dot{y}(x, 0) = v_0(x)$
(5.2)

Then, taking these initial conditions and replacing them in the Fourier coefficients shown in Equation (5.3), the values of A_n and B_n are obtained. These values are be replaced into Equation (5.1) to obtain Equation (5.4) where $\omega_n = \frac{n\pi c}{L}$ is the mode frequency, L is the string length, $v_0(x)$ is the initial velocity, n is the mode number, t is time, x is position in the string, and $c = \sqrt{\frac{T}{\mu}}$ is the speed of sound.

$$A_n = \frac{2}{\omega_n L} \int_0^L \dot{y}(x,0) \sin\left(\frac{n\pi x}{L}\right) dx$$
(5.3)

$$B_n = \frac{2}{L} \int_0^L y(x, 0) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$y(x,t) = \sum_{n=1}^{\infty} \left(\frac{2}{\omega_n L} \int_0^L v_0(x) \sin\left(\frac{n\pi x}{L}\right) dx \right) \Box in(\omega_n t) \sin\left(\frac{\omega_n}{c}x\right) \quad (5.4)$$

Now, assuming that the initial velocity is given to the string in a small portion (the area where the hammer first hits the string) while the rest of the string is kept at no velocity, a function for the initial velocity as a function of string position can be determined as seen in Equation (5.5).

$$v_0(x) = \begin{cases} v = 0 & 0 \le x < L_1 \\ v = v_0 & L_1 \le x \le L_2 \\ v = 0 & x > L_2 \end{cases}$$
(5.5)

And replacing Equation (5.5) into Equation (5.3) and integrating, Equation (5.6) which represents the Fourier coefficients for a struck string is obtained.

$$A_{n} = \frac{2}{\omega_{n}L} \left[\int_{L_{1}}^{L_{2}} v_{0} \sin\left(\frac{n\pi x}{L}\right) dx \right] = \frac{2v_{0}}{\omega_{n}L} \left(-\frac{L}{n\pi} \cos\left(\frac{n\pi x}{L}\right) \Big|_{L_{1}}^{L_{2}} \right)$$
(5.6)

$$A_n = \frac{2v_0}{n\pi\omega_n} \left(\cos\left(\frac{n\pi L_1}{L}\right) - \cos\left(\frac{n\pi L_2}{L}\right) \right)$$

Finally, replacing the expression from Equation (5.6) into Equation (5.4), Equation (5.7) which represents the behavior of a struck string, is obtained.

$$y(x,t) = \sum_{n=1}^{\infty} \left(\frac{2\nu_0}{n\pi\omega_n} \left(\cos\left(\frac{n\pi L_1}{L}\right) - \cos\left(\frac{n\pi L_2}{L}\right) \right) \right) \sin(\omega_n t) \sin\left(\frac{\omega_n}{c}x\right)$$
(5.7)

Using Equation (5.7), a Matlab function is created to obtain the numeric values for the string behavior as a function of time. This can be done by truncating the sum to a finite value of modes n to be able to find enough numeric values to accurately represent it. The Matlab function created can be appreciated in Appendix B. This function requires string length, diameter, material density, tension (to obtain the natural frequency $\omega = f(d, L, T, \rho)$, number

of modes requested, initial velocity, fraction of the string where velocity is applied, step time and total time to simulate; and returns modal frequencies (in radians per second and Hz), modal behavior and total behavior vectors. Additionally, it returns length and time vectors for plotting purposes.

With the results from the script, the first two modes of vibration of a C4 string with respect to different positions in the string and time is plotted as shown in Figures 5.6 and 5.7. The total behavior of the string, as seen in Equation 5.7, is the sum of different modes of vibration.



Figure 5.6.: First mode of vibration of a C4 String at several times



Figure 5.7.: Second mode of vibration of a C4 String at several times

After creating this function, a script that uses it to produce graphs and videos of the string behavior is made. The string's temporal behavior for a C4 string can be found in Figures 5.8 to 5.10. The initial velocity used is 5 [m/s] and it acts at 0.3 [m] in the string.



Figure 5.8.: Behavior of a C4 string at t=0.01s



Figure 5.9.: Behavior of a C4 string at t=0.1s



Figure 5.10.: Behavior of a C4 string at t=0.25s

Taking a look at struck string's behavior, the wave propagation along the string can be analyzed. The wave first starts forming where the velocity is applied, then it grows along the string until it reaches its maximum amplitude and starts moving along the string shifting when it reaches the edges. Some small oscillations outside the main wave can be appreciated, this happens because the wave is approximated by truncating the Fourier series to a limited value.

After verifying the correct behavior of the string, a model for obtaining the force excitation that the strings exert over the bridge is created. This is done with the assumption that the strings are ideal, therefore neglecting all material elasticity effects (tension is the only restoring force). By this assumption, it is logical to say that all the force the strings exert on the bridge comes directly from the vertical component of the tension T_y as the string vibrates, as shown in Figure 5.11. The vertical tension component is given by Equation (5.8).

$$T_{v} = Tsin\theta$$
 (5.8)



Figure 5.11.: Assumed directions in the analyzed string

And since the string's deviation angle θ is very small when it vibrates, $sin\theta$ can be approximated to $tan\theta = \frac{dy}{dx}$ and vertical Tension can be expressed as Equation (5.9).

$$T_y = T\left(\frac{dy}{dx}\right) \quad (5.9)$$

From Equation (5.9), it can be seen that the force exerted by the string into the bridge can be estimated using the total magnitude of the tension in the string, and the string slope.

Now, given that the bridge is one of the edges where the string is clamped, the total force exerted to the bridge is simply the vertical tension exerted by the string in point L, which is the bridge end of the string. Of course, the force is oscillatory and time dependent, since the string is vibrating. The force exerted over the bridge by the string is expressed in Equation (5.10).

$$F_{bridge}(t) = \left(T_y\right)_L = T\left(\frac{dy(x,t)}{dx}\right)_L \quad (5.10)$$

To find this force, it is necessary to find the string slope's behavior given that the tension magnitude is a constant. To do this, string's behavior found in Equation (5.7) is used. By deriving Equation (5.7) by x, a function of the string slope can be found as shown in equation (5.11).

$$\frac{\partial y(x,t)}{\partial x} = \frac{\partial}{\partial x} \left(\sum_{n=1}^{\infty} \left(\frac{2v_0}{n\pi\omega_n} \left(\cos\left(\frac{n\pi L_1}{L}\right) - \cos\left(\frac{n\pi L_2}{L}\right) \right) \right) \sin(\omega_n t) \sin(k_n x) \right)$$
(5.11)

$$\frac{\partial y(x,t)}{\partial x} = \sum_{n=1}^{\infty} \frac{n\pi}{L} \left(\frac{2v_0}{n\pi\omega_n} \left(\cos\left(\frac{n\pi L_1}{L}\right) - \cos\left(\frac{n\pi L_2}{L}\right) \right) \right) \sin(\omega_n t) \cos(k_n x)$$

Using the expression from Equation (5.11), a numerical estimation of the string's slope as a function of time can be obtained using the same procedure as the one used for string's temporal behavior discussed previously in this section. So, a Matlab function similar to the one created for string's behavior is made. This function can be appreciated in Appendix B. The same parameters as the string's behavior function are required, and it returns modal frequencies, modal behavior and total behavior of the slope as function of length and time. It also returns length and time vectors for plotting if required. String slope behavior for several times, as well as the corresponding string shape, can be found in Figures 5.12 to 5.14.



Figure 5.12.: String's slope behavior for t=0.01s



Figure 5.13.: String's slope behavior for t=0.1s



Figure 5.14.: String's slope behavior for t=0.25s

From Figures 5.12 to 5.14, it can be appreciated that the slope function has is zero on mostly all the string except on two points where two peaks are observed. These peaks correspond to the rise and fall of the wave. These peaks move as the wave runs through the string shifting when reaching the string edges which is consistent with the string behavior. Some minor oscillations can be observed too. These oscillations appear due to the truncation of the modes and would fade if more modes are used for the estimation.

The string slope behavior is consistent throughout all the string length, but, as can be seen in Equation (5.10), the force exerted in the bridge is the slope function evaluated in L (the point where the string is attached to the bridge) and multiplied by the tension magnitude. Therefore, only the variation of the slope in the last point of the string (where the string joins the bridge) is needed. So, by taking the vector of values of the string slope for point L and multiplying it by T a force profile of the string applied to the bridge as the one shown in Figure 5.15 can be obtained.


Figure 5.15.: Force profile for a C4 string as a function of time

Using this method, a good estimation of the forces exerted by the strings to the bridge can be obtained to provide a load input for the steady state dynamic analysis using FEM software called Abaqus. For this analysis, response for four strings: E2, C4, G5, and B7 was set. This is discussed later in this Chapter. The force profiles for these strings can be found in Figures 5.15 to 5.18.



Figure 5.16.: Force profile for E2 string as a function of time



Figure 5.17.: Force profile for G5 string as a function of time



Figure 5.18.: Force profile for B7 string as a function of time

After taking a look at Abaqus documentation [34] for a brief on steady state analyses, it is observed that the software uses frequency sweeps along a range to obtain the response and loads are applied with constant amplitude throughout the frequency sweeps unless a frequency depended amplitude profile is inserted. And, since the forces the strings exert on the bridge are frequency dependent (its amplitude is not the same for every frequency), a force spectrum is needed in order to obtain more accurate results for the FEM simulation.

To obtain the force spectrum that the strings would exert, a fast Fourier transform is made using the force time profiles previously obtained. But, in order to obtain a spectrum that ranges through the entire audible frequency spectrum (20 Hz to 20 KHz), the Nyquist Theorem states that a sampling frequency of at least double the max value of the desired range (in this case 20 kHz) is needed [3]. This requirement makes the previous slope function memory inefficient, since the step time needed is very small and it would be quite hard to determine the maximum final time allowed to not saturate CPU's memory allocation. To solve this problem, a new function to determine the slope is made using the previous one as reference. The Matlab function code can be found in Appendix B. This new function retrieves the same output as the previous one; but instead of needing the final time value, it needs the number of data values for time required. This allows the user to find a number of samples which is large enough to obtain an accurate FFT transform and ensuring that the CPU's memory is not saturated.

Using this new function, a script to obtain frequency responses of the forces set for the Steady state analysis is made. This script obtains the slope response with the necessary sampling frequency, calculates the necessary number of modes to accurately represent the force profile as a frequency function and then obtains the force amplitude spectrum using FFT algorithm. After running the script, the frequency responses for the desired strings are obtained. These responses are shown in Figures 5.19 to 5.22. They are created using a logarithmic scale for better appreciation. The script created is found in Appendix B.



Figure 5.19.: Frequency response Profile for E2 string



Figure 5.20.: Frequency response Profile for C4 string



Figure 5.21.: Frequency response Profile for G5 string



Figure 5.22.: Frequency response Profile for B7 string

As can be seen from Figures 5.19 to 5.22, the force exerted on the bridge by the string is a sum of various amplitudes at different frequencies, as was expected from the study of the string's behavior discussed previously in this chapter. Each peak in the Figures corresponds to

the fundamental (first peak) and overtone frequencies (following peaks). Each one of them contributes to the final force profile with certain amplitude values.

As it can be appreciated in Figures 5.19 to 5.22, as string's fundamental frequency is high, fewer overtones are present in the spectrum which is reasonable given the fact that the overtones are integer values of the fundamental and thus, a less number of modes are needed to accurately represent its behavior in the frequency range of interest.

5.3. Modal Analysis of the resonance box

In order to evaluate the efficiency for sound production of the resonance box, a modal analysis of the resonance box is made using Abaqus, FEM software [34]. The purpose of this analysis is to obtain the resonant frequencies of the system to ensure the proper amplification of the vibration in the soundboard. If the system has resonant frequencies along the string's excitation frequencies, the resonance box will be driven near its natural frequencies and good amplification of the vibrations due to resonance will be achieved.

First of all, the geometric features are exported from Autodesk Inventor as STEP (generic CAD) files and then imported into Abaqus. Air models of the inner cavities of the resonance box and speakers are also modeled since they can affect the natural frequencies of the system and results as accurate as possible are desired. After defining the geometric parts through the step files, material properties are determined. Four materials are used: air for the inner cavities which is defined as an acoustic medium with a bulk modulus of 0.134 MPa, white oak which is a hard wood used to model the rim, speaker boxes, and the bridge; Sitka

Spruce, which is a soft wood used to model the soundboard; and polyethylene which is a kind of plastic used to model the speaker cones.

To define all these materials, elasticity properties and density are defined [34].

Polyethylene is modeled as an isotropic material with density $\rho = 450 \frac{Kg}{m^3}$, Young Modulus E = 3.4 GPa and Poisson's Ratio $\mu = 0.34$. Wood is an orthotropic material, which means it has orthogonal properties [20]. The values used to model the elastic behavior of wood as well as wood's density were obtained from Literature [20]; they can be found in Table 5.1.

Table 5.1.: Density and elastic Properties of wood materials defined [20]

Material	Density [Kg/m3]	<i>E_L</i> [GPa]	E _t [Gpa]	E _r [Gpa]	μ_{Lt}	μ_{Lr}	μ_{tr}	<i>G_{Lt}</i> [Gpa]	<i>G_{Lr}</i> [Gpa]	<i>G_{tr}</i> [Gpa]
Sitka Spruce	380	10.89	0.4683	0.8494	0.467	0.372	0.245	0.6643	0.6970	0.0327
White Oak	755	13.53	0.9742	2.2054	0.428	0.369	0.3	1.0959	1.1636	0.3383

With materials defined, section definition and assignation are defined. Each section is first defined as solid and homogenous and then is assigned to each part to define its properties. After that the assembly of the components is made. Each part is brought to the assembly section as an independent instance which is then placed using position constraints. Figures 5.23 and 5.24 show the final assembly of the resonance box in the Assembly module.



Figure 5.23.: Isometric front view of the assembly of the Resonance box in Abaqus



Figure 5.24.: Isometric rear view of the assembly of the Resonance box in Abaqus

After geometrically assembling the parts, the interaction between each of them needs to be defined. All solid parts are fixed together, so one does not move without the other. For the interaction between air and solid parts, tie constraints are made and acoustic impedance with a non-reflecting boundary condition (NRBC) is applied. This is done since Abaqus Manuals state that it is the best way to model acoustic coupling [34]. Acoustic impedance constant used is obtained using Equation (5.12) with ρ being the air density and c being the speed of sound at 25°C [3].

$$Z_{acousti\Box} = \rho c = \left(1.2 \frac{Kg}{m^3}\right)(343 \, m/s) = 411.6 \, Rayls \quad (5.12)$$

After defining all the interaction properties, the assembly is complete. The next step is to mesh each part (since they were brought to assembly as independent instances) in order to create the finite elements of the components. To do this, global seeding is applied to each part, a meshing technique is chosen and element type is decided for each part. C3D4 tetrahedral elements are used for solid parts, and AC3D4 tetrahedral elements are used for air (acoustic medium). In the case of the present model, all parts are meshed using tetrahedral elements with a default algorithm provided by Abaqus. For the solid elements, stress linear tetrahedral elements are used. And for the air elements, acoustic tetrahedral linear elements are used.

The last module to define before analysis is the load module where loads and boundary conditions are set, as shown in Figure 5.25. Since the purpose of this analysis is to determine natural frequencies of the system, no loads or border conditions are needed. But, to obtain only the relevant modes and reducing computing needs, border conditions are applied. The base of the structure is anchored and all degrees of freedom restricted. This eliminates the rigid body modes which are calculated when no boundary conditions are applied.



