Higgs Phenomenology in the Two Higgs Doublet Model of type II (Personal Notes)

Vol. I

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CONTENTS

Note: In these three volumenes, we present the detailed calculations of the results that appear in the thesis: "Higgs Phenomenology in the Two Higgs Doublet Model of type II".

Vol. I : Limits on the Two Higgs Doublet Model from meson decay, mixing and CP violation.

Vol. II: Mass constraints, production cross sections, and decay rates in the Two Higgs Doublet Model of type II.

Vol. III:Higgs production at a muon collider in the Two Higgs Doublet Model of type II.

The Wigner-Weisskopf approximation for the desription of unstable particles and the $K^0 - \overline{K}^0$ system

The Wigner–Weisskopf approximation for the description of the decay of unstable particles and the $K^0 - \bar{K}^0$ system:

We consider a system described by the Hamiltonian:

$$H = H_0 + H' \qquad (1)$$

where H_0 is the unperturbed Hamiltonian, and H' is a small perturbation. (H_0 is the Hamiltonian of the strong interaction and H' that of the weak interaction). The eigenstates of H_0 are assumed to consist of n degenerate discrete states $|\alpha\rangle$ and a continuum of states $|\beta\rangle$.

$$H_0 |\alpha\rangle = E_0 |\alpha\rangle \qquad (\alpha = 1, \ldots n) \qquad (2)$$

$$H_0 |\beta\rangle = E_\beta |\beta\rangle \qquad (3)$$

When H' is switched on, it should be possible for the states $|\alpha\rangle$ to decay into the continuum states $|\beta\rangle$

In the Schrödinger picture:

$$|t\rangle_S = \sum_{\alpha=1}^{n} \psi_\alpha(t) |\alpha\rangle + \sum_\beta c_\beta(t) |\beta\rangle \qquad (4)$$

As an initial state at $t = 0$ sec we take:

$$|t=0\rangle_S = \sum_{\alpha=1}^{n} \psi_\alpha^{(0)} |\alpha\rangle \qquad (5)$$

The time evolution of the state vectors is given by:

$$i \frac{d}{dt} |t\rangle_S = H |t\rangle_S \qquad (6)$$

In the interaction picture:

$$|t\rangle_I = e^{iH_0 t} |t\rangle_S = \sum_{\alpha=1}^{n} a_\alpha(t) |\alpha\rangle + \sum_\beta b_\beta(t) |\beta\rangle \qquad (7)$$

$$|t=0\rangle_I = |t=0\rangle_S$$

$$i \frac{d}{dt} |t\rangle_I = H_I' |t\rangle_I = e^{iH_0 t} H' e^{-iH_0 t} |t\rangle_I \qquad (8)$$

$$i\frac{d}{dt}\Big[\sum_{\alpha=1}^{n} a_\alpha(t)|\alpha\rangle + \sum_\beta b_\beta(t)|\beta\rangle\Big] = e^{iH_0t}H' e^{-iH_0t}\Big(\sum_{\alpha=1}^{n} a_\alpha(t)|\alpha\rangle + \sum_\beta b_\beta(t)|\beta\rangle\Big)$$

$$i\sum_{\alpha'=1}^{n} \dot{a}_{\alpha'}(t)|\alpha'\rangle + i\sum_\beta \dot{b}_\beta(t)|\beta\rangle = e^{iH_0t}H' e^{-iE_0t}\sum_{\alpha'=1}^{n} a_{\alpha'}(t)|\alpha'\rangle + e^{iH_0t}H'\sum_\beta b_\beta(t)\, e^{-iE_\beta t}|\beta\rangle$$

$$(\langle\alpha|H_0 = \langle\alpha|E_0)$$

$$i\sum_{\alpha'=1}^{n} \dot{a}_{\alpha'}(t)\underbrace{\langle\alpha|\alpha'\rangle}_{\delta_{\alpha\alpha'}} + i\sum_\beta \dot{b}_\beta(t)\underbrace{\langle\alpha|\beta\rangle}_{0} = \langle\alpha|H'\cancel{e^{iE_0t}}\,\cancel{e^{-iE_0t}}\sum_{\alpha'=1}^{n} a_{\alpha'}(t)|\alpha'\rangle + \langle\alpha|e^{iE_0t}H'\sum_\beta b_\beta(t)\,e^{-iE_\beta t}|\beta\rangle$$

$$\boxed{i\,\dot{a}_\alpha(t) = \sum_{\alpha'=1}^{n}\langle\alpha|H'|\alpha'\rangle a_{\alpha'}(t) + \sum_\beta e^{i(E_0-E_\beta)t}\langle\alpha|H'|\beta\rangle b_\beta(t)} \quad (9)$$

on the other hand;

$$i\sum_{\alpha'=1}^{n} \dot{a}_{\alpha'}(t)\underbrace{\langle\beta|\alpha'\rangle}_{0} + i\sum_{\beta'} \dot{b}_{\beta'}(t)\underbrace{\langle\beta|\beta'\rangle}_{\delta_{\beta\beta'}} = \langle\beta|e^{iE_\beta t}H'e^{-iE_0t}\sum_{\alpha'=1}^{n} a_{\alpha'}(t)|\alpha'\rangle + \langle\beta|e^{iE_\beta t}H'\sum_{\beta'} b_{\beta'}(t)\,e^{-iE_{\beta'}t}|\beta'\rangle$$

$$\boxed{\begin{aligned} i\,\dot{b}_\beta(t) = &\sum_{\alpha'=1}^{n} e^{i(E_\beta-E_0)t}\langle\beta|H'|\alpha'\rangle a_{\alpha'}(t) \\ &+ \sum_{\beta'} e^{i(E_\beta-E_{\beta'})t}\langle\beta|H'|\beta'\rangle b_{\beta'}(t)\end{aligned}} \quad (10)$$

(We consider the π-mesons and muons that occur in K-decay as stable particles). Then we can neglect the second term in (10).

$\Rightarrow$

$$\boxed{b_\beta(t) = -i\sum_{\alpha'=1}^{n}\int_0^t dt'\, e^{i(E_\beta - E_0)t'} \langle\beta|H'|\alpha'\rangle\, a_{\alpha'}(t')} \qquad (11)$$

$(b_\beta(0)=0)$

From (7)

$$|t=0\rangle_I = |t=0\rangle_S = \sum_{\alpha=1}^{n}\psi_\alpha^{(0)}\,|\alpha\rangle = \sum_{\alpha=1}^{n} a_\alpha(0)\,|\alpha\rangle$$

$$\Rightarrow \boxed{a_\alpha(0) = \psi_\alpha^{(0)}} \qquad (12)$$

$$\int_{a_\alpha(0)}^{a_\alpha(t)} d a_\alpha(t) = -i\int_0^t dt'\left(\sum_{\alpha'=1}^{n}\langle\alpha|H'|\alpha'\rangle a_{\alpha'}(t') + \sum_\beta e^{i(E_0-E_\beta)t'}\langle\alpha|H'|\beta\rangle b_\beta(t')\right)$$

$$\boxed{\begin{aligned} a_\alpha(t) = \psi_\alpha^{(0)} &- i\int_0^t dt'\sum_{\alpha'=1}^{n}\langle\alpha|H'|\alpha'\rangle a_{\alpha'}(t') - \sum_{\beta,\alpha'}\int_0^t dt'\, e^{i(E_0-E_\beta)t'} \\ &\langle\alpha|H'|\beta\rangle\int_0^{t'} e^{i(E_\beta-E_0)t''}\langle\beta|H'|\alpha'\rangle\, a_{\alpha'}(t'')\,dt'' \end{aligned}} \qquad (13a)$$

$$\boxed{\begin{aligned} a_\alpha(t) = \psi_\alpha^{(0)} &- i\int_0^t dt'\sum_{\alpha'=1}^{n}\langle\alpha|H'|\alpha'\rangle a_{\alpha'}(t') - \sum_{\alpha',\beta}\int_0^t dt'\int_0^{t'} dt'' \\ &e^{i(E_0-E_\beta)(t'-t'')}\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle\, a_{\alpha'}(t'') \end{aligned}} \qquad (13b)$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = F(s)$$

$$\tilde{a}_\alpha(s) \equiv \int_0^\infty e^{-st} a_\alpha(t)\,dt \qquad (14)$$

$$\int_0^\infty e^{-st}\psi_\alpha^{(0)}\,dt = \psi_\alpha^{(0)}\left.\frac{e^{-st}}{-s}\right|_0^\infty = \frac{\psi_\alpha^{(0)}}{s} \qquad (15)$$

$$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$$

$$\mathcal{L}\left\{\int_0^t dt' \sum_{\alpha'=1}^{n} \langle\alpha|H'|\alpha'\rangle\, a_{\alpha'}(t')\right\}$$

$$= \sum_{\alpha'=1}^{n} \langle\alpha|H'|\alpha'\rangle \frac{\tilde{a}_{\alpha'}(s)}{s} \qquad (16)$$

$$\mathcal{L}\left\{\sum_{\alpha',\beta} \int_0^t dt' \int_0^{t'} dt''\, e^{i(E_0-E_\beta)(t'-t'')} \langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle\, a_{\alpha'}(t'')\right\}$$

$$= \sum_{\alpha',\beta} \langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle \frac{1}{s}\mathcal{L}\left\{\int_0^t dt''\, e^{i(E_0-E_\beta)(t-t'')}\, a_{\alpha'}(t'')\right\}$$

$$= \sum_{\alpha',\beta} \langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle \frac{1}{s}\mathcal{L}\left\{\int_0^t \underbrace{e^{i(E_0-E_\beta)(t-\tau)}}_{g(t-\tau)}\, \underbrace{a_{\alpha'}(\tau)}_{f(\tau)}\, d\tau\right\}$$

$$\left(\mathcal{L}\left\{\int_0^t f(\tau)\, g(t-\tau)\, d\tau\right\} = F(s)\, G(s)\right)$$

$$= \sum_{\alpha',\beta} \langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle \frac{1}{s}\int_0^\infty e^{-st} a_{\alpha'}(t)\,dt \int_0^\infty e^{-st} e^{i(E_0-E_\beta)t}\,dt$$

$$= \sum_{\alpha',\beta} \langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle \frac{1}{s}\tilde{a}_{\alpha'}(s) \int_0^\infty e^{-[s-i(E_0-E_\beta)]t}\,dt$$

$$= \sum_{\alpha',\beta} \langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle \frac{1}{s}\tilde{a}_{\alpha'}(s) \left.\frac{-e^{-[s-i(E_0-E_\beta)]t}}{[s-i(E_0-E_\beta)]}\right|_0^\infty$$

$$= \sum_{\alpha',\beta} \langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle \frac{1}{s}\tilde{a}_{\alpha'}(s) \left.\frac{(-i)\, e^{-st}\, \overbrace{e^{i(E_0-E_\beta)t}}^{\cos + i\sin}}{[E_0-E_\beta+is]}\right|_0^\infty$$

$$= \frac{i}{s}\sum_{\alpha',\beta} \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle\, \tilde{a}_{\alpha'}(s)}{(E_0-E_\beta+is)} \qquad (17)$$

Then (13b) can be written as: (after the laplace transform)

$$\tilde{a}_\alpha(s) = \frac{\psi_\alpha^{(0)}}{s} - i\sum_{\alpha'=1}^{n} \langle\alpha|H'|\alpha'\rangle \frac{\tilde{a}_{\alpha'}(s)}{s} - \frac{i}{s}\sum_{\alpha',\beta} \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle \tilde{a}_{\alpha'}(s)}{(E_0 - E_\beta + is)}$$

$$\tilde{a}_\alpha(s) = \frac{1}{s}\psi_\alpha^{(0)} - \frac{i}{s}\sum_{\alpha'=1}^{n} \tilde{a}_{\alpha'}(s)\left[\langle\alpha|H'|\alpha'\rangle + \sum_\beta \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle}{(E_0 - E_\beta + is)}\right]$$

$$\boxed{\tilde{a}_\alpha(s) = \frac{1}{s}\psi_\alpha^{(0)} - \frac{i}{s}\sum_{\alpha'=1}^{n} \tilde{a}_{\alpha'}(s)\, W_{\alpha\alpha'}(s)} \qquad (18)$$

where
$$\boxed{W_{\alpha\alpha'}(s) \equiv \langle\alpha|H'|\alpha'\rangle + \sum_\beta \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle}{(E_0 - E_\beta + is)}} \qquad (19)$$

$$\psi(t) \equiv \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \\ \vdots \\ \psi_n(t) \end{pmatrix} ; \quad \psi^0 = \begin{pmatrix} \psi_1^0 \\ \psi_2^0 \\ \vdots \\ \psi_n^0 \end{pmatrix} ; \quad a(t) = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

$$\tilde{a}(s) = \begin{pmatrix} \tilde{a}_1(s) \\ \tilde{a}_2(s) \\ \vdots \\ \tilde{a}_n(s) \end{pmatrix} ; \quad W(s) = (W_{\alpha\alpha'}(s))$$

$$s\,\tilde{a}(s) + i\,W(s)\,\tilde{a}(s) = \psi^0$$

$$\boxed{(s + iW(s))\,\tilde{a}(s) = \psi^0} \qquad (20)$$

$$\Rightarrow \boxed{\tilde{a}(s) = (s + iW(s))^{-1}\,\psi^0} \qquad (21)$$

$$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases} \qquad (22)$$

$$\frac{d}{dt} H(t) = \delta(t) \qquad (23)$$

$$f(x)\, e^{-cx} H(x)$$

$$F(z) = \int_{-\infty}^{+\infty} f'(u)\, e^{-izu}\, du = \int_{-\infty}^{+\infty} f'(x)\, e^{-izx}\, dx$$

$$F(z) = \int_{-\infty}^{+\infty} f(x)\, e^{-cx} H(x)\, e^{-izx}\, dx = \int_{0}^{+\infty} f(x)\, e^{-cx} e^{-izx}\, dx$$

$$f(x)\, e^{-cx} H(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(z)\, e^{izx}\, dz$$

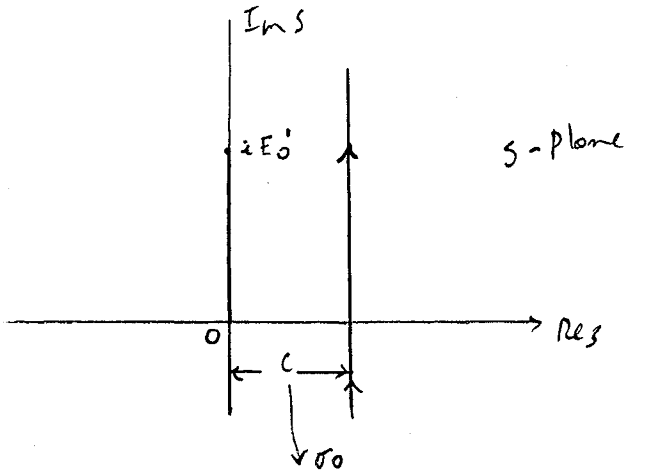
with $s = c + iz$, $ds = i\,dz$

$$F(z) = \int_0^{\infty} f(x)\, e^{-xs}\, dx = F(s)$$

$$\Rightarrow \quad \boxed{f(x) H(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(s)\, e^{xs}\, ds}$$

inverse Laplace transform. (24)

(c real)



$$a(t) = \frac{1}{2\pi i} \int_{\sigma_0 - i\infty}^{\sigma_0 + i\infty} \tilde{a}(s)\, e^{st}\, ds$$

$$a(t) = \frac{1}{2\pi i} \int_{\sigma_0 - i\infty}^{\sigma_0 + i\infty} (s + i w(s))^{-1} \psi^0 e^{st}\, ds = \frac{1}{2\pi i} \int_{\sigma_0 - i\infty}^{\sigma_0 + i\infty} ds\, (s + i w(s))^{-1} e^{st} \psi^0 \qquad (25)$$

$W(s)$ is analytic except at those points where

$$E_0 - E_\beta + is = 0 \qquad (s = i(E_0 - E_\beta))$$

is satisfied for some β.

$$E_0' = E_0 - \min_\beta E_\beta \qquad (26)$$

$W(s)$ has a cut from $-i\infty$ to iE_0'

$s = \mathrm{Re}\, s + i\, \mathrm{Im}\, s$

$$W_{\alpha\alpha'}(s) = \langle\alpha|H'|\alpha'\rangle + \sum_\beta \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle\,(E_0 - E_\beta - \mathrm{Im}\, s - i\,\mathrm{Re}\, s)}{[(E_0 - E_\beta - \mathrm{Im}\, s) + i\,\mathrm{Re}\, s][(E_0 - E_\beta - \mathrm{Im}\, s) - i\,\mathrm{Re}\, s]}$$

$$s\delta_{\alpha\alpha'} + iW_{\alpha\alpha'}(s) = s\delta_{\alpha\alpha'} + i\langle\alpha|H'|\alpha'\rangle + i\sum_\beta \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle(E_0 - E_\beta - \mathrm{Im}\, s)}{(E_0 - E_\beta - \mathrm{Im}\, s)^2 + (\mathrm{Re}\, s)^2}$$

$$+ \mathrm{Re}\, s \sum_\beta \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle}{(E_0 - E_\beta - \mathrm{Im}\, s)^2 + (\mathrm{Re}\, s)^2}$$

$$s\delta_{\alpha\alpha'} + iW_{\alpha\alpha'}(s) = \delta_{\alpha\alpha'}(\mathrm{Re}\, s + i\,\mathrm{Im}\, s) + i\langle\alpha|H'|\alpha'\rangle + i\sum_\beta \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle(E_0 - E_\beta - \mathrm{Im}\, s)}{(E_0 - E_\beta - \mathrm{Im}\, s)^2 + (\mathrm{Re}\, s)^2}$$

$$+ \mathrm{Re}\, s \sum_\beta \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle}{(E_0 - E_\beta - \mathrm{Im}\, s)^2 + (\mathrm{Re}\, s)^2}$$

$$\Rightarrow \boxed{s + iW(s) = X + iY} \qquad (27)$$

where

$$\boxed{X_{\alpha\alpha'} \equiv \mathrm{Re}\, s\left(\delta_{\alpha\alpha'} + \sum_\beta \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle}{(E_0 - E_\beta - \mathrm{Im}\, s)^2 + (\mathrm{Re}\, s)^2}\right)} \qquad (28)$$

$$\boxed{Y_{\alpha\alpha'} \equiv \delta_{\alpha\alpha'}\,\mathrm{Im}\, s + \langle\alpha|H'|\alpha'\rangle + \sum_\beta \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle\,(E_0 - E_\beta - \mathrm{Im}\, s)}{(E_0 - E_\beta - \mathrm{Im}\, s)^2 + (\mathrm{Re}\, s)^2}} \qquad (29)$$

X, Y are real

$$\langle\alpha|H'|\alpha'\rangle = H'_{\alpha\alpha'}$$

$$H'^{+} = H'$$

$$(\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle)^+ = (H'_{\alpha\beta} H'_{\beta\alpha'})^+ = H'_{\beta\alpha'} H'_{\alpha\beta} = H'_{\alpha\beta} H'_{\beta\alpha'}$$

$$\Rightarrow \quad \boxed{\begin{aligned} X^+ &= X \\ \text{and} \quad Y^+ &= Y \end{aligned}} \qquad (30)$$

If $\mathrm{Re}\,s > 0$

X is positive definite and then has an inverse.

$$(X+iY)\,X^{-1}(X-iY) = (X+iY)(I - iX^{-1}Y) = (X - \cancel{iY} + \cancel{iY} + YX^{-1}Y)$$

$$= X + YX^{-1}Y \quad \text{is positive definite}$$

$$\Rightarrow \det\left((X+iY)\,X^{-1}(X-iY)\right) > 0$$

$$\det(X+iY)\det(X-iY)\det(X^{-1}) > 0$$

$$|\det(X+iY)|^2 \det(X^{-1}) > 0$$

$$\Rightarrow \det(X+iY) \neq 0 \qquad (31)$$

$\therefore$ $(X+iY)$ has an inverse.

The same if $\mathrm{Re}\,s < 0$.

Then $(s+iW(s))$ is regular for $\mathrm{Re}\,s \neq 0$. Its singularities lie lie on the imaginary axis

$$\frac{1}{2\pi i}\int_\Gamma f(z)\,dz = \sum_{a\in A} \mathrm{Res}(f;a)\,\mathrm{ind}_\Gamma(a)$$

$$\mathrm{Res}(f;a) = \lim_{z\to a} \frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}\left[(z-a)^m f(z)\right]$$

If $H' = 0 \Rightarrow W = 0$ and we have a pole at $s = 0$

$$\Rightarrow \frac{1}{2\pi i}\int_{\sigma_0 - i\infty}^{\sigma_0 + i\infty} \overbrace{\frac{e^{st}}{s}}^{f(s)}\,ds\,\gamma^0 = e^0\gamma^0 = \gamma^0.$$

$$\therefore \quad \boxed{a(t) = \gamma^0 \quad \text{if } H' = 0} \qquad (32)$$

The second approximation of the Wigner-Weisskopf approach is to consider only the contribution of this pole when H' is switched on, that is, to replace $W(s)$ in the vecinity of $s=0$, $\mathrm{Re}\,s > 0$, by a constant:

$$W(s) \to W \equiv \lim_{s \to 0^+} W(s).$$

$$\Rightarrow a(t) = \frac{1}{2\pi i} \int_{\sigma_0 - i\infty}^{\sigma_0 + i\infty} \frac{\overset{\nearrow f(s)}{e^{st}}}{s + iW} ds \, \gamma^0 = \frac{\cancel{(s+iW)}\, e^{st}}{\cancel{(s+iW)}} \Big|_{s=-iW} \gamma^0$$

$$\boxed{a(t) = e^{-iWt} \gamma^0} \qquad (33)$$

$$|t\rangle_S = e^{-iH_0 t} |t\rangle_I = e^{-iH_0 t} \left(\sum_{\alpha=1}^{n} a_\alpha(t) |\alpha\rangle + \sum_\beta b_\beta(t) |\beta\rangle \right)$$

$$|t\rangle_S = e^{-iH_0 t} \left(\sum_{\alpha=1}^{n} e^{-iW_{\alpha\gamma} t} \gamma^0_\gamma \, |\alpha\rangle + \sum_\beta b_\beta(t) |\beta\rangle \right)$$

$$|t\rangle_S = \sum_{\alpha=1}^{n} e^{-iW_{\alpha\gamma} t} \gamma^0_\gamma \, e^{-iE_0 t} |\alpha\rangle + \sum_\beta b_\beta(t) \, e^{-iE_\beta t} |\beta\rangle$$

but $|t\rangle_S = \sum_{\alpha=1}^{n} \gamma_\alpha(t) \, |\alpha\rangle + \sum_\beta c_\beta(t) |\beta\rangle$

$$\Rightarrow \gamma_\alpha(t) = e^{-i(W_{\alpha\gamma} + E_0 \delta_{\alpha\gamma})t} \gamma^0_\gamma$$

$$\therefore \boxed{\gamma(t) = e^{-i(W + E_0)t} \gamma^0 = e^{-i\mu t} \gamma^0} \qquad (34)$$

$$\boxed{\mu = E_0 + W} \qquad (35)$$

Defining:

$$M \equiv \frac{1}{2}(\mu + \mu^+) \qquad \text{mass matrix} \qquad (36)$$

and $\Gamma \equiv i(\mu - \mu^+)$ decay matrix (37)

$$M^+ = \frac{1}{2}(\mu^+ + \mu) = M$$

$$\Gamma^+ = -i(\mu^+ - \mu) = i(\mu - \mu^+) = \Gamma$$

$$\therefore \boxed{M^+ = M \; ; \quad \Gamma^+ = \Gamma} \qquad (38)$$

$$\Rightarrow \quad 2M + \frac{\Gamma}{i}\,i = 2\mathcal{M} \quad ; \quad 2M - i\Gamma = 2\mathcal{M}$$

$$\therefore \quad \boxed{\mathcal{M} = M - i\frac{\Gamma}{2}} = H_{eff} \qquad (39)$$

$$\mathcal{M}_{\alpha\alpha'} = E_0\,\delta_{\alpha\alpha'} + W_{\alpha\alpha'}$$

$$\mathcal{M}_{\alpha\alpha'} = E_0\,\delta_{\alpha\alpha'} + \langle\alpha|H'|\alpha'\rangle + \sum_\beta \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle}{E_0 - E_\beta + i\varepsilon}$$

$$\mathcal{M}_{\alpha\alpha'} = E_0\,\delta_{\alpha\alpha'} + \langle\alpha|H'|\alpha'\rangle + P\left(\sum_\beta \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle}{E_0 - E_\beta}\right)$$

$$- i\pi \sum_\beta \langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle\,\delta(E_0 - E_\beta) \qquad (40)$$

$$\frac{1}{X - X_0 \pm i\varepsilon} = P\left(\frac{1}{X - X_0}\right) \mp i\pi\,\delta(X - X_0)$$

$$M = \frac{1}{2}(\mathcal{M} + \mathcal{M}^+)$$

$$\Rightarrow \boxed{M_{\alpha\alpha'} = E_0\,\delta_{\alpha\alpha'} + \langle\alpha|H'|\alpha'\rangle + P\left(\sum_\beta \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle}{E_0 - E_\beta}\right)} \qquad (41)$$

$$\Gamma = i(\mathcal{M} - \mathcal{M}^+)$$

$$\Rightarrow \boxed{\Gamma_{\alpha\alpha'} = 2\pi \sum_\beta \langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle\,\delta(E_0 - E_\beta)} \qquad (42)$$

for the $K^0 - \bar{K}^0$ system $(B^0 - \bar{B}^0)$

$$\boxed{\begin{aligned} M_{\alpha\alpha'} &= m\,\delta_{\alpha\alpha'} + \langle\alpha|H'|\alpha'\rangle + P\Bigg(\sum_{\substack{\beta \\ (E_\beta \neq m)}}{}' \frac{\langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha\rangle}{m - E_\beta}\Bigg) && (43) \\ \Gamma_{\alpha\alpha'} &= 2\pi \sum_{\beta\,(E_\beta \neq m)}{}' \langle\alpha|H'|\beta\rangle\langle\beta|H'|\alpha'\rangle\,\delta(m - E_\beta) && (44) \end{aligned}}$$

$$H' = H_w$$

the matrix element of the effective hamiltonian in the $K^0-\bar{K}^0$, $B^0-\bar{B}^0$, $D^0-\bar{D}^0$ systems is:

weak hamiltonian

$$\langle\alpha'|H_{eff}|\alpha\rangle = H_{\alpha\alpha'} = m\,\delta_{\alpha\alpha'} + \langle\alpha'|H_w|\alpha\rangle + \sum_\beta{}' \frac{\langle\alpha'|H_w|\beta\rangle\langle\beta|H_w|\alpha\rangle}{m - E_\beta + i\varepsilon}$$

but $\quad P\left(\int_a^b f(x)\,dx\right) = \lim_{\varepsilon\to 0^+}\left(\int_a^{p-\varepsilon} f(x)\,dx + \int_{p+\varepsilon}^b f(x)\,dx\right)$

$\downarrow$

principal part

where $f(x)$ is continuous for $a \le x < p$; $p < x \le b$.

then

$$\frac{1}{x - x_0 \pm i\varepsilon} = P\left(\frac{1}{x-x_0}\right) \mp i\pi\,\delta(x-x_0)$$

$$\therefore \quad \frac{1}{m - E_\beta + i\varepsilon} = P\left(\frac{1}{m-E_\beta}\right) - i\pi\,\delta(m-E_\beta)$$

$$\Rightarrow \langle\alpha'|H_{eff}|\alpha\rangle = m\,\delta_{\alpha\alpha'} + \langle\alpha'|H_w|\alpha\rangle + P\Bigg(\sum_{\substack{\beta \\ (E_\beta \neq m)}}{}' \frac{\langle\alpha'|H_w|\beta\rangle\langle\beta|H_w|\alpha\rangle}{(m-E_\beta)}\Bigg)$$

$$- i\pi \sum_\beta{}' \langle\alpha'|H_w|\beta\rangle\langle\beta|H_w|\alpha\rangle\,\delta(m-E_\beta)$$

$$H_{eff} = M - \frac{i\Gamma}{2}$$

$$\langle\alpha'|H_{eff}|\alpha\rangle = \langle\alpha'|M|\alpha\rangle - \frac{i}{2}\langle\alpha'|\Gamma|\alpha\rangle$$

$$= M_{\alpha\alpha'} - \frac{i}{2}\Gamma_{\alpha\alpha'}$$

so

$$\boxed{\begin{aligned} M_{\alpha\alpha'} &= m\,\delta_{\alpha\alpha'} + \langle\alpha'|H_w|\alpha\rangle + P\Bigg(\sum_{\substack{\beta \\ (E_\beta\neq m)}}{}' \frac{\langle\alpha'|H_w|\beta\rangle\langle\beta|H_w|\alpha\rangle}{(m-E_\beta)}\Bigg) \\ \Gamma_{\alpha\alpha'} &= 2\pi \sum_\beta{}' \langle\alpha'|H_w|\beta\rangle\langle\beta|H_w|\alpha\rangle\,\delta(m-E_\beta) \end{aligned}}$$

$M_{\alpha\alpha'} = M^*_{\alpha'\alpha}$ because $\langle\alpha|H_w|\alpha'\rangle^* = \langle\alpha'|H_w|\alpha\rangle$

$$\langle\alpha|H_w|\beta\rangle^*\langle\beta|H_w|\alpha'\rangle^* = \langle\alpha'|H_w|\beta\rangle\langle\beta|H_w|\alpha\rangle$$

also

$\Gamma_{\alpha\alpha'} = \Gamma^{*}_{\alpha'\alpha}$

So $\quad M = M^{+}$

and $\Gamma = \Gamma^{+}$

$$H_{eff} = M - i\frac{\Gamma}{2}$$

$$H^{+}_{eff} = M^{+} + i\frac{\Gamma^{+}}{2} = M + \frac{i}{2}\Gamma \neq H_{eff}$$

H can be written as:

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

M_{11} is the amplitude for a B^{0} to remain B^{0}. Aplying CPT we obtain the amplitude for a $\bar{B}^{0}$ to remain $\bar{B}^{0}$, i.e. M_{22}. So if CPT invariance holds than $M_{11} = M_{22} = m$. Γ_{11} is the probability per unit time for the decay $B^{0} \to \Sigma\xi$. Aplying CPT we obtain the probability per unit time for the decay $\Sigma\bar{\xi} \to \bar{B}^{0}$. Aplying

$$\Gamma_{\alpha\alpha'} = \Big(2\pi \sum_{\beta}{}' \langle \underset{\bar{B}^{0}}{\underset{B^{0}}{\alpha}} | H_w | \underset{\Sigma\bar{\xi}}{\underset{\Sigma\xi}{\beta}} \rangle \langle \underset{\Sigma\bar{\xi}}{\underset{\Sigma\xi}{\beta}} | H_w | \underset{\bar{B}^{0}}{\underset{B^{0}}{\alpha'}} \rangle \, \delta(m - E_\beta) \Big)$$

to the process $\Sigma\bar{\xi} \to \bar{B}^{0}$, we obtain $\bar{B}^{0} \to \Sigma\bar{\xi}$, i.e Γ_{22}

So if CPT holds $\Gamma_{11} = \Gamma_{22} = \gamma$.

$$\begin{vmatrix} H_{11} - \lambda & H_{12} \\ H_{21} & H_{22} - \lambda \end{vmatrix} = 0 \qquad (45)$$

$$(H_{11} - \lambda)(H_{22} - \lambda) - H_{12}H_{21} = 0$$

$$H_{11}H_{22} - \lambda(H_{11} + H_{22}) + \lambda^{2} - H_{12}H_{21} = 0$$

$$\lambda^{2} - \lambda(H_{11} + H_{22}) + [H_{11}H_{22} - H_{12}H_{21}] = 0$$

$$\lambda = \frac{H_{11} + H_{22} \pm [(H_{11} + H_{22})^{2} - 4(H_{11}H_{22} - H_{12}H_{21})]^{1/2}}{2}$$

$$\lambda = \frac{2H_{11} \pm \left[4H_{11}^2 - 4H_{11}^2 + 4H_{12}H_{21} \right]^{1/2}}{2} \qquad (46)$$

$$\lambda_L = H_{11} + (H_{12}H_{21})^{1/2} \quad ; \quad \lambda_L = m_L - \frac{i}{2}\gamma_L$$

$$\lambda_S = H_{11} - (H_{12}H_{21})^{1/2} \quad ; \quad \lambda_S = m_S - \frac{i}{2}\gamma_S$$

$$; \ \lambda_L - \lambda_S = \underbrace{(m_L - m_S)}_{\Delta m} - \frac{i}{2}\underbrace{(\gamma_L - \gamma_S)}_{\Delta\gamma}$$

$$\lambda_L - \lambda_S = 2(H_{12}H_{21})^{1/2} = +\Delta m - i\frac{\Delta\gamma}{2} \qquad (47)$$

$$\Rightarrow \Delta m = 2\,\mathrm{Re}\,(H_{12}H_{21})^{1/2} \qquad (48)$$

$$\Delta\gamma = -4\,\mathrm{Im}\,(H_{12}H_{21})^{1/2} \qquad (49)$$

$$H_{12} = M_{12} - \frac{i}{2}\Gamma_{12}$$

$$H_{21} = M_{21} - \frac{i}{2}\Gamma_{21}$$

$$M_{21} = M_{12}^*$$
$$\Gamma_{21} = \Gamma_{12}^*$$

$$\Rightarrow H_{21} = M_{12}^* - \frac{i}{2}\Gamma_{12}^*$$

So

$$\Delta m = 2\,\mathrm{Re}\left[\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)\right]^{1/2} \qquad (50)$$

$$\Delta\gamma = -4\,\mathrm{Im}\left[\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)\right]^{1/2} \qquad (51)$$

The $K^0 - \overline{K}^0$ and $B^0 - \overline{B}^0$ systems

$K^0 - \bar{K}^0 ; \; B^0 - \bar{B}^0$

$$H = M - \frac{i\Gamma}{2} \qquad (1)$$

M, Γ hermitianos

$$M - \frac{i\Gamma}{2} = \begin{pmatrix} m - \frac{i\gamma}{2} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*} & m - \frac{i\gamma}{2} \end{pmatrix} \qquad (2)$$

$$g'_{+}(t) = \langle \overset{K^0}{B^0} | \psi(t) \rangle \qquad (3)$$

$$g'_{-}(t) = \langle \underset{\bar{K}^0}{\bar{B}^0} | \psi(t) \rangle \qquad (4)$$

$$i\frac{d}{dt}\begin{pmatrix} g'_{+}(t) \\ g'_{-}(t) \end{pmatrix} = \begin{pmatrix} m - \frac{i\gamma}{2} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*} & m - \frac{i\gamma}{2} \end{pmatrix}\begin{pmatrix} g'_{+}(t) \\ g'_{-}(t) \end{pmatrix} \qquad (5)$$

$$\begin{vmatrix} m - \frac{i\gamma}{2} - \lambda & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*} & m - \frac{i\gamma}{2} - \lambda \end{vmatrix} = 0 \qquad (6)$$

$$m - \frac{i\gamma}{2} - \lambda = \pm\left[\frac{\Delta m}{2} - i\frac{\Delta\gamma}{4}\right] \qquad (7)$$

$$\Delta m = 2\,\mathrm{Re}\left\{(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^{*} - \tfrac{i}{2}\Gamma_{12}^{*})\right\}^{1/2} \qquad (8)$$

$$\Delta\gamma = -4\,\mathrm{Im}\left\{(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^{*} - \tfrac{i}{2}\Gamma_{12}^{*})\right\}^{1/2} \qquad (9)$$

$+ :\rightarrow$

$$\begin{pmatrix} \left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*})\right]^{1/2} & (M_{12} - \frac{i}{2}\Gamma_{12}) \\ (M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*}) & \left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*})\right]^{1/2} \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} (M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*}) & \left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*})\right]^{1/2} \\ (M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*}) & \left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*})\right]^{1/2} \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad x_1 = -\left(\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}\right)^{1/2} x_2$$

$$\text{with} \quad \frac{1+\varepsilon}{1-\varepsilon} = \left(\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}\right)^{1/2} \qquad (10)$$

$$\Rightarrow \quad x_1 = -\left(\frac{1+\varepsilon}{1-\varepsilon}\right) x_2$$

$$\begin{pmatrix} -\frac{(1+\varepsilon)}{(1-\varepsilon)} \\ 1 \end{pmatrix} \sim \begin{pmatrix} (1+\varepsilon) \\ -(1-\varepsilon) \end{pmatrix}$$

$$\Rightarrow \quad \boxed{\underset{\substack{\downarrow \\ 0.89\times 10^{-10}\,\text{sec}}}{|K_S^0\rangle} = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}}\left[(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\bar{K}^0\rangle\right]} \qquad CP=+1 \quad (11)$$

Similarly:

$$- : \rightarrow \begin{pmatrix} -\left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)\right]^{1/2} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & -\left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)\right]^{1/2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} M_{12}^* - \frac{i}{2}\Gamma_{12}^* & -\left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)\right]^{1/2} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & -\left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)\right]^{1/2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad x_1 = \frac{(1+\varepsilon)}{(1-\varepsilon)} x_2$$

$$\begin{pmatrix} \frac{(1+\varepsilon)}{(1-\varepsilon)} \\ 1 \end{pmatrix} \sim \begin{pmatrix} (1+\varepsilon) \\ (1-\varepsilon) \end{pmatrix}$$

$$\Rightarrow \boxed{\underset{5.17\times10^{-8}\,sec}{|K_2^0\rangle} = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}}\left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle\right]} \qquad (CP=-1) \quad (12)$$

$$\underset{P=-1}{CP}\,|K^0\rangle = -|\bar{K}^0\rangle \; ; \; CP|\bar{K}^0\rangle = -|K^0\rangle$$

Similarly :

$$|B^0_{1,2}\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}}\left[(1+\varepsilon)|B^0\rangle \pm (1-\varepsilon)|\bar{B}^0\rangle\right] \qquad (13)$$

$$i\frac{d}{dt}\begin{pmatrix}\langle B_1^0|\psi(t)\rangle \\ \langle B_2^0|\psi(t)\rangle\end{pmatrix} = \begin{pmatrix} m-\frac{i\gamma}{2}+\frac{\Delta m}{2}-\frac{i\Delta\gamma}{4} & 0 \\ 0 & m-\frac{i\gamma}{2}-\frac{\Delta m}{2}+\frac{i\Delta\gamma}{4}\end{pmatrix}\begin{pmatrix}\langle B_1^0|\psi(t)\rangle \\ \langle B_2^0|\psi(t)\rangle\end{pmatrix} \qquad (14)$$

$$i\frac{d}{dt}\begin{pmatrix}(1+\varepsilon^*)g_+' + (1-\varepsilon^*)g_-' \\ (1+\varepsilon^*)g_+' - (1-\varepsilon^*)g_-'\end{pmatrix} = \begin{pmatrix} m-\frac{i\gamma}{2}+\frac{\Delta m}{2}-\frac{i\Delta\gamma}{4} & 0 \\ 0 & m-\frac{i\gamma}{2}-\frac{\Delta m}{2}+\frac{i\Delta\gamma}{4}\end{pmatrix}\begin{pmatrix}(1+\varepsilon^*)g_+' + (1-\varepsilon^*)g_-' \\ (1+\varepsilon^*)g_+' - (1-\varepsilon^*)g_-'\end{pmatrix} \qquad (15)$$

setting :

$$g' \equiv (1+\varepsilon^*)g_+' + (1-\varepsilon^*)g_-' \qquad (16)$$

$$g'' \equiv (1+\varepsilon^*)g_+' - (1-\varepsilon^*)g_-' \qquad (17)$$

$$\Rightarrow \quad i\frac{d}{dt}g' = \left(m-\frac{i\gamma}{2}+\frac{\Delta m}{2}-\frac{i\Delta\gamma}{4}\right)g' \qquad (18)$$

and

$$i\frac{d}{dt}g'' = \left(m-\frac{i\gamma}{2}-\frac{\Delta m}{2}+\frac{i\Delta\gamma}{4}\right)g'' \qquad (19)$$

$$i\frac{d}{dt}f(t) = \lambda f(t) \quad \Rightarrow \quad f(t) = ce^{-i\lambda t} \qquad (20)$$

$$\therefore \quad g'(t) = A\, e^{-i\left(m - \frac{i\gamma}{2} + \frac{\Delta m}{2} - i\frac{\Delta\gamma}{4}\right)t} \qquad (21)$$

$$g''(t) = B\, e^{-i\left(m - \frac{i\gamma}{2} - \frac{\Delta m}{2} + i\frac{\Delta\gamma}{4}\right)t} \qquad (22)$$

$$\therefore \quad \boxed{\begin{aligned} (1+\check{\varepsilon})\, g_+' + (1-\check{\varepsilon})\, g_-' &= A\, e^{-imt}\, e^{-\frac{\gamma t}{2}}\, e^{-i\frac{\Delta m t}{2}}\, e^{-\frac{\Delta\gamma}{4}t} \\ (1+\check{\varepsilon})\, g_+' - (1-\check{\varepsilon})\, g_-' &= B\, e^{-imt}\, e^{-\frac{\gamma t}{2}}\, e^{i\frac{\Delta m t}{2}}\, e^{\frac{\Delta\gamma}{4}t} \end{aligned}} \qquad \begin{aligned}(23)\\(24)\end{aligned}$$

Writing :

$$S_+(t) \equiv e^{-imt}\, e^{-\frac{\gamma t}{2}}\, e^{-i\frac{\Delta m t}{2}}\, e^{-\frac{\Delta\gamma t}{4}} \qquad (25)$$

$$S_-(t) \equiv e^{-imt}\, e^{-\frac{\gamma t}{2}}\, e^{i\frac{\Delta m t}{2}}\, e^{\frac{\Delta\gamma t}{4}} \qquad (26)$$

$$\Rightarrow \begin{cases} (1+\check{\varepsilon})\, g_+' + (1-\check{\varepsilon})\, g_-' = A\, S_+(t) & (27) \\ (1+\check{\varepsilon})\, g_+' - (1-\check{\varepsilon})\, g_-' = B\, S_-(t) & (28) \end{cases}$$

$(27) + (28)$

$$2(1+\check{\varepsilon})\, g_+' = A\, S_+(t) + B\, S_-(t) \qquad (29)$$

If $g_+'(0) = \langle B^0 | \psi(0) \rangle = 1$ and $g_-'(0) = \langle \bar{B}^0 | \psi(0) \rangle = 0$

$$\Rightarrow \quad (1+\varepsilon) = A = B \qquad (30)$$

in (29)

$$2\cancel{(1+\check{\varepsilon})}\, g_+' = \cancel{(1+\check{\varepsilon})}\, S_+(t) + \cancel{(1+\check{\varepsilon})}\, S_-(t)$$

$$\Rightarrow \boxed{g_+' = \frac{1}{2}\left(S_+(t) + S_-(t)\right)} \qquad (31)$$

$(27) - (28)$

$$2(1-\check{\varepsilon})\, g_-' = (1+\check{\varepsilon})\left(S_+(t) - S_-(t)\right)$$

$$\Rightarrow \boxed{g_-' = \frac{1}{2}\,\frac{(1+\varepsilon)}{(1-\varepsilon)}\left(S_+(t) - S_-(t)\right)} \qquad (32)$$

$$|\psi\rangle = \sum_i c_i |i\rangle = \sum_i |i\rangle\langle i|\psi\rangle$$

$$\Rightarrow |B^0(t)\rangle = \underset{\substack{\downarrow \\ |B^0_0\rangle}}{|B^0\rangle} g_+' + \underset{\substack{\downarrow \\ |\bar{B}^0_0\rangle}}{|\bar{B}^0\rangle} g_-'$$

$$\boxed{|B^0(t)\rangle = |B^0\rangle \frac{1}{2}(S_+(t) + S_-(t)) + \frac{1}{2}\frac{(1+\epsilon^*)}{(1-\epsilon^*)}(S_+(t) - S_-(t))|\bar{B}^0\rangle} \quad (33)$$

If $g_+'(0) = 0$ and $g_-'(0) = 1$

$$\therefore \quad (1-\epsilon^*) = A$$

$$-(1-\epsilon^*) = B$$

$$\Rightarrow \quad A = -B = (1-\epsilon^*) \quad (34)$$

$(27) + (28) \Rightarrow$

$$2(1+\epsilon^*)\, g_+'(t) = (1-\epsilon^*)(S_+(t) - S_-(t))$$

$$\boxed{g_+'(t) = \frac{(1-\epsilon^*)}{2(1+\epsilon^*)}(S_+(t) - S_-(t))} \quad (35)$$

$(27) - (28) \Rightarrow$

$$2(\cancel{1-\epsilon^*})\, g_-' = (\cancel{1-\epsilon^*})(S_+(t) + S_-(t))$$

$$\therefore \quad \boxed{g_-'(t) = \frac{1}{2}(S_+(t) + S_-(t))} \quad (36)$$

$$\Rightarrow \boxed{|\bar{B}^0(t)\rangle = \frac{1}{2}|B^0\rangle \frac{(1-\epsilon^*)}{(1+\epsilon^*)}(S_+(t) - S_-(t)) + \frac{1}{2}|\bar{B}^0\rangle (S_+(t) + S_-(t))} \quad (37)$$

defining: $\quad f_+(t) = \frac{1}{2}(S_+(t) + S_-(t))$

$$f_-(t) = -\frac{1}{2}(S_+(t) - S_-(t))$$

we have:
$$\begin{cases} |B^0(t)\rangle = f_+(t)\,|B^0\rangle - \frac{(1+\epsilon^*)}{(1-\epsilon^*)} f_-(t)\,|\bar{B}^0\rangle \\ |\bar{B}^0(t)\rangle = \frac{(1-\epsilon^*)}{(1+\epsilon^*)} f_-(t)\,|B^0\rangle + f_+(t)\,|\bar{B}^0\rangle \end{cases}$$

Introducing :

$$\boxed{\frac{p}{q} = \left|\frac{1-\varepsilon^*}{1+\varepsilon^*}\right| = \left(\frac{M_{12} + \frac{i}{2}\Gamma_{12}}{M_{12}^* + \frac{i}{2}\Gamma_{12}^*}\right)^{1/2}} \qquad (38) \qquad \Rightarrow$$

$$\begin{cases} |B^0(t)\rangle = f_+(t)\,|B^0\rangle + \frac{q}{p} f_-(t)\,|\bar{B}^0\rangle \\ |\bar{B}^0(t)\rangle = \frac{p}{q} f_-(t)\,|B^0\rangle + f_+(t)\,|\bar{B}^0\rangle \end{cases} \qquad (39)$$

If $\Delta\gamma$ is very small

$$f_+(t) = \frac{1}{\cancel{2}} e^{-imt} e^{-\frac{\gamma t}{2}} \cancel{2} \cos\left(\frac{\Delta m t}{2}\right)$$

$$\boxed{f_+(t) = e^{-imt} e^{-\frac{\gamma t}{2}} \cos\left(\frac{\Delta m t}{2}\right)} \qquad (40)$$

$$f_-(t) = \frac{1}{\cancel{2}} e^{-imt} e^{-\frac{\gamma t}{2}} (-\cancel{2}i) \sin\left(\frac{\Delta m t}{2}\right)$$

$$\boxed{f_-(t) = i\, e^{-imt} e^{-\frac{\gamma t}{2}} \sin\left(\frac{\Delta m t}{2}\right)} \qquad (41)$$

f = non leptonic final state

$\bar{f}$ = CP conjugate state of f

$$A_f(t) = \frac{\Gamma(B^0 \to f) - \Gamma(\bar{B}^0 \to \bar{f})}{\Gamma(B^0 \to f) + \Gamma(\bar{B}^0 \to \bar{f})} \qquad (42)$$

$$\Gamma(B^0 \to f) = |\langle f | B^0\rangle|^2 \quad ; \quad \Gamma(\bar{B}^0 \to \bar{f}) = |\langle \bar{f} | \bar{B}^0\rangle|^2$$

$$\Gamma(B^0 \to f) = \left| f_+(t)\langle f | B^0\rangle + \frac{q}{p} f_-(t) \langle f | \bar{B}^0\rangle\right|^2$$

$$\alpha_f = \frac{q}{p}\rho_f \quad ; \quad \rho_f = \frac{\langle f | \bar{B}^0\rangle}{\langle f | B^0\rangle}$$

$$\bar{\alpha}_f = \frac{p}{q}\bar{\rho}_f \quad ; \quad \bar{\rho}_f = \frac{\langle \bar{f} | B^0\rangle}{\langle \bar{f} | \bar{B}^0\rangle} \qquad (43)$$

$$\Gamma(B^0 \to f) = \Big| e^{-imt} e^{-\frac{\gamma t}{2}} \cos\left(\frac{\Delta m t}{2}\right) \langle f | B^0\rangle + i\frac{q}{p} e^{-imt} e^{-\frac{\gamma t}{2}} \sin\left(\frac{\Delta m t}{2}\right) \rho_f \langle f | B^0\rangle\Big|^2$$

$$\Gamma(B^0 \to f) = e^{-\gamma t} \left|\cos\left(\frac{\Delta m t}{2}\right) + i\,\alpha_f \sin\left(\frac{\Delta m t}{2}\right)\right|^2 |\langle f | B^0\rangle|^2$$

$$\Gamma(B^0 \to f) = e^{-\gamma t}\left(\cos\left(\frac{\Delta m t}{2}\right) + i\, \alpha_f\, \sin\left(\frac{\Delta m t}{2}\right)\right)\left(\cos\left(\frac{\Delta m t}{2}\right) - i\, \alpha_f^*\, \sin\left(\frac{\Delta m t}{2}\right)\right)$$

$$= e^{-\gamma t}\left(\cos^2\left(\frac{\Delta m t}{2}\right) - i \sin\left(\frac{\Delta m t}{2}\right)\cos\left(\frac{\Delta m t}{2}\right)\alpha_f^*\right.$$

$$\left. + i \sin\left(\frac{\Delta m t}{2}\right)\cos\left(\frac{\Delta m t}{2}\right)\alpha_f + |\alpha_f|^2 \sin^2\left(\frac{\Delta m t}{2}\right)\right)$$

$$\boxed{\Gamma(B^0 \to f) = e^{-\gamma t}\left(\cos^2\left(\frac{\Delta m t}{2}\right) + |\alpha_f|^2 \sin^2\left(\frac{\Delta m t}{2}\right) - \sin(\Delta m t)\, \mathrm{Im}\, \alpha_f\right)} \quad (44)$$

$$\Gamma(\bar{B}^0 \to \bar{f}) = \left|\frac{p}{q} f_-(t) \langle \bar{f} | B^0 \rangle + f_+(t) \langle \bar{f} | \bar{B}^0 \rangle\right|^2$$

$$= \left|\frac{p}{q}(i)\, e^{-imt} e^{-\frac{\gamma t}{2}} \sin\left(\frac{\Delta m t}{2}\right)\bar{\rho}_f \langle \bar{f} | \bar{B}^0 \rangle + e^{-imt} e^{-\frac{\gamma t}{2}} \cos\left(\frac{\Delta m t}{2}\right)\langle \bar{f} | \bar{B}^0 \rangle\right|^2$$

$$= e^{-\gamma t}\left| i\, \bar{\alpha}_f \sin\left(\frac{\Delta m t}{2}\right) + \cos\left(\frac{\Delta m t}{2}\right)\right|^2 |\langle \bar{f} | \bar{B}^0 \rangle|^2$$

$$= e^{-\gamma t}\left(\cos\left(\frac{\Delta m t}{2}\right) + i\, \bar{\alpha}_f \sin\left(\frac{\Delta m t}{2}\right)\right)\left(\cos\left(\frac{\Delta m t}{2}\right) - i\, \bar{\alpha}_f^* \sin\left(\frac{\Delta m t}{2}\right)\right)|\langle \bar{f} | \bar{B}^0 \rangle|^2$$

$$= e^{-\gamma t}\left(\cos^2\left(\frac{\Delta m t}{2}\right) - i \sin\left(\frac{\Delta m t}{2}\right)\cos\left(\frac{\Delta m t}{2}\right)\bar{\alpha}_f^* + i \sin\left(\frac{\Delta m t}{2}\right)\cos\left(\frac{\Delta m t}{2}\right)\bar{\alpha}_f\right.$$

$$\left. + |\bar{\alpha}_f|^2 \sin^2\left(\frac{\Delta m t}{2}\right)\right)|\langle \bar{f} | \bar{B}^0 \rangle|^2$$

$$\boxed{\Gamma(\bar{B}^0 \to \bar{f}) = e^{-\gamma t}\left(\cos^2\left(\frac{\Delta m t}{2}\right) + |\bar{\alpha}_f|^2 \sin^2\left(\frac{\Delta m t}{2}\right) - \sin(\Delta m t)\, \mathrm{Im}\, \bar{\alpha}_f\right)} \quad (45)$$

If $|\langle f | B^0 \rangle| = |\langle \bar{f} | \bar{B}^0 \rangle|$

and $|\langle \bar{f} | B^0 \rangle| = |\langle f | \bar{B}^0 \rangle|$

$$\Rightarrow \quad |\rho_f| = |\bar{\rho}_f| \qquad (46)$$

$$\left|\frac{q}{p}\right|^2 \approx \left|\frac{p}{q}\right|^2 \approx 1 \qquad \left(\frac{q}{p} \approx -\left(\frac{M_{12}^*}{M_{12}}\right)^{1/2};\quad \Gamma_{12} \ll M_{12}\right)$$

(8)

$$\boxed{A_f(t) = \frac{-\sin(\Delta m t)\,(\operatorname{Im}\alpha_f - \operatorname{Im}\bar{\alpha}_f)}{2\cos^2\frac{\Delta m t}{2} + 2|\rho_f|^2 \sin^2\frac{\Delta m t}{2} - \sin(\Delta m t)\,(\operatorname{Im}\alpha_f + \operatorname{Im}\bar{\alpha}_f)}} \quad (47)$$

$$\alpha_f = \frac{q}{p}\rho_f$$

$$\frac{q}{p} = -\left(\frac{M_{12}^*}{M_{12}}\right)^{1/2} =$$

$$M_{12} \propto e^{i\theta}$$

$$M_{12}^* \propto e^{-i\theta}$$

$$\Rightarrow \quad \frac{q}{p} = -e^{-i\theta}$$

$$\rho_f = e^{i\theta'}$$

$$\bar{\rho}_f = e^{-i\theta'}$$

$$\alpha_f = -e^{-i\theta} e^{i\theta'} = -e^{i(\theta'-\theta)}$$

$$\alpha_f = -\cos(\theta'-\theta) - i\sin(\theta'-\theta) = -\cos(\theta-\theta') + i\sin(\theta-\theta')$$

$$\bar{\alpha}_f = \frac{p}{q}\bar{\rho}_f = -e^{i\theta} e^{-i\theta'} = -e^{i(\theta-\theta')}$$

$$\bar{\alpha}_f = -\cos(\theta-\theta') - i\sin(\theta-\theta')$$

$$\Rightarrow \boxed{\operatorname{Im}\alpha_f = -\operatorname{Im}\bar{\alpha}_f} \quad (48)$$

$$\therefore \quad A_f(t) = -(\operatorname{Im}\alpha_f)\sin(\Delta m t)$$

$$x_q = \frac{\Delta m}{\gamma_q} \quad ; \quad \tau_q = \frac{1}{\gamma_q} \quad (49)$$

$$\boxed{A_f(t) = -(\operatorname{Im}\alpha_f)\sin\left(\frac{x_q t}{\tau_q}\right)} \quad (50)$$

$$A_{f_{int}} = \frac{\int_0^\infty (\Gamma(B^0 \to f) - \Gamma(\bar{B}^0 \to \bar{f})\,dt}{\int_0^\infty (\Gamma(B^0 \to f) + \Gamma(\bar{B}^0 \to \bar{f})\,dt} \qquad (51)$$

$$A_{f_{int}} = \frac{\cancel{x}\int_0^\infty e^{-\gamma t}\, Im(\alpha_f)\, \sin(\Delta m t)\,dt}{\cancel{x}\int_0^\infty e^{-\gamma t}\,dt}$$

$$A_{f_{int}} = -\frac{Im(\alpha_f)\int_0^\infty \overset{v'}{e^{-\gamma t}}\, \overset{v}{\sin(\Delta m t)}\,dt}{\int_0^\infty e^{-\gamma t}\,dt}$$

$$A_{f_{int}} = -\frac{Im(\alpha_f)\left[\frac{\Delta m}{\gamma^2} / \left(1+\frac{\Delta m^2}{\gamma^2}\right)\right]}{\frac{1}{\gamma}}$$

because:

$$\int_0^\infty \underset{v'}{e^{-\gamma t}}\, \underset{v}{\sin(\Delta m t)}\,dt = -\frac{1}{\gamma} e^{-\gamma t} \sin(\Delta m t)\Big|_0^\infty + \int_0^\infty \frac{1}{\gamma} e^{-\gamma t}\, \Delta m \cos(\Delta m t)\,dt$$

$$= \frac{\Delta m}{\gamma}\int_0^\infty \underset{v'}{e^{-\gamma t}}\, \underset{v}{\cos \Delta m t}\,dt$$

$$= \frac{\Delta m}{\gamma}\left[-\frac{1}{\gamma} e^{-\gamma t} \cos \Delta m t\Big|_0^\infty - \int_0^\infty \frac{1}{\gamma} e^{-\gamma t}\, \Delta m \sin(\Delta m t)\,dt\right.$$

$$= \frac{\Delta m}{\gamma}\left[+\frac{1}{\gamma} - \frac{1}{\gamma}\Delta m \int_0^\infty e^{-\gamma t} \sin \Delta m t\,dt\right]$$

$$= \frac{\Delta m}{\gamma^2}\left(1 - \Delta m \int_0^\infty e^{-\gamma t} \sin \Delta m t\,dt\right)$$

$$= \frac{\Delta m}{\gamma^2} - \frac{\Delta m^2}{\gamma^2}\int_0^\infty e^{-\gamma t} \sin \Delta m t\,dt$$

$$\Rightarrow \int_0^\infty e^{-\gamma t} \sin \Delta m t\,dt = \frac{\frac{\Delta m}{\gamma^2}}{1+\frac{\Delta m^2}{\gamma^2}}$$

$$\therefore \boxed{A_{f_{int}} = -\frac{(Im\,\alpha_f)\; x_q}{(1+x_q^2)}} \qquad (52)$$

$$\langle K^0_S | K^0_L \rangle = \frac{1}{2(1+|\varepsilon|^2)} \left[(1+\varepsilon^*) \langle K^0| - (1-\varepsilon^*) \langle \bar{K}^0| \right] \left[(1+\varepsilon) |K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle \right]$$

$$= \frac{1}{2(1+|\varepsilon|^2)} \left[(1+\varepsilon^*)(1+\varepsilon) - (1-\varepsilon^*)(1-\varepsilon) \right]$$

$$= \frac{\cancel{1} + 2\,\mathrm{Re}\,\varepsilon + \cancel{|\varepsilon|^2} - \cancel{1} + 2\,\mathrm{Re}\,\varepsilon - \cancel{|\varepsilon|^2}}{2(1+|\varepsilon|^2)}$$

$$\boxed{\langle K^0_S | K^0_L \rangle = \frac{2\,\mathrm{Re}\,\varepsilon}{(1+|\varepsilon|^2)}} \qquad (53)$$

because: $|\psi\rangle = \sum_i c_i |i\rangle = \sum_i |i\rangle\langle i|\psi\rangle$

$$\Rightarrow \quad \langle\psi| = \sum_i \langle\psi|i\rangle\langle i| = \sum_i \langle i|\psi\rangle^* \langle i| = \sum_i c_i^* \langle i|$$

B_q^0 ($q \downarrow d,s$)

$$\rho_f = \frac{\langle \bar{f} | \bar{B}^0 \rangle}{\langle f | B^0 \rangle} = \frac{V_{ub}}{V_{ub}^*} \frac{V_{ud,s}^*}{V_{ud,s}} = e^{-2i\delta} \qquad b \to u$$

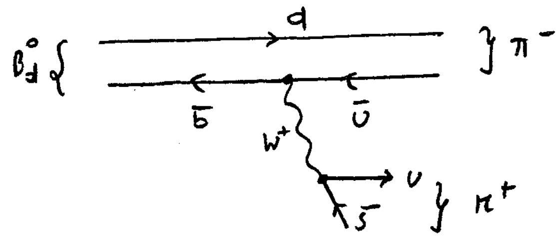
$$\rho_f = \frac{V_{cb}}{V_{cb}^*} \frac{V_{cs,d}^*}{V_{cs,d}} = 1 \qquad b \to c$$

$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$

$B_d^0 \to \pi^- K^+$ ($B_d^0 \sim 10^{-12}$ sec)

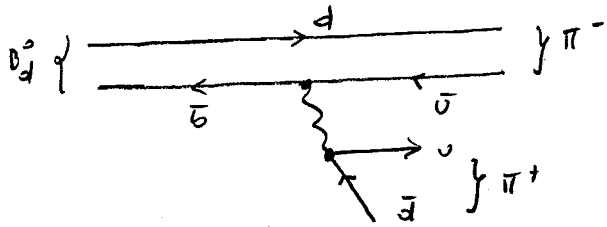
$d\bar{b} \to d\bar{u}\, u\bar{s}$

$(B_s^0 \sim 10^{-12}$ sec$)$



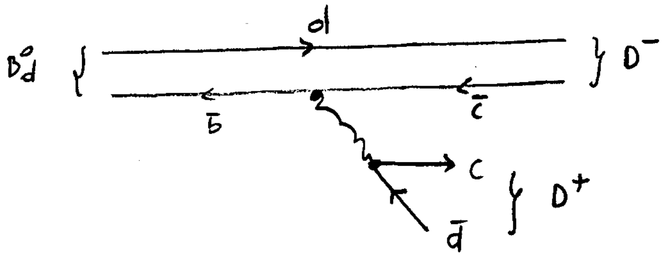
$B_d^0 \to \pi^- \pi^+$

$d\bar{b} \to d\bar{u}\, u\bar{d}$



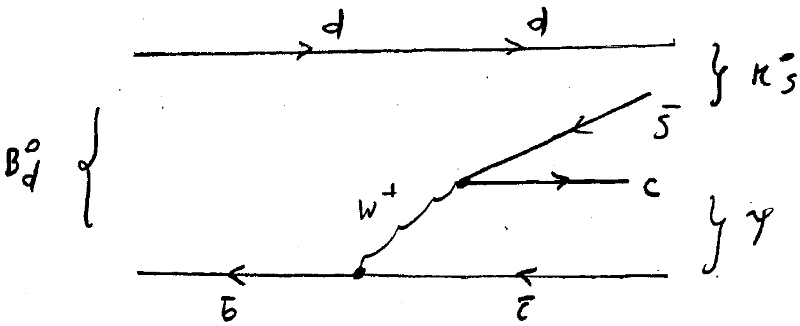
$B_d^0 \to D^+ D^-$

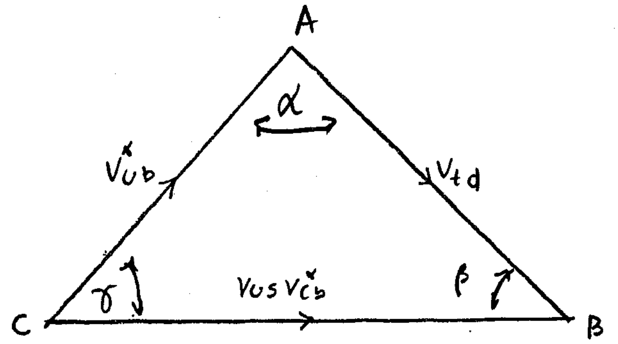
$d\bar{b} \to c\bar{d}\, d\bar{c}$



$B_d^0 \to \psi K_s^0$

$d\bar{b} \to c\bar{c}\, d\bar{s}$





$$V_{ub}^* + V_{td} - \overbrace{\underset{\downarrow \atop s_{12}}{V_{us}}\, V_{cb}^*}^{\lambda V_{ts}} = A\lambda^3 \cancel{\rho e^{-i\delta}} + A\lambda^3(1-\cancel{\rho e^{-i\delta}}) - \overbrace{\cancel{\lambda^3 A}}^{\lambda V_{ts}}$$

$$= 0$$

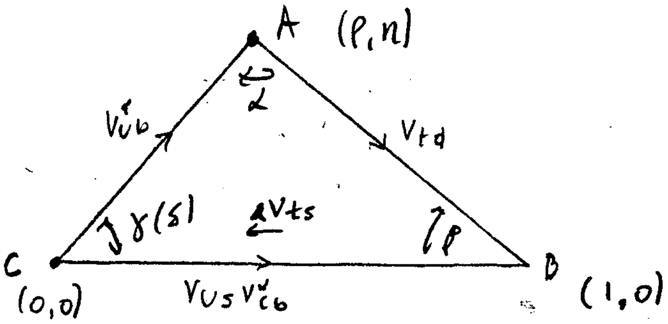
$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3\rho e^{-i\delta} \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho e^{i\delta}) & -A\lambda^2 & 1 \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3\rho(1-i\delta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho(1+i\delta)) & -A\lambda^2 & 1 \end{pmatrix}$$

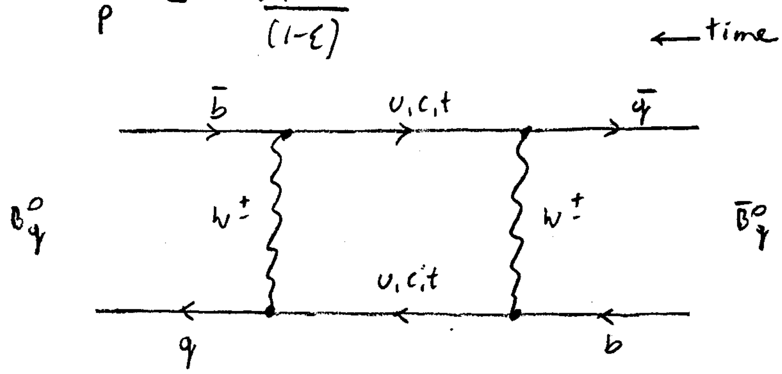
$$\rho\delta = \eta$$

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3((1-\rho)-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

(Wolfenstein Parametrization)



$$\frac{q}{p} = -\frac{(1+\varepsilon)}{(1-\varepsilon)}$$



$B_q^0 = q\bar{b}$; $\bar{B}_q^0 = b\bar{q}$

$$\frac{q}{p} = \frac{V_{tb}^* V_{tq}}{V_{tb} V_{tq}^*} = \begin{cases} \dfrac{e^{-i\beta}}{e^{i\beta}} = e^{-2i\beta} & q = d \\ 1 & q = s \end{cases}$$

$B_d^0 \quad (b \to u) \quad = -\operatorname{Im}\alpha_1 = -\operatorname{Im}(e^{-2i\beta} e^{-2i\delta}) = \sin 2(\beta+\delta) = -\sin 2\alpha$

$B_d^0 \quad (b \to c) \quad = -\operatorname{Im}\alpha_2 = -\operatorname{Im}(e^{-2i\beta}) = \sin 2\beta$

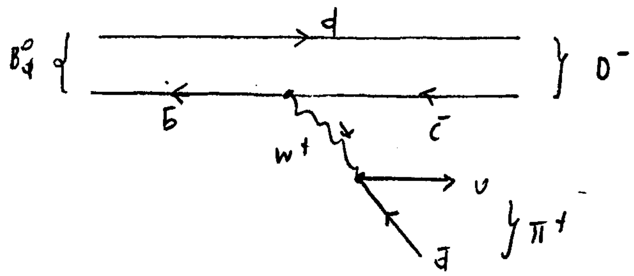
$B_s^0 \quad (b \to u) \quad = -\operatorname{Im}\alpha_3 = -\operatorname{Im}(e^{-2i\delta}) = \sin 2\underset{\gamma}{\underset{\downarrow}{\delta}} = -\sin 2(\alpha+\beta)$

$B_s^0 \quad (b \to c) \quad = -\operatorname{Im}\alpha_4 = 0$

$$A_f(t) = -\operatorname{Im}(\alpha_f)\sin\left(\frac{x_q t}{\tau}\right) = -\operatorname{Im}\left(\frac{q}{p}\frac{\langle f|\bar{B}^0\rangle}{\langle f|B^0\rangle}\right)\sin\left(\frac{x_q t}{\tau}\right)$$

$B_d^0 \to D^- \pi^+$

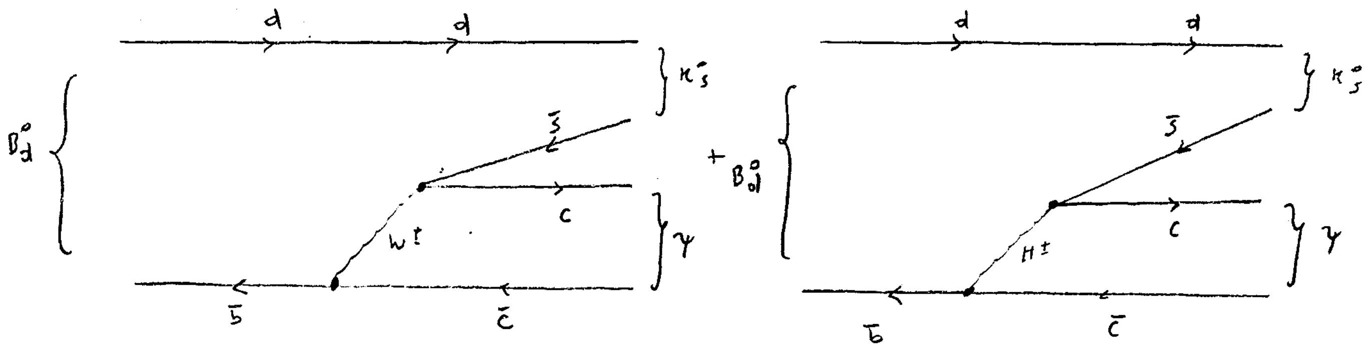
$d\bar{b} \to d\bar{c} \quad u\bar{d}$



$-\operatorname{Im}(\alpha) = -\operatorname{Im}(1 \cdot e^{-2i\beta}) = \sin 2\beta$

