Higgs Phenomenology in the Two Higgs Doublet Model of type II (Personal Notes)

Vol. II

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CONTENTS

Note: In these three volumenes, we present the detailed calculations of the results that appear in the thesis: "Higgs Phenomenology in the Two Higgs Doublet Model of type II".

Vol. I : Limits on the Two Higgs Doublet Model from meson decay, mixing and CP violation.

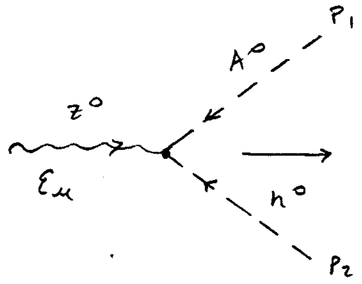
Vol. II: Mass constraints, production cross sections, and decay rates in the Two Higgs Doublet Model of type II.

Vol. III:Higgs production at a muon collider in the Two Higgs Doublet Model of type II.

Higgs Decays: $A^0 \to Z^0 h^0$, $H^\pm \to W^\pm A^0$, $H^\pm \to W^\pm h^0$, $H^\pm \to W^\pm H^0$, $H^0 \to 2h^0$, $H^0 \to 2A^0$, $H^0 \to 2Z^0$, $H^0 \to W^+ W^-$, $H^0 \to f\bar{f}$, $A^0 \to f\bar{f}$, $A^0 \to W^\pm H^\mp$, $H^0 \to W^\pm H^\mp$, $A^0 \to Z^0 H^0$, $H^0 \to H^+ H^-$, $h^0 \to 2g$, $H^0 \to 2g$, $A^0 \to 2g$, $A^0 \to 2\gamma$, $A^0 \to Z^0 \gamma$, $h^0 \to f\bar{f}$, $Z^0 \to h^0 \gamma$, $H^\pm \to W^\pm \gamma h^0$.

C. Martin

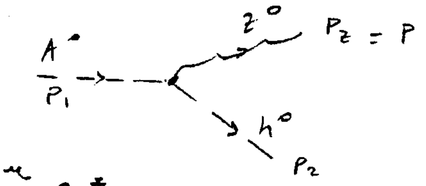
OK



$$\longrightarrow \frac{g}{2\cos\theta_W}(P_2 - P_1)^\mu \cos(\beta-\alpha)$$

$$\left(\text{for } H^0 : \frac{-g}{2\cos\theta_W}(P_2 - P_1)^\mu \sin(\beta-\alpha)\right)$$

The invariant amplitude for $\underline{A^0 \to Z^0 h^0}$



$$-iM = \frac{-g}{2\cos\theta_W}\cos(\beta-\alpha)\,(P_2+P_1)^\mu\,\varepsilon_\mu^*$$

$$\overline{|M|^2} = \frac{g^2}{4\cos^2\theta_W}\cos^2(\beta-\alpha)\,(P_2+P_1)^\mu(P_2+P_1)^\nu \sum_\lambda \varepsilon_\mu^{\lambda *}\varepsilon_\nu^\lambda$$

$$\overline{|M|^2} = \frac{g^2}{4\cos^2\theta_W}\cos^2(\beta-\alpha)\,(P_2+P_1)^\mu(P_2+P_1)^\nu\left[-g_{\mu\nu} + \frac{P_\mu P_\nu}{M_Z^2}\right]$$

$$\overline{|M|^2} = \frac{g^2}{4\cos^2\theta_W}\cos^2(\beta-\alpha)\left[-(P_2+P_1)^2 + \frac{(P.(P_1+P_2))^2}{M_Z^2}\right]$$

$$P_1 = P + P_2$$

$$P_1 - P_2 = P$$

$$M_{A^0}^2 + m_{h^0}^2 - 2P_1.P_2 = M_{Z^0}^2$$

$$P_1.P_2 = \frac{M_{A^0}^2 + m_{h^0}^2 - M_{Z^0}^2}{2}$$

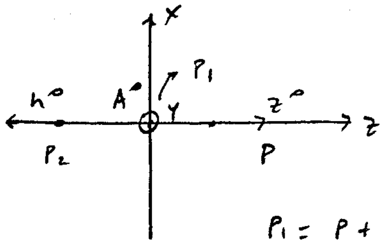
$$(P_1+P_2)^2 = M_{A^0}^{\cancel{2}} + m_{h^0}^2 + \cancel{M_{A^0}^2} + \cancel{m_{h^0}^2} - M_{Z^0}^2$$

$$(P_1+P_2)^2 = 2(M_{A^0}^2 + m_{h^0}^2) - M_{Z^0}^2$$

$$P.(P_1+P_2) = (P_1-P_2).(P_1+P_2) = M_{A^0}^2 - m_{h^0}^2$$

$$\Rightarrow \boxed{\overline{|M|^2} = \frac{g^2}{4\cos^2\theta_W}\cos^2(\beta-\alpha)\left[M_{Z^0}^2 - 2(M_{A^0}^2 + m_{h^0}^2) + \frac{(M_{A^0}^2 - m_{h^0}^2)^2}{M_{Z^0}^2}\right]}$$

In the rest frame of A^0 :



$$P_1 = P + P_2$$

$$P_1 = (M_{A^0}, 0, 0, 0) \; ; \quad P = (E, \vec{P}) \; ; \quad P_2 = (E', -\vec{P})$$

$$P_1 - P_2 = P$$

$$M_{A^0}^2 + m_{h^0}^2 - 2M_{A^0}E' = M_Z^2$$

$$E' = \frac{M_{A^0}^2 + m_{h^0}^2 - M_Z^2}{2M_{A^0}}$$

$$E = M_{A^0} - \frac{(M_{A^0}^2 + m_{h^0}^2 - M_Z^2)}{2M_{A^0}}$$

$$E = \frac{M_{A^0}^2 - m_{h^0}^2 + M_Z^2}{2M_{A^0}}$$

$$P_1 = P + P_2 \quad \Rightarrow$$

$$M_{A^0}^2 = M_{Z^0}^2 + m_{h^0}^2 + 2(EE' + |\vec{P}|^2)$$

$$M_{A^0}^2 = M_{Z^0}^2 + m_{h^0}^2 + \frac{1}{2}\frac{(M_{A^0}^2 - (m_{h^0}^2 - M_Z^2))(M_{A^0}^2 + (m_{h^0}^2 - M_Z^2))}{M_{A^0}^2} + 2|\vec{P}|^2$$

$$\frac{2\cancel{M_{A^0}^4} - 2M_Z^2M_{A^0}^2 - 2m_{h^0}^2M_{A^0}^2 - \cancel{M_{A^0}^4} + (\cancel{m_{h^0}^4} + \cancel{M_Z^4} - 2m_{h^0}^2M_Z^2)}{4M_{A^0}^2} = |\vec{P}|^2$$

$$|\vec{P}| = \frac{(M_{A^0}^4 + m_{h^0}^4 + M_Z^4 - 2M_Z^2M_{A^0}^2 - 2m_{h^0}^2M_{A^0}^2 - 2m_{h^0}^2M_Z^2)^{1/2}}{2M_{A^0}}$$

$$|\vec{P}| = \frac{\lambda^{1/2}(M_{A^0}^2, m_{h^0}^2, M_Z^2)}{2M_{A^0}}$$

$$\Rightarrow \quad \Gamma = \frac{g^2}{\cancel{4}\cos^2\theta_W}\cos^2(\beta-\alpha)\left[M_{Z^0}^2 - 2(M_{A^0}^2 + m_{h^0}^2) + \frac{(M_{A^0}^2 - m_{h^0}^2)^2}{M_Z^2}\right]\frac{\lambda^{1/2}(M_{A^0}^2, m_{h^0}^2, M_Z^2)}{\cancel{2}M_{A^0}\cdot 3\cancel{2}\pi^2M_{A^0}^2}\cancel{4}$$

$$\boxed{\Gamma(A^0 \to Z^0h^0) = \frac{g^2\cos^2(\beta-\alpha)\lambda^{3/2}(M_{A^0}^2, m_{h^0}^2, M_Z^2)}{64\pi M_{A^0}^3\cos^2\theta_W M_Z^2}}$$

$$\Gamma(A^0 \to Z^0 h^0) = \frac{\sqrt{2}\, G_F}{16\pi M_{A^0}^3}\, \lambda^{3/2}(M_{A^0}^2, m_{h^0}^2, M_Z^2)\, \cos^2(\beta-\alpha)$$

$$= \frac{\sqrt{2}\, G_F}{16\pi M_{A^0}^3}\, \lambda^{3/2}(M_{A^0}^2, m_{h^0}^2, M_Z^2)\, [\cos\beta \cos\alpha + \sin\beta \sin\alpha]^2$$

$$\boxed{\Gamma(A^0 \to Z^0 h^0) = \frac{\sqrt{2}\, G_F}{16\pi M_{A^0}^3}\, \lambda^{3/2}(M_{A^0}^2, m_{h^0}^2, M_Z^2)\, \frac{1}{(1+\tan^2\beta)} \cos^2\alpha\, [1 + \tan\beta \tan\alpha]^2}$$

if $M_{A^0} > M_{Z^0} + m_{h^0}$

where $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$

$$\lambda(M_{A^0}^2, m_{h^0}^2, M_Z^2) = M_{A^0}^4 + m_{h^0}^4 + M_Z^4 - 2M_{A^0}^2 m_{h^0}^2 - 2M_{A^0}^2 M_Z^2 - 2m_{h^0}^2 M_Z^2$$