Figure 5.25.: Resonance box with loads and boundary conditions applied

Finally, all modules are defined and revised to avoid all error possibilities, the step module is accessed to define the frequency extraction step to then create a job for analysis and run the procedure. In the frequency extraction step, which is a linear perturbation procedure, the frequency range in which the modes will be extracted is defined as well as the number of Eigen modes requested. The frequency range of interest is the audible range, but when run for the first time to extract all the Eigen frequency values in this range, it surpassed the maximum number that the program allows to obtain when it reached the 12500 modes and it crashed. So the frequency range of the analysis was reduced to values between 20 Hz and 4000 Hz which contain all the string fundamental frequencies in the instrument so it would provide enough insight for the frequency amplification which is the main objective.

After running the analysis, a total of 2223 Eigen frequencies were found along with the corresponding mode shapes. The mode shape for the Fundamental Frequency of 134.63 Hz is shown in Figure 5.26.



Figure 5.26.: Mode Shape for 134.63 Hz Eigen Frequency (Fundamental Mode)

From the range of frequencies obtained with the analysis, it is found that there are various natural frequencies from the structure lying in the range of the strings' fundamental frequencies. This means that the resonance box will provide effective vibration amplification from the strings which will translate into effective sound radiation. A complete list of all the Eigen frequencies found in the analysis as well as some of the mode shapes for some of these frequencies can be found in Appendix A.

5.4. Steady state behavior analysis

After the modal analysis for the system, four steady state analyses are made to determine the actual frequency response of the system in the audio range when driven by a certain force. Each analysis obtains the frequency response of the system to the forces applied by obtaining some points along the frequency range and linearly interpolating them [34]. The requested number of points for this procedure is 50 points per analysis.

As previously mentioned, four strings are selected to provide the load and make the analysis. These strings were selected to analyze how the system would respond to a low frequency, mid frequency and high frequency excitation. Along with these string forces, an additional mechanical force was set in the speaker cones to resemble electronic excitation. All the forces, with the exception of the speaker force were applied as frequency dependent loads with the amplitude spectra found previously in the String Behavior section. All the forces magnitudes can be found in Table 5.2.

Force Source	Magnituda [N]	Fundamental Frequency [Hz]		
Force Source				
E2 String	22.03	82.41		
C4 String	23.00	261.63		
G5 String	31.56	783.99		
B7 String	25.98	3951.07		

Table 5.2.: Force magnitudes applied in the analyses

Speaker cone

The model created for the resonance box modal analysis was used as base model for the steady state model. All the steps followed in the modal analysis model are followed in the same order, but some additional steps are made. First of all, four amplitude curves are defined. These amplitude curves are created using tabular data which is exported from Matlab in ASCII format and later read into Abaqus. These amplitude curves are created as relative values, which mean they represent a fraction of the actual value, since Abaqus CAE takes only relative amplitude values and it later multiplies it by the magnitude that is given to the load. The shape of these amplitude curves is the same as the one in Figures 5.19 to 5.22.

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After creating the amplitude curves, four steady state steps are created in the Step module, one for each string load procedure and the loads are specified in each one. The loads are applied to the points in the bridge where the strings would touch it, as shown in Figure 5.27. To define these points, some datum points are defined in the bridge features in the part module and then read when the load module asks for the point of application. Each step contains two loads, one coming from the string and the other one coming from the speaker cones. With all these taken care of, a job is created to analyze the model and then run.

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N/A



Figure 5.27.: Bridge with load applying points located



Figure 5.28.: System displacement response at 94.3 Hz with E2 string excitation

After running the procedure, frequency responses for different variables requested for each modeled frequency point are obtained. Figures 5.28 to 5.31 show 3D displacement plot of the system response for the different excitation loads at different frequencies. Other 3D frequency response plots can be found in Appendix A.



Figure 5.29.: System displacement response at 143.9 Hz with C4 String Excitation



Figure 5.30.: System displacement response at 678.6 Hz with G5 String Excitation



Figure 5.31.: System displacement response at 3684 Hz with B7 String Excitation

It can be seen from Figures 5.28 to 5.31 that the displacement generated by the forces applied by the strings is insignificant. Thus, the components would withstand these forces without failure. It is also observed that when frequency increases, the system has the tendency to become more chaotic. This is probably due to numerical error.

Using the obtained results from the model, some Frequency response data is obtained from randomly picked points in the system. Stress and displacement are plotted for each. Additionally, a point in the air cavity inside the resonance box is chosen and its pressure magnitude [dB SPL] response as a function of frequency plotted (for dB SPL, reference pressure of $p_0 = 2x10^{-5}Pa$ is used [3]). The same points are used to plot the responses for all the different force excitations and are plot in the same graphic. Figures 5.33 to 5.35 show the responses for one of the points chosen. The plots were made using logarithmic plotting for a better view. The plots made for the other specified points can be found in Appendix A.1. The location of the chosen nodes for the graphs is shown in Figure 5.32.



Figure 5.32.: Nodes chosen for Frequency Response Plotting



Figure 5.33.: Stress Frequency response of node 900 (soundboard upper left)



Figure 5.34.: Displacement Frequency response of node 900



Figure 5.35.: Pressure Frequency response of node 614 (air upper left)

As it can be appreciated from Figures 5.29 to 5.31, the system responds differently depending on the frequency it is excited and vibrates with enough amplitude to produce sound. Each frequency produces a certain response in the system which excites different resonant frequencies causing some parts of the system to deform and exhibit greater stress than others. These places would be the parts of the box that would amplify more effectively the vibrations in that frequency value.

Now, taking a look at Figures 5.33 to 5.35 and using the nodal positions in Figure 5.32, some things can be said from the frequency responses of the system. First of all, the responses of the system to different excitation loads appear to be very similar. Only small changes between the curves for stress, displacement and pressures for different loads appear. This could be due to the distance of the selected points from the places where loads are applied. If enough distance exists, it is reasonable to think that the specific node would respond similarly to different loads. This can also be verified when taking a look at the responses of the other

picked points. Curves for the loads applied closer to the analyzed note exhibit bigger difference with respect to the curves for loads applied further away from the node.

Another important thing noted from the plots is that the magnitudes for stress and displacement are relatively small. This means that the forces applied in the system are not big enough to permanently deform the resonance box. This verifies that the materials chosen will be able to support the excitation loads.

Additionally, by looking at Figure 5.35, it can be appreciated that the acoustic pressure to be produced by the resonance box reaches values as high as 70 dB SPL which means that the sound power that the instrument will be able to produce is quite high. This must be due to the combination of both sound producing sources. It is important to note though, that these values will be obtained in one of the openings that the resonance box presents and therefore the actual value to be heard by the user and listeners would be lower. Also, possible energy losses which were omitted in the model such as inner friction loss and sound reflection could alter the value in reality.

5.5. Sound Radiation Analysis

With the steady state response implying a good sound production, a sound radiation analysis is made to verify this. The sound radiation analysis is similar to the steady state analysis. Same loads are applied and same frequency range is used. The difference is that a hemisphere of air is added to surround the resonance box and coupled to the instrument using a tie constraint and acoustic impedance boundary conditions as shown in Figure 5.36. The air hemisphere has a diameter of 3 m.

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Figure 5.36.: Final model for sound radiation analysis

The air hemisphere is created by making a semi sphere with the revolve feature in the part module and then using the merge/cut option in the assembly module to produce the hole inside it where the resonance box will be placed. It is then coupled to the model using position constraints; and interactions (tie constraints) between the hemisphere and the box components are set [34]. The hemisphere part is modeled using air as material and acoustic finite elements. Additionally, a skin is created in the hemisphere and modeled with acoustic infinite elements. This is done to define far-field boundary conditions instead of using Non Reflecting Boundary Conditions. This optimizes the computational memory in the model and also allows the user to extract far-field results without need of rerunning the model (if the analysis done was a direct steady state analysis) [34]. It is important to note that in order to use acoustic infinite element,

a reference point is needed at the center of the hemisphere to set the normal vectors that these elements use for far-field interpolation. This reference point is set in the part module.

After getting the model assembled, it is run to obtain acoustic pressure magnitudes for the audible range for each set of force excitation, previously defined in the steady-state analysis. The responses obtained for some of the analyzed frequencies can be seen in Figures 5.37 to 5.40. Radiation responses for other frequencies can be found in Appendix A.



Figure 5.37.: Sound Radiation response for 81.9 Hz with E2 string excitation



Figure 5.38.: Sound Radiation response for 253 Hz with C4 string excitation



Figure 5.39.: Sound Radiation response for 781.4 Hz with G5 string excitation



Figure 5.40.: Sound Radiation response for 3684 Hz with B7 string excitation

After obtaining radiation response for each force excitation procedure a point is selected randomly in the outermost surface of the air hemisphere to graph its pressure magnitude in dB SPL as a function of frequency for each load. These responses are shown in Figures 5.41 to 5.44. Since dB SPL want to be used, reference pressure required to plot these graphs is $p_0 = 2x10^{-5}Pa$. It is important to note that the response around the hemisphere, as can be seen from Figures 5.37 to 5.40, is similar so it can be assumed that the results shown in Figure 5.41 to 5.44 are similar throughout the analyzed hemisphere surface.



Figure 5.41.: Acoustic response [dB] as a function of frequency for node 2420 with E2 string

excitation



Figure 5.42.: Acoustic response [dB] as a function of frequency for node 2420 with C4 string

excitation



Figure 5.43.: Acoustic response [dB] as a function of frequency for node 2420 with G5 string

excitation



Figure 5.44.: Acoustic response [dB] as a function of frequency for node 2420 with B7 string

excitation

As can be seen in the contours presented in Figures 5.37 to 5.44, the acoustic characteristics of the instruments are quite good. Sound pressures up to 47 dB are produced at distances of 1.5 m. Of course, different pressure magnitudes would be generated depending in the frequency analyzed. This happens since the resonance box amplifies some frequencies better than others. This is also due to the frequency dependent data used for load; some frequencies have higher amplitudes than others depending in the force excitation.

Pressure responses plotted for node 2420 in Figures 5.41 to 5.44, being node 2420 in top of the air hemisphere used and at a radial distance of approximately 1.5 m from the resonance box (this value would actually be less due to the box's height) are worth taking a look at. As can be noted, the response for all input loads is quite similar. This could be due to the distance of the point analyzed from the resonance box. Since it is at a considerable distance from the sound source, the pressure gradients would get to the point as a plane wave produced by an equivalent monopole source. This means that the point would respond similarly to the sound source regardless of where the force was applied.

Another important thing to be discussed are the negative pressures that appear in the response. This happens due to the logarithmic scale applied to the plot. The sound pressures calculated are variations around the atmospheric pressure created through the movements of the solid parts. Thus, these values are very small and go from values of mPa to values of μPa and smaller. Also, there are values of zero in some frequencies. So, when these are transformed into logarithmic scale, the result is a negative value.

Finally, the most important thing that can be noted in the pressure response plots of Figures 5.41 to 5.44 is the amplitudes reached at that distance. As it can be appreciated, the values of pressure at distances of 1.5 m are as high as 47 dB SPL which is very good with respect to the distance. This would mean that the hybrid instrument designed could be heard loud and clear at an almost 2m radius and in an open space. This ensures that the instrument in fact generates enough sound amplification to be heard at reasonable distance with no necessary extra amplifications and verifies the correct design of the resonance box.

5.6. Aluminum Frame Optimization

The final process made for the instrument design is a frame volume optimization procedure. As was previously discussed in Chapter 4, this component is essential since it is the responsible to withstand all the string tension forces which together can easily reach values over 10,000 Kgf. But, in order to make an optimal design for this component, it should be made as light as possible without compromising its tensile strength. That is the reason why this optimization is made.

In order to do this, a first prototype for the frame is created as the one shown in Figure 4.11 in previous Chapter. This prototype is made to match the shape of the outer rim which is the structure in which the frame will fit. The only parameters that were taken into account in the creation of this first prototype are geometric. This is to ensure that the structure would fit inside the rim; thus, no volume or weight characteristics are taken into account. The prototype is approximately 1.25m long and 0.90 m wide, with a 5 mm thickness giving an estimated weight of 16 Kg.

After creating this first prototype, it is exported into Abaqus environment using a STEP file and modeled using Aluminum density $\rho = 2700 \frac{Kg}{m^3}$, and elastic properties with a Young Modulus $E = 70 \ GPa$ and Poisson's Ratio $\mu = 0.35$. After specifying this model's parameters regarding material properties, the optimization module is started. Inside this module, three parameters need to be specified: The optimization technique, which in this case is a volume optimization; the restriction variables, which are the features that cannot be changed; and the optimization variable, which is the variable to be optimized as a function of the restriction variable. For this optimization process strain energy, which is the energy that the component is capable to store in the form of strain, is to be minimized restricting the volume feature to 20% of the initial volume which would lead to a frame of approximately 3Kg which is a good weight for the component. Thus, the process reduces the volume of the component to a 20 % of its original volume minimizing the deformation that the part would stand, therefore reducing its weight to a fifth of its initial value minimizing the strain the piece would suffer if a force is applied.

But before running this procedure, it is necessary to specify what the optimization regions of the component that cannot be touched are. This is necessary because there are parts of the component that need to keep intact since they are necessary to place other components such as pins to attach the strings. So if these regions are not restricted, it could take material off these necessary places which is not acceptable. The prototype frame with the zoned restricted from optimization is shown in Figure 5.45. After delimiting these regions, the procedure is started.



Figure 5.45.: Frame prototype with the zones restricted from optimization marked

After running the procedure, a new optimized model for the frame is obtained. This new model is shown in Figure 5.46. Using this new model as reference, the final model for the frame which is shown in Figure 4.12 in the previous chapter is made. The model that the program provides is not used directly because some inconsistent elements are left at the edges due to the recursive nature of the algorithm. The weight of the final model is estimated in 3.2Kg which is a good efficient value and proves that the optimization procedure is correct.



Figure 5.46.: Optimized model obtained from Abaqus

In order to verify the final model validity, a static analysis is performed in the final model of the frame using Autodesk Inventor. In this analysis, all forces and moments that the frame would have to withstand are applied. A 3D color graphic of the stress values, strain and Safety Factor obtained from the analysis are shown in Figures 5.47 to 5.49.



Figure 5.47.: Von Misses stress withstood by the frame



Figure 5.48.: Strain values in the frame



Figure 5.49.: Safety factor in the Frame

As it can be seen from Figure 5.47, the maximum stress that the frame would have to withstand is 3.19 MPa, which is much lower than the yield strength of aluminum which is 55 MPa proving that the frame would be able to stand all the tensions applied by the strings without major strain and with no possibility of failure since the minimum safety factor obtained is not under 15 as shown in Figure 5.49. Thus, the optimization is validated and a better design of the frame is obtained.

6. Discussion

Taking a look at the design and analyses made for the present thesis, some final points regarding the obtained results are worth discussing.

By taking a look at the final assembled design of the hybrid musical instrument, it is clear that the objectives proposed for the project are fulfilled. The design effectively combines the production of sound through two different sources, it is aesthetically attractive to the eyes of the user, and it is innovative in every aspect.

Regarding the acoustic part of the design, it is necessary to remark the effectivity of the excitation mechanism and the sound amplification devices. The achieved design for the mechanism would allow the user to play various notes at the same time, and play them quite fast due to the included repetition mechanism. As for the resonance box, it is clear that its design effectively amplifies the vibrations produced by the strings allowing higher sound pressures to be created, and thus providing the designed instrument with very good acoustic properties.

As for the electrical part of the design, a complex but effective design was achieved. The use of the Wien Bridge oscillator allows a low noise signal to be produced, and it is relatively easy to understand. Also, its simple configuration for the tuning of fundamental frequencies reduces the general complexity of the design. Still, a simpler design could have being achieved if the oscillator circuits would have being reduced, and a frequency regulating circuit implemented.