$B_d^0 \to J/\psi\, K_s^0$

$d\bar{b} \to c\bar{c} \quad d\bar{s}$



$$\lambda_f = \frac{q}{p}\rho_f = \frac{q}{p}\left[\frac{\langle f|W^\pm|\bar{B}^0\rangle + \langle f|H^\pm|\bar{B}^0\rangle}{\langle f|W^\pm|B^0\rangle + \langle f|H^\pm|B^0\rangle}\right]$$

$$\frac{q}{p} \quad q=d \quad e^{-2i\beta}$$

$$\langle f|W^\pm|\bar{B}^0\rangle + \langle f|H^\pm|\bar{B}^0\rangle \propto V_{cb}V_{cs}^*$$

$$\langle f|W^\pm|B^0\rangle + \langle f|H^\pm|B^0\rangle \propto V_{cb}^* V_{cs}$$

$$\Rightarrow \lambda_f = e^{-2i\beta}$$

$$-\mathrm{Im}\,\lambda_f = -\mathrm{Im}\,(e^{-2i\beta}) = \sin 2\beta \quad //$$

$$|\bar{B}^0(t)\rangle = \frac{1}{2}|B^0\rangle \frac{(1-\epsilon^*)}{(1+\epsilon^*)}\left(S_+(t) - S_-(t)\right) + \frac{1}{2}|\bar{B}^0\rangle \left(S_+(t) + S_-(t)\right)$$

$$\langle \bar{B}^0(t)| = \frac{1}{2}\frac{(1-\epsilon)}{(1+\epsilon)}\left(S_+^*(t) - S_-^*(t)\right)\langle B^0| + \frac{1}{2}\left(S_+^* + S_-^*\right)\langle \bar{B}^0|$$

$$\langle \bar{B}^0(t)|B^0\rangle = \frac{1}{2}\frac{(1-\epsilon)}{(1+\epsilon)}\left(S_+^* - S_-^*\right)$$

$$\boxed{|\langle \bar{B}^0(t)|B^0\rangle|^2 = \frac{1}{4}\left|\frac{1-\epsilon}{1+\epsilon}\right|^2 |S_+ - S_-|^2}$$

$$\langle B^0(t)| = \frac{1}{2}\left(S_+^* + S_-^*\right)\langle B^0| + \frac{1}{2}\frac{(1+\epsilon)}{(1-\epsilon)}\left(S_+^* - S_-^*\right)\langle \bar{B}^0|$$

$$\langle B^0(t)|\bar{B}^0\rangle = \frac{1}{2}\frac{(1+\epsilon)}{(1-\epsilon)}\left(S_+^* - S_-^*\right)$$

$$\boxed{|\langle B^0(t)|\bar{B}^0\rangle|^2 = \frac{1}{4}\left|\frac{1+\epsilon}{1-\epsilon}\right|^2 |S_+ - S_-|^2}$$

$$\langle B^0(t)|B^0\rangle = \frac{1}{2}\left(S_+^* + S_-^*\right)$$

$$\boxed{|\langle B^0(t)|B^0\rangle|^2 = \frac{1}{4}|S_+ + S_-|^2}$$

$$\langle \bar{B}^0(t)|\bar{B}^0\rangle = \frac{1}{2}\left(S_+^* + S_-^*\right)$$

$$\boxed{|\langle \bar{B}^0(t)|\bar{B}^0\rangle|^2 = \frac{1}{4}|S_+ + S_-|^2}$$

Probability of $\bar{B}^0 \to B^0$

$$\chi = \frac{\int_0^\infty \frac{1}{4}\left|\frac{1+\epsilon}{1-\epsilon}\right|^2 |S_+ - S_-|^2\,dt}{\int_0^\infty \frac{1}{4}\left|\frac{1+\epsilon}{1-\epsilon}\right|^2 |S_+ - S_-|^2\,dt + \int_0^\infty \frac{1}{4}|S_+ + S_-|^2\,dt}$$

$$S_+ - S_- = e^{-imt} e^{-\gamma t/2} \left[e^{-i\Delta m t/2} e^{-\Delta\gamma t/4} - e^{i\Delta m t/2} e^{\Delta\gamma t/4} \right]$$

$$|S_+ - S_-|^2 = e^{-imt} e^{-\gamma t/2} \left[e^{-i\Delta m t/2} e^{-\Delta\gamma t/4} - e^{i\Delta m t/2} e^{\Delta\gamma t/4} \right] \cdot$$

$$\cdot\, e^{imt} e^{-\gamma t/2} \left[e^{i\Delta m t/2} e^{-\Delta\gamma t/4} - e^{-i\Delta m t/2} e^{\Delta\gamma t/4} \right]$$

$$= e^{-\gamma t} \left[e^{-\Delta\gamma t/2} - e^{-i\Delta m t} - e^{i\Delta m t} + e^{\Delta\gamma t/2} \right]$$

$$|S_+ - S_-|^2 = e^{-(\gamma + \frac{\Delta\gamma}{2})t} - 2e^{-\gamma t}\cos\Delta m t + e^{-(\gamma - \frac{\Delta\gamma}{2})t}$$

$$\int_0^\infty |S_+ - S_-|^2 dt = \int_0^\infty e^{-(\gamma + \frac{\Delta\gamma}{2})t} dt - 2\int_0^\infty \underset{v}{e^{-\gamma t}} \underset{u'}{\cos\Delta m t}\, dt$$

$$+ \int_0^\infty e^{-(\gamma - \frac{\Delta\gamma}{2})t} dt$$

$$= - \frac{e^{-(\gamma + \frac{\Delta\gamma}{2})t}}{(\gamma + \frac{\Delta\gamma}{2})} \Big|_0^\infty - \frac{e^{-(\gamma - \frac{\Delta\gamma}{2})t}}{(\gamma - \frac{\Delta\gamma}{2})} \Big|_0^\infty$$

$$-2 \left[\frac{\gamma/(\Delta m)^2}{1 + \left(\frac{\gamma}{\Delta m}\right)^2} \right] = \frac{1}{\gamma + \frac{\Delta\gamma}{2}} + \frac{1}{\gamma - \frac{\Delta\gamma}{2}} - 2\frac{\gamma/(\Delta m)^2}{1 + \left(\frac{\gamma}{\Delta m}\right)^2}$$

$$\int_0^\infty \underset{v}{e^{-\gamma t}} \underset{u'}{\cos\Delta m t}\, dt = \frac{\sin\Delta m t}{\Delta m} e^{-\gamma t}\Big|_0^\infty - \int_0^\infty \frac{\sin\Delta m t}{\Delta m} (-\gamma) e^{-\gamma t} dt$$

$$= \frac{\gamma}{\Delta m} \int_0^\infty \underset{u'}{\sin\Delta m t}\, \underset{v}{e^{-\gamma t}} dt$$

$$= \frac{\gamma}{\Delta m} \left[-\frac{\cos\Delta m t}{\Delta m} e^{-\gamma t} \Big|_0^\infty - \int_0^\infty \left(-\frac{\cos\Delta m t}{\Delta m} \right) (-\gamma) e^{-\gamma t} dt \right]$$

$$\int_0^\infty e^{-\gamma t} \cos\Delta m t\, dt = \frac{\gamma}{\Delta m} \left[\frac{1}{\Delta m} - \frac{\gamma}{\Delta m} \int_0^\infty e^{-\gamma t} \cos\Delta m t\, dt \right]$$

$$\boxed{\int_0^\infty e^{-\gamma t} \cos\Delta m t\, dt = \frac{\gamma/\Delta m^2}{1 + \frac{\gamma^2}{\Delta m^2}}}$$

$$\int_0^\infty |S_+ - S_-|^2\,dt = \frac{2\gamma}{\gamma^2 - \left(\frac{\Delta\gamma}{2}\right)^2} - \frac{2\gamma/(\Delta m)^2}{1+(\gamma/\Delta m)^2}$$

$$|S_+ + S_-|^2 = \left| e^{-imt}\, e^{-\gamma t/2}\left(e^{-i\Delta m t/2}\, e^{-\Delta\gamma t/4} + e^{i\Delta m t/2}\, e^{\Delta\gamma t/4}\right)\right|^2$$

$$= e^{-\gamma t}\left(e^{-i\Delta m t/2}\, e^{-\Delta\gamma t/4} + e^{i\Delta m t/2}\, e^{\Delta\gamma t/4}\right)\cdot$$

$$\cdot\left(e^{i\Delta m t/2}\, e^{-\Delta\gamma t/4} + e^{-i\Delta m t/2}\, e^{\Delta\gamma t/4}\right)$$

$$= e^{-\gamma t}\left(e^{-\Delta\gamma t/2} + e^{-i\Delta m t} + e^{i\Delta m t} + e^{\Delta\gamma t/2}\right)$$

$$\Rightarrow \int_0^\infty |S_+ + S_-|^2 dt = \int_0^\infty e^{-(\gamma+\frac{\Delta\gamma}{2})t}\,dt + \int_0^\infty e^{-(\gamma-\frac{\Delta\gamma}{2})t}\,dt$$

$$+ 2\int_0^\infty e^{-\gamma t}\cos\Delta m t\, dt$$

$$= \frac{1}{\gamma+\frac{\Delta\gamma}{2}} + \frac{1}{\gamma-\frac{\Delta\gamma}{2}} + \frac{2\gamma/(\Delta m)^2}{1+(\gamma/\Delta m)^2}$$

$$\int_0^\infty |S_+ + S_-|^2 dt = \frac{2\gamma}{\gamma^2-\left(\frac{\Delta\gamma}{2}\right)^2} + \frac{2\gamma/(\Delta m)^2}{1+(\gamma/\Delta m)^2}$$

$$\chi = \frac{\cancel{\frac{1}{4}}\left|\frac{1+\varepsilon}{1-\varepsilon}\right|^2\left[\frac{2\gamma}{\gamma^2-\left(\frac{\Delta\gamma}{2}\right)^2} - \frac{2\gamma/(\Delta m)^2}{1+(\gamma/\Delta m)^2}\right]}{\cancel{\frac{1}{4}}\left|\frac{1+\varepsilon}{1-\varepsilon}\right|^2\left[\frac{2\gamma}{\gamma^2-\left(\frac{\Delta\gamma}{2}\right)^2} - \frac{2\gamma/(\Delta m)^2}{1+(\gamma/\Delta m)^2}\right] + \cancel{\frac{1}{4}}\left[\frac{2\gamma}{\gamma^2-\left(\frac{\Delta\gamma}{2}\right)^2} + \frac{2\gamma/(\Delta m)^2}{1+(\gamma/\Delta m)^2}\right]}$$

$$x = \frac{\Delta m}{\gamma} \;;\quad y = \frac{\Delta\gamma}{2\gamma} \;;\quad \alpha = \frac{Re(\varepsilon)}{1+|\varepsilon|^2}$$

$$\left|\frac{1+\varepsilon}{1-\varepsilon}\right|^2 = \frac{(1+\varepsilon)(1+\varepsilon^*)}{(1-\varepsilon)(1-\varepsilon^*)} = \frac{1+2\operatorname{Re}\varepsilon+|\varepsilon|^2}{1-2\operatorname{Re}\varepsilon+|\varepsilon|^2}$$

$$= \frac{1+\dfrac{2\operatorname{Re}\varepsilon}{1+|\varepsilon|^2}}{1-\dfrac{2\operatorname{Re}\varepsilon}{1+|\varepsilon|^2}} = \frac{1+2\alpha}{1-2\alpha}$$

$$\left|\frac{1-\varepsilon}{1+\varepsilon}\right|^2 = \frac{1-2\alpha}{1+2\alpha}$$

$$\Rightarrow \chi = \frac{\dfrac{(1+2\alpha)}{(1-2\alpha)}\left[\dfrac{2}{1-y^2} - \dfrac{2\left(\frac{1}{x}\right)^2}{1+\left(\frac{1}{x}\right)^2}\right]}{\dfrac{(1+2\alpha)}{(1-2\alpha)}\left[\dfrac{2}{1-y^2} - \dfrac{2\left(\frac{1}{x}\right)^2}{1+\left(\frac{1}{x}\right)^2}\right] + \left[\dfrac{2}{1-y^2} + \dfrac{2\left(\frac{1}{x}\right)^2}{1+\left(\frac{1}{x}\right)^2}\right]}$$

$$\chi = \frac{\dfrac{(1+2\alpha)}{(1-2\alpha)}\left[\dfrac{2}{1-y^2} - \dfrac{2}{x^2+1}\right]}{\dfrac{(1+2\alpha)}{(1-2\alpha)}\left[\dfrac{2}{1-y^2} - \dfrac{2}{x^2+1}\right] + \left[\dfrac{2}{1-y^2} + \dfrac{2}{x^2+1}\right]}$$

$$\chi = \frac{\dfrac{(1+2\alpha)}{(1-2\alpha)}\left[2x^2+2y^2\right]}{\dfrac{(1+2\alpha)}{(1-2\alpha)}\left[2x^2+2y^2\right] + \left[2x^2+4-2y^2\right]}$$

$$\chi = \frac{(1+2\alpha)(x^2+y^2)}{\cancel{x^2}+\cancel{y^2}+2\alpha\cancel{x^2}+2\cancel{\alpha y^2}+\cancel{x^2}-2\cancel{x^2}\alpha+\cancel{2}-\cancel{4\alpha}-\cancel{y^2}+2\cancel{\alpha y^2}}$$

$$\boxed{\chi = \frac{(1+2\alpha)(x^2+y^2)}{(2x^2+4\alpha y^2+2-4\alpha)} = \frac{(\alpha+\frac{1}{2})(x^2+y^2)}{(x^2+1-2\alpha(1-y^2))}}$$

Probability of $B^0 \to \bar{B}^0$

$$\bar{\chi} = \frac{\int_0^\infty \cancel{\frac{1}{4}} \left|\frac{1-\epsilon}{1+\epsilon}\right|^2 |S_+ - S_-|^2 \, dt}{\int_0^\infty \cancel{\frac{1}{4}} \left|\frac{1-\epsilon}{1+\epsilon}\right|^2 |S_+ - S_-|^2 dt + \int_0^\infty \cancel{\frac{1}{4}} |S_+ + S_-|^2 dt}$$

$$\bar{\chi} = \frac{\left(\frac{\cancel{2}\gamma}{\gamma^2 - (\Delta\gamma/2)^2} - \frac{\cancel{2}\gamma/(\Delta m)^2}{1+(\gamma/\Delta m)^2}\right)\left(\frac{1-2\alpha}{1+2\alpha}\right)\gamma}{\gamma\left\{\left(\frac{\cancel{2}\gamma}{\gamma^2 - (\Delta\gamma/2)^2} - \frac{\cancel{2}\gamma/(\Delta m)^2}{1+(\gamma/\Delta m)^2}\right)\frac{(1-2\alpha)}{(1+2\alpha)} + \left(\frac{\cancel{2}\gamma}{\gamma^2 - (\Delta\gamma/2)^2} + \frac{\cancel{2}\gamma/(\Delta m)^2}{1+(\gamma/\Delta m)^2}\right)\right\}}$$

$$\bar{\chi} = \frac{\left(\frac{1}{1-y^2} - \frac{(\frac{1}{x})^2}{1+(\frac{1}{x})^2}\right)\frac{(1-2\alpha)}{(1+2\alpha)}}{\left(\frac{1}{1-y^2} - \frac{(\frac{1}{x})^2}{1+(\frac{1}{x})^2}\right)\frac{(1-2\alpha)}{(1+2\alpha)} + \left(\frac{1}{1-y^2} + \frac{(\frac{1}{x})^2}{1+(\frac{1}{x})^2}\right)}$$

$$\bar{\chi} = \frac{\left(\frac{1}{1-y^2} - \frac{1}{x^2+1}\right)\frac{(1-2\alpha)}{(1+2\alpha)}}{\left(\frac{1}{1-y^2} - \frac{1}{x^2+1}\right)\frac{(1-2\alpha)}{(1+2\alpha)} + \left(\frac{1}{1-y^2} + \frac{1}{x^2+1}\right)}$$

$$\bar{\chi} = \frac{(x^2+y^2)(1-2\alpha)}{(x^2+y^2)(1-2\alpha) + (x^2+2-y^2)(1+2\alpha)}$$

$$\bar{\chi} = \frac{(x^2+y^2)(1-2\alpha)}{\cancel{x^2} - 2\alpha\cancel{x^2} + \cancel{y^2} - 2\alpha y^2 + \cancel{x^2} + 2\alpha\cancel{x^2} + \cancel{2} + 4\cancel{\alpha} - \cancel{y^2} - 2\cancel{\alpha} y^2}$$

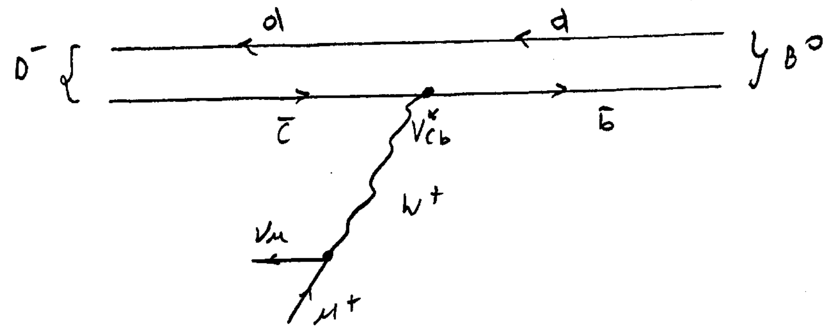
$$\bar{\chi} = \frac{(x^2+y^2)(\frac{1}{2}-\alpha)}{(x^2+1-2\alpha y^2+2\alpha)} = \frac{(x^2+y^2)(\frac{1}{2}-\alpha)}{(x^2+1-2\alpha(y^2-1))}$$

$$\boxed{\bar{\chi} = \frac{(x^2+y^2)(\frac{1}{2}-\alpha)}{(x^2+1+2\alpha(1-y^2))}}$$

Dimuon charge asymmetry:

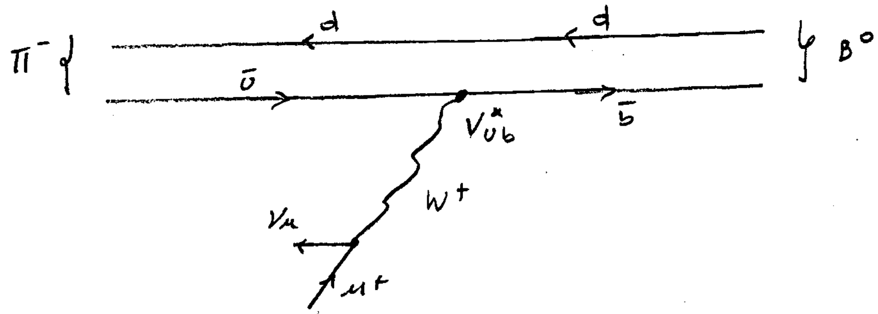
$$A_d \equiv \frac{N_{++} - N_{--}}{N_{++} + N_{--}} = \frac{\chi_d(1-\bar{\chi}_d) - \bar{\chi}_d(1-\chi_d)}{\chi_d(1-\bar{\chi}_d) + \bar{\chi}_d(1-\chi_d)}$$

$B^0 \to D^- \mu^+ \nu_\mu$ $\quad d\bar{b} \to d\bar{c}\,\mu^+\nu_\mu$ $\quad \leftarrow$ time



$B^0 \to \pi^- \mu^+ \nu_\mu$

$d\bar{b} \to d\bar{u}\,\mu^+\nu_\mu$



$A_d = ?$

$$\chi_d(1-\bar{\chi}_d) = \frac{(\alpha_d + \frac{1}{2})(x^2+y^2)}{((x^2+1) - 2\alpha_d(1-y^2))}\left[1 - \frac{(x^2+y^2)(\frac{1}{2}-\alpha_d)}{((x^2+1)+2\alpha_d(1-y^2))}\right]$$

neglecting y^2

$$\chi_d(1-\bar{\chi}_d) \simeq \frac{(\alpha_d + \frac{1}{2})x^2}{(x^2+1-2\alpha_d)}\left(1 - \frac{x^2(\frac{1}{2}-\alpha_d)}{(x^2+1+2\alpha_d)}\right)$$

$$= \frac{(\alpha_d + \frac{1}{2})x^2\left(\frac{x^2}{2} + 1 + 2\alpha_d + x^2\alpha_d\right)}{(x^2+1-2\alpha_d)(x^2+1+2\alpha_d)}$$

$$\bar{X}_d\,(1-X_d) \approx \frac{X^2(\frac{1}{2}-\alpha_d)}{(X^2+1+2\alpha_d)}\left(1-\frac{X^2(\alpha_d+\frac{1}{2})}{(X^2+1-2\alpha_d)}\right)$$

$$= \frac{X^2(\frac{1}{2}-\alpha_d)\,(\frac{X^2}{2}+1-2\alpha_d-X^2\alpha_d)}{(X^2+1+2\alpha_d)(X^2+1-2\alpha_d)}$$

$$\Rightarrow X_d\,(1-\bar{X}_d) - \bar{X}_d\,(1-X_d) = \frac{\left[(\alpha_d+\frac{1}{2})X^2(\frac{X^2}{2}+1+2\alpha_d+X^2\alpha_d) - (\frac{1}{2}-\alpha_d)X^2(\frac{X^2}{2}+1-2\alpha_d-X^2\alpha_d)\right]}{(X^2+1+2\alpha_d)\,(X^2+1-2\alpha_d)}$$

neglecting $\underline{\underline{\alpha_d X^4}}^{\downarrow \text{NO}}$, $\alpha_d^2 X^4$, $\alpha_d^2 X^2$

$(\alpha_d \approx -0.134;\ X_d \approx 0.73)$

$$X_d\,(1-\bar{X}_d) - \bar{X}_d(1-X_d) \approx [\,(\cancel{\alpha_d X^2} + \tfrac{1}{4}\cancel{X^4} + \tfrac{1}{2}\cancel{X^2} + \alpha_d X^2 + \alpha_d X^4$$

$$-\,(\tfrac{1}{4}X^4 + \tfrac{1}{2}\cancel{X^2} - \cancel{\alpha_d X^2} - \alpha_d X^2 - \alpha_d X^4)\,]/()()$$

$$= (4\alpha_d X^2 + 2\alpha_d X^4)/(\ )(\ )$$

$$X_d\,(1-\bar{X}_d) + \bar{X}_d\,(1-X_d) \approx (X^2 + \tfrac{1}{2}X^4)\,/\,(X^2+1+2\alpha_d)(X^2+1-2\alpha_d)$$

$$\Rightarrow \boxed{A_d \approx \frac{4\alpha_d \cancel{X^2}\cancel{(1+\frac{1}{2}X_d^2)}}{\cancel{(1+\frac{1}{2}X_d^2)}\cancel{X^2}}}$$

$$\Rightarrow \boxed{A_d \approx 4\alpha_d}$$

C. Marin

1

$$\Delta M = 2\,\mathrm{Re}\left\{ (M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*) \right\}^{1/2}$$

$$\Delta \Gamma = -4\,\mathrm{Im}\left\{ (M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*) \right\}^{1/2}$$

$$2\left[(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*) \right]^{1/2} = \Delta M - \tfrac{i}{2}\Delta\Gamma$$

$$4(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*) = (\Delta M)^2 - i\Delta M\Delta\Gamma - \tfrac{1}{4}(\Delta\Gamma)^2$$

$$\Rightarrow 4|M_{12}|^2 - 2i\,M_{12}\Gamma_{12}^* - 2i\,\Gamma_{12}M_{12}^* - |\Gamma_{12}|^2 = (\Delta M)^2 - i\Delta M\Delta\Gamma - \tfrac{1}{4}(\Delta\Gamma)^2$$

$$M_{12}\Gamma_{12}^* + \Gamma_{12}M_{12}^* = I$$

$$M_{12} = a + ib$$

$$\Gamma_{12} = c + id \quad \Rightarrow \quad I = (a+ib)(c-id) + (c+id)(a-ib)$$

$$I = ac - \cancel{iad} + \cancel{ibc} + bd + ac - \cancel{ibc} + \cancel{iad} + bd$$

$$I = 2ac + 2bd = \text{real number.}$$

$$M_{12}\Gamma_{12}^* = (a+ib)(c-id) = ac - iad + ibc + bd$$

$$= (ac+bd) + i(bc - ad)$$

$$\Rightarrow \mathrm{Re}(M_{12}\Gamma_{12}^*) = ac + bd$$

$$\Rightarrow \boxed{M_{12}\Gamma_{12}^* + \Gamma_{12}M_{12}^* = 2\,\mathrm{Re}(M_{12}\Gamma_{12}^*)} \qquad (1)$$

$$\Rightarrow 4|M_{12}|^2 - |\Gamma_{12}|^2 - 4i\,\mathrm{Re}(M_{12}\Gamma_{12}^*) = (\Delta M)^2 - i\Delta M\Delta\Gamma - \tfrac{1}{4}(\Delta\Gamma)^2$$

Then:

$$\boxed{4|M_{12}|^2 - |\Gamma_{12}|^2 = (\Delta M)^2 - \tfrac{1}{4}(\Delta\Gamma)^2} \qquad (2)$$

$$\boxed{\Delta M\Delta\Gamma = 4\,\mathrm{Re}(M_{12}\Gamma_{12}^*)} = 4|M_{12}||\Gamma_{12}|\cos\phi. \qquad (3)$$

$$M_{12} = |M_{12}|\,e^{i\phi_1}; \quad \Gamma_{12} = |\Gamma_{12}|\,e^{i\phi_2} \Rightarrow M_{12}\Gamma_{12}^* = |M_{12}||\Gamma_{12}|\,e^{i\overbrace{(\phi_1-\phi_2)}^{\phi}}$$

$$\Rightarrow \mathrm{Re}\,(M_{12}\Gamma_{12}^*) = |M_{12}||\Gamma_{12}|\cos\phi$$

$$\frac{1+\varepsilon}{1-\varepsilon} = \left(\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}\right)^{1/2} = X$$

$$\alpha \equiv \frac{\mathrm{Re}\,\varepsilon}{1+|\varepsilon|^2} \qquad (4)$$

$$\frac{1-2\alpha}{1+2\alpha} = \frac{1-\frac{2\,\mathrm{Re}(\varepsilon)}{|\varepsilon|^2+1}}{1+\frac{2\,\mathrm{Re}(\varepsilon)}{1+|\varepsilon|^2}} = \frac{|\varepsilon|^2+1-2\,\mathrm{Re}(\varepsilon)}{|\varepsilon|^2+1+2\,\mathrm{Re}(\varepsilon)}$$

$$1+\varepsilon = X - \varepsilon X$$

$$\varepsilon = \frac{X-1}{1+X}$$

$$\frac{(1+\varepsilon)(1+\varepsilon^*)}{(1-\varepsilon)(1-\varepsilon^*)} = |X|^2$$

$$\frac{1+2\,\mathrm{Re}\,\varepsilon + |\varepsilon|^2}{1-2\,\mathrm{Re}\,\varepsilon + |\varepsilon|^2} = |X|^2$$

$$\Rightarrow \frac{1-2\alpha}{1+2\alpha} = \frac{1}{|X|^2} = \left(\frac{(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)(M_{12} + \frac{i}{2}\Gamma_{12})}{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* + \frac{i}{2}\Gamma_{12}^*)}\right)^{1/2}$$

$$= \left(\frac{|M_{12}|^2 + \frac{i}{2}M_{12}^*\Gamma_{12} - \frac{i}{2}\Gamma_{12}^*M_{12} + \frac{1}{4}|\Gamma_{12}|^2}{|M_{12}|^2 + \frac{i}{2}M_{12}\Gamma_{12}^* - \frac{i}{2}\Gamma_{12}M_{12}^* + \frac{1}{4}|\Gamma_{12}|^2}\right)^{1/2}$$

$$M_{12}\Gamma_{12}^* - \Gamma_{12}M_{12}^* = (a+ib)(c-id) - (c+id)(a-ib)$$

$$= \cancel{ac} - iad + ibc + \cancel{bd} - \cancel{ac} + ibc - i\cancel{ad} - \cancel{bd}$$

$$= 2i(bc - ad) = 2i\,\mathrm{Im}(M_{12}\Gamma_{12}^*)$$

$$M_{12}\Gamma_{12}^* = (a+ib)(c-id) = ac - iad + ibc + bd$$
$$= (ac+bd) + i(bc-ad)$$

$$\Rightarrow \quad (bc-ad) = Im(M_{12}\Gamma_{12}^*)$$

$$\Rightarrow \boxed{M_{12}\Gamma_{12}^* - \Gamma_{12}M_{12}^* = 2i\, Im(M_{12}\Gamma_{12}^*)} \qquad (5)$$

so:

$$\frac{1-2\alpha}{1+2\alpha} = \left(\frac{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + \frac{i}{2}(-2i)\, Im(M_{12}\Gamma_{12}^*)}{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + \frac{i}{2}(2i)\, Im(M_{12}\Gamma_{12}^*)} \right)^{1/2}$$

$$= \left(\frac{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + Im(M_{12}\Gamma_{12}^*)}{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 - Im(M_{12}\Gamma_{12}^*)} \right)^{1/2}$$

$$\frac{1-2\alpha}{1+2\alpha} = \frac{\left(|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + Im(M_{12}\Gamma_{12}^*)\right)^{1/2}\left(|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 - Im(M_{12}\Gamma_{12}^*)\right)^{1/2}}{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 - Im(M_{12}\Gamma_{12}^*)} = \frac{1}{|x|^2} \qquad (*)$$

//

$$\Delta M = 2Re\left\{(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*)\right\}^{1/2}$$

$$= 2Re\left\{ |M|^2 - \tfrac{i}{2}M_{12}\Gamma_{12}^* - \tfrac{i}{2}\Gamma_{12}M_{12}^* - \tfrac{1}{4}|\Gamma_{12}|^2 \right\}^{1/2}$$

$$M_{12}\Gamma_{12}^* + \Gamma_{12}M_{12}^* = (a+ib)(c-id) + (c+id)(a-ib)$$
$$= \cancel{ac} - i\cancel{ad} + ibc + \cancel{bd} + \cancel{ac} - i\cancel{bc} + i\cancel{ad} + \cancel{bd}$$
$$= 2(ac+bd) = \text{real } \#$$

$$\Rightarrow \boxed{\Delta M = 2Re\left\{ (|M|^2 - \tfrac{1}{4}|\Gamma_{12}|^2) - i\, Re(M_{12}\Gamma_{12}^*) \right\}^{1/2}} \qquad (6)$$

//

$$2\left[(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*)\right]^{1/2} = \Delta M - \tfrac{i}{2}\Delta\Gamma$$
$$2\left[(M_{12}^* + \tfrac{i}{2}\Gamma_{12}^*)(M_{12} + \tfrac{i}{2}\Gamma_{12})\right]^{1/2} = \Delta M + \tfrac{i}{2}\Delta\Gamma \qquad (7)$$

multiplying:

$$\Rightarrow \left| (\Delta M)^2 + \tfrac{1}{4}(\Delta\Gamma)^2 = 4\left(|M_{12}|^2 + \tfrac{i}{2}M_{12}\Gamma_{12}^* - \tfrac{i}{2}\Gamma_{12}M_{12}^* + \tfrac{1}{4}|\Gamma_{12}|^2 \right)^{1/2}. \right.$$

$$\bullet \left(|M_{12}|^2 + \frac{i}{2} M_{12}^* \Gamma_{12} - \frac{i}{2} \Gamma_{12}^* M_{12} + \frac{1}{4} |\Gamma_{12}|^2 \right)^{1/2}$$

$$= 4 \left(|M_{12}|^2 + \frac{1}{4} |\Gamma_{12}|^2 - \mathrm{Im}\,(M_{12} \Gamma_{12}^*) \right)^{1/2} \left(|M_{12}|^2 + \frac{1}{4} |\Gamma_{12}|^2 + \mathrm{Im}\,(M_{12} \Gamma_{12}^*) \right)^{1/2}$$

then

$$\left(|M_{12}|^2 + \frac{1}{4} |\Gamma_{12}|^2 - \mathrm{Im}\,(M_{12} \Gamma_{12}^*) \right)^{1/2} \left(|M_{12}|^2 + \frac{1}{4} |\Gamma_{12}|^2 + \mathrm{Im}\,(M_{12} \Gamma_{12}^*) \right)^{1/2}$$

$$= \frac{1}{4} \left[(\Delta M)^2 + \frac{1}{4} (\Delta \Gamma)^2 \right]$$

Replacing in (*) we have:

$$\boxed{\frac{1-2\alpha}{1+2\alpha} = \frac{(\Delta M)^2 + \frac{1}{4} (\Delta \Gamma)^2}{4 |M_{12}|^2 + |\Gamma_{12}|^2 - 4\, \mathrm{Im}\,(M_{12} \Gamma_{12}^*)}} \qquad (8)$$

or:

$$\frac{1-2\alpha}{1+2\alpha} = \frac{|M_{12}|^2 + \frac{1}{4} |\Gamma_{12}|^2 + \mathrm{Im}\,(M_{12} \Gamma_{12}^*)}{\frac{1}{4} \left[(\Delta M)^2 + \frac{1}{4} (\Delta \Gamma)^2 \right]}$$

$$\boxed{\frac{1-2\alpha}{1+2\alpha} = \frac{4 |M_{12}|^2 + |\Gamma_{12}|^2 + 4\, \mathrm{Im}\,(M_{12} \Gamma_{12}^*)}{(\Delta M)^2 + \frac{1}{4} (\Delta \Gamma)^2}} = \frac{1}{|X|^2} \qquad (9)$$

$$\frac{(1+2\alpha)}{(1-2\alpha)} \frac{(1+2\alpha)}{(1+2\alpha)} = \frac{4 |M_{12}|^2 + |\Gamma_{12}|^2 - 4\, \mathrm{Im}\,(M_{12} \Gamma_{12}^*)}{(\Delta M)^2 + \frac{1}{4} (\Delta \Gamma)^2}$$

$$\frac{1 + 4\alpha + 4\alpha^2}{1 - 4\alpha^2} = \quad ''$$

Introducing

$$a \equiv \frac{4 |M_{12}|^2 + |\Gamma_{12}|^2}{(\Delta M)^2 + \frac{1}{4} (\Delta \Gamma)^2} \qquad (10)$$

$$b \equiv - \frac{4\, \mathrm{Im}\,(M_{12} \Gamma_{12}^*)}{(\Delta M)^2 + \frac{1}{4} (\Delta \Gamma)^2} \qquad (11)$$

We have:

$$\boxed{a + b = \frac{1+4\alpha+4\alpha^2}{1-4\alpha^2}} \qquad (12)$$

$$\frac{1+2\alpha}{1-2\alpha} = \frac{4\,|M_{12}|^2 + |\Gamma_{12}|^2 - 4\,\mathrm{Im}\,(M_{12}\Gamma_{12}^*)}{(\Delta M)^2 + \frac{1}{4}(\Delta\Gamma)^2}$$

$$\frac{1-2\alpha}{1+2\alpha} = \frac{4\,|M_{12}|^2 + |\Gamma_{12}|^2 + 4\,\mathrm{Im}\,(M_{12}\Gamma_{12}^*)}{(\Delta M)^2 + \frac{1}{4}(\Delta\Gamma)^2}$$

$$\Rightarrow \frac{1+2\alpha}{1-2\alpha} + \frac{1-2\alpha}{1+2\alpha} = \frac{2\left[4\,|M_{12}|^2 + |\Gamma_{12}|^2\right]}{(\Delta M)^2 + \frac{1}{4}(\Delta\Gamma)^2} = 2a$$

$$\Rightarrow \frac{(1+2\alpha)^2 + (1-2\alpha)^2}{1-4\alpha^2} = 2a$$

$$\frac{1+\cancel{4\alpha}+4\alpha^2+1-\cancel{4\alpha}+4\alpha^2}{1-4\alpha^2} = 2a$$

$$\frac{\cancel{2}(1+4\alpha^2)}{(1-4\alpha^2)} = \cancel{2}a$$

$$\therefore \quad \boxed{a = \frac{1+4\alpha^2}{1-4\alpha^2}} \qquad (13)$$

$$\Rightarrow b = \frac{1+4\alpha+4\alpha^2}{1-4\alpha^2} - \frac{(1+4\alpha^2)}{1-4\alpha^2} = \frac{4\alpha}{1-4\alpha^2}$$

$$\boxed{b = \frac{4\alpha}{1-4\alpha^2}} \qquad (14)$$

$$\Rightarrow \boxed{\frac{b}{a} = \frac{-4\,\mathrm{Im}\,(M_{12}\Gamma_{12}^*)}{4\,|M_{12}|^2 + |\Gamma_{12}|^2} = \frac{-4\,|M_{12}|\,|\Gamma_{12}|\,\sin\phi}{4\,|M_{12}|^2 + |\Gamma_{12}|^2}} \qquad (15)$$

because:

$$M_{12} = |M_{12}|\, e^{-i\phi_1}$$

$$\Gamma_{12} = |\Gamma_{12}|\, e^{-i\phi_2}$$

$$M_{12}\,\Gamma_{12}^{*} = |M_{12}|\,|\Gamma_{12}|\, e^{i\overbrace{(\phi_1-\phi_2)}^{\phi}}$$

$$\boxed{I_m\,(M_{12}\,\Gamma_{12}^{*}) = |M_{12}|\,|\Gamma_{12}|\,\sin\phi} \qquad (16)$$

$$\frac{\alpha}{1+4\alpha^2} = \frac{\dfrac{\mathrm{Re}\,\varepsilon}{1+|\varepsilon|^2}}{1+\dfrac{4(\mathrm{Re}\,\varepsilon)^2}{(1+|\varepsilon|^2)^2}} = \frac{\mathrm{Re}\,\varepsilon\,(1+|\varepsilon|^2)}{1+2|\varepsilon|^2+|\varepsilon|^4+4(\mathrm{Re}\,\varepsilon)^2}$$

$$\frac{1+\varepsilon}{1-\varepsilon} = x \Rightarrow 1+\varepsilon = x - x\varepsilon \Rightarrow \varepsilon = \frac{x-1}{1+x}$$

$$\frac{(1+\varepsilon)(1+\varepsilon^*)}{(1-\varepsilon)(1-\varepsilon^*)} = |x|^2$$

$$\frac{1+2\mathrm{Re}\,\varepsilon+|\varepsilon|^2}{1-2\mathrm{Re}\,\varepsilon+|\varepsilon|^2} = |x|^2$$

$$1+|\varepsilon|^2 = \beta$$

$$\frac{\beta+2\mathrm{Re}\,\varepsilon}{\beta-2\mathrm{Re}\,\varepsilon} = |x|^2$$

$$\beta+2\mathrm{Re}\,\varepsilon = \beta|x|^2 - 2\mathrm{Re}\,\varepsilon\,|x|^2$$

$$\boxed{\beta = \frac{-2\mathrm{Re}\,\varepsilon\,(1+|x|^2)}{(1-|x|^2)} = 1+|\varepsilon|^2} \qquad (17)$$

$$\Rightarrow \frac{\alpha}{1+4\alpha^2} = \frac{\cancel{\mathrm{Re}\,\varepsilon}\,(-2\cancel{\mathrm{Re}\,\varepsilon})\dfrac{(1+|x|^2)}{(1-|x|^2)}}{\dfrac{4\cancel{(\mathrm{Re}\,\varepsilon)^2}(1+|x|^2)^2}{(1-|x|^2)^2} + 4\cancel{(\mathrm{Re}\,\varepsilon)^2}}$$

$$\frac{\alpha}{1+4\alpha^2} = \frac{-2(1+|x|^2)(1-|x|^2)}{4(1+|x|^2)^2+4(1-|x|^2)^2}$$

$$= \frac{-(1-|x|^4)}{2[2+2|x|^4]} = -\frac{(1-|x|^4)}{4(1+|x|^4)}$$

$$|x|^4 = \frac{\left(|M_{12}|^2+\cancel{\tfrac14|\Gamma_{12}|^2}-\mathrm{Im}(M_{12}\Gamma_{12}^*)\right)^2}{\left(|M_{12}|^2+\tfrac14|\Gamma_{12}|^2+\mathrm{Im}(M_{12}\Gamma_{12}^*)\right)\left(|M_{12}|^2\cancel{+\tfrac14|\Gamma_{12}|^2}-\mathrm{Im}(M_{12}\Gamma_{12}^*)\right)}$$

$$|X|^4 = \frac{\left(|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 - \mathrm{Im}(M_{12}\Gamma_{12}^*)\right)}{\left(|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + \mathrm{Im}(M_{12}\Gamma_{12}^*)\right)}$$ (see *)

$$1 - |X|^4 = \frac{2\,\mathrm{Im}(M_{12}\Gamma_{12}^*)}{\left(|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + \mathrm{Im}(M_{12}\Gamma_{12}^*)\right)}$$

$$1 + |X|^4 = \frac{2|M_{12}|^2 + \frac{1}{2}|\Gamma_{12}|^2}{\left(|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + \mathrm{Im}(M_{12}\Gamma_{12}^*)\right)}$$

$$\Rightarrow \frac{\alpha}{1+4\alpha^2} = \frac{-2\,\mathrm{Im}(M_{12}\Gamma_{12}^*)}{4\left(2|M_{12}|^2 + \frac{1}{2}|\Gamma_{12}|^2\right)}$$

$$= -\frac{\mathrm{Im}(M_{12}\Gamma_{12}^*)}{4|M_{12}|^2 + |\Gamma_{12}|^2}$$

$$= \frac{-\mathrm{Im}\left(\frac{\cancel{M_{12}}\Gamma_{12}^*}{\cancel{M_{12}}M_{12}^*}\right)}{4 + \left|\frac{\Gamma_{12}}{M_{12}}\right|^2}$$

$$\boxed{\frac{\alpha}{1+4\alpha^2} = \frac{-\mathrm{Im}\left(\frac{\Gamma_{12}^*}{M_{12}^*}\right)}{4 + \left|\frac{\Gamma_{12}}{M_{12}}\right|^2} = \frac{\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)}{4 + \left|\frac{\Gamma_{12}}{M_{12}}\right|^2}} \quad (18)$$

(- in the other convention)

Using (2) and (9)

$$\frac{1-2\alpha}{1+2\alpha} = \frac{4|M_{12}|^2 + 4|M_{12}|^2 - (\Delta M)^2 + \frac{1}{4}(\Delta\Gamma)^2 + 4\,\mathrm{Im}(M_{12}\Gamma_{12}^*)}{(\Delta M)^2 + \frac{1}{4}(\Delta\Gamma)^2}$$

$$\frac{4\alpha}{1-4\alpha^2} = \frac{-4\,\mathrm{Im}(M_{12}\Gamma_{12}^*)}{(\Delta M)^2 + \frac{1}{4}(\Delta\Gamma)^2} \qquad (19)$$

$$\Rightarrow \frac{1-2\alpha}{1+2\alpha} = \frac{8|M_{12}|^2 - (\Delta M)^2 + \frac{1}{4}(\Delta\Gamma)^2 - \frac{4\alpha}{1-4\alpha^2}\left[(\Delta M)^2 + \frac{1}{4}(\Delta\Gamma)^2\right]}{(\Delta M)^2 + \frac{1}{4}(\Delta\Gamma)^2}$$

$$\Rightarrow (\Delta M)^2(1-2\alpha) + \frac{(1-2\alpha)}{4}(\Delta\Gamma)^2 = 8(1+2\alpha)|M_{12}|^2 - (1+2\alpha)(\Delta M)^2$$

$$+ \frac{1}{4}(1+2\alpha)(\Delta\Gamma)^2 - \frac{4\alpha}{(1-2\alpha)}(\Delta M)^2 - \frac{4\alpha}{(1-2\alpha)}\cdot\frac{1}{4}(\Delta\Gamma)^2$$

$$\Rightarrow |M_{12}|^2 = \Big[(\Delta M)^2\left[1-\cancel{2\alpha} + 1 + \cancel{2\alpha} + \frac{4\alpha}{1-2\alpha}\right]$$

$$+ (\Delta\Gamma)^2\left[\frac{(1-2\alpha)}{4} - \frac{1}{4}(1+2\alpha) + \frac{\alpha}{1-2\alpha}\right]\Big] / 8(1+2\alpha)$$

$$\Rightarrow |M_{12}|^2 = \frac{(\Delta M)^2\, 2\left[1 + \frac{2\alpha}{1-2\alpha}\right]}{8(1+2\alpha)} + (\Delta\Gamma)^2\left[-\alpha + \frac{\alpha}{1-2\alpha}\right]\cdot\frac{1}{8(1+2\alpha)}$$

$$\boxed{|M_{12}|^2 = \frac{1}{4}(\Delta M)^2\cdot\frac{1}{(1-4\alpha^2)} + \frac{1}{4}(\Delta\Gamma)^2\cdot\frac{\alpha^2}{(1-4\alpha^2)}} \qquad (20)$$

$$|M_{12}|^2 = (\Delta M)^2\frac{1}{1-4\alpha^2} + (\Delta\Gamma)^2\frac{\alpha^2}{1-4\alpha^2} - \cancel{\tfrac{1}{4}(\Delta M)^2} + \frac{1}{4}(\Delta\Gamma)^2$$

$$|\Gamma_{12}|^2 = (\Delta M)^2\left(\frac{1}{1-4\alpha^2} - 1\right) + (\Delta\Gamma)^2\left(\frac{\alpha^2}{1-4\alpha^2} + \frac{1}{4}\right)$$

$$\boxed{|\Gamma_{12}|^2 = (\Delta M)^2 \frac{4\alpha^2}{(1-4\alpha^2)} + (\Delta\Gamma)^2 \frac{1}{4}\frac{1}{(1-4\alpha^2)}} \qquad (21)$$

$$\frac{1+\varepsilon}{1-\varepsilon} = \left(\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}\right)^{1/2}$$

$$= \frac{(M_{12} - \frac{i}{2}\Gamma_{12})}{\left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)\right]^{1/2}}$$

$$\Delta M = 2\,\mathrm{Re}\left[(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*)\right]^{1/2}$$

$$\Delta\Gamma = -4\,\mathrm{Im}\left[(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*)\right]^{1/2}$$

$$\frac{\Delta M}{2} - i\frac{\Delta\Gamma}{4} = \left[(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*)\right]^{1/2}$$

$$\Rightarrow \boxed{\frac{1+\varepsilon}{1-\varepsilon} = \frac{2(M_{12} - \frac{i}{2}\Gamma_{12})}{\Delta M - i\frac{\Delta\Gamma}{2}}} \qquad (22)$$

$$\boxed{\frac{1+\varepsilon}{1-\varepsilon} = \frac{\frac{\Delta M}{2} - i\frac{\Delta\Gamma}{4}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*} = \frac{\Delta M - i\frac{\Delta\Gamma}{2}}{2(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}} \qquad (23)$$

$$\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) = \frac{\alpha}{1+4\alpha^2}\left[4 + \left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right]$$

$$= \frac{\alpha}{1+4\alpha^2}\left[4 + \frac{(\Delta M)^2 \dfrac{4\alpha^2}{(1-4\alpha^2)} + (\Delta\Gamma)^2 \dfrac{1}{4(1-4\alpha^2)}}{\dfrac{1}{4}(\Delta M)^2 \dfrac{1}{(1-4\alpha^2)} + \dfrac{1}{4}(\Delta\Gamma)^2 \dfrac{\alpha^2}{(1-4\alpha^2)}}\right]$$

$$= \frac{\alpha}{1+4\alpha^2}\left[\frac{4\left((\Delta M)^2 + \alpha^2(\Delta\Gamma)^2\right) + 16\alpha^2(\Delta M)^2 + (\Delta\Gamma)^2}{(\Delta M)^2 + \alpha^2(\Delta\Gamma)^2}\right]$$

$$= \frac{\alpha}{\cancel{(1+4\alpha^2)}}\left[\frac{4(\Delta M)^2\cancel{(1+4\alpha^2)} + (\Delta\Gamma)^2\cancel{(1+4\alpha^2)}}{(\Delta M)^2 + \alpha^2(\Delta\Gamma)^2}\right]$$

$$= \frac{\alpha\left[4(\Delta M)^2 + (\Delta\Gamma)^2\right]}{(\Delta M)^2 + \alpha^2(\Delta\Gamma)^2}$$

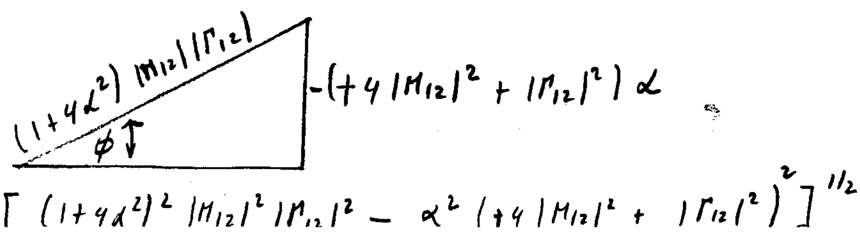
$$\boxed{\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) = 4\alpha\,\frac{\left(1 + \left(\dfrac{\Delta\Gamma}{2\Delta M}\right)^2\right)}{\left(1 + \left(\dfrac{\alpha\Delta\Gamma}{\Delta M}\right)^2\right)}} \qquad (24)$$

(− other convention)

Using (15)

$$\frac{b}{a} = \frac{4\alpha}{1+4\alpha^2} = \frac{-4|M_{12}||\Gamma_{12}|\sin\phi}{4|M_{12}|^2 + |\Gamma_{12}|^2}$$

$$\sin\phi = -\frac{\left(4|M_{12}|^2 + |\Gamma_{12}|^2\right)}{\cancel{4}|M_{12}||\Gamma_{12}|}\cdot\frac{\cancel{4}\alpha}{1+4\alpha^2}$$



$$\tan\phi = \frac{-\left(+4\,|M_{12}|^2 + |\Gamma_{12}|^2\right)\alpha}{\left[(1+4\alpha^2)^2\,|M_{12}|^2\,|\Gamma_{12}|^2 - \alpha^2\left(+4\,|M_{12}|^2 + |\Gamma_{12}|^2\right)^2\right]^{1/2}}$$

$$+4\,|M_{12}|^2 + |\Gamma_{12}|^2 = \frac{+(\Delta M)^2}{(1-4\alpha^2)} + \frac{(\Delta\Gamma)^2\alpha^2}{(1-4\alpha^2)} + \frac{(\Delta M)^2\,4\alpha^2}{(1-4\alpha^2)}$$

$$+ \frac{1}{4}\frac{(\Delta\Gamma)^2}{(1-4\alpha^2)}$$

$$= \frac{+4\,(\Delta M)^2 + 4\alpha^2(\Delta\Gamma)^2 + 16\alpha^2(\Delta M)^2 + (\Delta\Gamma)^2}{4\,(1-4\alpha^2)}$$

$$= \frac{4(\Delta M)^2\,(1+4\alpha^2) + (\Delta\Gamma)^2\,(1+4\alpha^2)}{4\,(1-4\alpha^2)}$$

$$4\,|M_{12}|^2 + |\Gamma_{12}|^2 = \frac{(1+4\alpha^2)\,\left(+4\,(\Delta M)^2 + (\Delta\Gamma)^2\right)}{4\,(1-4\alpha^2)} \qquad (25)$$

$$|M_{12}|^2\,|\Gamma_{12}|^2 = \left(\frac{1}{4}\,(\Delta M)^2\frac{1}{(1-4\alpha^2)} + \frac{1}{4}\,(\Delta\Gamma)^2\frac{\alpha^2}{(1-4\alpha^2)}\right)\cdot$$

$$\cdot\left(\frac{(\Delta M)^2}{4}\,\frac{16\alpha^2}{(1-4\alpha^2)} + (\Delta\Gamma)^2\frac{1}{4(1-4\alpha^2)}\right)$$

$$|M_{12}|^2\,|\Gamma_{12}|^2 = \frac{1}{16}\cdot\frac{1}{(1-4\alpha^2)^2}\left((\Delta M)^2 + \alpha^2(\Delta\Gamma)^2\right)\cdot\left(16\alpha^2(\Delta M)^2 + (\Delta\Gamma)^2\right) \qquad (26)$$

$$\text{then}\quad \tan\phi = \frac{-\alpha\,(1+4\alpha^2)\,\left(4(\Delta M)^2 + (\Delta\Gamma)^2\right)/\cancel{4\,(1-4\alpha^2)}}{\left[(1+4\alpha^2)^2\frac{1}{\cancel{16(1-4\alpha^2)^2}}\left(16\alpha^2(\Delta M)^4 + (\Delta M)^2(\Delta\Gamma)^2 + 16\alpha^4(\Delta M)^2(\Delta\Gamma)^2 + \alpha^2(\Delta\Gamma)^4\right) - \frac{\alpha^2\,(1+4\alpha^2)^2}{\cancel{16\,(1-4\alpha^2)^2}}\left(16(\Delta M)^4 + 8\,(\Delta M)^2(\Delta\Gamma)^2 + (\Delta\Gamma)^4\right)\right]^{1/2}}$$

$$\tan\phi = \frac{-\alpha\left(4(\Delta M)^2 + (\Delta\Gamma)^2\right)}{\left[\cancel{16\alpha^2(\Delta M)^4} + (\Delta M)^2(\Delta\Gamma)^2 + 16\alpha^4(\Delta M)^2(\Delta\Gamma)^2 + \cancel{\alpha^2(\Delta\Gamma)^4} - \cancel{16\alpha^2(\Delta M)^4} - 8\alpha^2(\Delta M)^2(\Delta\Gamma)^2 - \cancel{\alpha^2(\Delta\Gamma)^4}\right]^{1/2}}$$

$$= \frac{-\alpha\left(4(\Delta M)^2 + (\Delta\Gamma)^2\right)}{\left[(1-4\alpha^2)^2(\Delta M)^2(\Delta\Gamma)^2\right]^{1/2}}$$

$$\boxed{\tan\phi = \frac{-\alpha\left(4(\Delta M)^2 + (\Delta\Gamma)^2\right)}{(1-4\alpha^2)\,|\Delta M\,\Delta\Gamma|}} \qquad (27)$$

En el sistema $B^0 - \bar{B}^0$

$$\frac{1+\varepsilon}{1-\varepsilon} = \left(\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*} \right)^{1/2} =$$

$$\left| \frac{1+\varepsilon}{1-\varepsilon} \right| = \left(\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*} \right)^{1/4} \left(\frac{M_{12}^* + \frac{i}{2}\Gamma_{12}^*}{M_{12} + \frac{i}{2}\Gamma_{12}} \right)^{1/4}$$

$$\left| \frac{1+\varepsilon}{1-\varepsilon} \right| = \left(\frac{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + \frac{i}{2}M_{12}\Gamma_{12}^* - \frac{i}{2}\Gamma_{12}M_{12}^*}{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + \frac{i}{2}M_{12}^*\Gamma_{12} - \frac{i}{2}\Gamma_{12}^*M_{12}} \right)^{1/4}$$

$$\left| \frac{1+\varepsilon}{1-\varepsilon} \right| = \left(\frac{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + \frac{i}{2}\left(M_{12}\Gamma_{12}^* - \Gamma_{12}M_{12}^*\right)}{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + \frac{i}{2}\left(M_{12}^*\Gamma_{12} - \Gamma_{12}^*M_{12}\right)} \right)^{1/4}$$

$$M_{12} = a + ib \quad ; \quad |M_{12}|^2 = a^2 + b^2$$
$$\Gamma_{12} = c + id \quad ; \quad |\Gamma_{12}|^2 = c^2 + d^2$$

$$\begin{aligned} M_{12}\Gamma_{12}^* - \Gamma_{12}M_{12}^* &= (a+ib)(c-id) - (c+id)(a-ib) \\ &= \cancel{ac} - i\cancel{ad} + i\cancel{bc} + \cancel{bd} - \cancel{ac} + i\cancel{bc} - i\cancel{ad} - \cancel{bd} \\ &= 2i(\overbrace{bc - ad}^{f}) = 2if \end{aligned}$$

$$\Rightarrow \quad M_{12}^*\Gamma_{12} - \Gamma_{12}^*M_{12} = -2i(bc - ad) = -2if$$

$$\frac{\Gamma_{12}}{M_{12}} = \frac{c+id}{a+ib} = \frac{(c+id)(a-ib)}{a^2+b^2} = \frac{(ac+bd) + i(ad-bc)}{a^2+b^2}$$

$$I_m = \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) = \frac{ad - bc}{a^2+b^2}$$

$$|M_{12}|^2 \gg |\Gamma_{12}|^2$$

$$\Rightarrow \left|\frac{1+\varepsilon}{1-\varepsilon}\right| = \left(\frac{1+\frac{1}{4}\cancel{\frac{|\Gamma_{12}|^2}{|M_{12}|^2}}^{\to 0} + \frac{i}{2}\frac{(M_{12}\Gamma_{12}^* - \Gamma_{12}M_{12}^*)}{|M_{12}|^2}}{1+\frac{1}{4}\cancel{\left|\frac{\Gamma_{12}}{M_{12}}\right|^2}^{\simeq 0} + \frac{i}{2}\left(\frac{M_{12}^*\Gamma_{12} - \Gamma_{12}^* M_{12}}{|M_{12}|^2}\right)} \right)^{1/4}$$ (15)

$$\left|\frac{1+\varepsilon}{1-\varepsilon}\right| = \left(\frac{1+\frac{i}{\cancel{2}}\frac{(\cancel{2}if)}{a^2+b^2}}{1+\frac{i}{\cancel{2}}\frac{(-\cancel{2}if)}{a^2+b^2}} \right)^{1/4}$$

$$\left|\frac{1+\varepsilon}{1-\varepsilon}\right| = \left(\frac{1+I_m}{1-I_m} \right)^{1/4}$$

$$\left|\frac{1+\varepsilon}{1-\varepsilon}\right| \simeq (1+I_m)^{1/2} \simeq 1+\frac{1}{2}I_m$$

$$\boxed{\left|\frac{1+\varepsilon_B}{1-\varepsilon_B}\right| \simeq 1+\frac{1}{2}\,\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)}$$

$$\left|\frac{1+\varepsilon}{1-\varepsilon}\right| = \left(\frac{1+\varepsilon}{1-\varepsilon}\cdot\frac{1+\varepsilon^*}{1-\varepsilon^*}\right)^{1/2} = \left(\frac{1+I_m}{1-I_m}\right)^{1/4}$$

$$\Rightarrow \left[\frac{(1+\varepsilon+\varepsilon^*+|\varepsilon|^2)}{(1-\varepsilon-\varepsilon^*+|\varepsilon|^2)}\right]^2 = \frac{1+I_m}{1-I_m}$$

$$\varepsilon = g+ih$$

$$\varepsilon^* = g-ih$$

$$\varepsilon+\varepsilon^* = 2g = 2\,\mathrm{Re}\,\varepsilon$$

$$\left[\frac{(1+2\,\mathrm{Re}\,\varepsilon+|\varepsilon|^2)^2}{(1-2\,\mathrm{Re}\,\varepsilon+|\varepsilon|^2)^2}\right] = \frac{1+I_m}{1-I_m}$$

$$\frac{1+4(\mathrm{Re}\,\varepsilon)^2+4\,\mathrm{Re}\,\varepsilon+2|\varepsilon|^2}{1+4(\mathrm{Re}\,\varepsilon)^2-4\,\mathrm{Re}\,\varepsilon+2|\varepsilon|^2} \simeq \frac{1+I_m}{1-I_m}$$

$$\cancel{1} + \cancel{4(Re\,\varepsilon)^2} + 4 Re\,\varepsilon + \cancel{2|\varepsilon|^2} - \cancel{I_m} - 4(Re\,\varepsilon)^2 I_m - \cancel{4 Re\,\varepsilon\, I_m}$$
$$- 2|\varepsilon|^2 I_m = \cancel{1} + \cancel{4(Re\,\varepsilon)^2} - 4 Re\,\varepsilon + \cancel{2|\varepsilon|^2} + I_m + 4(Re\,\varepsilon)^2 I_m$$
$$- \cancel{4 Re\,\varepsilon\, I_m} + 2|\varepsilon|^2 I_m$$

$$8 Re\,\varepsilon - 2 I_m - 8(Re\,\varepsilon)^2 I_m - 4|\varepsilon|^2 I_m = 0$$

$$\boxed{I_m = \frac{4 Re\,\varepsilon}{1 + 2|\varepsilon|^2 + 4(Re\,\varepsilon)^2} = I_m\left(\frac{\Gamma_{12}}{M_{12}}\right)}$$ OK.

Si $(Re\,\varepsilon)$ es muy pequeño

$$I_m\left(\frac{\Gamma_{12}}{M_{12}}\right) \simeq \frac{4 Re\,\varepsilon}{1 + 2|\varepsilon|^2}.$$ ok.

$$\left|\frac{1+\varepsilon}{1-\varepsilon}\right| = \left(\frac{1 + \frac{1}{4}\left|\frac{\Gamma_{12}}{M_{12}}\right|^2 + \frac{i}{2}\left(\left(\frac{\Gamma_{12}}{M_{12}}\right)^* - \left(\frac{\Gamma_{12}}{M_{12}}\right)\right)}{1 + \frac{1}{4}\left|\frac{\Gamma_{12}}{M_{12}}\right|^2 + \frac{i}{2}\left(\left(\frac{\Gamma_{12}}{M_{12}}\right) - \left(\frac{\Gamma_{12}}{M_{12}}\right)^*\right)}\right)^{1/4}$$

$$\frac{\Gamma_{12}}{M_{12}} = s + ih$$

$$\Rightarrow \left(\frac{\Gamma_{12}}{M_{12}}\right)^* - \left(\frac{\Gamma_{12}}{M_{12}}\right) = -2i\, I_m\left(\frac{\Gamma_{12}}{M_{12}}\right)$$

$$\left(\frac{\Gamma_{12}}{M_{12}}\right)^* = s - ih$$

$$\left|\frac{1+\varepsilon}{1-\varepsilon}\right| = \left(\frac{1 + \frac{1}{4}\left|\cancel{\frac{\Gamma_{12}}{M_{12}}}\right|^2 + I_m}{1 + \frac{1}{4}\left|\cancel{\frac{\Gamma_{12}}{M_{12}}}\right|^2 - I_m}\right)^{1/4}$$

$$\Rightarrow \left|\frac{1+\varepsilon}{1-\varepsilon}\right| = \left(\frac{1+I_m}{1-I_m}\right)^{1/4}$$

Feynman rules in the Two Higgs Doublet Model of type II

Feynman Rules in the two Higgs doublet model

The lagrangian for VHH interaction is : (see reference [1])

$$\mathcal{L}_{VHH} = -\frac{ig}{2} W^+_\mu H^- \overleftrightarrow{\partial^\mu} [H^0 \sin(\alpha-\beta) + h^0 \cos(\alpha-\beta) + iA^0] + h.c.$$

$$-\frac{ig}{2\cos\theta_W} Z_\mu \left\{ iA^0 \overleftrightarrow{\partial^\mu} [H^0 \sin(\alpha-\beta) + h^0 \cos(\alpha-\beta)] - (2\sin^2\theta_W - 1) . \right.$$

$$\left. . H^- \overleftrightarrow{\partial^\mu} H^+ \right\}$$

where $A \overleftrightarrow{\partial^\mu} B = A(\partial^\mu B) - (\partial^\mu A) B$

The lagrangian for VVH interaction is :

$$\mathcal{L}_{VVH} = \left(g M_W W^+_\mu W^{-\mu} + \frac{g M_Z}{2\cos\theta_W} Z_\mu Z^\mu \right) [H^0 \cos(\beta-\alpha) + h^0 \sin(\beta-\alpha)]$$

There is no $Z H^0 H^0$ or $W^+ Z^0 H^-$ vertices

The interactions of neutral Higgs bosons with up and down type quarks are given by :

$$\mathcal{L} = -\frac{g m_f}{2 M_W \sin\beta} \left[\bar{U}_f V_{\bar{f}} (H^0 \sin\alpha + h^0 \cos\alpha) - i\cos\beta\, \bar{U}_f \gamma^5 V_{\bar{f}} A^0 \right]$$

$$-\frac{g m_{f'}}{2 M_W \cos\beta} \left[\bar{U}_{f'} V_{\bar{f}'} (H^0 \cos\alpha - h^0 \sin\alpha) - i\sin\beta\, \bar{U}_{f'} \gamma^5 V_{\bar{f}'} A^0 \right]$$

Where $f = u, c, t, \nu_e, \nu_\mu, \nu_\tau$

and $f' = d, s, b, e^-, \mu^-, \tau^-$

The Lagrangian corresponding to the $H^{\pm} f \bar{f}'$ Vertex is:

$$\mathcal{L} = \frac{g}{2\sqrt{2}\, M_W} \left\{ H^+ V_{ff'} \bar{U}_f (A + B\gamma^5) V_{\bar{f}'} + H^- V^*_{ff'} \bar{U}_{f'} (A - B\gamma^5) V_{\bar{f}} \right\}$$

$$A \equiv (m_{f'} \tan\beta + m_f \cot\beta)$$

$$B \equiv (m_{f'} \tan\beta - m_f \cot\beta)$$

$f = u, c, t, \nu_{e^-}, \nu_{\mu^-}, \nu_{\tau^-}$

$f' = d, s, b, e^-, \mu^-, \tau^-$

The Lagrangian corresponding to three Higgs bosons is:

$$\mathcal{L} = -g H^0 \left\{ H^+ H^- \left[M_W \cos(\beta - \alpha) - \frac{M_Z}{2\cos\theta_W} (\cos 2\beta) \cos(\beta + \alpha) \right] \right.$$

$$+ \frac{1}{4} H^0 H^0 \frac{M_Z}{\cos\theta_W} (\cos 2\alpha) \cos(\beta + \alpha)$$

$$+ \frac{h^0 h^0}{4} \frac{M_Z}{\cos\theta_W} \left[2(\sin 2\alpha) \sin(\beta + \alpha) - \cos(\beta + \alpha)(\cos 2\alpha) \right]$$

$$\left. - \frac{A^0 A^0}{4} \frac{M_Z}{\cos\theta_W} (\cos 2\beta) \cos(\beta + \alpha) \right\}$$

$$- g h^0 \left\{ H^+ H^- \left[M_W \sin(\beta - \alpha) + \frac{M_Z}{2\cos\theta_W} (\cos 2\beta) \sin(\beta + \alpha) \right. \right.$$

$$+ \frac{1}{4} h^0 h^0 \frac{M_Z}{\cos\theta_W} (\cos 2\alpha) \sin(\beta + \alpha) - \frac{H^0 H^0}{4} \frac{M_Z}{\cos\theta_W} \left[2(\sin 2\alpha) \cos(\beta + \alpha) \right.$$

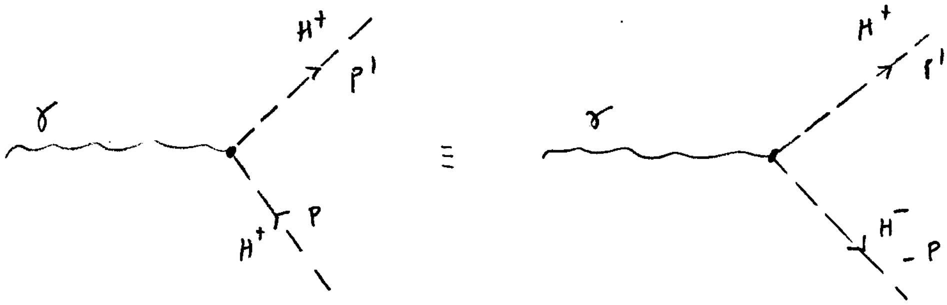
$$\left. \left. + \sin(\beta + \alpha)(\cos 2\alpha) \right] + \frac{A^0 A^0}{4} \frac{M_Z}{\cos\theta_W} (\cos 2\beta) \sin(\beta + \alpha) \right\}$$

The Lagrangian corresponding to vertices with a γ, a second gauge boson, and two Higgs bosons is:

$$\mathcal{L} = e^2 A_\mu A^\mu H^+ H^- + \frac{eg \cos 2\theta_W}{\cos\theta_W} A_\mu Z^\mu H^+ H^-$$

$$- \frac{eg}{2} \sin(\beta-\alpha) A_\mu W^{\pm\mu} H^0 H^\mp + \frac{eg}{2} \cos(\beta-\alpha) A_\mu W^{\pm\mu} h^0 H^\mp$$

$$\pm \frac{ige}{2} A_\mu W^{\pm\mu} A^0 H^\mp .$$

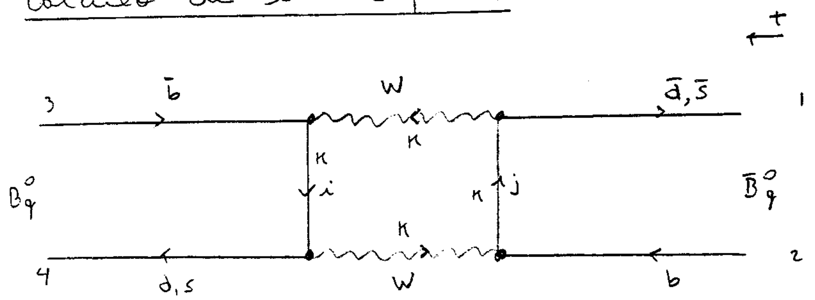


$$-ie\,(p+p')^{\mu}$$

$$\boxed{\mathcal{L}_{H^+H^-\gamma} = -ie\,A_\mu H^- \overleftrightarrow{\partial^\mu} H^+ \quad = -ig\sin\theta_W\,A_\mu H^- \overleftrightarrow{\partial^\mu} H^+}$$

Calculation of the box diagrams corresponding to charged Higgs contributions to $B^0 - \overline{B}^0$ mixing in the "Two Higgs Doublet Model of type II"

Cálculo de la amplitud :



$q = d o s$, $i, j = u, c, t$

$$A = \bar{V}_L(\bar{q})\gamma^\mu U_L(b).\bar{U}_L(q)\gamma_\mu V_L(\bar{b}) = \bar{V}_{1L}(\bar{q})\gamma^\mu U_{2L}(b).\bar{U}_{4L}(q)\gamma_\mu V_{3L}(\bar{b})$$

$$A = \bar{V}_1(\bar{q})\gamma^\mu \frac{1}{2}(1-\gamma^5)U_2(b)\,\bar{U}_4(q)\gamma_\mu \frac{1}{2}(1-\gamma^5)V_3(\bar{b})$$

Suponiendo $\theta = 0$, $\phi = 0$ $\qquad (P^\mu = (m, 0, 0, 0))$

$$U_R(0) = \sqrt{2m}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} ; \quad U_L(0) = \sqrt{2m}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} ; \quad V_R(0) = \sqrt{2m}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$V_L(0) = \sqrt{2m}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$\bar{V}_{1R}(\bar{q}) = \sqrt{2m_q}\,(0, 0, 0, -1) \; ; \; \bar{V}_{1L}(\bar{q}) = \sqrt{2m_q}\,(0, 0, -1, 0)$$

$$U_{2L}(b) = \sqrt{2m_b}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} ; \quad U_{1R}(b) = \sqrt{2m_b}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$\bar{U}_{4L}(q) = \sqrt{2m_q}\,(0, 1, 0, 0) \, , \; \bar{U}_{4R}(q) = \sqrt{2m_q}\,(1, 0, 0, 0)$$

$$V_{3R}(\bar{b}) = \sqrt{2m_b}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} ; \quad V_{3L}(\bar{b}) = \sqrt{2m_b}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