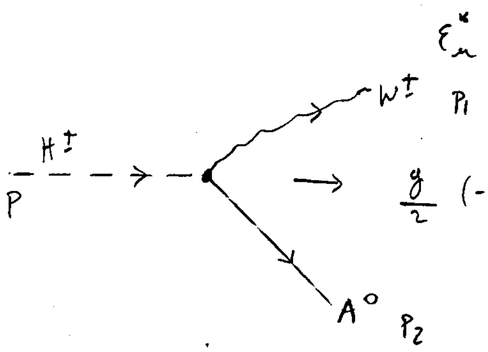
$$\tan\alpha = \left\{ \frac{1 + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)} \right]}{1 - \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)} \right]} \right\}^{1/2}$$

$$g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta) = \left[\left(1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}\right)^2 - 4\left(\frac{M_Z^2}{M_{H^\pm}^2}\right)\left(1 - \frac{M_W^2}{M_{H^\pm}^2}\right)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2 \right]^{1/2}$$

$$\cos^2\alpha = \frac{1}{2}\left\{ 1 - \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)} \right] \right\}$$

$(M_{A^0} < M_{H^\pm})$

$H^\pm \to W^\pm A^0$ decay   if $m_{H^\pm} > M_{W^\pm} + m_{A^0}$ OK



$$\longrightarrow \frac{g}{2}(-p_2-p)^\mu = -\frac{g}{2}(p_2+p)^\mu$$

$$P = P_1 + P_2$$

$$-iM = -\frac{g}{2}(p_2+p)^\mu \varepsilon^*_\mu$$

$$\overline{|M|^2} = \frac{g^2}{4}(p_2+p)^\mu (p_2+p)^\nu \sum_\lambda \varepsilon^{\lambda *}_\mu \varepsilon^\lambda_\nu$$

$$\overline{|M|^2} = \frac{g^2}{4}(p_2+p)^\mu (p_2+p)^\nu \left(-\eta_{\mu\nu} + \frac{p_{1\mu} p_{1\nu}}{M_W^2}\right)$$

$$\overline{|M|^2} = \frac{g^2}{4}\left[-(p_2+p)^2 + \frac{(p_1\cdot(p_2+p))^2}{M_W^2}\right]$$

$$p_1^2 = (p-p_2)^2 \Rightarrow M_W^2 = m_{H^\pm}^2 + m_{A^0}^2 - 2p\cdot p_2$$

$$p\cdot p_2 = \frac{m_{H^\pm}^2 + m_{A^0}^2 - M_W^2}{2}$$

$$p_1\cdot(p_2+p) = (p-p_2)\cdot(p+p_2) = p^2 - p_2^2 = m_{H^\pm}^2 - m_{A^0}^2$$

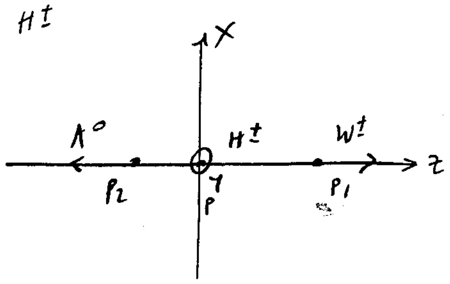
$$\overline{|M|^2} = \frac{g^2}{4}\left[-\cancel{m_{A^0}^2} - \cancel{m_{H^\pm}^2} - \cancel{m_{H^\pm}^2} - \cancel{m_{A^0}^2} + \cancel{M_W^2} + \frac{(m_{H^\pm}^2 - m_{A^0}^2)^2}{M_W^2}\right]$$

$$\overline{|M|^2} = \frac{g^2}{4}\left[-2m_{A^0}^2 - 2m_{H^\pm}^2 + M_W^2 + \frac{(m_{H^\pm}^2 - m_{A^0}^2)^2}{M_W^2}\right]$$

$$\overline{|M|^2} = \frac{g^2}{4}\left[M_W^4 + m_{H^\pm}^4 + m_{A^0}^4 - 2m_{A^0}^2 M_W^2 - 2m_{H^\pm}^2 M_W^2 - 2m_{H^\pm}^2 m_{A^0}^2\right]\frac{1}{M_W^2}$$

$$\boxed{\overline{|M|^2} = \frac{g^2}{4M_W^2}\lambda(m_{A^0}^2, M_W^2, m_{H^\pm}^2)}$$

In the rest frame of $H^\pm$



$$P = (m_{H^\pm}, \vec{0}) ; \quad p_1 = (E_1, \vec{p}_1) ; \quad p_2 = (E_2, -\vec{p}_1)$$

$$p_2^2 = (P - p_1)^2$$

$$\Rightarrow \quad m_{A^0}^2 = m_{H^\pm}^2 + M_{W^\pm}^2 - 2 m_{H^\pm} E_1$$

$$E_1 = \frac{m_{H^\pm}^2 + M_W^2 - m_{A^0}^2}{2 m_{H^\pm}}$$

$$E_1^2 = M_W^2 + |\vec{p}_1|^2$$

$$|\vec{p}_1|^2 = \frac{(m_{H^\pm}^2 + M_W^2 - m_{A^0}^2)^2}{4 m_{H^\pm}^2} - M_W^2$$

$$|\vec{p}_1|^2 = \frac{m_{H^\pm}^4 + M_W^4 + m_{A^0}^4 - 2 m_{H^\pm}^2 M_W^2 - 2 m_{H^\pm}^2 m_{A^0}^2 - 2 m_{A^0}^2 M_W^2}{4 m_{H^\pm}^2}$$

$$|\vec{p}_1|^2 = \frac{\lambda(m_{A^0}^2, M_W^2, m_{H^\pm}^2)}{4 m_{H^\pm}^2}$$

$$\boxed{|\vec{p}_1| = \frac{\lambda^{1/2}(m_{A^0}^2, M_W^2, m_{H^\pm}^2)}{2 m_{H^\pm}}}$$

∴

$$\Gamma = \frac{g^2}{4 M_W^2} \; \frac{\lambda^{3/2}(m_{A^0}^2, M_W^2, m_{H^\pm}^2)}{2 m_{H^\pm}} \; \frac{4\cancel{\pi}}{32 \pi^{\cancel{2}} m_{H^\pm}^2}$$

$$\searrow \quad \boxed{\Gamma(H^\pm \to W^\pm A^0) = \frac{\sqrt{2}\, G_F}{16\pi\, m_{H^\pm}^3} \, \lambda^{3/2}(m_{A^0}^2, M_W^2, m_{H^\pm}^2)} \quad \swarrow$$

Where $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$

$\Gamma(H^\pm \to W^\pm A^0)$ also can be obtained from $\Gamma(A^0 \to W^\pm H^\mp)$ with the substitution

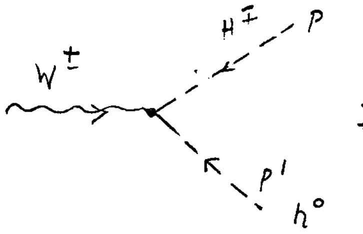
$$m_{A^0} \to m_{H^\pm}$$

$H^\pm \to W^\pm h^0$ decay $\quad (M_{H^+} > M_{W^+} + m_{h^0})$

ok

The Lagrangian for $W^\mp H H$ interactions is:

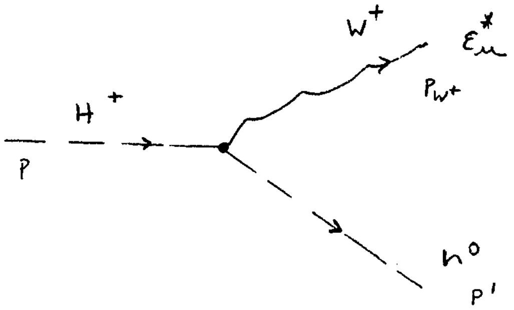
$$\mathcal{L} = -\frac{ig}{2} W_\mu^+ H^- \overleftrightarrow{\partial^\mu} \left[H^0 \sin(\alpha-\beta) + h^0 \cos(\alpha-\beta) + i A^0 \right] + h.c.$$



$\mp\frac{ig}{2} \cos(\alpha-\beta)\,(p'-p)^\mu$ ok.

$$-\frac{ig}{2} W_\mu^+ \left(H^- \partial^\mu h^0 - (\partial^\mu H^-)\, h^0 \right) \cos(\alpha-\beta)$$

$H^+ \to W^+ h^0$



$$-iM = -\frac{ig}{2} \cos(\beta-\alpha)\,(P'+P)^\mu\, \varepsilon_\mu^*$$

$$\overline{|M|^2} = \frac{g^2}{4} \cos^2(\beta-\alpha)\,(P'+P)^\mu (P'+P)^\nu \sum_\lambda \varepsilon_{\mu\lambda}^* \varepsilon_{\nu\lambda}$$

$$= \frac{g^2}{4} \cos^2(\beta-\alpha)\,(P'+P)^\mu (P'+P)^\nu \left[-g_{\mu\nu} + \frac{P_{W^+\mu} P_{W^+\nu}}{M_W^2} \right]$$

$$\overline{|M|^2} = \frac{g^2}{4} \cos^2(\beta-\alpha) \left[-(P'+P)^2 + \frac{((P'+P).P_{W^+})^2}{M_W^2} \right]$$

$$P = P' + P_{W^+}$$

$$(P'+P).P_{W^+} = (P'+P).(P-P') = \cancel{P.P'} - m_{h^0}^2 + M_{H^+}^2 - \cancel{P.P'}$$

$$(P'+P).P_{W^+} = M_{H^+}{}^2 - m_{h^0}^2$$

$$(P+P')^2 = M_{H^+}^2 + m_{h^0}^2 + 2\,P.P'$$

but $(P-P') = P_{W^+} \Rightarrow M_{H^+}^2 + m_{h^0}^2 - 2P.P' = M_{W^+}^2$

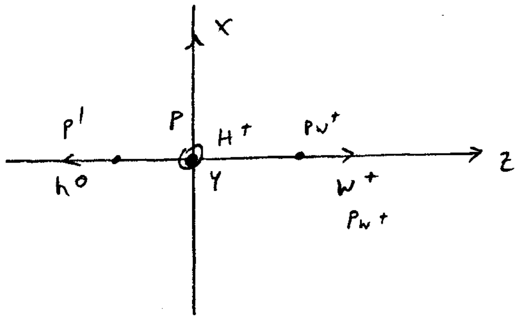
$$\Rightarrow +2P\cdot P' = -M_{W^+}^2 + M_{H^+}^2 + m_{h^0}^2$$

$$(P+P')^2 = 2M_{H^+}^2 + 2m_{h^0}^2 - M_{W^+}^2$$

$$\Rightarrow \quad \overline{|M|^2} = \frac{g^2}{4}\cos^2(\beta-\alpha)\left[-2M_{H^+}^2 - 2m_{h^0}^2 + M_{W^+}^2 + \frac{(M_{H^+}^2 - m_{h^0}^2)^2}{M_W^2}\right]$$

$$\overline{|M|^2} = \frac{g^2}{4}\cos^2(\beta-\alpha)\left[M_W^2 - 2(M_{H^+}^2 + m_{h^0}^2) + \frac{(M_{H^+}^2 - m_{h^0}^2)^2}{M_W^2}\right] = \frac{g^2}{4}\cos^2(\beta-\alpha)\frac{\lambda}{M_W^2}$$

In the rest frame of H^+



$$P = P_{H^+} = (M_{H^+}, 0, 0, 0) \; ; \quad P_{W^+} = (E, \vec{P}) \; ; \quad P' = (E', -\vec{P})$$

$$P = P_{W^+} + P'$$

$$P - P_{W^+} = P'$$

$$M_{H^+}^2 + M_{W^+}^2 - 2EM_{H^+} = m_{h^0}^2$$

$$E = \frac{M_{H^+}^2 + M_W^2 - m_{h^0}^2}{2M_{H^+}}$$

$$E' = M_{H^+} - E = M_{H^+} - \frac{(M_{H^+}^2 + M_W^2 - m_{h^0}^2)}{2M_{H^+}}$$

$$E' = \frac{M_{H^+}^2 - M_W^2 + m_{h^0}^2}{2M_{H^+}}$$

$$M_{H^+}^2 = M_{W^+}^2 + m_{h^0}^2 + 2(EE' + |\vec{P}|^2)$$

$$M_{H^+}^2 = M_{W^+}^2 + m_{h^0}^2 + \cancel{2}\frac{(M_{H^+}^2 + M_W^2 - m_{h^0}^2)}{\underset{2}{\cancel{4}}M_{H^+}^2}(M_{H^+}^2 - M_W^2 + m_{h^0}^2) + 2|\vec{P}|^2$$

$$2\cancel{M_{H^+}^4} = 2M_{W^+}^2 M_{H^+}^2 + 2m_{h^0}^2 M_{H^+}^2 + \cancel{M_{H^+}^4} - (M_W^2 - m_{h^0}^2)^2 + 4|\vec{P}|^2 M_{H^+}^2$$

$$|\vec{p}|^2 = \frac{M_{H^+}^4 + M_W^4 + m_{h^0}^4 - 2M_{W^+}^2 M_{H^+}^2 - 2m_{h^0}^2 M_{H^+}^2 - 2m_{h^0}^2 M_W^2}{4M_{H^+}^2}$$

$$\Rightarrow$$

$$|\vec{p}| = \frac{\lambda^{1/2}(M_{H^+}^2, M_W^2, m_{h^0}^2)}{2M_{H^+}}$$

with $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$

$$\Gamma = \frac{g^2}{4}\cos^2(\beta-\alpha)\left[M_W^2 - 2(M_{H^+}^2 + m_{h^0}^2) + \frac{(M_{H^+}^2 - m_{h^0}^2)^2}{M_W^2}\right]\frac{\lambda^{1/2}(M_{H^+}^2, M_W^2, m_{h^0}^2)}{2M_{H^+}}\cdot$$

$$\cdot\frac{4\pi}{32\pi^2 M_{H^+}^2}$$

$$\Gamma(H^+ \to W^+ h^0) = \frac{g^2\cos^2(\beta-\alpha)\,\lambda^{3/2}(M_{H^+}^2, M_W^2, m_{h^0}^2)}{64\pi M_{H^+}^3 M_W^2}$$

$$= \frac{\sqrt{2}\,G_F\,[\cos\beta\cos\alpha + \sin\beta\sin\alpha]^2\,\lambda^{3/2}(M_{H^+}^2, M_W^2, m_{h^0}^2)}{16\pi M_{H^+}^3}$$

$$\Gamma(H^+ \to W^+ h^0) = \frac{\sqrt{2}\,G_F\cos^2\beta\cos^2\alpha\,[1 + \tan\beta\tan\alpha]^2\,\lambda^{3/2}(M_{H^+}^2, M_W^2, m_{h^0}^2)}{16\pi M_{H^+}^3}$$

$$\boxed{\Gamma(H^\pm \to W^\pm h^0) = \frac{\sqrt{2}\,G_F\cos^2\alpha}{16\pi M_{H^\pm}^3(1+\tan^2\beta)}\,[1 + \tan\beta\tan\alpha]^2\,\lambda^{3/2}(M_{H^\pm}^2, M_W^2, m_{h^0}^2)}$$

$$M_{H^\pm} > M_{W^\pm} + m_{h^0} \qquad \left(\cos^2\alpha = \frac{1}{1+\tan^2\alpha}\right)$$