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Another important topic worth discussing is the string behavior simulations. These simulations were made using theoretical backgrounds and neglecting some effects that could change the way in which a real string would behave such as string elasticity and friction effects. Other effect worth mentioning is the string and hammer head interaction. In the models used, this effect was neglected by making the assumption that the hammer head was made of a solid material and the energy was transferred to the string in a single blow (as a velocity initial conditions). But in real life, the hammer head is made of wool or linen which are materials with special properties and variable rigidity. This would make the interaction between hammer and string much more complicated than the way it was modeled. Thus, this could be a possible source of error.

The excitation mechanism analysis is another point worth of discussion. The model created for the analysis provides a very good insight on how the actual model would behave. The total force profile used for excitation of the model comes from experimental data which means that the velocities obtained through the model resemble the ones that would be obtained in reality. Still, these values could vary due to factors that cannot be perfectly modeled such as damping properties due to friction.

As for the modal analysis, it was found that the structure created has a very high number of modes in the analyzed frequency range making it a very effective amplifier. The fact that it contains so many natural frequencies shows that it will amplify the vibrations in a proper way maintaining its spectral richness. It also shows that the materials chosen for this component are good options for a future construction of a prototype.

Now, taking a look at the steady state analysis for the resonance box, it can be appreciated that the forces exerted by the strings when vibrating are effectively withstood by the structure. The values of stress and strain obtained throughout the structure are very small compared to the actual values that the instrument would be able to resist, thus proving that the design is good and will not fail under these loading characteristics. As for the frequency response of the structure to different oscillatory forces, it is found that the structure has a similar response for the different applied forces. This is probably due to the high number of natural modes of the box in the analyzed frequency range making it amplify the vibrations regardless of the fundamental frequencies. Still, it was seen that there are some frequencies that are higher amplified than others. This is probably because there are some excitation frequencies that are closer to one of the structure's natural frequencies causing a greater amplification factor. Other interesting finding was the sound pressure levels that the instrument can generate. The maximum value found was of 70 dB SPL inside the resonance box of the instrument; this is equivalent to the sound produced by a radio at a relatively high volume. Therefore, it is shown that the acoustic properties of the Hybrid musical instrument are very good.

The fact that the designed instrument has very good sound properties can also be verified by taking a look at the sound radiation analysis. As can be appreciated from the results in Chapter 5, the instrument would be able to produce sound pressures of about 48 dB SPL at distances of 2m and in open places (without sound reflection) which is very good. This means that the instrument would be easily heard by an audience when played. It is important to remember that sound reflection could be a factor of variability for the actual sound pressures that would be produced in real life. If played in closed places, the sound pressure could increase. Additionally, it seems that the instrument would radiate sound evenly around a spherical hemisphere. There is very little variation in the sound response around the modeled hemisphere. With this being said, it is logical to infer that the instrument would behave as a monopole as the distance between the instrument and the person who hears it increases.

Finally, the last topic to be discussed is the optimization procedure. As it was verified by the static analysis conducted to the frame after the optimization, this procedure was valid and allowed a significant weight reduction in the frame. The final estimated weight of the frame was 3.29 Kg, which is a little higher than the theoretical value to be obtained. This happened because the optimization result was only used as a reference for the final frame model since it had some flaws due to numerical errors obtained in the analysis.

All in all, the final result of this project is very satisfactory. An innovative and effective design for a hybrid musical instrument was obtained. The obvious next step is the creation of a prototype to experimentally verify the results obtained through the theoretical models.

7. Conclusions and Recommendations

After reviewing and verifying all the results obtained through the realization of this thesis, some things can be concluded:

- A very good, creative and innovative design for a hybrid instrument that combined string acoustic sound generation with electronic sound synthesis was done.
- The instrument provides an intuitive mechanism to effectively produce sound from both acoustic and electronic sources accomplishing the objective of actively combine these two types of sound generation.
- The models created to analyze and verify the behavior of the designed instrument provided very promising results for the design.
- The design for the instrument was accomplished and precisely documented creating a valid base point for future construction and experimentation with the hybrid instrument.
- The parameters defined for the design of the hybrid instrument were successfully accomplished.

Also, some recommendations for future implementation of the design and improvements for the created models are suggested:

• More accurate damping features should be included to the present models in order to avoid wrong or altered responses. To do this, experimental measuring is suggested

since many of the components and materials to be used can show characteristic damping features due to non-linearity properties and other characteristic behaviors.

- Some omitted characteristics could be used to create more precise models.
 Characteristics such as string's elastic properties, internal friction and frequencydependent impedance conditions in the case of acoustic mediums could be included to create a more precise model.
- The materials chosen for construction in the present thesis are strongly recommended to maximize the properties of the instrument. Spruce is worldwide known for its outstanding acoustic properties as well as string wire is very popular for its elastic properties. As for the electronic components, metal film resistors and electrolytic capacitors are recommended since they have smaller tolerance values and are not too dependent in temperature as carbon film resistors and ceramic capacitors are. Precision potentiometers are recommended as well.
- Construction and experimentation on a prototype of the designed instrument is recommended to verify the correlation between the results in the present thesis and experimentation.

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Appendix A

A.1 Other Results

String Parameter Plots



Figure A.1.: Tension as a function of length for various diameters for C2



Figure A.2.: Tension as a function of length for various diameters for C#2



Figure A.3.: Tension as a function of length for various diameters for D2



Figure A.4.: Tension as a function of length for various diameters for D#2



Figure A.5.: Tension as a function of length for various diameters for E2



Figure A.6.: Tension as a function of length for various diameters for F2



Figure A.7.: Tension as a function of length for various diameters for F#2



Figure A.8.: Tension as a function of length for various diameters for G2



Figure A.9.: Tension as a function of length for various diameters for G#2



Figure A.10: Tension as a function of length for various diameters for A2



Figure A.11: Tension as a function of length for various diameters for A#2



Figure A.12: Tension as a function of length for various diameters for B2



Figure A.13: Tension as a function of length for various diameters for C3



Figure A.14: Tension as a function of length for various diameters for C#3



Figure A.15: Tension as a function of length for various diameters for D3



Figure A.16: Tension as a function of length for various diameters for D#3



Figure A.17: Tension as a function of length for various diameters for E3



Figure A.18: Tension as a function of length for various diameters for F3



Figure A.19: Tension as a function of length for various diameters for F#3



Figure A.20: Tension as a function of length for various diameters for G3



Figure A.21: Tension as a function of length for various diameters for G#3



Figure A.22: Tension as a function of length for various diameters for A3



Figure A.23: Tension as a function of length for various diameters for A#3



Figure A.24: Tension as a function of length for various diameters for B3



Figure A.25: Tension as a function of length for various diameters for C#4



Figure A.26: Tension as a function of length for various diameters for D4



Figure A.27: Tension as a function of length for various diameters for D#4



Figure A.28: Tension as a function of length for various diameters for E4



Figure A.29: Tension as a function of length for various diameters for F4



Figure A.30: Tension as a function of length for various diameters for F#4



Figure A.31: Tension as a function of length for various diameters for G4



Figure A.32: Tension as a function of length for various diameters for G#4



Figure A.33: Tension as a function of length for various diameters for A4



Figure A.34: Tension as a function of length for various diameters for A#4



Figure A.35: Tension as a function of length for various diameters for B4



Figure A.36: Tension as a function of length for various diameters for C5



Figure A.37: Tension as a function of length for various diameters for C#5



Figure A.38: Tension as a function of length for various diameters for D5



Figure A.39: Tension as a function of length for various diameters for D#5



Figure A.40: Tension as a function of length for various diameters for E5



Figure A.41: Tension as a function of length for various diameters for F5



Figure A.42: Tension as a function of length for various diameters for F#5



Figure A.43: Tension as a function of length for various diameters for G5



Figure A.44: Tension as a function of length for various diameters for G#5



Figure A.45: Tension as a function of length for various diameters for A5



Figure A.46: Tension as a function of length for various diameters for A#5



Figure A.47: Tension as a function of length for various diameters for B5



Figure A.48: Tension as a function of length for various diameters for C6



Figure A.49: Tension as a function of length for various diameters for C#6



Figure A.50: Tension as a function of length for various diameters for D6



Figure A.51: Tension as a function of length for various diameters for D#6



Figure A.52: Tension as a function of length for various diameters for E6



Figure A.53: Tension as a function of length for various diameters for F6



Figure A.54: Tension as a function of length for various diameters for F#6


Figure A.55: Tension as a function of length for various diameters for G6



Figure A.56: Tension as a function of length for various diameters for G#6



Figure A.57: Tension as a function of length for various diameters for A6



Figure A.58: Tension as a function of length for various diameters for A#6



Figure A.59: Tension as a function of length for various diameters for B6



Figure A.60: Tension as a function of length for various diameters for C7



Figure A.61: Tension as a function of length for various diameters for C#7



Figure A.62: Tension as a function of length for various diameters for D7



Figure A.63: Tension as a function of length for various diameters for D#7



Figure A.64: Tension as a function of length for various diameters for E7



Figure A.65: Tension as a function of length for various diameters for F7



Figure A.66: Tension as a function of length for various diameters for F#7



Figure A.67: Tension as a function of length for various diameters for G7



Figure A.68: Tension as a function of length for various diameters for G#7



Figure A.69: Tension as a function of length for various diameters for A7



Figure A.70: Tension as a function of length for various diameters for A#7



Figure A.71: Tension as a function of length for various diameters for B7



Figure A.72: Tension as a function of length for various diameters for C8

Equalizer Bode Plots



Figure A.73: Bode Plot for the Equalizer's Second Band



Figure A.74: Bode Plot for the Equalizer's Third Band



Figure A.75: Bode Plot for the Equalizer's Fourth Band



Figure A.76: Bode Plot for the Equalizer's Fifth Band



Figure A.77: Bode Plot for the Equalizer's Sixth Band

Mode Shapes of the Resonance Box



Figure A.78: Mode Shape of the resonance box for 146.87 Hz



Figure A.79: Mode Shape of the resonance box for 244.96 Hz



Figure A.80: Mode Shape of the resonance box for 450.06 Hz



Figure A.81: Mode Shape of the resonance box for 614.74 Hz



Figure A.82: Mode Shape of the resonance box for 688.79 Hz



Figure A.83: Mode Shape of the resonance box for 901.02 Hz



Figure A.84: Mode Shape of the resonance box for 1178.9 Hz



Figure A.85: Mode Shape of the resonance box for 2872.7 Hz



Figure A.86: Mode Shape of the resonance box for 2898.3 Hz



Figure A.87: Mode Shape of the resonance box for 2921 Hz

Steady State Frequency Response 3D Plots



Figure A.88: Stress Frequency Response at 81.9 Hz for E2 String Excitation





Figure A.89: Stress Frequency Response at 253 Hz for C4 String Excitation

Figure A.90: Stress Frequency Response at 1193 Hz for G5 String Excitation



Figure A.91: Stress Frequency Response at 3684 Hz for B7 String Excitation



Figure A.92: Strain Frequency Response at 81.9 Hz for E2 String Excitation



Figure A.93: Strain Frequency Response at 253 Hz for C4 String Excitation



Figure A.94: Strain Frequency Response at 1193 Hz for G5 String Excitation



Figure A.95: Strain Frequency Response at 3684 Hz for B7 String Excitation

Chosen Points Frequency Response Plots



Figure A.96: Stress Frequency Response for node 996



Figure A.97: Displacement Frequency Response for node 996



Figure A.98: Pressure Frequency Response for node 617



Figure A.99: Stress Frequency Response for node 413



Figure A.100: Displacement Frequency Response for node 413



Figure A.101: Pressure Frequency Response for node 616

Sound Radiation Response



Figure A.102: Sound Radiation response for 40.47 Hz with E2 string excitation



Figure A.103: Sound Radiation response for 125 Hz with C4 string excitation



Figure A.104: Sound Radiation response for 386.1 Hz with G5 string excitation



Figure A.105: Sound Radiation response for 1821 Hz with B7 string excitation



Figure A.106: Sound Radiation response for 335.4 Hz with E2 string excitation



Figure A.107: Sound Radiation response for 512 Hz with C4 string excitation



Figure A.108: Sound Radiation response for 1581 Hz with G5 string excitation



Figure A.109: Sound Radiation response for 7455 Hz with B7 string excitation

A.2 Tables of values

Table A.1: String Set Values of diameter, length and tension for fundamental Frequencies

Key	Musical Note	Frequency [Hz]	Length [m]	Diameter Number	Diameter [mm]	Tension [N]
1	C2	65,41	0,9	25	1,4986	191,92
2	C#2	69,30	0,9	25	1,4986	215,42
3	D2	73,42	0,9	25	1,4986	241,80
4	D#2	77,78	0,9	25	1,4986	271,41
5	E2	82,41	0,9	25	1,4986	304,65
6	F2	87,31	0,9	25	1,4986	341,96
7	F#2	92,50	0,9	25	1,4986	383,84
8	G2	98,00	0,9	25	1,4986	430,84
9	G#2	103,83	0,9	25	1,4986	483,60
10	A2	110,00	0,9	25	1,4986	542,83
11	A#2	116,54	0,9	25	1,4986	609,30

	Musical					Tension
Key	Note	Frequency [Hz]	Length [m]	Diameter Number	Diameter [mm]	[N]
12	B2	123,47	0,9	25	1,4986	683,92
13	C3	130,81	0,9	24	1,397	667,11
14	C#3	138,59	0,9	23	1,2954	643,85
15	D3	146,83	0,9	22	1,2446	667,13
16	D#3	155,56	0,89	22	1,2446	732,28
17	E3	164,81	0,875	21	1,1938	730,95
18	F3	174,61	0,845	20,5	1,1684	732,95
19	F#3	185,00	0,815	20	1,143	732,42
20	G3	196,00	0,805	19	1,0922	732,35
21	G#3	207,65	0,778	18,5	1,0668	732,52
22	A3	220,00	0,756	18	1,0414	739,85
23	A#3	233,08	0,729	17,5	1,016	734,99
24	B3	246,94	0,705	17	0,9906	733,47
25	C4	261,63	0,664	17	0,9906	730,32
26	C#4	277,18	0,628	17	0,9906	733,28
27	D4	293,66	0,607	16,5	0,9652	730,02
28	D#4	311,13	0,59	16	0,9398	733,96
29	E4	329,63	0,568	16	0,9398	763,54
30	F4	349,23	0,525	16	0,9398	732,20
31	F#4	369,99	0,495	16	0,9398	730,62
32	G4	392,00	0,467	16	0,9398	729,94
33	G#4	415,30	0,44	16	0,9398	727,33
34	A4	440,00	0,426	15,5	0,9144	724,46
35	A#4	466,16	0,404	15,5	0,9144	731,36
36	B4	493,88	0,381	15,5	0,9144	730,11
37	C5	523,25	0,36	15,5	0,9144	731,67
38	C#5	554,37	0,34	15,5	0,9144	732,56
39	D5	587,33	0,321	15,5	0,9144	732,94
40	D#5	622,25	0,302	15,5	0,9144	728,18
41	E5	659,26	0,286	15,5	0,9144	733,05
42	F5	698,46	0,27	15,5	0,9144	733,33
43	F#5	739,99	0,255	15,5	0,9144	734,21
44	G5	783,99	0,24	15,5	0,9144	730,02
45	G#5	830,61	0,232	15	0,889	723,76
46	A5	880,00	0,22	15	0,889	730,52
47	A#5	932,33	0,208	15	0,889	732,97
48	B5	987,77	0,202	14,5	0,8636	732,24
49	C6	1046,50	0,191	14,5	0,8636	734,84