R L L R :

$$\bar{V}_{1R}(\bar{q})\gamma^\mu \frac{1}{2}(1-\gamma^5)U_{2L}(b) = \sqrt{m_b m_q}\,(0, 0, 0, -1)\gamma^\mu \begin{pmatrix}1 & 0 & -1 & 0\\0 & 1 & 0 & -1\\-1 & 0 & 1 & 0\\0 & -1 & 0 & 1\end{pmatrix}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$= \sqrt{m_b m_q}\,(0, 0, 0, -1)\gamma^\mu \begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

si $\mu = 0$

$$= \sqrt{m_b m_q}\,(0, 0, 0, -1)\begin{pmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & -1 & 0\\0 & 0 & 0 & -1\end{pmatrix}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

$$= \sqrt{m_b m_q}\,(0, 0, 0, -1)\begin{pmatrix}0\\1\\0\\1\end{pmatrix} = -\sqrt{m_b m_q}$$

si $\mu = 1$

$$= \sqrt{m_b m_q}\ (0,0,0,-1)\begin{pmatrix} 0&0&0&1\\ 0&0&1&0\\ 0&-1&0&0\\ -1&0&0&0\end{pmatrix}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

$$= \sqrt{m_b m_q}\ (0,0,0,-1)\begin{pmatrix}-1\\0\\-1\\0\end{pmatrix} = 0$$

si $\mu = 2$

$$= \sqrt{m_b m_q}\ (0,0,0,-1)\begin{pmatrix} 0&0&0&-i\\ 0&0&i&0\\ 0&i&0&0\\ -i&0&0&0\end{pmatrix}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

$$= \sqrt{m_b m_q}\ (0,0,0,-1)\begin{pmatrix}i\\0\\i\\0\end{pmatrix} = \sqrt{m_b m_q}\cdot 0 = 0$$

si $\mu = 3$

$$= \sqrt{m_b m_q}\ (0,0,0,-1)\begin{pmatrix} 0&0&1&0\\ 0&0&0&-1\\ -1&0&0&0\\ 0&1&0&0\end{pmatrix}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

$$= \sqrt{m_b m_q}\ (0,0,0,-1)\begin{pmatrix}0\\1\\0\\1\end{pmatrix} = -\sqrt{m_b m_q}$$

$$\overline{U}_{4_L}(q)\,\gamma_\mu\,\frac{1}{2}(1-\gamma^5)\,V_{3_R}(\bar b) = \sqrt{m_b m_q}\ (0,1,0,0)\,\gamma_\mu\begin{pmatrix} 1&0&-1&0\\ 0&1&0&-1\\ -1&0&1&0\\ 0&-1&0&1\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$= \sqrt{m_b m_q}\ (0,1,0,0)\,\gamma_\mu\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}$$

si $\mu = 0$

$$= \sqrt{m_b m_q}\ (0,1,0,0)\begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&-1&0\\ 0&0&0&-1\end{pmatrix}\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}$$

$$= \sqrt{m_b m_q}\ (0,1,0,0)\begin{pmatrix}0\\-1\\0\\-1\end{pmatrix} = -\sqrt{m_b m_q}$$

$\mu = 1, 2 = 0$

si $\mu = 3$

$$= -\sqrt{m_b m_q}\ (0,1,0,0)\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix}\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}$$

$$= -\sqrt{m_b m_q}\ (0,1,0,0)\begin{pmatrix}0\\-1\\0\\-1\end{pmatrix} = +\sqrt{m_b m_q}$$

$$\Rightarrow A_{RLLR} = (m_b m_q - m_b m_q) = 0 \qquad 1$$

<u>LLLL :</u>

$$\bar{V}_{1_L}(\bar{q})\,\gamma^\mu \frac{1}{2}(1-\gamma^5)\,U_{2_L}(b) = \sqrt{m_b m_q}\ (0,0,-1,0)\,\gamma^\mu \begin{pmatrix}1&0&-1&0\\0&1&0&-1\\-1&0&1&0\\0&-1&0&1\end{pmatrix}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$= \sqrt{m_b m_q}\ (0,0,-1,0)\,\gamma^\mu \begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

si $\mu = 0$

$$= \sqrt{m_b m_q}\ (0,0,-1,0)\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

$$= \sqrt{m_b m_q}\ (0,0,-1,0)\begin{pmatrix}0\\1\\0\\1\end{pmatrix} = 0$$

si $\mu = 1$

$$= \sqrt{m_b m_q}\ (0,0,-1,0)\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

$$= \sqrt{m_b m_q}\ (0,0,-1,0)\begin{pmatrix}-1\\0\\-1\\0\end{pmatrix} = \sqrt{m_b m_q}$$

si $\mu = 2$

$$= \sqrt{m_b m_q}\ (0,0,-1,0) \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

$$= \sqrt{m_b m_q}\ (0,0,-1,0) \begin{pmatrix} i \\ 0 \\ i \\ 0 \end{pmatrix} = -i \sqrt{m_b m_q}$$

si $\mu = 3$

$$= \sqrt{m_b m_q}\ (0,0,-1,0) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

$$= \sqrt{m_b m_q}\ (0,0,-1,0) \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$\bar{U}_{q_L}(q)\, \gamma_\mu \frac{1}{2} (1-\gamma^5)\, V_{3_L}(\bar{b}) = \sqrt{m_b m_q}\ (0,1,0,0)\, \gamma_\mu \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= \sqrt{m_b m_q}\ (0,1,0,0)\, \gamma_\mu \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

si $\mu = 1$

$$= -\sqrt{m_b m_q}\ (0,1,0,0) \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= -\sqrt{m_b m_q}\ (0,1,0,0) \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = -\sqrt{m_b m_q}$$

si $\mu = 2$

$$= -\sqrt{m_b m_q}\ (0,1,0,0) \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= -\sqrt{m_b m_q}\ (0,1,0,0) \begin{pmatrix} 0 \\ i \\ 0 \\ i \end{pmatrix} = -i \sqrt{m_b m_q}$$

$$\Rightarrow \boxed{A_{LLLL} = -m_b m_q - m_b m_q = -2 m_b m_q}$$

RRRR:

$$\bar{V}_{1_R}(\bar{q})\,\gamma^\mu\,\frac{1}{2}(1-\gamma^5)\,U_{2R}(b) = \sqrt{m_b m_q}\,(0,0,0,-1)\,\gamma^\mu \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \sqrt{m_b m_q}\,(0,0,0,-1)\,\gamma^\mu \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

si $\mu = 0$:

$$= \sqrt{m_b m_q}\,(0,0,0,-1) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$= \sqrt{m_b m_q}\,(0,0,0,-1) \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = 0$$

si $\mu = 1$

$$= \sqrt{m_b m_q}\,(0,0,0,-1) \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$= \sqrt{m_b m_q}\,(0,0,0,-1) \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix} = \sqrt{m_b m_q}$$

si $\mu = 2$

$$= \sqrt{m_b m_q}\,(0,0,0,-1) \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$= \sqrt{m_b m_q}\,(0,0,0,-1) \begin{pmatrix} 0 \\ -i \\ 0 \\ -i \end{pmatrix} = i\sqrt{m_b m_q}$$

si $\mu = 3$

$$= \sqrt{m_b m_q}\,(0,0,0,-1) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$= \sqrt{m_b m_q}\,(0,0,0,-1) \begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = 0$$

$$\bar{U}_{4R}(q)\gamma_\mu \frac{1}{2}(1-\gamma^5) V_{3R}(\bar{b}) = \sqrt{m_b m_q}\,(1,0,0,0)\gamma_\mu \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0\\0\\0\\1\end{pmatrix}$$

$$= \sqrt{m_b m_q}\,(1,0,0,0)\gamma_\mu \begin{pmatrix} 0\\-1\\0\\1\end{pmatrix}$$

si $\mu = 1$:

$$= -\sqrt{m_b m_q}\,(1,0,0,0)\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0\\-1\\0\\1\end{pmatrix}$$

$$= -\sqrt{m_b m_q}\,(1,0,0,0)\begin{pmatrix} 1\\0\\1\\0\end{pmatrix} = -\sqrt{m_b m_q}$$

si $\mu = 2$:

$$= -\sqrt{m_b m_q}\,(1,0,0,0)\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0\\-1\\0\\1\end{pmatrix}$$

$$= -\sqrt{m_b m_q}\,(1,0,0,0)\begin{pmatrix} -i\\0\\-i\\0\end{pmatrix} = +i\sqrt{m_b m_q}$$

$$\Rightarrow \boxed{A_{RRRR} = -m_b m_q - m_b m_q = -2 m_b m_q}$$

3

<u>R L R L</u> :

$$\bar{U}_{4R}(q)\gamma_\mu \frac{1}{2}(1-\gamma^5) V_{3L}(\bar{b}) = \sqrt{m_b m_q}\,(1,0,0,0)\gamma_\mu \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0\\0\\1\\0\end{pmatrix}$$

$$= \sqrt{m_b m_q}\,(1,0,0,0)\gamma_\mu \begin{pmatrix} -1\\0\\1\\0\end{pmatrix}$$

si $\mu = 0$

$$= \sqrt{m_b m_q}\,(1,0,0,0)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} -1\\0\\1\\0\end{pmatrix}$$

$$= \sqrt{m_b m_q}\,(1,0,0,0)\begin{pmatrix} -1\\0\\-1\\0\end{pmatrix} = -\sqrt{m_b m_q}$$

si $\mu = 1$

$$= -\sqrt{m_b m_q}\,(1,0,0,0)\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} -1\\0\\1\\0\end{pmatrix}$$

$$= -\sqrt{m_b m_q}\,(1,0,0,0)\begin{pmatrix} 0\\1\\0\\1\end{pmatrix} = 0$$

si $\mu = 2$: $= -\sqrt{m_b m_q}\ (1,0,0,0) \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$$= -\sqrt{m_b m_q}\ (1,0,0,0) \begin{pmatrix} 0 \\ i \\ 0 \\ i \end{pmatrix} = 0$$

si $\mu = 3$: $= -\sqrt{m_b m_q}\ (1,0,0,0) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$$= -\sqrt{m_b m_q}\ (1,0,0,0) \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = -\sqrt{m_b m_q}$$

$$\Rightarrow \boxed{A_{RLRL} = \sqrt{m_b m_q}^2 + \sqrt{m_b m_q}^2 = 2\, m_b m_q} \quad (4)$$

<u>LRLR</u> : $\overline{V_{1L}}(\bar{q})\, \gamma^\mu \frac{1}{2}(1-\gamma^5)\, U_{2R}(b) = \sqrt{m_b m_q}\ (0,0,-1,0)\, \gamma^\mu \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$$= \sqrt{m_b m_q}\ (0,0,-1,0)\, \gamma^\mu \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

si $\mu = 0$: $= \sqrt{m_b m_q}\ (0,0,-1,0) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

$$= \sqrt{m_b m_q}\ (0,0,-1,0) \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = -\sqrt{m_b m_q}$$

si $\mu = 1$: $= \sqrt{m_b m_q}\ (0,0,-1,0) \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

$$= \sqrt{m_b m_q}\ (0,0,-1,0) \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix} = 0$$

si $\mu = 2$: $= \sqrt{m_b m_q}\ (0,0,-1,0) \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

$$= \sqrt{m_b m_q}\ (0,0,-1,0) \begin{pmatrix} 0 \\ -i \\ 0 \\ -i \end{pmatrix} = 0$$

si $\mu = 3$: $= \sqrt{m_b m_q} \; (0,0,-1,0) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

$$= \sqrt{m_b m_q} \; (0,0,-1,0) \begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \sqrt{m_b m_q}$$

$$\boxed{A_{LRLR} = m_b m_q + m_b m_q = 2 m_b m_q} \quad (5)$$

$\underline{LRRL}$: $\boxed{A_{LRRL} = m_b m_q - m_b m_q = 0}$ (6)

$\underline{RRRL}$: $\boxed{A_{RRRL} = 0}$ (7)

$\underline{RRLR}$: $\boxed{A_{RRLR} = 0}$ (8)

$\underline{RRLL}$: $\boxed{A_{RRLL} = -m_b m_q + m_b m_q = 0}$ (9)

$\underline{LLRL}$: $\boxed{A_{LLRL} = 0}$ (10)

$\underline{LLLR}$: $\boxed{A_{LLLR} = 0}$ (11)

$\underline{LLRR}$: $\boxed{A_{LLRR} = -m_b m_q + m_b m_q = 0}$ (12)

$\underline{LRLL}$: $\boxed{A_{LRLL} = 0}$ (13)

$\underline{RLRR}$: $\boxed{A_{RLRR} = 0}$ (14)

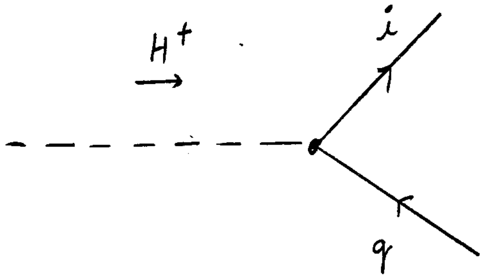
$\underline{LRRR}$: $\boxed{A_{LRRR} = 0}$ (15)

$\underline{RLLL}$: $\boxed{A_{RLLL} = 0}$ (16)

Polarización: 1 2 4 3	$A = \bar{V}_1(\bar{q}) \gamma^\mu \frac{1}{2}(1-\gamma^5) U_2(b) \bar{U}_4(q) \gamma_\mu \frac{1}{2}(1-\gamma^5) V_3(\bar{b})$
L L L L	$-2 m_b m_q$
L L L R	0
L L R L	0
L L R R	0
L R L L	0
L R L R	$2 m_b m_q$
L R R L	0
L R R R	0
R L L L	0
R L L R	0
R L R L	$2 m_b m_q$
R L R R	0
R R L L	0
R R L R	0
R R R L	0
R R R R	$-2 m_b m_q$

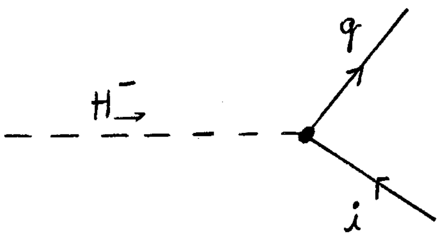
Feynman Rules for charged Higgs in the two-doublet Higgs model:

Higgs-quark-quark vertices:



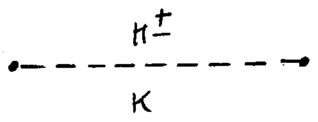
$i = u, c, t$
$q = d, s, b$

$$\frac{ig}{2\sqrt{2}M_W}\left[m_q \tan\beta\,(1+\gamma^5) + m_i \cot\beta\,(1-\gamma^5)\right] V_{iq}$$



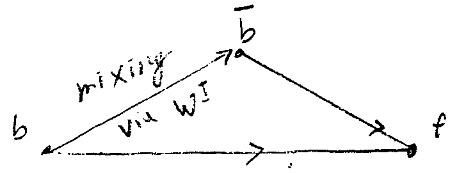
$$\frac{ig}{2\sqrt{2}M_W}\left[m_q \tan\beta\,(1-\gamma^5) + m_i \cot\beta\,(1+\gamma^5)\right] V^*_{iq}$$

Charged-Higgs propagator:



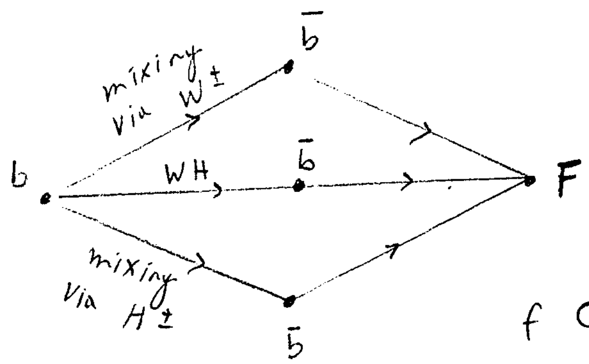
$$\frac{i}{K^2 - M_{H^\pm}^2 + i\epsilon}$$

CP violation via mixing:



(standard Model)

To obtain large inclusive CP violation:



Two doublet Higgs Model

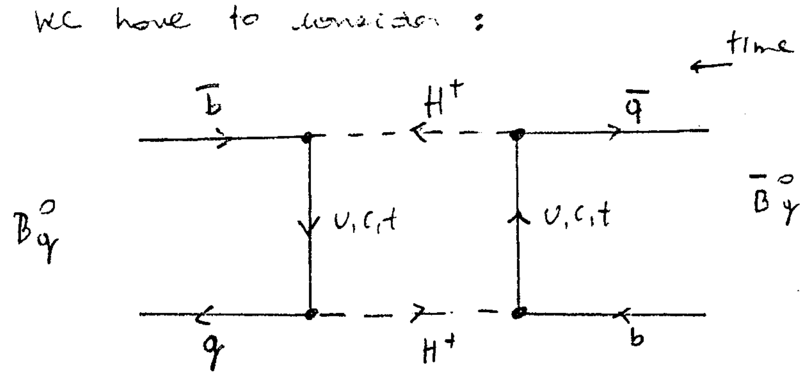
$f \subset F$

$$p + \bar{p} \rightarrow t + \bar{t}$$

$\left\{ \begin{array}{l} t \rightarrow W^+ b \\ t \rightarrow H^+ b \end{array} \right.$ $\rightarrow$ in D∅ a σ_t larger than the predicted by the standard Model is obtained. Possible solution: $H^\pm$ (D∅ Note 2896)

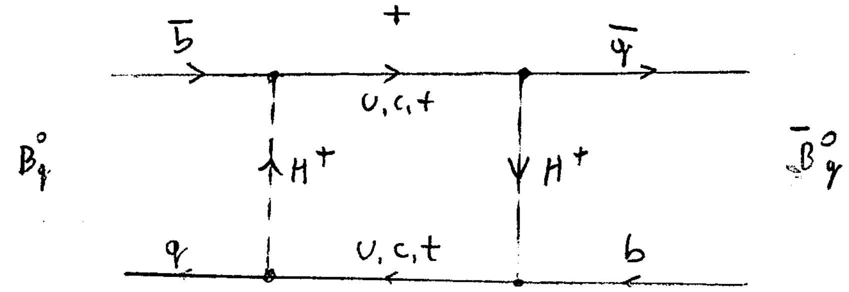
$\left\{ \begin{array}{l} \bar{t} \rightarrow W^- \bar{b} \\ \bar{t} \rightarrow H^- \bar{b} \end{array} \right.$

We have to consider:



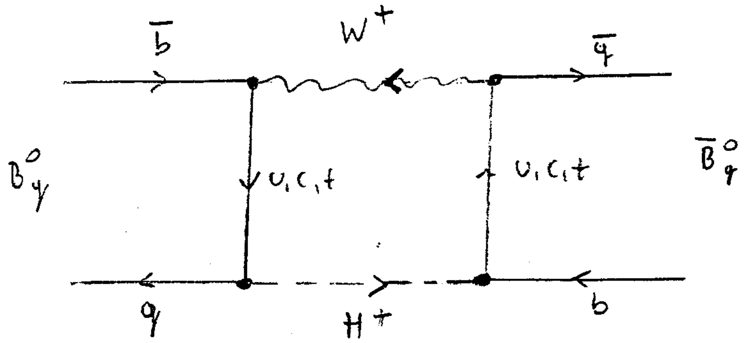
with $q = d, s$

+



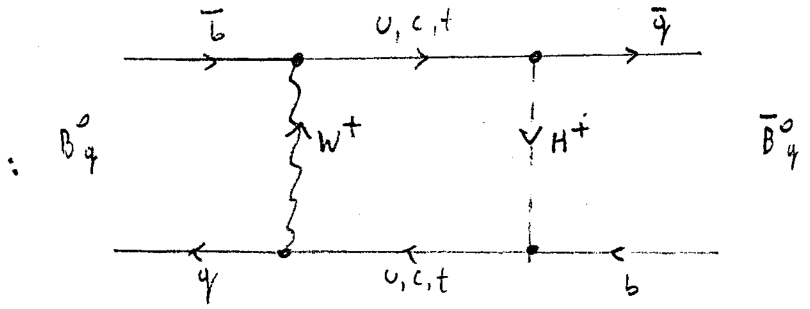
$+$

$\leftarrow$ time



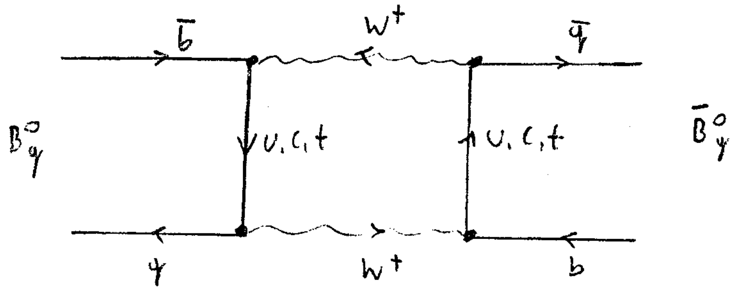
$+ \quad W^+ \longleftrightarrow H^+$

$+$

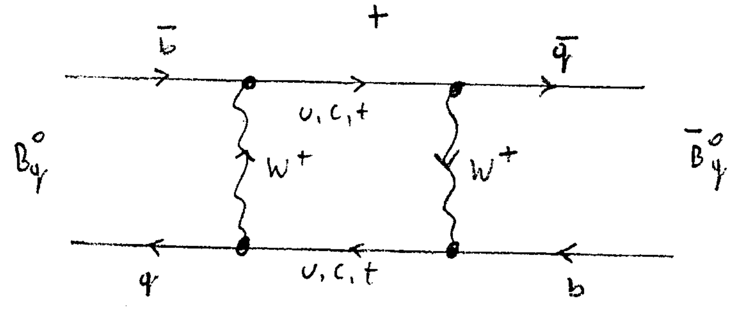


$+ \quad W^+ \longleftrightarrow H^+$

$+$



$+$



These box diagrams are evaluated in DØ Note 1372

Axial Currents

$$J^\mu = \bar\psi \gamma^\mu \psi$$

$$J^\mu_5 = \bar\psi \gamma^\mu \gamma^5 \psi$$

$\boxed{\partial_\mu J^\mu = 0}$ In fact:

$$\partial_\mu (\bar\psi \gamma^\mu \psi) = (\partial_\mu \bar\psi)\gamma^\mu \psi + \bar\psi \gamma^\mu (\partial_\mu \psi)$$

$$i\gamma^\mu(\partial_\mu \psi) = m\psi$$

$$i(\partial_\mu \bar\psi)\gamma^\mu + m\bar\psi = 0$$

$$\Rightarrow (\partial_\mu \bar\psi)\gamma^\mu \psi = im\bar\psi\psi$$

$$\Rightarrow \partial_\mu (\bar\psi\gamma^\mu\psi) = im\bar\psi\psi + \bar\psi(-im\psi) = 0.$$

$$\partial_\mu J^\mu_5 = (\partial_\mu\bar\psi)\gamma^\mu\gamma^5\psi + \bar\psi\gamma^\mu\gamma^5(\partial_\mu\psi)$$

$$(\partial_\mu\bar\psi)\gamma^\mu\gamma^5\psi = im\bar\psi\gamma^5\psi$$

$$\Rightarrow \partial_\mu J^\mu_5 = im\bar\psi\gamma^5\psi + \bar\psi\gamma^\mu\gamma^5(\partial_\mu\psi)$$

$$i\gamma^\mu(\partial_\mu\psi) = m\psi$$

$$\Rightarrow i\gamma^5\gamma^\mu(\partial_\mu\psi) = m\gamma^5\psi$$

$$-i\gamma^\mu\gamma^5(\partial_\mu\psi) = m\gamma^5\psi$$

$$\therefore \gamma^\mu\gamma^5(\partial_\mu\psi) = im\gamma^5\psi$$

Than $\partial_\mu J^\mu_5 = im\bar\psi\gamma^5\psi + im\bar\psi\gamma^5\psi$

$$\boxed{\partial_\mu J^\mu_5 = 2im\bar\psi\gamma^5\psi}$$

J^μ_5 is conserved only if $m=0$.
in this case we have exact
"chiral symmetry". (masses neglected)

Let us consider a massless Dirac Lagrangian for n quarks:

$$\mathcal{L} = \sum_{r=1}^{3} \sum_{i=1}^{n} \bar{\psi}_{ir}\, i\gamma^{\mu} D_{\mu} \psi_{ir}$$

r ↓ color

$$D_{\mu} = \partial_{\mu} - igM^{a} A^{a}_{\mu} \qquad (a=0,1,2,3) \qquad \left(M^{a} = \frac{\tau^{a}}{2}\right)$$

Pauli Matrices

$\mathcal{L}$ is invariant under $SU(2) \times U(1)$ transformations

$$\psi \to e^{-i\tau^{a} \frac{1}{2}\tau^{a}} \psi$$

$$\bar{\psi} \to \bar{\psi}\, e^{+i\alpha^{a} \frac{1}{2}\tau^{a}}$$

$a = 0,1,2,3$

$\tau^{0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\psi'^{\alpha}(x) = e^{i\Lambda^{a} M^{a}} \psi^{\alpha}(x) \qquad \text{(Global symmetry)}$$

$$\mathcal{L}\left(\psi^{\alpha}(x), \partial_{\mu}\psi^{\alpha}(x)\right) = \mathcal{L}\left(\psi'^{\alpha}(x), \partial_{\mu}\psi'^{\alpha}(x)\right)$$

For an infinitesimal Transformation $(\Lambda^{a}(x))$

$$\psi'^{\alpha}(x) = \psi^{\alpha}(x) + i\Lambda^{a} M^{a} \psi^{\alpha}(x)$$

$$\bar{\psi}'^{\alpha}(x) = \bar{\psi}^{\alpha}(x) + i\bar{\psi}^{\alpha}(x)\Lambda^{a} M^{a}$$

$$\partial_{\mu}\psi'^{\alpha}(x) = \partial_{\mu}\psi^{\alpha}(x) + i\Lambda^{a} M^{a}\left(\partial_{\mu}\psi^{\alpha}(x)\right)$$

$$\mathcal{L}\left(\psi'^{\alpha}(x), \partial_{\mu}\psi'^{\alpha}(x)\right) = \mathcal{L}\left(\psi^{\alpha}(x) + i\Lambda^{a} M^{a}\psi^{\alpha}(x),\ \partial_{\mu}\psi^{\alpha}(x) + i\Lambda^{a} M^{a}\left(\partial_{\mu}\psi^{\alpha}(x)\right)\right)$$

(*)

$$= \mathcal{L}(\psi^\alpha(x), \partial_\mu \psi^\alpha(x)) + \frac{\partial \mathcal{L}}{\partial \psi^\alpha(x)} i \Lambda^a M^a \psi^\alpha(x)$$

$$+ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^\alpha(x))} [i \Lambda^a M^a (\partial_\mu \psi^\alpha(x))] = \mathcal{L}(\psi^\alpha(x), \partial_\mu \psi^\alpha(x))$$

$$\frac{\partial \mathcal{L}}{\partial \psi^\alpha(x)} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^\alpha(x))} \right) = 0$$

$$D_\mu \psi_i = \partial_\mu \psi_i - i g M^a A_\mu^a \psi_i$$

$$\boxed{\mathcal{L} = \bar{\psi}_i i \gamma^\mu (\partial_\mu \psi_i) + g \bar{\psi}_i \gamma^\mu M^a A_\mu^a \psi_i}$$

$$\delta \psi^\alpha(x) = i \Lambda^a M^a \psi^\alpha(x)$$

$$\psi'^\alpha(x) = \psi^\alpha(x) + \delta \psi^\alpha(x)$$

$$\partial_\mu \psi'^\alpha(x) = \partial_\mu \psi^\alpha(x) + \partial_\mu(\delta \psi^\alpha(x))$$

$$\partial_\mu(\delta \psi^\alpha(x)) = i \Lambda^a M^a \partial_\mu \psi^\alpha(x)$$

* can be written as:

$\Rightarrow$

$$0 = \frac{\partial \mathcal{L}}{\partial \psi^\alpha(x)} \delta \psi^\alpha(x) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^\alpha(x))} \partial_\mu(\delta \psi^\alpha(x))$$

$$0 = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^\alpha(x))} \right) \delta \psi^\alpha(x) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^\alpha(x))} \partial_\mu(\delta \psi^\alpha(x))$$

$$0 = \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi_i^\alpha(x))} \delta \psi_i^\alpha(x) \right] = \Lambda^a \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi_i^\alpha(x))} \cdot (-i M^a) \psi_i^\alpha(x) \right]$$

$$\partial_\mu J_i^\mu = 0$$

$$\Rightarrow \boxed{J_i^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi_i^\alpha(x))} (-i) M^a \psi_i^\alpha(x)}$$

Then:

$$J_i^\mu = \bar{\psi}_i i \gamma^\mu (-i) M^a \psi_i$$

In our case $\Lambda^a = -\alpha^a$; $M^a = \frac{1}{2}\tau^a$

$$\Rightarrow \quad J^\mu_i = \bar{\psi}_i \, e \, \gamma^\mu \, (-i) \, \frac{1}{2} \tau^a \psi_i$$

$$\boxed{J^\mu_a = \bar{\psi}(x) \gamma^\mu \frac{1}{2} \tau^a \psi(x)}$$ (for each i and r)

$$Q^a = \int J^0_a \, d^3\vec{x}$$

$$\boxed{Q^a = \int d^3\vec{x} \, \bar{\psi}(x) \gamma^0 \frac{1}{2} \tau^a \psi(x)} \; ; \quad a = 0,1,2,3$$

$\mathcal{L}$ has also another global symmetry:

$$\psi'(x) = e^{-i\gamma_5 \alpha^a \frac{1}{2}\tau^a} \psi(x) .$$

$$\bar{\psi}'(x) = \bar{\psi}(x) \, e^{i\alpha_a \frac{1}{2}\tau^a \gamma^5} \qquad (\tau^a \gamma^5 = \gamma^5 \tau^a)$$

The corresponding current is:

$$\boxed{A^\mu_a = \bar{\psi}(x) \gamma^\mu \gamma^5 \frac{1}{2} \tau_a \psi(x)}$$

The axial charge is:

$$Q^5_a = \int A^0_a \, d^3\vec{x}$$

$$\boxed{Q^5_a = \int d^3\vec{x} \;\; \bar{\psi}(x) \gamma^0 \gamma^5 \frac{1}{2} \tau_a \psi(x)}$$

For a massive particle the corresponding Lagrangian is

$$\mathcal{L} = \sum_{r=1}^{3} \sum_{i=1}^{n} \bar{\psi}_{ir} \left(i\gamma^\mu D_\mu - m_i \right) \psi_{ir}$$

$$D_\mu = \partial_\mu - ig\, T^a A^a_\mu$$

$$a = 0, 1, 2, 3.$$

$\mathcal{L}$ is invariant under

$$\psi'(x) = e^{-i\alpha_a \frac{1}{2}\tau_a} \psi(x)$$

and $\psi'(x) = e^{-i\alpha_a \frac{1}{2}\tau_a \gamma^5} \psi(x)$

$$\mathcal{A}^\mu_a = \bar{\psi} \gamma^\mu \gamma^5 \left(\tfrac{1}{2}\tau_a\right) \psi$$

$$\partial_\mu \mathcal{A}^\mu_a = (\partial_\mu \bar{\psi}) \gamma^\mu \gamma^5 \left(\tfrac{1}{2}\tau_a\right)\psi + \bar{\psi}\gamma^\mu \gamma^5 \left(\tfrac{1}{2}\tau_a\right) \partial_\mu \psi$$

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0 \quad \Rightarrow \quad \gamma^\mu \partial_\mu \psi = -i m \psi$$

$$i(\partial_\mu \bar{\psi})\gamma^\mu + m\bar{\psi} = 0 \quad \Rightarrow \quad (\partial_\mu \bar{\psi})\gamma^\mu = i m \bar{\psi}$$

$$\partial_\mu \mathcal{A}^\mu_a = i m \bar{\psi} \gamma^5 \left(\tfrac{1}{2}\tau_a\right) \psi - \bar{\psi}\gamma^5 \left(\tfrac{1}{2}\tau_a\right)(-i m \psi)$$

$$\partial_\mu \mathcal{A}^\mu_a = 2 i m \bar{\psi} \gamma^5 \left(\tfrac{1}{2}\tau_a\right) \psi$$

if $m = 0$

$$\partial_\mu \mathcal{A}^\mu_a = 0$$

The pion decay amplitude is:

$$\langle 0 | \mathcal{A}^a_\mu (x) | \pi^b(q) \rangle$$

$$\mathcal{A}^a_\mu(x) = e^{iqx}\, \mathcal{A}^a_\mu(0)\, e^{-iqx}$$

$$\Rightarrow \langle 0 | \mathcal{A}^a_\mu(x) | \pi^b(q)\rangle = \langle 0 | e^{iqx}\, \mathcal{A}^a_\mu(0)\, e^{-iqx} | \pi^b(q)\rangle$$

$$q^\mu = \int \frac{d^3\vec{k}}{(2\pi)^3\, 2k^0}\, k^\mu\, a^+(k)\, a(k)$$

$$k^\mu = (k^0, \vec{k})$$

$$a(k)|0\rangle = 0 \quad \Rightarrow \quad \langle 0 | a^+(k) = 0$$

$$a^+(k)|0\rangle = |k\rangle$$

$$\therefore \quad \langle 0 | e^{iqx} = \langle 0 |$$

$$e^{iqx} = 1 + iqx + \frac{(iqx)^2}{2!} + \cdots$$

Then

$$\langle 0 | \mathcal{A}^a_\mu(x) | \pi^b(q)\rangle = e^{-iqx} \langle 0 | \mathcal{A}^a_\mu(0) | \pi^b(q)\rangle$$

$$a, b = 1, 2, 3$$

$$\pi^\pm = \frac{1}{\sqrt{2}}(\pi^1 \mp i\pi^2) \; ; \quad \pi^0 = \pi^3$$

$$\sqrt{2}\,\pi^+ = \pi^1 - i\pi^2$$

$$\sqrt{2}\,\pi^- = \pi^1 + i\pi^2$$

$$\Rightarrow \begin{cases} \pi^1 = \frac{1}{\sqrt{2}}(\pi^+ + \pi^-) \\ \pi^2 = \frac{1i}{2ii}\sqrt{2}(\pi^- - \pi^+) = -\frac{i}{\sqrt{2}}(\pi^- - \pi^+) = \frac{i}{\sqrt{2}}(\pi^+ - \pi^-) \\ \pi^3 = \pi^0 \end{cases}$$

$$\Rightarrow f_{\pi^b} = f_\pi \; (93\ \text{MeV}) = \frac{1}{\sqrt{2}} f_{\pi^\pm}$$

$$f_{\pi^\pm} \searrow (131.52)\ \text{MeV}$$

$$\boxed{\begin{aligned} \langle 0 | \mathcal{A}_\mu^a(0) | |\pi^b(q)\rangle &= i f_\pi\, q_\mu\, \delta^{ab} \\ \langle 0 | \mathcal{A}_\mu^a(0) | \pi^b(q)\rangle &= \frac{i}{\sqrt{2}} f_{\pi^\pm}\, q_\mu\, \delta^{ab} \end{aligned}}$$

$$\partial^\mu \mathcal{A}_\mu^a(x) = \partial^\mu \left[e^{iq^\mu x_\mu} \mathcal{A}_\mu^a(0)\, e^{-iq^{\mu'} x_{\mu'}} \right]$$

$$= i q^\mu e^{iqx} \mathcal{A}_\mu^a(0)\, e^{-iqx} + e^{iqx} \mathcal{A}_\mu^a(0)(-i q^\mu)\, e^{-iqx}$$

$$\langle 0 | \partial^\mu \mathcal{A}_\mu^a(x) = \langle 0 | i q^\mu e^{iqx} \mathcal{A}_\mu^a(0)\, e^{-iqx} + \langle 0 | e^{iqx} \mathcal{A}_\mu^a(0)(-iq^\mu)\, e^{-iqx}$$

$\langle 0 | q^\mu = 0$ because $\langle 0 | a^\dagger(k) = 0$

$$\langle 0 | e^{iqx} = \langle 0 |$$

$$\Rightarrow \langle 0 | \partial^\mu \mathcal{A}_\mu^a(x) = \langle 0 | \mathcal{A}_\mu^a(0)(-iq^\mu)\, e^{-iqx}$$

then $\langle 0 | \partial^\mu \mathcal{A}_\mu^a(x) | \pi^b(q)\rangle = \langle 0 | \mathcal{A}_\mu^a(0)(-iq^\mu)\, e^{-iqx} |\pi^b(q)\rangle$

$$= -iq^\mu e^{-iqx} \langle 0 | \mathcal{A}_\mu^a(0) | \pi^b(q)\rangle$$

$$= -iq^\mu e^{-iqx}\, i f_\pi\, q_\mu\, \delta^{ab}$$

So:

$$\boxed{\langle 0 | \partial^\mu \mathcal{A}_\mu^a(x) | \pi^b(q)\rangle = m_\pi^2 f_\pi\, \delta^{ab}\, e^{-iqx}}$$

If the Axial current is conserved:

$$\partial^\mu \mathcal{A}_\mu^a(x) = 0$$

Then because $f_\pi \neq 0$

$$\Rightarrow m_\pi = 0$$

(This is the Goldstone's Theorem)

"The exact chiral symmetry (neglecting the pion mass) is spontaneously broken"

$$\langle \pi^a(p) | \pi^b(x) | 0 \rangle = \langle \pi^a(p) | e^{ipx} \pi^b(0) e^{-ipx} | 0 \rangle$$

$$e^{-ipx} |0\rangle = |0\rangle$$

$$\Rightarrow \langle \pi^a(p) | \pi^b(x) | 0 \rangle = e^{ipx} \delta^{ab} \qquad \big(e^{ipx} \underbrace{\langle \pi^a(p) | \pi^b(0) | 0 \rangle}_{\delta^{ab}} \big)$$

More clearly:

$$\langle 0 | \pi(x) | p \rangle = \langle 0 | \int \frac{d^3\vec{k}}{(2\pi)^3 2k^0} \left(a(k) e^{-ikx} + a^+(k) e^{ikx} \right) | p \rangle$$

$$= \langle 0 | \int \frac{d^3\vec{k}}{(2\pi)^3 2k^0} a(k) e^{-ikx} | p \rangle$$

$$a^+(k) |0\rangle = |k\rangle$$

$$\Rightarrow \langle 0 | a(k) = \langle k |$$

$$= \int \frac{d^3\vec{k}}{(2\pi)^3 2k^0} \langle k | p \rangle e^{-ikx}$$

but $\langle k | p \rangle = (2\pi)^3 2k^0 \delta^3(\vec{k} - \vec{p})$

then

$$\langle 0 | \pi(x) | p \rangle = e^{-ipx} \Rightarrow \langle p | \pi(x) | 0 \rangle = e^{ipx}$$

$$\Rightarrow \boxed{\langle \pi^a(p) | \pi^b(x) | 0 \rangle = e^{ipx} \delta^{ab}}$$

or: $\boxed{\langle 0 | \pi^b(x) | \pi^a(p) \rangle = e^{-ipx} \delta^{ab}}$

If we define: $\partial^\mu A_\mu^a(x) = m_\pi^2 f_\pi \pi^a(x)$

(operator equation)

$$\Rightarrow \langle 0 | \partial^\mu A_\mu^a(x) | \pi^b(q) \rangle = m_\pi^2 f_\pi \langle 0 | \pi^a(x) | \pi^b(q) \rangle$$

$$\boxed{\langle 0 | \partial^\mu A_\mu^a(x) | \pi^b(q) \rangle = m_\pi^2 f_\pi e^{-iqx} \delta^{ab}}$$

with $f_\pi = \dfrac{f_{\pi^\pm}}{\sqrt{2}}$

That again is the Goldstone's theorem.

Let's consider the matrix element:

$$A = \langle B^0 | \bar{V}_L(\bar{q}) \gamma^\mu U_L(b)\ \bar{U}_L(q) \gamma_\mu V_L(\bar{b}) | \bar{B}^0 \rangle$$

$$A = \langle B^0 | \bar{V}(\bar{q}) \gamma^\mu \tfrac{1}{2}(1-\gamma^5)\ \tfrac{1}{2}(1-\gamma^5) U(b).\ \bar{U}(q) \gamma_\mu \tfrac{1}{2}(1-\gamma^5)\ \tfrac{1}{2}(1-\gamma^5) V(\bar{b}) | \bar{B}^0 \rangle$$

$$A = \langle B^0 | \bar{V}(\bar{q}) \gamma^\mu \tfrac{1}{2}(1-\gamma^5) U(b).\ \bar{U}(q) \gamma_\mu \tfrac{1}{2}(1-\gamma^5) V(\bar{b}) | \bar{B}^0 \rangle$$

$$A = \frac{1}{4} \langle B^0 | \bar{V}(\bar{q}) (\gamma^\mu - \gamma^\mu \gamma^5) U(b)\ \bar{U}(q) (\gamma_\mu - \gamma_\mu \gamma^5) V(\bar{b}) | \bar{B}^0 \rangle$$

$$A = \frac{1}{4} \langle B^0 | \bar{V}(\bar{q}) (\gamma^\mu - \gamma^\mu \gamma^5) U(b) | 0 \rangle \langle 0 | \bar{U}(q) (\gamma_\mu - \gamma_\mu \gamma^5) V(\bar{b}) | \bar{B}^0 \rangle$$

Here we have used the approximation of "vacuum insertion"

$$A = \frac{1}{4} \langle B^0 | \bar{V}(\bar{q}) \gamma^\mu \gamma^5 U(b) | 0 \rangle \langle 0 | \bar{U}(q) \gamma_\mu \gamma^5 V(\bar{b}) | \bar{B}^0 \rangle$$

$$(J^\mu = \bar{\psi} \gamma^\mu \psi \qquad \partial_\mu J^\mu = 0)$$

$$\langle 0 | \bar{U}(q) \gamma_\mu \gamma^5 V(\bar{b}) | \bar{B}^0 \rangle = \frac{i f'_{B^0} q_\mu}{\sqrt{2 m_B}} \qquad \left(f'_{B^0} = \frac{f_{B^0}}{\sqrt{2}}\right)$$

$$\langle B^0 | \bar{V}(\bar{q}) \gamma^\mu \gamma^5 U(b) | 0 \rangle = \frac{-i f'_{B^0} q^\mu}{\sqrt{2 m_B}}$$

$$\Rightarrow A = \frac{1}{4} \times \frac{4}{3} \times \frac{f_{B^0}^2}{2} \frac{q^\mu q_\mu}{2 m_B} = \frac{1}{6} \frac{f_{B^0}^2 m_{B^0}^2}{2 m_B} = \frac{1}{12} f_B^2 m_B$$

$$\boxed{A = \frac{1}{12} f_B^2 m_B}$$

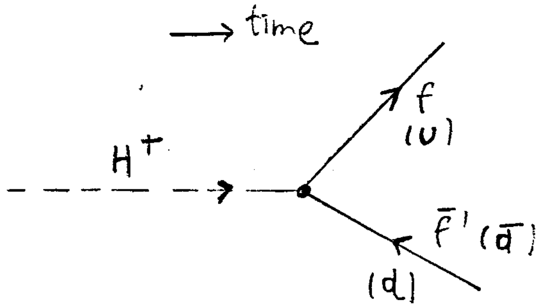
$(2 m_B)^{-1}$ is a normalization factor

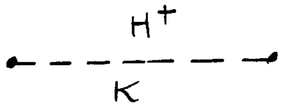
the vacuum state can be inserted in two different ways, corresponding to the two box diagrams.

$B^0 - \bar{B}^0$ Mixing

Carlos A. Moura

Feynman Rules $(H^\pm)$





$$\frac{i}{K^2 - M_{H^+}^2 + i\varepsilon}$$

spin 0 boson

The corresponding Lagrangian is:

$$\mathcal{L} = \frac{g}{2\sqrt{2}\,M_W}\left\{ H^+ V_{ff'}\,\bar{U}_f\,(A + B\gamma^5)\,V_{\bar{f}'} + H^- V^*_{ff'}\,\bar{U}_{f'}(A - B\gamma^5)V_{\bar{f}} \right\}$$

↓ CKM matrix element

$$A \equiv m_{\bar{f}'}\, tg\beta + m_f \cot\beta$$

$$B \equiv m_{\bar{f}'}\, tg\beta - m_f \cot\beta$$

$$A + B\gamma^5 = m_{\bar{f}'}\, tg\beta + m_f \cot\beta + (m_{\bar{f}'}\, tg\beta - m_f \cot\beta)\gamma^5$$

$$\boxed{A + B\gamma^5 = \underset{(d)}{m_{\bar{f}'}}\, tg\beta\,(1+\gamma^5) + \underset{(u)}{m_f} \cot\beta\,(1-\gamma^5)} \qquad (1.2a)$$

$$tg\beta = \frac{V_2}{V_1} \quad ; \quad 0 \le \beta \le \pi/2$$

Two doublets Higgs Model:

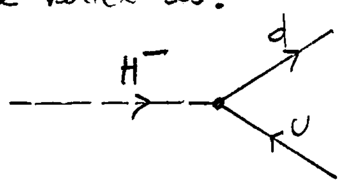
$$\Phi_1 = \begin{pmatrix} \phi_1^{0*} \\ -\phi_1^- \end{pmatrix} \quad ; \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} \qquad (1.3)$$

$$V_1 = \langle \phi_1^{0*} \rangle$$

$$V_2 = \langle \phi_2^0 \rangle$$

(neutral components of the doublets)

For H^- the vertex is:

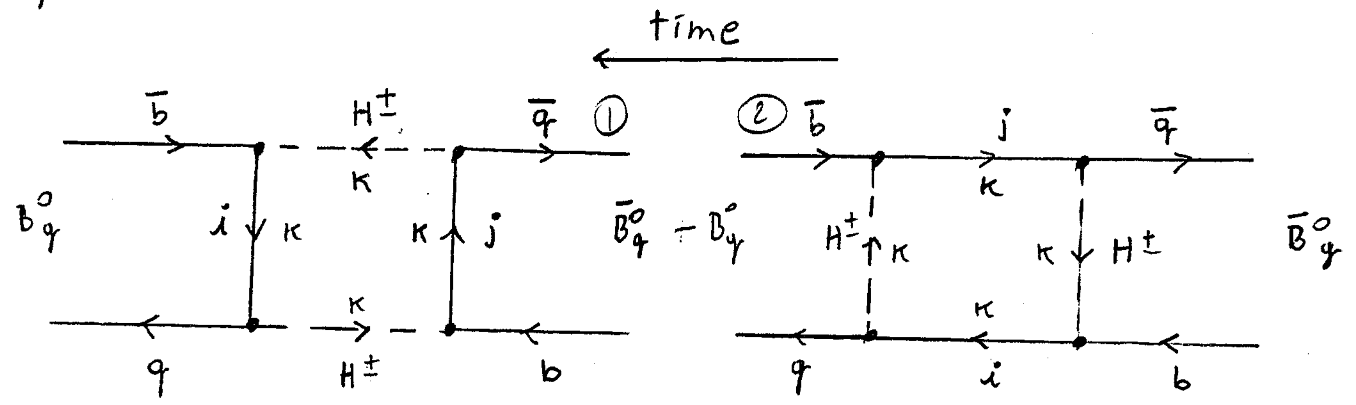


$$\boxed{A + B\gamma^5 = m_d\, tg\beta\,(1-\gamma^5) + m_u \cot\beta\,(1+\gamma^5)}$$

(1.2b)

Let's consider the box diagrams :

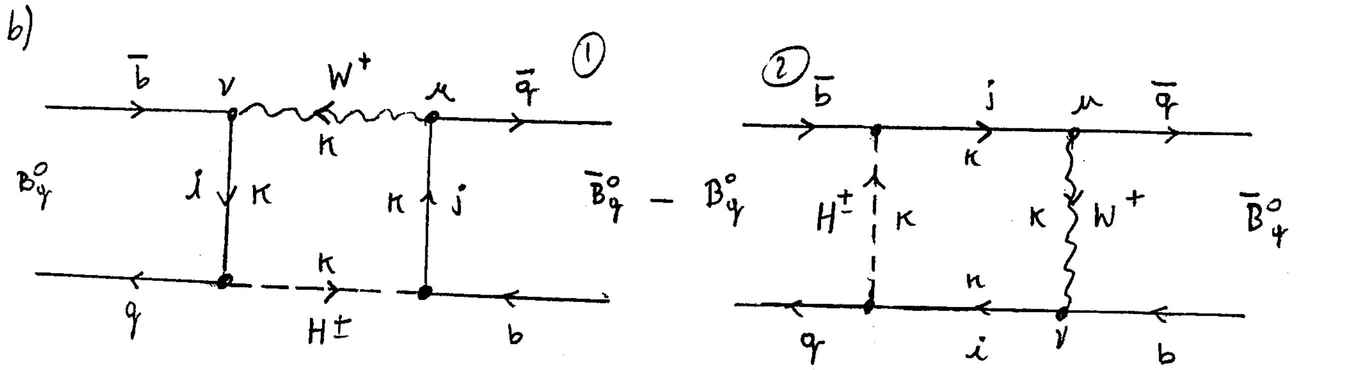
a)



With $q = d, s$

$i, j = u, c, t$

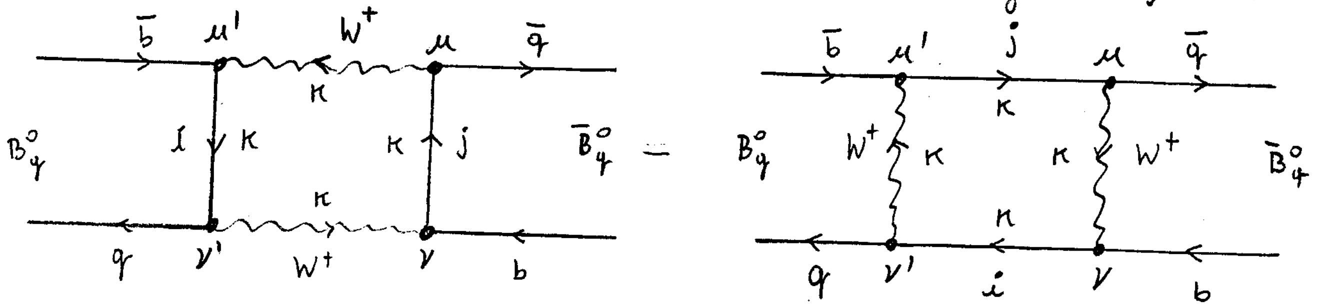
b)



+ Crossed diagrams ($W^{\pm} \leftrightarrow H^{\pm}$)

change $i \leftrightarrow j$ (not necessary)

c)



The last two box diagrams are calculated in DØ Note 1372

In the approximation of taking all external momenta to be zero, because they are small compared with M_{H^+}, M_W and M_t the invariant amplitude for a) can be written as:

$$\left(\xi_i = \underset{\substack{\downarrow \\ \text{CKM elements}}}{V_{ib} V_{iq}^*} \ ; \ \xi_j = V_{jb} V_{jq}^* \ ; \ \sum_i \xi_i = \sum_j \xi_j = 0 \right)$$

For the box diagrams a)

$$-iM_a^{(1)} = \sum_{i,j} \xi_i \xi_j \int \frac{d^4K}{(2\pi)^4} \bar{V}(\bar{q}) \frac{ig}{2\sqrt{2}M_W} [m_q \, tg\beta (1-\gamma^5) + m_j \cot\beta (1+\gamma^5)]$$

$$\frac{i(\not{K} + m_j)}{(K^2 - m_j^2)} \frac{ig}{2\sqrt{2}M_W} [m_b \, tg\beta (1+\gamma^5) + m_j \cot\beta (1-\gamma^5)] U(b) .$$

$$\cdot \bar{U}(q) \frac{ig}{2\sqrt{2}M_W} [m_q \, tg\beta (1-\gamma^5) + m_i \cot\beta (1+\gamma^5)] \frac{i(\not{K} + m_i)}{(K^2 - m_i^2)} .$$

$$\cdot \frac{ig}{2\sqrt{2}M_W} [m_b \, tg\beta (1+\gamma^5) + m_i \cot\beta (1-\gamma^5)] V(\bar{b}) .$$

$$\cdot \frac{i^2}{(K^2 - M_{H^+}^2)^2} \qquad (1.4)$$

$$M_a^{(1)} = i \left(\frac{g}{2\sqrt{2}M_W} \right)^4 \sum_{i,j} \xi_i \xi_j \int \frac{d^4K}{(2\pi)^4} \bar{V}(\bar{q}) [m_q \, tg\beta (1-\gamma^5) + m_j \cot\beta (1+\gamma^5)]$$

$$(\not{K} + m_j) [m_b \, tg\beta (1+\gamma^5) + m_j \cot\beta (1-\gamma^5)] U(b) . \bar{U}(q) [m_q \, tg\beta (1-\gamma^5)$$

$$+ m_i \cot\beta (1+\gamma^5)] (\not{K} + m_i) [m_b \, tg\beta (1+\gamma^5) + m_i \cot\beta (1-\gamma^5)] V(\bar{b})$$

$$\cdot \frac{1}{(K^2 - m_j^2)(K^2 - m_i^2)(K^2 - M_{H^+}^2)^2} \qquad (1.5)$$

$$\not{K} = \gamma^\mu K_\mu$$

① $[m_q \, tg\beta (1-\gamma^5) + m_j \cot\beta (1+\gamma^5)] (\gamma^\mu K_\mu + m_j) [m_b \, tg\beta (1+\gamma^5)$
$+ m_j \cot\beta (1-\gamma^5)] = [m_q \, tg\beta (1-\gamma^5) + m_j \cot\beta (1+\gamma^5)]$

$$[m_b \, tg\beta \, \gamma^\mu (1+\gamma^5) K_\mu + m_j \cot\beta \, \gamma^\mu (1-\gamma^5) K_\mu + m_j m_b \, tg\beta (1+\gamma^5)$$
$$+ m_j^2 \cot\beta (1-\gamma^5)]$$

$$= 2 m_q m_b \, tg^2\beta (1-\gamma^5) \not{K} + 2 m_q m_j^2 (1-\gamma^5) + 2 m_j^2 \cot^2\beta (1+\gamma^5) \not{K}$$
$$+ 2 m_j^2 m_b (1+\gamma^5)$$

because of: $\left\{\begin{array}{l}(1+\gamma^5)(1-\gamma^5) = 1-\gamma^{5^2} = 1-1 = 0. \\ \gamma^\mu\gamma^5+\gamma^5\gamma^\mu = 0\end{array}\right.$

(2) $[m_q \, tg\beta(1-\gamma^5) + m_i \cot\beta(1+\gamma^5)]\,(\gamma^\mu K_\mu + m_i)\,[m_b tg\beta(1+\gamma^5) + m_i \cot\beta\,(1-\gamma^5)]$

$$= [m_q \, tg\beta(1-\gamma^5) + m_i \cot\beta(1+\gamma^5)]\,[m_b tg\beta\,\gamma^\mu(1+\gamma^5)K_\mu + m_i\cot\beta\,\gamma^\mu(1-\gamma^5)K_\mu + m_i m_b tg\beta(1+\gamma^5) + m_i^2\cot\beta(1-\gamma^5)]$$

$$= 2m_q m_b tg^2\beta\,(1-\gamma^5)\not{K} + 2m_i^2 m_q(1-\gamma^5) + 2m_i^2\cot^2\beta\,(1+\gamma^5)\not{K} + 2m_i^2 m_b(1+\gamma^5)$$

$\Rightarrow$

$$M_a^{(1)} = (2)^2 i\left(\frac{g}{2\sqrt{2}M_W}\right)^4 \sum_{i,j}\xi_i\xi_j\int\frac{d^4K}{(2\pi)^4}\,\bar{V}(\bar{q})\,[m_q m_b tg^2\beta(1-\gamma^5)\not{K} + m_q m_j^2(1-\gamma^5) + m_j^2\cot^2\beta\,(1+\gamma^5)\not{K} + m_b m_j^2(1+\gamma^5)]\,U(b)$$
$$\bar{U}(q)\,[m_q m_b tg^2\beta(1-\gamma^5)\not{K} + m_i^2 m_q(1-\gamma^5) + m_i^2\cot^2\beta(1+\gamma^5)\not{K} + m_i^2 m_b(1+\gamma^5)]\,V(\bar{b})\;.\;\frac{1}{(K^2-m_j^2)(K^2-m_i^2)(K^2-M_{H^+}^2)^2} \quad (1.6)$$

Let's consider the following integrals:

$$I_{\alpha\beta}(i,j) = \int\frac{d^4K}{(2\pi)^4}\,\frac{K_\alpha K_\beta}{(K^2-M_{H^+}^2)^2(K^2-m_i^2)(K^2-m_j^2)} \quad (1.7)$$

$$I_\alpha(i,j) = \int\frac{d^4K}{(2\pi)^4}\,\frac{K_\alpha}{(K^2-M_{H^+}^2)^2(K^2-m_i^2)(K^2-m_j^2)} \quad (1.8)$$

$$I''(i,j) = \int\frac{d^4K}{(2\pi)^4}\,\frac{1}{(K^2-M_{H^+}^2)^2(K^2-m_i^2)(K^2-m_j^2)} \quad (1.9)$$

$$\frac{1}{abcd} = 3!\int_0^1\int_0^1\int_0^1\frac{dx\,dy\,dz\;x^2y}{[(1-x)d + x(1-y)c + xy(1-z)b + xyza]^4} \quad (1.10)$$

$a = K^2-M_{H^+}^2$
$b = K^2-M_{H^+}^2$
$c = K^2-m_i^2$
$d = K^2-m_j^2$

$$[(1-x)d + x(1-y)c + xy(1-z)b + xyza]^4 =$$

$$[(1-x)(k^2-m_j^2) + x(1-y)(k^2-m_i^2) + xy(1-z)(k^2-M_{H^+}^2) + xyz(k^2-M_{H^+}^2)]^4$$

$$= [k^2-m_j^2 - \cancel{xk^2} + xm_j^2 + \cancel{xk^2} - xm_i^2 - \cancel{xyk^2} + xym_i^2 + \cancel{xyk^2} - xyM_{H^+}^2$$
$$- \cancel{xyzk^2} + \cancel{xyzM_{H^+}^2} + \cancel{xyzk^2} - \cancel{xyzM_{H^+}^2}]^4$$

$$= [k^2 - m_j^2 + xm_j^2 - xm_i^2 + xym_i^2 - xyM_{H^+}^2]^4$$

$$= [k^2 + x(m_j^2 - m_i^2) + xy(m_i^2 - M_{H^+}^2) - m_j^2]^4$$

$$\Rightarrow \boxed{\frac{1}{(k^2-M_{H^+}^2)^2(k^2-m_i^2)(k^2-m_j^2)} = 3!\int_0^1\int_0^1\int_0^1 \frac{dx\,dy\,dz\;x^2y}{[k^2+x(m_j^2-m_i^2)+xy(m_i^2-M_{H^+}^2)-m_j^2]^4}} \quad (1.11)$$

using this relation we can evaluate $I_{\alpha\beta}(i,j)$ (See DØ Note 1372 and personal notes of Carlos Martin) replacing M_W by M_{H^+}

$$\boxed{\begin{aligned} I_{\alpha\beta}(i,j) &= \frac{-i\pi^2 g_{\alpha\beta}}{4(2\pi)^4 M_{H^+}^2}\left[\frac{J(X_i)-J(X_j)}{X_i-X_j}\right] \\ \text{with}\quad J(X_i) &= \frac{1}{1-X_i} + \frac{X_i^2\ln(X_i)}{(1-X_i)^2} \\ X_i &= \frac{m_i^2}{M_{H^+}^2}\end{aligned}}$$

(Page A12, 8)

(1.12)

$$\boxed{\begin{aligned} I''(i,j) &= \frac{i\pi^2}{(2\pi)^4 M_{H^+}^4 (1-X_i)(1-X_j)}\left[F(i,j)+F(j,i)-1\right] \\ F(i,j) &= \frac{-X_i\ln(X_i)(1-X_j)}{(1-X_i)(X_i-X_j)} \qquad X_i = m_i^2/M_{H^+}^2 \\ F(j,i) &= \frac{-X_j\ln(X_j)(1-X_i)}{(1-X_j)(X_j-X_i)} \qquad X_j = m_j^2/M_{H^+}^2\end{aligned}}$$

(Page 12 eq. 35)

(1.13)

Let's consider the integral:

$$I_\mu = \int \frac{d^n p}{(2\pi)^n} \frac{p_\mu}{(p^2 + 2K\cdot p + M^2 + i\varepsilon)^\alpha} = -K_\mu I_0$$

$$I_0 = \int \frac{d^n p}{(2\pi)^n} \cdot \frac{1}{(p^2 + 2K\cdot p + M^2 + i\varepsilon)^\alpha} = \frac{i(-\pi)^{n/2}}{(2\pi)^n} \cdot \frac{\Gamma(\alpha - \frac{1}{2}n)}{\Gamma(\alpha)} \cdot$$

$$\cdot \frac{1}{(M^2 - K^2 + i\varepsilon)^{\alpha - \frac{n}{2}}} \qquad (1.14)$$

(Stefan Pokorski "Gauge Field Theories")

For $n = 4, \alpha = 4$ $\quad K \leftrightarrow P$ $\quad I_\alpha(i,j) = 3! \iiint_0^1 dx\,dy\,dz\, x^2 y \int \frac{d^4K}{(2\pi)^4} \frac{K_\alpha}{[K^2 + M^2]^4}$

$\Rightarrow$

$$I_\alpha(i,j) \propto -P_\alpha \int \frac{d^4K}{(2\pi)^4} \frac{1}{(K^2 + 2P\cdot K + M^2 + i\varepsilon)^4} = \frac{-i}{2^4 \pi^2} \cdot \frac{1}{3!} \cdot$$

$$\boxed{M^2 = X(m_j^2 - m_i^2) + XY(m_i^2 - M_{H^+}^2) - m_j^2} \cdot \frac{1}{(M^2 - K^2 + i\varepsilon)^2} P_\alpha$$

In our case: $P_\alpha = 0$

$$\Rightarrow \boxed{I_\alpha(i,j) = 0} \qquad (1.15)$$

Introducing (12, 13.15) in (6) we have:

$$M_a^{(1)} = 2^2 i \left(\frac{g}{2\sqrt{2}M_W}\right)^4 \sum_{i,j} \xi_i \xi_j \Big[m_q^2 m_b^2 \, tg^4\beta \, \bar{V}(\bar{q})(1-\gamma^5)\gamma^\alpha U(b) . \bar{U}(q)(1-\gamma^5)\gamma^\beta$$

$$V(\bar{b}) I_{\alpha\beta}(i,j) + m_i^2 m_q m_b \bar{V}(\bar{q})(1-\gamma^5)\gamma^\alpha U(b) . \bar{U}(q)(1+\gamma^5)\gamma^\beta V(\bar{b}) I_{\alpha\beta}(i,j)$$

$$+ m_i^2 m_j^2 m_q^2 \bar{V}(\bar{q})(1-\gamma^5) U(b) . \bar{U}(q)(1-\gamma^5) V(\bar{b}) I''(i,j)$$

$$+ m_i^2 m_j^2 m_q m_b \bar{V}(\bar{q})(1-\gamma^5) U(b) . \bar{U}(q)(1+\gamma^5) V(\bar{b}) I''(i,j)$$

$$+ m_j^2 m_q m_b \bar{V}(\bar{q})(1+\gamma^5)\gamma^\alpha U(b) . \bar{U}(q)(1-\gamma^5)\gamma^\beta V(\bar{b}) I_{\alpha\beta}(i,j)$$

$$+ m_i^2 m_j^2 \cot^4\beta \, \bar{V}(\bar{q})(1+\gamma^5)\gamma^\alpha U(b) . \bar{U}(q)(1+\gamma^5)\gamma^\beta V(\bar{b}) I_{\alpha\beta}(i,j)$$

$$+ m_i^2 m_j^2 m_q m_b \bar{V}(\bar{q}) (1+\gamma^5) U(b) . \bar{U}(q) (1-\gamma^5) V(\bar{b}) I''(i,j)$$

$$+ m_i^2 m_j^2 m_b^2 \bar{V}(\bar{q}) (1+\gamma^5) U(b) . \bar{U}(q) (1+\gamma^5) V(\bar{b}) I''(i,j) \Big]$$

In the limit $m_q \to 0$ $(q = d \text{ or } s)$ (1.16)

$$M_a^{(1)} = 2^2 i \left(\frac{g}{2\sqrt{2} M_W}\right)^4 \sum_{i,j} \xi_i \xi_j \Big\{ m_i^2 m_j^2 \cot^4\beta \, \bar{V}(\bar{q}) (1+\gamma^5) \gamma^\alpha U(b) . \bar{U}(q) (1+\gamma^5) \gamma^\beta V(\bar{b}) I_{\alpha\beta}(i,j) + m_i^2 m_j^2 m_b^2 \bar{V}(\bar{q}) (1+\gamma^5) U(b) . \bar{U}(q) (1+\gamma^5) V(\bar{b}) I''(i,j) \Big\} \quad (1.17)$$

$$\boxed{M_a^{(1)} = 2^2 i \left(\frac{g}{2\sqrt{2} M_W}\right)^4 \sum_{i,j} \xi_i \xi_j \Big\{ m_i^2 m_j^2 \cot^4\beta \, \bar{V}(\bar{q}) \gamma^\alpha (1-\gamma^5) U(b) . \bar{U}(q) \gamma^\beta (1-\gamma^5) V(\bar{b}) I_{\alpha\beta}(i,j) + m_i^2 m_j^2 m_b^2 \bar{V}(\bar{q}) (1+\gamma^5) U(b) . \bar{U}(q) (1+\gamma^5) V(\bar{b}) I''(i,j) \Big\}} \quad (1.18)$$

$\downarrow$ M^{HH}

$$\boxed{I_{\alpha\beta}(i,i) = \frac{-i\pi^2 g_{\alpha\beta}}{4(2\pi)^4 M_{H^+}^2} \left[\frac{(1-X_i^2) + 2X_i \ln X_i}{(1-X_i)^3} \right]} \quad (1.19)$$

In fact: $\lim_{X_j \to X_i} \left[\frac{J(X_i) - J(X_j)}{X_i - X_j} \right] = \lim_{X_j \to X_i} \left[\frac{d}{dX_j} J(X_j) \right]$

$$= \lim_{X_j \to X_i} \left[+ \frac{1}{(1-X_j)^2} + \left[(2X_j \ln X_j + X_j)(1-X_j)^2 + X_j^2 \ln X_j \, 2(1-X_j) \right] / (1-X_j)^4 \right]$$

$$= \lim_{X_j \to X_i} \left[\frac{1}{(1-X_j)^2} + \frac{X_j(2\ln X_j + 1)(1-X_j) + 2X_j^2 \ln X_j}{(1-X_j)^3} \right]$$

$$= \frac{(1-\cancel{X_i}) + 2X_i \ln X_i - \cancel{2X_i^2 \ln X_i} + \cancel{X_i} - X_i^2 + \cancel{2X_i^2 \ln X_i}}{(1-X_i)^3}$$

$$= \frac{(1-X_i^2) + 2X_i \ln X_i}{(1-X_i)^3}$$

$$\boxed{I''(i,i) = \frac{-i\pi^2}{(2\pi)^4 M_{H+}^4 (1-x_i)^2}\left[2 + \frac{(1+x_i)}{(1-x_i)} \ln x_i\right]} \qquad (1.20)$$

In fact:

$$F(i,j) + F(j,i) = \frac{1}{(x_i - x_j)}\left[-\frac{x_i \ln(x_i)(1-x_j)}{(1-x_i)} + \frac{x_j \ln(x_j)(1-x_i)}{(1-x_j)}\right]$$

$$\lim_{x_j \to x_i}\left[F(i,j) + F(j,i)\right] = \lim_{x_j \to x_i}\left\{-\frac{x_i \ln(x_i)}{(1-x_i)} - (1-x_i)\cdot\left[\frac{(\ln x_j + 1)\cdot(1-x_j) + x_j \ln x_j}{(1-x_j)^2}\right]\right\}$$

$$= -\left\{\frac{x_i \ln x_i}{(1-x_i)} + \frac{(\ln x_i + 1 - \cancel{x_i \ln x_i} - x_i + \cancel{x_i \ln x_i})}{(1-x_i)}\right\}$$

$$= -1 - \ln x_i \frac{(1+x_i)}{(1-x_i)}$$

$$\therefore \lim_{x_j \to x_i}\left[F(i,j) + F(j,i) - 1\right] = -\left[2 + \frac{(1+x_i)}{(1-x_i)} \ln x_i\right]$$

neglecting the second term in (1.18) we have:

$$\boxed{M^{HH}_{Total} = 2^3 i \left(\frac{g}{2\sqrt{2}}\right)^4 \frac{1}{M_W^2} \sum_{i,j} \xi_i \xi_j \cot^4\beta\ \bar{V}(q)\gamma^\mu(1-\gamma^5)U(b)\cdot \bar{U}(q)\gamma_\mu(1-\gamma^5)V(\bar{b})\, I_{1*}^{HH}(i,j)} \qquad (1.21)$$

(Pages 129-132)

$$I_{1*}^{HH}(i,j) = \frac{-i\pi^2 (x_W^H)^{-1}}{4(2\pi)^4} x_i^H x_j^H \left[\frac{J(x_i^H) - J(x_j^H)}{x_i^H - x_j^H}\right] \qquad (1.22)$$

with $$J(x_i^H) = \frac{1}{1-x_i^H} + \frac{{x_i^H}^2 \ln(x_i^H)}{(1-x_i^H)^2} \qquad (1.23)$$

$$x_i^H = \frac{m_i^2}{M_{H+}^2}$$

$$\boxed{I_{1*}^{HH}(i,i) = \frac{-i\pi^2 (X_W^H)^{-1} (X_i^H)^2}{4(2\pi)^4}\left[\frac{(1-X_i^{H2}) + 2X_i^H \ln X_i^H}{(1-X_i^H)^3}\right]} \qquad (1.24)$$

$$\boxed{\lim_{X_i^H \to 0} I_{1*}^{HH}(i,i) = 0} \qquad (1.25)$$

because :

$$\lim_{X_i^H \to 0} X_i^H \ln X_i^H = \lim_{X_i^H \to 0} \frac{\ln X_i^H}{\frac{1}{X_i^H}} = \frac{\frac{1}{X_i^H}}{-\frac{1}{X_i^{H2}}}$$

$$= \lim_{X_i^H \to 0} (-X_i^H) = 0$$

$$C_1 = \frac{\cancel{2}^3 i \, g^4}{2^4 \cancel{2^2}} \frac{M_W^2}{M_W^4} \frac{(-\cancel{i})\pi^{\cancel{2}}}{\cancel{4} 2^4 \pi^4} = \frac{1}{2^9 \pi^2} \frac{g^4 M_W^2}{M_W^4}$$

but $\dfrac{G_F}{\sqrt{2}} = \dfrac{g^2}{8 M_W^2}$

$$\Rightarrow \frac{G_F^2}{2} = \frac{g^4}{64 M_W^4}$$

$$G_F^2 = \frac{g^4}{2^5 M_W^4}$$

$$\Rightarrow \boxed{C_1 = \frac{1}{2^4 \pi^2} G_F^2 M_W^2} \qquad (1.26)$$

$\Rightarrow$ See next page

$$\Rightarrow \boxed{M^{HH} = \frac{G_F^2 M_W^2}{16\pi^2} \cot^4\beta \sum_{i,j} \xi_i \xi_j \bar{v}(\bar{q}) \gamma^\mu (1-\gamma^5) u(b) \cdot \bar{u}(q) \gamma_\mu (1-\gamma^5) v(\bar{b})\, I_1^{HH}(i,j)} \qquad (1.27)$$

$$I_1^{HH}(i,j) = \frac{X_i^H X_j^H}{X_W^H} \left[\frac{J(X_i^H) - J(X_j^H)}{X_i^H - X_j^H} \right] \qquad (1.28)$$

with
$$J(X_i^H) = \frac{1}{1-X_i^H} + \frac{(X_i^H)^2 \ln(X_i^H)}{(1-X_i^H)^2}$$

$$X_i^H = \frac{m_i^2}{M_{H^+}^2} \; ; \quad X_W^H = \frac{M_W^2}{M_{H^+}^2}$$

$$I_1^{HH}(i,i) = \frac{(X_i^H)^2}{X_W^H} \left[\frac{(1-(X_i^H)^2) + 2X_i^H \ln X_i^H}{(1-X_i^H)^3} \right] \qquad (1.29)$$

$$\lim_{X_i^H \to 0} I_1^{HH}(i,i) = 0 \qquad (1.30)$$

Let's consider:

$$\langle B^0 | M^{HH} | \bar{B}^0 \rangle = \frac{G_F^2 M_W^2}{16\pi^2} \cot^4\beta \sum_{i,j} \xi_i \xi_j\, A'\, I_1^{HH}(i,j)$$

$$A' \equiv \langle B^0 | \bar{v}(\bar{q}) \gamma^\mu (1-\gamma^5) u(b) \cdot \bar{u}(q) \gamma_\mu (1-\gamma^5) v(\bar{b}) | \bar{B}^0 \rangle$$

$$A' = 4A = \frac{1}{3} f_B^2 m_B$$ (See Axial Currents)

$$\Rightarrow$$

$$\boxed{\langle B^0 | M^{HH} | \bar{B}^0 \rangle = \frac{G_F^2 M_W^2 \cot^4\beta\, f_B^2 m_B}{48\pi^2} \sum_{i,j} \xi_i \xi_j I_1^{HH}(i,j)} \cdot B_B \qquad (1.31)$$