Where:

$$\tan\alpha = \left\{\frac{1 + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)}\right]}{1 - \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)}\right]}\right\}^{1/2};$$

$$g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta) = \left[\left(1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}\right)^2 - 4\left(\frac{M_Z^2}{M_{H^\pm}^2}\right)\left(1 - \frac{M_W^2}{M_{H^\pm}^2}\right)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2\right]^{1/2};$$

$$\cos^2\alpha = \frac{1}{2}\left\{1 - \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)}\right]\right\};$$

$$\lambda(M_{H^\pm}^2, M_W^2, m_{h^0}^2) = M_{H^\pm}^4 + M_W^4 + m_{h^0}^4 - 2M_{H^\pm}^2 M_W^2 - 2M_{H^\pm}^2 m_{h^0}^2 - 2M_W^2 m_{h^0}^2.$$

$H^\pm \rightarrow W^\pm H^0$ $\quad (M_{H^\pm} > M_{W^\pm} + M_{H^0})$ $\quad$ OK

The decay width for $H^\pm \rightarrow W^\pm H^0$ is obtained from the decay width corresponding to $H^\pm \rightarrow W^\pm h^0$ replacing $\cos(\beta-\alpha) \rightarrow \sin(\beta-\alpha)$

$\Rightarrow$

$$\Gamma(H^\pm \longrightarrow W^\pm H^0) = \frac{g^2 \sin^2(\beta-\alpha)\, \lambda^{3/2}(M_{H^\pm}^2, M_W^2, m_{H^0}^2)}{64\pi M_{H^\pm}^3 M_W^2}$$

$$= \frac{g^2 (\sin\beta\cos\alpha - \sin\alpha\cos\beta)^2 \lambda^{3/2}}{64\pi M_{H^\pm}^3 M_W^2}$$

$$= \frac{\sqrt{2}\, G_F}{16\pi M_{H^\pm}^3} \cos^2\beta\, (\cos\alpha \tan\beta - \sin\alpha)^2 \lambda^{3/2}$$

$$\boxed{\Gamma(H^\pm \rightarrow W^\pm H^0) = \frac{\sqrt{2}\, G_F}{16\pi M_{H^\pm}^3 (1+\tan^2\beta)} (\cos\alpha \tan\beta - \sin\alpha)^2 \lambda^{3/2}(M_{H^\pm}^2, M_W^2, m_{H^0}^2)}$$

$\therefore$

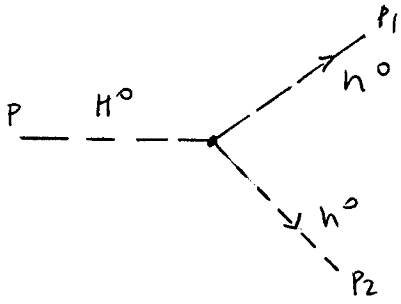
$$\boxed{\Gamma(H^\pm \rightarrow W^\pm H^0) = \frac{\sqrt{2}\, G_F (\tan\beta - \tan\alpha)^2}{16\pi M_{H^\pm}^3 (1+\tan^2\beta)(1+\tan^2\alpha)} \lambda^{3/2}(M_{H^\pm}^2, M_W^2, m_{H^0}^2)}$$

OK

where :

$$\lambda(M_{H^\pm}^2, M_W^2, m_{H^0}^2) = M_{H^\pm}^4 + M_W^4 + m_{H^0}^4 - 2 M_{H^\pm}^2 M_W^2 - 2 M_{H^\pm}^2 m_{H^0}^2 - 2 M_W^2 m_{H^0}^2$$

$H^0 \rightarrow 2h^0$ $\quad (M_{H^0} > 2 m_{h^0})$ $\quad$ OK

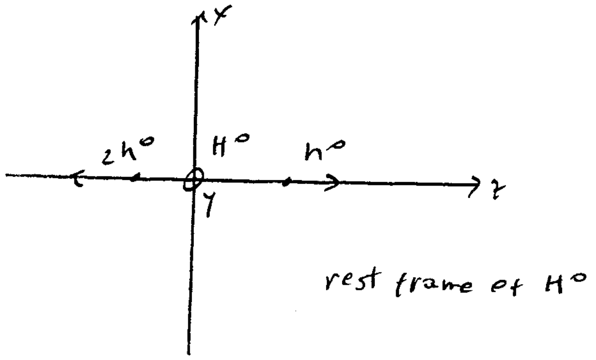


$$-\frac{i g M_Z}{2\cos\theta_W}\left[2\sin 2\alpha \sin(\alpha+\beta) - \cos(\alpha+\beta) \cdot \cos 2\alpha\right]$$

$$-iM = -\frac{i g M_Z}{2\cos\theta_W}\left[2\sin 2\alpha \sin(\alpha+\beta) - \cos(\alpha+\beta)\cos 2\alpha\right]$$

$$\overline{|M|^2} = \frac{g^2 M_Z^2}{4\cos^2\theta_W}\left[2\sin 2\alpha \sin(\alpha+\beta) - \cos(\alpha+\beta)\cos 2\alpha\right]^2$$

$P = P_1 + P_2$



$$P = (m_{H^\circ}, 0, 0, 0)$$

$$P_1 = (E_1, \vec{P}_1) \; ; \quad P_2 = (E_1, -\vec{P}_1)$$

$$m_{H^\circ}^2 = 2m_{h^\circ}^2 + 2P_1 \cdot P_2$$

$$m_{H^\circ}^2 = 2m_{h^\circ}^2 + 2(E_1^2 + |\vec{P}_1|^2)$$

but $E_1 = \frac{m_{H^\circ}}{2}$

$$\Rightarrow \quad m_{H^\circ}^2 = 2m_{h^\circ}^2 + \frac{1}{2}m_{H^\circ}^2 + 2|\vec{P}_1|^2$$

$$|\vec{P}_1|^2 = \left(\frac{1}{2}m_{H^\circ}^2 - 2m_{h^\circ}^2\right)\frac{1}{2}$$

$$|\vec{P}_1| = \frac{1}{2}\left(m_{H^\circ}^2 - 4m_{h^\circ}^2\right)^{1/2}$$

$$\Gamma = \frac{g^2 M_Z^2}{4\cos^2\theta_W}\,\frac{[2\sin 2\alpha\,\sin(\alpha+\beta) - \cos(\alpha+\beta)\cos 2\alpha]^2}{8\pi\, m_{H^\circ}^2}\,\overset{\text{identical particles}}{\overset{\uparrow}{\left(\frac{1}{2}\right)}}\,\frac{1}{2}m_{H^\circ}\left(1 - \frac{4m_{h^\circ}^2}{m_{H^\circ}^2}\right)^{1/2}$$

$$\Gamma(H^\circ \rightarrow 2h^\circ) = \frac{g^2 M_Z^4}{128\, M_W^2\, \pi\, m_{H^\circ}}\left(1 - \frac{4m_{h^\circ}^2}{m_{H^\circ}^2}\right)^{1/2}\left[2\sin 2\alpha\,\sin(\alpha+\beta) - \cos(\alpha+\beta)\cos 2\alpha\right]^2$$

$$\Gamma(H^\circ \rightarrow 2h^\circ) = \frac{\sqrt{2}\, G_F\, M_Z^4}{32\pi\, m_{H^\circ}}\left(1 - \frac{4m_{h^\circ}^2}{m_{H^\circ}^2}\right)^{1/2}\Big[2\sin 2\alpha\,(\sin\alpha\cos\beta + \sin\beta\cos\alpha)$$
$$- (\cos\alpha\cos\beta - \sin\alpha\sin\beta)\cos 2\alpha\Big]^2$$

$$= \frac{\sqrt{2}\, G_F\, M_Z^4}{32\pi\, m_{H^\circ}}\left(1 - \frac{4m_{h^\circ}^2}{m_{H^\circ}^2}\right)^{1/2}\cos^2(2\alpha)\Big[2\tan 2\alpha\,(\sin\alpha\cos\beta + \sin\beta\cos\alpha)$$
$$- (\cos\alpha\cos\beta - \sin\alpha\sin\beta)\Big]^2$$

$$\Gamma(H^0 \to 2h^0) = \frac{\sqrt{2}\, G_F M_Z^4}{32\pi m_{H^0}} \left(1 - 4\frac{m_{h^0}^2}{m_{H^0}^2}\right)^{1/2} \frac{1}{1+\tan^2(2\alpha)} \cos^2\alpha \cos^2\beta \Big[2\tan 2\alpha \; (\tan\alpha + \tan\beta) - (1 - \tan\alpha\tan\beta)\Big]^2$$

$$\Gamma(H^0 \to 2h^0) = \frac{\sqrt{2}\, G_F M_Z^4}{32\pi m_{H^0}} \left(1 - 4\frac{m_{h^0}^2}{m_{H^0}^2}\right)^{1/2} \frac{1}{(1+\tan^2\alpha)(1+\tan^2\beta)\left[1 + \frac{4\tan^2\alpha}{(1-\tan^2\alpha)^2}\right]} \cdot$$