Key	Musical Note	Frequency [Hz]	Length [m]	Diameter Number	Diameter [mm]	Tension
50	C#6	1108 73	0.18	14 5	0.8636	732.56
51	D6	1174.66	0.17	14.5	0.8636	733,44
52	D#6	1244.51	0.16	14.5	0.8636	729.26
53	E6	1318.51	0.152	14.5	0.8636	738,75
54	F6	1396,91	0,142	14,5	0,8636	723,70
55	F#6	1479,98	0,139	14	0,8382	733,25
56	G6	1567,98	0,131	14	0,8382	731,04
57	G#6	1661,22	0,124	14	0,8382	735,21
58	A6	1760,00	0,117	14	0,8382	734,70
59	A#6	1864,66	0,11	14	0,8382	728,95
60	B6	1975,53	0,108	13,5	0,8128	741,66
61	C7	2093,00	0,101	13,5	0,8128	728,06
62	C#7	2217,46	0,098	13	0,7874	722,06
63	D7	2349,32	0,094	13	0,7874	745,68
64	D#7	2489,02	0,088	13	0,7874	733,55
65	E7	2637,02	0,083	13	0,7874	732,48
66	F7	2793,83	0,084	12	0,7366	736,95
67	F#7	2959,96	0,079	12	0,7366	731,66
68	G7	3135,96	0,0718	12	0,7366	678,38
69	G#7	3322,44	0,071	12	0,7366	744,58
70	A7	3520,00	0,0666	12	0,7366	735,39
71	A#7	3729,31	0,063	12	0,7366	738,62
72	B7	3951,07	0,06	12	0,7366	751,99
73	C8	4186,01	0,054	12	0,7366	683,71

Table A.2: XY Values used for bridge design after linear regression

	String Length
Bridge Position [m]	[m]
0,000	0,900
0,005	0,900
0,010	0,900
0,015	0,900
0,020	0,900
0,025	0,900
0,030	0,900
0,035	0,900

	String Length
Bridge Position [m]	[m]
0,040	0,900
0,045	0,900
0,050	0,900
0,055	0,900
0,060	0,900
0,065	0,900
0,070	0,900
0,075	0,900
0,080	0,900
0,085	0,900
0,090	0,900
0,095	0,900
0,100	0,900
0,105	0,900
0,110	0,900
0,115	0,900
0,120	0,900
0,125	0,900
0,130	0,900
0,135	0,900
0,140	0,900
0,145	0,900
0,150	0,900
0,155	0,900
0,160	0,900
0,165	0,900
0,170	0,900
0,175	0,894
0,180	0,889
0,185	0,882
0,190	0,875
0,195	0,868
0,200	0,860
0,205	0,851
0,210	0,842
0,215	0,832
0,220	0,823
0,225	0,812

	String Length
Bridge Position [m]	[m]
0,230	0,802
0,235	0,791
0,240	0,779
0,245	0,768
0,250	0,756
0,255	0,745
0,260	0,733
0,265	0,721
0,270	0,708
0,275	0,696
0,280	0,684
0,285	0,671
0,290	0,659
0,295	0,647
0,300	0,634
0,305	0,622
0,310	0,610
0,315	0,597
0,320	0,585
0,325	0,573
0,330	0,561
0,335	0,549
0,340	0,538
0,345	0,526
0,350	0,515
0,355	0,503
0,360	0,492
0,365	0,481
0,370	0,470
0,375	0,460
0,380	0,449
0,385	0,439
0,390	0,429
0,395	0,419
0,400	0,409
0,405	0,399
0,410	0,390
0,415	0,381

	String Length
Bridge Position [m]	[m]
0,420	0,372
0,425	0,363
0,430	0,354
0,435	0,346
0,440	0,338
0,445	0,330
0,450	0,322
0,455	0,314
0,460	0,307
0,465	0,299
0,470	0,292
0,475	0,285
0,480	0,279
0,485	0,272
0,490	0,266
0,495	0,259
0,500	0,253
0,505	0,247
0,510	0,242
0,515	0,236
0,520	0,231
0,525	0,225
0,530	0,220
0,535	0,215
0,540	0,210
0,545	0,206
0,550	0,201
0,555	0,197
0,560	0,192
0,565	0,188
0,570	0,184
0,575	0,180
0,580	0,176
0,585	0,172
0,590	0,168
0,595	0,165
0,600	0,161
0,605	0,158

	String Length
Bridge Position [m]	[m]
0,610	0,154
0,615	0,151
0,620	0,148
0,625	0,145
0,630	0,142
0,635	0,139
0,640	0,136
0,645	0,133
0,650	0,130
0,655	0,127
0,660	0,125
0,665	0,122
0,670	0,120
0,675	0,117
0,680	0,115
0,685	0,112
0,690	0,110
0,695	0,108
0,700	0,105
0,705	0,103
0,710	0,101
0,715	0,099
0,720	0,097
0,725	0,095
0,730	0,093
0,735	0,091
0,740	0,089
0,745	0,087
0,750	0,085
0,755	0,083
0,760	0,081
0,765	0,080
0,770	0,078
0,775	0,076
0,780	0,075
0,785	0,073
0,790	0,072
0,795	0,070

	String Length
Bridge Position [m]	[m]
0,800	0,069
0,805	0,067
0,810	0,066
0,815	0,064
0,820	0,063
0,825	0,062
0,830	0,061
0,835	0,059
0,840	0,058

Table A.3: List of values of the components for the Oscillators

		Capacitance
Frequency [Hz]	Resistance [ohm]	[F]
Low Freq [0-		
33]	100 - 10000	4,70E-05
65,406	5177,285	4,70E-07
69,296	4886,706	4,70E-07
73,416	4612,437	4,70E-07
77,782	5246,599	3,90E-07
82,407	4952,130	3,90E-07
87,307	4674,188	3,90E-07
92,499	5214,000	3,30E-07
97,999	4921,360	3,30E-07
103,826	4645,146	3,30E-07
110,000	5358,752	2,70E-07
116,541	5057,989	2,70E-07
123,471	4774,106	2,70E-07
130,813	4506,156	2,70E-07
138,591	5219,891	2,20E-07
146,832	4926,921	2,20E-07
155,563	4650,394	2,20E-07
164,814	5364,807	1,80E-07
174,614	5063,704	1,80E-07
		Capacitance
----------------	------------------	-------------
Frequency [Hz]	Resistance [ohm]	[F]
184,997	4779,500	1,80E-07
195,998	5413,496	1,50E-07
207,652	5109,660	1,50E-07
220,000	4822,877	1,50E-07
233,082	4552,190	1,50E-07
246,942	5370,869	1,20E-07
261,626	5069,425	1,20E-07
277,183	4784,900	1,20E-07
293,665	5419,613	1,00E-07
311,127	5115,434	1,00E-07
329,628	4828,326	1,00E-07
349,228	4557,333	1,00E-07
369,994	5245,792	8,20E-08
391,995	4951,369	8,20E-08
415,305	4673,470	8,20E-08
440,000	5319,350	6,80E-08
466,164	5020,798	6,80E-08
493,883	4739,002	6,80E-08
523,251	5431,527	5,60E-08
554,365	5126,679	5,60E-08
587,330	4838,940	5,60E-08
622,254	5441,951	4,70E-08
659,255	5136,517	4,70E-08
698,456	4848,227	4,70E-08
739,989	4576,117	4,70E-08
783,991	5205,285	3,90E-08
830,609	4913,135	3,90E-08
880,000	5480,542	3,30E-08
932,328	5172,943	3,30E-08
987,767	4882,608	3,30E-08
1046,502	4608,568	3,30E-08
1108,731	5316,556	2,70E-08
1174,659	5018,160	2,70E-08
1244,508	4736,513	2,70E-08
1318,510	5486,735	2,20E-08
1396,913	5178,788	2,20E-08
1479,978	4888,125	2,20E-08
1567,982	4613,775	2,20E-08

		Capacitance
Frequency [Hz]	Resistance [ohm]	[F]
1661,219	5322,563	1,80E-08
1760,000	5023,830	1,80E-08
1864,655	4741,864	1,80E-08
1975,533	5370,869	1,50E-08
2093,005	5069,425	1,50E-08
2217,461	4784,900	1,50E-08
2349,318	4516,344	1,50E-08
2489,016	5328,577	1,20E-08
2637,020	5029,507	1,20E-08
2793,826	4747,222	1,20E-08
2959,955	5376,937	1,00E-08
3135,963	5075,153	1,00E-08
3322,438	4790,306	1,00E-08
3520,000	4521,447	1,00E-08
3729,310	5204,485	8,20E-09
3951,066	4912,380	8,20E-09
4186,009	4636,669	8,20E-09

Table A.4: List of Eigen Frequencies of the Resonance Box found in Modal Analysis

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
1	134,627	742	2664,63	1483	3337,9
2	146,866	743	2666,73	1484	3339,45
3	153,418	744	2667,34	1485	3340,42
4	189,828	745	2668,22	1486	3340,51
5	244,958	746	2669,08	1487	3341,07
6	254,991	747	2670,86	1488	3341,7
7	278,661	748	2672,02	1489	3341,72
8	295,329	749	2674,64	1490	3341,79
9	300,009	750	2676,13	1491	3342,74
10	300,025	751	2677,07	1492	3343,63
11	305,268	752	2677,44	1493	3344,09
12	320,332	753	2678,69	1494	3344,32
13	366,304	754	2680,71	1495	3344,92
14	380,569	755	2681,83	1496	3346,27
15	392,379	756	2682,59	1497	3346,39

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
16	407,236	757	2683,88	1498	3346,82
	427,405	758	2684,97	1499	3347,83
18	450,06	759	2686,03	1500	3348,38
19	462,122	760	2687,56	1501	3349,1
20	468,952	761	2688,07	1502	3349,46
21	485,306	762	2689,01	1503	3350,28
22	493,057	763	2689,94	1504	3350,83
23	524,744	764	2691,58	1505	3351,13
24	533,058	765	2693,22	1506	3351,97
25	537,411	766	2695,64	1507	3352,69
26	555,736	767	2697,08	1508	3353,11
27	556,229	768	2697,74	1509	3353,67
28	565,19	769	2699,43	1510	3354,56
29	569,265	770	2700,33	1511	3355,83
30	580,762	771	2701,17	1512	3356,3
31	588,445	772	2702,19	1513	3357,37
32	590,943	773	2703,71	1514	3359,28
33	591,999	774	2704,57	1515	3359,92
34	604,663	775	2705,79	1516	3360,6
35	614,739	776	2706,49	1517	3362,19
36	633,802	777	2707,52	1518	3362,54
37	654,062	778	2707,66	1519	3363,63
38	660,299	779	2709,06	1520	3364,66
39	673,598	780	2710,16	1521	3365,41
40	688,794	781	2712,67	1522	3366,55
41	702,387	782	2713,83	1523	3367,6
42	702,619	783	2714,99	1524	3367,92
43	703,022	784	2716,19	1525	3369,19
44	711,501	785	2716,58	1526	3370,37
45	725,213	786	2718,66	1527	3371,05
46	730,169	787	2719,87	1528	3371,86
47	735,468	788	2720,58	1529	3373,75
48	740,131	789	2721	1530	3374,72
49	763,733	790	2722,45	1531	3375,48
50	765,292	791	2723,33	1532	3376,79
51	766,557	792	2724,02	1533	3377,64
52	768,644	793	2725,69	1534	3378,07
53	783,935	794	2726,21	1535	3379,49

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
54	793,585	795	2726,4	1536	3380,6
55	805,744	796	2729,55	1537	3382,07
56	816,668	797	2730,75	1538	3382,59
57	830,326	798	2731,3	1539	3383,9
58	835,069	799	2731,51	1540	3384,22
59	844,422	800	2733,15	1541	3384,99
60	850,907	801	2733,31	1542	3385,73
61	858,451	802	2735,24	1543	3387,05
62	863,56	803	2735,73	1544	3387,18
63	885,581	804	2736,7	1545	3387,7
64	895,343	805	2739,47	1546	3388,01
65	896,022	806	2740,57	1547	3388,87
66	897,134	807	2741,82	1548	3389,3
67	901,016	808	2742,21	1549	3390,28
68	904,652	809	2742,69	1550	3390,86
69	912,352	810	2743,44	1551	3392,06
70	919,68	811	2743,61	1552	3392,94
71	923,204	812	2743,87	1553	3393,42
72	925,032	813	2744,17	1554	3395,16
73	927,465	814	2745,2	1555	3396,98
74	928,184	815	2745,68	1556	3397,72
75	933,155	816	2746,63	1557	3398,28
76	936,704	817	2747,7	1558	3400,02
77	943,229	818	2748,18	1559	3400,41
78	944,567	819	2750,43	1560	3400,98
79	973,765	820	2750,77	1561	3402,42
80	982,228	821	2752,11	1562	3403,42
81	1006,7	822	2753,69	1563	3403,98
82	1018,02	823	2754,61	1564	3404,29
83	1021,58	824	2755,21	1565	3406,64
84	1026,54	825	2756,72	1566	3407,53
85	1029,92	826	2758,97	1567	3409,19
86	1036,73	827	2759,32	1568	3409,64
87	1050,78	828	2761,82	1569	3410,24
88	1056,69	829	2762,85	1570	3410,41
89	1062,97	830	2763,25	1571	3410,8
90	1063,32	831	2763,79	1572	3411,91
91	1073,76	832	2764,55	1573	3412,25

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
92	1078,7	833	2765,66	1574	3413,16
93	1088,54	834	2768,01	1575	3414,32
94	1094,69	835	2769,57	1576	3415,39
95	1105,52	836	2770,49	1577	3415,51
96	1114,26	837	2771,33	1578	3416,7
97	1119,7	838	2771,73	1579	3417,29
98	1122,25	839	2772,12	1580	3418,6
99	1124,85	840	2773,25	1581	3418,94
100	1137,05	841	2774,13	1582	3419,43
101	1141,21	842	2774,22	1583	3420,48
102	1141,27	843	2775,25	1584	3421,84
103	1147,16	844	2776,95	1585	3422,84
104	1149,97	845	2777,47	1586	3423,87
105	1156,81	846	2778,12	1587	3424,36
106	1163,16	847	2779,7	1588	3424,82
107	1169,52	848	2780,35	1589	3425,92
108	1174,91	849	2780,93	1590	3427,31
109	1176,39	850	2783,35	1591	3427,42
110	1178,9	851	2784,46	1592	3428,45
111	1183,18	852	2785,6	1593	3429,25
112	1200,06	853	2786,65	1594	3429,53
113	1201,62	854	2787,57	1595	3430,83
114	1201,81	855	2788,58	1596	3431,43
115	1205,83	856	2788,88	1597	3432,64
116	1208,98	857	2789,63	1598	3433,45
117	1212,3	858	2789,97	1599	3433,64
118	1217,93	859	2790,75	1600	3434,16
119	1220,09	860	2791,15	1601	3435,09
120	1223,3	861	2792,25	1602	3435,16
121	1230,63	862	2792,74	1603	3435,97
122	1238,74	863	2793,57	1604	3436,67
123	1249,89	864	2794,13	1605	3438,34
124	1254,24	865	2794,25	1606	3439,2
125	1262,96	866	2797,18	1607	3439,83
126	1270,28	867	2798,33	1608	3440,14
127	1286,2	868	2798,98	1609	3441,18
128	1289,8	869	2799,3	1610	3443,18
129	1293,36	870	2799,67	1611	3443,68