($B_B = 1$ corresponds to the saturation by the vacuum intermediate state)

For our model considering free particles inside the Meson, we have: $A' = 4A = \frac{\eta\, m_B f_B^2}{4}$

$$\boxed{\langle B^0 | H^{HH} | \bar{B}^0 \rangle = \frac{G_F^2 M_W^2 \cot^4\beta\, f_B^2 m_B}{64\pi^2} \sum_{i,j} \xi_i \xi_j \overbrace{I_1^{HH}(i,j)}^{S^{HH}(i,j)}\, \eta \qquad (\eta = B_B)} \quad (1.32)$$

$$A = \frac{\bar{V}_L(\bar{q})\gamma^\mu U_L(b) \,.\, \bar{U}_L(q)\gamma_\mu V_L(\bar{b})}{\sqrt{2E_1 2E_2 2E_3 2E_4}\; V} \rightarrow \eta\, \frac{m_B f_B^2}{16}$$

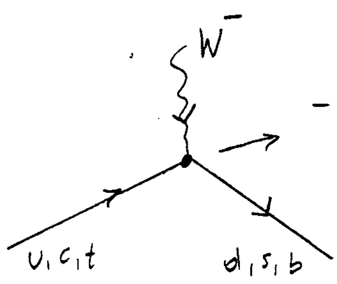
$$\sum_{i,j} \xi_i \xi_j I_1^{HH}(i,j) = \xi_u^2 \cancelto{0}{I_1^{HH}(u,u)} + \xi_c^2 \cancelto{0}{I_1^{HH}(c,c)}$$
$$+ \xi_t^2 I_1^{HH}(t,t) + 2\xi_u \xi_c \cancelto{0}{I_1^{HH}(u,c)} + 2\xi_u \xi_t \cancelto{0}{I_1^{HH}(u,t)}$$
$$+ 2\xi_c \xi_t I_1^{HH}(c,t)$$

$$\boxed{\sum_{i,j} \xi_i \xi_j I_1^{HH}(i,j) \approx 2\xi_c \xi_t I_1^{HH}(c,t) + \xi_t^2 I_1^{HH}(t,t)} \quad (1.33)$$

$$\boxed{\approx \xi_t^2 I_1^{HH}(t,t)} \quad (1.33\text{ a})$$

Let's call: $\boxed{I_1^{HH}(i,j) \equiv S^{HH}(i,j)}$

To evaluate the box diagrams b)



CKM matrix element

$$-\frac{ig}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)V_{pn}^{*}$$

$p = u, c, t$

$n = d, s, b$

$$W^{\mp}, Z^0 \quad \mu \sim\!\!\sim_{K}\!\!\sim \nu \qquad \frac{-i\left[\eta_{\mu\nu} + \dfrac{(\xi-1)K_\mu K_\nu}{K^2-\xi M_{W,Z}^2}\right]}{K^2 - M_{W,Z}^2 + i\varepsilon} \tag{2.1}$$

where $\xi = 1$ in the 't Hooft-Feynman gauge

$\xi = 0$ in the Landau gauge

$\xi = \infty$ in the unitary gauge

In the unitary gauge the propagator for $W^{\pm}, Z^0$ reads:

$$\frac{-i\left[\eta_{\mu\nu} - \dfrac{K_\mu K_\nu}{M_{W,Z}^2}\right]}{K^2 - M_{W,Z}^2 + i\varepsilon} \tag{2.2}$$

In this gauge ghosts diagrams disappear because the propagators are:

$$\phi^{\pm}, w^{\pm} \quad \xrightarrow{K} \qquad \frac{\pm i}{K^2 - \xi M_W^2 + i\varepsilon} \tag{2.3}$$

The invariant amplitude corresponding to diagrams b) is:

$$-iM_b^{(1)} = \sum_{i,j} \xi_i \xi_j \int \frac{d^4k}{(2\pi)^4} \bar{V}(\bar{q}) \left[\frac{-ig}{\sqrt{2}} \gamma^\mu \frac{1}{2}(1-\gamma^5)\right] \frac{i(\not{k}+m_j)}{k^2-m_j^2} \cdot$$

$$\frac{ig}{2\sqrt{2}M_W}\left[m_b\, tg\beta(1+\gamma^5) + m_j \cot\beta(1-\gamma^5)\right] U(b) \cdot \bar{U}(q)$$

$$\frac{ig}{2\sqrt{2}M_W}\left[m_q\, tg\beta(1-\gamma^5) + m_i \cot\beta(1+\gamma^5)\right] \frac{i(\not{k}+m_i)}{k^2-m_i^2} \cdot$$

$$\cdot\left[-\frac{ig}{\sqrt{2}}\gamma^\nu \frac{1}{2}(1-\gamma^5)\right] V(\bar{b})\,(-i)\left[n_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}\right] \cdot \frac{1}{(k^2-M_W^2)}$$

$$\frac{i}{k^2-M_{H^+}^2} \qquad (2.4)$$

$$M_b = -i\left(\frac{g}{2\sqrt{2}}\right)^4 \frac{1}{M_W^2}\sum_{i,j}\xi_i\xi_j \int\frac{d^4k}{(2\pi)^4}\bar{V}(\bar{q})\gamma^\mu(1-\gamma^5)(\not{k}+m_j) \cdot$$

$$\left[m_b\, tg\beta(1+\gamma^5) + m_j\cot\beta(1-\gamma^5)\right]U(b) \cdot \bar{U}(q)\left[m_q\, tg\beta(1-\gamma^5) +\right.$$

$$\left.+ m_i\cot\beta(1+\gamma^5)\right](\not{k}+m_i)\gamma^\nu(1-\gamma^5)V(\bar{b})\left[n_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}\right] \cdot$$

$$\cdot \frac{1}{(k^2-M_W^2)(k^2-M_{H^+}^2)(k^2-m_i^2)(k^2-m_j^2)} \qquad (2.5)$$

$$M_b = -i\left(\frac{g}{2\sqrt{2}}\right)^4 \frac{1}{M_W^2}\sum_{i,j}\xi_i\xi_j\int\frac{d^4k}{(2\pi)^4}\left\{\bar{V}(\bar{q})\gamma^\mu(1-\gamma^5)(\not{k}+m_j)\right.$$

$$\left[m_b\, tg\beta(1+\gamma^5) + m_j\cot\beta(1-\gamma^5)\right]U(b)\cdot\bar{U}(q)\left[m_q\, tg\beta(1-\gamma^5)+\right.$$

$$\left.+m_i\cot\beta(1+\gamma^5)\right](\not{k}+m_i)\gamma_\mu(1-\gamma^5)V(\bar{b}) - \frac{1}{M_W^2}\bar{V}(\bar{q})\not{k}(1-\gamma^5)(\not{k}+m_j)\cdot$$

$$\left[m_b\, tg\beta(1+\gamma^5) + m_j\cot\beta(1-\gamma^5)\right]U(b)\cdot\bar{U}(q)\left[m_q\, tg\beta(1-\gamma^5) + m_i\cot\beta\cdot\right.$$

$$\left.\left.\cdot(1+\gamma^5)\right](\not{k}+m_i)\not{k}(1-\gamma^5)V(\bar{b})\right\} \cdot \frac{1}{(k^2-M_W^2)(k^2-M_{H^+}^2)(k^2-m_i^2)(k^2-m_j^2)}$$

$$(2.6)$$

$$M_b = -i\left(\frac{g}{2\sqrt{2}}\right)^4 \frac{1}{M_W^2}\sum_{i,j}\xi_i\xi_j\int\frac{d^4k}{(2\pi)^4}\Big\{\bar{V}(\bar{q})\gamma^\mu(1-\gamma^5).$$

$$[m_b\, tg\beta\,(1-\gamma^5)\not{k} + m_j m_b\, tg\beta\,(1+\gamma^5) + m_j\cot\beta\,(1+\gamma^5)\not{k}$$

$$+ m_j^2\cot\beta\,(1-\gamma^5)]\,U(b).\,\bar{U}(q)\,[m_q\, tg\beta\,(1-\gamma^5) + m_i\cot\beta\,(1+\gamma^5)]$$

$$[(1-\gamma^5)\not{k} + m_i(1+\gamma^5)]\,\gamma_\mu V(\bar{b}) - \frac{1}{M_W^2}\bar{V}(\bar{q})\not{k}(1-\gamma^5).$$

$$[m_b\, tg\beta\,(1-\gamma^5)\not{k} + m_j m_b\, tg\beta\,(1+\gamma^5) + m_j\cot\beta\,(1+\gamma^5)\not{k} + m_j^2\cot\beta\,(1-\gamma^5)]$$

$$U(b).\,\bar{U}(q)\,[m_q\, tg\beta\,(1-\gamma^5) + m_i\cot\beta\,(1+\gamma^5)]\,[(1-\gamma^5)\not{k} + m_i(1+\gamma^5)]$$

$$\not{k}\,V(\bar{b})\Big\}.\ \frac{1}{(k^2-M_W^2)(k^2-M_{H^+}^2)(k^2-m_i^2)(k^2-m_j^2)} \qquad (2.7)$$

$$M_b = -i\left(\frac{g}{2\sqrt{2}}\right)^4\frac{1}{M_W^2}\sum_{i,j}\xi_i\xi_j\int\frac{d^4k}{(2\pi)^4}\Big\{[2m_b\, tg\beta\,\bar{V}(\bar{q})\gamma^\mu(1-\gamma^5)\not{k}$$

$$+2m_j^2\cot\beta\,\bar{V}(\bar{q})\gamma^\mu(1-\gamma^5)]\,U(b).\,\bar{U}(q)\,[2m_q\, tg\beta\,(1-\gamma^5)\not{k} +$$

$$+2m_i^2\cot\beta\,(1+\gamma^5)\ ]\,\gamma_\mu V(\bar{b}) - \frac{1}{M_W^2}[2m_b\, tg\beta\,\bar{V}(\bar{q})\not{k}(1-\gamma^5)\not{k}$$

$$+2m_j^2\cot\beta\,\bar{V}(\bar{q})\not{k}(1-\gamma^5)]\,U(b).\,\bar{U}(q)\,[2m_q\, tg\beta\,(1-\gamma^5)\not{k}$$

$$+2m_i^2\cot\beta\,(1+\gamma^5)\ ]\not{k}\,V(\bar{b})\Big\}.\ \frac{1}{(k^2-M_W^2)(k^2-M_{H^+}^2)(k^2-m_i^2)(k^2-m_j^2)}$$

$$(2.8)$$

$$M_b = -(2)^2 i\left(\frac{g}{2\sqrt{2}}\right)^4\frac{1}{M_W^2}\sum_{i,j}\xi_i\xi_j\int\frac{d^4k}{(2\pi)^4}\Big\{[m_b\, tg\beta\,\bar{V}(\bar{q})\gamma^\mu(1-\gamma^5)\not{k}$$

$$+m_j^2\cot\beta\,\bar{V}(\bar{q})\gamma^\mu(1-\gamma^5)]\,U(b).\,\bar{U}(q)\,[m_q\, tg\beta\,(1-\gamma^5)\not{k} +$$

$$+m_i^2\cot\beta\,(1+\gamma^5).\ ]\,\gamma_\mu V(\bar{b}) - \frac{1}{M_W^2}[\ m_b\, tg\beta\, k^2\,\bar{V}(\bar{q})(1+\gamma^5)$$

$$+m_j^2\cot\beta\,\bar{V}(\bar{q})\not{k}(1-\gamma^5)]\,U(b).\,\bar{U}(q)\,[m_q\, tg\beta\,(1-\gamma^5)\,k^2$$

$$+m_i^2\cot\beta\,(1+\gamma^5)\not{k}\,]\,V(\bar{b})\Big\}.\ \frac{1}{(k^2-M_W^2)(k^2-M_{H^+}^2)(k^2-m_i^2)(k^2-m_j^2)}$$

$$(2.9)$$

We have used: $\gamma^\mu\gamma^5+\gamma^5\gamma^\mu=0$; $(1+\gamma^5)^2 = 2(1+\gamma^5)$;
$(1-\gamma^5)^2 = 2(1-\gamma^5)$; $\not{k}\not{k} = k^2$.

We need to evaluate the integrals:

$$I_{\alpha}^{HW\,(1)}(i,j) \equiv \int \frac{d^4K}{(2\pi)^4} \frac{K_\alpha}{(K^2-M_W^2)(K^2-M_{H^+}^2)(K^2-m_i^2)(K^2-m_j^2)} \qquad (2.10)$$

$$I_{\alpha\beta}^{HW}(i,j) \equiv \int \frac{d^4K}{(2\pi)^4} \frac{K_\alpha K_\beta}{(K^2-M_W^2)(K^2-M_{H^+}^2)(K^2-m_i^2)(K^2-m_j^2)} \qquad (2.11)$$

$$I^{HW}(i,j) \equiv \int \frac{d^4K}{(2\pi)^4} \cdot \frac{1}{(K^2-M_W^2)(K^2-M_{H^+}^2)(K^2-m_i^2)(K^2-m_j^2)} \qquad (2.12)$$

$$I_{\alpha}^{HW*\,(2)}(i,j) \equiv \int \frac{d^4K}{(2\pi)^4} \cdot \frac{K^2 K_\alpha}{(K^2-M_W^2)(K^2-M_{H^+}^2)(K^2-m_i^2)(K^2-m_j^2)} \qquad (2.13)$$

$$I^{HW*}(i,j) \equiv \int \frac{d^4K}{(2\pi)^4} \cdot \frac{K^4}{(K^2-M_W^2)(K^2-M_{H^+}^2)(K^2-m_i^2)(K^2-m_j^2)} \qquad (2.14)$$

For the last integral let's consider:

$$\frac{m_i^2 m_j^2}{K^4(K^2-m_i^2)(K^2-m_j^2)} - \frac{1}{K^4} + \frac{1}{K^2(K^2-m_i^2)} + \frac{1}{K^2(K^2-m_j^2)}$$

$$= \frac{m_i^2 m_j^2 - (K^2-m_i^2)(K^2-m_j^2) + K^2(K^2-m_j^2) + K^2(K^2-m_i^2)}{K^4(K^2-m_i^2)(K^2-m_j^2)}$$

$$= \frac{\cancel{m_i^2 m_j^2} - \cancel{K^4} + \cancel{K^2 m_j^2} + \cancel{K^2 m_i^2} - \cancel{m_i^2 m_j^2} + \cancel{K^4} - \cancel{K^2 m_j^2} + K^4 - \cancel{K^2 m_i^2}}{K^4(K^2-m_i^2)(K^2-m_j^2)}$$

$$= \frac{K^4}{K^4(K^2-m_i^2)(K^2-m_j^2)} = \frac{1}{(K^2-m_i^2)(K^2-m_j^2)} \qquad (2.15)$$

Let's evaluate:

$$I_{HW}^{**}(i,j) \equiv \sum_{i,j} \xi_i \xi_j m_i \int \frac{d^4K}{(2\pi)^4} \cdot \frac{K^4}{(K^2-M_W^2)(K^2-M_{H^+}^2)(K^2-m_i^2)(K^2-m_j^2)}$$

Using (2.15) we have:

$$\sum_{i,j} \frac{\xi_i \xi_j m_i}{(K^2-m_i^2)(K^2-m_j^2)} = \sum_{i,j} \frac{m_i^3 m_j^2 \xi_i \xi_j}{K^4(K^2-m_i^2)(K^2-m_j^2)} - \sum_{i,j} \cancelto{0}{\frac{m_i \xi_i \xi_j}{K^4}}$$

$$+ \sum_{i,j} \cancelto{0}{\frac{\xi_i \xi_j m_i}{K^2(K^2-m_i^2)}} + \sum_{i,j} \frac{\xi_i \xi_j m_i}{K^2(K^2-m_j^2)}$$

Remember that $\sum_i \xi_i = 0 \;;\; \sum_j \xi_j = 0$

$$\Rightarrow \quad I^{xx}_{HW}(i,j) = \sum_{i,j} m_i^3 m_j^2 \xi_i \xi_j \int \frac{d^4K}{(2\pi)^4} \frac{1}{(K^2-M_W^2)(K^2-M_{H^+}^2)(K^2-m_i^2)(K^2-m_j^2)}$$

$$+ \sum_{i,j} m_i \xi_i \xi_j \int \frac{d^4K}{(2\pi)^4} \frac{K^2}{(K^2-M_W^2)(K^2-M_{H^+}^2)(K^2-m_j^2)}$$

$$\boxed{I^{xx}_{HW}(i,j) = \sum_{i,j} m_i^3 m_j^2 \xi_i \xi_j \, I^{HW}(i,j) + \sum_{i,j} m_i \xi_i \xi_j \, I^r_{HW}(i,j)} \qquad (2.16)$$

where $$\boxed{I^r_{HW}(i,j) = \int \frac{d^4K}{(2\pi)^4} \frac{K^2}{(K^2-M_W^2)(K^2-M_{H^+}^2)(K^2-m_j^2)}} \qquad (2.17)$$

To evaluate (2.10) we will consider the identity:

$$\frac{1}{abcd} = 3! \int_0^1 \int_0^1 \int_0^1 \frac{X^2 Y \, dx\, dY\, dz}{[(1-x)d + x(1-Y)c + XY(1-z)b + XYza]^4}$$

Let be

$$a = K^2 - M_{H^+}^2$$
$$b = K^2 - M_W^2$$
$$c = K^2 - m_i^2$$
$$d = K^2 - m_j^2$$

$$\Rightarrow \quad (1-x)d + x(1-Y)c + XY(1-z)b + XYza =$$

$$(1-x)(K^2-m_j^2) + x(1-Y)(K^2-m_i^2) + XY(1-z)(K^2-M_W^2) + XYz(K^2-M_{H^+}^2)$$

$$= \cancel{K^2} - \cancel{m_j^2} - X\cancel{K^2} + X\cancel{m_j^2} + X\cancel{K^2} - X\cancel{m_i^2} - XY\cancel{K^2} + XY\cancel{m_i^2} + XY\cancel{K^2}$$
$$- XY\cancel{M_W^2} - XYz\cancel{K^2} + XYz\cancel{M_W^2} + XY\cancel{z}K^2 - XYz\cancel{M_{H^+}^2} =$$

$$= K^2 + X(m_j^2 - m_i^2) + XY(m_i^2 - M_W^2) + XYz(M_W^2 - M_{H^+}^2) - m_j^2$$

$$= K^2 + M^2$$

$$\therefore \quad \boxed{(1-x)d + x(1-Y)c + XY(1-z)b + XYza = K^2 + M^2} \qquad (2.18)$$

$$\boxed{\text{with } M^2 = X(m_j^2 - m_i^2) + XY(m_i^2 - M_W^2) + XYz(M_W^2 - M_{H^+}^2) - m_j^2} \qquad (2.19)$$

Then:

$$I_\alpha^{HW}(i,j) = 3!\int \frac{d^4K}{(2\pi)^4}\, K_\alpha \int_0^1\int_0^1\int_0^1 \frac{x^2y\,dx\,dy\,dz}{[K^2+M^2]^4}$$

$$= 3!\int_0^1\int_0^1\int_0^1 x^2y\,dx\,dy\,dz \int\frac{d^4K}{(2\pi)^4}\,\frac{K_\alpha}{(K^2+M^2)^4}$$

Using:

$$I_\alpha = \int\frac{d^nK}{(2\pi)^n}\,\frac{K_\alpha}{(K^2+2P\cdot K+M^2+i\varepsilon)^\alpha} = -P_\alpha I_0 \qquad (2.20)$$

$$I_0 = \int\frac{d^nK}{(2\pi)^n}\,\frac{1}{(K^2+2P\cdot K+M^2+i\varepsilon)^\alpha} = \frac{i(-\pi)^{n/2}}{(2\pi)^n}\,\frac{\Gamma(\alpha-\frac{1}{2}n)}{\Gamma(\alpha)}\cdot\frac{1}{(M^2-P^2+i\varepsilon)^{\alpha-\frac{n}{2}}} \qquad (2.21)$$

$$\int\frac{d^4K}{(2\pi)^4}\,\frac{K_\alpha}{(K^2+M^2)^4} = 0 \qquad (n=4,\ \alpha=4)$$

$$\Rightarrow \boxed{I_\alpha^{HW}(i,j) = 0} \qquad (2.22)$$

$$I_\alpha^{HW^*}(i,j) = 3!\int\frac{d^4K}{(2\pi)^4}\,\not K\,\not K\,K_\alpha \int_0^1\int_0^1\int_0^1 \frac{x^2y\,dx\,dy\,dz}{[K^2+M^2]^4}$$

$$= 3!\,\gamma^\mu\gamma^\nu\int_0^1\int_0^1\int_0^1 x^2y\,dx\,dy\,dz\int\frac{d^4K}{(2\pi)^4}\,\frac{K_\mu K_\nu K_\alpha}{[K^2+M^2]^4} \qquad (2.23)$$

Using:

$$I_{\mu\nu\alpha} = \int\frac{d^nK}{(2\pi)^n}\,\frac{K_\mu K_\nu K_\alpha}{[K^2+2P\cdot K+M^2+i\varepsilon]^\alpha}$$

$$= -I_0\left[P_\mu P_\nu P_\alpha + \frac{1}{2}(\eta_{\mu\nu}P_\alpha+\eta_{\mu\alpha}P_\nu+\eta_{\nu\alpha}P_\mu)\frac{(M^2-P^2)}{\alpha-\frac{1}{2}n-1}\right] \qquad (2.24)$$

with $n=4,\ \alpha=4$

we get: $\boxed{I_\alpha^{HW^*}(i,j) = 0}$ (2.25)

$$I^{HW}_{\alpha\beta}(i,j) = 3! \int_0^1\int_0^1\int_0^1 x^2 y\, dx\, dy\, dz \int \frac{d^4k}{(2\pi)^4} \frac{k_\alpha k_\beta}{(k^2+M^2)^4}$$

Using :

$$I_{\alpha\beta} = \int \frac{d^n k}{(2\pi)^n} \frac{k_\alpha k_\beta}{(k^2+2p\cdot k + M^2 + i\varepsilon)^\alpha} = I_0\left[P_\alpha P_\beta + \frac{1}{2} n_{\alpha\beta}(M^2-p^2) \cdot \frac{1}{\alpha - \frac{1}{2}n - 1}\right] \quad (2\text{-}26)$$

$$\Rightarrow \int \frac{d^4k}{(2\pi)^4} \frac{k_\alpha k_\beta}{(k^2+M^2)^4} = \frac{i\cancel{\pi^2}}{(2\cancel{\pi})^4} \cdot \frac{1}{3!} \cdot \frac{1}{\cancel{(M^2)}^2} \cdot \frac{1}{2} n_{\alpha\beta} \,\cancel{(M^2)}$$

$$= \frac{i\pi^2}{2(2\pi)^4} \cdot \frac{1}{3!} \frac{n_{\alpha\beta}}{M^2} \quad (2.27)$$

$$\therefore I^{HW}_{\alpha\beta}(i,j) = \frac{i\pi^2}{2(2\pi)^4} n_{\alpha\beta} \int_0^1\int_0^1\int_0^1 \frac{x^2 y\, dx\, dy\, dz}{x(m_j^2-m_i^2) + xy(m_i^2-M_W^2) + xyz(M_W^2-M_H^2) - m_j^2} \quad (2\text{-}28)$$

Putting $a = x(m_j^2 - m_i^2) + xy(m_i^2 - M_W^2) - m_j^2$

$b = xy(M_W^2 - M_{H^+}^2)$

$$\int_0^1 \frac{dz}{a+bz} = \frac{1}{b} \ln|a+bz|\Big|_0^1$$

$$= \frac{1}{b}\ln\left|\frac{a+b}{a}\right|$$

$$= \frac{1}{xy(M_W^2 - M_{H^+}^2)} \ln\left|\frac{x(m_j^2-m_i^2)+xy(m_i^2-M_W^2)-m_j^2+xy(M_W^2-M_H^2)}{x(m_j^2-m_i^2)+xy(m_i^2-M_W^2)-m_j^2}\right|$$

$X_i = \frac{m_i^2}{M_{H^+}^2}$; $X_W = \frac{M_W^2}{M_{H}^{+2}}$ ← ojo

$$\int_0^1 \frac{dz}{a+bz} = \frac{1}{xy M_H^2 (X_W - 1)} \ln\left|\frac{x(X_j - X_i) + xy(X_i - X_W) - X_j + xy(X_W - 1)}{x(X_j - X_i) + xy(X_i - X_W) - X_j}\right|$$

$$= \frac{1}{xy M_H^2 (X_W-1)} \ln\left|\frac{x(X_j - X_i) - X_j + yx(X_i - 1)}{x(X_j-X_i) - X_j + yx(X_i - X_W)}\right| \quad (2.29)$$

$$J = \int_0^1 \frac{dY}{XM_H^2(X_W-1)} \ln\left|\frac{X(X_j-X_i)-X_j+YX(X_i-1)}{X(X_j-X_i)-X_j+YX(X_i-X_W)}\right|$$

$$\int_0^1 \ln(c+dY)\,dY = \frac{1}{d}\left[(c+dY)\ln(c+dY)-(c+dY)\right]$$

$$= \frac{1}{d}\left[(c+d)\ln(c+d)-d-c\ln c\right]$$

$\Rightarrow$

$$J = \frac{1}{XM_H^2(X_W-1)}\left\{\int_0^1 dY \ln\left[X(X_j-X_i)-X_j+YX(X_i-1)\right]\right.$$
$$\left.-\int_0^1 dY \ln\left[X(X_j-X_i)-X_j+YX(X_i-X_W)\right]\right\}$$

$$J = \frac{1}{X^2M_H^2(X_W-1)}\left\{\frac{1}{(X_i-1)}\left[(X_j(X-1)-X)\ln(X_j(X-1)-X)-X(X_i-1)\right.\right.$$
$$\left.-(X(X_j-X_i)-X_j)\ln(X(X_j-X_i)-X_j)\right]$$
$$-\frac{1}{(X_i-X_W)}\left[(X_j(X-1)-XX_W)\ln(X_j(X-1)-XX_W)-X(X_i-X_W)\right.$$
$$\left.\left.-(X(X_j-X_i)-X_j)\ln(X(X_j-X_i)-X_j)\right]\right\}$$

$$J = \frac{1}{X^2M_H^2(X_W-1)}\left\{\frac{1}{(X_i-1)}\left[(X(X_j-1)-X_j)\ln(X(X_j-1)-X_j)-X(X_i-1)\right.\right.$$
$$\left.-(X(X_j-X_i)-X_j)\ln(X(X_j-X_i)-X_j)\right]$$
$$-\frac{1}{(X_i-X_W)}\left[(X(X_j-X_W)-X_j)\ln(X(X_j-X_W)-X_j)-X(X_i-X_W)\right.$$
$$\left.\left.-(X(X_j-X_i)-X_j)\ln(X(X_j-X_i)-X_j)\right]\right\} \qquad (2.30)$$

Now we have to calculate:

$$\int_0^1 X^2 J(X)\,dX = \int_0^1\int_0^1\int_0^1 \frac{X^2Y\,dX\,dY\,dZ}{X(m_j^2-m_i^2)+XY(m_i^2-M_W^2)+XYZ(M_W^2-M_H^2)-m_j^2}$$

$$J_1 = \int_0^1 [(x(x_j-1)-x_j] \ln (x(x_j-1)-x_j)\,dx$$

Let be $\quad x(x_j-1)-x_j = u$

$$du = (x_j-1)\,dx$$

$$\frac{1}{(x_j-1)}\int_{-x_j}^{-1} u \ln u \, du = \frac{1}{(x_j-1)} \cdot \frac{u^2}{2}\left(\ln|u| - \frac{1}{2}\right)\Bigg|_{-x_j}^{-1}$$

$$= \frac{1}{(x_j-1)}\left[\frac{1}{2}\left(-\frac{1}{2}\right) - \frac{x_j^2}{2}\left(\ln x_j - \frac{1}{2}\right)\right]$$

$$J_1 = -\frac{1}{4(x_j-1)}\left[1 + x_j^2(2\ln x_j - 1)\right] \qquad (2.31)$$

$$J_2 = \int_0^1 [x(x_j-x_i)-x_j] \ln (x(x_j-x_i)-x_j)\,dx$$

Putting $\quad x(x_j-x_i)-x_j = u$

$$du = (x_j-x_i)\,dx$$

$$\Rightarrow \quad J_2 = \frac{1}{(x_j-x_i)}\int_{-x_j}^{-x_i} u \ln u \, du$$

$$= \frac{1}{(x_j-x_i)} \frac{u^2}{2}\left(\ln|u| - \frac{1}{2}\right)\Bigg|_{-x_j}^{-x_i}$$

$$= \frac{1}{(x_j-x_i)}\left[\frac{x_i^2}{2}\left(\ln x_i - \frac{1}{2}\right) - \frac{x_j^2}{2}\left(\ln x_j - \frac{1}{2}\right)\right]$$

$$J_2 = \frac{1}{4(x_j-x_i)}\left[x_i^2(2\ln x_i - 1) - x_j^2(2\ln x_j - 1)\right] \qquad (2.32)$$

$$J_3 = \int_0^1 [x(x_j - x_w) - x_j] \ln [x(x_j - x_w) - x_j]\, dx$$

is the same as I_2 replacing $x_i \to x_w$

$$J_3 = \frac{1}{4(x_j - x_w)} \left[x_w^2 (2\ln x_w - 1) - x_j^2 (2 \ln x_j - 1)\right] \qquad (2\text{-}33)$$

$$\Rightarrow \int_0^1 x^2 I(x)\, dx = \frac{1}{M_H^2 (x_w - 1)} \left\{ \frac{-1}{4(x_i - 1)(x_j - 1)} [\cancel{1} + x_j^2 (2\ln x_j - 1)] \right.$$

$$- \cancel{\tfrac{1}{2}} - \frac{1}{4(x_i - 1)(x_j - x_i)} [x_i^2 (2 \ln x_i - 1) - x_j^2 (2 \ln x_j - 1)]$$

$$- \frac{1}{4(x_i - x_w)(x_j - x_w)} [x_w^2 (2\ln x_w - 1) - x_j^2 (2\ln x_j - 1)] + \cancel{\tfrac{1}{2}}$$

$$\left. + \frac{1}{4(x_i - x_w)(x_j - x_i)} [x_i^2 (2 \ln x_i - 1) - x_j^2 (2\ln x_j - 1)] \right\}$$

$\therefore$

$$I^{HW}_{\alpha\beta}(i,j) = \frac{i\pi^2}{2^3 (2\pi)^4} \eta_{\alpha\beta} \cdot \frac{1}{M_H^2 (x_w - 1)} \left\{ -\frac{1}{(x_j - x_i)} [x_i^2 (2\ln x_i - 1) - x_j^2 (2\ln x_j - 1)] \right.$$

$$\cdot \left[\frac{1}{(x_i - 1)} - \frac{1}{(x_i - x_w)} \right] - \frac{1}{(x_i - 1)(x_j - 1)} [1 + x_j^2 (2\ln x_j - 1)]$$

$$\left. - \frac{1}{(x_i - x_w)(x_j - x_w)} [x_w^2 (2\ln x_w - 1) - x_j^2 (2 \ln x_j - 1)] \right\}$$

$$\boxed{\begin{aligned} I^{HW}_{\alpha\beta}(i,j) = -\frac{i\pi^2}{2^3 (2\pi)^4} \eta_{\alpha\beta} \cdot \frac{1}{M_H^2 (x_w - 1)} \Bigg\{ & -\frac{(x_w - 1)[x_i^2 (2\ln x_i - 1) - x_j^2 (2\ln x_j - 1)]}{(x_j - x_i)(x_i - 1)(x_i - x_w)} \\ & + \frac{1}{(x_i - 1)(x_j - 1)} [1 + x_j^2 (2\ln x_j - 1)] \\ & + \frac{1}{(x_i - x_w)(x_j - x_w)} [x_w^2 (2 \ln x_w - 1) - x_j^2 (2\ln x_j - 1)] \Bigg\} \end{aligned}} \qquad (2.34)$$

$$\left[\frac{(x_w-1)}{(x_j-x_i)(x_i-1)(x_i-x_w)} + \frac{1}{(x_i-1)(x_j-1)} - \frac{1}{(x_i-x_w)(x_j-x_w)} \right] x_j^2\,(2\ln x_j - 1)$$

$$[\ ] = \frac{(x_w-1)(x_j-1)(x_j-x_w) + (x_j-x_i)(x_i-x_w)(x_j-x_w) - (x_j-x_i)(x_i-1)(x_j-1)}{(x_j-x_i)(x_i-1)(x_j-1)(x_i-x_w)(x_j-x_w)}$$

$$= \big[(x_w-1)(x_j^2 - x_j x_w - x_j + x_w) + (x_j-x_i)(x_i x_j - x_i x_w - x_w x_j + x_w^2)$$
$$- (x_j-x_i)(x_i x_j - x_i - x_j + 1)\big] / (\ )(\ )(\ )(\ )(\ )$$

$$= \big[\cancel{x_j^2 x_w} - \cancel{x_j x_w^2} - \cancel{x_j x_w} + x_w^2 - \cancel{x_j^2} + \cancel{x_j x_w} + \cancel{x_j} - x_w + \cancel{x_i x_j^2} - \cancel{x_i x_j x_w}$$
$$- \cancel{x_w x_j^2} + \cancel{x_j x_w^2} - \cancel{x_i^2 x_j} + x_i^2 x_w + \cancel{x_i x_j x_w} - x_i x_w^2 - \cancel{x_i x_j^2}$$
$$+ \cancel{x_i x_j} + \cancel{x_j^2} - \cancel{x_j} + \cancel{x_i^2 x_j} - x_i^2 - \cancel{x_i x_j} + x_i \big] / (\ )(\ )(\ )(\ )(\ )$$

$$= \big[x_w^2 - x_w + x_i^2 x_w - x_i x_w^2 - x_i^2 + x_i \big] / (\ )(\ )(\ )(\ )(\ )$$

$$= \frac{x_w^2(1-x_i) - x_w(1-x_i)(1+x_i) - x_i(-1+x_i)}{(x_j-x_i)(x_i-1)(x_j-1)(x_i-x_w)(x_j-x_w)}$$

$$= \frac{-x_w^2 + x_w(1+x_i) - x_i}{(x_j-x_i)(x_j-1)(x_i-x_w)(x_j-x_w)}$$

$$= \frac{x_w(1-x_w) + x_i(x_w-1)}{(x_j-x_i)(x_j-1)(x_i-x_w)(x_j-x_w)} = \frac{(x_w-1)\cancel{(x_i-x_w)}}{(x_j-x_i)(x_j-1)\cancel{(x_i-x_w)}(x_j-x_w)}$$

$$= \frac{(x_w-1)}{(x_j-x_i)(x_j-1)(x_j-x_w)}$$

ok.

$$\Rightarrow \boxed{\begin{aligned} I^{HW}_{\alpha\beta}(i,j) = \frac{-i\pi^2}{2^3(2\pi)^4}\,\eta_{\alpha\beta}\,\frac{1}{M_{H^+}^2(x_w-1)} \Bigg\{ & -\frac{(x_w-1)\,x_i^2\,(2\ln x_i-1)}{(x_j-x_i)(x_i-1)(x_i-x_w)} \\ & - \frac{(x_w-1)}{(x_i-x_j)(x_j-1)(x_j-x_w)}\,x_j^2\,(2\ln x_j-1) + \frac{1}{(x_i-1)(x_j-1)} \\ & + \frac{x_w^2\,(2\ln x_w-1)}{(x_i-x_w)(x_j-x_w)} \Bigg\} \qquad (2.35) \end{aligned}}$$

Let be

$$G(X_i) = \frac{X_i^2\,(2\ln X_i - 1)}{(X_i - 1)(X_i - X_w)} \qquad (2.35\,a)$$

$$G(X_j) = \frac{X_j^2\,(2\ln X_j - 1)}{(X_j - 1)(X_j - X_w)} \qquad (2.35\,b)$$

$$\frac{G(X_i) - G(X_j)}{(X_i - X_j)}$$

$$\lim_{X_j \to X_i} \frac{G(X_i) - G(X_j)}{(X_i - X_j)} = \lim_{X_j \to X_i} \left(\frac{d\,G(X_j)}{d\,X_j} \right)$$

$$= \lim_{X_j \to X_i} \left[\frac{\left(2X_j\,(2\ln X_j - 1) + 2X_j\right)(X_j - 1)(X_j - X_w) - X_j^2\,(2\ln X_j - 1)\,(2X_j - X_w - 1)}{(X_j - 1)^2\,(X_j - X_w)^2} \right]$$

$$= \lim_{X_j \to X_i} \left[\frac{4X_j \ln X_j\,(X_j - 1)(X_j - X_w) - X_j^2\,(2\ln X_j - 1)\,(2X_j - X_w - 1)}{(X_j - 1)^2\,(X_j - X_w)^2} \right]$$

$$= \lim_{X_j \to X_i} \Big[4X_j \ln X_j\,(X_j^2 - X_j X_w - X_j + X_w) - X_j^2\,(4X_j \ln X_j - 2X_w \ln X_j - 2\ln X_j - 2X_j + X_w + 1 \Big] \Big/ (X_j - 1)^2\,(X_j - X_w)^2$$

$$= \lim_{X_j \to X_i} \Big[\cancel{4X_j^3 \ln X_j} - 4X_j^2 \cancel{X_w} \ln X_j - 4X_j^2 \cancel{\ln} X_j + 4X_j \cancel{X_w} \ln X_j - \cancel{4X_j^3 \ln X_j} + \cancel{2X_j^2} X_w \ln X_j + 2\cancel{X_j^2} \ln X_j + 2\cancel{X_j^3} - X_j^2 \cancel{X_w} - \cancel{X_j^2} \Big] \Big/ (X_j - 1)^2\,(X_j - X_w)^2$$

$$= \lim_{X_j \to X_i} \Big[-2X_j^2 \ln X_j\,X_w - 2X_j^2 \ln X_j + 4X_j \ln X_j\,X_w + 2X_j^3 - X_j^2 X_w - X_j^2 \Big] \Big/ (X_j - 1)^2\,(X_j - X_w)^2$$

$$= \lim_{X_j \to X_i} \left[\frac{-2X_j \ln X_j (X_j X_w + X_j - 2X_w) + X_j^2 (2X_j - 1 - X_w)}{(X_j-1)^2 \; (X_j - X_w)^2} \right]$$

$$= X_i \left[\frac{-2 \ln X_i \; (X_i X_w + X_i - 2X_w) + X_i (2X_i - 1 - X_w)}{(X_i-1)^2 (X_i - X_w)^2} \right]$$

$$\Rightarrow \boxed{\begin{aligned} I^{HW}_{\alpha\beta}(i,i) = & -\frac{i\pi^2}{2^3 (2\pi)^4} \, n_{\alpha\beta} \frac{1}{M_{H^+}^2 (X_w - 1)} \left\{ \frac{X_i (X_w - 1)}{(X_i-1)^2 (X_i - X_w)^2} \cdot \right. \\ & \cdot \left[-2 \ln X_i \, (X_i X_w + X_i - 2X_w) + X_i (2X_i - 1 - X_w) \right] \\ & \left. + \frac{1}{(X_i - 1)^2} + \frac{X_w^2 (2 \ln X_w - 1)}{(X_i - X_w)^2} \right\} \qquad (2.35\,c) \end{aligned}}$$

If we begin with (2.34)

$$\lim_{X_j \to X_i} \left[\frac{X_i^2 (2 \ln X_i - 1) - X_j^2 (2 \ln X_j - 1)}{(X_j - X_i)} \right]$$

$$= \lim_{X_j \to X_i} \left[-2X_j (2 \ln X_j - 1) - 2X_j \right]$$

$$= -4 X_i \ln X_i$$

$$\Rightarrow I^{HW}_{\alpha\beta}(i,i) = \frac{-i\pi^2}{2^3 (2\pi)^4} \, n_{\alpha\beta} \frac{1}{M_{H^2} (X_w - 1)} \left\{ \frac{(X_w - 1) \, 4 X_i \ln X_i}{(X_i - 1)(X_i - X_w)} \right.$$

$$+ \frac{X_i^2 (2 \ln X_i - 1)}{(X_i - 1)^2} - \frac{X_i^2 (2 \ln X_i - 1)}{(X_i - X_w)^2}$$

$$\left. + \frac{1}{(X_i - 1)^2} + \frac{X_w^2 (2 \ln X_w - 1)}{(X_i - X_w)^2} \right\}$$

The first three terms in { } can be written as:

$$\frac{+4X_i \ln X_i (X_w-1)(X_i-1)(X_i-X_w) + X_i^2(2\ln X_i-1)(X_i-X_w)^2 -}{(X_i-1)^2(X_i-X_w)^2}$$

$$\underline{-X_i^2(2\ln X_i-1)(X_i-1)^2}$$

$$= [(X_w-1)(+4X_i\ln X_i)(X_i^2-X_iX_w-X_i+X_w)$$
$$+ X_i^2(2\ln X_i-1)(-2X_iX_w+X_w^2+2X_i-1)]/(\ )^2(\ )^2$$

$$= [(X_w-1)(4X_i\ln X_i)(X_i^2-X_iX_w-X_i+X_w) + X_i^2(2\ln X_i-1)\cdot$$
$$(2X_i(1-X_w)+(X_w-1)(X_w+1))]/(\ )^2(\ )^2$$

$$= (X_w-1)X_i[4\ln X_i(X_i^2-X_iX_w-X_i+X_w) + X_i(2\ln X_i-1)\cdot$$
$$\cdot(-2X_i+X_w+1)]/(X_i-1)^2(X_i-X_w)^2$$

$$= X_i(X_w-1)[\cancel{4X_i^2\ln X_i} - 4X_i\ln X_i X_w - \cancel{4X_i\ln X_i} + 4\ln X_i X_w$$
$$- \cancel{4X_i^2\ln X_i} + 2X_iX_w\ln X_i + \cancel{2X_i\ln X_i} + 2X_i^2$$
$$- X_iX_w - X_i]/(X_i-1)^2(X_i-X_w)^2$$

$$= X_i(X_w-1)[-2X_iX_w\ln X_i - 2X_i\ln X_i + 4(\ln X_i)X_w + 2X_i^2$$
$$- X_iX_w - X_i]/(X_i-1)^2(X_i-X_w)^2$$

$$= X_i(X_w-1)[-2\ln X_i(X_iX_w+X_i-2X_w) + X_i(2X_i-X_w-1)]/(\ )^2(\ )^2$$

$\Rightarrow$ Again we arrive to eq. (2.35c)

$$I^{HW}_{\alpha\beta}(i,i) = \frac{-i\pi^2}{2^3\,(2\pi)^4}\, n_{\alpha\beta} \cdot \frac{1}{M_{H^+}^2\,(X_W-1)} \left\{ \frac{X_i\,(X_W-1)}{(X_i-1)^2\,(X_i-X_W)^2} \cdot \right.$$

$$\cdot \Big[-2\ln X_i \left(X_i X_W + X_i - 2X_W \right) + X_i \left(2X_i - X_W - 1 \right) \Big]$$

$$\left. + \frac{1}{(X_i-1)^2} + \frac{X_W^2\,(2\ln X_W - 1)}{(X_i-X_W)^2} \right\} \qquad (2.35\,c)$$

$$I^{HW}_{\alpha\beta}(i,i) = n_{\alpha\beta}\, I^{HW}_{**}(i,j)$$

Next, we will evaluate the following integral:

$$I^{HW}(i,j) = \int \frac{d^4K}{(2\pi)^4}\; 3! \int_0^1\int_0^1\int_0^1 \frac{x^2 y\, dx\, dy\, dz}{[K^2+M^2]^4}$$

$$= 3! \int_0^1\int_0^1\int_0^1 x^2 y\, dx\, dy\, dz \cdot \int \frac{d^4K}{(2\pi)^4}\cdot\frac{1}{[K^2+M^2]^4}$$

$$= \cancel{3!}\int_0^1\int_0^1\int_0^1 x^2 y\, dx\, dy\, dz\; \frac{i\pi^2}{(2\pi)^4}\cdot\frac{1}{\cancel{3!}}\cdot\frac{1}{(M^2)^2}$$

$$I^{HW}(i,j) = \frac{i\pi^2}{(2\pi)^4}\int_0^1\int_0^1\int_0^1 \frac{x^2 y\, dx\, dy\, dz}{[x(m_j^2-m_i^2)+xy(m_i^2-M_W^2)+xyz(M_W^2-M_H^2)-m_j^2]^2} \qquad (2.36)$$

Putting

$a = x(m_j^2 - m_i^2) + xy(m_i^2 - M_W^2) - m_j^2$

$b = xy(M_W^2 - M_H^2)$

$$\int_0^1 \frac{dz}{[a+bz]^2} = -\frac{1}{b}\cdot\frac{1}{(a+bz)}\Big|_0^1 = -\frac{1}{b}\cdot\frac{1}{(a+b)} + \frac{1}{ab}$$

$$= \frac{1}{\cancel{b}}\;\frac{\cancel{b}}{a(a+b)} = \frac{1}{a(a+b)}$$

$$= \frac{1}{[x(m_j^2-m_i^2)+xy(m_i^2-M_W^2)-m_j^2][x(m_j^2-m_i^2)+xy(m_i^2-M_H^2)-m_j^2]}$$

$$\int_0^1 \frac{dz}{[a+bz]^2} = \frac{1}{M_{H^+}^4[x(x_j-x_i)+xy(x_i-x_W)-x_j][x(x_j-x_i)+xy(x_i-1)-x_j]} \qquad (2.37)$$

$$\frac{1}{M_{H^+}^2}\int_0^1 \frac{y\, dy}{(C_1+yC_2)(C_1+yC_3)} = ?$$

$C_1 = x(x_j - x_i) - x_j$; $C_2 = x(x_i - x_W)$

$C_3 = x(x_i - 1)$

$$\frac{Y}{(C_1+YC_2)(C_1+YC_3)} = \frac{A}{(C_1+YC_2)} + \frac{B}{(C_1+YC_3)}$$

$$= \frac{AC_1 + AY\cancel{C_3} + BC_1 + BY\cancel{C_2}}{(C_1+YC_2)(C_1+YC_3)}$$

$$= \frac{Y(AC_3+BC_2) + (AC_1+BC_1)}{(C_1+YC_2)(C_1+YC_3)}$$

$$\Rightarrow \begin{array}{l} AC_3 + BC_2 = 1 \\ C_1(A+B) = 0 \quad \Rightarrow \quad A = -B \end{array}$$

$$A(C_3-C_2) = 1$$

$$A = \frac{1}{C_3-C_2} = -B$$

$$\frac{1}{H_{H^+}^4}\int_0^1 \frac{Y\,dY}{(C_1+YC_2)(C_1+YC_3)} = \frac{1}{(C_3-C_2)}\int_0^1 \frac{dY}{C_1+YC_2} - \frac{1}{(C_3-C_2)}\int_0^1 \frac{dY}{C_1+YC_3}$$

$$= \frac{1}{C_2(C_3-C_2)} \ln|C_1+YC_2|\Big|_0^1$$

$$- \frac{1}{C_3(C_3-C_2)} \ln|C_1+YC_3|\Big|_0^1$$

$$= \frac{1}{C_2(C_3-C_2)} \ln\left|\frac{C_1+C_2}{C_1}\right|$$

$$- \frac{1}{C_3(C_3-C_2)} \ln\left|\frac{C_1+C_3}{C_1}\right|$$

$$L(x) = \frac{1}{H_{H^+}^4}\int_0^1 \frac{Y\,dY}{(C_1+YC_2)(C_1+YC_3)} = \frac{1}{H_{H^+}^4}\frac{1}{(C_3-C_2)}\left[\frac{1}{C_2}\ln\left|\frac{C_1+C_2}{C_1}\right| - \frac{1}{C_3}\ln\left|\frac{C_1+C_3}{C_1}\right|\right] \tag{2.38}$$

$$= \frac{1}{H_{H^+}^4}\frac{1}{x^2(X_w-1)}\left[\frac{1}{(X_i-X_w)}\ln\left|\frac{x(X_j-X_w)-X_j}{x(X_j-X_i)-X_j}\right| - \frac{1}{(X_i-1)}\cdot \ln\left|\frac{x(X_j-1)-X_j}{x(X_j-X_i)-X_j}\right|\right] \tag{2.39}$$

$$\int_0^1 x^2 L(x)\, dx = \frac{1}{M_{H^+}^4}\cdot\frac{1}{(x_w-1)}\left\{\frac{1}{(x_i-x_w)}\int_0^1 \ln\left[\frac{x(x_j-x_w)-x_j}{x(x_j-x_i)-x_j}\right]dx\right.$$

$$\left.-\frac{1}{(x_i-1)}\int_0^1 \ln\left[\frac{x(x_j-1)-x_j}{x(x_j-x_i)-x_j}\right]dx\right\} \quad (2.40)$$

$$\int_0^1 \ln\left[x(x_j-x_w)-x_j\right]dx = \frac{1}{(x_j-x_w)}\left[u\ln|u|-u\right]\Big|_{-x_j}^{-x_w}$$

$$=\frac{1}{(x_j-x_w)}\left[-x_w\ln x_w + x_w + x_j\ln x_j - x_j\right] \quad (2.41)$$

$$\int_0^1 \ln\left[x(x_j-x_i)-x_j\right]dx = \frac{1}{(x_j-x_i)}\left[u\ln|u|-u\right]\Big|_{-x_j}^{-x_i}$$

$$=\frac{1}{(x_j-x_i)}\left[-x_i\ln x_i + x_i + x_j\ln x_j - x_j\right] \quad (2.42)$$

$$\int_0^1 \ln\left[x(x_j-1)-x_j\right]dx = \frac{1}{(x_j-1)}\left[u\ln|u|-u\right]\Big|_{-x_j}^{-1}$$

$$=\frac{1}{(x_j-1)}\left[1 + x_j\ln x_j - x_j\right] \quad (2.43)$$

$\Rightarrow$

$$\int_0^1 x^2 L(x)\, dx = \frac{1}{M_{H^+}^4(x_w-1)}\left\{\frac{1}{(x_i-x_w)}\left[\frac{1}{(x_j-x_w)}(-x_w\ln x_w + x_w + x_j\ln x_j - x_j)\right.\right.$$

$$\left.-\frac{1}{(x_j-x_i)}(-x_i\ln x_i + x_i + x_j\ln x_j - x_j)\right]$$

$$-\frac{1}{(x_i-1)}\left[\frac{1}{(x_j-1)}(1 + x_j\ln x_j - x_j)\right.$$

$$\left.\left.-\frac{1}{(x_j-x_i)}(-x_i\ln x_i + x_i + x_j\ln x_j - x_j)\right]\right\}$$

$$(2.44)$$

$$\left[\frac{1}{(x_i-x_w)(x_j-x_w)}-\frac{1}{(x_i-x_w)(x_j-x_i)}-\frac{1}{(x_i-1)(x_j-1)}+\frac{1}{(x_i-1)(x_j-x_i)}\right]\cdot x_j(\ln x_j-1)$$

$$=\left[\frac{(x_j-x_i)(x_i-1)(x_j-1)-(x_j-x_w)(x_i-1)(x_j-1)-(x_i-x_w)(x_j-x_w)(x_j-x_i)+(x_i-x_w)(x_j-x_w)(x_j-1)}{(x_i-x_w)(x_j-x_w)(x_j-x_i)(x_i-1)(x_j-1)}\right]x_j(\ln x_j-1)$$

$$[\;]=\Big[(x_j-x_i)(x_ix_j-x_i-x_j+1)-(x_j-x_w)(x_ix_j-x_i-x_j+1)$$
$$-(x_i-x_w)(x_j^2-x_ix_j-x_wx_j+x_wx_i)$$
$$+(x_i-x_w)(x_j^2-x_j-x_wx_j+x_w)\Big]\Big/(\;)(\;)(\;)(\;)(\;)$$

$$=\Big[\cancel{x_ix_j^2}-\cancel{x_ix_j}-\cancel{x_j^2}+\cancel{x_j}-\cancel{x_i^2x_j}+x_i^2+\cancel{x_ix_j}-\cancel{x_i}-\cancel{x_ix_j^2}+\cancel{x_ix_j}+\cancel{x_j^2}$$
$$-\cancel{x_j}+x_ix_wx_j-\cancel{x_ix_w}-\cancel{x_jx_w}+\cancel{x_w}-\cancel{x_ix_j^2}+\cancel{x_i^2x_j}+x_ix_wx_j$$
$$-\cancel{x_i^2x_w}+\cancel{x_j^2x_w}-x_ix_jx_w-\cancel{x_w^2x_j}+x_w^2x_i+\cancel{x_ix_j^2}-\cancel{x_ix_j}$$
$$-x_ix_wx_j+\cancel{x_ix_w}-\cancel{x_wx_j^2}+\cancel{x_wx_j}+\cancel{x_w^2x_j}-x_w^2\Big]\Big/(\;)(\;)(\;)(\;)(\;)$$

$$=\frac{(x_i^2-\cancel{x_i}+x_w-\cancel{x_i^2x_w}+x_ix_w^2-x_w^2)}{(x_i-x_w)(x_j-x_w)(x_j-x_i)(x_i-1)(x_j-1)}$$

$$=\frac{x_i(x_i-1)+x_w(1-x_i)(1+x_i)+x_w^2(x_i-1)}{(x_i-x_w)(x_j-x_w)(x_j-x_i)(x_i-1)(x_j-1)}$$

$$=\frac{x_i-x_w(1+x_i)+x_w^2}{(x_i-x_w)(x_j-x_w)(x_j-x_i)(x_j-1)}=\frac{(x_i-x_w)+x_w(x_w-x_i)}{(x_i-x_w)(x_j-x_w)(x_j-x_i)(x_j-1)}$$

$$[\;]=\frac{(1-x_w)}{(x_j-x_w)(x_j-x_i)(x_j-1)}\qquad(2.45)$$

$$\Rightarrow \int_0^1 X^2 L(X)\,dX = \frac{1}{M_{H^+}^4 (X_w - 1)} \left\{ \frac{- X_w (\ln X_w - 1)}{(X_i - X_w)(X_j - X_w)} \right.$$

$$+ \frac{(1 - X_w)}{(X_j - X_w)(X_j - X_i)(X_j - 1)} X_j (\ln X_j - 1)$$

$$+ \left[- \frac{1}{(X_i - X_w)(X_j - X_i)} + \frac{1}{(X_i - 1)(X_j - X_i)} \right] X_i (1 - \ln X_i) \tag{2.46}$$

$$\left. - \frac{1}{(X_i - 1)(X_j - 1)} \right\}$$

$$\frac{1}{(X_i - 1)(X_j - X_i)} - \frac{1}{(X_j - X_i)(X_i - X_w)} = \frac{1}{(X_j - X_i)} \left[\frac{1}{(X_i - 1)} - \frac{1}{(X_i - X_w)} \right]$$

$$= \frac{(1 - X_w)}{(X_j - X_i)(X_i - 1)(X_i - X_w)} \tag{2.47}$$

$$\therefore \int_0^1 X^2 L(X)\,dX = \frac{1}{M_{H^+}^4 (X_w - 1)} \left\{ \frac{- X_w (\ln X_w - 1)}{(X_i - X_w)(X_j - X_w)} - \frac{1}{(X_i - 1)(X_j - 1)} \right.$$

$$+ \frac{(1 - X_w)}{(X_j - X_w)(X_j - X_i)(X_j - 1)} X_j (\ln X_j - 1)$$

$$\left. + \frac{(1 - X_w)}{(X_i - X_j)(X_i - 1)(X_i - X_w)} X_i (\ln X_i - 1) \right\} \tag{2.48}$$

Then

$$\boxed{\begin{aligned} I^{HW}(i,j) = -\frac{i\pi^2}{(2\pi)^4} \frac{1}{M_{H^+}^4 (X_w - 1)} \Bigg\{ & \frac{X_w (\ln X_w - 1)}{(X_i - X_w)(X_j - X_w)} + \\ & + \frac{1}{(X_i - 1)(X_j - 1)} + \frac{(X_w - 1)}{(X_j - X_w)(X_j - X_i)(X_j - 1)} X_j (\ln X_j - 1) \\ & + \frac{(X_w - 1)}{(X_i - X_w)(X_i - X_j)(X_i - 1)} X_i (\ln X_i - 1) \Bigg\} \end{aligned}} \tag{2.49}$$

Let be

$$h(x_i) \equiv \frac{x_i(\ln x_i - 1)}{(x_i - x_w)(x_i - 1)} \qquad (2.49\ a)$$

$$h(x_j) = \frac{x_j(\ln x_j - 1)}{(x_j - x_w)(x_j - 1)} \qquad (2.49\ b)$$

$$\lim_{x_j \to x_i} \frac{h(x_i) - h(x_j)}{x_i - x_j} = \lim_{x_j \to x_i} \frac{d}{dx_j} h(x_j) \qquad (\text{L'Hopital})$$

$$= \lim_{x_j \to x_i} \Big[\big((\ln x_j - \cancel{1}) + \cancel{1} \big)(x_j - x_w)(x_j - 1) - x_j(\ln x_j - 1) \cdot (2x_j - 1 - x_w) \Big] \Big/ (x_j - x_w)^2 (x_j - 1)^2$$

$$= \lim_{x_j \to x_i} \Big[\ln x_j \,(\cancel{x_j^2} - \cancel{x_j} - x_j\cancel{x_w} + \cancel{x_w}) - 2x_j^2 \cancel{\ln} x_j + x_j \cancel{\ln} x_j + x_j x_w \cancel{\ln} x_j + 2x_j^2 - x_j - x_j x_w \Big] \Big/ (\ )^2 (\ )^2$$

$$= \lim_{x_j \to x_i} \frac{\left[-x_j^2 \ln x_j + x_w \ln x_j + 2x_j^2 - x_j - x_j x_w \right]}{(x_j - x_w)^2 (x_j - 1)^2}$$

$$= \frac{\left[\ln x_i (x_w - x_i^2) + x_i (2x_i - 1 - x_w) \right]}{(x_i - x_w)^2 (x_i - 1)^2}$$

$$\Rightarrow \boxed{ I^{HW}(i,i) = \frac{-i\pi^2}{(2\pi)^4} \cdot \frac{1}{M_{H^+}^4 (x_w - 1)} \left\{ \frac{(x_w - 1)}{(x_i - x_w)^2 (x_i - 1)^2} \cdot \left[\ln x_i (x_w - x_i^2) + x_i (2x_i - 1 - x_w) \right] + \frac{x_w (\ln x_w - 1)}{(x_i - x_w)^2} + \frac{1}{(x_i - 1)^2} \right\} }$$

(2.49 c)

Going back to (2.41) If $x_j = x_i$

$$\int_0^1 \ln[\,x(x_i - x_w) - x_i\,]\,dx = \frac{1}{(x_i - x_w)}\left[-x_w \ln x_w + x_w + x_i \ln x_i - x_i\right]$$

$$\int_0^1 \ln x_i \, dx = \ln x_i$$

$$\int_0^1 \ln[\,x(x_i - 1) - x_i\,]\,dx = \frac{1}{(x_i - 1)}\left[1 + x_i \ln x_i - x_i\right]$$

$$\Rightarrow \int_0^1 x^2 L(x)\,dx = \frac{1}{M_{H^+}^4}\cdot\frac{1}{(x_w - 1)}\left\{\frac{1}{(x_i - x_w)}\left[\frac{1}{(x_i - x_w)}(-x_w \ln x_w + x_w + x_i \ln x_i - x_i) - \ln x_i\right] - \frac{1}{(x_i - 1)}\left[\frac{1}{(x_i - 1)}(1 + x_i \ln x_i - x_i) - \ln x_i\right]\right\}$$

$$= \frac{-1}{M_{H^+}^4 (x_w - 1)}\left\{\frac{x_w(\ln x_w - 1)}{(x_i - x_w)^2} + \frac{x_i(1 - \ln x_i)}{(x_i - x_w)^2} + \frac{\ln x_i}{(x_i - x_w)} + \frac{1}{(x_i - 1)^2}(1 + x_i \ln x_i - x_i) - \frac{\ln x_i}{(x_i - 1)}\right\}$$

$$\frac{x_i(1 - \ln x_i)}{(x_i - x_w)^2} + \ln x_i\left(\frac{1}{x_i - x_w} - \frac{1}{x_i - 1}\right) - \frac{x_i(1 - \ln x_i)}{(x_i - 1)^2}$$

$$= \frac{x_i(1 - \ln x_i)}{(x_i - x_w)^2} + \frac{(x_w - 1)\ln x_i}{(x_i - x_w)(x_i - 1)} - \frac{x_i(1 - \ln x_i)}{(x_i - 1)^2}$$

$$= x_i(1 - \ln x_i)\left[\frac{\cancel{x_i^2} - 2x_i + 1 - \cancel{x_i^2} + 2x_i x_w - x_w^2}{(x_i - x_w)^2 (x_i - 1)^2}\right] + \frac{(x_w - 1)\ln x_i}{(x_i - x_w)(x_i - 1)}$$

$$= \frac{x_i(1 - \ln x_i)\left[2x_i(x_w - 1) - (x_w - 1)(x_w + 1)\right]}{(x_i - x_w)^2 (x_i - 1)^2} + \frac{(x_w - 1)\ln x_i}{(x_i - x_w)(x_i - 1)}$$

$$= \frac{(X_W-1)\, X_i\,(1-\ln X_i)\,(2X_i - X_W - 1)}{(X_i - X_W)^2\,(X_i-1)^2} + \frac{(X_W-1)\ln X_i}{(X_i-X_W)(X_i-1)}$$

$$= \frac{(X_W-1)\left[\, X_i\,(2X_i - X_W - 1) - 2X_i^2 \cancel{\ln X_i} + X_i X_W \cancel{\ln X_i} + X_i \cancel{\ln X_i} + \right.}{(X_i-X_W)^2\,(X_i-1)^2}$$

$$\left. + \ln X_i\,(\cancel{X_i^2} - \cancel{X_i} - X_W \cancel{X_i} + X_W)\,\right]$$

$$= \frac{(X_W-1)\left[\, X_i\,(2X_i - X_W - 1) - X_i^2 \ln X_i + X_W \ln X_i \,\right]}{(X_i-X_W)^2\,(X_i-1)^2}$$

$$= \frac{(X_W-1)\left[\, \ln X_i\,(X_W - X_i^2) + X_i\,(2X_i - X_W - 1)\,\right]}{(X_i-X_W)^2\,(X_i-1)^2}$$

$$\therefore \boxed{\begin{aligned} I^{HW}(i,i) = \frac{-i\pi^2}{(2\pi)^4}\cdot\frac{1}{M_{H^+}^4\,(X_W-1)} \Bigg\{ & \frac{X_W(\ln X_W - 1)}{(X_i - X_W)^2} + \frac{1}{(X_i-1)^2} \\ & + (X_W-1)\frac{\left[\ln X_i\,(X_W - X_i^2) + X_i\,(2X_i - X_W - 1)\right]}{(X_i-X_W)^2\,(X_i-1)^2} \Bigg\} \end{aligned}}$$

that again is (2.49 c)

(2.49 c)

The invariant amplitude (2.9) can be written in terms of I_α^{HW}, $I_{\alpha\beta}^{HW}$, I^{HW}, I_α^{HW*}, I^{HW*} as:

$$M_b = -(2)^2 i \left(\frac{g}{2\sqrt{2}}\right)^4 \frac{1}{M_W^2} \sum_{i,j} \xi_i \xi_j \Big\{ m_q m_b \, tg^2\beta \, \bar{V}(\bar{q}) \gamma^\mu (1-\gamma^5) \gamma^\alpha$$

$$U(b) . \bar{U}(q) (1-\gamma^5) \gamma^\beta \gamma_\mu V(\bar{b}) \, I_{\alpha\beta}^{HW}$$

$$+ m_i^2 m_j^2 \cot^2\beta \, \bar{V}(\bar{q}) \gamma^\mu (1-\gamma^5) U(b) . \bar{U}(q) \gamma_\mu (1-\gamma^5) V(\bar{b}) \, I^{HW}(i,j)$$

$$- \frac{1}{M_W^2} m_q m_b \, tg^2\beta \, \bar{V}(\bar{q}) (1+\gamma^5) U(b) . \bar{U}(q) (1-\gamma^5) V(\bar{b}) \, I^{HW*}(i,j)$$

$$- \frac{1}{M_W^2} m_i^2 m_j^2 \cot^2\beta \, \bar{V}(\bar{q}) \gamma^\alpha (1-\gamma^5) U(b) . \bar{U}(q) \gamma^\beta (1-\gamma^5) V(\bar{b}) .$$

$$. I_{\alpha\beta}^{HW}(i,j) \Big\} \qquad (2.50)$$

In the limit $m_q \to 0$ $(q = d \text{ or } s)$

$$M_b = -(2)^2 i \left(\frac{g}{2\sqrt{2}}\right)^4 \frac{1}{M_W^2} \sum_{i,j} \xi_i \xi_j \Big\{ m_i^2 m_j^2 \cot^2\beta \, \bar{V}(\bar{q}) \gamma^\mu (1-\gamma^5) U(b) .$$

$$. \bar{U}(q) \gamma_\mu (1-\gamma^5) V(\bar{b}) \, I^{HW}(i,j) - \frac{1}{M_W^2} m_i^2 m_j^2 \cot^2\beta \, \bar{V}(\bar{q}) \gamma^\mu (1-\gamma^5) U(b) .$$

$$. \bar{U}(q) \gamma^\nu (1-\gamma^5) V(\bar{b}) \, I_{\mu\nu}^{HW}(i,j) \Big\}$$

Let's put: $\boxed{I_{\mu\nu}^{HW}(i,j) = \eta_{\mu\nu} I_{**}^{HW}(i,j)}$ (see 2.35)

$$\Rightarrow \boxed{\begin{aligned} M_{b\,\text{Total}} = -(2)^4 i \left(\frac{g}{2\sqrt{2}}\right)^4 \frac{\cot^2\beta}{M_W^2} \sum_{i,j} \xi_i \xi_j m_i^2 m_j^2 \, \bar{V}(\bar{q}) \gamma^\mu (1-\gamma^5) U(b) . \\ . \bar{U}(q) \gamma_\mu (1-\gamma^5) V(\bar{b}) \left[I^{HW}(i,j) - \frac{1}{M_W^2} I_{**}^{HW}(i,j) \right] \end{aligned}} \qquad (2.51)$$

see pages: 133-136

$I^{HW}(i,j)$; $I_{**}^{HW}(i,j)$ are given in: (2,49); (2.35)

$I^{HW}(i,i)$ is given in (2.49 c)

$I_{**}^{HW}(i,i)$ is given in (2.35 c)