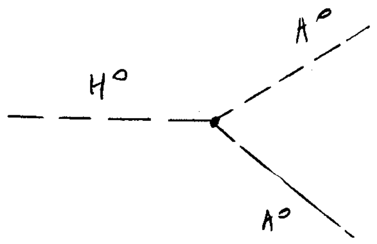
$$\cdot \left[\frac{4\tan\alpha}{1-\tan^2\alpha}(\tan\alpha + \tan\beta) - (1 - \tan\alpha\tan\beta)\right]^2$$

$\downarrow$

$$\boxed{\begin{aligned}\Gamma(H^0 \to 2h^0) = \frac{\sqrt{2}\, G_F M_Z^4}{32\pi m_{H^0}} \left(1 - 4\frac{m_{h^0}^2}{m_{H^0}^2}\right)^{1/2} \frac{(1-\tan^2\alpha)^2}{(1+\tan^2\alpha)^3(1+\tan^2\beta)} \cdot \\ \cdot \left[\frac{4\tan\alpha}{(1-\tan^2\alpha)}(\tan\alpha + \tan\beta) - (1 - \tan\alpha\tan\beta)\right]^2\end{aligned}} \swarrow$$

note: (we don't have $A^0 \to 2h^0$)

$\underline{H^0 \to 2A^0}$: $(m_{H^0} > 2m_{A^0})$ $\underline{\text{OK}}$



$$\frac{i g M_Z}{2\cos\theta_W} \cos 2\beta \cos(\alpha+\beta)$$

For analogy

$$\Gamma(H^0 \to 2A^0) = \frac{g^2 M_Z^4}{128 M_W^2 \pi m_{H^0}} \left(1 - 4\frac{m_{A^0}^2}{m_{H^0}^2}\right)^{1/2} [\cos 2\beta \; \cos(\alpha+\beta)]^2$$

$$\Gamma(H^0 \to 2A^0) = \frac{\sqrt{2}\, G_F M_Z^4}{32\pi m_{H^0}} \left(1 - \frac{4 m_{A^0}^2}{m_{H^0}^2}\right)^{1/2} \frac{1}{1+\tan^2 2\beta} [\cos\alpha\cos\beta - \sin\alpha\sin\beta]^2$$

$$\Gamma(H^0 \to 2A^0) = \frac{\sqrt{2}\, G_F M_Z^4}{32\pi m_{H^0}} \left(1 - \frac{4 m_{A^0}^2}{m_{H^0}^2}\right)^{1/2} \frac{\cos^2\alpha\cos^2\beta}{\left(1 + \frac{4\tan^2\beta}{(1-\tan^2\beta)^2}\right)} [1 - \tan\alpha\tan\beta]^2$$

⑫

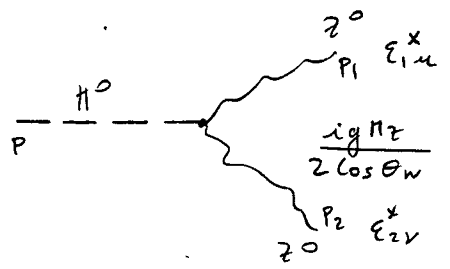
$$\boxed{\Gamma(H^0 \to 2A^0) = \frac{\sqrt{2}\, G_F M_Z^4}{32\pi m_{H^0}} \left(1 - \frac{4 m_{A^0}^2}{m_{H^0}^2}\right)^{1/2} \frac{(1-\tan^2\beta)^2}{(1+\tan^2\alpha)(1+\tan^2\beta)^3} \left[1 - \tan\alpha \tan\beta\right]^2}$$

$H^0 \to ZZ^0$ decay $\quad (M_{H^0} > 2M_{Z^0})$ $\qquad$ OK

The lagrangian for VVH interactions is:

$$\mathcal{L} = \left(g M_W W^+_\mu W^{-\mu} + \frac{g M_Z}{2\cos\theta_W} Z_\mu Z^\mu\right)\left(H^0 \cos(\beta-\alpha) + h^0 \sin(\beta-\alpha)\right) \qquad (1)$$

$P = P_1 + P_2$



$$\frac{ig M_Z}{2\cos\theta_W}\cos(\beta-\alpha)\,\eta^{\mu\nu}\cdot 2!$$ (identical particles in the final state)

$$-iM = \frac{ig M_Z}{\cos\theta_W}\cos(\beta-\alpha)\,\eta^{\mu\nu}\,\varepsilon^*_{1\mu}\,\varepsilon^*_{2\nu} \qquad (2)$$

$$\overline{|M|^2} = \frac{g^2 M_Z^2}{\cos^2\theta_W}\cos^2(\beta-\alpha)\,\eta^{\mu\nu}\eta^{\rho\sigma}\sum_{\lambda,\lambda'}\overset{\lambda}{\varepsilon}{}^*_{1\mu}\,\overset{\lambda'}{\varepsilon}{}^*_{2\nu}\,\varepsilon^\lambda_{1\rho}\,\varepsilon^{\lambda'}_{2\sigma}$$

$$\overline{|M|^2} = \frac{g^2 M_Z^2}{\cos^2\theta_W}\cos^2(\beta-\alpha)\,\eta^{\mu\nu}\eta^{\rho\sigma}\left(-\eta_{\mu\rho} + \frac{P_{1\mu}P_{1\rho}}{M_{Z^0}^2}\right)\left(-\eta_{\nu\sigma} + \frac{P_{2\nu}P_{2\sigma}}{M_{Z^0}^2}\right)$$

$$\overline{|M|^2} = \frac{g^2 M_Z^2}{\cos^2\theta_W}\cos^2(\beta-\alpha)\left(-\delta^\nu_\rho + \frac{P_1^\nu P_{1\rho}}{M_{Z^0}^2}\right)\left(-\delta^\rho_\nu + \frac{P_{2\nu}P_2^\rho}{M_{Z^0}^2}\right)$$

$$\overline{|M|^2} = \frac{g^2 M_Z^2}{\cos^2\theta_W}\cos^2(\beta-\alpha)\left[4 - 1 - 1 + \frac{(P_1\cdot P_2)^2}{M_{Z^0}^4}\right]$$

$$\overline{|M|^2} = \frac{g^2 M_Z^2}{\cos^2\theta_W}\cos^2(\beta-\alpha)\left[2 + \frac{(P_1\cdot P_2)^2}{M_{Z^0}^4}\right] \qquad (3)$$

$$m_{H^0}^2 = 2M_{Z^0}^2 + 2P_1\cdot P_2$$

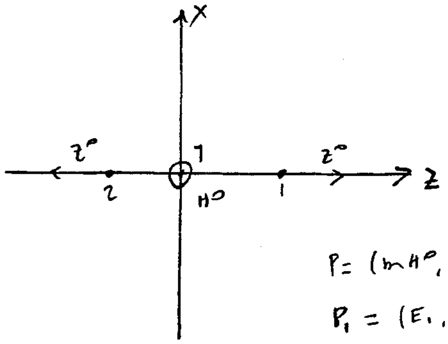
$$P_1\cdot P_2 = \frac{1}{2}(m_{H^0}^2 - 2M_{Z^0}^2)$$

$$\overline{|M|^2} = \frac{g^2 M_Z^2}{\cos^2\theta_W}\cos^2(\beta-\alpha)\left[2 + \frac{1}{4}\frac{(m_{H^0}^2 - 2M_{Z^0}^2)^2}{M_{Z^0}^4}\right]$$

$$\overline{|M|^2} = \frac{g^2 \cancel{M_Z^2}\cos^2(\beta-\alpha)}{4\cos^2\theta_W \cancel{M_Z^4}}\left[\cancel{8}M_Z^4 + m_{H^0}^4 - 4m_{H^0}^2M_Z^2 + \cancel{4}M_Z^4\right]$$

$$\boxed{\overline{|M|^2} = \frac{g^2\cos^2(\beta-\alpha)}{4\cos^2\theta_W M_Z^2}\left[12M_Z^4 + m_{H^0}^4 - 4m_{H^0}^2M_Z^2\right]} \qquad (4)$$