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
130	1299,17	871	2801,02	1612	3444,5
131	1301,34	872	2801,51	1613	3445,82
132	1309,45	873	2803	1614	3447,01
133	1318,24	874	2805,33	1615	3447,58
134	1320,89	875	2806,04	1616	3449,48
135	1332,46	876	2807,36	1617	3449,79
136	1342,21	877	2808,12	1618	3451,99
137	1343,84	878	2808,86	1619	3453,02
138	1354,82	879	2810,45	1620	3454,85
139	1359,42	880	2810,65	1621	3455,26
140	1360,6	881	2812,47	1622	3456
141	1369,53	882	2813,54	1623	3456,53
142	1374,74	883	2814,29	1624	3457,31
143	1387,9	884	2815,31	1625	3457,71
144	1387,94	885	2815,62	1626	3458,64
145	1390,95	886	2815,99	1627	3458,69
146	1395,06	887	2816,53	1628	3459,36
147	1401,77	888	2817,4	1629	3460,29
148	1406,37	889	2817,91	1630	3461,57
149	1408,97	890	2819,4	1631	3462,61
150	1411,38	891	2820,19	1632	3463,52
151	1413,67	892	2820,89	1633	3464,34
152	1416,01	893	2821,64	1634	3464,68
153	1418,33	894	2822,12	1635	3466,97
154	1418,78	895	2823,34	1636	3468,4
155	1422,75	896	2823,6	1637	3469,61
156	1429,31	897	2824,62	1638	3469,69
157	1433,93	898	2825,74	1639	3470,25
158	1441,65	899	2826,6	1640	3470,44
159	1450,69	900	2827,37	1641	3471,4
160	1451,05	901	2829,85	1642	3472,14
161	1461,15	902	2830,97	1643	3473,96
162	1463,37	903	2831,18	1644	3474,21
163	1468,14	904	2833,13	1645	3474,62
164	1470,85	905	2833,45	1646	3475,13
165	1475,53	906	2834,1	1647	3476,11
166	1479,43	907	2834,44	1648	3476,48
167	1492,24	908	2835,88	1649	3476,58

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
168	1495,69	909	2836,56	1650	3478,83
169	1496,08	910	2837,2	1651	3478,9
170	1496,99	911	2838,19	1652	3479,9
171	1500,61	912	2839,19	1653	3480,84
172	1510,51	913	2839,71	1654	3481,4
173	1513,57	914	2840,22	1655	3482,34
174	1514,74	915	2842,2	1656	3482,75
175	1519,03	916	2842,91	1657	3482,82
176	1521,68	917	2843,35	1658	3483,36
177	1531,27	918	2844,04	1659	3484,45
178	1532,45	919	2844,48	1660	3484,92
179	1532,72	920	2845,76	1661	3485,1
180	1542,98	921	2846,55	1662	3485,94
181	1552,3	922	2847,51	1663	3486,47
182	1556,27	923	2848,18	1664	3487,85
183	1559,12	924	2850,4	1665	3488,37
184	1562,33	925	2850,71	1666	3490,08
185	1570,93	926	2852,14	1667	3492,49
186	1574,42	927	2852,85	1668	3493,24
187	1583,73	928	2856,55	1669	3493,77
188	1590,08	929	2857,39	1670	3494,72
189	1594,03	930	2857,61	1671	3495,6
190	1600,63	931	2857,8	1672	3496,44
191	1604,38	932	2859,94	1673	3497,13
192	1611,48	933	2861,46	1674	3498,12
193	1616,16	934	2862,82	1675	3498,72
194	1617,23	935	2863,72	1676	3499,43
195	1626,09	936	2864,07	1677	3500,12
196	1629,21	937	2865,69	1678	3500,27
197	1634,21	938	2866,5	1679	3501,16
198	1637,12	939	2867,34	1680	3502,49
199	1641,91	940	2868,64	1681	3502,95
200	1646,12	941	2869,46	1682	3504
201	1647,1	942	2869,95	1683	3504,72
202	1649,04	943	2871,04	1684	3505,3
203	1649,23	944	2871,96	1685	3506,56
204	1651,31	945	2872,69	1686	3507,7
205	1652,51	946	2874,15	1687	3509,07

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
206	1657,02	947	2874,38	1688	3510,08
207	1659,68	948	2875,92	1689	3510,8
208	1664,96	949	2876,23	1690	3511,87
209	1665,39	950	2877,41	1691	3512,96
210	1668,1	951	2878,28	1692	3513,55
211	1676,33	952	2879,58	1693	3514,03
212	1676,47	953	2880,12	1694	3515,7
213	1676,57	954	2880,89	1695	3516,02
214	1680,43	955	2881,5	1696	3517,56
215	1683,55	956	2881,96	1697	3519,22
216	1685,14	957	2883,88	1698	3519,61
217	1691,14	958	2884,86	1699	3519,96
218	1698,16	959	2885,72	1700	3520,7
219	1702,42	960	2886,96	1701	3520,97
220	1705,06	961	2889,1	1702	3521,51
221	1709,54	962	2891,04	1703	3522,96
222	1713,71	963	2891,39	1704	3524,08
223	1715,24	964	2891,89	1705	3524,66
224	1715,85	965	2892,2	1706	3524,9
225	1720,09	966	2893,53	1707	3525,52
226	1725,91	967	2893,8	1708	3525,95
227	1730,54	968	2895,41	1709	3526,4
228	1735,79	969	2895,76	1710	3526,87
229	1737,37	970	2897,9	1711	3528,95
230	1741,22	971	2898,32	1712	3530,3
231	1749,18	972	2899,26	1713	3530,85
232	1752,63	973	2899,53	1714	3532,78
233	1754,91	974	2900,31	1715	3533,49
234	1759,96	975	2900,47	1716	3534,74
235	1765,67	976	2901,21	1717	3535,41
236	1766,69	977	2902,12	1718	3535,53
237	1767,96	978	2903,33	1719	3535,85
238	1773,63	979	2903,87	1720	3536,76
239	1776,15	980	2905,12	1721	3537,42
240	1778,48	981	2905,54	1722	3538,85
241	1781,46	982	2905,73	1723	3539,6
242	1782,36	983	2907,29	1724	3540,21
243	1786,48	984	2908,69	1725	3541,5

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
244	1787,26	985	2910,42	1726	3542,71
245	1789,43	986	2911,92	1727	3544,01
246	1791,88	987	2912,58	1728	3544,87
247	1792,59	988	2913,35	1729	3545,34
248	1797,25	989	2913,62	1730	3545,73
249	1798,1	990	2914,3	1731	3546,69
250	1801,8	991	2914,95	1732	3548,5
251	1804,99	992	2915,92	1733	3549,38
252	1806,49	993	2916,7	1734	3551,28
253	1809,26	994	2917,5	1735	3551,76
254	1814,39	995	2918,2	1736	3552,83
255	1817,7	996	2918,83	1737	3554,58
256	1820,09	997	2920,19	1738	3554,86
257	1823,25	998	2920,66	1739	3555,04
258	1825,32	999	2921	1740	3556,06
259	1826,1	1000	2921,43	1741	3557,12
260	1833,18	1001	2922,32	1742	3558,2
261	1837,24	1002	2923,71	1743	3559,13
262	1838,71	1003	2925,06	1744	3560,46
263	1844,07	1004	2926,01	1745	3561,15
264	1846,01	1005	2926,4	1746	3562,17
265	1851,51	1006	2927,86	1747	3563,2
266	1851,95	1007	2929,3	1748	3564,22
267	1853,27	1008	2929,74	1749	3565,04
268	1854,81	1009	2930,24	1750	3566,36
269	1855,69	1010	2930,29	1751	3566,92
270	1856,95	1011	2930,9	1752	3568,57
271	1860,07	1012	2931,5	1753	3569,68
272	1861,75	1013	2932,13	1754	3570,24
273	1864,32	1014	2933,58	1755	3571,1
274	1868,95	1015	2934,8	1756	3571,15
275	1870,41	1016	2935,71	1757	3571,67
276	1874,09	1017	2936,85	1758	3574,05
277	1877,47	1018	2936,95	1759	3574,4
278	1884,06	1019	2938	1760	3574,72
279	1885,76	1020	2939,16	1761	3575,43
280	1887,32	1021	2939,49	1762	3575,68
281	1889,98	1022	2940,69	1763	3575,94

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
282	1893,06	1023	2942,29	1764	3576,66
283	1895,97	1024	2943,02	1765	3577,17
284	1896,95	1025	2945,2	1766	3577,92
285	1901,04	1026	2946,14	1767	3578,73
286	1903,07	1027	2947,24	1768	3581,56
287	1904,82	1028	2947,91	1769	3582,13
288	1906,96	1029	2949,6	1770	3583,52
289	1908,78	1030	2950,68	1771	3585,05
290	1913,36	1031	2951,62	1772	3586,27
291	1913,97	1032	2953,05	1773	3588,18
292	1916,27	1033	2954,35	1774	3588,73
293	1916,38	1034	2955,02	1775	3589,04
294	1916,82	1035	2955,21	1776	3589,91
295	1919,27	1036	2957,56	1777	3591,04
296	1921,11	1037	2958,11	1778	3592,39
297	1924,49	1038	2958,85	1779	3593,12
298	1926,63	1039	2959,69	1780	3593,73
299	1930,01	1040	2960,13	1781	3595
300	1933,96	1041	2960,73	1782	3595,28
301	1935,11	1042	2962,56	1783	3596,44
302	1937,38	1043	2963,04	1784	3597,12
303	1938,65	1044	2964,26	1785	3598,61
304	1939,83	1045	2965,44	1786	3600,31
305	1943,01	1046	2967,34	1787	3600,66
306	1944,43	1047	2967,5	1788	3600,76
307	1948,73	1048	2968,59	1789	3600,81
308	1950,21	1049	2969,87	1790	3602,01
309	1951,87	1050	2970,84	1791	3603,27
310	1956,57	1051	2971,16	1792	3604,4
311	1957,44	1052	2972,67	1793	3605,04
312	1964,43	1053	2973,21	1794	3607,04
313	1969,2	1054	2974,41	1795	3607,88
314	1970,28	1055	2974,79	1796	3609,87
315	1972,76	1056	2975,57	1797	3610,9
316	1975,86	1057	2976,47	1798	3611,15
317	1976,54	1058	2977,7	1799	3611,94
318	1981,17	1059	2978,53	1800	3612,71
319	1982,3	1060	2979,29	1801	3613,56

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
320	1986,62	1061	2980,48	1802	3613,69
321	1989,3	1062	2982,26	1803	3614,38
322	1991,72	1063	2982,4	1804	3614,93
323	1993,33	1064	2983,98	1805	3615,19
324	1996,92	1065	2984,98	1806	3617,32
325	2001,55	1066	2985,35	1807	3617,61
326	2002,8	1067	2985,58	1808	3617,94
327	2007,29	1068	2986,19	1809	3619,6
328	2009,39	1069	2986,78	1810	3620,64
329	2010,49	1070	2987,11	1811	3621,24
330	2013,23	1071	2988,41	1812	3622,86
331	2013,55	1072	2990,52	1813	3623,97
332	2016,08	1073	2991,06	1814	3624,35
333	2020,6	1074	2993,02	1815	3626,18
334	2023,54	1075	2994,14	1816	3627,08
335	2024,07	1076	2995,33	1817	3627,23
336	2026,2	1077	2995,8	1818	3627,87
337	2028,27	1078	2996,29	1819	3628,29
338	2033,21	1079	2996,38	1820	3630,5
339	2035,62	1080	2997,6	1821	3631,85
340	2037,38	1081	2998,58	1822	3632,38
341	2038,28	1082	2999,34	1823	3634,02
342	2041,16	1083	2999,85	1824	3635
343	2043,32	1084	3000,64	1825	3635,84
344	2045,8	1085	3001,63	1826	3636,03
345	2046,17	1086	3002,85	1827	3637,1
346	2049,25	1087	3003,2	1828	3637,69
347	2051,2	1088	3003,51	1829	3638,91
348	2052,52	1089	3004,05	1830	3639,39
349	2052,69	1090	3005,6	1831	3640,07
350	2058,25	1091	3005,92	1832	3640,59
351	2060,42	1092	3006,29	1833	3641,02
352	2061,94	1093	3006,65	1834	3642,28
353	2064,18	1094	3007,19	1835	3642,46
354	2065,35	1095	3007,8	1836	3643,07
355	2068,22	1096	3008,45	1837	3644,66
356	2070,75	1097	3009,84	1838	3645,72
357	2071,91	1098	3011,7	1839	3646,19

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
358	2076,53	1099	3013,21	1840	3646,5
359	2076,71	1100	3013,66	1841	3647,02
360	2078,51	1101	3014,02	1842	3647,69
361	2079	1102	3014,99	1843	3650,34
362	2079,03	1103	3015,8	1844	3650,83
363	2081,89	1104	3017,19	1845	3651,89
364	2082,76	1105	3017,49	1846	3652,39
365	2085,26	1106	3018,65	1847	3652,9
366	2086,36	1107	3019,22	1848	3654,15
367	2088,15	1108	3020,56	1849	3654,69
368	2090,88	1109	3020,96	1850	3655,1
369	2093,63	1110	3021,71	1851	3655,48
370	2096,12	1111	3021,92	1852	3657,45
371	2096,59	1112	3022,75	1853	3657,79
372	2101,92	1113	3024,6	1854	3658,29
373	2103,28	1114	3024,81	1855	3660,65
374	2104,69	1115	3026,96	1856	3661,42
375	2106,17	1116	3028,34	1857	3662,35
376	2108,14	1117	3029,22	1858	3662,97
377	2110,22	1118	3030,92	1859	3663,36
378	2112,26	1119	3031,57	1860	3664,27
379	2113,09	1120	3032,67	1861	3665,46
380	2113,58	1121	3033,45	1862	3666,57
381	2118,77	1122	3035,16	1863	3668,58
382	2121,54	1123	3035,93	1864	3670,58
383	2123,88	1124	3036,35	1865	3671,31
384	2124,78	1125	3037,06	1866	3672,13
385	2126,65	1126	3038,26	1867	3672,94
386	2128,11	1127	3039,9	1868	3673,67
387	2128,17	1128	3040,69	1869	3674,15
388	2130,74	1129	3041,55	1870	3676,04
389	2133,04	1130	3042,86	1871	3677,26
390	2135,57	1131	3043,16	1872	3677,8
391	2136,61	1132	3043,64	1873	3679,42
392	2140,11	1133	3045,1	1874	3680,56
393	2142,77	1134	3046,36	1875	3681,13
394	2143,26	1135	3047,41	1876	3682,08
395	2148,46	1136	3047,75	1877	3682,6

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
396	2148,86	1137	3048,73	1878	3683,52
397	2151,13	1138	3049,7	1879	3684,72
398	2152,32	1139	3050,61	1880	3686,2
399	2154,05	1140	3051,77	1881	3686,94
400	2155,94	1141	3052,28	1882	3688,32
401	2156,77	1142	3053,34	1883	3688,9
402	2158	1143	3054,56	1884	3690,4
403	2160,43	1144	3055,11	1885	3691,14
404	2162,77	1145	3055,3	1886	3691,65
405	2164,57	1146	3055,64	1887	3692,2
406	2166,37	1147	3057,7	1888	3692,49
407	2168,91	1148	3058,34	1889	3694,25
408	2171,48	1149	3058,45	1890	3695,03
409	2174,18	1150	3060,38	1891	3695,53
410	2175,95	1151	3060,75	1892	3696,26
411	2178,53	1152	3061,34	1893	3697,02
412	2180,13	1153	3062,56	1894	3697,66
413	2183,59	1154	3062,89	1895	3697,94
414	2185,79	1155	3063,92	1896	3699,04
415	2187,58	1156	3064,3	1897	3700,28
416	2187,68	1157	3065,18	1898	3700,82
417	2188,11	1158	3067,31	1899	3701,68
418	2189,09	1159	3068,43	1900	3702,81
419	2190,14	1160	3068,74	1901	3704,07
420	2190,41	1161	3069,12	1902	3705,31
421	2190,46	1162	3070,24	1903	3705,84
422	2193,86	1163	3071,59	1904	3706,16
423	2194,35	1164	3073,54	1905	3708,04
424	2194,74	1165	3073,87	1906	3708,37
425	2198,4	1166	3074,43	1907	3708,86
426	2199,39	1167	3074,48	1908	3709,6
427	2201,14	1168	3074,62	1909	3709,97
428	2203	1169	3075,02	1910	3711,11
429	2206,34	1170	3075,65	1911	3711,45
430	2209,82	1171	3076,26	1912	3711,79
431	2211,31	1172	3077,61	1913	3712,95
432	2212,27	1173	3078,29	1914	3714,06
433	2213,59	1174	3079,98	1915	3714,38