We will write:

$$M^{HW} = -(2)^4 i \left(\frac{g}{2\sqrt{2}}\right)^4 \frac{\cot^2\beta}{M_W^2} \sum_{i,j} \xi_i \xi_j \bar{V}(\bar{q}) \gamma^\mu (1-\gamma^5) U(b) \cdot \bar{U}(q) \gamma_\mu (1-\gamma^5) V(\bar{b}) \left[I_{1*}^{HW}(i,j) - I_{2*}^{HW}(i,j) \right] \tag{2.52}$$

$$I_{1*}^{HW}(i,j) = \frac{-i\pi^2}{(2\pi)^4} \cdot \frac{X_i^H X_j^H}{(X_W^H - 1)} \left\{ \frac{X_W^H (\ln X_W^H - 1)}{(X_i^H - X_W^H)(X_j^H - X_W^H)} + \frac{1}{(X_i^H - 1)(X_j^H - 1)} + \frac{(X_W^H - 1)}{(X_j^H - X_W^H)(X_j^H - X_i^H)(X_j^H - 1)} X_j^H (\ln X_j^H - 1) + \frac{(X_W^H - 1)}{(X_i^H - X_W^H)(X_i^H - X_j^H)(X_i^H - 1)} X_i^H (\ln X_i^H - 1) \right\} \tag{2.53}$$

$$I_{1*}^{HW}(i,i) = \frac{-i\pi^2}{(2\pi)^4} \cdot \frac{(X_i^H)^2}{(X_W^H - 1)} \left\{ \frac{X_W^H (\ln X_W^H - 1)}{(X_i^H - X_W^H)^2} + \frac{1}{(X_i^H - 1)^2} + (X_W^H - 1) \frac{\left[\ln X_i^H (X_W^H - X_i^{H\,2}) + X_i^H (2X_i^H - X_W^H - 1) \right]}{(X_i^H - X_W^H)^2 (X_i^H - 1)^2} \right\} \tag{2.54}$$

$$\boxed{\lim_{X_i^H \to 0} I_{1*}^{HW}(i,i) = 0} \tag{2.55}$$

$$I_{2*}^{HW}(i,j) = \frac{-i\pi^2 (X_W^H)^{-1}}{2^3 (2\pi)^4} \frac{X_i^H X_j^H}{(X_W^H - 1)} \Bigg\{ - \frac{(X_W^H - 1)(X_i^H)^2 (2 \ln X_i^H - 1)}{(X_j^H - X_i^H)(X_i^H - 1)(X_i^H - X_W^H)}$$

$$- \frac{(X_W^H - 1)(X_j^H)^2 (2 \ln X_j^H - 1)}{(X_i^H - X_j^H)(X_j^H - 1)(X_j^H - X_W^H)} + \frac{1}{(X_i^H - 1)(X_j^H - 1)}$$

$$+ \frac{(X_W^H)^2 (2 \ln X_W^H - 1)}{(X_i^H - X_W^H)(X_j^H - X_W^H)} \Bigg\} \qquad (2.56)$$

$$I_{2*}^{HW}(i,i) = \frac{-i\pi^2 (X_W^H)^{-1}}{2^3 (2\pi)^4} \frac{(X_i^H)^2}{(X_W^H - 1)} \Bigg\{ \frac{X_i^H (X_W^H - 1)}{(X_i^H - 1)^2 (X_i^H - X_W^H)^2} \cdot$$

$$\cdot \left[-2 \ln X_i^H (X_i^H X_W^H + X_i^H - 2 X_W^H) + X_i^H (2 X_i^H - 1 - X_W^H) \right]$$

$$+ \frac{1}{(X_i^H - 1)^2} + \frac{(X_W^H)^2 (2 \ln X_W^H - 1)}{(X_i^H - X_W^H)^2} \Bigg\} \qquad (2.57)$$

$$\boxed{\lim_{X_i^H \to 0} I_{2*}^{HW}(i,i) = 0} \qquad (2.58)$$

$$\boxed{X_i^H = \frac{m_i^2}{M_{H^+}^2} \; ; \quad X_W^H = \frac{M_W^2}{M_{H^+}^2}} \qquad (2.59)$$

$$C_2 = -(2)^{\cancel{4}} i \frac{g^4}{\cancel{2^4} 2^2} M_W^2 \cdot \frac{1}{M_W^4} \frac{(-i)\pi^2}{2^4 \pi^4} = - \frac{g^4 M_W^2}{2^6 M_W^4 \pi^2} = - \frac{G_F^2 M_W^2}{2\pi^2}$$

$$\boxed{C_2 = - \frac{G_F^2 M_W^2}{2\pi^2}} \qquad (2.60)$$

$\Rightarrow$

$$\boxed{M^{HW} = -\frac{G_F^2 M_W^2}{2\pi^2} \cot^2\beta \sum_{i,j} \xi_i \xi_j \bar{V}(\bar{q}) \gamma^\mu (1-\gamma^5) U(b) . \bar{U}(q) \gamma_\mu (1-\gamma^5) V(\bar{b}) . \left[I_1^{HW}(i,j) - \frac{1}{8} I_2^{HW}(i,j) \right]} \quad (2.61)$$

$$\boxed{\begin{aligned} I_1^{HW}(i,j) = \frac{X_i^H X_j^H}{(X_W^H - 1)} \Bigg\{ & \frac{X_W^H (\ln X_W^H - 1)}{(X_i^H - X_W^H)(X_j^H - X_W^H)} + \frac{1}{(X_i^H - 1)(X_j^H - 1)} + \\ & + \frac{(X_W^H - 1) X_j^H (\ln X_j^H - 1)}{(X_j^H - X_W^H)(X_j^H - X_i^H)(X_j^H - 1)} + \frac{(X_W^H - 1) X_i^H (\ln X_i^H - 1)}{(X_i^H - X_W^H)(X_i^H - X_j^H)(X_i^H - 1)} \Bigg\} \end{aligned}} \quad (2.62)$$

$$\boxed{\begin{aligned} I_1^{HW}(i,i) = \frac{(X_i^H)^2}{(X_W^H - 1)} \Bigg\{ & \frac{X_W^H (\ln X_W^H - 1)}{(X_i^H - X_W^H)^2} + \frac{1}{(X_i^H - 1)^2} + \\ & + (X_W^H - 1) \frac{\left[\ln(X_i^H)(X_W^H - X_i^{H\,2}) + X_i^H (2 X_i^H - X_W^H - 1) \right]}{(X_i^H - X_W^H)^2 (X_i^H - 1)^2} \Bigg\} \end{aligned}} \quad (2.63)$$

$$\boxed{\lim_{X_i^H \to 0} I_1^{HW}(i,i) = 0} \quad (2.64)$$

$$\boxed{X_i^H = \frac{m_i^2}{M_{H^+}^2} \quad ; \quad X_W^H = \frac{M_W^2}{M_{H^+}^2}}$$

$$\lim_{x \to 0} x^2 \ln x = \lim_{x \to 0} \frac{\ln x}{\frac{1}{x^2}} = \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{2}{x^3}} = \lim_{x \to 0} -\frac{1}{2} x^2 = 0$$

$$I_2^{HW}(i,j) = \frac{X_i^H X_j^H}{(X_w^H)(X_w^H-1)}\left\{ \frac{(X_w^H)^2(2\ln X_w^H - 1)}{(X_i^H - X_w^H)(X_j^H - X_w^H)} + \frac{1}{(X_i^H-1)(X_j^H-1)} - \frac{(X_w^H-1)(X_i^H)^2(2\ln X_i^H - 1)}{(X_j^H - X_i^H)(X_i^H-1)(X_i^H - X_w^H)} - \frac{(X_w^H-1)(X_j^H)^2(2\ln X_j^H-1)}{(X_i^H - X_j^H)(X_j^H-1)(X_j^H - X_w^H)} \right\} \tag{2.65}$$

$$I_2^{HW}(i,i) = \frac{(X_i^H)^2}{X_w^H(X_w^H-1)}\left\{ \frac{(X_w^H)^2(2\ln X_w^H-1)}{(X_i^H - X_w^H)^2} + \frac{1}{(X_i^H-1)^2} + \frac{X_i^H(X_w^H-1)}{(X_i^H-1)^2(X_i^H-X_w^H)^2}\cdot\left[-2(\ln X_i^H)(X_i^H X_w^H + X_i^H - 2X_w^H) + X_i^H(2X_i^H - 1 - X_w^H)\right]\right\} \tag{2.66}$$

$$\lim_{X_i^H \to 0} I_2^{HW}(i,i) = 0 \tag{2.67}$$

Let be $S^{HW}(i,j) = I_1^{HW}(i,j) - \frac{1}{8} I_2^{HW}(i,j)$

$$S^{HW}(i,j) = \frac{X_i^H X_j^H}{(X_w^H-1)} \frac{X_w^H}{(X_i^H - X_w^H)(X_j^H - X_w^H)}\left[\ln X_w^H - 1 - \frac{1}{8}(2\ln X_w^H - 1)\right]$$
$$+ \frac{X_i^H X_j^H}{(X_w^H-1)} \cdot \frac{1}{(X_i^H-1)(X_j^H-1)}\left[1 - \frac{1}{8}\left(\frac{1}{X_w^H}\right)\right]$$
$$+ \frac{(X_i^H)^2 X_j^H}{(X_i^H - X_w^H)(X_i^H - X_j^H)(X_i^H-1)}\left[\ln X_i^H - 1 - \frac{1}{8}\left(\frac{X_i^H}{X_w^H}(2\ln X_i^H - 1)\right)\right]$$
$$+ \frac{(X_i^H)(X_j^H)^2}{(X_j^H - X_w^H)(X_j^H - X_i^H)(X_j^H-1)}\left[\ln X_j^H - 1 - \frac{1}{8}\left(\frac{X_j^H}{X_w^H}(2\ln X_j^H - 1)\right)\right]$$

$$\Rightarrow \boxed{\begin{aligned} S^{HW}(i,j) = {} & \frac{x_i^H x_j^H}{(x_W^H-1)} \frac{x_W^H}{(x_i^H - x_W^H)(x_j^H - x_W^H)} \left[\frac{3}{4} \ln x_W^H - \frac{7}{8} \right] \\ & + \frac{x_i^H x_j^H}{(x_W^H - 1)} \cdot \frac{1}{(x_i^H-1)(x_j^H-1)} \left[1 - \frac{1}{8 x_W^H} \right] \\ & + \frac{(x_i^H)^2 (x_j^H)}{(x_i^H - x_W^H)(x_i^H - x_j^H)(x_i^H - 1)} \left[\ln x_i^H \left(1 - \frac{1}{4} \overbrace{\frac{x_i^H}{x_W^H}}^{x_i^W}\right) + \left(\frac{1}{8} \overbrace{\frac{x_i^H}{x_W^H}}^{x_i^W} - 1 \right) \right] \\ & + \frac{(x_i^H)(x_j^H)^2}{(x_j^H - x_W^H)(x_j^H - x_i^H)(x_j^H - 1)} \left[\ln x_j^H \left(1 - \frac{1}{4} \overbrace{\frac{x_j^H}{x_W^H}}^{x_j^W}\right) + \left(\frac{1}{8} \overbrace{\frac{x_j^H}{x_W^H}}^{x_j^W} - 1 \right) \right] \qquad (2.68) \end{aligned}}$$

$$\Rightarrow \boxed{\begin{aligned} M^{HW} = {} & - \frac{G_F^2 M_W^2 \cot^2\beta}{2\pi^2} \sum_{i,j} \xi_i \xi_j \, \bar{v}(\bar{q}) \gamma^\mu (1-\gamma^5) v(b) . \bar{u}(q) \gamma_\mu (1-\gamma^5) v(\bar{b}) \, . \\ & . \, S^{HW}(i,j) \qquad (2.69) \end{aligned}}$$

Let's return to (2.66)

$$\frac{-(x_W^H)^2}{(x_i^H - x_W^H)^2} + \frac{1}{(x_i^H - 1)^2} + \frac{(x_i^H)^2 (x_W^H - 1)(2x_i^H - 1 - x_W^H)}{(x_i^H-1)^2 (x_i^H - x_W^H)^2} =$$

$$= \frac{-(x_W^H)^2 (x_i^H - 1)^2 + (x_i^H - x_W^H)^2 + (x_i^H)^2 (x_W^H - 1)(2 x_i^H - 1 - x_W^H)}{(x_i^H - 1)^2 (x_i^H - x_W^H)^2}$$

$$\begin{aligned} = \Big[& -(x_W^H)^2 (x_i^H)^2 + 2 (x_W^H)^2 x_i^H - \cancel{(x_W^H)^2} + (x_i^H)^2 - 2 x_i^H x_W^H + \cancel{(x_W^H)^2} \\ & + 2 (x_i^H)^3 x_W^H - 2 (x_i^H)^3 - \cancel{(x_i^H)^2 x_W^H} + (x_i^H)^2 - (x_i^H)^2 (x_W^H)^2 \\ & + \cancel{(x_i^H)^2 x_W^H} \Big] \Big/ (x_i^H - 1)^2 (x_i^H - x_W^H)^2 \end{aligned}$$

$$= \frac{-2 (x_i^H)^2 (x_W^H)^2 + 2 (x_i^H)^2 + 2 (x_W^H)^2 x_i^H - 2 x_i^H x_W^H + 2 (x_i^H)^3 x_W^H - 2 (x_i^H)^3}{(x_i^H - 1)^2 (x_i^H - x_W^H)^2}$$

$$= 2 \frac{\left[- x_i^H (x_W^H)^2 (x_i^H - 1) - (x_i^H)^2 (x_i^H - 1) + x_i^H x_W^H ((x_i^H)^2 - 1) \right]}{(x_i^H - 1)^2 (x_i^H - x_W^H)^2}$$

$$= 2\,\frac{\left[-X_i^H (X_w^H)^2 - (X_i^H)^2 + X_i^H X_w^H (X_i^H + 1)\right]}{(X_i^H - 1)(X_i^H - X_w^H)^2}$$

$$= 2\,\frac{\left[-X_i^H \cancel{(X_w^H)^2} - (X_i^H)^2 + \cancel{(X_i^H)^2} X_w^H + X_i^H X_w^H\right]}{(X_i^H - 1)(X_i^H - X_w^H)^2}$$

$$= 2\,\frac{\left[X_i^H X_w^H (X_i^H - X_w^H) - X_i^H (X_i^H - X_w^H)\right]}{(X_i^H - 1)(X_i^H - X_w^H)^2}$$

$$= \frac{2 X_i^H (X_w^H - 1)}{(X_i^H - 1)(X_i^H - X_w^H)}$$

$\therefore$

$$\boxed{I_2^{H\,W}(i,i) = \frac{2(X_i^H)^2}{X_w^H (X_w^H - 1)} \left\{ \frac{(X_w^H)^2 \ln X_w^H}{(X_i^H - X_w^H)^2} + \frac{X_i^H (X_w^H - 1)}{(X_i^H - 1)(X_i^H - X_w^H)} - \frac{X_i^H (X_w^H - 1)(\ln X_i^H)(X_i^H X_w^H + X_i^H - 2X_w^H)}{(X_i^H - 1)^2 (X_i^H - X_w^H)^2} \right\}} \qquad (2.70)$$

Let's go back into eq. (2.63)

$$\frac{-X_w^H}{(X_i^H - X_w^H)^2} + \frac{1}{(X_i^H - 1)^2} + \frac{X_i^H (X_w^H - 1)(2X_i^H - X_w^H - 1)}{(X_i^H - X_w^H)^2 (X_i^H - 1)^2}$$

$$= \frac{-X_w^H (X_i^H - 1)^2 + (X_i^H - X_w^H)^2 + X_i^H (X_w^H - 1)(2X_i^H - X_w^H - 1)}{(X_i^H - 1)^2 (X_i^H - X_w^H)^2}$$

$$= \Big(-X_w^H \cancel{(X_i^H)^2} + \cancel{2 X_w^H X_i^H} - \cancel{X_w^H} + \cancel{(X_i^H)^2} - 2X_i^H \cancel{X_w^H} + \cancel{(X_w^H)^2} + 2\cancel{(X_i^H)^2} X_w^H - 2\cancel{(X_i^H)^2} - X_i^H \cancel{(X_w^H)^2} + X_i^H \cancel{X_w^H} - \cancel{X_i^H X_w^H} + \cancel{X_i^H}\Big) \Big/ (X_i^H - 1)^2 (X_i^H - X_w^H)^2$$

$$= \Big((X_i^H)^2 X_w^H - \cancel{X_w^H} - (\cancel{X_i^H})^2 + (\cancel{X_w^H})^2 - X_i^H \cancel{(X_w^H)^2} + \cancel{X_i^H}\Big) \Big/ (\ )^2 (\ )^2$$

$$= \frac{(x_W^H-1)(x_i^H)^2 + x_W^H(x_W^H-1) - x_i^H(x_W^H-1)(x_W^H+1)}{(x_i^H-1)^2(x_i^H-x_W^H)^2}$$

$$= \frac{(x_W^H-1)\left[\cancel{(x_i^H)^2} + \cancel{x_W^H} - x_i^H \cancel{x_W^H} - \cancel{x_i^H}\right]}{(x_i^H-1)^2(x_i^H-x_W^H)^2}$$

$$= \frac{(x_W^H-1)\left[x_i^H(x_i^H-x_W^H) - (x_i^H-x_W^H)\right]}{(x_i^H-1)^2(x_i^H-x_W^H)^2}$$

$$= \frac{(x_W^H-1)\cancel{(x_i^H-x_W^H)}\cancel{(x_i^H-1)}}{(x_i^H-1)^2(x_i^H-x_W^H)^2}$$

$$= \frac{(x_W^H-1)}{(x_i^H-1)(x_i^H-x_W^H)}$$

$$\Rightarrow \boxed{I_1^{HW}(i,i) = \frac{(x_i^H)^2}{(x_W^H-1)}\left\{\frac{x_W^H \ln x_W^H}{(x_i^H-x_W^H)^2} + \frac{(x_W^H-1)}{(x_i^H-1)(x_i^H-x_W^H)} + \frac{(x_W^H-1)\ln(x_i^H)(x_W^H-(x_i^H)^2)}{(x_i^H-x_W^H)^2(x_i^H-1)^2}\right\} \qquad (2.71)}$$

$$S^{HW}(i,i) = I_1^{HW}(i,i) - \frac{1}{8} I_2^{HW}(i,i)$$

$$= \frac{3}{4}\frac{(x_i^H)^2 x_W^H \ln x_W^H}{(x_W^H-1)(x_i^H-x_W^H)^2} + \frac{(x_i^H)^2}{(x_i^H-1)(x_i^H-x_W^H)}\left(1-\frac{1}{4}\frac{x_i^H}{x_W^H}\right)$$

$$+ \frac{\ln(x_i^H)(x_i^H)^2}{(x_i^H-1)^2(x_i^H-x_W^H)^2}\left[x_W^H-(x_i^H)^2+\frac{1}{4}\frac{x_i^H}{x_W^H}(x_i^H x_W^H + x_i^H - 2x_W^H)\right]$$

$$\boxed{S^{HW}(i,i) = \frac{3}{4}\frac{(x_i^H)^2 x_W^H \ln x_W^H}{(x_W^H-1)(x_i^H-x_W^H)^2} + \frac{(x_i^H)^2}{(x_i^H-1)(x_i^H-x_W^H)}\left(1-\frac{1}{4}\frac{x_i^H}{x_W^H}\right) + \frac{\ln(x_i^H)(x_i^H)^2}{4(x_i^H-1)^2(x_i^H-x_W^H)^2 x_W^H}\left[4(x_W^H)^2 + (x_i^H)(x_i^H - 3x_i^H x_W^H - 2x_W^H)\right] \qquad (2.72)}$$

Let's evaluate the matrix element:

$$\langle B^0 | H^{HW} | \bar{B}^0 \rangle = -\frac{G_F^2 M_W^2 \cot^2\beta}{2\pi^2} \sum_{i,j} S^{HW}(i,j)\, \xi_i \xi_j\, A'$$

$$A' = \langle B^0 | \bar{V}(\bar{q})\, \gamma^\mu (1-\gamma^5) U(b) \,.\, \bar{U}(q)\, \gamma_\mu (1-\gamma^5) V(\bar{b}) | \bar{B}^0 \rangle$$

$$= 4A = \frac{1}{3} f_B^2 m_B \qquad \text{(see Axial Currents)}$$

$$\Rightarrow \boxed{\langle B^0 | H^{HW} | \bar{B}^0 \rangle = -\frac{G_F^2 M_W^2 \cot^2\beta}{6\pi^2} f_B^2 m_B \sum_{i,j} S^{HW}(i,j)\, \xi_i \xi_j} \cdot B_B \qquad (2.73)$$

$$(B_B \simeq 1)$$

In our model (free particles inside a Meson):

$$A' = 4A = 4\frac{n\, m_B f_B^2}{16} = \frac{n\, m_B f_B^2}{4}$$

$$\Rightarrow \boxed{\langle B^0 | H^{HW} | \bar{B}^0 \rangle = -\frac{G_F^2 M_W^2 \cot^2\beta}{8\pi^2} f_B^2 m_B \sum_{i,j} \xi_i \xi_j S^{HW}(i,j) \cdot \eta \qquad (\eta = B_B)} \qquad (2.74)$$

η = correction factor

$$\boxed{\lim_{X_i^H \to 0} S^{HW}(i,i) = 0}$$

$S^{HW}(i,i)$ is given in A2 (3) (Appendix)

$$\sum_{i,j} \xi_i \xi_j S^{HW}(i,j) = \xi_u^2 \cancelto{0}{S^{HW}(u,u)} + \xi_c^2 \cancelto{0}{S^{HW}(c,c)} + \xi_t^2 S^{HW}(t,t) + 2\xi_u \xi_c \cancelto{0}{S^{HW}(u,c)} + 2\xi_u \xi_t \cancelto{0}{S^{HW}(u,t)} + 2\xi_c \xi_t S^{HW}(c,t)$$

$$\boxed{\begin{aligned} \sum_{i,j} \xi_i \xi_j S^{HW}(i,j) &\approx 2\xi_c \xi_t S^{HW}(c,t) + \xi_t^2 S^{HW}(t,t) && (2.75) \\ &\approx \xi_t^2 S^{HW}(t,t) && (2.75\,a) \end{aligned}}$$

Appendix A2 : Let's consider (2.71)

$$\ln x_W^H = \ln M_W^2 - \ln M_{H^+}^2 = \ln M_W^2 - \ln m_i^2 + \ln m_i^2 - \ln M_{H^+}^2$$
$$= -\ln x_i^W + \ln x_i^H$$

$$\Rightarrow$$

$$I_1^{HW}(i,i) = \frac{(x_i^H)^2}{(x_W^H - 1)} \left\{ \frac{x_W^H (\ln x_i^H - \ln x_i^W)}{(x_i^H - x_W^H)^2} + \frac{(x_W^H - 1)}{(x_i^H - 1)(x_i^H - x_W^H)} + \frac{(x_W^H - 1)\ln(x_i^H)(x_W^H - (x_i^H)^2)}{(x_i^H - x_W^H)^2 (x_i^H - 1)^2} \right\}$$

$$\frac{x_W^H \ln x_i^H}{(x_i^H - x_W^H)^2} + \frac{(x_W^H - 1)\ln(x_i^H)(x_W^H - (x_i^H)^2)}{(x_i^H - x_W^H)^2 (x_i^H - 1)^2}$$

$$= \frac{(x_i^H)^2 x_W^H \cancel{\ln x_i^H} - 2 x_i^H x_W^H \ln x_i^H + x_W^H \cancel{\ln x_i^H} + (x_W^H)^2 \ln x_i^H + - (x_W^H)\cancel{(x_i^H)^2} \ln x_i^H - x_W^H \cancel{\ln x_i^H} + (x_i^H)^2 \ln x_i^H}{(x_i^H - x_W^H)^2 (x_i^H - 1)^2}$$

$$= \frac{\cancel{(x_W^H - x_i^H)^2} \ln x_i^H}{\cancel{(x_i^H - x_W^H)^2} (x_i^H - 1)^2} = \frac{\ln x_i^H}{(x_i^H - 1)^2}$$

$$\Rightarrow I_1^{HW}(i,i) = \frac{(x_i^H)^2 \ln x_i^H}{(x_W^H - 1)(x_i^H - 1)^2} - \frac{(x_i^H)^2 x_W^H \ln x_i^W}{(x_W^H - 1)(x_i^H - x_W^H)^2} + \frac{(x_i^H)^2}{(x_i^H - 1)(x_i^H - x_W^H)}$$

$$\boxed{I_1^{HW}(i,i) = (x_i^H)^2 \left[\frac{\ln x_i^H}{(x_W^H - 1)(x_i^H - 1)^2} - \frac{x_W^H \ln x_i^W}{(x_W^H - 1)(x_i^H - x_W^H)^2} + \frac{1}{(x_i^H - 1)(x_i^H - x_W^H)} \right]}$$

A2 (1)

Let's consider now $I_2^{HW}(i,i)$

$$\frac{(X_W^H)^2 \ln X_W^H}{(X_i^H - X_W^H)^2} - \frac{X_i^H (X_W^H - 1) \ln X_i^H (X_i^H X_W^H + X_i^H - 2X_W^H)}{(X_i^H - 1)^2 (X_i^H - X_W^H)^2}$$

$$= \frac{(X_W^H)^2 (\ln X_i^H - \ln X_i^W)}{(X_i^H - X_W^H)^2} - \frac{X_i^H (X_W^H - 1) \ln X_i^H (X_i^H X_W^H + X_i^H - 2X_W^H)}{(X_i^H - 1)^2 (X_i^H - X_W^H)^2}$$

Considering the terms containing $\ln X_i^H$ only we have:

$$= \ln X_i^H \frac{[(X_W^H)^2 \cancel{(X_i^H)^2} - 2X_i^H \cancel{(X_W^H)^2} + \cancel{(X_W^H)^2} - (X_i^H)^2 \cancel{(X_W^H)^2} - \cancel{(X_i^H)^2} X_W^H}{(X_i^H - X_W^H)^2 (X_i^H - 1)^2}$$

$$\frac{+ 2X_i^H \cancel{(X_W^H)^2} + \cancel{(X_i^H)^2 X_W^H} + \cancel{(X_i^H)^2} - 2X_i^H X_W^H\,]}{}$$

$$= \ln X_i^H \frac{[\underbrace{-2X_i^H X_W^H + (X_W^H)^2 + (X_i^H)^2}_{(X_i^H - X_W^H)^2}]}{(X_i^H - X_W^H)^2 (X_i^H - 1)^2}$$

$$= \frac{\ln X_i^H}{(X_i^H - 1)^2}$$

$$\Rightarrow \boxed{I_2^{HW}(i,i) = \frac{2(X_i^H)^2}{X_W^H}\left[\frac{\ln X_i^H}{(X_i^H - 1)^2 (X_W^H - 1)} - \frac{(X_W^H)^2 \ln X_i^W}{(X_i^H - X_W^H)^2 (X_W^H - 1)} + \frac{X_i^H}{(X_i^H - 1)(X_i^H - X_W^H)}\right]} \qquad \text{A2 (2)}$$

$$\boxed{S^{HW}(i,i) = I_1^{HW}(i,i) - \frac{1}{8} I_2^{HW}(i,i)}$$

$$S^{HW}(i,i) = (\chi_i^H)^2 \left[\frac{\ln \chi_i^H}{(\chi_W^H - 1)(\chi_i^H - 1)^2} \left(1 - \frac{1}{4\chi_W^H}\right) - \frac{3}{4} \frac{\chi_W^H \ln \chi_i^W}{(\chi_W^H - 1)(\chi_i^H - \chi_W^H)^2} + \frac{1}{(\chi_i^H - 1)(\chi_i^H - \chi_W^H)} \left(1 - \frac{\chi_i^H}{4\chi_W^H}\right) \right]$$

$$\boxed{S^{HW}(i,i) = (\chi_i^H)^2 \left[\frac{\ln \chi_i^H}{(\chi_W^H - 1)(\chi_i^H - 1)^2} \left(1 - \frac{1}{4\chi_W^H}\right) - \frac{3}{4} \frac{\chi_W^H \ln \chi_i^W}{(\chi_W^H - 1)(\chi_i^H - \chi_W^H)^2} + \frac{1}{(\chi_i^H - 1)(\chi_i^H - \chi_W^H)} \left(1 - \frac{\chi_i^W}{4}\right) \right] \qquad A2\ (3)}$$

For the box diagrams c) (D∅ Note 1372)

$$M^{WW} = 2\left(\frac{g}{\sqrt{2}}\right)^4 \frac{\pi^2}{(2\pi)^4 M_W^2} \sum_{i,j} \xi_i \xi_j S^{WW}(i,j)\, \bar{V}_L(\bar{q})\gamma^\mu U_L(b).$$

$\bar{U}_L(q)\gamma_\mu V_L(\bar{b})$

$$\boxed{S^{WW}(i,j) \equiv \left(1 + \frac{X_i^W X_j^W}{4}\right)\left[\frac{J(X_i^W) - J(X_j^W)}{X_i^W - X_j^W}\right] + \frac{2X_i^W X_j^W}{(1-X_i^W)(1-X_j^W)}\left[F(X_i^W, X_j^W) + F(X_j^W, X_i^W) - 1\right]} \quad (3.1)$$

for $i \neq j$, and

$$\boxed{S^{WW}(i,i) = \left(1 + \frac{(X_i^W)^2}{4}\right)\left[\frac{1 - (X_i^W)^2 + 2X_i^W \ln(X_i^W)}{(1-X_i^W)^3}\right] - \frac{2(X_i^W)^2}{(1-X_i^W)^2}\left[2 + \frac{(1+X_i^W)}{(1-X_i^W)}\ln(X_i^W)\right]} \quad (3.2)$$

Si $X_i^W \ll 1$ $\boxed{S^{WW}(i,i) \approx 1}$

$$\boxed{J(X_i^W) = \frac{1}{1-X_i^W} + \frac{(X_i^W)^2 \ln(X_i^W)}{(1-X_i^W)^2}} \quad (3.3)$$

$$\boxed{F(X_i^W, X_j^W) = -\frac{X_i^W \ln(X_i^W)\,(1-X_j^W)}{(1-X_i^W)(X_i^W - X_j^W)}} \quad (3.4)$$

$$\boxed{X_i^W = \frac{m_i^2}{M_W^2}}$$

$$\boxed{\xi_i = V_{ib} V_{iq}^* \quad (q = d \text{ or } s)}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \Rightarrow 2^5 G_F^2 = \frac{g^4}{M_W^4}$$

If $i = j$

$$S^{WW}(i,i) = \left(1 + \frac{(X_i^W)^2}{4}\right)\frac{(1+X_i^W)}{(1-X_i^W)^2} - \frac{4\,(X_i^W)^2}{(1-X_i^W)^2}$$

$$+ \frac{\ln(X_i^W)}{(1-X_i^W)^3}\left[\, 2X_i^W + \frac{1}{2}(X_i^W)^3 - 2(X_i^W)^2 - 2(X_i^W)^3 \,\right]$$

$$= \frac{4 + 4X_i^W + \cancel{(X_i^W)^2} + (X_i^W)^3 - 16\cancel{(X_i^W)^2}}{4\,(1-X_i^W)^2} + \frac{\ln(X_i^W)\,(X_i^W)}{(1-X_i^W)^3}\cdot\left[2 - \frac{3}{2}(X_i^W)^2 - 2X_i^W\right]$$

$$= \frac{4 + 4X_i^W - 15(X_i^W)^2 + (X_i^W)^3}{4\,(1-X_i^W)^2} + \frac{2X_i^W\ln(X_i^W)}{(1-X_i^W)^3}\left[(1-X_i^W) - \frac{3}{4}(X_i^W)^2\right]$$

Because $\sum_i \xi_i = 0$ we can subtract 1 from $S^{WW}(i,i)$ and $S^{WW}(i,j)$ in general.

$$S^{WW}(i,i) \longrightarrow \frac{\cancel{4} + 4\cancel{X_i^W} - 15\cancel{(X_i^W)^2} + (\cancel{X_i^W})^3 - \cancel{4} + 8X_i^W - 4(\cancel{X_i^W})^2}{4\,(1-X_i^W)^2} + \frac{2X_i^W\ln(X_i^W)}{(1-X_i^W)^3}\cdot\left[(1-X_i^W) - \frac{3}{4}(X_i^W)^2\right]$$

$$\boxed{S^{WW}(i,i) = \frac{X_i^W\left(3 - \frac{19}{4}X_i^W + \frac{1}{4}(X_i^W)^2\right)}{(1-X_i^W)^2} + \frac{2X_i^W\ln(X_i^W)}{(1-X_i^W)^2}\left[1 - \frac{3}{4}\frac{(X_i^W)^2}{(1-X_i^W)}\right]}$$

For $i \neq j$

$$S^{ww}(i,j) = \left(1+\frac{x_i^w x_j^w}{4}\right)\left[\frac{1}{(1-x_i^w)(x_i^w-x_j^w)} - \frac{1}{(1-x_j^w)(x_i^w-x_j^w)}\right]$$

$$- \frac{2x_i^w x_j^w}{(1-x_i^w)(1-x_j^w)} - 1 + \left(1+\frac{x_i^w x_j^w}{4}\right)\left[\frac{(x_i^w)^2 \ln(x_i^w)}{(1-x_i^w)^2}\right.$$

$$\left. - \frac{(x_j^w)^2 \ln(x_j^w)}{(1-x_j^w)^2}\right] \cdot \frac{1}{(x_i^w - x_j^w)}$$

$$+ \frac{2x_i^w x_j^w}{(1-x_i^w)(1-x_j^w)}\left[-\frac{x_i^w \ln(x_i^w)(1-x_j^w)}{(1-x_i^w)(x_i^w-x_j^w)} - \frac{x_j^w \ln(x_j^w)(1-x_i^w)}{(1-x_j^w)(x_j^w-x_i^w)}\right]$$

$$= \frac{\left(\cancel{1+}\frac{x_i^w x_j^w}{4}\right) - 2x_i^w x_j^w - \cancel{1} + x_j^w + x_i^w - x_i^w x_j^w}{(1-x_i^w)(1-x_j^w)}$$

$$+ \frac{1}{(x_i^w - x_j^w)}\left[\frac{(x_i^w)^2 \ln(x_i^w)}{(1-x_i^w)^2} + \frac{(x_i^w)^3 (x_j^w) \ln(x_i^w)}{4(1-x_i^w)^2}\right.$$

$$- \frac{(x_j^w)^2 \ln(x_j^w)}{(1-x_j^w)^2} - \frac{(x_i^w)(x_j^w)^3 \ln(x_j^w)}{4(1-x_j^w)^2}$$

$$\left. - 2\frac{(x_i^w)^2 (x_j^w) \ln(x_i^w)\cancel{(1-x_j^w)}}{(1-x_i^w)^2\cancel{(1-x_j^w)}} + 2\frac{(x_i^w)(x_j^w)^2 \ln(x_j^w)\cancel{(1-x_i^w)}}{\cancel{(1-x_i^w)}(1-x_j^w)^2}\right]$$

$$= \frac{x_i^w + x_j^w - \frac{11}{4}x_i^w x_j^w}{(1-x_i^w)(1-x_j^w)} + \frac{1}{(x_i^w - x_j^w)}\left\{\frac{(x_i^w)^2 \ln(x_i^w)}{(1-x_i^w)^2}\left[1 + \frac{1}{4}x_i^w x_j^w\right.\right.$$

$$\left.\left. - 2(x_j^w)\right] - \frac{(x_j^w)^2 \ln(x_j^w)}{(1-x_j^w)^2}\left[1 + \frac{1}{4}x_i^w x_j^w - 2(x_i^w)\right]\right\}$$

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$$S^{WW}(i,j) = \frac{X_i^W + X_j^W - \frac{11}{4} X_i^W X_j^W}{(1-X_i^W)(1-X_j^W)} + \frac{1}{(X_i^W - X_j^W)} \Big\{ \overbrace{\frac{(X_i^W)^2 \ln (X_i^W)}{(1-X_i^W)^2} \Big[1 - 2X_j^W + \frac{1}{4} X_i^W X_j^W \Big]}^{G(i,j)}$$

$$- \underbrace{\frac{(X_j^W)^2 \ln (X_j^W)}{(1-X_j^W)^2} \Big[1 - 2X_i^W + \frac{1}{4} X_j^W X_i^W \Big]}_{G(j,i)} \Big\}$$

$$G = G(X_W^i, X_W^j)$$

$$\therefore \boxed{H^{WW} = \frac{G_F^2 M_W^2}{\pi^2} \sum_{i,j} \xi_i \xi_j S^{WW}(i,j) \bar{V}_L(\bar{q}) \gamma^\mu U_L(b) \cdot \bar{U}_L(q) \gamma_\mu V_L(\bar{b})} \quad (3\text{-}5)$$

Let's consider :

$$\langle B^0 | H^{WW} | \bar{B}^0 \rangle = \frac{G_F^2 M_W^2}{4\pi^2} \sum_{i,j} \xi_i \xi_j A' S^{WW}(i,j)$$

$$A' \equiv \langle B^0 | \bar{V}(\bar{q}) \gamma^\mu (1-\gamma^5) U(b) \cdot \bar{U}(q) \gamma_\mu (1-\gamma^5) V(\bar{b}) | \bar{B}^0 \rangle$$

$$A' = 4A = \frac{1}{3} f_B^2 m_B$$

$$\Rightarrow \boxed{\langle B^0 | H^{WW} | \bar{B}^0 \rangle = \frac{G_F^2 M_W^2 f_B^2 m_B}{12\pi^2} \sum_{i,j} \xi_i \xi_j S^{WW}(i,j)} \cdot B_B \quad (3.6)$$

($B_B = 1$ corresponds to the saturation by the vacuum intermediate state)

For our model considering free particles inside the Meson, we have :

$$A' = 4A = \frac{\underset{B_B}{\eta} m_B f_B^2}{4} \quad (3\text{-}7)$$

$$\Rightarrow \boxed{\langle B^0 | H^{WW} | \bar{B}^0 \rangle = \frac{G_F^2 M_W^2}{16\pi^2} f_B^2 m_B \eta \sum_{i,j} \xi_i \xi_j S^{WW}(i,j)} \quad (3.8)$$

$$\boxed{\eta = B_B}$$

$$A = \frac{\bar{V}_L(\bar{q}) \gamma^\mu U_L(b) \cdot \bar{U}_L(q) \gamma_\mu V_L(\bar{b})}{\sqrt{2E_1\, 2E_2\, 2E_3\, 2E_4}\; V} \rightarrow \frac{\eta m_B f_B^2}{16} \quad (3\text{-}9)$$

(3-2) can be written as :

$$\boxed{S^{WW}(i,i) = 1 + \frac{(12 X_i^W - 19 (X_i^W)^2 + (X_i^W)^3}{4(1-X_i^W)^2} + \frac{2X_i^W \ln(X_i^W)}{(1-X_i^W)^3} \cdot \left[1 - X_i^W - \frac{3}{4}(X_i^W)^2\right]} \quad (3.10)$$

Using unitarity : $\sum_i \xi_i = 0$ we have :

$$\sum_{i,j} \xi_i \xi_j S^{WW}(i,j) \approx \alpha(X_t^W)\xi_t^2 + \beta(X_c^W)\xi_c^2 + \gamma(X_c^W, X_t^W)\xi_t\xi_c + \delta(X_u^W)\xi_u^2 + \phi(X_c^W)\xi_u\xi_c \qquad (3.11)$$

where :

$$\alpha(X_t^W) \equiv \frac{(X_t^W)^3 - 11(X_t^W)^2 + 4(X_t^W)}{4(1-X_t^W)^2} - \frac{3}{2}\frac{(X_t^W)^3 . \ln(X_t^W)}{(1-X_t^W)^3} \qquad (3.12)$$

$$\beta(X_c^W) \equiv \frac{(X_c^W)^3 - 19(X_c^W)^2 + 12(X_c^W)}{4(1-X_c^W)^2} + \frac{2X_c^W . \ln(X_c^W)}{(1-X_c^W)^3}\left[1 - X_c^W - \frac{3}{4}(X_c^W)^2\right] \qquad (3.13)$$

$$\gamma(X_c^W, X_t^W) \equiv \frac{X_c^W X_t^W}{2(1-X_t^W)}\left[-7 + \frac{(X_t^W - 8)\ln(X_t^W)}{(1-X_t^W)}\right] \qquad (3.14)$$

$$\delta(X_u^W) \equiv X_u^W \left[3 + 2\ln(X_u^W)\right] \qquad (3.15)$$

$$\phi(X_c^W) \equiv \frac{2X_c^W}{(1-X_c^W)^2}\left[1 - X_c^W + \ln(X_c^W)\right] \qquad (3.16)$$

The dominant terms in (3.11) are $\alpha(X_t^W)$, $\phi(X_c^W)$, $\beta(X_c^W)$, $\gamma(X_c^W, X_t^W)$

$$\Delta m_{B_q^0} = m_{B_q^0 H} - m_{B_q^0 L} \simeq 2|M_{12}| = 2|\langle B^0 | \mathcal{L}_{ef} | \bar{B}^0\rangle| \qquad (3.17)$$

$$\boxed{X_q = \frac{\Delta m_{B_q^0}}{\Gamma_{B_q^0}}} \qquad (3.18)$$

H and L stands for heavy and light respectively

$$\Delta m_{B_q^0} = 2|\langle B^0 | M^{WW} + M^{HW} + M^{HH} | \bar{B}^0\rangle|$$

$$\Delta m_{B_q^0} = 2|\langle B^0 | M^{WW} | \bar{B}^0\rangle + \langle B^0 | M^{HW} | \bar{B}^0\rangle + \langle B^0 | M^{HH} | \bar{B}^0\rangle| \qquad (3.19)$$

$\langle B^0 | M^{WW} | \bar{B}^0 \rangle$, $\langle B^0 | M^{HW} | \bar{B}^0 \rangle$, $\langle B^0 | M^{HH} | \bar{B}^0 \rangle$

are given in: (3.8), (2.74), (1.32) respectively.

$$\boxed{\Delta m_{B_q^0} = \frac{G_F^2 M_W^2}{24\pi^2} f_B^2 m_B B_B \left| \sum_{i,j} \xi_i \xi_j \left[4 S^{WW}(i,j) - 8\cot^2\beta\, S^{HW}(i,j) + \cot^4\beta\, S^{HH}(i,j) \right| \right. \quad (3\text{-}20)}$$

(PCAC)

In our model considering free particles inside the Meson, we have:

$$\boxed{\Delta m_{B_q^0} = \frac{G_F^2 M_W^2}{32\pi^2} f_B^2 m_B \eta_B \left| \sum_{i,j} \xi_i \xi_j \left[4 S^{WW}(i,j) - 8\cot^2\beta\, S^{HW}(i,j) + \cot^4\beta\, S^{HH}(i,j) \right] \right| \quad (3\text{-}21)}$$

$$\Delta m_{B_q^0} \simeq \frac{G_F^2 M_W^2}{\underset{\updownarrow\atop 32}{24\pi^2}} f_B^2 m_B \underset{\updownarrow\atop \eta_B}{B_B} \left| 4\left(\alpha(X_t^W)\xi_t^2 + \beta(X_c^W)\xi_c^2 + \phi(X_c^W)\xi_u\xi_c + \gamma(X_c^W, X_t^W)\xi_t\xi_c \right) - 8\cot^2\beta \cdot \left(2\xi_c\xi_t \cdot S^{HW}(c,t) + \xi_c^2 S^{HW}(c,c) + \xi_t^2 S^{HW}(t,t) \right) + \cot^4\beta \left(2\xi_c\xi_t S^{HH}(c,t) + \xi_c^2 S^{HH}(c,c) + \xi_t^2 S^{HH}(t,t) \right) \right|$$

$$\boxed{\begin{aligned}\Delta m_{B_q^0} = \frac{G_F^2 M_W^2}{\underset{\updownarrow\atop 32}{24\pi^2}} f_B^2 m_B \underset{\updownarrow\atop \eta_B}{B_B} \Big| &\left(4\alpha(X_t^W) - 8\cot^2\beta\, S^{HW}(t,t) + \cot^4\beta\, S^{HH}(t,t)\right) \cdot \xi_t^2 + 2\big(2\gamma(X_c^W, X_t^W) - 8\cot^2\beta\, S^{HW}(c,t) + \\ &+ \cot^4\beta\, S^{HH}(c,t)\big)\xi_c\xi_t + \xi_c^2\left(4\beta(X_c^W) - 8\cot^2\beta\, S^{HW}(c,c) + \cot^4\beta\, S^{HH}(c,c)\right) + 4\phi(X_c^W)\xi_u\xi_c \Big| \quad (3.22.a)\end{aligned}}$$

$q = d$ or s

Setting :

$$\alpha(X_t^W) \equiv \alpha^{WW}(t)$$

$$\beta(X_c^W) \equiv \beta^{WW}(c)$$

$$\gamma(X_c^W, X_t^W) \equiv \gamma^{WW}(c,t)$$

$$\delta(X_u^W) \equiv \delta^{WW}(u)$$

$$\phi(X_c^W) \equiv \phi^{WW}(c)$$

$\Rightarrow$

$$\boxed{\begin{aligned}\Delta m_{B_q^0} = \frac{G_F^2 M_W^2}{\underset{\substack{\updownarrow \\ 32}}{24}\,\pi^2} f_B^2 m_B \underset{\substack{\updownarrow \\ \eta_B}}{B_B} \Big| &\left(4\alpha^{WW}(t) - 8\cot^2\beta\, S^{HW}(t,t) + \cot^4\beta\, S^{HH}(t,t)\right) \cdot \xi_t^2 + 2\left(2\gamma^{WW}(c,t) - 8\cot^2\beta\, S^{HW}(c,t) + \right. \\ &\left. + \cot^4\beta\, S^{HH}(c,t)\right)\xi_c \xi_t + \left(4\beta^{WW}(c) - 8\cot^2\beta \cdot \right. \\ &\left. \cdot S^{HW}(c,c) + \cot^4\beta\, S^{HH}(c,c)\right)\xi_c^2 + 4\phi^{WW}(c)\,\xi_u \xi_c \Big|\end{aligned}} \quad (3.22\text{ b})$$

(PCAC)

$q = d$ or s

We use the Wolfenstein parametrization for the C-K-M Matrix :

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3 \rho e^{-i\delta} \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho e^{i\delta}) & -A\lambda^2 & 1 \end{pmatrix} \qquad (3.23)$$

$$\xi_i = V_{ib} V_{iq}^* \Rightarrow \quad \xi_c = V_{cb} V_{cq}^*$$

$$\xi_t = V_{tb} V_{tq}^* = V_{tq}^*$$

$$\therefore \ \xi_t^2 = (V_{tq}^*)^2 = \begin{cases} A^2\lambda^6 (1-\rho e^{-i\delta})^2 & \text{if } q = d \\ A^2 \lambda^4 & \text{if } q = s \end{cases}$$

$$\xi_t^2 = \begin{cases} \left(1-2\rho\cos\delta+\rho^2\cos 2\delta+2i\rho\sin\delta\,(1-\rho\cos\delta)\right)A^2\lambda^6 & q=d \\ A^2\lambda^4 & q=s \end{cases}$$

$$|\xi_t^2| = \left[\left(1-2\rho\cos\delta+\rho^2\cos 2\delta\right)^2+4\rho^2\sin^2\delta\,(1-\rho\cos\delta)^2\right]^{1/2}A^2\lambda^6$$

$$= \left[1+4\rho^2\cos^2\delta+\rho^4\cos^2 2\delta-4\rho\cos\delta+2\rho^2\cos 2\delta-4\rho^3\cos\delta\cos 2\delta \right.$$
$$\left. +4\rho^2\sin^2\delta-8\rho^3\sin^2\delta\cos\delta+4\rho^4\sin^2\delta\cos^2\delta\right]^{1/2}A^2\lambda^6$$

$$= \left[1+4\rho^2\cos^2\delta+\rho^4(1-4\sin^2\delta+4\sin^4\delta)-4\rho\cos\delta+2\rho^2(1-2\sin^2\delta)\right.$$
$$\left. -4\rho^3\cos\delta\,(1-2\sin^2\delta)+4\rho^2\sin^2\delta-8\rho^3\sin^2\delta\cos\delta+4\rho^4\sin^2\delta\cos^2\delta\right]^{1/2}A^2\lambda^6$$

$$= \left[1+\rho^4+2\rho^2+4\rho^2\cos^2\delta-4\rho\cos\delta-4\rho^3\cos\delta\right]^{1/2}A^2\lambda^6$$

$$= \left(\left(1+\rho^2-2\rho\cos\delta\right)^2\right)^{1/2}A^2\lambda^6$$

$$= \left(1+\rho^2-2\rho\cos\delta\right)A^2\lambda^6$$

$$\boxed{\begin{array}{l} |\xi_t^2| = (1+\rho^2-2\rho\cos\delta)\,A^2\lambda^6 = \lambda^2 V_{cb}^2\, f(\delta) \\ \text{with} \quad f(\delta) = 1+\rho^2-2\rho\cos\delta \end{array}} \quad \text{for } q=d \quad (3.24)$$

$$|\xi_t|^2 = |V_{tq}^*|^2 = (V_{tq}V_{tq}^*) = A^2\lambda^6\,(1-\rho e^{i\delta})(1-\rho e^{-i\delta}) \qquad q=d$$
$$= A^2\lambda^6\,(1-\rho e^{-i\delta}-\rho e^{i\delta}+\rho^2)$$
$$= A^2\lambda^6\left(1+\rho^2-2\rho\,\tfrac{1}{2}(e^{i\delta}+e^{-i\delta})\right)$$
$$= A^2\lambda^6\,(1+\rho^2-2\rho\cos\delta) \qquad (\text{as in } 3.24)$$

Then:

$$\boxed{\begin{array}{l} |\xi_t|^2 = |V_{tq}^*|^2 = \begin{cases} \lambda^2 V_{cb}^2\, f(\delta) & \text{if } q=d \\ V_{cb}^2 & \text{if } q=s \end{cases} \\ \text{with} \quad f(\delta) = 1+\rho^2-2\rho\cos\delta \\ \qquad\quad\; V_{cb}^2 = A^2\lambda^4 \end{array}} \qquad (3.25)$$

returning to eq (3-12), if we take $m_t = 180$ Gev
$M_W = 80.33$ Gev
$m_c = 1.3$ Gev
$m_u = 5\times 10^{-3}$ Gev

we get:

$$\alpha(X_t^w) = 2.69250021 \qquad (3.26)$$

$$\beta(X_c^w) = -3.536513\times 10^{-3} \qquad (3.27)$$

$$\gamma(X_c^w, X_t^w) = 9.491260\times 10^{-4} \qquad (3.28)$$

$$\delta(X_u^w) = -1.384563\times 10^{-7} \qquad (3.29)$$

$$\phi(X_c^w) = -3.798359\times 10^{-3} \qquad (3.30)$$

$$\boxed{|\xi_t|^2 = \begin{cases} \lambda^2 V_{cb}^2 f(\delta) & \text{if } q=d \\ V_{cb}^2 & \text{if } q=s \end{cases}} \qquad (V_{cb}^2 = A^2\lambda^4)$$

$$f(\delta) = 1+\rho^2-2\rho\cos\delta$$

$$\boxed{|\xi_c|^2 = \begin{cases} A^2\lambda^6 = \lambda^2 V_{cb}^2 & \text{if } q=d \\ A^2\lambda^4(1-\frac{1}{2}\lambda^2)^2 = V_{cb}^2(1-\frac{1}{2}\lambda^2)^2 & \text{if } q=s \end{cases}} \qquad (3.31)$$

$$\xi_u = V_{ub}V_{uq}^* = \begin{cases} V_{ub}V_{ud}^* & \text{if } q=d \\ V_{ub}V_{us}^* & \text{if } q=s \end{cases}$$

$$= \begin{cases} A\lambda^3\rho e^{-i\delta}(1-\frac{1}{2}\lambda^2) = A\lambda^3(1-\frac{1}{2}\lambda^2)\rho e^{-i\delta} & \text{if } q=d \\ A\lambda^3\rho e^{-i\delta}\lambda = A\lambda^4\rho e^{-i\delta} & \text{if } q=s \end{cases}$$

$$\boxed{|\xi_u|^2 = \begin{cases} A^2\lambda^6(1-\frac{1}{2}\lambda^2)^2\rho^2 & \text{if } q=d \\ A^2\lambda^8\rho^2 & \text{if } q=s \end{cases}} \qquad (3.32)$$

$$\xi_t\xi_c = V_{tb}V_{tq}^*V_{cb}V_{cq}^* = \begin{cases} -A\lambda^3(1-\rho e^{-i\delta})A\lambda^2\lambda = -A^2\lambda^6(1-\rho e^{-i\delta}) \\ -A\lambda^2 A\lambda^2(1-\frac{1}{2}\lambda^2) = -A^2\lambda^4(1-\frac{1}{2}\lambda^2) \end{cases}$$

$$\Rightarrow \boxed{|\xi_t \xi_c| = \begin{cases} A^2\lambda^6 \left(|f(\delta)|\right)^{1/2} & \text{if } q=d \\ A^2\lambda^4\left(1-\frac{1}{2}\lambda^2\right) & \text{if } q=s \end{cases}} \quad f(\delta) = 1+\rho^2-2\rho\cos\delta \qquad (3.33)$$

$$\xi_u \xi_c = V_{ub}V_{uq}^* \; V_{cb}V_{cq}^* = \begin{cases} -A\lambda^3\rho e^{-i\delta}\left(1-\frac{1}{2}\lambda^2\right) A\lambda^2 \lambda & \text{if } q=d \\ A\lambda^3\rho e^{-i\delta} \lambda A\lambda^2\left(1-\frac{1}{2}\lambda^2\right) & \text{if } q=s \end{cases}$$

$$= \begin{cases} -A^2\lambda^6\left(1-\frac{1}{2}\lambda^2\right)\rho e^{-i\delta} & \text{if } q=d \\ A^2\lambda^6\left(1-\frac{1}{2}\lambda^2\right)\rho e^{-i\delta} & \text{if } q=s \end{cases}$$

$$\boxed{|\xi_u \xi_c| = A^2\lambda^6\left(1-\frac{1}{2}\lambda^2\right)\rho \quad \text{if } q=d,s} \qquad (3.34)$$

For $A \simeq 1$; $\lambda = 0.221$; $\rho = 0.52$; $\delta = 49.583°$

$$|\xi_t|^2 = \begin{cases} 6.945245\times10^{-5} & \text{if } q=d \\ 2.385443\times10^{-3} & \text{if } q=s \end{cases} \qquad (3.35)$$

$$|\xi_c|^2 = \begin{cases} 1.165074\times10^{-4} & \text{if } q=d \\ 2.270358\times10^{-3} & \text{if } q=s \end{cases} \qquad (3.36)$$

$$|\xi_u|^2 = \begin{cases} 2.998373\times10^{-5} & \text{if } q=d \\ 1.538667\times10^{-6} & \text{if } q=s \end{cases} \qquad (3.37)$$

$$|\xi_t \xi_c| = \begin{cases} 8.995402\times10^{-5} & \text{if } q=d \\ 2.327189\times10^{-3} & \text{if } q=s \end{cases} \qquad (3.38)$$

$$|\xi_u \xi_c| = \left\{ 5.910437\times10^{-5} \quad \text{if } q=d \text{ or } s \right. \qquad (3.39)$$

important $$|\alpha^{WW}(t)\,\xi_t^2| = \begin{cases} 1.870007\times10^{-4} & \text{if } q=d \\ 6.422806\times10^{-3} & \text{if } q=s \end{cases} \qquad (3.40)$$

Small $$|\beta^{WW}(c)\,\xi_c^2| = \begin{cases} 4.120301\times10^{-7} & \text{if } q=d \\ 8.029151\times10^{-6} & \text{if } q=s \end{cases} \qquad (3.41)$$

Small $$|\gamma^{WW}(c,t)\,\xi_t\xi_c| = \begin{cases} 8.53777\times10^{-8} & \text{if } q=d \\ 2.208796\times10^{-6} & \text{if } q=s \end{cases} \qquad (3.42)$$

$$\text{very small} \quad \left| S^{WW}(U)\, \varepsilon_U^2 \right| = \begin{cases} 4.151436 \times 10^{-12} \\ 2.130381 \times 10^{-13} \end{cases} \tag{3.43}$$

$$\text{small} \quad \left| \phi^{WW}(c)\, \varepsilon_U \varepsilon_c \right| = \begin{cases} 2.244996 \times 10^{-7} \end{cases} \tag{3.44}$$

$$S^{HH}(t,t) = 2.720956 \tag{3.45}$$

$$S^{HH}(c,c) = 4.415310 \times 10^{-8} \tag{3.46}$$

$$S^{HH}(c,t) = 2.652857 \times 10^{-4} \tag{3.47}$$

$$S^{HW}(t,t) = -1.275614 \tag{3.48}$$

$$S^{HW}(c,c) = -2.994183 \times 10^{-7} \tag{3.49}$$

$$S^{HW}(c,t) = -2.233976 \times 10^{-4} \tag{3.50}$$

$$\text{important} \quad \left| S^{HH}(t,t)\, \varepsilon_t^2 \right| = \begin{cases} 1.889770 \times 10^{-4} & \text{if } q = d \\ 6.363076 \times 10^{-3} & \text{if } q = s \end{cases} \tag{3.51}$$

$$\text{very small} \quad \left| S^{HH}(c,c)\, \varepsilon_c^2 \right| = \begin{cases} 5.144162 \times 10^{-12} & \text{if } q = d \\ 1.002433 \times 10^{-10} & \text{if } q = s \end{cases} \tag{3.52}$$

$$\text{small} \quad \left| S^{HH}(c,t)\, \varepsilon_c \varepsilon_t \right| = \begin{cases} 2.386352 \times 10^{-8} & \text{if } q = d \\ 6.1737 \times 10^{-7} & \text{if } q = s \end{cases} \tag{3.53}$$

$$\text{important} \quad \left| S^{HW}(t,t)\, \varepsilon_t^2 \right| = \begin{cases} 8.854452 \times 10^{-5} & \text{if } q = d \\ 3.042904 \times 10^{-3} & \text{if } q = s \end{cases} \tag{3.54}$$

$$\text{very small} \quad \left| S^{HW}(c,c)\, \varepsilon_c^2 \right| = \begin{cases} 3.488445 \times 10^{-11} & \text{if } q = d \\ 6.797867 \times 10^{-10} & \text{if } q = s \end{cases} \tag{3.55}$$

$$\text{small} \quad \left| S^{HW}(c,t)\, \varepsilon_c \varepsilon_t \right| = \begin{cases} 2.009551 \times 10^{-8} & \text{if } q = d \\ 5.198884 \times 10^{-7} & \text{if } q = s \end{cases} \tag{3.56}$$

$\Rightarrow$

$$\boxed{\Delta m_{B_q^0} \simeq \frac{G_F^2 M_W^2}{\underset{\underset{32}{\updownarrow}}{24\pi^2}} f_B^2 m_B \underset{\underset{\eta_B}{\updownarrow}}{B_B} \left| \left(4\alpha^{WW}(t) - 8\cot^2_\beta S^{HW}(t,t) + \cot^4_\beta S^{HH}(t,t)\right)\right| |\xi_t|^2}$$

PCAC

$$|\xi_t|^2 = \begin{cases} \lambda^2 V_{cb}^2 f(\delta) & \text{if } q = d \\ V_{cb}^2 & \text{if } q = s \end{cases}$$

$q = d$ or s

$$f(\delta) = 1+\rho^2 - 2\rho\cos\delta \qquad (3.57)$$

$$V_{cb}^2 = A^2\lambda^4$$

$$X_q = \frac{\Delta m_{B_q^0}}{\Gamma_{B_q^0}}$$

$$X_d = \frac{\Delta m_{B_d^0}}{\Gamma_{B_d^0}} \quad ; \quad X_s = \frac{\Delta m_{B_s^0}}{\Gamma_{B_s^0}}$$

$$\Gamma_{B_d^0} \simeq \Gamma_{B_s^0}$$

$$\Rightarrow \boxed{\frac{X_s}{X_d} = \frac{\Delta m_{B_s^0}}{\Delta m_{B_d^0}} = \frac{|V_{ts}^*|^2}{|V_{td}^*|^2} = \left|\frac{V_{ts}^*}{V_{td}^*}\right|^2} = \frac{A^2 \cancel{\lambda^4}}{\cancel{A^2}\lambda^{\cancel{4}} f(\delta)} \qquad (3.58)$$

$$\boxed{\frac{X_s}{X_d} \simeq \frac{1}{\lambda^2 f(\delta)} = \frac{1}{\lambda^2(1+\rho^2-2\rho\cos\delta)}} \qquad (3.59)$$

$m_{B_d^0} = 5.2792$ GeV ; $f_B = \sqrt{2}\,(0.11 \pm 0.07)$ GeV $= 155.56$ MeV

Taking $tg\,\beta = 1$, $M_{H^\pm} = 100$ GeV , $M_W = 80.33$ GeV, $m_t = 180$ GeV

$\eta = 1$

$$\left| \left(4\alpha^{WW}(t) - 8\cot^2\beta\, S^{HW}(t,t) + \cot^4\beta\, S^{HH}(t,t)\right)\right|$$

$$= 23.69587$$

$$\Delta m_{B_d^0} = \begin{cases} 7.792509 \times 10^{-13} \text{ GeV} & \text{PCAC} \\ 5.844382 \times 10^{-13} \text{ GeV} & \text{our model} \end{cases}$$

$$\Delta m_{B_s^0} = \begin{cases} 2.722127 \times 10^{-11} \text{ Gev} & \text{PCAC} \\ 2.041595 \times 10^{-11} \text{ Gev} & \text{OUR MODEL} \end{cases}$$

$\Gamma_{B_s^0} \approx ?$

$$\boxed{\begin{aligned} \tau_{B_s^0} &= 1.61 \times 10^{-12}\, s \\ \Rightarrow \Gamma_{B_s^0} &= 4.088275 \times 10^{-13} \text{ Gev} \end{aligned}} \simeq \Gamma_{B_d^0}$$

$$\Rightarrow \quad x_d \approx \begin{cases} 1.846869 & \text{PCAC} \\ 1.385151 & \text{OUR MODEL} \end{cases}$$

$$x_s \approx \begin{cases} 66.58375 & \text{PCAC} \\ 49.93781 & \text{OUR MODEL} \end{cases}$$

From (3.59) $\dfrac{x_s}{x_d} \approx 34.35$

More generally:

$$\boxed{\frac{x_s}{x_d} = \frac{\Delta m_{B_s^0}}{\Delta m_{B_d^0}} \frac{\Gamma_{B_d^0}}{\Gamma_{B_s^0}} = \frac{f_{B_s^0}^2 B_{B_s^0}}{f_{B_d^0}^2 B_{B_d^0}} \left|\frac{V_{ts}^*}{V_{td}^*}\right|^2 \frac{\Gamma_{B_d^0}}{\Gamma_{B_s^0}} \frac{m_{B_s^0}}{m_{B_d^0}}} \quad (3.60)$$

1

with $\left|\dfrac{V_{ts}^*}{V_{td}^*}\right|^2 = \dfrac{1}{\lambda^2(1+\rho^2-2\rho\cos\delta)} \Rightarrow \boxed{\dfrac{x_s}{x_d} \approx 36.052}$ (3.61)

<u>If we take $\tan\beta = 2$</u> $\Rightarrow \cot\beta = \frac{1}{2}$

$$\left|\left(4\alpha^{WW}(t) - 8\cot^2\beta\, S^{HW}(t,t) + \cot^4\beta\, S^{HH}(t,t)\right)\right|$$

$$= 13.49129$$

$$\Rightarrow \quad \Delta m_{B_d^0} = \begin{cases} 4.436688 \times 10^{-13} \text{ Gev} & \text{PCAC} \\ 3.327510 \times 10^{-13} \text{ Gev} & \text{OUR MODEL} \end{cases}$$

$$\boxed{\begin{aligned} \tau_{B_d^0} &= 1.56 \times 10^{-12}\, s. \\ \Gamma_{B_d^0} &= 4.219309 \times 10^{-13} \text{ Gev} \end{aligned}} \simeq \Gamma_{B_s^0}$$

$$\Rightarrow \quad \boxed{x_d = \begin{cases} 1.051520 & \text{PCAC} \\ 0.78864 & \text{OUR MODEL} \end{cases};} \quad (3.62)$$

$$\Delta m_{B_s^0} = \begin{cases} 1.549848 \times 10^{-11} \text{ Gev} & \text{PCAC} \\ 1.162386 \times 10^{-11} \text{ Gev} & \text{OUR MODEL} \end{cases}$$

$$\Rightarrow \boxed{X_s = \begin{cases} 37.27345 & \text{PCAC} \\ 27.95509 & \text{OUR MODEL} \end{cases}} \qquad (3.63)$$

because $\alpha^{WW}(t) > 0$; $-S^{HW}(t,t) > 0$; $S^{HH}(t,t) > 0$

$$\therefore \quad \Delta m^{HH}_{B_q^0} < \Delta m_{B_q^0}\, \text{exp.}$$

$$\Delta m^{HH}_{B_q^0} = 2\,|\langle B^0 | M^{HH} | \bar{B}^0 \rangle| \sim \cot^4\beta$$

$$\Delta m^{HW}_{B_q^0} = 2\,|\langle B^0 | M^{HW} | \bar{B}^0 \rangle| \sim \cot^2\beta$$

$$\Delta m^{HH}_{B_q^0} = \frac{G_F^2 M_W^2}{\underbrace{24\pi^2}_{32}}\, f^2_{B_q^0}\, m_{B_q^0}\, \underbrace{B_{B_q^0}}_{\eta} \cot^4\beta\, |S^{HH}(t,t)|\, |\xi_t|^2$$

PCAC

$$\Rightarrow \boxed{\tan^4\beta > \frac{\dfrac{G_F^2 M_W^2}{\underbrace{24\pi^2}_{32}}\, f^2_{B_q^0}\, m_{B_q^0}\, \underbrace{B_{B_q^0}}_{\eta}\, |S^{HH}(t,t)\, \xi_t^2|}{\Delta m_{B_q^0}\, \text{exp}} \quad \text{with } |\xi_t|^2 = \begin{cases} A^2\lambda^6(1+\rho^2-2\rho\cos\delta) & \text{if } q = d \\ A^2\lambda^4 & \text{if } q = s \end{cases}} \qquad (3.64)$$

$m_{B_d^0} = 5.2792$ Gev ; $\Delta m_{B_d^0\,\text{exp}} = (0.474 \pm 0.031) \times 10^{12}\, \hbar\, s^{-1}$

$m_{B_s^0} = 5.3693$ Gev $\qquad = 3.119926 \times 10^{-13}$ Gev

for $q = d$ $\quad \tan^4\beta > \begin{cases} 0.2868 \\ 0.2151 \end{cases} \quad \therefore \tan^2\beta > \begin{cases} 0.5355 \\ 0.46379 \end{cases}$

$$\therefore \quad \boxed{\tan\beta > \begin{cases} 0.7318 & \text{PCAC} \\ 0.6810 & \text{OUR MODEL} \end{cases} \quad q = d} \qquad (3.65)$$

for $q = s$ $\quad \Delta m_{B_s^0}\ exp > 5.9 \times 10^{12}\ \hbar\, s^{-1}$

$= 3.883 \times 10^{-12}\ GeV$

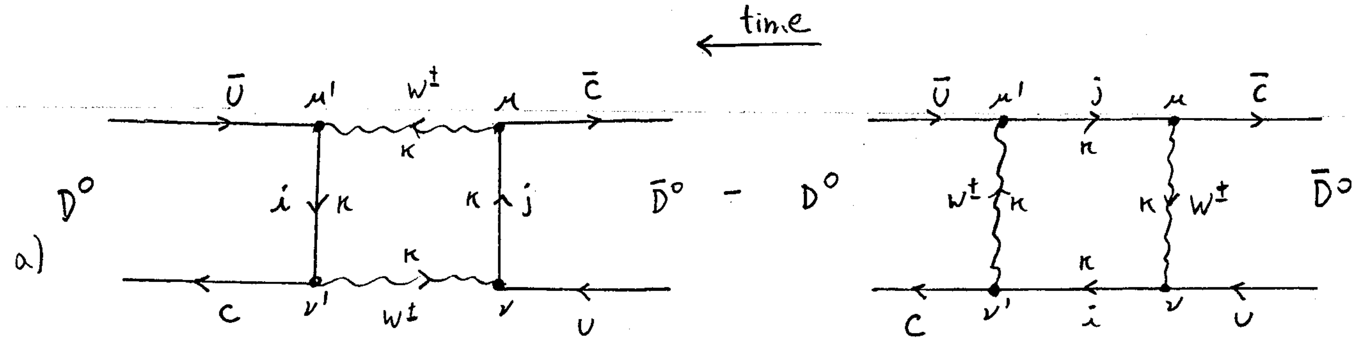
$\tan^4\beta > \{ \quad ?$

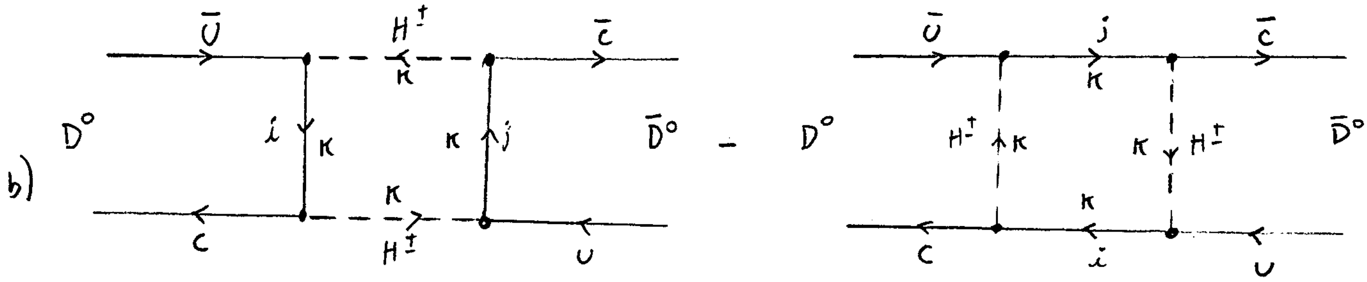
we can't establish a limit.