In the rest frame of H^0:



$$P = (m_{H^0}, \vec{0})$$
$$P_1 = (E_1, \vec{P}_1)$$
$$P_2 = (E_1, -\vec{P}_1)$$

$$m_{H^0} = 2E_1$$

$$E_1^2 = M_Z^2 + |\vec{P}_1|^2$$

$$|\vec{P}_1|^2 = \frac{m_{H^0}^2}{4} - M_Z^2 = \frac{m_{H^0}^2}{4}\left(1 - \frac{4M_Z^2}{m_{H^0}^2}\right)$$

$$|\vec{P}_1| = \frac{m_{H^0}}{2}\left(1 - \frac{4M_Z^2}{m_{H^0}^2}\right)^{1/2} \qquad (5)$$

$$\Rightarrow \Gamma = \frac{g^2 \cos^2(\beta-\alpha)}{4M_W^2}\,\frac{[12M_Z^4 - 4m_{H^0}^2 M_Z^2 + m_{H^0}^4]}{32\cancel{\pi}^2 m_{\cancel{H^0}}^2}\,\frac{m_{\cancel{H^0}}}{\cancel{2}}\left(1 - \frac{4M_Z^2}{m_{H^0}^2}\right)^{1/2} \cancel{4\pi}\,\left(\frac{1}{\cancel{2}}\right)^{\nearrow \text{identical particle}} \qquad (6)$$

$$\Gamma(H^0 \to 2Z^0) = \frac{\sqrt{2}\,G_F \cos^2(\beta-\alpha)}{32\pi\, m_{H^0}}\left[12M_Z^4 - 4m_{H^0}^2M_Z^2 + m_{H^0}^4\right]\left(1 - \frac{4M_Z^2}{m_{H^0}^2}\right)^{1/2}$$

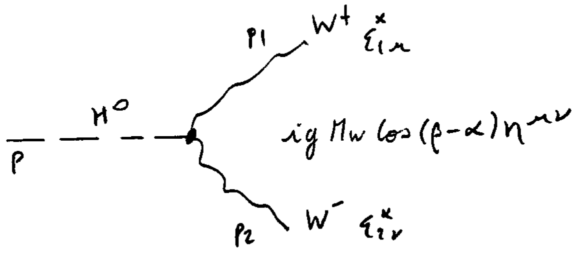
$$= \frac{\sqrt{2}\,G_F\,[\cos\beta\cos\alpha + \sin\beta\sin\alpha]^2}{32\pi}\, m_{H^0}^3\left[12\left(\frac{M_Z}{m_{H^0}}\right)^4 - 4\left(\frac{M_Z}{m_{H^0}}\right)^2 + 1\right]\left(1-4\left(\frac{M_Z}{m_{H^0}}\right)^2\right.$$

$$\Gamma(H^0 \to 2Z^0) = \frac{\sqrt{2}\,G_F \cos^2\beta\cos^2\alpha\,[1 + \tan\beta\tan\alpha]^2}{32\pi}\, m_{H^0}^3\left[12X^2 - 4X + 1\right](1-4X)^{1/2}$$

$$\boxed{\begin{aligned} & X \equiv \left(\frac{M_Z}{m_{H^0}}\right)^2 \\ \Rightarrow\ & \Gamma(H^0 \to 2Z^0) = \frac{\sqrt{2}\,G_F\,[1+\tan\beta\tan\alpha]^2\, m_{H^0}^3}{32\pi\,(1+\tan^2\beta)(1+\tan^2\alpha)}\left[12X^2 - 4X + 1\right](1-4X)^{1/2} \end{aligned}} \qquad (7)$$

OK

$H^0 \to W^+W^-$



$$-i\mathcal{M} = i g M_W \cos(\beta-\alpha)\, \eta^{\mu\nu}\, \varepsilon^*_{1\mu}\, \varepsilon^*_{2\nu} \qquad (8)$$

$$\Rightarrow \overline{|\mathcal{M}|^2} = g^2 M_W^2 \cos^2(\beta-\alpha) \left[2 + \frac{(p_1\cdot p_2)^2}{M_W^4} \right] \qquad (9)$$

$$m_{H^0}^2 = 2M_W^2 + 2p_1\cdot p_2$$

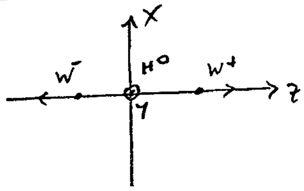
$$p_1\cdot p_2 = \frac{1}{2}(m_{H^0}^2 - 2M_W^2)$$

$$\overline{|\mathcal{M}|^2} = g^2 M_W^2 \cos^2(\beta-\alpha) \left[2 + \frac{1}{4} \frac{(m_{H^0}^2 - 2M_W^2)^2}{M_W^4} \right]$$

$$\overline{|\mathcal{M}|^2} = \frac{g^2 \cancel{M_W^2} \cos^2(\beta-\alpha)}{4 M_W^{\cancel{4}\,2}} \left[12M_W^4 - 4M_W^2 m_{H^0}^2 + m_{H^0}^4 \right]$$

$$\boxed{\overline{|\mathcal{M}|^2} = \frac{g^2 \cos^2(\beta-\alpha)}{4 M_W^2} \left[12M_W^4 - 4M_W^2 m_{H^0}^2 + m_{H^0}^4 \right]} \qquad (10)$$

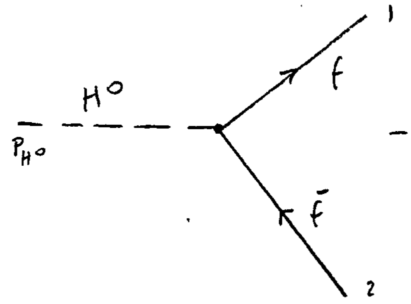
$$|\vec{p}_1| = \frac{m_{H^0}}{2} \left(1 - \frac{4M_W^2}{m_{H^0}^2} \right)^{1/2}$$



$$\Gamma(H^0 \to W^+W^-) = \frac{g^2 \cos^2(\beta-\alpha)}{4M_W^2} \left[\frac{12M_W^4 - 4M_W^2 m_{H^0}^2 + m_{H^0}^4}{32\pi^2 m_{H^0}^2} \right] \frac{m_{H^0}}{2} \left(1 - \frac{4M_W^2}{m_{H^0}^2} \right)^{1/2} 4\pi$$

$$\boxed{\begin{gathered} \Gamma(H^0 \to W^+W^-) = \frac{\sqrt{2}\, G_F \left[1 + \tan\beta \tan\alpha \right]^2 m_{H^0}^3}{16\pi (1+\tan^2\beta)(1+\tan^2\alpha)} \left[12Y^2 - 4Y + 1 \right] (1-4Y)^{1/2} \\ Y \equiv \left(\frac{M_W}{m_{H^0}} \right)^2 \end{gathered}} \qquad (11)$$

$H^0 \to f\bar{f}$ decay   ($M_{H^0} \geq M_Z$; $M_{H^0} \leq (M_Z^2 + m_A^2)^{1/2}$)   ok



$$-\frac{igm_f}{2M_W} B_f$$

$$B_f = \begin{cases} \dfrac{\sin\alpha}{\sin\beta} & f = u, c \\ \dfrac{\cos\alpha}{\cos\beta} & f = d, s, b, e^-, \mu^-, \tau^- \end{cases}$$

if $f = u, c$, : $-\dfrac{igm_f}{2M_W}\dfrac{\sin\alpha}{\sin\beta}$

if $f = d, s, b, e^-, \mu^-, \tau^-$: $-\dfrac{igm_f}{2M_W}\dfrac{\cos\alpha}{\cos\beta}$

$$-iM = \overline{U_1}\left(-\frac{igm_f}{2M_W}B_f\right)V_2$$

$$M = \frac{gm_f}{2M_W} B_f \overline{U_1} V_2$$

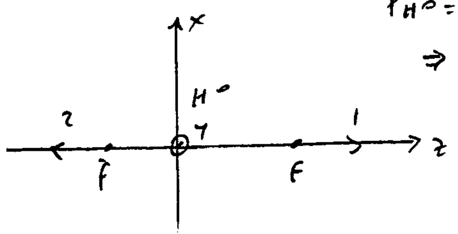
$$|M|^2 = M^+M = \frac{g^2 m_f^2}{4M_W^2} B_f^2 \sum_s (\bar{U_1} V_2)(\bar{V_2} U_1)$$

$$= \frac{g^2 m_f^2}{4M_W^2} B_f^2 \, Tr\left((\not{P}_1 + m_f)(\not{P}_2 - m_f)\right)$$

$$= \frac{g^2 m_f^2}{4M_W^2} B_f^2 \left[Tr(\not{P}_1 \not{P}_2) - m_f^2 4 \right]$$

$$\overline{|M|^2} = \frac{g^2 m_f^2}{M_W^2} B_f^2 \left[P_1 \cdot P_2 - m_f^2 \right]$$