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
434	2216,28	1175	3080,96	1916	3715,41
435	2216,69	1176	3081,52	1917	3716,14
436	2218,13	1177	3082,81	1918	3718,11
437	2219,45	1178	3083,81	1919	3719,22
438	2221,95	1179	3084,09	1920	3720,15
439	2222,55	1180	3084,89	1921	3720,51
440	2224,51	1181	3085,36	1922	3720,64
441	2227,82	1182	3085,7	1923	3720,92
442	2228,94	1183	3087,58	1924	3721,38
443	2229,22	1184	3087,82	1925	3723,82
444	2231,19	1185	3089,29	1926	3724,9
445	2235,01	1186	3090,31	1927	3726,43
446	2237,54	1187	3091,56	1928	3726,98
447	2239,42	1188	3092,4	1929	3727,56
448	2242,09	1189	3093,19	1930	3727,88
449	2242,56	1190	3094,03	1931	3728,08
450	2247,45	1191	3095,11	1932	3728,79
451	2248,73	1192	3095,65	1933	3729,9
452	2250,28	1193	3096,1	1934	3732,01
453	2251,94	1194	3096,84	1935	3733,01
454	2253,58	1195	3098,32	1936	3734,31
455	2257,35	1196	3099,03	1937	3734,9
456	2259,46	1197	3100,67	1938	3735,46
457	2260,81	1198	3101,03	1939	3736,55
458	2261,18	1199	3102,06	1940	3737,39
459	2261,75	1200	3102,43	1941	3737,64
460	2265,33	1201	3102,48	1942	3739,08
461	2265,8	1202	3104,64	1943	3739,67
462	2266,73	1203	3106,42	1944	3739,71
463	2270,75	1204	3106,8	1945	3740
464	2273,25	1205	3108,54	1946	3740,93
465	2273,82	1206	3108,92	1947	3741,62
466	2274,28	1207	3109,45	1948	3744,04
467	2274,37	1208	3109,54	1949	3744,61
468	2277,03	1209	3110,22	1950	3745,41
469	2278,84	1210	3111	1951	3746,66
470	2281,36	1211	3111,46	1952	3747,74
471	2281,94	1212	3112,27	1953	3749,85

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
472	2284,78	1213	3112,96	1954	3750,14
473	2286,16	1214	3114,06	1955	3751,36
474	2287,06	1215	3114,23	1956	3752,65
475	2289,12	1216	3115,84	1957	3753,05
476	2289,6	1217	3116,72	1958	3753,26
477	2291,5	1218	3117,83	1959	3754,22
478	2295,16	1219	3118,15	1960	3756,58
479	2295,43	1220	3119,19	1961	3756,89
480	2297,49	1221	3120,07	1962	3756,97
481	2299,21	1222	3120,68	1963	3758,51
482	2301,13	1223	3121,45	1964	3758,97
483	2304,76	1224	3121,94	1965	3759,25
484	2305,44	1225	3123,02	1966	3760,96
485	2307,99	1226	3123,47	1967	3761,4
486	2310,5	1227	3124,71	1968	3762,35
487	2312,15	1228	3125,72	1969	3762,9
488	2313,28	1229	3126,43	1970	3764,25
489	2313,45	1230	3127,59	1971	3765,23
490	2314,76	1231	3128,35	1972	3765,81
491	2318,08	1232	3129,18	1973	3767,64
492	2320,62	1233	3130,18	1974	3767,93
493	2325,53	1234	3131,28	1975	3768,67
494	2326,43	1235	3131,93	1976	3769,29
495	2328,51	1236	3133,33	1977	3770,76
496	2330,03	1237	3133,61	1978	3771,57
497	2331,11	1238	3133,91	1979	3772,11
498	2332,34	1239	3134,14	1980	3773,13
499	2336,73	1240	3134,6	1981	3773,17
500	2337,54	1241	3135,2	1982	3773,56
501	2339,65	1242	3135,47	1983	3775,19
502	2341,29	1243	3135,76	1984	3775,98
503	2343,23	1244	3136,73	1985	3777,54
504	2344,83	1245	3137,53	1986	3777,89
505	2345,79	1246	3138,29	1987	3779,04
506	2347,22	1247	3138,99	1988	3779,19
507	2350,18	1248	3139,54	1989	3779,73
508	2351,35	1249	3141,08	1990	3781,13
509	2354,45	1250	3141,53	1991	3781,62

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
510	2354,99	1251	3143,7	1992	3783,77
511	2356,16	1252	3144,23	1993	3784,11
512	2358,92	1253	3145,74	1994	3784,25
513	2360,75	1254	3146,19	1995	3784,63
514	2361,23	1255	3147,94	1996	3785,48
515	2361,36	1256	3148,59	1997	3786,59
516	2363,25	1257	3149,12	1998	3786,77
517	2364,56	1258	3149,7	1999	3787,92
518	2366,37	1259	3150,73	2000	3788,3
519	2367,62	1260	3152,07	2001	3788,56
520	2370,08	1261	3152,91	2002	3788,93
521	2372,14	1262	3154,15	2003	3789,12
522	2373,48	1263	3154,79	2004	3789,29
523	2375,66	1264	3154,97	2005	3790,25
524	2376,93	1265	3155,37	2006	3791,35
525	2378,34	1266	3156,35	2007	3792,26
526	2379,35	1267	3157,04	2008	3793,52
527	2381,93	1268	3158,02	2009	3794,99
528	2382,17	1269	3158,57	2010	3795,42
529	2383,7	1270	3158,91	2011	3795,47
530	2386,32	1271	3160,35	2012	3797,25
531	2387,64	1272	3160,65	2013	3799,21
532	2389,31	1273	3162,07	2014	3799,53
533	2393,22	1274	3163,3	2015	3800,14
534	2394,79	1275	3164,9	2016	3801,09
535	2396,34	1276	3165,83	2017	3802,37
536	2396,89	1277	3166,24	2018	3802,8
537	2397,87	1278	3167,53	2019	3804,37
538	2401,28	1279	3168,37	2020	3804,6
539	2402,31	1280	3169,81	2021	3806,35
540	2404,22	1281	3170,69	2022	3806,88
541	2405,35	1282	3170,93	2023	3809,21
542	2407,98	1283	3172,69	2024	3811,72
543	2409,73	1284	3172,84	2025	3812,33
544	2411,19	1285	3173,27	2026	3812,96
545	2413,67	1286	3174,5	2027	3813,74
546	2415,01	1287	3175,53	2028	3815,7
547	2415,78	1288	3176,11	2029	3816,54

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
548	2418,52	1289	3176,39	2030	3816,81
549	2419,96	1290	3177,38	2031	3818,14
550	2420,49	1291	3178	2032	3819,3
551	2422,03	1292	3178,38	2033	3820,02
552	2423,08	1293	3179,25	2034	3821,03
553	2424,27	1294	3181,49	2035	3821,4
554	2425,5	1295	3181,71	2036	3822,48
555	2428,54	1296	3182,13	2037	3823,6
556	2429,6	1297	3182,83	2038	3824,65
557	2429,99	1298	3183,32	2039	3825,07
558	2434,33	1299	3184,28	2040	3825,91
559	2434,41	1300	3185,1	2041	3827,38
560	2434,94	1301	3185,21	2042	3827,74
561	2436,46	1302	3185,61	2043	3828,48
562	2438,18	1303	3186,56	2044	3828,66
563	2438,85	1304	3187,31	2045	3828,82
564	2441,52	1305	3188,83	2046	3829,24
565	2442,02	1306	3190,03	2047	3830,78
566	2444,02	1307	3191,42	2048	3832,44
567	2445,43	1308	3191,85	2049	3833,81
568	2447,82	1309	3192,08	2050	3834,13
569	2451	1310	3193,14	2051	3835,83
570	2451,22	1311	3194,98	2052	3836,11
571	2453,25	1312	3197,52	2053	3836,88
572	2454,21	1313	3197,97	2054	3837
573	2455,46	1314	3199	2055	3838,7
574	2457,29	1315	3199,55	2056	3839,57
575	2459,34	1316	3200,41	2057	3840,45
576	2460,1	1317	3201,48	2058	3841,48
577	2461	1318	3201,76	2059	3842,24
578	2461,85	1319	3202,59	2060	3843,26
579	2461,88	1320	3203,36	2061	3844,87
580	2463	1321	3204,23	2062	3845,14
581	2464,72	1322	3204,82	2063	3846,93
582	2465,28	1323	3205,1	2064	3848,87
583	2467,21	1324	3205,4	2065	3850,36
584	2468,34	1325	3205,88	2066	3850,44
585	2469,62	1326	3206,04	2067	3850,86

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
586	2472,67	1327	3207,12	2068	3851,15
587	2473,33	1328	3207,4	2069	3852,72
588	2474,66	1329	3208,46	2070	3853,88
589	2475,76	1330	3209,12	2071	3855,01
590	2476,66	1331	3210,4	2072	3856,29
591	2476,89	1332	3212,24	2073	3857,1
592	2478,06	1333	3212,63	2074	3857,69
593	2478,8	1334	3213,58	2075	3858,56
594	2479,94	1335	3214,01	2076	3858,75
595	2480,69	1336	3215,04	2077	3858,99
596	2483,04	1337	3215,26	2078	3860,08
597	2485,26	1338	3216,57	2079	3861,35
598	2485,73	1339	3217,69	2080	3862,48
599	2487,45	1340	3218,14	2081	3863,21
600	2489,47	1341	3218,66	2082	3864,66
601	2491,52	1342	3220,04	2083	3866,43
602	2491,79	1343	3220,57	2084	3866,49
603	2492,4	1344	3221,07	2085	3868,25
604	2493,56	1345	3222,02	2086	3868,85
605	2494,85	1346	3222,75	2087	3870,22
606	2497,93	1347	3223,68	2088	3870,82
607	2500,24	1348	3224,97	2089	3871,44
608	2502,17	1349	3225,19	2090	3872,55
609	2503,07	1350	3226,27	2091	3873,33
610	2504,97	1351	3226,99	2092	3874,65
611	2505,2	1352	3228,83	2093	3875,18
612	2505,38	1353	3229,83	2094	3876,7
613	2506,56	1354	3230,65	2095	3877,24
614	2506,65	1355	3231,46	2096	3877,87
615	2506,72	1356	3232,12	2097	3878,52
616	2508,67	1357	3232,66	2098	3881,19
617	2509,97	1358	3233,38	2099	3881,4
618	2512,21	1359	3234,33	2100	3882,14
619	2512,66	1360	3234,62	2101	3883,78
620	2515,59	1361	3234,99	2102	3885,84
621	2516,63	1362	3236,51	2103	3886,28
622	2517,75	1363	3237,01	2104	3886,56
623	2519,04	1364	3237,56	2105	3887,63

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
624	2520,88	1365	3238,44	2106	3889,37
625	2522,07	1366	3239,1	2107	3890,16
626	2523,35	1367	3240,05	2108	3891,13
627	2524,05	1368	3240,48	2109	3891,28
628	2524,86	1369	3240,76	2110	3892,58
629	2526,04	1370	3242,49	2111	3892,94
630	2527,95	1371	3242,64	2112	3894,49
631	2528,45	1372	3243,37	2113	3895,62
632	2530,93	1373	3244,66	2114	3896,96
633	2531,73	1374	3244,88	2115	3898,3
634	2532,72	1375	3247,98	2116	3898,45
635	2533,78	1376	3248,55	2117	3899,86
636	2535,36	1377	3249,24	2118	3902,13
637	2537,01	1378	3250,1	2119	3902,66
638	2537,2	1379	3250,49	2120	3903,67
639	2538,72	1380	3251,06	2121	3905,34
640	2539,55	1381	3251,44	2122	3906,75
641	2540,69	1382	3251,66	2123	3909,21
642	2541,69	1383	3252,44	2124	3909,88
643	2542,9	1384	3253,7	2125	3910,8
644	2544,27	1385	3254,33	2126	3912,42
645	2546,72	1386	3254,39	2127	3913,58
646	2547,42	1387	3255,12	2128	3914,32
647	2548,01	1388	3256,73	2129	3915,79
648	2549,04	1389	3257,39	2130	3916,56
649	2550,05	1390	3258,34	2131	3917,95
650	2552,03	1391	3259,05	2132	3918,58
651	2552,23	1392	3259,63	2133	3921,1
652	2554,71	1393	3260,03	2134	3922,18
653	2555,89	1394	3260,66	2135	3923,38
654	2556,53	1395	3261,37	2136	3925,25
655	2559,44	1396	3262,47	2137	3925,87
656	2559,83	1397	3263,1	2138	3927,67
657	2562,35	1398	3264,13	2139	3928,02
658	2564	1399	3264,27	2140	3929,49
659	2564,15	1400	3265,54	2141	3930,89
660	2566,96	1401	3266,06	2142	3931,61
661	2568,48	1402	3266,24	2143	3932,18

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
662	2568,93	1403	3267,03	2144	3933,74
663	2570,66	1404	3267,82	2145	3934,64
664	2572,26	1405	3268,47	2146	3935,05
665	2573,6	1406	3269,03	2147	3935,2
666	2573,68	1407	3270,49	2148	3935,24
667	2574,39	1408	3271,49	2149	3935,41
668	2575,27	1409	3272,09	2150	3936,78
669	2575,8	1410	3273,09	2151	3937,28
670	2578,84	1411	3273,32	2152	3937,85
671	2579,86	1412	3273,76	2153	3940,7
672	2580,27	1413	3274,43	2154	3941,17
673	2581,89	1414	3275,78	2155	3942,55
674	2582,57	1415	3275,96	2156	3943,45
675	2583,72	1416	3276,87	2157	3944,62
676	2585,1	1417	3277,74	2158	3945,29
677	2586,06	1418	3278,62	2159	3946,63
678	2587,64	1419	3279,6	2160	3946,79
679	2589,13	1420	3280,07	2161	3946,92
680	2589,78	1421	3280,17	2162	3947,98
681	2591,52	1422	3281,09	2163	3948,41
682	2591,6	1423	3282,53	2164	3949
683	2591,99	1424	3283,46	2165	3951,62
684	2593,6	1425	3284,08	2166	3952,05
685	2594,46	1426	3284,81	2167	3952,77
686	2594,86	1427	3286,13	2168	3954,13
687	2596,58	1428	3287,2	2169	3955,1
688	2600,01	1429	3287,57	2170	3958,09
689	2600,48	1430	3289,07	2171	3958,95
690	2602,38	1431	3289,31	2172	3959,85
691	2604,23	1432	3291,21	2173	3960,7
692	2605,1	1433	3292,89	2174	3961,06
693	2606,52	1434	3294,57	2175	3962,14
694	2607,92	1435	3295,99	2176	3962,63
695	2608,57	1436	3296,29	2177	3962,7
696	2609,57	1437	3297,86	2178	3963,31
697	2610,76	1438	3298,72	2179	3964,68
698	2613,54	1439	3299	2180	3965,96
699	2615,41	1440	3300,26	2181	3968,24