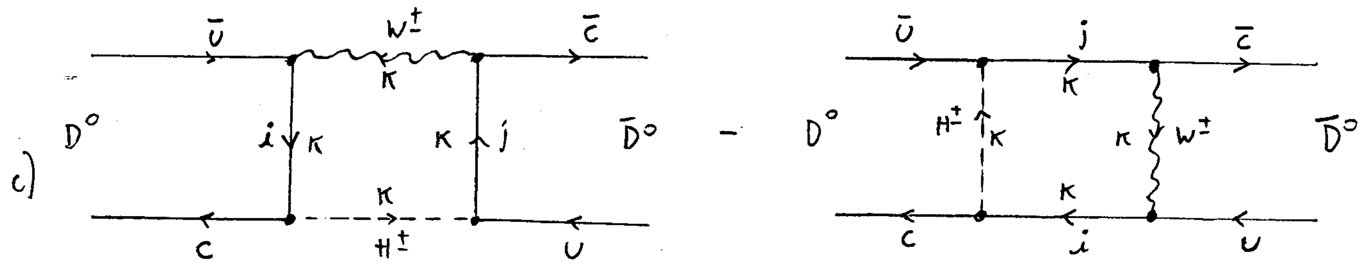
$D^0 - \bar{D}^0$ Mixing

$D^0 = c\bar{u}$; $\bar{D}^0 = u\bar{c}$

The corresponding box diagrams are :







+ crossed diagrams ($W^\pm \leftrightarrow H^\pm$)

with $i, j = d, s, b$

$$\xi_i = V_{ci} V_{ui}^*$$
$$\xi_j = V_{cj} V_{uj}^* \qquad \sum_i \xi_i = 0$$

$$\Delta m_{D^0} = m_{\underset{D_1^0}{D_H^0}} - m_{\underset{D_2^0}{D_L^0}} = 2\,|M_{12}|$$

$$= 2\,\left|\langle D^0 | M^{WW} | \bar{D}^0\rangle + \langle D^0 | M^{HW} | \bar{D}^0\rangle + \langle D^0 | M^{HH} | \bar{D}^0\rangle\right| \tag{4.1}$$

$$M^{WW} = \frac{G_F^2 M_W^2}{\pi^2} \sum_{i,j} \xi_i \xi_j \, S^{WW}(i,j) \; \bar{V}_L(\bar{c}) \gamma^\mu U_L(u) \,.\, \bar{U}_L(c) \gamma_\mu V_L(\bar{u}) \qquad (4.2)$$

Comparing with $B^0 - \bar{B}^0$

$\bar{q} \leftrightarrow \bar{c}$ $\qquad b \leftrightarrow u$ $\qquad tg\beta \leftrightarrow \cot\beta$

$\bar{b} \leftrightarrow \bar{u}$ $\qquad q \leftrightarrow c$ $\qquad$ (see 4.47)

$$\langle D^0 | M^{WW} | \bar{D}^0 \rangle = \frac{G_F^2 M_W^2 f_D^2 m_D}{\underset{\underset{16}{\updownarrow}}{12}\, \pi^2} \; \underset{\underset{\eta_D}{\updownarrow}}{B_D} \sum_{i,j} \xi_i \xi_j \, S^{WW}(i,j) \qquad (4.3)$$

$$\langle D^0 | M^{HW} | \bar{D}^0 \rangle = - \frac{G_F^2 M_W^2 \, tg^2\beta}{\underset{\underset{8}{\updownarrow}}{6}\, \pi^2} \, f_D^2 m_D \; \underset{\underset{\eta_D}{\updownarrow}}{B_D} \sum_{i,j} \xi_i \xi_j \, S^{HW}(i,j) \qquad (4.4)$$

$(m_u \to 0)$

$$\langle D^0 | M^{HH} | \bar{D}^0 \rangle = \frac{G_F^2 M_W^2 \, tg^4\beta}{\underset{\underset{64}{\updownarrow}}{48}\, \pi^2} \, f_D^2 m_D \; \underset{\underset{\eta_D}{\updownarrow}}{B_D} \sum_{i,j} \xi_i \xi_j \, S^{HH}(i,j) \qquad (4.5)$$

($m_c \to 0$ the term with: $m_i^2 m_j^2 m_c^2$ neglected)

if $tg\beta \approx 1$

$$A' \equiv \langle D^0 | \bar{V}(\bar{c}) \gamma^\mu (1-\gamma^5) U(u) \,.\, \bar{U}(c) \gamma_\mu (1-\gamma^5) V(\bar{u}) | \bar{D}^0 \rangle \qquad (4.6)$$

$$A' = 4A = \begin{cases} \frac{1}{3} f_D^2 m_D B_D & \text{PCAC} \\ \frac{f_D^2 m_D}{4} \eta_D & \text{free particles} \end{cases} \qquad (4.7)$$

$$\boxed{\Delta m_{D^0} = \frac{G_F^2 M_W^2}{\underset{\underset{32}{\updownarrow}}{24}\, \pi^2} \, f_D^2 m_D \; \underset{\underset{\eta_D}{\updownarrow}}{B_D} \left| \sum_{i,j} \xi_i \xi_j \left[4 S^{WW}(i,j) - 8\, tg^2\beta \, S^{HW}(i,j) + tg^4\beta \, S^{HH}(i,j) \right] \right|}$$

(4.8)

Let's consider $S^{WW}(i,i)$

$$S^{WW}(i,i) = \frac{1}{4}\left(4+(x_i^w)^2\right)\frac{\left(1-(x_i^w)^2\right)}{(1-x_i^w)^3} - \frac{4(x_i^w)^2}{(1-x_i^w)^2}$$

$$+\frac{1}{4}\frac{\left(4+(x_i^w)^2\right)2x_i^w \ln(x_i^w)}{(1-x_i^w)^3} - \frac{2(x_i^w)^2(1+x_i^w)}{(1-x_i^w)^3}\ln(x_i^w)$$

$$= \frac{\left(4+(x_i^w)^2\right)(1+x_i^w) - 16(x_i^w)^2}{4(1-x_i^w)^2} + 1 - 1$$

$$+ \frac{2x_i^w \ln(x_i^w)}{4(1-x_i^w)^3}\left[4+(\cancel{x_i^w})^2 - 4x_i^w - 4(\cancel{x_i^w})^2\right]$$

$$= 1 + \frac{\cancel{4}+4(\cancel{x_i^w}) + (\cancel{x_i^w})^2 + (x_i^w)^3 - 16(\cancel{x_i^w})^2 - \cancel{4} + 8(\cancel{x_i^w}) - 4(\cancel{x_i^w})^2}{4(1-x_i^w)^2}$$

$$+ \frac{2x_i^w \ln(x_i^w)}{4(1-x_i^w)^3}\left[4 - 4x_i^w - 3(x_i^w)^2\right]$$

$$\boxed{S^{WW}(i,i) = 1 + \frac{\left(12x_i^w - 19(x_i^w)^2 + (x_i^w)^3\right)}{4(1-x_i^w)^2} + \frac{2x_i^w \ln(x_i^w)}{(1-x_i^w)^3}\left[1 - x_i^w - \frac{3}{4}(x_i^w)^2\right]} \quad (4.9)$$

$$\sum_{i,j} \xi_i \xi_j S^{WW}(i,j) = \xi_d^2 S^{WW}(d,d) + \xi_s^2 S^{WW}(s,s) + \xi_b^2 S^{WW}(b,b)$$

$$+ 2\xi_d\xi_s S^{WW}(d,s) + 2\xi_d\xi_b S^{WW}(d,b) + 2\xi_s\xi_b S^{WW}(s,b)$$

$$= \xi_d^2\left[1 + \frac{3x_d^w}{(1-x_d^w)^2} + \cancel{\frac{2x_d^w \ln x_d^w}{(1-x_d^w)^2}}\right]$$

$$+ \xi_s^2\left[1 + \frac{3x_s^w}{(1-x_s^w)^2} + \cancel{\frac{2x_s^w \ln x_s^w}{(1-x_s^w)^2}}\right]$$

$$+ \xi_b^2\left[1 + \frac{\left(12x_b^w - 19(x_b^w)^2 + \cancel{(x_b^w)^3}\right)}{4(1-x_b^w)^2} + \frac{2x_b^w \ln(x_b^w)}{(1-x_b^w)^3}\left[1 - x_b^w - \frac{3}{4}(x_b^w)^2\right]\right]$$

$$+ 2\xi_d \xi_s \left(1 + \frac{X_d^W X_s^W}{4}\right) \frac{\left(\frac{1}{1-X_s^W} + \frac{(X_s^W)^2 \ln (X_s^W)}{(1-X_s^W)^2} - 1\right)}{X_s^W}$$

$$+ 2\xi_d \xi_b \left(1 + \frac{X_d^W X_b^W}{4}\right) \frac{\left(\frac{1}{1-X_b^W} + \frac{(X_b^W)^2 \ln (X_b^W)}{(1-X_b^W)^2} - 1\right)}{X_b^W}$$

$$+ 2\xi_s \xi_b \left(1 + \frac{X_s^W X_b^W}{4}\right) \frac{\left(\frac{1}{1-X_b^W} + \frac{(X_b^W)^2 \ln (X_b^W)}{(1-X_b^W)^2} - \frac{1}{(1-X_s^W)}\right)}{(X_b^W - X_s^W)}$$

$$+ 2\xi_d \xi_s \frac{2 X_d^W X_s^W}{(1-X_s^W)} \left[-1 - \frac{\cancel{X_s^W} \ln (X_s^W)}{(1-X_s^W)\cancel{X_s^W}} \right]$$

$$+ 2\xi_d \xi_b \frac{2 X_d^W X_b^W}{(1-X_b^W)} \left[-1 - \frac{\cancel{X_b^W} \ln (X_b^W)}{(1-X_b^W)\cancel{X_b^W}} \right]$$

$$+ 2\xi_s \xi_b \frac{2 X_s^W X_b^W}{(1-X_b^W)(1-X_s^W)} \left[\frac{-X_b^W \ln (X_b^W)(1-X_s^W)}{(1-X_b^W)(X_b^W - X_s^W)} - \frac{X_s^W \ln (X_s^W)(1-X_b^W)}{(1-X_s^W)(X_s^W - X_b^W)} - 1 \right]$$

$$\xi_d + \xi_s + \xi_b = 0$$

$$\xi_d^2 + \xi_s^2 = \xi_b^2 - 2\xi_d \xi_s$$

$$\Rightarrow \sum_{i,j} \xi_i \xi_j S^{WW}(i,j) = \xi_b^2 \left[2 + \frac{(12 X_b^W - 19 (X_b^W)^2 + (X_b^W)^3)}{4(1-X_b^W)^2} + \frac{2 X_b^W \ln (X_b^W)}{(1-X_b^W)^3} \left(1 - X_b^W - \tfrac{3}{4}(X_b^W)^2\right) \right]$$

$$+ \xi_d^2 \frac{X_d^W}{(1-X_d^W)^2} \left[3 + 2 \ln X_d^W \right]$$

$$+ \xi_s^2 \frac{X_s^W}{(1-X_s^W)^2} \left[3 + 2 \ln X_s^W \right]$$

$$+ 2\xi_d \xi_s \left(1 + \frac{X_d^W X_s^W}{4}\right)\left(\frac{1}{1-X_s^W} + \frac{X_s^W \ln X_s^W}{(1-X_s^W)^2}\right)$$

$$+ 2\xi_d \xi_b \left(1 + \frac{X_d^W X_b^W}{4}\right)\left(\frac{1}{1-X_b^W} + \frac{X_b^W \ln X_b^W}{(1-X_b^W)^2}\right)$$

$$+ 2\xi_s \xi_b \left(1 + \frac{X_s^W X_b^W}{4}\right)\left(\frac{X_b^W}{1-X_b^W} + \frac{(X_b^W)^2 \ln X_b^W}{(1-X_b^W)^2} - \frac{X_s^W}{(1-X_s^W)}\right) \Big/ (X_b^W - X_s^W)$$

$$+ 4\xi_d \xi_s \frac{X_d^W X_s^W}{(1-X_s^W)}\left(-1 - \frac{\ln X_s^W}{(1-X_s^W)}\right) + 4\xi_d \xi_b \frac{X_d^W X_b^W}{(1-X_b^W)}\left(-1 - \frac{\ln X_b^W}{(1-X_b^W)}\right)$$

$$+ 4\xi_s \xi_b \frac{X_s^W X_b^W}{(1-X_b^W)}\left(\frac{-X_b^W \ln (X_b^W)}{(1-X_b^W)(X_b^W - X_s^W)} - 1\right) - 2\xi_d \xi_s$$

$$= \xi_b^2 \left[2 + \frac{(12X_b^W - 19(X_b^W)^2 + (X_b^W)^3)}{4(1-X_b^W)^2} + \frac{2X_b^W \ln (X_b^W)}{(1-X_b^W)^3}\left(1 - X_b^W - \frac{3}{4}(X_b^W)^2\right)\right]$$

$$+ \xi_d^2 X_d^W (3 + 2\ln X_d^W) + \xi_s^2 X_s^W (\cancel{3} + 2\ln X_s^W)/(1-X_s^W)^2$$

$$+ 2\xi_d \xi_s \left[\frac{1}{1-X_s^W} + \frac{X_d^W X_s^W}{4(1-X_s^W)} - 1 - \frac{2X_d^W X_s^W}{(1-X_s^W)} + \frac{X_s^W \ln X_s^W}{(1-X_s^W)^2}\right.$$

$$\left. - \frac{2X_d^W X_s^W \ln X_s^W}{(1-X_s^W)^2}\right]$$

$$+ 2\xi_d \xi_b \left[\frac{1}{1-X_b^W} + \frac{X_d^W X_b^W}{4(1-X_b^W)} - \frac{2X_d^W X_b^W}{(1-X_b^W)} + \frac{X_b^W \ln X_b^W}{(1-X_b^W)^2} - \frac{2X_d^W X_b^W \ln X_b^W}{(1-X_b^W)^2}\right.$$

$$\left. + \frac{X_d^W (X_b^W)^2 \ln X_b^W}{4(1-X_b^W)^2}\right]$$

$$+ 2\xi_s \xi_b \left[\frac{1}{1-X_b^W} + \frac{X_s^W X_b^W}{4(1-X_b^W)} - \frac{2X_s^W X_b^W}{(1-X_b^W)} + \frac{X_b^W \ln X_b^W}{(1-X_b^W)^2} + \frac{X_s^W (X_b^W)^2 \ln X_b^W}{4(1-X_b^W)^2}\right.$$

$$\left. - \frac{2X_s^W X_b^W \ln X_b^W}{(1-X_b^W)^2}\right]$$

with

$$\alpha(X_b^w) = \left[\frac{(X_b^w)^3 - 11\,(X_b^w)^2 + 4(X_b^w)}{4\,(1-X_b^w)^2} - \frac{3}{2}\,\frac{(X_b^w)^3 \ln(X_b^w)}{(1-X_b^w)^3} \right]$$

$$\delta(X_q^w) = \frac{X_q^w(3 + 2\ln X_q^w)}{(1-X_q^w)^2} \qquad q = d \text{ or } s$$

$$\phi(X_s^w) = \frac{2X_s^w}{(1-X_s^w)}\left(1 + \frac{\ln X_s^w}{(1-X_s^w)}\right)$$

$$\gamma(X_d^w, X_b^w) = \frac{X_d^w X_b^w}{2(1-X_b^w)}\left[-7 + \frac{(X_b^w - 8)\ln X_b^w}{(1-X_b^w)}\right]$$

$$\updownarrow$$

$$\gamma(X_s^w, X_b^w) \qquad d \leftrightarrow s$$

(4.12)

$$X_i^w = \frac{m_i^2}{M_w^2}$$

We will take:

m_d = 10 Mev = 0.01 Gev

m_s = 200 Mev = 0.2 Gev

m_b = 4.3 Gev

M_W = 80.33 Gev

$$\alpha(X_b^w) = 2.85937 \times 10^{-3} \qquad (4.13)$$

$$\delta(X_d^w) = -5.108588 \times 10^{-7} \qquad (4.14)$$

$$\delta(X_s^w) = -1.30064 \times 10^{-4} \qquad (4.15)$$

$$\phi(X_s^w) = -1.362648 \times 10^{-4} \qquad (4.16)$$

$$\gamma(X_d^w, X_b^w) = 8.897095 \times 10^{-10} \qquad (4.17)$$

$$\gamma(X_s^w, X_b^w) = 3.558838 \times 10^{-7} \qquad (4.18)$$

$$\boxed{\begin{aligned}
(\xi_b)^2 &= (V_{cb}V_{ub}^*)^2 = (A^2\lambda^5\rho e^{-i\delta})^2 = A^4\lambda^{10}\rho^2 e^{2i\delta} \\
(\xi_d)^2 &= (V_{cd}V_{ud}^*)^2 = (-\lambda(1-\tfrac{1}{2}\lambda^2))^2 = \lambda^2(1-\tfrac{1}{2}\lambda^2)^2 \\
(\xi_s)^2 &= (V_{cs}V_{us}^*)^2 = (\lambda(1-\tfrac{1}{2}\lambda^2))^2 = \lambda^2(1-\tfrac{1}{2}\lambda^2)^2 \\
\xi_d\xi_s &= V_{cd}V_{ud}^*V_{cs}V_{us}^* = -\lambda(1-\tfrac{1}{2}\lambda^2)\lambda(1-\tfrac{1}{2}\lambda^2) = -\lambda^2(1-\tfrac{1}{2}\lambda^2)^2 \\
\xi_d\xi_b &= V_{cd}V_{ud}^*V_{cb}V_{ub}^* = -\lambda(1-\tfrac{1}{2}\lambda^2)A^2\lambda^5\rho e^{i\delta} = -A^2\lambda^6(1-\tfrac{1}{2}\lambda^2)\rho e^{i\delta} \\
\xi_s\xi_b &= V_{cs}V_{us}^*V_{cb}V_{ub}^* = \lambda(1-\tfrac{1}{2}\lambda^2)A^2\lambda^5\rho e^{i\delta} = A^2\lambda^6(1-\tfrac{1}{2}\lambda^2)\rho e^{i\delta}
\end{aligned}}$$

taking $A \approx 1$; $\lambda = 0.221$; $-0.7 < \rho < 0.7$ $(\rho = 0.52)$ (4.19)

$$|(\xi_b)^2| = 7.515008 \times 10^{-8}$$

$$|(\xi_d)^2| = 4.648468 \times 10^{-2}$$

$$|(\xi_s)^2| = 4.648468 \times 10^{-2}$$

$$|\xi_d\xi_s| = \text{''}$$

$$|\xi_d\xi_b| = 5.910438 \times 10^{-5}$$

$$|\xi_s\xi_b| = \text{''}$$

$$\Rightarrow |\alpha(X_b^w)\xi_b^2| = 2.148819 \times 10^{-10}$$

$$|\delta(X_d^W)\xi_d^2| = 2.374707 \times 10^{-8}$$

$$|\delta(X_s^w)\xi_s^2| = 6.045983 \times 10^{-6}$$

$$|\phi(X_s^w)\xi_d\xi_s| = 6.334226 \times 10^{-6}$$

$$|\gamma(X_d^w, X_b^w)\xi_d\xi_b| = 5.258573 \times 10^{-14}$$

$$|\gamma(X_s^W, X_b^w)\xi_s\xi_b| = 2.103429 \times 10^{-11}$$

$$\Rightarrow \boxed{\sum_{i,j}\xi_i\xi_j S^{ww}(i,j) \approx \delta(X_d^w)\xi_d^2 + \delta(X_s^w)\xi_s^2 + \phi(X_s^w)\xi_d\xi_s} \quad (4.20)$$

$$S^{WW}(d,d) = 9.99999489 \times 10^{-1}$$
$$S^{WW}(s,s) = 9.998699 \times 10^{-1}$$
$$S^{WW}(b,b) = 9.74859614 \times 10^{-1}$$
$$S^{WW}(d,s) = 1.00254592$$
$$S^{WW}(d,b) = 9.860052 \times 10^{-1}$$
$$S^{WW}(s,b) = 9.858997 \times 10^{-1}$$

$$\mathcal{E}_b^2 = 7.515008 \times 10^{-8} \qquad (\delta = 0)$$
$$\mathcal{E}_d^2 = 4.648468 \times 10^{-2}$$
$$\mathcal{E}_s^2 = \quad ''$$
$$\mathcal{E}_d \mathcal{E}_s = -4.648468 \times 10^{-2}$$
$$\mathcal{E}_d \mathcal{E}_b = -5.910438 \times 10^{-5} \qquad (\delta = 0)$$
$$\mathcal{E}_s \mathcal{E}_b = + \quad '' \qquad (\delta = 0)$$

$$\Rightarrow \sum_{i,j} \mathcal{E}_i \mathcal{E}_j S^{WW}(i,j) = -2.427 \times 10^{-4}$$

$$\mathcal{E}_i = V_{ci} V_{ui}^*$$

$$\mathcal{E}_d + \mathcal{E}_s + \mathcal{E}_b = V_{cd} V_{ud}^* + V_{cs} V_{us}^* + V_{cb} V_{ub}^* = 0$$

In the wolfenstein parametrization:

$$\mathcal{E}_d + \mathcal{E}_s + \mathcal{E}_b = -\cancel{\lambda} + \cancel{\tfrac{1}{2}\lambda^3} + \cancel{\lambda} - \cancel{\tfrac{1}{2}\lambda^3} + A^2 \lambda^5 \rho e^{-i\delta}$$

$$\boxed{\mathcal{E}_d + \mathcal{E}_s + \mathcal{E}_b = A^2 \lambda^5 \rho \, e^{-i\delta}}$$

For $\delta = 0$
$\lambda = 0.221$
$\rho = 0.52$

$$\mathcal{E}_d + \mathcal{E}_s + \mathcal{E}_b = 2.74135 \times 10^{-4}$$

$m_t = 180$ GeV $\quad S^{WW}(t,t) = 2.197315163$; $S^{WW}(\substack{u \\ c}, t) \simeq 0.2524$;

$$S^{WW}\begin{pmatrix} c,c \\ u,u \\ u,c \end{pmatrix} = 1 \qquad \mathcal{E}_t^2 = \begin{cases} 6.945245 \times 10^{-5} & q = d \\ 2.385443 \times 10^{-3} & q = s \end{cases}$$

$$\simeq \Big[X_d^w (3 + 2 \ln X_d^w) + X_s^w (3 + 2 \ln X_s^w) - \frac{2 X_s^w}{(1 - X_s^w)} \Big(1 + \frac{\ln X_s^w}{(1 - X_s^w)} \Big) \Big] \lambda^2 (1 - \tfrac{1}{2}\lambda^2)^2$$

$$\simeq \Big[X_d^w (3 + 2 \ln X_d^w) + X_s^w (\cancel{3} + 2 \cancel{\ln X_s^w}) - \cancel{2 X_s^w} - 2 X_s^w \cancel{\ln} X_s^w \Big] \lambda^2 (1 - \tfrac{1}{2}\lambda^2)^2$$

$$\simeq \Big[X_s^w + X_d^w (3 + 2 \ln X_d^w) \Big] \lambda^2 (1 - \tfrac{1}{2}\lambda^2)^2$$

$(\sim 2.644 \times 10^{-7})$

$$\boxed{\sum_{i,j} \xi_i \xi_j S^{ww}(i,j) \simeq \Big[X_s^w + X_d^w (3 + 2 \ln X_d^w) \Big] \lambda^2 (1 - \tfrac{1}{2}\lambda^2)^2} \tag{4.21}$$

If we don't consider charged Higgs contributions

$$\Delta m_{D^0} \simeq \frac{G_F^2 M_W^2}{6\pi^2} f_D^2\, m_D\, B_D \cdot \Big[X_s^w + X_d^w (3 + 2 \ln X_d^w) \Big] \lambda^2 (1 - \tfrac{1}{2}\lambda^2)^2 \tag{4.22}$$

$6\pi^2 \updownarrow 8$; $B_D \updownarrow \eta_D$

or:

$$\Delta m_{D^0} \simeq \frac{G_F^2 f_D^2 m_D B_D}{6\pi^2} \Big[m_s^2 + m_d^2 \Big(3 + 2 \ln \Big(\frac{m_d^2}{M_W^2} \Big) \Big) \Big] \lambda^2 (1 - \tfrac{1}{2}\lambda^2)^2 \tag{4.23}$$

$6\pi^2 \updownarrow 8$; $B_D \updownarrow \eta_D$

taking $f_D = 0.2$ GeV ; $\eta \simeq 1$; $m_{D^0} = 1.8645$ GeV

$G_F = 1.166392 \times 10^{-5}$ GeV^{-2} ; $m_s = 0.2$ GeV ; $m_d = 10^{-2}$ GeV

$M_W = 80.33$ GeV ; $\lambda = 0.221$

we get $\Delta m_{D^0} = (2.923328 \times 10^{-16})$ GeV $\xrightarrow{\text{PCAC}}$ $= 2.923328 \times 10^{-13}$ MeV $\xrightarrow{\text{PCAC}}$ (4.24)

$(2.192496 \times 10^{-16})$ GeV $\quad (2.192496 \times 10^{-13}$ MeV$)$

experiment : $|m_{D_1^0} - m_{D_2^0}| < 21 \times 10^{10}\ \hbar\, s^{-1} = 1.3822 \times 10^{-13}$ GeV

$= 1.3822 \times 10^{-10}$ MeV

$$\tau_{D^0} = (0.415 \pm 0.004) \times 10^{-12}\ s$$

$$\Rightarrow \quad \Gamma_{D^0} = 1.585542 \times 10^{-12}\ \text{GeV}$$

$$\therefore \quad \boxed{X_{D^0} = \frac{\Delta m_{D^0}}{\Gamma_{D^0}} \approx \begin{cases} 1.84374 \times 10^{-4} \\ 1.3828 \times 10^{-4} \end{cases}} \qquad (4.25)$$

Let's evaluate

$$\sum_{i,j} \xi_i \xi_j S^{HH}(i,j) = \xi_d^2 S^{HH}(d,d) + \xi_s^2 S^{HH}(s,s) + \xi_b^2 S^{HH}(b,b) + 2\xi_d \xi_s S^{HH}(d,s) + 2\xi_d \xi_b S^{HH}(d,b) + 2\xi_s \xi_b S^{HH}(s,b)$$

$$\boxed{S^{HH}(d,d) \approx (X_d^H)^2 / X_W^H} \qquad (4.26)$$

$$\boxed{S^{HH}(s,s) \approx \frac{(X_s^H)^2}{X_W^H} \left(\frac{1 + 2X_s^H \ln X_s^H}{(1-X_s^H)^3} \right)} \qquad (4.27)$$

$$\boxed{S^{HH}(b,b) = \frac{(X_b^H)^2}{X_W^H} \left[\frac{1 - (X_b^H)^2 + 2X_b^H \ln X_b^H}{(1-X_b^H)^3} \right]} \qquad (4.28)$$

$$S^{HH}(d,s) = \frac{X_d^H X_s^H}{X_W^H} \left[\frac{1}{1-X_s^H} + \frac{\frac{(X_s^H)^2 \ln(X_s^H)}{(1-X_s^H)^2} - 1}{X_s^H} \right]$$

$$= \frac{X_d^H X_s^H}{X_W^H} \left[\frac{1}{1-X_s^H} + \frac{X_s^H \ln(X_s^H)}{(1-X_s^H)^2} \right]$$

$$\boxed{S^{HH}(d,s) = \frac{X_d^H X_s^H}{X_W^H (1-X_s^H)} \left[1 + \frac{X_s^H \ln(X_s^H)}{(1-X_s^H)} \right]} \qquad (4.29)$$

$$\boxed{S^{HH}(d,b) = \frac{x_d^H\, x_b^H}{x_w^H\,(1-x_b^H)}\left[1 + \frac{x_b^H \ln(x_b^H)}{(1-x_b^H)}\right]} \qquad (4.30)$$

$$\boxed{S^{HH}(s,b) = \frac{x_s^H\, x_b^H}{x_w^H}\left[\frac{\dfrac{1}{1-x_b^H} + \dfrac{(x_b^H)^2 \ln(x_b^H)}{(1-x_b^H)^2} - \dfrac{1}{1-x_s^H}}{(x_b^H - x_s^H)}\right]} \qquad (4.31)$$

$$S^{HH}(d,d) = 1.549689 \times 10^{-16} \qquad (4.32)$$

$$S^{HH}(s,s) = 2.479285 \times 10^{-11} \qquad (4.33)$$

$$S^{HH}(b,b) = 5.203573 \times 10^{-6} \qquad (4.34)$$

$$S^{HH}(d,s) = 6.198471 \times 10^{-14} \qquad (4.35)$$

$$S^{HH}(d,b) = 2.837217 \times 10^{-11} \qquad (4.36)$$

$$S^{HH}(s,b) = 1.134862 \times 10^{-8} \qquad (4.37)$$

Taking ($M_{H^+} = 100$ GeV)

$$\Rightarrow \quad \left|\xi_d^2\, S^{HH}(d,d)\right| = 7.20368 \times 10^{-18}$$

$$\left|\xi_s^2\, S^{HH}(s,s)\right| = 1.152488 \times 10^{-12}$$

$$\left|\xi_b^2\, S^{HH}(b,b)\right| = 3.910489 \times 10^{-13}$$

$$\left|2\,\xi_d\,\xi_s\, S^{HH}(d,s)\right| = 5.762679 \times 10^{-15}$$

$$\left|2\,\xi_d\,\xi_b\, S^{HH}(d,b)\right| = 3.353839 \times 10^{-15}$$

$$\left|2\,\xi_s\,\xi_b\, S^{HH}(s,b)\right| = 1.341506 \times 10^{-12}$$

$$\therefore \quad \boxed{\sum_{i,j} \xi_i\,\xi_j\, S^{HH}(i,j) \approx \xi_s^2\, S^{HH}(s,s) + \xi_b^2\, S^{HH}(b,b) + 2\,\xi_s\,\xi_b\, S^{HH}(s,b)} \qquad (4.38)$$

$$\sum_{i,j} \xi_i \xi_j S^{HW}(i,j) = \xi_d^2 S^{HW}(d,d) + \xi_s^2 S^{HW}(s,s) + \xi_b^2 S^{HW}(b,b)$$
$$+ 2\xi_d \xi_s S^{HW}(d,s) + 2\xi_d \xi_b S^{HW}(d,b) + 2\xi_s \xi_b S^{HW}(s,b) \quad (4.39)$$

$$S^{HW}(d,d) = -6.138628 \times 10^{-15} \quad (4.40)$$

$$S^{HW}(s,s) = -2,604266 \times 10^{-10} \quad (4.41)$$

$$S^{HW}(b,b) = -2.335539 \times 10^{-5} \quad (4.42)$$

$$S^{HW}(d,s) = -7.121177 \times 10^{-13} \quad (4.43)$$

$$S^{HW}(d,b) = -1.544265 \times 10^{-10} \quad (4.44)$$

$$S^{HW}(s,b) = -6.161946 \times 10^{-8} \quad (4.45)$$

$$|\xi_d^2 S^{HW}(d,d)| = 2.853522 \times 10^{-16}$$

$$|\xi_s^2 S^{HW}(s,s)| = 1.210585 \times 10^{-11}$$

$$|\xi_b^2 S^{HW}(b,b)| = 1.755159 \times 10^{-12}$$

$$|2\xi_d \xi_s S^{HW}(d,s)| = 6.620513 \times 10^{-14}$$

$$|2\xi_d \xi_b S^{HW}(d,b)| = 1.825457 \times 10^{-14}$$

$$|2\xi_s \xi_b S^{HW}(s,b)| = 7.283960 \times 10^{-12}$$

$$\Rightarrow \boxed{\sum_{i,j} \xi_i \xi_j S^{HW}(i,j) \approx \xi_s^2 S^{HW}(s,s) + \xi_b^2 S^{HW}(b,b) + 2\xi_s \xi_b S^{HW}(s,b)} \quad (4.46)$$

So charged Higgs contributions are negligible for the $D^0-\bar{D}^0$ system.

The invariant amplitude is for diagram (b):

$$-iM^{HH} = 2 \sum_{i,j} \xi_i \xi_j \int \frac{d^4 K}{(2\pi)^4} \bar{V}(\bar{c}) \frac{ig}{2\sqrt{2} M_W} [m_j \, tg\beta (1+\gamma^5) + m_c \cot\beta (1-\gamma^5)]$$

$$\cdot \frac{i(\not K + m_j)}{(K^2 - m_j^2)} \frac{ig}{2\sqrt{2} M_W} [m_j \, tg\beta (1-\gamma^5) + m_u \cot\beta (1+\gamma^5)] U(u)$$

$$\cdot \bar{U}(c) \frac{ig}{2\sqrt{2} M_W} [m_i \, tg\beta (1+\gamma^5) + m_c \cot\beta (1-\gamma^5)] \frac{i(\not K + m_i)}{(K^2 - m_i^2)} \cdot$$

$$\frac{ig}{2\sqrt{2} M_W} [m_i \, tg\beta (1-\gamma^5) + m_u \cot\beta (1+\gamma^5)] V(\bar{u}) \frac{i^2}{(K^2 - M_{H^+}^2)^2}$$

(4.47)

$\cot\beta \leftrightarrow tg\,\beta$

$c \leftrightarrow q$ (comparing with 1.4)

$u \leftrightarrow b$

Similarly:

For diagrams c):

$$-iM^{HW} = 4 \sum_{i,j} \xi_i \xi_j \int \frac{d^4 K}{(2\pi)^4} \bar{V}(\bar{c}) \left[-\frac{ig}{\sqrt{2}} \gamma^\mu \frac{1}{2} (1-\gamma^5) \right] \frac{i(\not K + m_j)}{(K^2 - m_j^2)}$$

$$\frac{ig}{2\sqrt{2} M_W} [m_j \, tg\beta (1-\gamma^5) + m_u \cot\beta (1+\gamma^5)] U(u). \ \bar{U}(c).$$

$$\cdot \frac{ig}{2\sqrt{2} M_W} [m_i \, tg\beta (1+\gamma^5) + m_c \cot\beta (1-\gamma^5)] \frac{i(\not K + m_i)}{(K^2 - m_i^2)} \cdot$$

$$\left[-\frac{ig}{\sqrt{2}} \gamma^\nu \frac{1}{2} (1-\gamma^5) \right] V(\bar{u}) \, (-i) \frac{\left[\eta_{\mu\nu} - \frac{K_\mu K_\nu}{M_W^2} \right]}{(K^2 - M_W^2)} \frac{i}{(K^2 - M_{H^+}^2)}$$

(4.48)

If $tg\beta \gg 1$

$$\Delta m_{D^0}^{HH} \gg \Delta m_{D^0}^{HW}, \Delta m_{D^0}^{WW}$$ (In this case $\Delta m_{D^0}^{HH}$ is the leading contribution to $\Delta m_{D^0\,exp}$)

because:

$$\Delta m_{D^0}^{HH} = 2\,|\langle D^0|\, M^{HH}\,|\bar{D}^0\rangle| \qquad (\sim tg^4\beta)$$

$$\Delta m_{D^0}^{HW} = 2\,|\langle D^0|\, M^{HW}\,|\bar{D}^0\rangle| \qquad (\sim tg^2\beta)$$

on the other hand:

$$\Delta m_{D^0\,exp} > \Delta m_{D^0}^{HH} \tag{4.49}$$

$$\Delta m_{D^0\,exp} < 1.3822\times 10^{-13}\ \text{Gev}$$

$$\therefore \quad \boxed{1.3822\times 10^{-13}\ \text{Gev} > \Delta m_{D^0}^{HH}}$$

$$\Delta m_{D^0}^{HH} = \frac{G_F^2 M_W^2}{\underset{\updownarrow\atop 32\ \text{(our model)}}{24\pi^2}} f_D^2\, m_D\, \underset{\updownarrow\atop \eta_D}{B_D} \left|\sum_{i,j} \xi_i \xi_j S^{HH}(i,j)\right| tg^4\beta \tag{4.50}$$

$$= \frac{G_F^2 M_W^2}{\underset{\updownarrow\atop 32}{24\pi^2}} f_D^2\, m_D\, \underset{\updownarrow\atop \eta_D}{B_D}\, tg^4\beta \left|\xi_s^2 S^{HH}(s,s) + \xi_b^2 S^{HH}(b,b) + 2\xi_s\xi_b S^{HH}(s,b)\right| \tag{4.51}$$

$$|\xi_s^2 S^{HH}(s,s) + \xi_b^2 S^{HH}(b,b) + 2\xi_s\xi_b S^{HH}(s,b)| \equiv ABS$$

$$= \Big\{\big[\xi_s^2 S^{HH}(s,s) + \xi_b^2 S^{HH}(b,b) + 2\xi_s\xi_b S^{HH}(s,b)\big]\big[(\xi_s^*)^2 S^{HH}(s,s) + (\xi_b^*)^2 S^{HH}(b,b) + 2\xi_s^*\xi_b^* S^{HH}(s,b)\big]\Big\}^{1/2}$$

$$= \Big\{ |\xi_s|^4 (S^{HH}(s,s))^2 + (\xi_s\xi_b^*)^2 S^{HH}(s,s)\, S^{HH}(b,b) + 2|\xi_s|^2 \xi_s\xi_b^* S^{HH}(s,s)\, S^{HH}(s,b) + |\xi_b|^4 (S^{HH}(b,b))^2 + (\xi_b\xi_s^*)^2 S^{HH}(b,b)\, S^{HH}(s,s) + 2\xi_b\xi_s^*|\xi_b|^2 \cdot S^{HH}(b,b)\, S^{HH}(s,b) + 2|\xi_s|^2 \xi_b\xi_s^* S^{HH}(s,b)\, S^{HH}(s,s) + 2|\xi_b|^2 \xi_s\xi_b^* S^{HH}(s,b)\, S^{HH}(b,b) + 4|\xi_s|^2|\xi_b|^2 (S^{HH}(s,b))^2 \Big\}^{1/2}$$

$$ABS = \Big[\ |\xi_s|^4 \left(S^{HH}(s,s)\right)^2 + |\xi_b|^4 \left(S^{HH}(b,b)\right)^2 + 4|\xi_s|^2|\xi_b|^2 \left(S^{HH}(s,b)\right)^2$$
$$+ 2\left(\mathrm{Re}^2(\xi_s\xi_b^*) - \mathrm{Im}^2(\xi_s\xi_b^*)\right) S^{HH}(s,s)\, S^{HH}(b,b)$$
$$+ 4|\xi_s|^2 \mathrm{Re}(\xi_s\xi_b^*)\, S^{HH}(s,s)\, S^{HH}(s,b) + 4|\xi_b|^2 \mathrm{Re}(\xi_s\xi_b^*)\, S^{HH}(s,b)\, S^{HH}(b,b) \tag{4.52}$$

with

$$|\xi_s|^2 = \lambda^2\left(1-\tfrac{1}{2}\lambda^2\right)^2 \tag{4.53}$$
$$|\xi_b|^2 = A^4\lambda^{10}\rho^2 \tag{4.54}$$
$$\mathrm{Re}(\xi_s\xi_b^*) = A^2\lambda^6\left(1-\tfrac{1}{2}\lambda^2\right)\rho\cos\delta \tag{4.55}$$
$$\mathrm{Im}(\xi_s\xi_b^*) = -A^2\lambda^6\left(1-\tfrac{1}{2}\lambda^2\right)\rho\sin\delta \tag{4.56}$$

$$\boxed{tg^4\beta < \frac{\overset{32}{\overset{\updownarrow}{24}}\pi^2}{G_F^2 M_W^2 f_D^2 m_D \underset{\underset{\eta_D}{\updownarrow}}{B_D}} \cdot \frac{1}{\left|\xi_s^2 S^{HH}(s,s) + \xi_b^2 S^{HH}(b,b) + 2\xi_s\xi_b S^{HH}(s,b)\right|} \cdot 1.3822\times10^{-13}}$$

Taking $A \simeq 1$ $\quad (\rho^2+\eta^2 = (0.46 \pm 0.23)^2)$ (4.57)

$\lambda = 0.221$; $\rho = 0.52$; $\eta = 0.45$ $\quad \rho\delta = \eta \Rightarrow \delta = 0.8653846$

$\eta = 1$; $f_D = 0.2$ GeV $\quad \delta = 49.583°$

$m_{D^0} = 1.8645$ GeV

$M_W = 80.33$ GeV , $M_{H^+} = 100$ GeV

$G_F = 1.166392 \times 10^{-5}$ GeV^{-2}

$|\xi_s|^2 = 4.648468 \times 10^{-2}$

$|\xi_b|^2 = 7.515 \times 10^{-8}$

$\mathrm{Re}(\xi_s\xi_b^*) = 3.832 \times 10^{-5}$

$\mathrm{Im}(\xi_s\xi_b^*) = 4.4998 \times 10^{-5}$

$$\therefore ABS = \Big[1.328228\times10^{-24} + 1.529189\times10^{-25} + \underset{\overset{\wedge}{1.799637\times10^{-24}}}{2\times8.998185\times10^{-25}}$$
$$- 1.435632\times10^{-25} + 2.004771\times10^{-24} + 6.802353\times10^{-25}\Big]^{1/2}$$

$$\boxed{ABS = 2.412929 \times 10^{-12}} \qquad (4.58)$$

$$tg^4\beta < 2.763179 \times 10^8 \qquad (2.072384 \times 10^8)$$

$$\Rightarrow \boxed{tg^2\beta < 1.662281 \times 10^4} \qquad (4.59) \qquad (1.439571 \times 10^4)$$

$$\therefore \boxed{tg\beta < 128.929} \qquad (4.60)$$

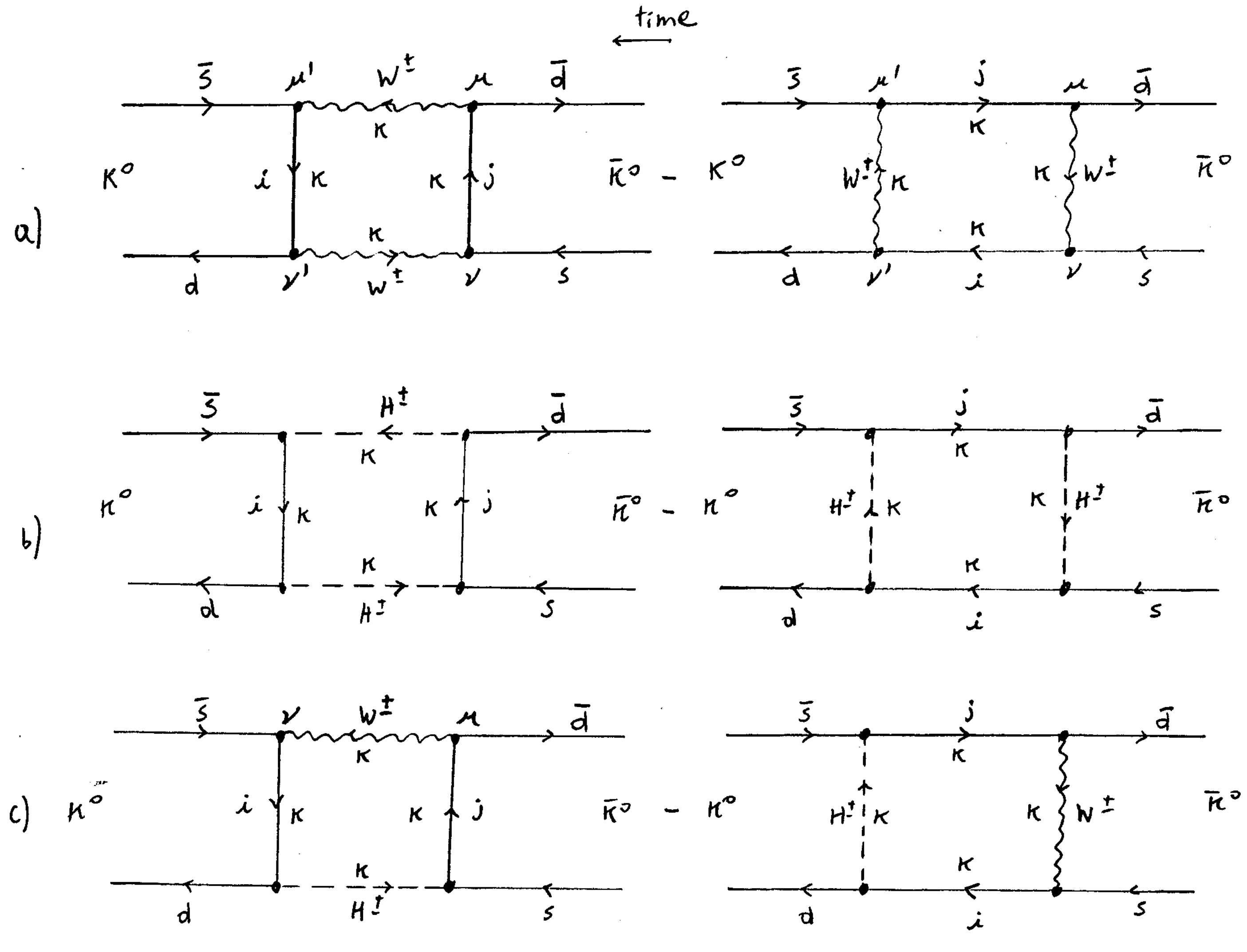
OUR MODEL

and $$\boxed{tg\beta < 119.9824} \quad \text{PCAC} \qquad (4.61)$$

$K^0 - \bar{K}^0$ Mixing

$K^0 = d\bar{s}$; $\bar{K}^0 = s\bar{d}$

The corresponding box diagrams are:



$+$ crossed diagrams $(W^\pm \leftrightarrow H^\pm)$

with $i, j = u, c, t$

$\xi_i = V_{is} V_{id}^*$; $\xi_j = V_{js} V_{jd}^*$; $\sum_i \xi_i = 0$

Comparing with $B^0 - \bar{B}^0$: $\bar{b} \leftrightarrow \bar{s}$

$q \leftrightarrow d$

$$\boxed{M^{WW} = 2\left(\frac{g}{\sqrt{2}}\right)^4 \frac{\pi^2}{(2\pi)^4 M_W^2} \sum_{i,j} \xi_i \xi_j S^{WW}(i,j)\, \bar{V}_L(\bar{d})\gamma^\mu V_L(s) . \bar{U}_L(d)\gamma_\mu V_L(\bar{s})} \tag{5.1}$$

The invariant amplitude corresponding to diagram b) is:

$$-iM^{HH} = 2\sum_{i,j} \xi_i \xi_j \int \frac{d^4K}{(2\pi)^4}\, \bar{V}(\bar{d})\, \frac{ig}{2\sqrt{2}\,M_W}\left[m_d \overset{\text{tan}}{tg\beta}(1-\gamma^5) + m_j \cot\beta\,(1+\gamma^5)\right] \cdot$$

$$\cdot\, \frac{i\,(\not{K}+m_j)}{(K^2-m_j^2)}\, \frac{ig}{2\sqrt{2}\,M_W}\left[m_s\, tg\beta\,(1+\gamma^5) + m_j \cot\beta\,(1-\gamma^5)\right] U(s)\,.$$

$$\bar{U}(d)\, \frac{ig}{2\sqrt{2}\,M_W}\left[m_d\, tg\beta\,(1-\gamma^5) + m_i \cot\beta\,(1+\gamma^5)\right] \frac{i\,(\not{K}+m_i)}{(K^2-m_i^2)}\,.$$

$$\frac{ig}{2\sqrt{2}\,M_W}\left[m_s\, tg\beta\,(1+\gamma^5) + m_i \cot\beta\,(1-\gamma^5)\right] V(\bar{s})\, \frac{i^2}{(K^2-M_{H^+}^2)^2} \qquad (5.2)$$

and for diagram c) is:

$$-iM^{HW} = 4\sum_{i,j} \xi_i \xi_j \int \frac{d^4K}{(2\pi)^4}\, \bar{V}(\bar{d})\left[-\frac{ig}{\sqrt{2}}\gamma^\mu \frac{1}{2}(1-\gamma^5)\right] \frac{i\,(\not{K}+m_j)}{(K^2-m_j^2)}\,.$$

$$\frac{ig}{2\sqrt{2}\,M_W}\left[m_s\, tg\beta\,(1+\gamma^5) + m_j \cot\beta\,(1-\gamma^5)\right] U(s)\,.\ \bar{U}(d)$$

$$\frac{ig}{2\sqrt{2}\,M_W}\left[m_d\, tg\beta\,(1-\gamma^5) + m_i \cot\beta\,(1+\gamma^5)\right] \frac{i\,(\not{K}+m_i)}{(K^2-m_i^2)}\,.$$

$$\left[-\frac{ig}{\sqrt{2}}\gamma^\nu \frac{1}{2}(1-\gamma^5)\right] V(\bar{s}) \left[\frac{i}{K^2-M_{H^+}^2}\right](-i)\left(\eta_{\mu\nu} - \frac{K_\mu K_\nu}{M_W^2}\right) \cdot \frac{1}{(K^2-M_W^2)}$$

(5.3)

$$\boxed{\begin{aligned}\Delta m_{K^0} &= m_{K_L^0} - m_{K_S^0} = 2\,|M_{12}| \qquad (5.4)\\ &= 2\left|\langle K^0|M^{WW}|\bar{K}^0\rangle + \langle K^0|M^{HW}|\bar{K}^0\rangle + \langle K^0|M^{HH}|\bar{K}^0\rangle\right|\end{aligned}}$$

$$\langle K^0|M^{WW}|\bar{K}^0\rangle = \frac{G_F^2 M_W^2 f_K^2 m_K}{\underset{\updownarrow\ 16}{12\pi^2}} \sum_{i,j} \xi_i \xi_j\, S^{WW}(i,j)\,.\ \underset{\updownarrow\ \eta_K}{B_K} \qquad (5.5)$$

$$\langle K^0|M^{HW}|\bar{K}^0\rangle = -\frac{G_F^2 M_W^2 \cot^2\beta}{\underset{\updownarrow\ 8}{6\pi^2}}\, f_K^2 m_K \sum_{i,j} \xi_i \xi_j\, S^{HW}(i,j)\,.\ \underset{\updownarrow\ \eta_K}{B_K} \qquad (5.6)$$

$$\langle K^0 | H^{HH} | \bar{K}^0 \rangle = \frac{G_F^2 M_W^2 \cot^4\beta}{\underset{\underset{64}{\updownarrow}}{48\pi^2}} f_K^2 m_K \sum_{i,j} \mathcal{E}_i \mathcal{E}_j S^{HH}(i,j) \underset{\underset{\eta_K}{\updownarrow}}{B_K} \qquad (5\text{-}7)$$

$$\Rightarrow \boxed{\Delta m_{K^0} = \frac{G_F^2 M_W^2}{\underset{\underset{32}{\updownarrow}}{24\pi^2}} f_K^2 m_K \underset{\underset{\eta_K}{\updownarrow}}{B_K} \left| \sum_{i,j} \mathcal{E}_i \mathcal{E}_j \left[4 S^{WW}(i,j) - 8\cot^2\beta\, S^{HW}(i,j) + \cot^4\beta\, S^{HH}(i,j) \right] \right|} \qquad (5\text{-}8)$$

$i,j = u,c,t$ (see 3-22 b)

$$\boxed{\begin{aligned} \Delta m_{K^0} = \frac{G_F^2 M_W^2}{\underset{\underset{32}{\updownarrow}}{24\pi^2}} f_K^2 m_K \underset{\underset{\eta_K}{\updownarrow}}{B_K} \Big| & \left(4\alpha^{WW}(t) - 8\cot^2\beta\, S^{HW}(t,t) + \cot^4\beta\, S^{HH}(t,t)\right) . \mathcal{E}_t^2 \\ & + 2\left(2\gamma^{WW}(c,t) - 8\cot^2\beta\, S^{HW}(c,t) + \cot^4\beta\, S^{HH}(c,t)\right) . \\ & \cdot \mathcal{E}_c \mathcal{E}_t + \left(4\beta^{WW}(c) - 8\cot^2\beta\, S^{HW}(c,c) + \cot^4\beta\, S^{HH}(c,c)\right) . \\ & \mathcal{E}_c^2 + 4\phi^{WW}(c)\, \mathcal{E}_u \mathcal{E}_c \Big| \end{aligned}}$$

PCAC

(5.9)

$$\boxed{\mathcal{E}_c = V_{cs} V_{cd}^* = -\lambda\left(1 - \tfrac{1}{2}\lambda^2\right)} \Rightarrow \boxed{\mathcal{E}_c^2 = \lambda^2\left(1 - \tfrac{1}{2}\lambda^2\right)^2} \qquad (5.10)$$

$$\boxed{\mathcal{E}_t = V_{ts} V_{td}^* = -A^2\lambda^5\left(1 - \rho e^{-i\delta}\right)}$$

$$\Rightarrow \boxed{\mathcal{E}_t^2 = A^4\lambda^{10}\left(1 - \rho e^{-i\delta}\right)^2 = A^4\lambda^{10}\left(1 - 2\rho e^{-i\delta} + \rho^2 e^{-2i\delta}\right)} \qquad (5.11)$$

$$\boxed{\mathcal{E}_u = V_{us} V_{ud}^* = \lambda\left(1 - \tfrac{1}{2}\lambda^2\right)} \Rightarrow \boxed{\mathcal{E}_u^2 = \lambda^2\left(1 - \tfrac{1}{2}\lambda^2\right)^2} \qquad (5.12)$$

$$|\mathcal{E}_c^2| = \lambda^2\left(1 - \tfrac{1}{2}\lambda^2\right)^2 = |\mathcal{E}_c|^2 \qquad (5.13)$$

$$|\mathcal{E}_u \mathcal{E}_c| = \lambda^2\left(1 - \tfrac{1}{2}\lambda^2\right)^2 \qquad (5.14)$$

$$|\mathcal{E}_c \mathcal{E}_t| = A^2\lambda^6\left(1 - \tfrac{1}{2}\lambda^2\right)\left(1 - 2\rho\cos\delta + \rho^2\right)^{1/2} = A^2\lambda^6\left(1 - \tfrac{1}{2}\lambda^2\right) f^{1/2}(\delta) \qquad (5.15)$$

$$|\mathcal{E}_t|^2 = |\mathcal{E}_t^2| = A^4\lambda^{10}\left(1 + \rho^2 - 2\rho\cos\delta\right) = A^4\lambda^{10} f(\delta) \qquad (5.16)$$

Let's show that $f(\delta) \geq 0$

$$\cos\delta \leq 1$$

If $\rho > 0$

$$2\rho\cos\delta \leq 2\rho$$

$$-2\rho\cos\delta \geq -2\rho$$

$$\Rightarrow 1+\rho^2-2\rho\cos\delta \geq 1+\rho^2-2\rho = (1-\rho)^2 \geq 0$$

If $\rho < 0$

$$\cos\delta \geq -1$$

$$2\rho\cos\delta \leq -2\rho$$

$$\Rightarrow -2\rho\cos\delta \geq 2\rho$$

$$\Rightarrow 1+\rho^2-2\rho\cos\delta \geq 1+\rho^2+2\rho = (1+\rho)^2 \geq 0$$

$$\therefore \boxed{f(\delta) \geq 0} \qquad (5.17)$$

Taking $\lambda = 0.221$; $\rho = 0.52$; $\eta = 0.45$; $\delta = 49.583^\circ$; $A = 1$

$$|\xi_c^2| = 4.648468 \times 10^{-2}$$

$$|\xi_u \xi_c| = 4.648468 \times 10^{-2}$$

$$|\xi_c \xi_t| = 8.775731 \times 10^{-5}$$

$$|\xi_t^2| = 1.656749 \times 10^{-7}$$

$|\alpha^{WW}(t)\,\xi_t^2| = 4.460797 \times 10^{-7}$; $|S^{HW}(c,c)\,\xi_c^2| = 1.391836 \times 10^{-8}$

$|S^{HW}(t,t)\,\xi_t^2| = 2.113372 \times 10^{-7}$; $|S^{HH}(c,c)\,\xi_c^2| = 2.052443 \times 10^{-9}$

$|S^{HH}(t,t)\,\xi_t^2| = 4.507941 \times 10^{-7}$; $|\phi^{WW}(c)\,\xi_u\xi_c| = 1.765655 \times 10^{-4}$

$|\gamma^{WW}(c,t)\,\xi_c\xi_t| = 8.329274 \times 10^{-8}$

$|S^{HW}(c,t)\,\xi_c\xi_t| = 1.960477 \times 10^{-8}$ (5.18)

$|S^{HH}(c,t)\,\xi_c\xi_t| = 2.328076 \times 10^{-8}$

$|\rho^{WW}(c)\,\xi_c^2| = 1.643937 \times 10^{-4}$

So charged Higgs contributions are small $\Rightarrow$

$\Rightarrow$

$$\Delta m_{K^0} \approx \underset{\updownarrow \atop 32}{\frac{G_F^2 M_W^2}{24\pi^2}} f_K^2 m_K \underset{\updownarrow \atop \eta_K}{B_K}\, 4\left|\beta^{WW}(c)\,\xi_c^2 + \phi^{WW}(c)\,\xi_u \xi_c\right| \qquad (5.19)$$

2.618460×10^{-4}

$$\boxed{\Delta m_{K^0} \approx \underset{\updownarrow \atop 8}{\frac{G_F^2 M_W^2}{6\pi^2}} f_K^2 m_K \underset{\updownarrow \atop \eta_K}{B_K}\, \lambda^2 \left(1-\tfrac{1}{2}\lambda^2\right)^2 \left|\beta^{WW}(c) - \phi^{WW}(c)\right|} \qquad (5.20)$$

$$\beta^{WW}(c) \approx \frac{3X_c^W}{(1-X_c^W)^2} \underbrace{}_{-\frac{19}{4}\frac{(X_c^W)^2}{(1-X_c^W)^2}} + \frac{2X_c^W \ln X_c^W}{(1-X_c^W)^2} = \frac{X_c^W}{(1-X_c^W)^2}\left[3 + 2\ln(X_c^W) - \frac{19}{4}X_c^W\right]$$

$$\phi^{WW}(c) = \frac{2X_c^W}{(1-X_c^W)^2}\left[1 - X_c^W + \ln(X_c^W)\right]$$

$$\beta^{WW}(c) - \phi^{WW}(c) = \frac{X_c^W}{(1-X_c^W)^2}\left[\cancel{3} + 2\cancel{\ln(X_c^W)} - \cancel{2} + 2X_c^W - 2\cancel{\ln(X_c^W)} - \frac{19}{4}X_c^W\right]$$

$$\boxed{\beta^{WW}(c) - \phi^{WW}(c) = \frac{X_c^W}{(1-X_c^W)^2}\left[1 - \frac{11}{4}X_c^W\right]} \qquad (5.21)$$

$$\boxed{\Delta m_{K^0} \approx \frac{G_F^2 M_W^2}{6\pi^2} f_K^2 m_K B_K \lambda^2\left(1-\tfrac{1}{2}\lambda^2\right)^2 \frac{X_c^W}{(1-X_c^W)^2}\left[1 - \frac{11}{4}X_c^W\right]} \qquad (5.22)$$

PCAC ← 6; 8 → OUR MODEL; $B_K \leftrightarrow \eta_K$

$$\Delta m_{K^0\,exp} = m_{K^0_L} - m_{K^0_S} = (10.5333 \pm 0.0027) \times 10^{10}\ \hbar\, s^{-1}$$
$$= 3.510246 \times 10^{-15}\ GeV$$

$f_K = 0.1606$ GeV ; $M_W = 80.33$ GeV

$G_F = 1.166392 \times 10^{-5}$ GeV^{-2}

$m_K = .497672$ GeV

$B_K = ?$

$m_c = 1.3$ GeV

$\lambda = 0.221$

From (5.20) or (5.22) we deduce :

$$\boxed{B_K = \begin{cases} 1.5155 & \text{PCAC} \\ 2.0207 & \text{OUR MODEL} \end{cases}} \qquad (5.23)$$

$$\Delta m_{K^0\,exp} > \Delta m_{K^0}^{HH} \qquad (5.24)$$

$$\Delta m_{K^0}^{HH} = \underbrace{\frac{G_F^2 M_W^2}{24\pi^2}}_{32} f_K^2 m_K \underbrace{B_K}_{n_K} \cot^4\beta \left| \sum_{i,j} \xi_i \xi_j S^{HH}(i,j) \right| \qquad (5.25)$$

$$\sum_{i,j} \xi_i \xi_j S^{HH}(i,j) = \xi_u^2 \cancel{S^{HH}(u,u)}^{\,0} + \xi_c^2 S^{HH}(c,c) + \xi_t^2 S^{HH}(t,t) + 2\xi_u \xi_c \cancel{S^{HH}(u,c)}^{\,0} + 2\xi_u \xi_t \cancel{S^{HH}(u,t)}^{\,0} + 2\xi_c \xi_t S^{HH}(c,t)$$

Let be $ABS' = \left| \sum_{i,j} \xi_i \xi_j S^{HH}(i,j) \right| = \left| \xi_c^2 S^{HH}(c,c) + \xi_t^2 S^{HH}(t,t) + 2\xi_c \xi_t S^{HH}(c,t) \right|$ (5.26)

$$\boxed{\begin{aligned} ABS' = \Big[& |\xi_c|^4 (S^{HH}(c,c))^2 + |\xi_t|^4 (S^{HH}(t,t))^2 + 4|\xi_c|^2|\xi_t|^2 (S^{HH}(c,t))^2 + 2\left(Re^2(\xi_c \xi_t^*) - Im^2(\xi_c \xi_t^*)\right) . S^{HH}(c,c) . \\ & S^{HH}(t,t) + 4|\xi_c|^2 Re(\xi_c \xi_t^*) S^{HH}(c,c) S^{HH}(c,t) + 4|\xi_t|^2 Re(\xi_c \xi_t^*) . \\ & . S^{HH}(c,t) S^{HH}(t,t) \Big]^{1/2} \end{aligned}} \qquad (5.27)$$

$$Re(\xi_c \xi_t^*) = A^2 \lambda^6 (1 - \tfrac{1}{2}\lambda^2)(1 - \rho\cos\delta) = 7.534219 \times 10^{-5} \qquad (5.28)$$

$$Im(\xi_c \xi_t^*) = -A^2 \lambda^6 (1 - \tfrac{1}{2}\lambda^2) \rho \sin\delta = -4.499888 \times 10^{-5} \qquad (5.29)$$

$$|\xi_c|^2 = \lambda^2 (1 - \tfrac{1}{2}\lambda^2)^2 = 4.648468 \times 10^{-2} \qquad (5.30)$$

$$|\xi_t|^2 = A^4 \lambda^{10} (1 + \rho^2 - 2\rho\cos\delta) = 1.656749 \times 10^{-7} \qquad (5.31)$$

$$ABS' = [4.212521 \times 10^{-18} + 2.032153 \times 10^{-13} + 2.167975 \times 10^{-15} + 8.773837 \times 10^{-16} + 1.640904 \times 10^{-16} + 3.604046 \times 10^{-14}]^{1/2} = 4.924118 \times 10^{-7}$$

$$tg^4\beta > \frac{G_F^2 M_W^2}{\underset{\updownarrow}{\overset{}{24}}\,\pi^2}\;\frac{f_K^2\, m_K\, \overset{n_K}{\overset{\updownarrow}{B_K}}\; ABS'}{\Delta m_{K^0_{exp}}} \qquad (5.32)$$

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$$tg^4\beta > 0.01011381 \Rightarrow \boxed{tg\beta > 0.317124} \qquad \text{PCAC} \quad (5.33)$$

$$tg^4\beta > 0.01011398 \Rightarrow \boxed{tg\beta > 0.317125} \qquad \text{OUR MODEL} \quad (5.34)$$

$$\frac{1+\varepsilon}{1-\varepsilon} = \left(\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*} \right)^{1/2} = \left(\frac{a}{b}\right)^{1/2} \qquad (5.35)$$

$$a = M_{12} - \frac{i}{2}\Gamma_{12}$$
$$b = M_{12}^* - \frac{i}{2}\Gamma_{12}^*$$

$$1 + \varepsilon = \left(\frac{a}{b}\right)^{1/2} - \varepsilon\left(\frac{a}{b}\right)^{1/2}$$

$$\Rightarrow \quad \varepsilon = \frac{\left(\frac{a}{b}\right)^{1/2} - 1}{\left(\frac{a}{b}\right)^{1/2} + 1} = \frac{(a^{1/2} - b^{1/2})}{(a^{1/2} + b^{1/2})} \cdot \frac{(a^{1/2} + b^{1/2})}{(a^{1/2} + b^{1/2})}$$

$$\varepsilon = \frac{(a-b)}{a+b+2a^{1/2}b^{1/2}}$$

$$\varepsilon = \frac{2i\,\mathrm{Im}(M_{12}) + \mathrm{Im}(\Gamma_{12})}{2\,\mathrm{Re}(M_{12}) - i\,\mathrm{Re}(\Gamma_{12}) + 2\left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)\right]^{1/2}}$$

$$\Delta M = 2\,\mathrm{Re}\left[(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*)\right]^{1/2}$$

$$\Delta\Gamma = -4\,\mathrm{Im}\left[(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{11}^*)\right]^{1/2}$$

$$\Rightarrow \left[(M_{12} - \tfrac{i}{2}\Gamma_{12})(M_{12}^* - \tfrac{i}{2}\Gamma_{12}^*)\right]^{1/2} = \frac{\Delta M}{2} - \frac{i}{4}\Delta\Gamma$$

We can choose a phase convention in which

$$\left.\begin{array}{l}\mathrm{Re}(M_{12}) \gg \mathrm{Im}(M_{12}) \\ \mathrm{Re}(\Gamma_{12}) \gg \mathrm{Im}(\Gamma_{12})\end{array}\right. \Rightarrow \begin{cases}\Delta M \simeq 2\,\mathrm{Re}\left[(M_{12} - \frac{i}{2}\Gamma_{12})^2\right]^{1/2} = 2\,\mathrm{Re}\,M_{12} \\ \Delta\Gamma \simeq -4\,\mathrm{Im}\left[M_{12} - \frac{i}{2}\Gamma_{12}\right] = 2\,\mathrm{Re}\,\Gamma_{12}\end{cases}$$

$$\Rightarrow \quad \Delta M \simeq 2\,\mathrm{Re}(M_{12}) \quad ; \quad \Delta\Gamma \simeq 2\,\mathrm{Re}(\Gamma_{12})$$

$$\Rightarrow \quad \varepsilon = \frac{2i\,\mathrm{Im}(M_{12}) + \mathrm{Im}(\Gamma_{12})}{\Delta M - \frac{i}{2}\Delta\Gamma + 2\left(\frac{\Delta M}{2} - \frac{i}{4}\Delta\Gamma\right)} = \frac{2i\,\mathrm{Im}(M_{12}) + \mathrm{Im}(\Gamma_{12})}{2\Delta M - i\Delta\Gamma}$$

$$\boxed{\varepsilon_K \approx \frac{2i\,\mathrm{Im}(M_{12}) + \mathrm{Im}(\Gamma_{12})}{2\Delta M - i\Delta\Gamma}} \qquad (5.36)$$

$$\mathrm{Im}(M_{12}) \gg \mathrm{Im}(\Gamma_{12})$$

$$\Rightarrow \quad \varepsilon_K \approx \frac{2i\,\mathrm{Im}(M_{12})}{2\Delta M - i\Delta\Gamma}$$

Experimentally

$$\Delta M_{K^0_{exp}} = m_{K^0_L} - m_{K^0_S} = (3.510 \pm 0.018) \times 10^{-15}\ \text{GeV}$$

$$\Delta\Gamma = \Gamma_{K^0_L} - \Gamma_{K^0_S}$$

$$\tau_{K^0_S} = 0.8926 \times 10^{-10}\ \text{s.} \Rightarrow \Gamma_{K^0_S} = 7.3741 \times 10^{-15}\ \text{GeV}$$

$$\tau_{K^0_L} = 5.17 \times 10^{-8}\ \text{s.} \quad \Rightarrow \Gamma_{K^0_L} = 1.273138 \times 10^{-17}\ \text{GeV}$$

$$\Rightarrow \quad \Delta\Gamma = -7.361369 \times 10^{-15}\ \text{GeV}$$

$$\therefore \quad \frac{\Delta\Gamma}{\Delta M} \approx -2 \qquad \boxed{\Delta\Gamma \approx -2\Delta M}$$

$$\varepsilon_K \approx \frac{\cancel{2}i\,\mathrm{Im}(M_{12})}{\cancel{2}\Delta M + \cancel{2}i\Delta M}$$

$$\boxed{\varepsilon_K \approx \frac{i}{(1+i)}\,\frac{\mathrm{Im}(M_{12})}{\Delta M} = \frac{e^{i\pi/4}}{\sqrt{2}}\,\frac{\mathrm{Im}(M_{12})}{\Delta M}} \qquad (5.37)$$

$$\boxed{|\varepsilon_K| \approx \frac{1}{\sqrt{2}}\,\frac{\mathrm{Im}(M_{12})}{\Delta M}} \qquad (5.38)$$

$$M_{12} = \langle K^0 | M^{WW} | \bar{K}^0 \rangle + \langle K^0 | M^{HW} | \bar{K}^0 \rangle + \langle K^0 | M^{HH} | \bar{K}^0 \rangle \qquad (5.39)$$

$$M_{12} = \frac{G_F^2 M_W^2}{\underset{\substack{\updownarrow \\ 64}}{48\pi^2}} f_K^2 m_K \underset{\substack{\updownarrow \\ \eta_K}}{B_K} \sum_{i,j} \xi_i \xi_j \left[4 S^{WW}(i,j) - 8 \cot^2\beta\, S^{HW}(i,j) + \cot^4\beta\, S^{HH}(i,j) \right] \qquad (5.40)$$

$$i,j = u, c, t$$

$$\boxed{\begin{aligned} \mathrm{Im}(M_{12}) = \frac{G_F^2 M_W^2}{\underset{\substack{\updownarrow \\ 64}}{48\pi^2}} f_K^2 m_K \underset{\substack{\updownarrow \\ \eta_K}}{B_K} \Big\{ & \left(4 \alpha^{WW}(t) - 8 \cot^2\beta\, S^{HW}(t,t) + \cot^4\beta\, S^{HH}(t,t)\right) \cdot \\ & \mathrm{Im}(\xi_t^2) + 2\left(2 \gamma^{WW}(c,t) - 8 \cot^2\beta\, S^{HW}(c,t) \right. \\ & \left. + \cot^4\beta\, S^{HH}(c,t)\right) \mathrm{Im}(\xi_c \xi_t) \Big\} \end{aligned}} \qquad (5.41)$$

$$\mathrm{Im}(\xi_t^2) = A^4 \lambda^{10} (2\rho \sin\delta - \rho^2 \sin 2\delta) = 2 A^4 \lambda^{10} \rho \sin\delta\, (1 - \rho\cos\delta) \qquad (5.42)$$

$$\mathrm{Im}(\xi_c \xi_t) = A^2 \lambda^6 (1 - \tfrac{1}{2}\lambda^2) \rho \sin\delta \qquad (5.43)$$

$$\boxed{\Delta m_{K^0} = \frac{G_F^2 M_W^2}{\underset{\substack{\updownarrow \\ 8 \text{ OUR MODEL}}}{6\pi^2}} f_K^2 m_K \underset{\substack{\updownarrow \\ \eta_K}}{B_K} \lambda^2 (1 - \tfrac{1}{2}\lambda^2)^2 \frac{x_c^W}{(1 - x_c^W)^2} \left[1 - \frac{11}{4} x_c^W \right]} \qquad (5.44) \text{ or } (5.22)$$

$$\boxed{\varepsilon_K = \frac{e^{i\pi/4}}{8\sqrt{2}} \cdot \left\{ \frac{\left[\left(4\alpha^{WW}(t) - 8\cot^2\beta\, S^{HW}(t,t) + \cot^4\beta\, S^{HH}(t,t)\right) \mathrm{Im}(\xi_t^2) + 2\left(2\gamma^{WW}(c,t) - 8\cot^2\beta\, S^{HW}(c,t) + \cot^4\beta\, S^{HH}(c,t)\right) \mathrm{Im}(\xi_c \xi_t)\right]}{\lambda^2 (1 - \frac{1}{2}\lambda^2)^2 \frac{x_c^W}{(1 - x_c^W)^2} \left[1 - \frac{11}{4} x_c^W\right]} \right\}} \qquad (5.45)$$

Let be: $\boxed{\alpha_1(t) \equiv 4\alpha^{WW}(t) - 8\cot^2\beta\, S^{HW}(t,t) + \cot^4\beta\, S^{HH}(t,t)}$ (5.46)

$$\boxed{\alpha_2(c,t) \equiv 2\left(2\gamma^{WW}(c,t) - 8\cot^2\beta\, S^{HW}(c,t) + \cot^4\beta\, S^{HH}(c,t)\right)} \qquad (5.47)$$

$$\Rightarrow \quad \varepsilon_K = \frac{e^{i\pi/4}}{\sqrt{2}} \; \frac{G_F^2 M_W^2}{\underset{\substack{\updownarrow \\ 64}}{48\pi^2}} \; \frac{f_K^2 m_K}{\Delta m_K} \; \underset{\substack{\updownarrow \\ n_K}}{B_K} \left\{ \alpha_1(t) \, \mathrm{Im}(\xi_t^2) + \alpha_2(c,t) \, \mathrm{Im}(\xi_c \xi_t) \right\} \tag{5-48}$$

$$\varepsilon_K = \frac{e^{i\pi/4}}{\sqrt{2}} \; \frac{G_F^2 M_W^2}{\underset{\substack{\updownarrow \\ 64}}{48\pi^2}} \; \frac{f_K^2 m_K}{\Delta m_K} \; \underset{\substack{\updownarrow \\ n_K}}{B_K} \, \alpha_1(t) \left\{ \mathrm{Im}(\xi_t^2) + \frac{\alpha_2(c,t)}{\alpha_1(t)} \, \mathrm{Im}(\xi_c \xi_t) \right\} \tag{5.49}$$

$$\mathrm{Im}(\xi_t^2) = 2 V_{cb}^4 \lambda^2 \rho \sin\delta \, (1 - \rho\cos\delta)$$

$$\mathrm{Im}(\xi_c \xi_t) = V_{cb}^2 \lambda^2 \left(1 - \tfrac{1}{2}\lambda^2\right) \rho \sin\delta$$

$$\varepsilon_K = \frac{e^{i\pi/4}}{\sqrt{2}} \; \frac{G_F^2 M_W^2}{\underset{\substack{\updownarrow \\ 64}}{48\pi^2}} \; \frac{f_K^2 m_K}{\Delta m_K} \; \underset{\substack{\updownarrow \\ n_K}}{B_K} \, \alpha_1(t) \lambda^2 V_{cb}^4 \rho \left\{ 2\sin\delta \, (1 - \rho\cos\delta) + \right.$$

$$\left. + \frac{\alpha_2(c,t)}{\alpha_1(t)} \, \frac{(1 - \frac{1}{2}\lambda^2)}{V_{cb}^2} \sin\delta \right\}$$

$$\boxed{\varepsilon_K = \frac{e^{i\pi/4}}{\sqrt{2}} \; \frac{G_F^2 M_W^2}{\underset{\substack{\updownarrow \\ 32}}{24\pi^2}} \; \frac{f_K^2 m_K}{\Delta m_K} \; \underset{\substack{\updownarrow \\ n_K}}{B_K} \lambda^2 V_{cb}^4 \rho \, \alpha_1(t) \cdot \left\{ \left[1 + \frac{\alpha_2(c,t)}{2\alpha_1(t)} \, \frac{(1 - \frac{1}{2}\lambda^2)}{V_{cb}^2} \right] \sin\delta - \frac{\rho}{2} \sin 2\delta \right\}} \tag{5.50}$$

$$\boxed{\varepsilon_K = \frac{e^{i\pi/4}}{\sqrt{2}} \; \frac{G_F^2 M_W^2}{\underset{\substack{\updownarrow \\ 32}}{24\pi^2}} \; \frac{f_K^2 m_K}{\Delta m_K} \; \underset{\substack{\updownarrow \\ n_K}}{B_K} \lambda^2 V_{cb}^4 \rho \, \alpha_1(t) \cdot g(\delta)} \tag{5.51}$$

with
$$\boxed{g(\delta) = \left[1 + \frac{\alpha_2(c,t)}{2\alpha_1(t)} \, \frac{(1 - \frac{1}{2}\lambda^2)}{V_{cb}^2} \right] \sin\delta - \frac{\rho}{2} \sin 2\delta} \tag{5.51 a}$$

Experimentally $|\varepsilon_K| = 2.26 \times 10^{-3}$

remember:

$$\Delta m_{B_d^0} = \frac{G_F^2 M_W^2}{\underset{\substack{\updownarrow \\ 32}}{24}\,\pi^2} f_{B_d}^2\, m_{B_d^0}\, B_{B_d^0}\, \lambda^2 V_{cb}^2\, f(\delta)\, |\alpha_1(t)|$$

$$f(\delta) = 1 + \rho^2 - 2\rho\cos\delta$$

$$V_{cb}^2 = A^2\lambda^4$$

$$\frac{|g(\delta)|}{f(\delta)} = \frac{\sqrt{2}\,|\varepsilon_K|\, \cancel{\underset{\substack{\updownarrow \\ 32}}{24}\pi^2}\, \Delta m_K \Big/ \cancel{G_F^2} \cancel{M_W^2} f_K^2\, m_K\, \underset{\substack{\updownarrow \\ n_K}}{B_K}\, \cancel{\lambda^2} V_{cb}^4\, \rho\, \cancel{|\alpha_1(t)|}}{\cancel{\underset{\substack{\updownarrow \\ 32}}{24}\pi^2}\, \Delta m_{B_d^0} \Big/ \cancel{G_F^2} \cancel{M_W^2} f_{B_d^0}^2\, m_{B_d^0}\, B_{B_d^0}\, \cancel{\lambda^2} \cancel{V_{cb}^2}\, \cancel{|\alpha_1(t)|}}$$

$$\boxed{\frac{|g(\delta)|}{f(\delta)} = \frac{\sqrt{2}\,|\varepsilon_K|\, \Delta m_K\, f_{B_d^0}^2\, m_{B_d^0}\, B_{B_d^0}}{\Delta m_{B_d^0}\, f_{K^0}^2\, m_{K^0}\, B_{K^0}\, V_{cb}^2\, \rho}} \qquad (5.52)$$

because $\quad x_d = \dfrac{\Delta m_{B_d^0}}{\Gamma_{B_d^0}}$

$$\Rightarrow \boxed{\frac{|g(\delta)|}{f(\delta)} = \frac{\sqrt{2}\,|\varepsilon_K|\, \Delta m_K\, f_{B_d^0}^2\, m_{B_d^0}\, B_{B_d^0}}{x_d\, \Gamma_{B_d^0}\, f_{K^0}^2\, m_{K^0}\, B_{K^0}\, V_{cb}^2\, \rho}} \qquad (5.53)$$

$$\rho\delta = \eta \quad \Rightarrow$$

$$\varepsilon_K = \frac{e^{i\pi/4}}{\sqrt{2}}\, \frac{G_F^2 M_W^2}{\underset{\substack{\updownarrow \\ 32}}{24}\,\pi^2}\, \frac{f_K^2\, m_K}{\Delta m_K}\, \underset{\substack{\updownarrow \\ n_K}}{B_K}\, \lambda^2 V_{cb}^4\, \eta\, \alpha_1(t) \Big[(1-\rho) + \frac{\alpha_2(c,t)}{2\alpha_1(t)}\, \frac{(1-\frac{1}{2}\lambda^2)}{V_{cb}^2} \Big] \qquad (5.54)$$

$$\Rightarrow \boxed{\eta = \frac{\sqrt{2}\,|\varepsilon_K|\, \underset{\substack{\updownarrow \\ 32}}{24}\,\pi^2\, \Delta m_K}{G_F^2 M_W^2 f_K^2\, m_K\, \underset{\substack{\updownarrow \\ n_K}}{B_K}\, \lambda^2 V_{cb}^4\, \alpha_1(t) \Big[(1-\rho) + \dfrac{\alpha_2(c,t)}{2\alpha_1(t)}\, \dfrac{(1-\frac{1}{2}\lambda^2)}{V_{cb}^2} \Big]}} \qquad (5.55)$$

Constraints

$$V_{ub} \approx A\lambda^3(\rho - i\eta) \qquad (6.1)$$

$$V_{cb} = A\lambda^2 \qquad (6.2)$$

$$V_{td} \approx A\lambda^3(1-\rho-i\eta) \qquad (6.3)$$

$$\rho\delta = \eta \qquad (6.4)$$

$$\left|\frac{V_{ub}}{V_{cb}}\right| = \lambda\,(\rho^2+\eta^2)^{1/2} \qquad (6.5)$$

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.08 \pm 0.02 \qquad (6.6)$$

$$\lambda = 0.221$$

$$\Rightarrow \boxed{(\rho^2+\eta^2) = (0.36 \pm 0.09)^2}$$

Review of Particle Properties

(6.7)

Returning to (3.57):

$$|\xi_t|^2 = |V_{tq}^*|^2$$

if $q = d$

$$|\xi_t|^2 = |V_{td}^*|^2 = A^2\lambda^6\,(1-\rho-i\eta)(1-\rho+i\eta)$$

$$|\xi_t|^2 = A^2\lambda^6\,[(1-\rho)^2+\eta^2] \qquad (6.8)$$

$$\boxed{[(1-\rho)^2+\eta^2] = \frac{\begin{smallmatrix}24\\ \Updownarrow\\ 32\end{smallmatrix}\,\pi^2 X_d\,\Gamma_{B_d^0}}{A^2\lambda^6 G_F^2 M_W^2 f_{B_d^0}^2 m_{B_d^0} \begin{smallmatrix}B_{B_d^0}\\ \Downarrow\\ \eta_{B_d^0}\end{smallmatrix} \left|\left(4\alpha^{WW}(t) - 8\cot^2\beta\, S^{HW}(t,t) + \cot^4\beta\, S^{HH}(t,t)\right)\right|}}$$

(6.9)

See also page 110

Fierz Theorem

Fierz Theorem

Carlos Marín

Let's consider the set of 4x4 matrices:

$$I, \gamma^5, \gamma^\mu, i\gamma^\mu\gamma^5, \sigma^{\mu\nu}$$

with $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$ (6 antisymmetric matrices)
$(\underset{\mu\nu}{01}, 02, 03, 12, 13, 23)$

In total we have: $1 + 1 + 4 + 4 + 6 = 16$ matrices 4x4

We will denote these matrices by Γ^α $(\alpha = 1, \ldots 16)$.