In the rest frame of H^0:



$P_{H^0} = P_1 + P_2$

$\Rightarrow M_{H^0}^2 = 2m_f^2 + 2 P_1 \cdot P_2$

$P_1 \cdot P_2 = \dfrac{M_{H^0}^2 - 2m_f^2}{2}$

$$\overline{|M|^2} = \frac{g^2 m_f^2}{M_W^2} B_f^2 \left[\frac{M_{H^0}^2}{2} - m_f^2 - m_f^2 \right] = \frac{g^2 m_f^2}{2M_W^2} B_f^2 M_{H^0}^2 \left(1 - \frac{4m_f^2}{M_{H^0}^2} \right)$$

$$d\Gamma = \frac{\overline{|M|^2}\,|\vec{P_1}|\,d\Omega}{32\pi^2 M_{H^0}^2}$$

$$P_{H^0} = (M_{H^0}, 0, 0, 0)$$

$$P_1 = (E_1, 0, 0, P_1)$$

$$P_2 = (E_1, 0, 0, -P_1)$$

$$E_1 = \frac{M_{H^0}}{2}$$

$$E_1^2 = m_f^2 + |\vec{P_1}|^2$$

$$|\vec{P_1}|^2 = \frac{M_{H^0}^2}{4} - m_f^2$$

$$|\vec{P_1}|^2 = \frac{1}{4}(M_{H^0}^2 - 4m_f^2) = \frac{1}{4} M_{H^0}^2 \left(1 - 4\frac{m_f^2}{M_{H^0}^2}\right)$$

$$|\vec{P_1}| = \frac{M_{H^0}}{2}\left(1 - \frac{4m_f^2}{M_{H^0}^2}\right)^{1/2}$$

$$\Gamma(H^0 \to f\bar{f}) = \frac{\frac{g^2 m_f^2}{2M_W^2} B_f^2 \cancel{M_{H^0}^2}\left(1 - \frac{4m_f^2}{M_{H^0}^2}\right)\frac{M_{H^0}}{2}\left(1 - \frac{4m_f^2}{M_{H^0}^2}\right)^{1/2}}{8\pi \cancel{M_{H^0}^2}} N_f$$

$$\Gamma(H^0 \to f\bar{f}) = \frac{g^2 m_f^2 M_{H^0} B_f^2}{32\pi M_W^2}\left(1 - \frac{4m_f^2}{M_{H^0}^2}\right)^{3/2} N_f$$

$$\boxed{\Gamma(H^0 \to f\bar{f}) = \frac{\sqrt{2}\,G_F}{8\pi} m_f^2 M_{H^0}\left(1 - \frac{4m_f^2}{M_{H^0}^2}\right)^{3/2} B_f^2 N_f}$$

where $N_f = \begin{cases} 3 & \text{for } f = u, c, d, s, b \quad \text{quark} \\ 1 & \text{for } f = e^-, \mu^-, \tau^- \quad \text{lepton} \end{cases}$

$$B_f = \begin{cases} \dfrac{\sin\alpha}{\sin\beta} & \text{for } f = u, c \\[2ex] \dfrac{\cos\alpha}{\cos\beta} & \text{for } f = d, s, b, e^-, \mu^-, \tau^- \end{cases}$$

i) for $f = u, c$

$$B_f^2 = \frac{\sin^2\alpha}{\sin^2\beta} = \sin^2\alpha(1 + \cot^2\beta)$$

ii) for $f = d, s, b, e^-, \mu^-, \tau^-$ $\qquad B_f^2 = \dfrac{\cos^2\alpha}{\cos^2\beta} = (1 - \sin^2\alpha)(1 + \tan^2\beta)$

$\therefore$

$$\boxed{\Gamma(H^0 \rightarrow f\bar{f}) = \frac{\sqrt{2}\, G_F}{8\pi}\, m_f^2\, M_{H^0} \left(1 - \frac{4 m_f^2}{M_{H^0}^2}\right)^{3/2} N_f\, B_f^2}$$

$$\text{where} \quad N_f = \begin{cases} 3 & \text{for quarks} \\ 1 & \text{for leptons} \end{cases}$$

$$B_f^2 = \begin{cases} \sin^2\alpha\,(1+\cot^2\beta) & \text{for } f = u, c \\ (1-\sin^2\alpha)\,(1+\tan^2\beta) & \text{for } f = d, s, b, e^-, \mu^-, \tau^- \end{cases}$$

$$\sin^2\alpha = \frac{1}{2}\left\{ 1 + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)} \right] \right]\right\}$$

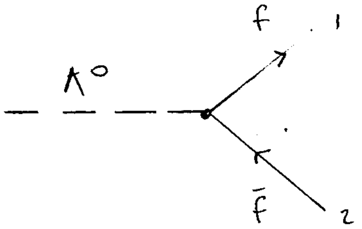
$$g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta) = \left[\left(1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}\right)^2 - 4\left(\frac{M_Z^2}{M_{H^\pm}^2}\right)\left(1 - \frac{M_W^2}{M_{H^\pm}^2}\right)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2 \right]^{1/2}$$

OK

$A^0 \to f\bar{f}$ $\quad (M_{A^0} < M_{H^\pm})$

if $f = u, c$

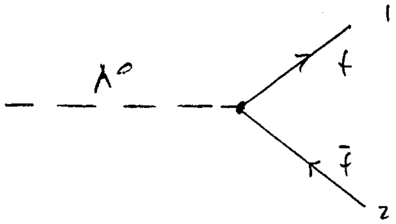
a)



$-\frac{g\, m_f \cot\beta}{2M_W}\gamma^5$

if $f = d, s, b$

b)



$-\frac{g\, m_f \tan\beta}{2M_W}\gamma^5$

$$-iM_a = \bar{U}_1\left(-\frac{g\, m_f A_f}{2M_W}\gamma^5\right)V_2 \qquad \text{where } A_f = \begin{cases}\cot\beta & f = u, c\\ \tan\beta & f = d, s, b\end{cases}$$

$$M_a = -\frac{i g\, m_f A_f}{2M_W}\,\bar{U}_1\gamma^5 V_2$$

$$\overline{|M_a|^2} = \frac{g^2 m_f^2 A_f^2}{4M_W^2}\sum_s[U_1^+\gamma^0\gamma^5 V_2]^+[\bar{U}_1\gamma^5 V_2]$$

$$= \frac{g^2 m_f^2 A_f^2}{4M_W^2}\sum_s[V_2^+\gamma^5\gamma^0 U_1][\bar{U}_1\gamma^5 V_2]$$

$$= -\frac{g^2 m_f^2 A_f^2}{4M_W^2}\sum_s[\bar{V}_2\gamma^5 U_1][\bar{U}_1\gamma^5 V_2]$$

$$= -\frac{g^2 m_f^2 A_f^2}{4M_W^2}\,\mathrm{Tr}[(\not{p}_1 + m_f)\gamma^5(\not{p}_2 - m_f)\gamma^5]$$

$$= +\frac{g^2 m_f^2 A_f^2}{4M_W^2}\,\mathrm{Tr}[(\not{p}_1 + m_f)\overbrace{\gamma^5\gamma^5}^{I}(\not{p}_2 + m_f)]$$