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
700	2615,96	1441	3301,21	2182	3968,29
701	2617,27	1442	3303,26	2183	3969,8
702	2617,91	1443	3303,89	2184	3970,71
703	2618,54	1444	3304,7	2185	3971,11
704	2619,13	1445	3305,62	2186	3971,17
705	2619,88	1446	3305,89	2187	3971,68
706	2622,07	1447	3306,59	2188	3971,96
707	2623,73	1448	3307,25	2189	3973,74
708	2624,34	1449	3307,87	2190	3974,46
709	2624,97	1450	3308,7	2191	3976,86
710	2627,2	1451	3309,11	2192	3977,17
711	2628,46	1452	3309,76	2193	3977,93
712	2630,03	1453	3310,18	2194	3978,31
713	2631,09	1454	3312,41	2195	3978,82
714	2632	1455	3312,58	2196	3980,07
715	2634,86	1456	3313,72	2197	3980,21
716	2635,32	1457	3314,39	2198	3980,25
717	2636,84	1458	3315,42	2199	3980,69
718	2637,81	1459	3315,76	2200	3981,56
719	2638,45	1460	3316,56	2201	3982,2
720	2640,29	1461	3318,26	2202	3984,31
721	2642,46	1462	3318,48	2203	3984,38
722	2643,57	1463	3321,66	2204	3985,13
723	2644,33	1464	3322,55	2205	3987,05
724	2646,17	1465	3322,76	2206	3989,66
725	2646,41	1466	3323,77	2207	3991,25
726	2646,83	1467	3324,52	2208	3993,54
727	2647,54	1468	3325,55	2209	3994,66
728	2648,43	1469	3326,43	2210	3995,29
729	2649,22	1470	3327,1	2211	3995,79
730	2651,52	1471	3328,85	2212	3996,38
731	2651,91	1472	3329,31	2213	3997,14
732	2652,82	1473	3329,63	2214	3998,68
733	2653,81	1474	3330,7	2215	3999,49
734	2655,38	1475	3331,45	2216	4000,93
735	2655,84	1476	3332,33	2217	4001,45
736	2657,11	1477	3332,64	2218	4001,92
737	2658,31	1478	3333,53	2219	4002,02

	Frequency		Frequency		Frequency
Mode Number	[Hz]	Mode Number	[Hz]	Mode Number	[Hz]
738	2658,99	1479	3334,86	2220	4002,79
739	2660,18	1480	3336,21	2221	4003,01
740	2662,49	1481	3336,65	2222	4003,13
741	2663,39	1482	3337,08	2223	4003,8

A. 3 Enlargements



Figure A.110: Enlargement of the Synthesizer's flow chart



Figure A.111: Enlargement of the Block Diagram created for mechanism Simulation

Appendix B

Scripts

Code B.1: String Parameter Script

%% String selection according to parameters criteria

% With this script, plots for the selection of string diameter, tension, % and length are made. Restrictions for tension, length and diameter are % applied regarding instrument dimensions (for length), maximum stress % withstanding (in the case of tension) and comercial values (regarding % diameter). This selection is done for the strings to be tuned to their % fundamental frequency.

clearall% All previous values are erased as caution

p=7850; % String volumetric density (steel)
k=sqrt(1/(p*pi)); % Constant from the equation to be used of tension as
% a function of length

% Comercial diameter values in inches
D=[0.029;0.030;0.031;0.032;0.033;0.034;0.035;0.036;0.037; 0.038;0.039;0.040;0.041;0.042;0.043;0.044;0.045;0.046; 0.047;0.048;0.049;0.051;0.055;0.059];
D=D*(25.4/1000); % conversion to milimeters
l=linspace(0,0.9,125); % Length vector in m (restricted to 0.9 m)
freq=zeros(73,1); % preallocation of the frequencies
Tmax=0.63*(2755e6*pi*D.^2)/4; % Maximum tension value
Tm=ones(24,125); % Matrix for preallocation of max tension for each diameter
nd=size(freq); % counter for frequency
n=(-33:1:39); % values of exponent for frequency obtention using fn=fref^(n/12)
a=size(D); % number of diameters available
b=size(1); % number of length data

datos=zeros(b(1,2),a(1,1),nd(1,1)); % preallocation for data storage

```
% Loop to get all the fundamental frequencies for j=1:nd freq(j)=440*(2^(n(j)/12)); end
```

% Loop to get tension as function of length for the different diameters and % frequencies fori=1:nd(1,1) x=k/freq(i); for h=1:a(1,1) T=(D(h).*(1/x)*l).^2; datos(:,h,i)=T; end end % Loop to get horizontal lines to mark the tension restriction in plots for c=1:a(1,1) Tm(c,:)=Tmax(c)*Tm(c,:); end % Loop to plot the different curves for each fundamental frequency. Each % plot consists in a set of curves of tension as function of length for % different diameters. **for** u=1:73 h=figure; for m=1:24 plot(l,datos(:,m,u),'Linewidth',1.5,'Markersize',10), hold on, grid on, title('String Tension as a function of length for different Diameter Sizes', 'FontSize', 18), xlabel('String Length [m]', 'Fontsize', 14), ylabel('String Tension [N]','Fontsize',14) xlim([0 0.09]),ylim([0 1000]) plot(1,Tm(1,:),'r','Linewidth',1.5,'Markersize',10) end

end

Code B.2: Matlab Function created to find string displacement behavior

%% String Behavior obtention

% In this code, a function for obtaining a struck string behavior is made.

% It recovers natural frequency in Hz and in rad/s; it also recovers the

% behavior of every mode as a function of string position and time.

% Finally, it recovers the total string behavior function as a function of

% string position and time. Additionally, it retrieves the vectors of

% string length and time for plotting purposes.

% It asks for string length (l), diameter (d), volumetric density (p),

% tension (T), number of modes (n), total time (t), iteration time (it),

% fraction of the length of the string where it is struck (beta),

% and initial velocity (since it is a struck string, no initial

% displacement appears).

% All values should be placed using related units in order to get accurate % results. SI units are recommended.

function [fn,wn,yn,y,long,time]=string_behavior(l,d,p,T,n,t,it,beta,v0)

A=pi*(d/2)^2; %Transversal Area of the String u=p*A; %Linear density c=(T/u)^0.5; %Wave velocity in the string time=0:it:t; %Array of the period of time to analyze long=linspace(0,l,length(time)); %Array of values of string longitude tm=length(time); %counter for number of time data yn=zeros(n,length(long),length(time)); %preallocation for modal disp. data %in number of used modes, length of the string and time y=zeros(length(long),length(time));% preallocation of string disp. L1=beta*l; L2=L1+0.01; % Defines the portion of string where the initial struck happens fn=zeros(n,1); wn=zeros(n,1); % preallocation of the vectors for the frequencies of each

% mode: the fundamental and the overtones of each mode

b=zeros(n,1); % preallocation for fouriercoeficients of the modes

fori=1:n %loop to extract fourier coefficients and modal frequencies b(i)=((2*v0*l)/(i^2*pi^2*c))*(-cos((i*pi*L2)/l)+cos((i*pi*L1)/l)); wn(i)=i*pi*(c/l); fn(i)=wn(i)/(2*pi); end%data is stored in three vectors (coeff., angfreq and real freq.)

for j=1:tm %Loop to extract data of behavior. It is stored in an mxnxh matrix
for a=1:n %Extracts a vector corresponding to the behavior of each mode
mod=b(a)*sin(a*pi*(c/l)*time(j))*sin((a/l)*pi.*long);
yn(a,:,j)=mod;
end

ysum=sum(yn(:,:,j)); % Sums all the mode contributions for each point y(j,:)=ysum;% and gets a general string behavior with respect to time matrix end end% The final data matrix yn shows modal behavior across the length and % with respect to time.

% Matrix y shows general behavior

Code B.3: Matlab Function created to find String Slope Behavior

%% String Slope behavior obtention

% In this code, a function for obtaining a struck string's slope is made.

% It recovers natural frequency in Hz and in rad/s; it also recovers the

% slope behavior of every mode as a function of string position and time.

% Finally, it recovers the total string slope behavior as function of

% string position and time. Additionally, it retrieves the vectors of

% string length and time for plotting purposes.

% It asks for string length (l), diameter (d), volumetric density (p),

% tension (T), number of modes (n), total time (t), iteration time (it),

% fraction of the length of the string where it is struck (beta),

% and initial velocity (since it is a struck string, no initial

% displacement appears).

% All values should be placed using related units in order to get accurate % results. SI units are recommended.

function [fn,wn,yn,y,long,time]=string_slope(l,d,p,T,n,t,it,beta,v0)

A=pi*(d/2)^2; % Transversal Area of the String

u=p*A; %Linear density c=(T/u)^0.5; %Wave velocity in the string time=0:it:t; %Array of the period of time to analyze long=linspace(0,l,length(time)); %Array of values of string longitude tm=length(time); %counter for number of time data yn=zeros(n,length(long),length(time)); %preallocation for modal disp. data %in number of used modes, length of the string and time y=zeros(length(long),length(time));%preallocation of string disp. L1=beta*l; L2=L1+0.01; %Defines the portion of string where the initial struck happens fn=zeros(n,1); wn=zeros(n,1); %preallocation of the vectors for the frequencies of each %mode: the fundamental and the overtones of each mode

b=zeros(n,1); % preallocation for fouriercoeficients of the modes

fori=1:n %loop to extract fourier coefficients and modal frequencies b(i)=((2*v0*l)/(i^2*pi^2*c))*(-cos((i*pi*L2)/l)+cos((i*pi*L1)/l)); wn(i)=i*pi*(c/l); fn(i)=wn(i)/(2*pi); end%data is stored in three vectors (coeff., angfreq and real freq.)

for j=1:tm %Loop to extract data of behavior. It is stored in an mxnxh matrix
for a=1:n %Extracts a vector corresponding to the behavior of each mode
mod=(a/l)*pi*b(a)*sin(a*pi*(c/l)*time(j))*cos((a/l)*pi.*long);
yn(a,:,j)=mod;
end

ysum=sum(yn(:,:,j)); % Sums all the mode contributions for each point y(j,:)=ysum;% and gets a general string behavior with respect to time matrix end

end%The final data matrix yn shows modal behavior across the length and %with respect to time.

% Matrix y shows general behavior of slope

Code B.4: Matlab Function created to find String Slope Behavior optimizing Memory Allocation

%% String Slope behavior obtention (alternative)

% In this code, a function for obtaining a struck string's slope is made.

% It recovers natural frequency in Hz and in rad/s; it also recovers the

% slope behavior of every mode as a function of string position and time.

% Finally, it recovers the total string slope behavior as function of

% string position and time. Additionally, it retrieves the vectors of

% string length and time for plotting purposes.

% The difference in this one is that it recovers a specific number of data

% regarding time so instead of retrieving data using a time span defined by

% the ending time, it obtains the specified number of data points and stops

% when this number is reached regardless of the time span obtained. It was

% made for memory optimization purposes.

% It asks for string length (l), diameter (d), volumetric density (p),

% tension (T), number of modes (n), number of data points to obtain,

% iteration time (it), fraction of the length of the string where it is

% struck (beta), and initial velocity (since it is a struck string,

% no initial displacement appears).

% All values should be placed using related units in order to get accurate % results. SI units are recommended.

function [fn,wn,yn,y,long,time]=string_slope2(l,d,p,T,n,L,it,beta,v0)

A=pi*(d/2)^2; %Transversal Area of the String

u=p*A; %Linear density

 $c=(T/u)^0.5$; % Wave velocity in the string

time=(0:L-1)*it; % Array of the period of time to analyze

long=linspace(0,l,length(time)); % Array of values of string longitude
tm=length(time); % counter for number of time data

yn=zeros(n,length(long),length(time)); %preallocation for modal disp. data %in number of used modes, length of the string and time

y=zeros(length(long),length(time));% preallocation of string disp.

L1=beta*l;

L2=L1+0.01; % Defines the portion of string where the initial struck happens fn=zeros(n,1);

wn=zeros(n,1); % preallocation of the vectors for the frequencies of each % mode: the fundamental and the overtones of each mode

b=zeros(n,1); % preallocation for fouriercoeficients of the modes

fori=1:n %loop to extract fourier coefficients and modal frequencies b(i)=((2*v0*l)/(i^2*pi^2*c))*(-cos((i*pi*L2)/l)+cos((i*pi*L1)/l)); wn(i)=i*pi*(c/l); fn(i)=wn(i)/(2*pi); end%data is stored in three vectors (coeff., angfreq and real freq.)

for j=1:tm %Loop to extract data of behavior. It is stored in an mxnxh matrix
for a=1:n %Extracts a vector corresponding to the behavior of each mode
mod=(a/l)*pi*b(a)*sin(a*pi*(c/l)*time(j))*cos((a/l)*pi.*long);
yn(a,:,j)=mod;
end

ysum=sum(yn(:,:,j)); % Sums all the mode contributions for each point y(j,:)=ysum;% and gets a general string behavior with respect to time matrix end

end% The final data matrix yn shows modal behavior across the length and % with respect to time.

% Matrix y shows general behavior of slope

Code B.5: Matlab Script used to plot different modes of vibration and final transversal displacement and slope behavior

%% Animated graphic obtention for string behavior

% In this script, animated plots of the first five modes of vibration of % the string, and total behavior of its displacement and slope are % obtained.

clearmi%Clears any previous data c=size(yn); m=c(1,3); %obtains and stores the number of time matrixes available

% Each loop plots the behavior of one of the modes with respect to time. % The final loop plots the general behavior of the string with respect to % time.

fori=1:m plot(l,yn(1,:,i)), grid on,xlim([0 0.664]), ylim([-0.0003 0.0003]), title('Temporal behavior of the first mode of vibration'), xlabel('Position in the string [m]'), ylabel('Transveral displacement [m]') M(i)=getframe(gcf); %Stores the current plot end

movie2avi(M, 'Mode1_behavior', 'fps', 10, 'compression', 'None')
% The movie2avi function transforms the stored frames into a movie file for
% outside matlab playing.

fori=1:m
plot(l,yn(2,:,i)), grid on,xlim([0 0.664]), ylim([-0.0003 0.0003]),
title('Temporal behavior of the second mode of vibration'),
xlabel('Position in the string [m]'),
ylabel('Transveral displacement [m]')
M2(i)=getframe(gcf);
end

movie2avi(M2, 'Mode2_behavior', 'fps', 10, 'compression', 'None')

fori=1:m
plot(l,yn(3,:,i)), grid on,xlim([0 0.664]), ylim([-0.003 0.003]),
title("Temporal behavior of the third mode of vibration'),
xlabel('Position in the string [m]'),
ylabel("Transveral displacement [m]')
M3(i)=getframe(gcf);
end

movie2avi(M3,'Mode3_behavior','fps',10,'compression','None')

fori=1:m

plot(l,yn(4,:,i)), grid on,xlim([0 0.664]), ylim([-0.003 0.003]), title('Temporal behavior of the fourth mode of vibration'), xlabel('Position in the string [m]'), ylabel('Transveral displacement [m]') M4(i)=getframe(gcf); end

movie2avi(M4, 'Mode4_behavior', 'fps', 10, 'compression', 'None')

fori=1:m
plot(l,yn(5,:,i)), grid on,xlim([0 0.664]), ylim([-0.003 0.003]),
title('Temporal behavior of the fifth mode of vibration'),
xlabel('Position in the string [m]'),
ylabel('Transveral displacement [m]')
M5(i)=getframe(gcf);
end

movie2avi(M5, 'Mode5_behavior', 'fps', 10, 'compression', 'None')

fori=1:m subplot(2,1,1); plot(1,y(i,:),'LineWidth',2.5),xlim([0 0.664]), ylim([-0.0005 0.0005]), gridon,title('Temporal behavior of the String'), xlabel('Position in the string [m]'), ylabel('Transveral displacement [m]'), subplot(2,1,2); plot(1,yp(i,:),'r','LineWidth',2.5),grid on, xlim([0 0.664]), ylim([-0.008 0.008]), title('Temporal behavior of the String Slope'), xlabel('Position in the String [m]'), ylabel('Slope of the string [m/m]'), Mt(i)=getframe(gcf); end

movie2avi(Mt,'String_Behavior and slope','fps',8,'compression','None')