On the other hand we define Γ_α: $I, \gamma^5, \gamma_\mu, i\gamma_\mu\gamma_5, \sigma_{\mu\nu}$.

The Γ^α matrices satisfy:

(1.1) $\boxed{Tr(\Gamma^\alpha) = 0}$ $(\alpha \neq 1 \quad \underset{\text{trace}}{\underset{\downarrow}{Tr}}(I) = 4)$

$\boxed{\Gamma^\alpha \Gamma_\alpha = I}$ (1.2) (not summing over α)

In fact: not summing over k $\quad ((\gamma^k)^2 = -I)$

$$(\gamma^5)^2 = I \; ; \; (\gamma^0)^2 = I \; ; \; \gamma^k\gamma_k = I \quad (\gamma_k = -\gamma^k)$$

$$(i\gamma^0\gamma^5)(i\gamma^0\gamma^5) = 1 \; ; \quad (i\gamma^k\gamma^5)(i\gamma_k\gamma^5) = 1$$

(because $\gamma^\mu\gamma^5 + \gamma^5\gamma^\mu = 0$)

Let be: fixed

$\mu = k; \nu = l$: $\sigma^{kl}\,\sigma_{kl} = -\frac{1}{4}(\gamma^k\gamma^l - \gamma^l\gamma^k)(\gamma_k\gamma_l - \gamma_l\gamma_k)$

$$= -\frac{1}{4}(\gamma^k\gamma^l - \gamma^l\gamma^k)(\gamma^k\gamma^l - \gamma^l\gamma^k) = -\frac{1}{4}(\gamma^k\gamma^l\gamma^k\gamma^l - \gamma^k\gamma^l\gamma^l\gamma^k$$

$$- \gamma^l\gamma^k\gamma^k\gamma^l + \gamma^l\gamma^k\gamma^l\gamma^k) = -\frac{1}{4}(\gamma^k\gamma^l(2\overset{0\; k\neq l}{\cancel{\eta^{kl}}} - \gamma^l\gamma^k) - I$$

$$- I + \gamma^l\gamma^k(2\overset{0\; k\neq l}{\cancel{\eta^{kl}}} - \gamma^k\gamma^l)) = -\frac{1}{4}[-I - I - I - I]$$

$$= I.$$

If $\mu = 0$; $\nu = k$ $(\neq 0)$

$$\sigma^{0k}\sigma_{0k} = -\frac{1}{4}(\gamma^0\gamma^k - \gamma^k\gamma^0)(\gamma_0\gamma_k - \gamma_k\gamma_0)$$

$$= -\frac{1}{4}(2\gamma^0\gamma^k)(2\gamma_0\gamma_k) = +\gamma^0\gamma^k\gamma_k\gamma^0 = I$$

$$(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu})$$

Additionally:

$$\boxed{\frac{1}{4}Tr[\Gamma^\alpha\Gamma_\beta] = \delta^\tau_\beta} \qquad C(1.3)$$

For example:

a) $$\gamma^5\gamma^5 = I$$

$$\frac{1}{4}Tr(I) = 1$$

$$\Rightarrow \frac{1}{4}Tr(\gamma^5\gamma^5) = 1$$

b) $$\gamma^5\gamma_0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$

$$\frac{1}{4}Tr(\gamma^5\gamma_0) = 0$$

c) $$\gamma^5\gamma_k = -\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix} = \begin{pmatrix} -\sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}$$

$$\Rightarrow \frac{1}{4}Tr(\gamma^5\gamma_k) = 0$$

d) $$\gamma^5(i\gamma^k\gamma^5) = -i\gamma^k$$

$$\Rightarrow \frac{1}{4}Tr[\gamma^5(i\gamma^k\gamma^5)] = 0$$

e) $$\gamma^k\sigma_{em} = \frac{i}{2}\gamma^k(\gamma_e\gamma_m - \gamma_m\gamma_e)$$

$$\Rightarrow \frac{1}{4}Tr[\gamma^k\sigma_{em}] = 0$$ (Trace of an odd number of γ matrices $= 0$)

f)

$$\frac{1}{4}\,Tr\left(\sigma^{\ell m}\,\sigma_{\ell m}\right) = -\frac{1}{16}\,Tr\,(2\gamma^{\ell}\gamma^{m})(2\gamma_{\ell}\gamma_{m})$$
$\ell \neq m$
$$= \frac{1}{4}\,Tr\left(\gamma^{\ell}\gamma^{m}\gamma_{m}\gamma_{\ell}\right) = 1$$

g) $$\frac{1}{4}\,Tr\left(i\gamma^{\kappa}\gamma^{5}\, i\gamma_{\kappa}\gamma^{5}\right) = +\frac{1}{4}\,Tr\left(\gamma^{\kappa}\gamma^{5}\gamma^{5}\gamma_{\kappa}\right) = 1$$

and so on

Then: $$\boxed{\frac{1}{4}\,Tr\left[\Gamma^{\alpha}\,\Gamma_{\beta}\right] = \delta^{\alpha}_{\beta}} \qquad (1.3)$$

The last expression tell us that the 16 Γ^{α} matrices are linear independent and then, they are a complete set. This means than any arbitrary matrix Γ^{*} can be written as a linear combination of these matrices:

$$\boxed{\Gamma^{*} = \sum_{\alpha} C_{\alpha}\,\Gamma^{\alpha}} \qquad (1.4)$$

$$\Rightarrow \quad \Gamma_{\beta}\,\Gamma^{*} = \sum_{\alpha} C_{\alpha}\,\Gamma_{\beta}\,\Gamma^{\alpha}$$

$$\Rightarrow \quad Tr\left(\Gamma_{\beta}\,\Gamma^{*}\right) = \sum_{\alpha} C_{\alpha}\,Tr\left(\Gamma_{\beta}\,\Gamma^{\alpha}\right) \qquad \left(Tr\left(\Gamma_{\beta}\Gamma^{\alpha}\right) = Tr\left(\Gamma^{\alpha}\Gamma_{\beta}\right)\right)$$

$$= 4\sum_{\alpha} C_{\alpha}\,\delta_{\alpha\beta} = 4\,C_{\beta}$$

$$\Rightarrow \quad \boxed{C_{\beta} = \frac{1}{4}\,Tr\left(\Gamma_{\beta}\,\Gamma^{*}\right)} \qquad (1.5)$$

Then $$\boxed{\Gamma^{*} = \frac{1}{4}\sum_{\alpha} Tr\left(\Gamma_{\alpha}\,\Gamma^{*}\right)\Gamma^{\alpha}} \qquad (1.6)$$

(1.6) also can be written as:

$$\boxed{\Gamma^{*}_{ij} = \frac{1}{4} \sum_{\alpha} \Gamma^{*}_{\ell m} \Gamma^{\alpha}_{m\ell} \Gamma_{\alpha\, ij}} \qquad (1.7)$$

Supposing that Γ^{*} contains only one element different of zero : $\Gamma^{*}_{\ell m} \equiv r$ we have

$$\boxed{\delta_{i\ell}\, \delta_{jm} = \frac{1}{4} \sum_{\alpha} \Gamma_{\alpha\, ij}\, \Gamma^{\alpha}_{m\ell}} \qquad (1.8)$$

If we multiply the both sides of (1.8) by

$$S = \bar{\psi}^{a}_{i}\, \psi^{b}_{j}\, \bar{\psi}^{c}_{m}\, \psi^{d}_{\ell} \qquad (1.9)$$

we get:

$$(\bar{\psi}^{a}_{i}\, \psi^{d}_{i}).(\bar{\psi}^{c}_{j}\, \psi^{b}_{j}) = \frac{1}{4} \sum_{\alpha} \Gamma_{\alpha\, ij}\, \Gamma^{\alpha}_{m\ell}\, \bar{\psi}^{a}_{i}\, \psi^{b}_{j}\, \bar{\psi}^{c}_{m}\, \psi^{d}_{\ell}$$

or:

$$\boxed{(\bar{\psi}^{a} \psi^{d})\, (\bar{\psi}^{c} \psi^{b}) = \frac{1}{4} \sum_{\alpha} (\bar{\psi}^{a} \Gamma_{\alpha} \psi^{b})\, (\bar{\psi}^{c} \Gamma^{\alpha} \psi^{d})} \qquad (1.9)$$

Let's multiply now equation (1.8) by

$$P = \bar{\psi}^{a}_{i}\, \gamma^{5}_{jk}\, \psi^{b}_{k} \,.\, \bar{\psi}^{c}_{m}\, \gamma^{5}_{\ell n}\, \psi^{d}_{n}$$

$$\Rightarrow \bar{\psi}^{a}_{i}\, \gamma^{5}_{jk}\, \psi^{b}_{k}\, \bar{\psi}^{c}_{j}\, \gamma^{5}_{in}\, \psi^{d}_{n} = \frac{1}{4} \sum_{\alpha} \Gamma_{\alpha\, ij}\, \Gamma^{\alpha}_{m\ell}\, \bar{\psi}^{a}_{i}\, \gamma^{5}_{jk}\, \psi^{b}_{k}\, \bar{\psi}^{c}_{m}\, \gamma^{5}_{\ell n}\, \psi^{d}_{n}$$

$$\bar{\psi}^{a}_{i}\, \gamma^{5}_{in}\, \psi^{d}_{n} \,.\, \bar{\psi}^{c}_{j}\, \gamma^{5}_{jk}\, \psi^{b}_{k} = \frac{1}{4} \sum_{\alpha} \bar{\psi}^{a}_{i}\, \Gamma_{\alpha\, ij}\, \gamma^{5}_{jk}\, \psi^{b}_{k} \,.\, \bar{\psi}^{c}_{m}\, \Gamma^{\alpha}_{m\ell}\, \gamma^{5}_{\ell n}\, \psi^{d}_{n}$$

Then:

$$\boxed{(\bar{\psi}^a \gamma^5 \psi^d)(\bar{\psi}^c \gamma^5 \psi^b) = \frac{1}{4}\sum_\alpha (\bar{\psi}^a \Gamma_\alpha \gamma^5 \psi^b)(\bar{\psi}^c \Gamma^\alpha \gamma^5 \psi^d)}$$

(1-10)

Multiplying (1.8) by

$$V = \bar{\psi}_i^a \gamma^\mu{}_{jk} \psi_k^b \, \bar{\psi}_m^c \gamma_{\mu l n} \psi_n^d$$

we have:

$$\bar{\psi}_i^a \gamma^\mu_{jk} \psi_k^b \, \bar{\psi}_j^c \gamma_{\mu i n} \psi_n^d = \frac{1}{4}\sum_\alpha \Gamma_{\alpha ij} \Gamma^\alpha_{ml} \bar{\psi}_i^a \gamma^\mu_{jk} \psi_k^b \cdot \bar{\psi}_m^c \gamma_{\mu l n} \psi_n^d$$

$$\bar{\psi}_i^a \gamma^\mu_{in} \psi_n^d \cdot \bar{\psi}_j^c \gamma_{\mu jk} \psi_k^b = \frac{1}{4}\sum_\alpha \bar{\psi}_i^a \Gamma_{\alpha ij} \gamma^\mu_{jk} \psi_k^b \cdot \cdot \bar{\psi}_m^c \Gamma^\alpha_{ml} \gamma_{\mu l n} \psi_n^d$$

$\therefore$

$$\boxed{(\bar{\psi}^a \gamma^\mu \psi^d)(\bar{\psi}^c \gamma_\mu \psi^b) = \frac{1}{4}\sum_\alpha (\bar{\psi}^a \Gamma_\alpha \gamma^\mu \psi^b)(\bar{\psi}^c \Gamma^\alpha \gamma_\mu \psi^d)}$$

(1-11)

Multiplying (1.8) by

$$A = \bar{\psi}_i^a \, i \gamma^\mu{}_{jk} \gamma^5_{kr} \psi_r^b \, \bar{\psi}_m^c \, i \gamma_{\mu l n} \gamma^5_{ns} \psi_s^d$$

we get:

$$\bar{\psi}_i^a \, i \gamma_{\mu jk} \gamma^5_{kr} \psi_r^b \, \bar{\psi}_j^c \, i \gamma^\mu{}_{in} \gamma^5_{ns} \psi_s^d$$

$$= \frac{1}{4}\sum_\alpha \Gamma_{\alpha ij} \Gamma^\alpha_{ml} \bar{\psi}_i^a \, i \gamma^\mu_{jk} \gamma^5_{kr} \psi_r^b \, \bar{\psi}_m^c \, i \gamma_{\mu l n} \gamma^5_{ns} \psi_s^d$$

$$\bar{\psi}_i^a \, i \gamma^\mu{}_{in} \gamma^5_{ns} \psi_s^d \cdot \bar{\psi}_j^c \, i \gamma_{\mu jk} \gamma^5_{kr} \psi_r^b =$$

$$\frac{1}{4}\sum_\alpha \bar{\psi}_i^a \Gamma_{\alpha ij}\, i\gamma^\mu_{jk}\gamma^5_{kr}\psi^b_r \cdot \bar{\psi}^c_m \Gamma^\alpha_{m\ell}\, i\gamma_{\mu\ell n}\gamma^5_{ns}\psi^d_s$$

$\Rightarrow$

$$\boxed{(\bar{\psi}^a i\gamma^\mu\gamma^5\psi^d)(\bar{\psi}^c i\gamma_\mu\gamma^5\psi^b) = \frac{1}{4}\sum_\alpha(\bar{\psi}^a\Gamma_\alpha i\gamma^\mu\gamma^5\psi^b)\cdot(\bar{\psi}^c\Gamma^\alpha i\gamma_\mu\gamma^5\psi^d)} \tag{1.12}$$

Similarly multiplying (1.8) by

$$T = \bar{\psi}_i^a(\sigma^{\mu\nu})_{jk}\psi_k^b \cdot \bar{\psi}_m^c(\sigma_{\mu\nu})_{\ell n}\psi_n^d$$

we get:

$$\boxed{(\bar{\psi}^a\sigma^{\mu\nu}\psi^d)(\bar{\psi}^c\sigma_{\mu\nu}\psi^b) = \frac{1}{4}\sum_\alpha(\bar{\psi}^a\Gamma_\alpha\sigma^{\mu\nu}\psi^b)\cdot(\bar{\psi}^c\Gamma^\alpha\sigma_{\mu\nu}\psi^d)} \tag{1.13}$$

Writting:

$$J_S = (\bar{\psi}^a\psi^b)(\bar{\psi}^c\psi^d)$$

$$J_P = (\bar{\psi}^a\gamma^5\psi^b)(\bar{\psi}^c\gamma^5\psi^d)$$

$$J_V = (\bar{\psi}^a\gamma^\mu\psi^b)(\bar{\psi}^c\gamma_\mu\psi^d)$$

$$J_A = (\bar{\psi}^a i\gamma^\mu\gamma^5\psi^b)(\bar{\psi}^c i\gamma_\mu\gamma^5\psi^d)$$

$$J_T = (\bar{\psi}^a\sigma^{\mu\nu}\psi^b)(\bar{\psi}^c\sigma_{\mu\nu}\psi^d)$$

and

$$J_S' = (\bar{\psi}^a \psi^d)(\bar{\psi}^c \psi^b) \qquad (J_S \quad b \leftrightarrow d)$$

$$J_P' = (\bar{\psi}^a \gamma^5 \psi^d)(\bar{\psi}^c \gamma^5 \psi^b)$$

$$J_V' = (\bar{\psi}^a \gamma^\mu \psi^d)(\bar{\psi}^c \gamma_\mu \psi^b)$$

$$J_A' = (\bar{\psi}^a i \gamma^\mu \gamma^5 \psi^d)(\bar{\psi}^c i \gamma_\mu \gamma^5 \psi^b)$$

$$J_T' = (\bar{\psi}^a \sigma^{\mu\nu} \psi^d)(\bar{\psi}^c \sigma_{\mu\nu} \psi^b)$$

From (1.9) we obtain:

$$\boxed{J_S' = \frac{1}{4}\left[J_S + J_P + J_V + J_A + J_T \right]} \qquad (1.14)$$

From (1.10) we obtain:

$$\boxed{J_P' = \frac{1}{4}\left[J_P + J_S - J_A - J_V + J_T \right]} \qquad (1.15)$$

To get the last term observe that:

a) $\sigma^{01}\gamma^5 = i\gamma^0\gamma^1 i \gamma^0\gamma^1\gamma^2\gamma^3 = -\gamma^2\gamma^3$
$= i(\sigma^{23})$

$\sigma_{01}\gamma^5 = i\gamma^0\gamma_1 i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma^0\gamma^1 i\gamma^0\gamma^1\gamma^2\gamma^3$
$= \gamma^2\gamma^3 = -i\sigma_{23}$

Then we have:

$$(\bar{\psi}^a \sigma^{23} \psi^b)(\bar{\psi}^c \sigma_{23} \psi^d)$$

b) $\sigma^{02}\gamma^5 = i\gamma^0\gamma^2 i\gamma^0\gamma^1\gamma^2\gamma^3 = +\gamma^2\gamma^1\gamma^2\gamma^3$
$= \gamma^1\gamma^3 = -i\sigma^{13}$

$\sigma_{02}\gamma^5 = i\sigma_{13}$

So, we have another term

$$(\bar{\psi}^a \sigma^{13} \psi^b)(\bar{\psi}^c \sigma_{13} \psi^d)$$

c) $$\sigma^{03}\gamma^5 = i\gamma^0\gamma^3 i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma^3\gamma^1\gamma^2\gamma^3 = -\gamma^1\gamma^2$$
$$= i\sigma^{12}$$

$$\sigma_{03}\gamma^5 = -i\sigma_{12}$$

Then I have the following term:

$$(\bar{\psi}^a \sigma^{12}\psi^b)(\bar{\psi}^c \sigma_{12}\psi^d)$$

d) $$\sigma^{12}\gamma^5 = i\gamma^1\gamma^2 i\gamma^0\gamma^1\gamma^2\gamma^3 = -\gamma^1\gamma^2\gamma^0\gamma^1\gamma^2\gamma^3$$
$$= -\gamma^0\gamma^1\gamma^2\gamma^1\gamma^2\gamma^3$$
$$= -\gamma^0\gamma^2\gamma^2\gamma^3 = \gamma^0\gamma^3$$
$$= -i\sigma^{03}$$

$$\sigma_{12}\gamma^5 = -i\sigma^{03} = i\sigma_{03}$$

Then we have:

$$(\bar{\psi}^a \sigma^{03}\psi^b)(\bar{\psi}^c \sigma_{03}\psi^d)$$

e) $$\sigma^{13}\gamma^5 = i\gamma^1\gamma^3 i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma^3\gamma^1\gamma^0\gamma^1\gamma^2\gamma^3$$
$$= \gamma^3\gamma^0\gamma^2\gamma^3 = -\gamma^0\gamma^2$$
$$= i\sigma^{02}$$

$$\sigma_{13}\gamma^5 = i\sigma^{02} = -i\sigma_{02}$$

Then we have:

$$(\bar{\psi}^a \sigma^{02}\psi^b)(\bar{\psi}^c \sigma_{02}\psi^d)$$

f) $$\sigma^{23}\gamma^5 = i\gamma^2\gamma^3 i\gamma^0\gamma^1\gamma^2\gamma^3 = -\gamma^2\gamma^3\gamma^0\gamma^1\gamma^2\gamma^3$$
$$= -\gamma^3\gamma^0\gamma^1\gamma^3$$
$$= \gamma^0\gamma^1 = -i\sigma^{01}$$

$$\sigma_{23}\gamma^5 = -i\sigma^{01} = i\sigma_{01}$$

Then we have:

$$(\bar{\psi}^a \sigma^{01}\psi^b)(\bar{\psi}^c \sigma_{01}\psi^d)$$

We have used:
$[(\gamma^0)^2 = 1;\ (\gamma^k)^2 = -1]$

$$\Rightarrow \boxed{(\bar{\psi}^a \sigma_{\mu\nu}\gamma^5\psi^b)(\bar{\psi}^c \sigma^{\mu\nu}\gamma^5\psi^d) = (\bar{\psi}^a \sigma^{\mu\nu}\psi^b)(\bar{\psi}^c \sigma_{\mu\nu}\psi^d)}$$

Let's consider (1.11) :

$$(\bar\psi^a \gamma^\mu \psi^d)(\bar\psi^c \gamma_\mu \psi^b) = \frac{1}{4}(\bar\psi^a \gamma^\mu \psi^b)(\bar\psi^c \gamma_\mu \psi^d)$$

$$+ \frac{1}{4}(\bar\psi^a \gamma^5 \gamma^\mu \psi^b)(\bar\psi^c \gamma^5 \gamma_\mu \psi^d) + \frac{1}{4}(\bar\psi^a \gamma_\nu \gamma^\mu \psi^b) \cdot$$

$$\cdot (\bar\psi^c \gamma^\nu \gamma_\mu \psi^d) + \frac{1}{4}(\bar\psi^a i\gamma_\nu \gamma^5 \gamma^\mu \psi^b)(\bar\psi^c i\gamma^\nu \gamma^5 \gamma_\mu \psi^d)$$

$$+ \frac{1}{4}(\bar\psi^a \sigma_{\nu\rho} \gamma^\mu \psi^b)(\bar\psi^c \sigma^{\nu\rho} \gamma_\mu \psi^d)$$

$$= \frac{1}{4}\Big\{ (\bar\psi^a \gamma^\mu \psi^b)(\bar\psi^c \gamma_\mu \psi^d) - (\bar\psi^a i\gamma^\mu \gamma^5 \psi^b)(\bar\psi^c i\gamma_\mu \gamma^5 \psi^d)$$

$$+ (\bar\psi^a \psi^b)(\bar\psi^c \psi^d) + (\bar\psi^a \gamma_0 \gamma^1 \psi^b) \cdot (\bar\psi^c \gamma^0 \gamma_1 \psi^d)$$

$$+ (\bar\psi^a \gamma_0 \gamma^2 \psi^b) \cdot (\bar\psi^c \gamma^0 \gamma_2 \psi^d) + (\bar\psi^a \gamma_0 \gamma^3 \psi^b) \cdot (\bar\psi^c \gamma^0 \gamma_3 \psi^d)$$

$$+ (\bar\psi^a \gamma_1 \gamma^0 \psi^b)(\bar\psi^c \gamma^1 \gamma_0 \psi^d) + (\bar\psi^a \psi^b)(\bar\psi^c \psi^d)$$

$$+ (\bar\psi^a \gamma_1 \gamma^2 \psi^b)(\bar\psi^c \gamma^1 \gamma_2 \psi^d) + (\bar\psi^a \gamma_1 \gamma^3 \psi^b)(\bar\psi^c \gamma^1 \gamma_3 \psi^d)$$

$$+ (\bar\psi^a \gamma_2 \gamma^0 \psi^b)(\bar\psi^c \gamma^2 \gamma_0 \psi^d) + (\bar\psi^a \gamma_2 \gamma^1 \psi^b)(\bar\psi^c \gamma^2 \gamma_1 \psi^d)$$

$$+ (\bar\psi^a \psi^b)(\bar\psi^c \psi^d) + (\bar\psi^a \gamma_3 \gamma^0 \psi^b)(\bar\psi^c \gamma^3 \gamma_0 \psi^d) + (\bar\psi^a \gamma_2 \gamma^3 \psi^b)$$

$$(\bar\psi^c \gamma^2 \gamma_3 \psi^d) + (\bar\psi^a \gamma_3 \gamma^1 \psi^b)(\bar\psi^c \gamma^3 \gamma_1 \psi^d) + (\bar\psi^a \gamma_3 \gamma^2 \psi^b)(\bar\psi^c \gamma^3 \gamma_2 \psi^d)$$

$$+ (\bar\psi^a \psi^b)(\bar\psi^c \psi^d) - 4(\bar\psi^a \gamma^5 \psi^b)(\bar\psi^c \gamma^5 \psi^d)$$

$$+ (\bar\psi^a i\gamma_0 \gamma^5 \gamma^1 \psi^b)(\bar\psi^c i\gamma^0 \gamma^5 \gamma_1 \psi^d) + (\bar\psi^a i\gamma_0 \gamma^5 \gamma^2 \psi^b)(\bar\psi^c i\gamma^0 \gamma^5 \cdot$$

$$\cdot \gamma_2 \psi^d) + (\bar\psi^a i\gamma_0 \gamma^5 \gamma^3 \psi^b)(\bar\psi^c i\gamma^0 \gamma^5 \gamma_3 \psi^d)$$

$$+ (\bar\psi^a i\gamma_1 \gamma^5 \gamma^0 \psi^b)(\bar\psi^c i\gamma^1 \gamma^5 \gamma_0 \psi^d) + (\bar\psi^a i\gamma_1 \gamma^5 \gamma^2 \psi^b)(\bar\psi^c i\gamma^1 \gamma^5 \gamma_2 \psi^d)$$

$$+ (\bar\psi^a i\gamma_1 \gamma^5 \gamma^3 \psi^b)(\bar\psi^c i\gamma^1 \gamma^5 \gamma_3 \psi^d)$$

$$+ (\bar\psi^a i\gamma_2 \gamma^5 \gamma^0 \psi^b)(\bar\psi^c i\gamma^2 \gamma^5 \gamma_0 \psi^d) + (\bar\psi^a i\gamma_2 \gamma^5 \gamma^1 \psi^b)(\bar\psi^c i\gamma^2 \gamma^5 \gamma_1 \psi^d)$$

$$+ (\bar\psi^a i\gamma_2 \gamma^5 \gamma^3 \psi^b)(\bar\psi^c i\gamma^2 \gamma^5 \gamma_3 \psi^d)$$

$$+ (\bar\psi^a i\gamma_3 \gamma^5 \gamma^0 \psi^b)(\bar\psi^c i\gamma^3 \gamma^5 \gamma_0 \psi^d) + (\bar\psi^a i\gamma_3 \gamma^5 \gamma^1 \psi^b)(\bar\psi^c i\gamma_3 \gamma^5 \gamma_1 \psi^d)$$

$$+ (\bar\psi^a i\gamma_3 \gamma^5 \gamma^2 \psi^b)(\bar\psi^c i\gamma^3 \gamma^5 \gamma_2 \psi^d)$$

(observe that: $(\bar\psi^a i\gamma_\nu \gamma^5 \gamma^\mu \psi^b)(\bar\psi^c i\gamma^\nu \gamma^5 \gamma_\mu \psi^d) = -4(\bar\psi^a \gamma^5 \psi^b)(\bar\psi^c \gamma^5 \psi^d)$
$+ 2(\bar\psi^a \sigma^{\mu\nu} \psi^b)(\bar\psi^c \sigma_{\mu\nu} \psi^d)$ (1.16))

$$-3\,(\bar\psi^a \gamma^\mu \psi^b)(\bar\psi^c \gamma_\mu \psi^d) \quad +$$

$$\Downarrow$$

$$\gamma^1 \left\{ \begin{array}{ccc} 0 & 1 & 0 \\ 1 & 2 & 2 \\ 1 & 3 & 3 \end{array} \right. \qquad (\bar\psi^a \sigma_{01}\gamma^0\psi^b)(\bar\psi^c \sigma^{01}\gamma_0\psi^d) = -(\bar\psi^a \gamma^1 \psi^b)(\bar\psi^c \gamma_1 \psi^d)$$

$$\gamma^2 \left\{ \begin{array}{ccc} 0 & 2 & 0 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{array} \right.$$

$$\gamma^3 \left\{ \begin{array}{ccc} 0 & 3 & 0 \\ 1 & 3 & 1 \\ 2 & 3 & 2 \end{array} \right.$$

$$\gamma^0 \left\{ \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \end{array} \right.$$

$$+3\,(\bar\psi^a i\gamma^\mu\gamma^5\psi^b)(\bar\psi^c i\gamma_\mu\gamma^5\psi^d)$$

$$\Downarrow$$

$$i\gamma^3\gamma^5 \left\{ \begin{array}{ccc} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 0 & 2 & 1 \end{array} \right.$$

$$i\gamma^2\gamma^5 \left\{ \begin{array}{ccc} 0 & 1 & 3 \\ 0 & 3 & 1 \\ 1 & 3 & 0 \end{array} \right.$$

$$i\gamma^1\gamma^5 \left\{ \begin{array}{ccc} 2 & 3 & 0 \\ 0 & 2 & 3 \\ 0 & 3 & 2 \end{array} \right. \qquad (\bar\psi^a \sigma_{23}\gamma^0\psi^b)(\bar\psi^c \sigma^{23}\gamma_0\psi^d)$$

$$= i(\bar\psi^a \gamma^0\gamma^2\gamma^3\gamma^1\gamma_1\psi^b)\, i(\bar\psi^c \gamma^0\gamma^2\gamma^3\gamma^1\gamma_1\psi^d)$$

$$= (\bar\psi^a \gamma^5\gamma_1\psi^b)(\bar\psi^c \gamma^5\gamma_1\psi^d)$$

$$= (\bar\psi^a \gamma^1\gamma^5\psi^b)(-\bar\psi^c \gamma_1\gamma^5\psi^d)$$

$$= (\bar\psi^a i\gamma^1\gamma^5\psi^b)(\bar\psi^c i\gamma_1\gamma^5\psi^d)$$

$$i\gamma^0\gamma^5 \left\{ \begin{array}{ccc} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 3 & 1 \end{array} \right.$$

Note that the last two terms are:

$$\boxed{(\bar\psi^a \sigma_{\nu\rho}\gamma^\mu\psi^b)(\bar\psi^c \sigma^{\nu\rho}\gamma_\mu\psi^d) = -3(\bar\psi^a \gamma^\mu\psi^b)(\bar\psi^c \gamma_\mu\psi^d) + 3(\bar\psi^a i\gamma^\mu\gamma^5\psi^b)(\bar\psi^c i\gamma_\mu\gamma^5\psi^d)} \quad (1.17)$$

additionally:

$$\boxed{(\bar\psi^a \gamma_\nu\gamma^\mu\psi^b)(\bar\psi^c \gamma^\nu\gamma_\mu\psi^d) = 4(\bar\psi^a\psi^b)(\bar\psi^c\psi^d) - 2(\bar\psi^a \sigma^{\mu\nu}\psi^b)(\bar\psi^c \sigma_{\mu\nu}\psi^d)} \quad (1.18)$$

(see last page)

$$\therefore\ (\bar{\psi}^a \gamma^\mu \psi^d)(\bar{\psi}^c \gamma_\mu \psi^b) = \frac{1}{4}\Big[\cancel{(\bar{\psi}^a \gamma^\mu \psi^b)(\bar{\psi}^c \gamma_\mu \psi^d)}$$

$$- \cancel{(\bar{\psi}^a i\gamma^\mu \gamma^5 \psi^b)(\bar{\psi}^c i\gamma_\mu \gamma^5 \psi^d)} + 4\,(\bar{\psi}^a \psi^b)(\bar{\psi}^c \psi^d)$$

$$- 4\,(\bar{\psi}^a \gamma^5 \psi^b)(\bar{\psi}^c \gamma^5 \psi^d) - 3\,\cancel{(\bar{\psi}^a \gamma^\mu \psi^b)(\bar{\psi}^c \gamma_\mu \psi^d)}$$

$$+ 3\,\cancel{(\bar{\psi}^a i\gamma^\mu \gamma^5 \psi^b)(\bar{\psi}^c i\gamma_\mu \gamma^5 \psi^d)}$$

$$- 2\,\cancel{(\bar{\psi}^a \sigma^{\mu\nu} \psi^b)(\bar{\psi}^c \sigma_{\mu\nu} \psi^d)} \rightarrow (\text{Ex: } \bar{\psi}^a \gamma_0\gamma^1 \psi^b)(\bar{\psi}^c \gamma^0\gamma_1 \psi^d))$$

$$+ 2\,\cancel{(\bar{\psi}^a \sigma^{\mu\nu} \psi^b)(\bar{\psi}^c \sigma_{\mu\nu} \psi^d)} \rightarrow (\text{Example}: (\bar{\psi}^a i\gamma_0\gamma^5\gamma^1 \psi^b)\cdot(\bar{\psi}^c i\gamma^0\gamma^5\gamma_1 \psi^d))$$

$$\Rightarrow \boxed{\begin{aligned}(\bar{\psi}^a \gamma^\mu \psi^d)(\bar{\psi}^c \gamma_\mu \psi^b) &= \frac{1}{4}\Big[4\cdot(\bar{\psi}^a \psi^b)(\bar{\psi}^c \psi^d) \\ &- 2\,(\bar{\psi}^a \gamma^\mu \psi^b)(\bar{\psi}^c \gamma_\mu \psi^d) - 4\,(\bar{\psi}^a \gamma^5 \psi^b)(\bar{\psi}^c \gamma^5 \psi^d) \\ &+ 2\,(\bar{\psi}^a i\gamma^\mu\gamma^5 \psi^b)(\bar{\psi}^c i\gamma_\mu \gamma^5 \psi^d)\Big]\end{aligned}} \quad (1.19)$$

Then:

$$\boxed{J_V' = \frac{1}{4}\Big[\,4J_S - 4J_P - 2J_V + 2J_A\,\Big]} \quad (1.20)$$

From (1.12):

$$(\bar{\psi}^a i\gamma^\mu\gamma^5 \psi^d)(\bar{\psi}^c i\gamma_\mu\gamma^5 \psi^b) = \frac{1}{4}\sum_\alpha (\bar{\psi}^a \Gamma_\alpha (i\gamma^\mu\gamma^5)\psi^b)\cdot(\bar{\psi}^c \Gamma^\alpha (i\gamma_\mu\gamma^5)\psi^d)$$

$$= \frac{1}{4}\Big\{(\bar{\psi}^a\, i\gamma^\mu\gamma^5\psi^b)(\bar{\psi}^c i\gamma_\mu\gamma^5\psi^d)$$

$$-(\bar{\psi}^a \gamma^\mu \psi^b)(\bar{\psi}^c \gamma_\mu \psi^d) + (\bar{\psi}^a i\gamma_\nu\gamma^5\gamma^\mu\psi^b)(\bar{\psi}^c i\gamma^\nu\gamma^5\gamma_\mu\psi^d)$$

$$+ (\bar{\psi}^a \gamma_\nu\gamma^\mu\psi^b)(\bar{\psi}^c\gamma^\nu\gamma_\mu\psi^d) + (\bar{\psi}^a \sigma_{\nu\rho}\, i\gamma^\mu\gamma^5\psi^b)\cdot$$

$$\cdot(\bar{\psi}^c \sigma^{\nu\rho} i\gamma_\mu\gamma^5\psi^d)\Big\}$$

$$= \frac{1}{4}\Big\{(\bar{\psi}^a i\gamma^\mu\gamma^5\psi^b)(\bar{\psi}^c i\gamma_\mu\gamma^5\psi^d) - (\bar{\psi}^a\gamma^\mu\psi^b)\cdot$$

$$\cdot(\bar{\psi}^c\gamma_\mu\psi^d) + 2\,\cancel{(\bar{\psi}^a\sigma^{\mu\nu}\psi^b)(\bar{\psi}^c\sigma_{\mu\nu}\psi^d)}$$

$$-4(\bar{\psi}^a\gamma^5\psi^b)(\bar{\psi}^c\gamma^5\psi^d) + 4(\bar{\psi}^a\psi^b)(\bar{\psi}^c\psi^d) - 2\,\cancel{(\bar{\psi}^a\sigma^{\mu\nu}\psi^b)(\bar{\psi}^c\sigma_{\mu\nu}\psi^d)}$$

$$+ \left(\bar\psi^a \sigma_{\nu\rho}\, i\gamma^\mu\gamma^5\psi^b\right).\left(\bar\psi^c \sigma^{\nu\rho}\, i\gamma_\mu\gamma^5\psi^d\right)\Big\}$$

Let's consider the last term:

$$\left(\bar\psi^a \sigma_{\nu\rho}\, i\gamma^\mu\gamma^5\psi^b\right).\left(\bar\psi^c \sigma^{\nu\rho}\, i\gamma_\mu\gamma^5\psi^d\right)$$

$$\gamma^1\gamma^5 \begin{cases} 0 & 1 & 0 \\ 1 & 2 & 2 \\ 1 & 3 & 3 \end{cases}$$

$$\gamma^2\gamma^5 \begin{cases} 0 & 2 & 0 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{cases}$$

$$\gamma^3\gamma^5 \begin{cases} 0 & 3 & 0 \\ 1 & 3 & 1 \\ 2 & 3 & 2 \end{cases}$$

$$\gamma^0\gamma^5 \begin{cases} 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \end{cases}$$

$$-3\left(\bar\psi^a i\gamma^\mu\gamma^5\psi^b\right).\left(\bar\psi^c i\gamma_\mu\gamma^5\psi^d\right)$$

$$i\gamma^3 \begin{cases} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 0 & 2 & 1 \end{cases}$$

$$i\gamma^2 \begin{cases} 0 & 1 & 3 \\ 0 & 3 & 1 \\ 1 & 3 & 0 \end{cases}$$

$$i\gamma^1 \begin{cases} 2 & 3 & 0 \\ 0 & 2 & 3 \\ 0 & 3 & 2 \end{cases}$$

$$i\gamma^0 \begin{cases} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 3 & 1 \end{cases}$$

$$3\left(\bar\psi^a \gamma^\mu\psi^b\right)\left(\bar\psi^c\gamma_\mu\psi^d\right)$$

$$\Rightarrow \boxed{\begin{aligned} &\left(\bar\psi^a \sigma_{\nu\rho}\, i\gamma^\mu\gamma^5\psi^b\right).\left(\bar\psi^c \sigma^{\nu\rho}\, i\gamma_\mu\gamma^5\psi^d\right) \\ &= -3\left(\bar\psi^a i\gamma^\mu\gamma^5\psi^b\right)\left(\bar\psi^c i\gamma_\mu\gamma^5\psi^d\right) \\ &\quad + 3\left(\bar\psi^a\gamma^\mu\psi^b\right)\left(\bar\psi^c\gamma_\mu\psi^d\right) \end{aligned}} \qquad (1\text{-}21)$$

$$\Rightarrow \left(\bar\psi^a i\gamma^\mu\gamma^5\psi^d\right)\left(\bar\psi^c i\gamma_\mu\gamma^5\psi^b\right) = \frac{1}{4}\Big\{\left(\bar\psi^a i\gamma^\mu\gamma^5\psi^b\right)\left(\bar\psi^c i\gamma_\mu\gamma^5\psi^d\right)$$

$$-\left(\bar\psi^a\gamma^\mu\psi^b\right)\left(\bar\psi^c\gamma_\mu\psi^d\right) + 4\left(\bar\psi^a\psi^b\right)\left(\bar\psi^c\psi^d\right) - 3\left(\bar\psi^a i\gamma^\mu\gamma^5\psi^b\right).$$

$$.\left(\bar\psi^c i\gamma_\mu\gamma^5\psi^d\right) + 3\left(\bar\psi^a\gamma^\mu\psi^b\right)\left(\bar\psi^c\gamma_\mu\psi^d\right) - 4\left(\bar\psi^a\gamma^5\psi^b\right)\left(\bar\psi^c\gamma^5\psi^d\right)$$

Then

$$\boxed{\begin{aligned}(\bar\psi^a i\gamma^\mu\gamma^5\psi^d)(\bar\psi^c i\gamma_\mu\gamma^5\psi^b) &= \tfrac14\Big\{-2(\bar\psi^a i\gamma^\mu\gamma^5\psi^b)(\bar\psi^c i\gamma_\mu\gamma^5\psi^d)\\ &+2(\bar\psi^a\gamma^\mu\psi^b)(\bar\psi^c\gamma_\mu\psi^d)+4(\bar\psi^a\psi^b)(\bar\psi^c\psi^d)\\ &-4(\bar\psi^a\gamma^5\psi^b)(\bar\psi^c\gamma^5\psi^d)\Big\}\end{aligned}}$$

(1.22)

$\therefore$

$$\boxed{J_A' = \frac14\left\{4J_S + 2J_V - 2J_A - 4J_P\right\}} \qquad (1.23)$$

Let's return to (1.13):

$$(\bar\psi^a\sigma^{\mu\nu}\psi^d)(\bar\psi^c\sigma_{\mu\nu}\psi^b) = \frac14\sum_\alpha(\bar\psi^a\Gamma_\alpha\sigma^{\mu\nu}\psi^b).(\bar\psi^c\Gamma^\alpha\sigma_{\mu\nu}\psi^d)$$

$$=\frac14\Big\{(\bar\psi^a\sigma^{\mu\nu}\psi^b)(\bar\psi^c\sigma_{\mu\nu}\psi^d)+(\bar\psi^a\gamma^5\sigma^{\mu\nu}\psi^b)(\bar\psi^c\gamma^5\sigma_{\mu\nu}\psi^d)$$

$$+(\bar\psi^a\gamma_\rho\sigma^{\mu\nu}\psi^b)(\bar\psi^c\gamma^\rho\sigma_{\mu\nu}\psi^d)+(\bar\psi^a i\gamma_\rho\gamma^5\sigma^{\mu\nu}\psi^b).$$

$$.(\bar\psi^c i\gamma^\rho\gamma^5\sigma^{\mu\nu}\psi^d)+(\bar\psi^a\sigma_{\rho\delta}\sigma^{\mu\nu}\psi^b)(\bar\psi^c\sigma^{\rho\delta}\sigma_{\mu\nu}\psi^d)\Big\}$$

$$(\bar\psi^a\gamma^5\sigma^{\mu\nu}\psi^b)(\bar\psi^c\gamma^5\sigma_{\mu\nu}\psi^d)$$

$$=-(\bar\psi^a\gamma^5\gamma^\mu\gamma^\nu\psi^b)(\bar\psi^c\gamma^5\gamma_\mu\gamma_\nu\psi^d) \qquad (\mu\neq\nu)$$

$$=+(\bar\psi^a i\gamma^5\gamma^\mu\gamma^\nu\psi^b)(\bar\psi^c i\gamma^5\gamma_\mu\gamma_\nu\psi^d)$$

$$=+(\bar\psi^a i\gamma_\nu\gamma^5\gamma^\mu\psi^b)(\bar\psi^c i\gamma^\nu\gamma^5\gamma_\mu\psi^d)$$

$$=(\bar\psi^a\sigma^{\mu\nu}\psi^b)(\bar\psi^c\sigma_{\mu\nu}\psi^d)$$

$$\Rightarrow\ \boxed{(\bar\psi^a\gamma^5\sigma^{\mu\nu}\psi^b)(\bar\psi^c\gamma^5\sigma_{\mu\nu}\psi^d) = (\bar\psi^a\sigma^{\mu\nu}\psi^b)(\bar\psi^c\sigma_{\mu\nu}\psi^d)}$$

(1.24)

$$(\bar\psi^a\, i\gamma^5\gamma^\mu\gamma^\nu\,\psi^b)(\bar\psi^c\, i\gamma^5\,\gamma_\mu\gamma_\nu\,\psi^d)$$

$\mu\neq\nu$

$$i\gamma^5\gamma^\mu\gamma^\nu \qquad\qquad\qquad\qquad i\gamma^5\gamma_\mu\gamma_\nu$$

$$\left\{\begin{array}{ll} 0\ 1 & \to +i\,\sigma^{23} \\ 0\ 2 & \to i\,\sigma^{13} \\ 0\ 3 & \to i\,\sigma^{12} \\ 1\ 2 & \to -i\,\sigma^{03} \\ 1\ 3 & \to -i\,\sigma^{02} \\ 2\ 3 & \to -i\,\sigma^{01} \end{array}\right. \qquad \left\{\begin{array}{ll} 0\ 1 & \to -i\,\sigma_{23} \\ 0\ 2 & \to -i\,\sigma_{13} \\ 0\ 3 & \to -i\,\sigma_{12} \\ 1\ 2 & \to +i\,\sigma_{03} \\ 1\ 3 & \to i\,\sigma_{02} \\ 2\ 3 & \to i\,\sigma_{01} \end{array}\right.$$

$\Rightarrow (\bar\psi^a\, i\gamma^5\gamma^\mu\gamma^\nu\,\psi^b)(\bar\psi^c\, i\gamma^5\,\gamma_\mu\gamma_\nu\,\psi^d)$ with $\mu\neq\nu$

$$= (\bar\psi^a\,\sigma^{\mu\nu}\,\psi^b)(\bar\psi^c\,\sigma_{\mu\nu}\,\psi^d)$$

For

$(\bar{\psi}^a \gamma_\rho \sigma^{\mu\nu} \psi^b)(\bar{\psi}^c \gamma^\rho \sigma_{\mu\nu} \psi^d)$ we have, using (1.17):

$$\boxed{\begin{aligned}(\bar{\psi}^a \gamma_\rho \sigma^{\mu\nu} \psi^b)(\bar{\psi}^c \gamma^\rho \sigma_{\mu\nu} \psi^d) &= -3(\bar{\psi}^a \gamma^\mu \psi^b)(\bar{\psi}^c \gamma_\mu \psi^d) \\ &\quad + 3(\bar{\psi}^a i \gamma^\mu \gamma^5 \psi^b)(\bar{\psi}^c i \gamma_\mu \gamma^5 \psi^d)\end{aligned}} \quad (1.25)$$

For

$(\bar{\psi}^a i \gamma_\rho \gamma^5 \sigma^{\mu\nu} \psi^b).(\bar{\psi}^c i \gamma^\rho \gamma^5 \sigma^{\mu\nu} \psi^d)$ using (1.21) we get:

$$\boxed{\begin{aligned}&(\bar{\psi}^a i \gamma_\rho \gamma^5 \sigma^{\mu\nu} \psi^b).(\bar{\psi}^c i \gamma^\rho \gamma^5 \sigma^{\mu\nu} \psi^d) = \\ &\quad -3(\bar{\psi}^a i \gamma^\mu \gamma^5 \psi^b)(\bar{\psi}^c i \gamma_\mu \gamma^5 \psi^d) + 3(\bar{\psi}^a \gamma^\mu \psi^b)(\bar{\psi}^c \gamma_\mu \psi^d)\end{aligned}}$$

(1.26)

To finish we will consider:

$$(\bar{\psi}^a \sigma_{\rho\delta} \sigma^{\mu\nu} \psi^b)(\bar{\psi}^c \sigma^{\rho\delta} \sigma_{\mu\nu} \psi^d) =$$

$$6(\bar{\psi}^a \psi^b)(\bar{\psi}^c \psi^d) \longrightarrow \left\{\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 3 & 0 & 3 \\ 1 & 2 & 1 & 2 \\ 1 & 3 & 1 & 3 \\ 2 & 3 & 2 & 3 \end{array}\right.$$

$+$

$$6(\bar{\psi}^a \gamma^5 \psi^b)(\bar{\psi}^c \gamma^5 \psi^d) \longrightarrow \left\{\begin{array}{cccc} 0 & 1 & 2 & 3 \\ 1 & 2 & 0 & 3 \\ 0 & 2 & 1 & 3 \\ 1 & 3 & 0 & 2 \\ 0 & 3 & 1 & 2 \\ 2 & 3 & 0 & 1 \end{array}\right.$$

$+$

$$-4(\bar{\psi}^a \sigma^{\mu\nu} \psi^b)(\bar{\psi}^c \sigma_{\mu\nu} \psi^d) \longrightarrow \left\{\begin{array}{ccccl} 0 & 1 & 1 & 2 & \\ 1 & 2 & 0 & 1 & > \sigma_{02} \\ 0 & 1 & 1 & 3 & \\ 1 & 3 & 0 & 1 & > \sigma_{03} \\ 0 & 2 & 2 & 3 & \\ 2 & 3 & 0 & 2 & > \sigma_{03} \\ 0 & 3 & 2 & 3 & \\ 2 & 3 & 0 & 3 & > \sigma_{02} \\ 0 & 2 & 1 & 2 & \\ 1 & 2 & 0 & 2 & > \sigma_{01} \\ 0 & 3 & 1 & 3 & \\ 1 & 3 & 0 & 3 & > \sigma_{01} \end{array}\right.$$

$\sigma_{01}\sigma^{12} = -\gamma_0 \cancel{\gamma_1 \gamma^1}^{\,1} \gamma^2 = -\gamma^0\gamma^2 = i\sigma^{02}$

$\sigma^{01}\sigma_{12} = -\gamma^0\gamma^1\gamma^1\gamma^2 = \gamma^0\gamma^2 = -\gamma^0\gamma_2$

$= i\sigma_{02}$

$$\rightarrow \left\{ \begin{array}{llll} 0\,1 & 0\,2 & \multirow{2}{*}{$> \sigma_{12}$} \\ 0\,2 & 0\,1 & \\ 1\,3 & 2\,3 & > \sigma_{12} \\ 2\,3 & 1\,3 & \\ 0\,1 & 0\,3 & > \sigma_{13} \\ 0\,3 & 0\,1 & \\ 1\,2 & 2\,3 & > \sigma_{13} \\ 2\,3 & 1\,2 & \\ 0\,2 & 0\,3 & > \sigma_{23} \\ 0\,3 & 0\,2 & \\ 1\,2 & 1\,3 & > \sigma_{23} \\ 1\,3 & 1\,2 & \end{array} \right.$$

So:

$$\boxed{\begin{aligned}(\bar{\psi}^a \sigma_{\rho\delta} \sigma^{\mu\nu} \psi^b)(\bar{\psi}^c \sigma^{\rho\delta} \sigma_{\mu\nu} \psi^d) &= 6(\bar{\psi}^a \psi^b)(\bar{\psi}^c \psi^d) + 6(\bar{\psi}^a \gamma^5 \psi^b)(\bar{\psi}^c \gamma^5 \psi^d) \\ &\quad - 4(\bar{\psi}^a \sigma^{\mu\nu} \psi^b)(\bar{\psi}^c \sigma_{\mu\nu} \psi^d)\end{aligned}} \quad (1.27)$$

Introducing (1.24), (1.25), (1.26), (1.27) in (1.13) we have:

$$\boxed{\begin{aligned}(\bar{\psi}^a \sigma^{\mu\nu} \psi^d)(\bar{\psi}^c \sigma_{\mu\nu} \psi^b) &= \frac{1}{4}\Big[-2(\bar{\psi}^a \sigma^{\mu\nu} \psi^b)(\bar{\psi}^c \sigma_{\mu\nu} \psi^d) \\ &\quad + 6(\bar{\psi}^a \psi^b)(\bar{\psi}^c \psi^d) + 6(\bar{\psi}^a \gamma^5 \psi^b)(\bar{\psi}^c \gamma^5 \psi^d)\Big]\end{aligned}} \quad (1.28)$$

$\therefore$

$$\boxed{J_T' = \frac{1}{4}\left[6 J_S + 6 J_P - 2 J_T\right]} \quad (1.29)$$

Putting together: (1.14), (1.15), (1.20), (1.23), (1.29)
we have:

$$\boxed{\begin{pmatrix} J_S' \\ J_V' \\ J_T' \\ J_A' \\ J_P' \end{pmatrix} = \frac{1}{4} \begin{array}{c} \begin{array}{ccccc} S & V & T & A & P \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & -2 & 0 & 2 & -4 \\ 6 & 0 & -2 & 0 & 6 \\ 4 & 2 & 0 & -2 & -4 \\ 1 & -1 & 1 & -1 & 1 \end{pmatrix} \end{array} \begin{pmatrix} J_S \\ J_V \\ J_T \\ J_A \\ J_P \end{pmatrix}} \quad (1.30)$$

Then we can write: (Fierz Theorem)

$$\boxed{(\bar{\psi}^a \Gamma^i \psi^d)(\bar{\psi}^c \Gamma_i \psi^b) = \sum_{j=1}^{5} \lambda_{ij} (\bar{\psi}^a \Gamma^j \psi^b)(\bar{\psi}^c \Gamma_j \psi^d)} \quad (1.31)$$

(We are not summing over i)

λ_{ij} is the matrix given in (1.30)

Because the spins are arbitrary, one may replace $\psi^b \to \gamma^5 \psi^b$ and recover the same result for matrix elements of the form:

$$(\bar{\psi}^a \Gamma^j \psi^b)(\bar{\psi}^c \Gamma_j \gamma^5 \psi^d)$$

Let's evaluate:

$$(\bar{\psi}^a \gamma^\mu (1-\gamma^5) \psi^d)(\bar{\psi}^c \gamma_\mu (1-\gamma^5) \psi^b)$$

$$= (\bar{\psi}^a \gamma^\mu \psi^d)(\bar{\psi}^c \gamma_\mu \psi^b) - (\bar{\psi}^a \gamma^\mu \psi^d)(\bar{\psi}^c \gamma_\mu \gamma^5 \psi^b)$$

$$- (\bar{\psi}^a \gamma^\mu \gamma^5 \psi^d)(\bar{\psi}^c \gamma_\mu \psi^b) + (\bar{\psi}^a \gamma^\mu \gamma^5 \psi^d)(\bar{\psi}^c \gamma_\mu \gamma^5 \psi^b)$$

$$-(\bar\psi^a \gamma^\mu \gamma^5 \psi^d)(\bar\psi^c \gamma_\mu \psi^b) = -(\bar\psi^a \gamma^\mu \gamma^5 \psi^d)(\bar\psi^c \gamma_\mu \gamma^5 \gamma^5 \psi^b)$$

$$= (\bar\psi^a i \gamma^\mu \gamma^5 \psi^d)(\bar\psi^c i \gamma_\mu \gamma^5 \gamma^5 \psi^b)$$

$$= \frac{1}{4}\left(4 J_{S5} + 2 J_{V5} - 2 J_{A5} - 4 J_{P5}\right) \qquad (1.32)$$

$$-(\bar\psi^a \gamma^\mu \psi^d)(\bar\psi^c \gamma_\mu \gamma^5 \psi^b) = \frac{1}{4}\left(-4 J_{S5} + 2 J_{V5} - 2 J_{A5} + 4 J_{P5}\right) \qquad (1.33)$$

Where:

$$\boxed{\begin{aligned}
J_{S5} &\equiv (\bar\psi^a \psi^b)(\bar\psi^c \gamma^5 \psi^d) \\
J_{P5} &\equiv (\bar\psi^a \gamma^5 \psi^b)(\bar\psi^c \psi^d) \\
J_{V5} &\equiv (\bar\psi^a \gamma^\mu \psi^b)(\bar\psi^c \gamma_\mu \gamma^5 \psi^d) \\
J_{A5} &\equiv (\bar\psi^a i \gamma^\mu \gamma^5 \psi^b)(\bar\psi^c i \gamma_\mu \psi^d) \\
J_{T5} &\equiv (\bar\psi^a \sigma^{\mu\nu} \psi^b)(\bar\psi^c \sigma_{\mu\nu} \gamma^5 \psi^d)
\end{aligned}} \qquad (1.34)$$

$$\Rightarrow \quad (\bar\psi^a \gamma^\mu (1-\gamma^5) \psi^d)(\bar\psi^c \gamma_\mu (1-\gamma^5) \psi^b) =$$

$$= \frac{1}{4}\Big(\cancel{4 J_S} - 2 J_V + 2 J_A - \cancel{4 J_P} + 4 J_{V5} - 4 J_{A5}$$
$$\cancel{-4 J_S} - 2 J_V + 2 J_A + \cancel{4 J_P}\Big)$$

$$= -J_V + J_A + J_{V5} - J_{A5}$$

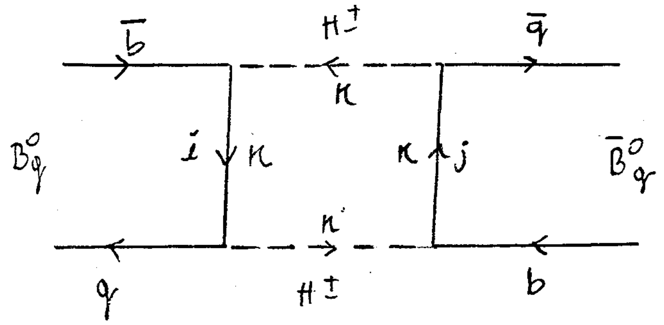
$$= -(\bar\psi^a \gamma^\mu \psi^b)(\bar\psi^c \gamma_\mu \psi^d) - (\bar\psi^a \gamma^\mu \gamma^5 \psi^b)(\bar\psi^c \gamma_\mu \gamma^5 \psi^d)$$
$$+ (\bar\psi^a \gamma^\mu \psi^b)(\bar\psi^c \gamma_\mu \gamma^5 \psi^d) + (\bar\psi^a \gamma^\mu \gamma^5 \psi^b)(\bar\psi^c \gamma_\mu \psi^d)$$

$$= -(\bar\psi^a \gamma^\mu (1-\gamma^5) \psi^b)(\bar\psi^c \gamma_\mu (1-\gamma^5) \psi^d)$$

$$\therefore \quad \boxed{\begin{aligned}
&[\bar\psi^a \gamma^\mu (1-\gamma^5) \psi^d][\bar\psi^c \gamma_\mu (1-\gamma^5) \psi^b] \\
&\quad = -[\bar\psi^a \gamma^\mu (1-\gamma^5) \psi^b][\bar\psi^c \gamma_\mu (1-\gamma^5) \psi^d]
\end{aligned}} \qquad (1.35)$$

$$= -[\bar\psi^c \gamma^\mu (1-\gamma^5) \psi^d][\bar\psi^a \gamma_\mu (1-\gamma^5) \psi^b]$$

The invariant amplitude for:



$q = d$ or s

is: (taking $m_q = 0$)

$$M_{a_1} = 2^2 i \left(\frac{g}{2\sqrt{2}\, M_W}\right)^4 \sum_{i,j} \xi_i \xi_j \Big\{ m_i^2 m_j^2 \cot^4\beta\, \bar{V}(\bar{q}) \gamma^\alpha (1-\gamma^5) U(b) . \bar{U}(q) \gamma_\alpha (1-\gamma^5) V(\bar{b})\, I_{\kappa\kappa}(i,j) + m_i^2 m_j^2 m_b^2\, \bar{V}(\bar{q})(1+\gamma^5) U(b) . \bar{U}(q)(1+\gamma^5) V(\bar{b}) . I''(i,j) \quad (1.36)$$

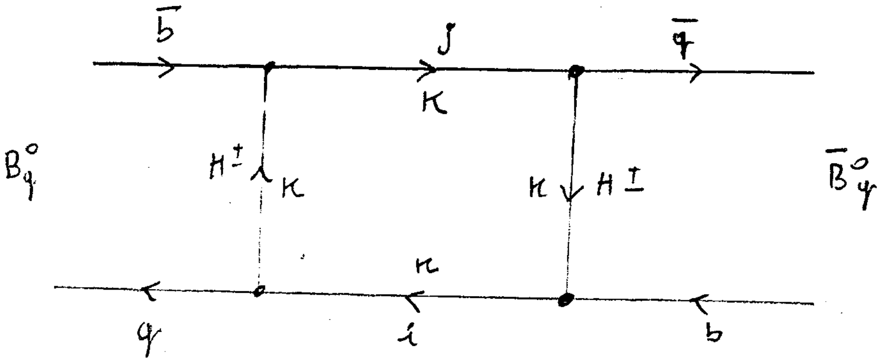
In the general expression (1.16)

we have amplitudes like

1. $\bar{V}(\bar{q})\, (1-\gamma^5) \gamma^\alpha\, U(b) . \bar{U}(q)\, (1-\gamma^5) \gamma_\alpha\, V(\bar{b})$
2. $\bar{V}(\bar{q})\, (1-\gamma^5) \gamma^\alpha\, U(b) . \bar{U}(q)\, (1+\gamma^5) \gamma_\alpha\, V(\bar{b})$
3. $\bar{V}(\bar{q})\, (1-\gamma^5)\, U(b) . \bar{U}(q)\, (1-\gamma^5) V(\bar{b})$
4. $\bar{V}(\bar{q})(1+\gamma^5) \gamma^\alpha\, U(b) . \bar{U}(q)\, (1-\gamma^5) \gamma_\alpha V(\bar{b})$
5. $\bar{V}(\bar{q})\, (1+\gamma^5) \gamma^\alpha\, U(b) . \bar{U}(q) (1+\gamma^5) \gamma_\alpha\, V(\bar{b})$
6. $\bar{V}(\bar{q}) (1+\gamma^5)\, U(b) . \bar{U}(q)\, (1-\gamma^5) V(\bar{b})$
7. $\bar{V}(\bar{q})\, (1+\gamma^5)\, U(b) . \bar{U}(q)\, (1+\gamma^5) V(\bar{b})$
8. $\bar{V}(\bar{q})\, (1-\gamma^5)\, U(b) . \bar{U}(q) (1+\gamma^5)\, V(\bar{b})$

$$I_{\alpha\beta}(i,j) = \eta_{\alpha\beta}\, I_{xx}(i,j)$$

The invariant amplitude for:



is:

$$-iM_{a_2} = \sum_{i,j} \xi_i \xi_j \int \frac{d^4k}{(2\pi)^4}\, \bar V(\bar q)\, \frac{ig}{2\sqrt{2} M_W}\left(m_q\, tg\beta\,(1-\gamma^5) + m_j \cot\beta\,(1+\gamma^5)\right)$$

$$\frac{i(\not k + m_j)}{k^2 - m_j^2}\, \frac{ig}{2\sqrt{2} M_W}\left(m_b\, tg\beta\,(1+\gamma^5) + m_j \cot\beta\,(1-\gamma^5)\right) V(\bar b).$$

$$\bar U(q)\, \frac{ig}{2\sqrt{2} M_W}\left(m_q\, tg\beta\,(1-\gamma^5) + m_i \cot\beta\,(1+\gamma^5)\right) \frac{i(\not k + m_i)}{k^2 - m_i^2}.$$

$$\frac{ig}{2\sqrt{2} M_W}\left(m_b\, tg\beta\,(1+\gamma^5) + m_i \cot\beta\,(1-\gamma^5)\right) U(b).$$

$$\cdot \frac{i^2}{(k^2 - M_{H^+}^2)^2} \qquad (1\text{-}37)$$

$$\Rightarrow M_{a_2} = 2^2 i \left(\frac{g}{2\sqrt{2} M_W}\right)^4 \sum_{i,j} \xi_i \xi_j \Big[m_q^2 m_b^2\, tg^4\beta\; \bar V(\bar q)(1-\gamma^5)\gamma^\alpha V(\bar b).$$

$$\cdot \bar U(q)(1-\gamma^5)\gamma^\beta U(b)\, I_{\alpha\beta}(i,j) + m_i^2 m_q m_b\, \bar V(\bar q)(1-\gamma^5)\gamma^\alpha V(\bar b)$$

$$\bar U(q)(1+\gamma^5)\gamma^\beta U(b)\, I_{\alpha\beta}(i,j) + m_i^2 m_j^2 m_q^2\, \bar V(\bar q)(1-\gamma^5) V(\bar b).$$

$$\bar U(q)(1-\gamma^5) U(b)\, I''(i,j) + m_i^2 m_j^2 m_q m_b\, \bar V(\bar q)(1-\gamma^5) V(\bar b). \bar U(q)(1+\gamma^5).$$

$$U(b)\, I''(i,j) + m_j^2 m_q m_b\, \bar V(\bar q)(1+\gamma^5)\gamma^\alpha V(\bar b). \bar U(q)(1-\gamma^5)\gamma^\beta U(b)\, I_{\alpha\beta}(i,j)$$

$$+ m_i^2 m_j^2 \cot^4\beta\; \bar V(\bar q)(1+\gamma^5)\gamma^\alpha V(\bar b). \bar U(q)(1+\gamma^5)\gamma^\beta U(b)\, I_{\alpha\beta}(i,j)$$

$+ m_i^2 m_j^2 m_q m_b \bar{V}(\bar{q})(1+\gamma^5) V(\bar{b}) . \bar{U}(q)(1-\gamma^5) U(b) I''(i,j)$

$+ m_i^2 m_j^2 m_b^2 \bar{V}(\bar{q})(1+\gamma^5) V(\bar{b}) . \bar{U}(q)(1+\gamma^5) U(b) I''(i,j) \Big]$

(1.38)

We have amplitudes like :

① $\bar{V}(\bar{q})(1-\gamma^5)\gamma^\alpha V(\bar{b}) . \bar{U}(q)(1-\gamma^5)\gamma_\alpha U(b)$

② $\bar{V}(\bar{q})(1-\gamma^5)\gamma^\alpha V(\bar{b}) . \bar{U}(q)(1+\gamma^5)\gamma_\alpha U(b)$

③ $\bar{V}(\bar{q})(1-\gamma^5) V(\bar{b}) . \bar{U}(q)(1-\gamma^5) U(b)$

④ $\bar{V}(\bar{q})(1-\gamma^5) V(\bar{b}) . \bar{U}(q)(1+\gamma^5) U(b)$

⑤ $\bar{V}(\bar{q})(1+\gamma^5)\gamma^\alpha V(\bar{b}) . \bar{U}(q)(1-\gamma^5)\gamma_\alpha U(b)$

⑥ $\bar{V}(\bar{q})(1+\gamma^5)\gamma^\alpha V(\bar{b}) . \bar{U}(q)(1+\gamma^5)\gamma_\alpha U(b)$

⑦ $\bar{V}(\bar{q})(1+\gamma^5) V(\bar{b}) . \bar{U}(q)(1-\gamma^5) U(b)$

⑧ $\bar{V}(\bar{q})(1+\gamma^5) V(\bar{b}) . \bar{U}(q)(1+\gamma^5) U(b)$

in the limit $m_q \to 0$

$$\boxed{\begin{aligned} M_{d_2} = 2^2 i \left(\frac{g}{2\sqrt{2}\, M_W}\right)^4 \sum_{i,j} \xi_i \xi_j \Big\{ & m_i^2 m_j^2 \cot^4\beta\, \bar{V}(\bar{q}) \gamma^\alpha (1-\gamma^5) V(\bar{b}) . \\ & . \bar{U}(q) \gamma_\alpha (1-\gamma^5) U(b)\, I_{xx}(i,j) \\ & + m_i^2 m_j^2 m_b^2 \bar{V}(\bar{q})(1+\gamma^5) V(\bar{b}) . \bar{U}(q)(1+\gamma^5) U(b)\, I''(i,j) \Big\} \end{aligned}}$$

(1.39)

Using the Fierz Theorem (1.35) we have :

$$\boxed{\begin{aligned} & \bar{V}(\bar{q}) \gamma^\alpha (1-\gamma^5) V(\bar{b}) . \bar{U}(q) \gamma_\alpha (1-\gamma^5) U(b) \\ & = - \bar{V}(\bar{q}) \gamma^\alpha (1-\gamma^5) U(b) . \bar{U}(q) \gamma_\alpha (1-\gamma^5) V(\bar{b}) \end{aligned}} \quad (1.40)$$

The second term in (1.36) and (1.39) is negligible.

$$\Rightarrow M_{a_1} = 2^2 i \left(\frac{g}{2\sqrt{2}M_W}\right)^4 \sum_{i,j} \xi_i \xi_j \left\{ m_i^2 m_j^2 \cot^4\beta \cdot \right.$$

$$\left. \cdot \bar{V}(\bar{q}) \gamma^\alpha (1-\gamma^5) U(b) \cdot \bar{U}(q) \gamma_\alpha (1-\gamma^5) V(\bar{b}) I_{xx}(i,j) \right\} \quad (1.41)$$

$$M_{a_2} = 2^2 i \left(\frac{g}{2\sqrt{2}M_W}\right)^4 \sum_{i,j} \xi_i \xi_j \left\{ m_i^2 m_j^2 \cot^4\beta \cdot \right.$$

$$\left. \cdot \bar{V}(\bar{q}) \gamma^\alpha (1-\gamma^5) V(\bar{b}) \cdot \bar{U}(q) \gamma_\alpha (1-\gamma^5) U(b) I_{xx}(i,j) \right\} \quad (1.42)$$

The total amplitude is then

$$M_a = M_{a_1} - M_{a_2}$$

And because of (1.40)

$$\boxed{M_a = 2 M_{a_1}} \quad (1.43)$$

The invariant amplitude corresponding to diagram ② in b) is:

$$-iM_b^{(2)} = \sum_{i,j} \xi_i \xi_j \int \frac{d^4k}{(2\pi)^4}\, \bar{V}(\bar{q}) \left[-\frac{ig}{\sqrt{2}}\gamma^\mu \frac{1}{2}(1-\gamma^5)\right] \frac{i(\not{k}+m_j)}{(k^2-m_j^2)}$$

$$\frac{ig}{2\sqrt{2}M_W}\left[m_b \tan\beta(1+\gamma^5) + m_j \cot\beta(1-\gamma^5)\right] V(\bar{b}) \cdot \bar{U}(q) \cdot$$

$$\cdot \frac{ig}{2\sqrt{2}M_W}\left[m_q \tan\beta(1-\gamma^5) + m_i \cot\beta(1+\gamma^5)\right] \frac{i(\not{k}+m_i)}{(k^2-m_i^2)} \cdot$$

$$\left[-\frac{ig}{\sqrt{2}}\gamma^\nu \frac{1}{2}(1-\gamma^5)\right] U(b)\; (-i)\frac{\left[\eta_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}\right]}{(k^2-M_W^2)} \frac{i}{(k^2-M_{H^\pm}^2)}$$

In the limit $m_q \to 0$

$$M_b^{(2)} = -4i\left(\frac{g}{2\sqrt{2}}\right)^4 \frac{\cot^2\beta}{M_W^2} \sum_{i,j} \xi_i \xi_j m_i^2 m_j^2\, \bar{V}(\bar{q})\gamma^\mu(1-\gamma^5)V(\bar{b}) \cdot$$

$$\cdot \bar{U}(q)\gamma_\mu(1-\gamma^5)U(b)\left[I^{HW}(i,j) - \frac{1}{M_W^2} I^{HW}_{xx}(i,j)\right]$$

$$M_b^{(1)} = -4i\left(\frac{g}{2\sqrt{2}}\right)^4 \frac{\cot^2\beta}{M_W^2} \sum_{i,j} \xi_i \xi_j m_i^2 m_j^2\, \bar{V}(\bar{q})\gamma^\mu(1-\gamma^5)U(b) \cdot$$

$$\cdot \bar{U}(q)\gamma_\mu(1-\gamma^5)V(\bar{b})\left[I^{HW}(i,j) - \frac{1}{M_W^2} I^{HW}_{xx}(i,j)\right]$$

Using the Fierz Theorem:

$$\bar{V}(\bar{q})\gamma^\mu(1-\gamma^5)V(\bar{b}) \cdot \bar{U}(q)\gamma_\mu(1-\gamma^5)U(b)$$

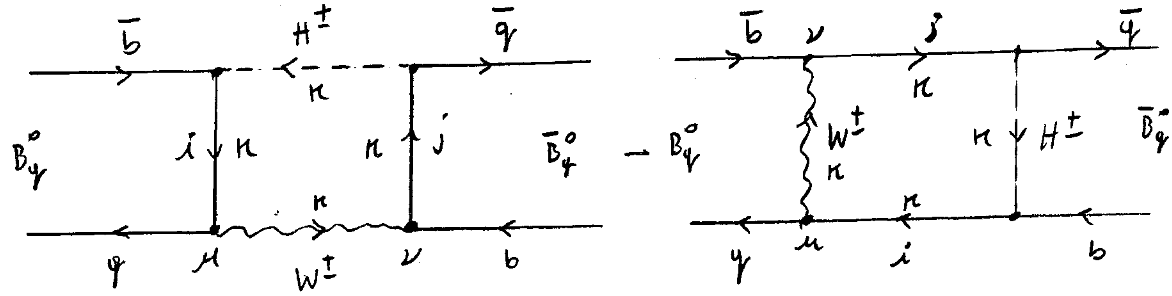
$$= -\bar{V}(\bar{q})\gamma^\mu(1-\gamma^5)U(b) \cdot \bar{U}(q)\gamma_\mu(1-\gamma^5)V(\bar{b})$$

$$\Rightarrow M_b^{(2)} = -M_b^{(1)}$$

but $\boxed{M_b = M_b^{(1)} - M_b^{(2)} = 2M_b^{(1)}}$

The crossed diagrams are:

b) ③ ④



$$-iM_b^{(3)} = \sum_{i,j} \xi_i \xi_j \int \frac{d^4k}{(2\pi)^4} \bar{V}(\bar{q}) \left(\frac{ig}{2\sqrt{2}M_W}\right)\left[m_q \, tg\beta\,(1-\gamma^5) + m_j \cot\beta\,(1+\gamma^5)\right]$$

$$\frac{i(\not{k}+m_j)}{(k^2-m_j^2)} \left[-\frac{ig}{\sqrt{2}}\gamma^\nu \frac{1}{2}(1-\gamma^5)\right] U(b) \, . \, \bar{U}(q) \left[-\frac{ig}{\sqrt{2}}\gamma^\mu \frac{1}{2}(1-\gamma^5)\right]$$

$$i\frac{(\not{k}+m_i)}{(k^2-m_i^2)} \left(\frac{ig}{2\sqrt{2}M_W}\right)\left[m_b \, tg\beta\,(1+\gamma^5) + m_i \cot\beta\,(1-\gamma^5)\right] V(\bar{b})$$

$$(-i)\left[\eta_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}\right] \cdot \frac{1}{(k^2-M_W^2)} \cdot \frac{i}{(k^2-M_{H^\pm}^2)}$$

$i \leftrightarrow j$:

$$-iM_b^{(3)} = \sum_{i,j} \xi_i \xi_j \int \frac{d^4k}{(2\pi)^4} \bar{U}(q) \left[-\frac{ig}{\sqrt{2}}\gamma^\mu \frac{1}{2}(1-\gamma^5)\right] \frac{i(\not{k}+m_j)}{(k^2-m_j^2)}$$

$$\left(\frac{ig}{2\sqrt{2}M_W}\right)\left[m_b \, tg\beta\,(1+\gamma^5) + m_j \cot\beta\,(1-\gamma^5)\right] V(\bar{b}) \, . \, \bar{V}(\bar{q}) \, .$$

$$\left(\frac{ig}{2\sqrt{2}M_W}\right)\left[m_q \, tg\beta\,(1-\gamma^5) + m_i \cot\beta\,(1+\gamma^5)\right] i\frac{(\not{k}+m_i)}{(k^2-m_i^2)} \, .$$

$$\cdot \left[-\frac{ig}{\sqrt{2}}\gamma^\nu \frac{1}{2}(1-\gamma^5)\right] U(b) \, . \, (-i)\left[\eta_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}\right] \cdot \frac{1}{(k^2-M_W^2)} \cdot \frac{i}{(k^2-M_{H^\pm}^2)}$$

In the limit $m_q \to 0$ (see page 57)

$$M_b^{(3)} = -4i\left(\frac{g}{2\sqrt{2}}\right)^4 \frac{\cot^2\beta}{M_W^2} \sum_{i,j} \xi_i \xi_j \, m_i^2 m_j^2 \, \bar{U}(q)\gamma^\mu(1-\gamma^5)V(\bar{b}) \, .$$

$$\bar{V}(\bar{q})\gamma_\mu(1-\gamma^5)U(b)\left[I^{HW}(i,j) - \frac{1}{M_W^2} I^{HW}_{xx}(i,j)\right]$$

$$\bar{U}(q)\,\gamma^{\mu}(1-\gamma^5)V(\bar{b})\,.\,\bar{V}(\bar{q})\gamma_{\mu}(1-\gamma^5)U(b)$$

$$= \bar{V}(\bar{q})\,\gamma^{\mu}(1-\gamma^5)U(b)\,.\,\bar{U}(q)\,\gamma_{\mu}(1-\gamma^5)V(\bar{b})$$

$$\Rightarrow \boxed{M_b^{(3)} = M_b^{(1)}}\,.$$

$$-iM_b^{(4)} = \sum_{i,j} \xi_i \xi_j \int \frac{d^4k}{(2\pi)^4} \bar{V}(\bar{q}) \left(\frac{ig}{2\sqrt{2}M_W}\right)\left[m_q \, tg\beta(1-\gamma^5) + m_j \cot\beta(1+\gamma^5)\right]$$

$$\frac{i(\not k + m_j)}{(k^2-m_j^2)} \left[-\frac{ig}{\sqrt{2}}\gamma^{\nu}\frac{1}{2}(1-\gamma^5)\right]V(\bar{b})\,.\,\bar{U}(q)\left[-\frac{ig}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right].$$

$$i\frac{(\not k + m_i)}{(k^2-m_i^2)}\left(\frac{ig}{2\sqrt{2}M_W}\right)\left[m_b \, tg\beta(1+\gamma^5) + m_i \cot\beta(1-\gamma^5)\right]U(b)$$

$$(-i)\left[\eta_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}\right]\cdot\frac{1}{(k^2-M_W^2)}\cdot\frac{i}{(k^2-M_{H^\pm}^2)}$$

$$i \leftrightarrow j:$$

$$= \sum_{i,j} \xi_i \xi_j \int \frac{d^4k}{(2\pi)^4} \bar{U}(q)\left[-\frac{ig}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right] i\frac{(\not k + m_j)}{(k^2-m_j^2)}\,.$$

$$\cdot\left(\frac{ig}{2\sqrt{2}M_W}\right)\left[m_b \, Tg\beta(1+\gamma^5) + m_j \cot\beta(1-\gamma^5)\right]U(b)\,.\,\bar{V}(\bar{q})\,.$$

$$\left(\frac{ig}{2\sqrt{2}M_W}\right)\left[m_q \, tg\beta(1-\gamma^5) + m_i \cot\beta(1+\gamma^5)\right] \frac{i(\not k + m_i)}{(k^2-m_i^2)}\,.$$

$$\left[-\frac{ig}{\sqrt{2}}\gamma^{\nu}\frac{1}{2}(1-\gamma^5)\right]V(\bar{b})\;(-i)\left[\eta_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}\right]\cdot\frac{1}{(k^2-M_W^2)}\cdot\frac{i}{(k^2-M_{H^\pm}^2)}$$

In the limit $m_q \to 0$ (see p. 57)

$$M_b^{(4)} = -4i\left(\frac{g}{2\sqrt{2}}\right)^4 \frac{\cot^2\beta}{M_W^2}\sum_{i,j}\xi_i\xi_j\, m_i^2 m_j^2\, \bar{U}(q)\gamma^{\mu}(1-\gamma^5)U(b)\,.$$

$$\cdot\,\bar{V}(q)\gamma_{\mu}(1-\gamma^5)V(\bar{b})\left[I^{HW}(i,j) - \frac{1}{M_W^2}I^{HW}_{xx}(i,j)\right]$$

$$\bar{U}(q)\,\gamma^{\mu}(1-\gamma^5)U(b)\,.\,\bar{V}(\bar{q})\,\gamma_{\mu}(1-\gamma^5)V(\bar{b})$$

$$= \bar{V}(\bar{q})\gamma^{\mu}(1-\gamma^5)V(\bar{b})\,.\,\bar{U}(q)\gamma_{\mu}(1-\gamma^5)U(b)$$

Using the Fierz Theorem:

$$= -\bar{V}(\bar{q})\,\gamma^{\mu}(1-\gamma^5)\,U(b)\,.\,\bar{U}(q)\gamma_{\mu}(1-\gamma^5)V(\bar{b})$$

$$\therefore \quad M_b^{(4)} = -M_b^{(1)}$$

$$M_b^{Total} = M_b^{(1)} - M_b^{(2)} + M_b^{(3)} - M_b^{(4)}$$

$$M_b^{Total} = 2M_b^{(1)} + 2M_b^{(1)} = 4M_b^{(1)}$$

$$\boxed{M_b^{Total} = 4\,M_b^{(1)}}$$

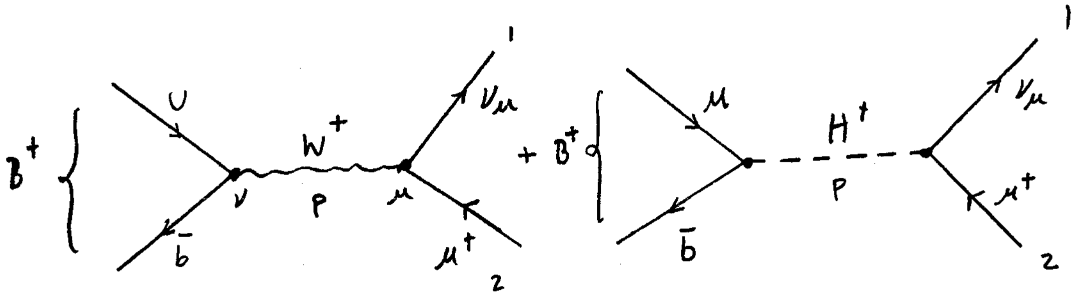
Then in this limit, all four diagrams contribute equally. //

Charged Higgs contribution to meson decay

①

$B^+ \to \mu^+ \nu_\mu$ Charged Higgs contribution

$\longrightarrow$ time



If $p^2 << M_W^2, M_{H^\pm}^2$

$$-iM = \bar U_1 \left(-\frac{ig}{\sqrt 2}\gamma^\mu \frac{1}{2}(1-\gamma^5)\right) V_2 \cdot \left(\frac{-i\eta_{\mu\nu}}{-M_W^2}\right) \bar V(\bar b)\left(-\frac{ig}{\sqrt 2}\gamma^\nu \frac{1}{2}(1-\gamma^5)\right) U(u) V_{ub}^*$$

$$+ \bar U_1 \left(\frac{ig}{2\sqrt 2 M_W} m_\mu \tan\beta\,(1+\gamma^5)\right) V_2 \cdot \frac{i}{-M_{H^+}^2} \bar V(\bar b)\left(\frac{ig}{2\sqrt 2 M_W}\left[m_b \tan\beta(1-\gamma^5) + m_u \cot\beta\,(1+\gamma^5)\right]\right) U(u) V_{ub}^*$$

$$\frac{G_F}{\sqrt 2} = \frac{g^2}{8 M_W^2}$$

$$-iM = -i\frac{G_F}{\sqrt 2} \bar U_1 \gamma^\mu (1-\gamma^5) V_2 \cdot \bar V(\bar b)\gamma_\mu (1-\gamma^5) U(u) V_{ub}^*$$

$$+ i\frac{G_F}{\sqrt 2} \frac{m_\mu \tan\beta}{M_{H^+}^2} \bar U_1 (1+\gamma^5) V_2 \cdot \bar V(\bar b)\left[m_b \tan\beta\,(1-\gamma^5) + m_u \cot\beta\,(1+\gamma^5)\right] \cdot U(u) V_{ub}^*$$

In the limit $m_u \to 0$

$$M = \frac{G_F}{\sqrt 2}\Big[\bar U_1 \gamma^\mu (1-\gamma^5) V_2 \cdot \overbrace{\bar V(\bar b)\gamma_\mu (1-\gamma^5) U(u)}^{p_\mu f_B}$$

$$- \frac{m_\mu m_b \tan^2\beta}{M_H^2} \bar U_1 (1+\gamma^5) V_2 \cdot \overbrace{\bar V(\bar b)(1-\gamma^5) U(u)}^{-g_B^{\prime 2}}\Big] V_{ub}^*$$

2

$$\not p_1 U_1 = 0 \quad \Rightarrow \quad \bar{U}_1 \not p_1 = 0$$

$$(\not p_2 + m_\mu) V_2 = 0 \quad \Rightarrow \quad \not p_2 V_2 = - m_\mu V_2$$

$$M = \frac{G_F}{\sqrt{2}} [\bar{U}_1 \underbrace{\gamma^\mu (1-\gamma^5) V_2 P_\mu}_{\not P = \not p_1 + \not p_2} f_B + \frac{m_\mu m_b}{M_H^2} \tan^2\beta \, \bar{U}_1 (1+\gamma^5) V_2 \, g_B'^2] V_{ub}^*$$

$$M = \frac{G_F}{\sqrt{2}} [\bar{U}_1 (\not p_1 + \not p_2)(1-\gamma^5) V_2 f_B + \frac{m_\mu m_b}{M_H^2} \tan^2\beta \, \bar{U}_1 (1+\gamma^5) V_2 \, g_B'^2] V_{ub}^*$$

$$M = \frac{G_F}{\sqrt{2}} [\bar{U}_1 \not p_2 (1-\gamma^5) V_2 f_B + \frac{m_\mu m_b}{M_H^2} \tan^2\beta \, \bar{U}_1 (1+\gamma^5) V_2 \, g_B'^2] V_{ub}^*$$

$$(\gamma^\mu \gamma^5 + \gamma^5 \gamma^\mu = 0)$$

$$= \frac{G_F}{\sqrt{2}} [-\bar{U}_1 (1+\gamma^5) m_\mu V_2 f_B + \frac{m_\mu m_b}{M_H^2} \tan^2\beta \, \bar{U}_1 (1+\gamma^5) V_2 \, g_B'^2] V_{ub}^*$$

$$M = -\frac{G_F}{\sqrt{2}} m_\mu [\bar{U}_1 (1+\gamma^5) V_2 f_B - \frac{m_b}{M_H^2} \tan^2\beta \, \bar{U}_1 (1+\gamma^5) V_2 \, g_B'^2] V_{ub}^*$$

$$M = -\frac{G_F}{\sqrt{2}} m_\mu \bar{U}_1 (1+\gamma^5) V_2 [f_B - \frac{m_b}{M_H^2} \tan^2\beta \, g_B'^2] V_{ub}^*$$

$$\overline{|M|^2} = \frac{G_F^2}{2} m_\mu^2 [f_B - \frac{m_b}{M_H^2} \tan^2\beta \, g_B'^2]^2 |V_{ub}|^2 \sum_{\text{Spins}} [\bar{U}_1 (1+\gamma^5) V_2][V_2^+ (1+\gamma^5) \gamma^0 U_1]$$

$$= \frac{G_F^2}{2} m_\mu^2 [f_B - \frac{m_b}{M_H^2} \tan^2\beta \, g_B'^2]^2 |V_{ub}|^2 \sum_{\text{Spins}} [\bar{U}_1 (1+\gamma^5) V_2][\bar{V}_2 (1-\gamma^5) U_1]$$

$$= \frac{G_F^2}{2} m_\mu^2 [f_B - \frac{m_b}{M_H^2} \tan^2\beta \, g_B'^2]^2 |V_{ub}|^2 \, Tr [(\not p_2 - m_\mu)(1-\gamma^5)(\not p_1)(1+\gamma^5)]$$

$$Tr [(\not p_2 - m_\mu)(1-\gamma^5)(\not p_1)(1+\gamma^5)] = Tr [(\not p_2 - m_\mu)(1-\gamma^5)^2 \not p_1]$$

$$= 2 \, Tr [(\not p_2 - m_\mu)(1-\gamma^5) \not p_1]$$

because : $\gamma^\mu \gamma^5 + \gamma^5 \gamma^\mu = 0$

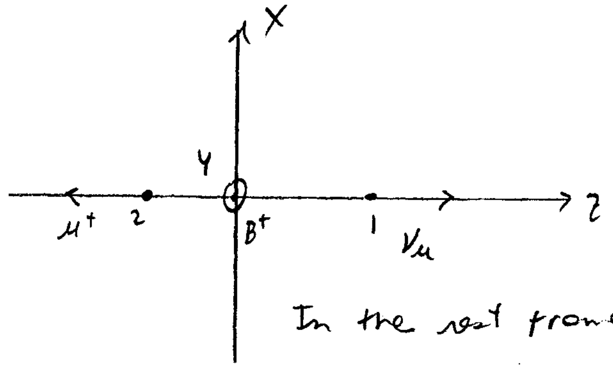
$(1-\gamma^5)^2 = 2(1-\gamma^5)$

$$\Rightarrow \quad Tr = 2\left[Tr(\not{p}_2 \not{p}_1) - \underbrace{Tr(\not{p}_2 \gamma^5 \not{p}_1)}_{Tr(\gamma^5 \not{p}_2 \not{p}_1) = 0} - m_\mu \cancel{Tr(\not{p}_1)}^{\,0} + m_\mu \cancel{Tr(\gamma^5 \not{p}_1)}^{\,0} \right]$$

$$Tr = 2\,Tr(\not{p}_1 \not{p}_2) = 8(P_1 \cdot P_2)$$

(Trace of an odd of $\gamma_5 = 0$)

$$\therefore \quad \boxed{\overline{|M|}^2 = 4 G_F^2 m_\mu^2 \left[f_B - \frac{m_b}{M_H^2} \tan^2\beta \, g_B^2 \right]^2 (P_1 \cdot P_2)\, |V_{ub}|^2}$$



In the rest frame of B^+

$$P_{B^+} = (m_{B^+}, 0, 0, 0)$$

$$P_1 = (E_1, \vec{P}_1) \; ; \; P_2 = (E_2, -\vec{P}_1)$$

$$P_{B^+} = P_1 + P_2 \quad \Rightarrow \quad P_{B^+}^2 = (P_1 + P_2)^2$$

$$\therefore \quad m_B^2 = m_\mu^2 + 2(P_1 \cdot P_2)$$

$$\boxed{(P_1 \cdot P_2) = \frac{m_{B^+}^2 - m_\mu^2}{2}}$$

$$P_1^2 = (P_{B^+} - P_2)^2$$

$$\Rightarrow \quad 0 = m_{B^+}^2 + m_\mu^2 - 2 P_{B^+} \cdot P_2$$

$$0 = m_{B^+}^2 + m_\mu^2 - 2 m_{B^+} E_2$$

$$E_2 = \frac{m_{B^+}^2 + m_\mu^2}{2 m_{B^+}}$$

$$E_2^2 = m_\mu^2 + |\vec{P}_1|^2$$

$$|\vec{P}_1|^2 = \left(\frac{m_{B^+}^2 + m_\mu^2}{2 m_{B^+}} \right)^2 - m_\mu^2 = \frac{m_{B^+}^4 + m_\mu^4 - 2 m_{B^+}^2 m_\mu^2}{4 m_{B^+}^2}$$

Then:

$$|\vec{P}_1|^2 = \frac{(m_{B^+}^2 - m_\mu^2)^2}{4 m_{B^+}^2} \Rightarrow |\vec{P}_1| = \frac{m_{B^+}^2 - m_\mu^2}{2 m_{B^+}}$$

$$d\Gamma = \frac{\overline{|M|^2}\, |\vec{P}_1|\, d\Omega}{32\pi^2 m_{B^+}^2}$$

$$d\Gamma = \cancel{4} G_F^2 m_\mu^2 \left[f_B - \frac{m_b}{M_H^2} \tan^2\beta\, g_B'^2 \right]^2 \frac{(m_{B^+}^2 - m_\mu^2)}{\cancel{2}\cdot 2 m_{B^+}} |V_{ub}|^2 \frac{2\pi\, d\cos\theta}{32\pi^2 m_{B^+}^2}$$

Performing the integral ($\int d\Omega = 4\pi$)

$$\boxed{\Gamma = \frac{1}{8\pi} |V_{ub}|^2 G_F^2 m_{B^+} m_\mu^2 \left[f_B - \frac{m_b}{M_H^2} \tan^2\beta\, g_B'^2 \right]^2 \left(1 - \frac{m_\mu^2}{m_{B^+}^2} \right)^2}$$

taking $m_u = 3$ MeV ; $m_b = 4.3$ GeV

at rest: $$f_B \propto \frac{2\sqrt{2}\,(m_b m_u)^{1/2}}{m_B}$$

$$g_B \propto 2\sqrt{2}\,(m_b m_u)^{1/2} \frac{m_b}{m_{B^+}^2}$$

$$\frac{g_B}{f_B} = \left(\frac{m_{B^+}}{m_b} \right)^{-1}$$

$$\bar{v}(\bar{b})(1-\gamma^5)\,u(u) \rightarrow -\frac{m_{B^+}^2 g_B}{m_b} \Rightarrow g_B'^2 = \frac{m_{B^+}^2 g_B}{m_b}$$

$$\Rightarrow \boxed{\Gamma = \frac{1}{8\pi} |V_{ub}|^2 G_F^2 m_{B^+} m_\mu^2 \left[f_B - \frac{m_{B^+}^2}{M_H^2} \tan^2\beta\, g_B \right]^2 \left(1 - \frac{m_\mu^2}{m_{B^+}^2} \right)^2}$$

OK

From * and ** $f_B \simeq g_B$ (because $m_b \simeq m_{B^+}$)

$B^+ \to \mu^+ \nu_\mu$

$$M = \frac{G_F}{\sqrt{2}} \Big[\bar{U}_1 \gamma^\mu (1-\gamma^5) V_2 \,.\, \overbrace{\bar{V}(\bar{b})\, \gamma_\mu (1-\gamma^5)\, U(u)}^{P_\mu f_B}$$

$$- \frac{m_\mu m_b}{M_{H^+}^2} \tan^2\beta\, \bar{U}_1 (1+\gamma^5) V_2 \,.\, \overbrace{\bar{V}(\bar{b}) (1-\gamma^5) U(u)}^{-\frac{m_{B^+}^2 g_B}{m_b}}$$

$$- \frac{m_\mu m_u}{M_{H^+}^2} \bar{U}_1 (1+\gamma^5) V_2 \,.\, \underbrace{\bar{V}(\bar{b}) (1+\gamma^5) U(u)}_{+\frac{m_{B^+}^2 h_B}{m_u}} \Big] V_{ub}^*$$

$$\gamma^\mu P_\mu = \not{P} = \not{p}_1 + \not{p}_2$$

$$\bar{U}_1 \not{p}_1 = 0$$

$$(\not{p}_2 + m_\mu) V_2 = 0 \quad \Rightarrow \quad \not{p}_2 V_2 = -m_\mu V_2$$

$$M = \frac{G_F}{\sqrt{2}} \Big[\bar{U}_1 \not{p}_2 (1-\gamma^5) V_2 \,.\, f_B + m_\mu \frac{m_{B^+}^2}{M_{H^+}^2} g_B \tan^2\beta\, \bar{U}_1 (1+\gamma^5) V_2$$

$$- \frac{m_\mu m_{B^+}^2}{M_H^2} h_B \bar{U}_1 (1+\gamma^5) V_2 \Big] V_{ub}^*$$

$$\bar{U}_1 \not{p}_2 (1-\gamma^5) V_2 = \bar{U}_1 (1+\gamma^5)(-m_\mu V_2) = -m_\mu \bar{U}_1 (1+\gamma^5) V_2$$

$$M = -\frac{G_F}{\sqrt{2}} m_\mu \bar{U}_1 (1+\gamma^5) V_2 \Big[f_B - g_B \frac{m_{B^+}^2}{M_{H^+}^2} \tan^2\beta + h_B \frac{m_{B^+}^2}{M_{H}^2} \Big] V_{ub}^*$$

$$\Rightarrow \overline{|M|^2} = 4 G_F^2 m_\mu^2 \Big[f_B - g_B \frac{m_{B^+}^2}{M_{H^+}^2} \tan^2\beta + h_B \frac{m_{B^+}^2}{M_H^2} \Big]^2 |V_{ub}|^2 (P_1 . P_2)$$

$$\Rightarrow \Gamma = \frac{1}{8\pi} |V_{ub}|^2 G_F^2 m_{B^+} m_\mu^2 \left[f_B - g_B \frac{m_{B^+}^2}{M_{H^+}^2} \tan^2\beta + h_B \frac{m_{B^+}^2}{M_{H^2}} \right]^2 \cdot$$

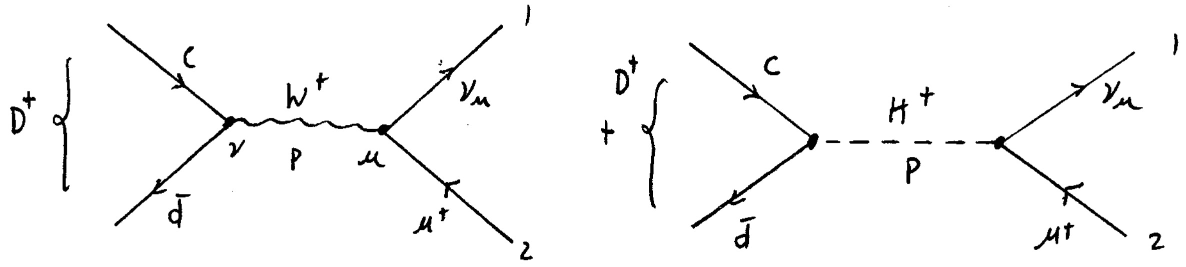
$$\cdot \left(1 - \frac{m_\mu^2}{m_{B^+}^2} \right)^2$$

with $\boxed{\begin{aligned} h_B &= g_B \frac{m_u}{m_b} \\ f_B &= g_B \frac{m_{B^+}}{m_b} \end{aligned}}$ $\rightarrow 0 \quad (m_u \rightarrow 0)$

$D^+ \to \mu^+ \nu_\mu$

$c\bar{d}$

time $\longrightarrow$



$$p^2 \ll M_W^2, M_{H^\pm}^2$$

$$-iM = \bar{U}_1 \left(-\frac{ig}{\sqrt{2}} \gamma^\mu \frac{1}{2}(1-\gamma^5) \right) V_2 \left(\frac{-i\eta_{\mu\nu}}{-M_W^2} \right) \bar{V}(\bar{d}) \left(\frac{-ig}{\sqrt{2}} \gamma^\nu \frac{1}{2}(1-\gamma^5) \right) U(c)\, V_{cd}^*$$

$$+ \bar{U}_1 \left(\frac{ig}{2\sqrt{2} M_W} m_\mu \tan\beta (1+\gamma^5) \right) V_2 \left(\frac{i}{-M_{H^2}} \right) \cdot \bar{V}(\bar{d}) \cdot$$

$$\cdot \left[\frac{ig}{2\sqrt{2} M_W} \left[m_d \tan\beta (1-\gamma^5) + m_c \cot\beta (1+\gamma^5) \right] U(c)\, V_{cd}^* \right.$$

$$-iM = -i \frac{G_F}{\sqrt{2}} \bar{U}_1 \gamma^\mu (1-\gamma^5) V_2\ \bar{V}(\bar{d}) \gamma_\mu (1-\gamma^5) U(c)\ V_{cd}^*$$

$$+ i \frac{G_F}{\sqrt{2}} \frac{m_\mu m_d}{M_H^2} \tan^2\beta\ \bar{U}_1 (1+\gamma^5) V_2 \cdot \bar{V}(\bar{d}) (1-\gamma^5) U(c)\ V_{cd}^*$$

$$+ i \frac{G_F}{\sqrt{2}} \frac{m_\mu m_c}{M_H^2} \bar{U}_1 (1+\gamma^5) V_2 \cdot \bar{V}(\bar{d}) (1+\gamma^5) U(c)\ V_{cd}^*$$

$$M = \frac{G_F}{\sqrt{2}} \Big[\bar{U}_1 \gamma^\mu (1-\gamma^5) V_2 \cdot \bar{V}(\bar{d}) \gamma_\mu (1-\gamma^5) U(c)$$

$$- \frac{m_\mu m_d}{M_H^2} \tan^2\beta\ \bar{U}_1 (1+\gamma^5) V_2 \cdot \bar{V}(\bar{d}) (1-\gamma^5) U(c)$$

$$- \frac{m_\mu m_c}{M_H^2} \bar{U}_1 (1+\gamma^5) V_2 \cdot \bar{V}(\bar{d}) (1+\gamma^5) U(c) \Big] V_{cd}^*$$

Let be $\bar{v}(\bar{d})\gamma_\mu(1-\gamma^5)u(c) = P_\mu f_{D^+}$

$$\bar{v}(\bar{d})(1-\gamma^5)u(c) = -\frac{m_{D^+}^2}{m_d} g_{D^+}$$

$$\bar{v}(\bar{d})(1+\gamma^5)u(c) = \frac{m_{D^+}^2 h_{D^+}}{m_c}$$

$$\Rightarrow M = \frac{G_F}{\sqrt{2}}\Big[\bar{U}_1\gamma^\mu(1-\gamma^5)V_2 P_\mu f_{D^+} + m_\mu \frac{m_{D^+}^2}{M_{H^+}^2}\tan^2\beta\, g_{D^+}\bar{U}_1(1+\gamma^5)V_2$$
$$- m_\mu \frac{m_{D^+}^2}{M_{H^+}^2} h_{D^+}\bar{U}_1(1+\gamma^5)V_2\Big] V_{cd}^*$$

$$\bar{U}_1\gamma^\mu(1-\gamma^5)V_2 P_\mu = \bar{U}_1(\not{P}_1+\not{P}_2)(1-\gamma^5)V_2 = -\bar{U}_1(1+\gamma^5)V_2 m_\mu$$

$$\bar{U}_1\not{P}_1 = 0$$

$$(\not{P}_2 + m_\mu)V_2 = 0$$

$$\not{P}_2 V_2 = -m_\mu V_2$$

$$\Rightarrow M = -\frac{G_F}{\sqrt{2}} m_\mu \bar{U}_1(1+\gamma^5)V_2\Big[f_{D^+} - \frac{m_{D^+}^2}{M_H^2}\tan^2\beta\, g_{D^+}$$
$$+ \frac{m_{D^+}^2}{M_H^2} h_{D^+}\Big] V_{cd}^*$$

$$\Rightarrow \boxed{\Gamma = \frac{1}{8\pi}|V_{cd}|^2 G_F^2 m_{D^+} m_\mu^2\Big[f_{D^+} - g_{D^+}\frac{m_{D^+}^2}{M_H^2}\tan^2\beta + h_{D^+}\frac{m_{D^+}^2}{M_H^2}\Big]^2 \cdot \left(1-\frac{m_\mu^2}{m_{D^+}^2}\right)^2}$$

$$f_{D^+} \propto \frac{2\sqrt{2}\sqrt{m_d m_c}}{m_{D^+}}$$

$$g_{D^+} \propto 2\sqrt{2}\sqrt{m_d m_c}\,\frac{m_d}{m_{D^+}^2}$$

$$h_{D^+} \propto 2\sqrt{2}\sqrt{m_d m_c}\,\frac{m_c}{m_{D^+}^2}$$

$$\therefore \quad \frac{f_{D^+}}{g_{D^+}} = \frac{m_{D^+}}{m_d}$$

$$\Rightarrow \quad \boxed{g_{D^+} = f_{D^+}\frac{m_d}{m_{D^+}}} \qquad \text{If } m_d = 0 \; g_{D^+} = 0$$

$$\therefore \quad \frac{f_{D^+}}{h_{D^+}} = \frac{m_{D^+}}{m_c}$$

$$\Rightarrow \quad \boxed{h_{D^+} = f_{D^+}\frac{m_c}{m_{D^+}}}$$

//

C. Merin

$$1+2 \rightarrow 3+4$$

For 3:

$$U_R(\vec{p}) = \sqrt{2m}\begin{pmatrix} \cos(\theta/2)\cosh(\Phi/2) \\ -\sin(\theta/2)\cosh(\Phi/2) \\ \cos(\theta/2)\sinh(\Phi/2) \\ -\sin(\theta/2)\sinh(\Phi/2) \end{pmatrix}$$

$$U_L(\vec{p}) = \sqrt{2m}\begin{pmatrix} \sin(\theta/2)\cosh(\Phi/2) \\ \cos(\theta/2)\cosh(\Phi/2) \\ -\sin(\theta/2)\sinh(\Phi/2) \\ -\cos(\theta/2)\sinh(\Phi/2) \end{pmatrix}$$

$$V_R(\vec{p}) = \sqrt{2m}\begin{pmatrix} -\sin(\theta/2)\sinh(\Phi/2) \\ -\cos(\theta/2)\sinh(\Phi/2) \\ \sin(\theta/2)\cosh(\Phi/2) \\ \cos(\theta/2)\cosh(\Phi/2) \end{pmatrix}$$

$$V_L(\vec{p}) = \sqrt{2m}\begin{pmatrix} \cos(\theta/2)\sinh(\Phi/2) \\ -\sin(\theta/2)\sinh(\Phi/2) \\ \cos(\theta/2)\cosh(\Phi/2) \\ -\sin(\theta/2)\cosh(\Phi/2) \end{pmatrix}$$

For particles 1, 2, 4 we replace θ by $0, \pi, \pi+\theta$. //

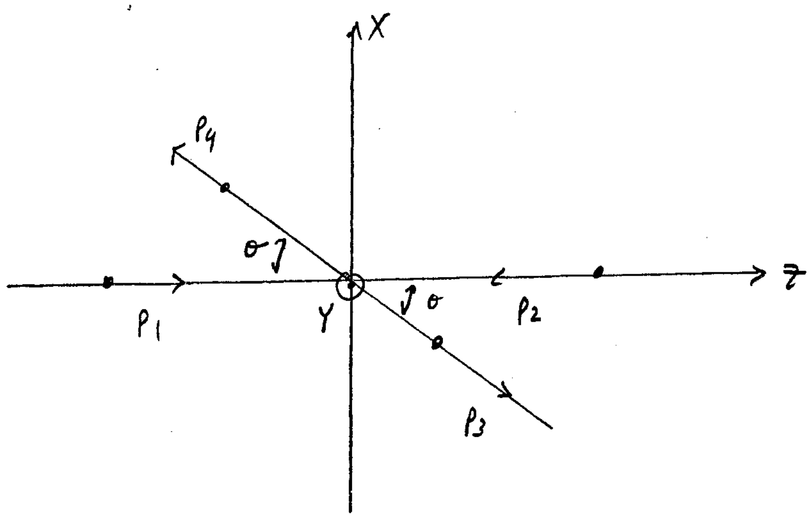
$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \qquad \bar{\psi} = \psi^\dagger \gamma^0 = (\psi_1^* \ \psi_2^* \ \psi_3^* \ \psi_4^*)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\bar{\psi} = (\psi_1^*, \psi_2^*, -\psi_3^*, -\psi_4^*)$$

Let be $C = \cos(\theta/2)$; $s = \sin(\theta/2)$

$$S_i = \sinh(\phi_i/2) \; ; \; C_i = \cosh(\phi_i/2)$$

$$S^{\pm}_{ik} = \sinh\left(\frac{\phi_i \pm \phi_k}{2}\right) \; ; \; C^{\pm}_{ik} = \cosh\left(\frac{\phi_i \pm \phi_k}{2}\right)$$



$$p^\mu = (m\cosh\phi, -m\sin\theta\sinh\phi, 0, m\cos\theta\sinh\phi)$$

$$U_{R_3}(\vec p) = \sqrt{2m}\begin{pmatrix} c\, C_3 \\ -s\, C_3 \\ c\, S_3 \\ -s\, S_3 \end{pmatrix} \; ; \; U_{L_3}(\vec p) = \sqrt{2m}\begin{pmatrix} s\, C_3 \\ c\, C_3 \\ -s\, S_3 \\ -c\, S_3 \end{pmatrix}$$

$$V_{R_3}(\vec p) = \sqrt{2m}\begin{pmatrix} -s\, S_3 \\ -c\, S_3 \\ s\, C_3 \\ c\, C_3 \end{pmatrix} \; ; \; V_{L_3}(\vec p) = \sqrt{2m}\begin{pmatrix} c\, S_3 \\ -s\, S_3 \\ c\, C_3 \\ -s\, C_3 \end{pmatrix}$$

$$(1,2,4 \quad \theta \to 0, \pi, \theta+\pi)$$

$$\tilde V_2 \gamma^\mu \frac{1}{2}(1-\gamma^5) U_1 \cdot \frac{1}{\sqrt{m_1 m_2}}$$

$$U_{1R} = \sqrt{2m_1}\begin{pmatrix} C_1 \\ 0 \\ S_1 \\ 0 \end{pmatrix} \; ; \; U_{1L} = \sqrt{2m_1}\begin{pmatrix} 0 \\ C_1 \\ 0 \\ -S_1 \end{pmatrix}$$

$$\tilde V_{2R} = \sqrt{2m_2}\,(-S_2, 0, -C_2, 0) \; ; \; \tilde V_{2L} = \sqrt{2m_2}\,(0, -S_2, 0, C_2)$$

$$\gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \Rightarrow \frac{1}{2}(1-\gamma^5) = \frac{1}{2}\left[\begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{pmatrix} - \begin{pmatrix} 0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0 \end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix} 1&0&-1&0\\0&1&0&-1\\-1&0&1&0\\0&-1&0&1 \end{pmatrix}$$

$$\tilde{V}_{2_R}\, \gamma^\mu \frac{1}{2}(1-\gamma^5) U_{1_L} \cdot \frac{1}{\sqrt{m_1 m_2}}$$

$$(-S_2, 0, -C_2, 0)\, \gamma^\mu \begin{pmatrix} 1&0&-1&0\\0&1&0&-1\\-1&0&1&0\\0&-1&0&1 \end{pmatrix}\begin{pmatrix} 0\\C_1\\0\\-S_1 \end{pmatrix}$$

$$(-S_2\ 0, -C_2, 0)\, \gamma^\mu \begin{pmatrix} 0\\C_1+S_1\\0\\-C_1-S_1 \end{pmatrix}$$

If $\mu = 0$

$$(-S_2, 0, -C_2, 0)\begin{pmatrix} 1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1 \end{pmatrix}\begin{pmatrix} 0\\C_1+S_1\\0\\-C_1-S_1 \end{pmatrix}$$

$$(-S_2, 0, -C_2, 0)\begin{pmatrix} 0\\C_1+S_1\\0\\-C_1-S_1 \end{pmatrix} = 0 \quad //$$

If $\mu = 1$

$$(-S_2\ 0, -C_2, 0)\begin{pmatrix} 0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0 \end{pmatrix}\begin{pmatrix} 0\\C_1+S_1\\0\\-C_1-S_1 \end{pmatrix}$$

$$= (-S_2, 0, -C_2, 0)\begin{pmatrix} -C_1-S_1\\0\\-C_1-S_1\\0 \end{pmatrix} = S_2(C_1+S_1) + C_2(C_1+S_1)$$

$$= (C_1+S_1)(C_2+S_2)$$

$$\therefore \quad (C_1+S_1)(C_2+S_2) =$$

$$\left(\cosh\frac{\phi_1}{2} + \sinh\frac{\phi_1}{2}\right)\left(\cosh\frac{\phi_2}{2} + \sinh\frac{\phi_2}{2}\right)$$

$$\left(\frac{e^{\frac{\phi_1}{2}} + \cancel{e^{-\frac{\phi_1}{2}}}}{2} + \frac{e^{\frac{\phi_1}{2}} - \cancel{e^{-\frac{\phi_1}{2}}}}{2}\right)\left(\frac{e^{\frac{\phi_2}{2}} + \cancel{e^{-\frac{\phi_2}{2}}}}{2} + \frac{e^{\frac{\phi_2}{2}} - \cancel{e^{-\frac{\phi_2}{2}}}}{2}\right)$$

$$= e^{\phi_1/2}\, e^{\phi_2/2} = e^{\frac{\phi_1+\phi_2}{2}}$$

$$C_{12}^{+} + S_{12}^{+} = \cosh\left(\frac{\phi_1+\phi_2}{2}\right) + \sinh\left(\frac{\phi_1+\phi_2}{2}\right)$$

$$= \frac{e^{\frac{\phi_1+\phi_2}{2}} + \cancel{e^{-\frac{(\phi_1+\phi_2)}{2}}}}{2} + \frac{e^{\frac{\phi_1+\phi_2}{2}} - \cancel{e^{-\frac{(\phi_1+\phi_2)}{2}}}}{2}$$

$$= e^{\frac{\phi_1+\phi_2}{2}}$$

$$\Rightarrow \quad (C_1+S_1)(C_2+S_2) = \boxed{C_{12}^{+} + S_{12}^{+}} \quad //$$

If $\mu = 2$

$$(-S_2,\ 0,\ -C_2,\ 0)\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ C_1+S_1 \\ 0 \\ -C_1-S_1 \end{pmatrix}$$

$$= (-S_2,\ 0,\ -C_2,\ 0)\begin{pmatrix} i(C_1+S_1) \\ 0 \\ i(C_1+S_1) \\ 0 \end{pmatrix} = -iS_2(C_1+S_1) - iC_2(C_1+S_1)$$

$$= -i(C_1+S_1)(C_2+S_2)$$

$$= -i(C_{12}^{+} + S_{12}^{+}) \quad //$$

If $\mu = 3$

$$(-S_2,\ 0,\ -C_2,\ 0)\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ C_1+S_1 \\ 0 \\ -C_1-S_1 \end{pmatrix}$$

$$= (-S_2,\ 0,\ -C_2,\ 0)\begin{pmatrix} 0 \\ C_1+S_1 \\ 0 \\ C_1+S_1 \end{pmatrix} = 0 \quad //$$

$$\tilde{V}_{2L}\,\gamma^\mu\,\frac{1}{2}(1-\gamma^5)\,U_{1_R}\cdot\frac{1}{\sqrt{m_1 m_2}}$$

$$= (0,\,-s_2,\,0,\,c_2)\,\gamma^\mu\begin{pmatrix}1&0&-1&0\\0&1&0&-1\\-1&0&1&0\\0&-1&0&1\end{pmatrix}\begin{pmatrix}c_1\\0\\s_1\\0\end{pmatrix}$$

$$= (0,\,-s_2,\,0,\,c_2)\,\gamma^\mu\begin{pmatrix}c_1-s_1\\0\\-c_1+s_1\\0\end{pmatrix}$$

If $\mu=0$

$$= (0,\,-s_2,\,0,\,c_2)\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}c_1-s_1\\0\\-c_1+s_1\\0\end{pmatrix}$$

$$= (0,\,-s_2,\,0,\,c_2)\begin{pmatrix}c_1-s_1\\0\\c_1-s_1\\0\end{pmatrix} = 0 \quad //$$

If $\mu=1$

$$= (0,\,-s_2,\,0,\,c_2)\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}\begin{pmatrix}c_1-s_1\\0\\-c_1+s_1\\0\end{pmatrix}$$

$$= (0,\,-s_2,\,0,\,c_2)\begin{pmatrix}0\\-c_1+s_1\\0\\-c_1+s_1\end{pmatrix} = (s_1-c_1)(-s_2+c_2)$$

$$= -(c_1-s_1)(c_2-s_2)$$

$$-(c_1-s_1)(c_2-s_2) = -\left(\frac{\cancel{e^{\frac{\phi_1}{2}}}+e^{-\frac{\phi_1}{2}}}{2} - \frac{(\cancel{e^{\frac{\phi_1}{2}}}-e^{-\frac{\phi_1}{2}})}{2}\right)\cdot$$

$$\cdot\left(\frac{\cancel{e^{\frac{\phi_2}{2}}}+e^{-\frac{\phi_2}{2}}}{2} - \frac{(\cancel{e^{\frac{\phi_2}{2}}}-e^{-\frac{\phi_2}{2}})}{2}\right)$$

$$= -e^{-\frac{\phi_1}{2}}\,e^{-\frac{\phi_2}{2}} = -e^{-\frac{(\phi_1+\phi_2)}{2}}$$

but:

$$-\left(C_{12}^{+}-S_{12}^{+}\right)=-\left(\cosh\left(\frac{\phi_1+\phi_2}{2}\right)-\sinh\left(\frac{\phi_1+\phi_2}{2}\right)\right)$$

$$=-\left(\frac{e^{\frac{\phi_1+\phi_2}{2}}+e^{-\left(\frac{\phi_1+\phi_2}{2}\right)}}{2}-\frac{\left(e^{\frac{\phi_1+\phi_2}{2}}-e^{-\frac{(\phi_1+\phi_2)}{2}}\right)}{2}\right)$$

$$=-e^{-\frac{(\phi_1+\phi_2)}{2}}$$

$$\Rightarrow \boxed{-(C_1-S_1)(C_2-S_2)=-\left(C_{12}^{+}-S_{12}^{+}\right)}$$

//

If $\mu=2$

$$=(0,-S_2,0,C_2)\begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix}\begin{pmatrix}C_1-S_1\\0\\-C_1+S_1\\0\end{pmatrix}$$

$$=(0,-S_2,0,C_2)\begin{pmatrix}0\\(-C_1+S_1)i\\0\\-i(C_1-S_1)\end{pmatrix}=i(C_1-S_1)S_2-i(C_1-S_1)C_2$$

$$=i(C_1-S_1)(S_2-C_2)$$

$$=-i(C_1-S_1)(C_2-S_2)$$

$$=-i\left(C_{12}^{+}-S_{12}^{+}\right)$$

//

If $\mu=3$

$$=(0,-S_2,0,C_2)\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix}\begin{pmatrix}C_1-S_1\\0\\-C_1+S_1\\0\end{pmatrix}$$

$$=(0,-S_2,0,C_2)\begin{pmatrix}-C_1+S_1\\0\\-C_1+S_1\\0\end{pmatrix}=0$$

//

$$\tilde{V}_{2_R}\, \gamma^{\mu}\, \frac{1}{2}(1-\gamma^5)\, U_{1_R} \cdot \frac{1}{\sqrt{m_1 m_2}}$$

$$(-s_2,\ 0,\ -c_2,\ 0)\ \gamma^{\mu} \begin{pmatrix} c_1 - s_1 \\ 0 \\ -c_1 + s_1 \\ 0 \end{pmatrix}$$

If $\mu = 0$

$$(-s_2,\ 0,\ -c_2,\ 0) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} c_1 - s_1 \\ 0 \\ -c_1 + s_1 \\ 0 \end{pmatrix}$$

$$= (-s_2,\ 0,\ -c_2,\ 0) \begin{pmatrix} c_1 - s_1 \\ 0 \\ c_1 - s_1 \\ 0 \end{pmatrix} = -s_2(c_1 - s_1) - c_2(c_1 - s_1)$$

$$= -(c_1 - s_1)(c_2 + s_2)$$

$$-(c_1 - s_1)(c_2 + s_2) = -\left(\frac{\cancel{e^{\frac{\phi_1}{2}}} + e^{-\frac{\phi_1}{2}}}{2} - \frac{\left(\cancel{e^{\frac{\phi_1}{2}}} - e^{-\frac{\phi_1}{2}}\right)}{2} \right)$$

$$\cdot \left(\frac{e^{\frac{\phi_2}{2}} + \cancel{e^{-\frac{\phi_2}{2}}}}{2} + \frac{\left(e^{\frac{\phi_2}{2}} - \cancel{e^{-\frac{\phi_2}{2}}}\right)}{2} \right)$$

$$= -e^{-\frac{\phi_1}{2}}\, e^{\frac{\phi_2}{2}} = -e^{-\frac{(\phi_1 - \phi_2)}{2}}$$

$$-(c_{\bar{12}} - s_{\bar{12}}) = -\left(\cosh\left(\frac{\phi_1 - \phi_2}{2}\right) - \sinh\left(\frac{\phi_1 - \phi_2}{2}\right) \right)$$

$$= -\left(\frac{\cancel{e^{\frac{\phi_1 - \phi_2}{2}}} + e^{-\frac{(\phi_1 - \phi_2)}{2}}}{2} - \frac{\left(\cancel{e^{\frac{\phi_1 - \phi_2}{2}}} - e^{-\frac{(\phi_1 - \phi_2)}{2}}\right)}{2} \right)$$

$$= -e^{-\frac{(\phi_1 - \phi_2)}{2}}$$

$$\Rightarrow \boxed{-(c_1 - s_1)(c_2 + s_2) = -(c_{\bar{12}} - s_{\bar{12}})}$$

//

If $\mu = 1$

$$(-S_2, 0, -C_2, 0)\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} C_1 - S_1 \\ 0 \\ -C_1 + S_1 \\ 0 \end{pmatrix}$$

$$= (-S_2, 0, -C_2, 0)\begin{pmatrix} 0 \\ -C_1 + S_1 \\ 0 \\ C_1 - S_1 \end{pmatrix} = 0 \quad //$$

If $\mu = 2$

$$(-S_2, 0, -C_2, 0)\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} C_1 - S_1 \\ 0 \\ -C_1 + S_1 \\ 0 \end{pmatrix}$$

$$= (-S_2, 0, -C_2, 0)\begin{pmatrix} 0 \\ i(-C_1 + S_1) \\ 0 \\ -i\,(C_1 - S_1) \end{pmatrix} = 0 \quad //$$

If $\mu = 3$

$$(-S_2, 0, -C_2, 0)\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} C_1 - S_1 \\ 0 \\ -C_1 + S_1 \\ 0 \end{pmatrix}$$

$$= (-S_2, 0, -C_2, 0)\begin{pmatrix} -C_1 + S_1 \\ 0 \\ -C_1 + S_1 \\ 0 \end{pmatrix} = (C_1 - S_1) S_2 + C_2 (C_1 - S_1)$$

$$= (C_1 - S_1)(C_2 + S_2)$$

$$= (C_{12}^- - S_{12}^-) \quad //$$

$$\tilde{V}_{2_L} \gamma^\mu \frac{1}{2}(1 - \gamma^5) U_{1L} \cdot \frac{1}{\sqrt{m_1 m_2}}$$

$$(0, -S_2, 0, C_2)\,\gamma^\mu \begin{pmatrix} 0 \\ C_1 + S_1 \\ 0 \\ -C_1 - S_1 \end{pmatrix}$$

If $\mu = 0$ $\quad (0, -S_2, 0, C_2)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ C_1 + S_1 \\ 0 \\ -C_1 - S_1 \end{pmatrix}$

$$= (0, -S_2, 0, C_2)\begin{pmatrix} 0 \\ C_1 + S_1 \\ 0 \\ C_1 + S_1 \end{pmatrix} = (C_1 + S_1)(C_2 - S_2)$$

$$(c_1+s_1)(c_2-s_2) = \left(\frac{\cancel{e^{\frac{\phi_1}{2}}}+e^{-\frac{\phi_1}{2}}}{2} + \frac{(\cancel{e^{\frac{\phi_1}{2}}}-e^{-\frac{\phi_1}{2}})}{2}\right)\left(\frac{\cancel{e^{\frac{\phi_2}{2}}}+e^{-\frac{\phi_2}{2}}}{2} - \frac{(\cancel{e^{\frac{\phi_2}{2}}}-e^{-\frac{\phi_2}{2}})}{2}\right)$$

$$= e^{+\frac{\phi_1}{2}}\, e^{-\frac{\phi_2}{2}} = e^{\frac{\phi_1-\phi_2}{2}}$$

$$c_{\bar{12}} + s_{\bar{12}} = \cosh\left(\frac{\phi_1-\phi_2}{2}\right) + \sinh\left(\frac{\phi_1-\phi_2}{2}\right)$$

$$= \frac{e^{\frac{\phi_1-\phi_2}{2}} + \cancel{e^{-\frac{(\phi_1-\phi_2)}{2}}}}{2} + \frac{(e^{\frac{\phi_1-\phi_2}{2}} - \cancel{e^{-\frac{(\phi_1-\phi_2)}{2}}})}{2}$$

$$= e^{\frac{\phi_1-\phi_2}{2}}$$

$$\Rightarrow \boxed{(c_1+s_1)(c_2-s_2) = c_{\bar{12}} + s_{\bar{12}}} \quad //$$

If $\mu = 1$

$$(0,-s_2,0,c_2)\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}\begin{pmatrix}0\\c_1+s_1\\0\\-c_1-s_1\end{pmatrix}$$

$$= (0,-s_2,0,c_2)\begin{pmatrix}-c_1-s_1\\0\\-c_1-s_1\\0\end{pmatrix} = 0 \;//$$

If $\mu = 2$

$$(0,-s_2,0,c_2)\begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix}\begin{pmatrix}0\\c_1+s_1\\0\\-c_1-s_1\end{pmatrix}$$

$$= (0,-s_2,0,c_2)\begin{pmatrix}+i(c_1+s_1)\\0\\i(c_1+s_1)\\0\end{pmatrix} = 0 \;//$$

If $\mu = 3$

$$(0,-s_2,0,c_2)\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix}\begin{pmatrix}0\\c_1+s_1\\0\\-c_1-s_1\end{pmatrix}$$

$$= (0,-s_2,0,c_2)\begin{pmatrix}0\\c_1+s_1\\0\\c_1+s_1\end{pmatrix} = (c_1+s_1)(c_2-s_2) = c_{\bar{12}} + s_{\bar{12}} \;//$$

$\Rightarrow$

Caso General o dispersión elástica

ρ	$\tilde{V}_2 \gamma^\mu \frac{1}{2}(1-\gamma^5) U_1 \cdot \frac{1}{\sqrt{m_1 m_2}}$			
2 1	$\mu=0$	$\mu=1$	$\mu=2$	$\mu=3$
$R^- \; R^+$	$-(C_{12}^- - S_{12}^-)$	0	0	$(C_{12}^- - S_{12}^-)$
$R^- \; L^-$	0	$C_{12}^+ + S_{12}^+$	$-i(C_{12}^+ + S_{12}^+)$	0
$L^+ \; R^+$	0	$-(C_{12}^+ - S_{12}^+)$	$-i(C_{12}^+ - S_{12}^+)$	0
$L^+ \; L^-$	$C_{12}^- + S_{12}^-$	0	0	$C_{12}^- + S_{12}^-$

$\tilde{V}_{2_R} \frac{1}{2}(1-\gamma^5) U_{1_R} \cdot \frac{1}{\sqrt{m_1 m_2}}$:

$$(-S_2, 0, -C_2, 0)\begin{pmatrix} C_1 - S_1 \\ 0 \\ -C_1+S_1 \\ 0 \end{pmatrix} = -(C_1-S_1)S_2 + (C_1-S_1)C_2$$
$$= (C_1-S_1)(C_2-S_2) = (C_{12}^+ - S_{12}^+) \quad //$$

$\tilde{V}_{2_R} \frac{1}{2}(1-\gamma^5) U_{1_L} \cdot \frac{1}{\sqrt{m_1 m_2}}$:

$$(-S_2, 0, -C_2, 0)\begin{pmatrix} 0 \\ C_1+S_1 \\ 0 \\ -C_1-S_1 \end{pmatrix} = 0 \quad //$$

$\tilde{V}_{2_L} \frac{1}{2}(1-\gamma^5) U_{1_R} \cdot \frac{1}{\sqrt{m_1 m_2}}$:

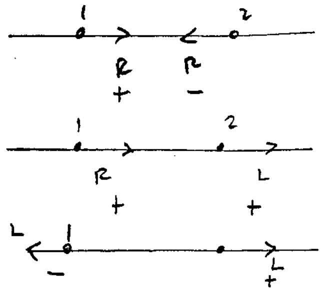
$$(0, -S_2, 0, C_2)\begin{pmatrix} C_1 - S_1 \\ 0 \\ -C_1+S_1 \\ 0 \end{pmatrix} = 0 \quad //$$

$\tilde{V}_{2_L} \frac{1}{2}(1-\gamma^5) U_{1_L} \cdot \frac{1}{\sqrt{m_1 m_2}}$:

$$(0, -S_2, 0, C_2)\begin{pmatrix} 0 \\ C_1+S_1 \\ 0 \\ -C_1-S_1 \end{pmatrix} = -S_2(C_1+S_1) - C_2(C_1+S_1)$$
$$= -(C_1+S_1)(C_2+S_2) = -(C_{12}^+ + S_{12}^+) \quad //$$

$\Rightarrow$

	ℓ	$\underset{\downarrow b}{\tilde{V}_2}\ \frac{1}{2}(1-\gamma^5)\ \underset{\downarrow \nu}{U_1} \cdot \frac{1}{\sqrt{m_1 m_2}}$
2	1	
R^-	R^+	$(C_{12}^+ - S_{12}^+)$
R^-	L^-	0
L^+	R^+	0
L^+	L^-	$-(C_{12}^+ + S_{12}^+)$



$|S=1, M_S=+1\rangle = \uparrow\uparrow$

$1 \rightarrow |S=1, M_S=0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$

$|S=1, M_S=-1\rangle = \downarrow\downarrow$

$0 \rightarrow |S=0, M_S=0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$

At rest $\phi = 0 \Rightarrow$ $S_{12} = 0$, $C_{12} = 1$

$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$

$$\frac{1}{\sqrt{2}}\Big(\underset{L^+L^-}{\overset{2\ 1}{\uparrow\downarrow}} - \underset{R^-R^+}{\overset{2\ 1}{\downarrow\uparrow}}\Big) \Rightarrow$$

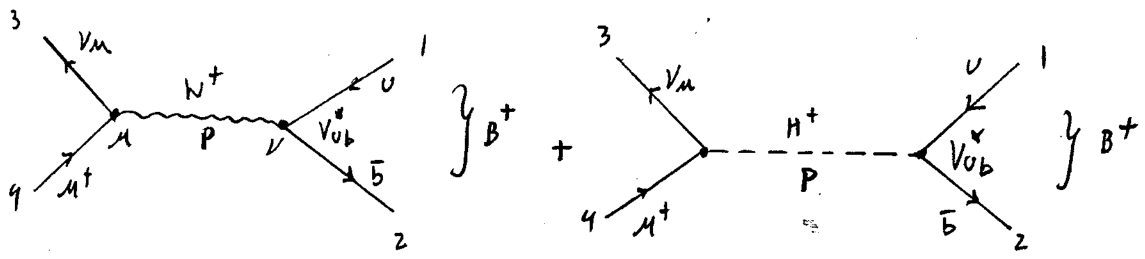
$$\tilde{V}_2\, \gamma^\mu\, (1-\gamma^5)\, U_1 = 2\sqrt{2}\,(1,0,0,0)\sqrt{m_b m_\nu} \ \propto\ P^\mu f_B = (m_B,0,0,0) f_B$$

$$\Rightarrow \boxed{f_B \propto \frac{2\sqrt{2}\sqrt{m_b m_\nu}}{m_B}}$$

$$\tilde{V}_2 (1-\gamma^5) U_1 = -2\sqrt{2}\sqrt{m_b m_\nu} \ \propto\ -\frac{m_B^2\, g_B}{m_b}$$

$$\Rightarrow \boxed{g_B \propto +2\sqrt{2}\sqrt{m_b m_\nu}\,\frac{m_b}{m_{B^+}^2}}$$

$$\therefore \boxed{g_B = +f_B\,\frac{m_b}{m_{B^+}}}$$



$P^2 \ll M_W^2, M_{H^+}^2$

$$\tilde{V}_{2R}\ \frac{1}{2}(1+\gamma^5)\,U_{1R}\cdot\frac{1}{\sqrt{m_1m_2}}\ :$$

$$(1+\gamma^5)=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}+\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix}$$

$$=\begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&1&0\\0&1&0&1\end{pmatrix}$$

$$=(-S_2,\ 0,\ -C_2,\ 0)\begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&1&0\\0&1&0&1\end{pmatrix}\begin{pmatrix}C_1\\0\\S_1\\0\end{pmatrix}$$

$$=(-S_2,\ 0,\ -C_2,\ 0)\begin{pmatrix}C_1+S_1\\0\\C_1+S_1\\0\end{pmatrix}=-(C_1+S_1)(C_2+S_2)$$

$$=-(C_{12}^{+}+S_{12}^{+})$$

$$\tilde{V}_{2R}\ \frac{1}{2}(1+\gamma^5)\,U_{1L}\cdot\frac{1}{\sqrt{m_1m_2}}\ :$$

$$=(-S_2,\ 0,\ -C_2,\ 0)\begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&1&0\\0&1&0&1\end{pmatrix}\begin{pmatrix}0\\C_1\\0\\-S_1\end{pmatrix}$$

$$=(-S_2,\ 0,\ -C_2,\ 0)\begin{pmatrix}0\\C_1-S_1\\0\\C_1-S_1\end{pmatrix}=0$$

$$\tilde{V}_{2L}\ \frac{1}{2}(1+\gamma^5)\,U_{1R}\cdot\frac{1}{\sqrt{m_1 m_2}}$$

$$(0,-S_2,0,C_2)\begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&1&0\\0&1&0&1\end{pmatrix}\begin{pmatrix}C_1\\0\\S_1\\0\end{pmatrix}$$

$$=(0,-S_2,0,C_2)\begin{pmatrix}C_1+S_1\\0\\C_1+S_1\\0\end{pmatrix}=0$$

$$\tilde{V}_{2L}\ \frac{1}{2}(1+\gamma^5)\,U_{1L}\cdot\frac{1}{\sqrt{m_1 m_2}}$$

$$(0,-S_2,0,C_2)\begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&1&0\\0&1&0&1\end{pmatrix}\begin{pmatrix}0\\C_1\\0\\-S_1\end{pmatrix}$$

$$=(0,-S_2,0,C_2)\begin{pmatrix}0\\C_1-S_1\\0\\C_1-S_1\end{pmatrix}=(C_1-S_1)(C_2-S_2)=(C_{12}^{+}-S_{12}^{+})$$

ℓ (2, 1)	$\tilde{V}_2\frac{1}{2}(1+\gamma^5)U_1\cdot\frac{1}{\sqrt{m_1m_2}}$
$^-R\ \ R^+$	$-(C_{12}^{+}+S_{12}^{+})$
$^-R\ \ L^-$	0
$^+L\ \ R^+$	0
$^+L\ \ L^-$	$(C_{12}^{+}-S_{12}^{+})$

$$\begin{cases}S_{12}=0\\C_{12}=1\end{cases}\quad \text{At rest}$$

$$\frac{1}{\sqrt{2}}\underset{(+\,-)-(-\,+)}{(\uparrow\downarrow-\downarrow\uparrow)}\ \Rightarrow$$

$$\tilde{V}_2(1+\gamma^5)U_1=2\sqrt{2}\,(m_b m_u)^{1/2}\qquad\propto\ \frac{m_{B^+}^2 h_B}{m_u}$$

$$\Rightarrow\ \boxed{h_B\propto 2\sqrt{2}\sqrt{m_b m_u}\,\frac{m_u}{m_{B^+}^2}}\ \Rightarrow\ \boxed{\frac{h_B}{g_B}=\frac{m_u}{m_b}}$$