Code B.6: Matlab Script used to find Frequency responses of the Force Profiles

%% Frequency response for the string excitation to the Soundboard

% In this section, Force profiles for different musical notes are found as

% a function of time which represent the string excitation from the bridge

% to the soundboard. With this force profiles, fast fourier transforms are

% made to obtain the Frequency-domain force profiles that will be used as

% force excitation in the Finite Element models

%% Frequency response for E2

% Setting of the parameters corresponding to the musical note Lp=1000; % Number of data points Fs=40e3; % Sampling Frequency it=1/Fs; %Period between points l=0.9; % String Length d=1.4986e-3; % String Diameter p=7850; % String volumetric density T=304.651; % String Tension A=pi*(d/2)^2; % Transversal area u=p*A; %Linear density c=(T/u)^0.5; % Constant c wn=pi*(c/l); % Angular fundamental frequency fn=wn/(2*pi); % Fundamental Freq [Hz] n=floor(20e3/fn); % Number of overtones in the audio range. [~,~,~,yp,~,t]=string_slope2(1,d,p,T,n,Lp,it,0.1,5); % String slope

% Force Exerted by the strings to the bridge when excited. Force obtained % is transversal tension T(dy/dx) F=2*T*yp(:,length(t));

 $nfft=2^nextpow2(Lp); \%$ Padding number fot FFT Y = fft(F,nfft)/Lp; % Amplitude FFT from the data (Complex values) $Yamp=2^abs(Y); \%$ Obtention of the amplitude of the FFT $f = Fs/2^alinspace(0,1,nfft/2+1); \%$ Frequency range vector

% It is important to remember that FFT is symmetric so, half the obtained % values are the ones necessary. The other half is just a reflection.

% Plot of the Single sided FFT plot(f,Yamp(1:nfft/2+1),'LineWidth',2),grid on, title('Frequency Response for E2'), xlabel('Frequency [Hz]'), ylabel('Amplitude [N]')

% Since the same procedure is done for the other strings only varying the % magnitudes from the parameters, commenting of the rest of the code is % ommited.

%% Frequency response for C4

Lp=1000; Fs=40e3; it=1/Fs; l=0.664; d=0.0009906; p=7850; T=730.319; A=pi*(d/2)^2; u=p*A; c=(T/u)^0.5; wn=pi*(c/l); fn=wn/(2*pi); n=floor(20e3/fn);

[~,~,~,yp,~,t]=string_slope2(l,d,p,T,n,Lp,it,0.1,5);

F=2*T*yp(:,length(t));

$$\begin{split} nfft=&2^nextpow2(Lp);\\ Y&=fft(F,nfft)/Lp;\\ Yamp=&2^*abs(Y);\\ f&=Fs/2^*linspace(0,1,nfft/2+1); \end{split}$$

plot(f,Yamp(1:nfft/2+1),'LineWidth',2),grid on, title('Frequency Response for C4'), xlabel('Frequency [Hz]'), ylabel('Amplitude [N]')

%% Frequency Response for G5

Lp=1000; Fs=40e3; it=1/Fs; l=0.24; d=0.9144e-3; p=7850; T=730.023; A=pi*(d/2)^2; u=p*A; c=(T/u)^0.5; wn=pi*(c/l); fn=wn/(2*pi); n=floor(20e3/fn);

[~,~,~,yp,~,t]=string_slope2(l,d,p,T,n,Lp,it,0.1,5);

F=3*T*yp(:,length(t));

$$\label{eq:constraint} \begin{split} nfft=&2^nextpow2(Lp);\\ Y&=fft(F,nfft)/Lp;\\ Yamp=&2^*abs(Y);\\ f&=Fs/2^*linspace(0,1,nfft/2+1); \end{split}$$

plot(f,Yamp(1:nfft/2+1),'LineWidth',2),grid on, title('Frequency Response for G5'), xlabel('Frequency [Hz]'), ylabel('Amplitude [N]')

%% Frequency Response for B7

Lp=1000; Fs=40e3; it=1/Fs; l=0.06; d=0.7366e-3; p=7850; T=751.994; A=pi*(d/2)^2; u=p*A; c=(T/u)^0.5; wn=pi*(c/l); fn=wn/(2*pi); n=floor(20e3/fn);

[~,~,~,yp,~,t]=string_slope2(l,d,p,T,n,Lp,it,0.1,5);

F=3*T*yp(:,length(t));

 $nfft=2^nextpow2(Lp);$ Y = fft(F,nfft)/Lp; $Yamp=2^*abs(Y);$ $f = Fs/2^*linspace(0,1,nfft/2+1);$

plot(f,Yamp(1:nfft/2+1),'LineWidth',2),grid on, title('Frequency Response for B7'), xlabel('Frequency [Hz]'), ylabel('Amplitude [N]')

Code B.7: Matlab script created for electric devices behavior modeling

%% Electric Devices Analysis

% In this script a complete analysis for the different electronic devices% that will be implemented is made. These analyses include the amplifier,% the equalizer and the oscillators that will be used for sound synthesis% and manipulation

%% Amplifier

% This section makes an analysis of the features of the amplifier used, how % much it amplifies the input and how it varies with resistance.

Rp=10000; %Set the values for the components that will be used R1=2200; R2=1000; Vineff=0.5; % Effective voltage input R=linspace(0,10000,10000); Vo=(1./(1+(R./(Rp-R))))*(1+(R1/R2))*Vineff;

 $\begin{array}{l} G=inline('(1/(1+(x/(10000-x))))*(1+(2200/1000))',x'); \ \% \ Gain \ Functions \\ GdB=inline('20*log10((1/(1+(x/(10000-x))))*(1+(2200/1000)))',x'); \ \% \ dB \end{array}$

figure(1); % Create Figure

fplot(G,[0 10000]), grid on, %Plot linear gain function against R title('Gain with Respect to Resistance'), xlabel('Potenciometer Resistance [ohm]'), ylabel('Voltage gain of the amplifier [V/V]') figure(2); %Create another Fig fplot(GdB,[0 10000]),grid on%Plot dB gain versus R title('Gain in dB with Respect to Resistance'), xlabel('Potenciometer Resistance [ohm]'), ylabel('Voltage dB gain of the amplifier [dB]') figure(3); % Create Third Figure Voltage output versus R plot(R,Vo,'r','LineWidth',2),grid on, title('Output voltage with Respect to Resistance'), xlabel('Potenciometer Resistance [ohm]'), ylabel('Potenciometer Resistance [ohm]'), ylabel('Output Voltage of the Amplifier [V]')

%% Equalizer

% In this section, a frquency analysis of the filters that compose the % equalizer is made. A six-band equalizer with six independent filters is % created and Bode plots are made for each of the pass-band Filters. The % Filters used are MFB Pass-band filters.

```
% Create Matrix with values of resistors and capacitors
Z=[57e3 27e3 6.8e3 3.9e3 3.9e3 2.2e3;
100e3 57e3 10e3 6.8e3 7.5e3 3.9e3;
6.8e3 3.9e3 1e3 440 560 270;
100e-9 100e-9100e-910e-9 10e-9];
% Preallocatememoryforthedifferent variables to be found
fo=zeros(6,1);
wo=zeros(6,1);
Q=zeros(6,1);
df=zeros(6,1);
K=zeros(6,1);
nums=zeros(3,6);
```

% Loop used to find the different needed values for each of the six bands fori=1:6 % These equations are obtained by a frequency response analysis % previously made. fo(i) = (1/(2*pi*Z(4,i)))*sqrt((Z(1,i)+Z(3,i))/(Z(1,i)*Z(2,i)*Z(3,i)));wo(i)=2*pi*fo(i); Q(i)=pi*Z(2,i)*Z(4,i)*fo(i); df(i)=fo(i)/Q(i);K(i) = -(2*Z(3,i))/(Z(1,i)+Z(3,i));% The transfer function coefficients are stored for later creation of % the dynamic systemm nums(1,i)=K(i)*Q(i)/wo(i); $nums(2,i)=1/wo(i)^{2};$ nums(3,i)=1/(Q(i)*wo(i));end % Changes the frequency units from rad/s (default) to Hz for better % visualization

P=bodeoptions; P.FreqUnits='Hz';

% After storing the necessary values, the band dynamic system is modeled % through a transfer function using the obtained values and a Bode plot of % its magnitude and phase is created

% 1st Band

sys1=tf([nums(1,1) 0],[nums(2,1) nums(3,1) 1]); % Creates system
figure(1); % Creates Figure
% makes bodeplot of the system with the previously defined options
bodeplot(sys1,P), grid on

% 2nd Band

sys2=tf([nums(1,2) 0],[nums(2,2) nums(3,2) 1]);
figure(2);
bodeplot(sys2,P), grid on

% 3rd Band

sys3=tf([nums(1,3) 0],[nums(2,3) nums(3,3) 1]);
figure(3);
bodeplot(sys3,P), grid on

% 4th Band

sys4=tf([nums(1,4) 0],[nums(2,4) nums(3,4) 1]);
figure(4);
bodeplot(sys4,P), grid on

% 5th Band

sys5=tf([nums(1,5) 0],[nums(2,5) nums(3,5) 1]);
figure(5);
bodeplot(sys5,P), grid on

% 6th Band

sys6=tf([nums(1,6) 0],[nums(2,6) nums(3,6) 1]);
figure(6);
bodeplot(sys6,P), grid on

%% Oscillator

% In this section, the behavior of the oscillator is simulated;% especifically how the gain and the phase work. Since an oscillator% commonly provides a sinusoidal signal, it is not necessary to show this.
% Wien Bridge oscillator normalized transfer function c=tf([1 0],[1 3 1]);

% Options for bode plot P=bodeoptions; P.FreqUnits='Hz';

% Normalized bode plot for the oscillator, this behavior is the same % regardless of the components used. What changes is the resonance freq % which in this case is normalized to 1 (w=1/RC). figure(1); bodeplot(c,P), grid on

%% Envelope Generator

% This section models the behavior of the envelope generator, which in this % case is just a simple RC circuit using DC current. This voltage signal % will be used to modulate the oscillator's output.

% Setting of the applied values for the generator R=100:1000:250000; % potenciometer with variable resistance C=2.2e-6; % Capacitor Vin=6; % Voltage supply tao=C*R; % Time constant t=0:0.001:5*tao(length(R)); % Time

% Figure of the time constant relationship with variable resistance figure(1); plot(R,tao,'LineWidth',2), grid on, title('Time constant as a function of Resistance'), xlabel('Resistance [Ohm]'), ylabel('Time constant [s]')

% Up and down voltage signals (key pressing and releasing) Vup=Vin*(1-exp(-t/tao(length(R))));

Vdwn=Vup(length(Vup))*exp(-t/tao(length(R)));

%Get the whole voltage signal in one vector V=[VupVdwn]; time=linspace(0,10*tao(length(R)),length(V));

% Plot of the envelope signal as a function of time figure(2); plot(time,V,'LineWidth',2), grid on, title('Envelope Generator behavior for a key hit'), xlabel('Time [s]'), ylabel('Amplitude [V]')

Code B.8: Script used to define Resistance values for oscillators

%% Choosing of Oscillator's Values

% This code obtains the resistance and capacitance values for the

% oscillators to be used. It calculates a value for frequency between a

% specified range in accordance to a capacitance commercial value

% Capacitance commercial numerical values (this values can be obtained in % pF, nF and uF

Ccm=[1;1.2;1.5;1.8;2.2;2.7;3.3;3.9;4.7;5.6;6.8;8.2;10;12;15;18;22;27;33;39; 47;56;68;82;100;120;150;180;220;270;330;390;470;560;680;820]; % Ctot is a vector with capacitance commercial values in nF and uF Ctot=[1e-9*Ccm;1e-6*Ccm]; % Allocation of vectors for component values R=zeros(73,1);C = zeros(73,1);% Frequency memory allocation freq=zeros(73,1); % Exponent to be used to find frequency by using $f=fref^{(n/12)}$ n=(-33:1:39); % Determination of the counter's lenghts nd=length(freq); cd=length(Ctot); % Allocation for possible resistance values found with the different % capacitance values Rps=zeros(nd,cd);

```
%Loop to find all frequencies to use
for j=1:nd
freq(j)=440*(2^(n(j)/12));
end
```

% Loop to find the matrix of possible resistance values for each frequency % and for each capacitance value available fori=1:nd for l=1:cd Rps(i,l)=1/(2*pi*freq(i)*Ctot(l)); end end

% Loop used to find the value of resistance inside the determined range and % storage of selected capacitance and resistance values for a=1:nd m=1;% Flag for while loop b=1;% Counter for resistance values while m>0 % While loop to search for the resistance value in the range ifRps(a,b)>=4500 &&Rps(a,b)<=5500 % Selection criterion R(a)=Rps(a,b); %Store of the value and exit flag for loop C(a)=Ctot(b); m=-1; else b=b+1; %if criterion is not satisfied, counter increases end% and loop restarts to look for next value end end Appendix C

C.1 Electric Circuits Schematics

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C.2 Design Drawings





		PARTS LIST		
ITEM	QTY	PART NUMBER	DESCRIPTION	
1	1	Percussion Mechanism	To see a detail of the components, refer to percussion mechanism	D
			drawing.	
2	1	Outer Rim		
3	2	Speaker		
4	1	pinned block		
5	180	Tuning pin		
6	1	Damper_rail		
7	1	Back Leg		
8	3	Front Leg		
9	1	Soundboard		
10	1	Bridge		
11	144	Bridge pins		C
12	1	Cast Frame		Ŭ
13	36	2 hole agraffe		
14	36	3 hole agraffe		
15	180	Hitch pins		
16	1	Keyboard box		
17	1	Central Panel		┢

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USFQ	Designed by Carlos Gudiño C. Gudiño C. Gudiño	$1,00 + 0,50 \\ -0,00 $
Percussion Mechanism Spoon Edition	by Date Date Chiriboga 26/05/2015	13±1,00
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_		Percussion Me	Date String Date	Damper	Damper lever	Base stand	Spoon	Lever Screw	Capstan Screw	Repetition Lever	Jack	Wippen hammer support	Wippen	Top base	Regulating Screw	Metal Stand	Base	Wood rail Hammer	Key button	Keytop	Frontrail	Felt backrail	Backcheck	Key	Bearing Balance Rail Keypin	Balance Rail	ARTS LIST PART NUMBER
	Edition Sheet	echanism	5/2015 Sound Source	Stops string vibration	To transmit key movement to damper	To place components	Serves as rest for the jack	To stop repetition lever	Contact between wippen and key, regulable	Special lever for allowing double blow	Lever that triggers the hammer	Cushion for hammer rest	Lever that transmits key movement	To hold hammer	For regulation of the mechanism	To hold rails and give support	To hold wippen	Hits the string	Prevents sideway movement	Stetical appearance for key	To stop the key when pushed	Cushion	Stops hammer in check	Principal lever	Permits rotation	For Key balancing	DESCRIPTION

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