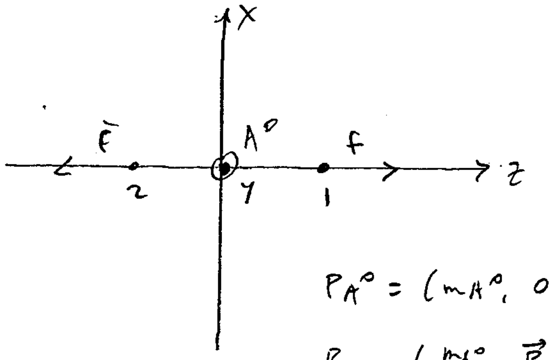
$$= \frac{g^2 m_f^2 A_f^2}{4M_W^2}\,\mathrm{Tr}[(\not{p}_1 + m_f)(\not{p}_2 + m_f)]$$

$$\mathrm{Tr}((\not{p}_1 + m_f)(\not{p}_2 + m_f)) = \mathrm{Tr}(\not{p}_1\not{p}_2) + 4m_f^2 = 4(p_1\cdot p_2) + 4m_f^2$$

$$\overline{|M_a|^2} = \frac{g^2 m_f^2 A_f^2 \cancel{4}}{\cancel{4} M_W^2} [p_1 \cdot p_2 + m_f^2]$$

$$\overline{|M_a|^2} = \frac{g^2 m_f^2 A_f^2}{M_W^2} (p_1 \cdot p_2 + m_f^2)$$

In the rest frame of A^0



$$P_{A^0} = (m_{A^0}, 0, 0, 0)$$

$$p_1 = \left(\frac{m_{A^0}}{2}, \vec{p}_1 \right)$$

$$p_2 = \left(\frac{m_{A^0}}{2}, -\vec{p}_1 \right)$$

$$P_{A^0} = p_1 + p_2$$

$$m_{A^0}^2 = 2 m_f^2 + 2 p_1 \cdot p_2$$

$$p_1 \cdot p_2 = \frac{m_{A^0}^2 - 2 m_f^2}{2} = \frac{m_{A^0}^2}{2} - m_f^2$$

$$\Rightarrow \overline{|M_a|^2} = \frac{g^2 m_f^2 A_f^2}{M_W^2} \frac{m_{A^0}^2}{2}$$

$$d\Gamma = \frac{\overline{|M_a|^2} \, |\vec{p}_1| \, d\Omega}{32 \pi^2 m_{A^0}^2}$$

$$m_f^2 = p_1^2 = \frac{m_{A^0}^2}{4} - |\vec{p}_1|^2$$

$$\Rightarrow |\vec{p}_1| = \left(\frac{m_{A^0}^2}{4} - m_f^2 \right)^{1/2} = \frac{(m_{A^0}^2 - 4 m_f^2)^{1/2}}{2}$$

$$\Gamma = \frac{g^2 m_f^2 A_f^2 \cancel{m_{A^0}^2}}{\cancel{2} M_W^2} \frac{(m_{A^0}^2 - 4 m_f^2)^{1/2}}{\cancel{2}} \frac{\cancel{4\pi}}{32 \cancel{\pi^2} \cancel{m_{A^0}^2}} \overset{\nearrow \text{color factor}}{N_f}$$

$$\Gamma = \frac{g^2 m_f^2 A_f^2 (m_{A^0}^2 - 4 m_f^2)^{1/2} N_f}{32 \pi M_W^2}$$

$\Rightarrow$

$$\boxed{\Gamma(A^0 \to f\bar{f}) = \frac{g^2 m_f^2 A_f^2 (m_{A^0}^2 - 4m_f^2)^{1/2} N_f}{32\pi M_W^2}}$$

Where $N_f = \begin{cases} 3 & \text{for quarks} \\ 1 & \text{for leptons} \end{cases}$

$A_f = \begin{cases} \cot\beta & \text{for } f = u, c \\ \tan\beta & \text{for } f = d, s, b, e^-, \mu^-, \tau^- \end{cases}$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

$$\Rightarrow \Gamma(A^0 \to f\bar{f}) = \frac{G_F}{4\sqrt{2}\pi} m_f^2 A_f^2 (m_{A^0}^2 - 4m_f^2)^{1/2} N_f$$

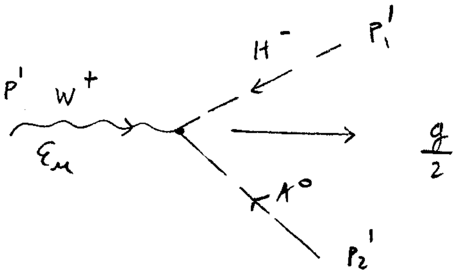
$$\boxed{\Gamma(A^0 \to f\bar{f}) = \frac{\sqrt{2}\, G_F}{8\pi} m_f^2 A_f^2 (m_{A^0}^2 - 4m_f^2)^{1/2} N_f}$$

or $$\boxed{\Gamma(A^0 \to f\bar{f}) = \frac{\sqrt{2}\, G_F}{8\pi} m_{A^0} m_f^2 A_f^2 \left(1 - 4\frac{m_f^2}{m_{A^0}^2}\right)^{1/2} N_f}$$

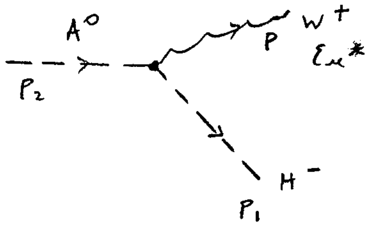
$A^0 \rightarrow W^\pm H^\mp$ decay: $\quad (M_{A^0} > M_{W^+} + M_{H^-})$

OK

$$\mathcal{L} = \frac{g}{2} W_\mu^+ H^- \overleftrightarrow{\partial^\mu} A^0 = \frac{g}{2} W_\mu^+ \left(H^-(\partial^\mu A^0) - (\partial^\mu H^-) A^0\right)$$



$$\longrightarrow \quad \frac{g}{2}\left(P_2'^\mu - P_1'^\mu\right) = \frac{g}{2}\left(P_2' - P_1'\right)^\mu$$



$$-iM = \frac{g}{2}(P_2 + P_1)^\mu \varepsilon_\mu^*$$

$$\overline{|M|^2} = \frac{g^2}{4}(P_2 + P_1)^\mu (P_2 + P_1)^\nu \sum_\lambda \varepsilon_\mu^\lambda \varepsilon_\nu^{\lambda *}$$

$$\overline{|M|^2} = \frac{g^2}{4}(P_2 + P_1)^\mu (P_2 + P_1)^\nu \left[-\eta_{\mu\nu} + \frac{P_\mu P_\nu}{M_W^2} \right]$$

$$\overline{|M|^2} = \frac{g^2}{4}\left[-(P_1 + P_2)^2 + \frac{(P.(P_1 + P_2))^2}{M_W^2} \right]$$

$$P_2 = P + P_1$$

$$P = P_2 - P_1$$

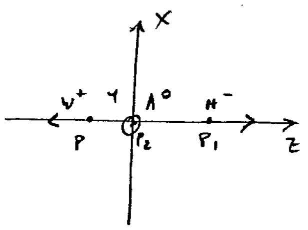
$$M_W^2 = M_{A^0}^2 + M_{H^-}^2 - 2 P_1 . P_2$$

$$P_1 . P_2 = \frac{M_{A^0}^2 + M_{H^-}^2 - M_W^2}{2}$$

$$P.(P_1 + P_2) = (P_2 - P_1).(P_2 + P_1) = M_{A^0}^2 - M_H^2 .$$

$$\Rightarrow \overline{|M|^2} = \frac{g^2}{4}\left[-\cancel{M_H^2} - \cancel{M_{A^0}^2} - \cancel{M_{A^0}^2} - \cancel{M_H^2} + \cancel{M_W^2} + \frac{(M_{A^0}^2 - M_H^2)^2}{M_W^2} \right]$$

$$\therefore \boxed{\overline{|M|}^2 = \frac{g^2}{4}\left[M_W^2 - 2\left(M_{H^-}^2 + M_{A^0}^2\right) + \frac{\left(M_{A^0}^2 - M_{H^-}^2\right)^2}{M_W^2}\right]}$$



In the rest frame of A^0:

$$P_2 = (M_{A^0}, 0, 0, 0) \ ; \quad P_1 = (E_1, \vec{P}_1) \ ; \quad P = (E, -\vec{P}_1)$$

$$M_{A^0} = E_1 + E$$

$$P_2 = P + P_1$$

$$M_{A^0}^2 = M_W^2 + M_H^2 + 2\left[E_1 E + |\vec{P}_1|^2\right] \qquad (*)$$

$$P_1 = P_2 - P$$

$$M_H^2 = M_{A^0}^2 + M_W^2 - 2 E M_{A^0}$$

$$E = \frac{M_{A^0}^2 + M_W^2 - M_H^2}{2 M_{A^0}}$$

$$E_1 = M_{A^0} - \frac{(M_{A^0}^2 + M_W^2 - M_H^2)}{2 M_{A^0}}$$

$$E_1 = \frac{M_{A^0}^2 - M_W^2 + M_H^2}{2 M_{A^0}}$$

$$M_{A^0}^2 = M_W^2 + M_H^2 + \frac{1}{2 M_{A^0}^2}\left(M_{A^0}^2 + (M_W^2 - M_H^2)\right)\left(M_{A^0}^2 - (M_W^2 - M_H^2)\right) + 2|\vec{P}_1|^2$$

$$\frac{2 M_{A^0}^4 - 2 M_W^2 M_{A^0}^2 - 2 M_H^2 M_{A^0}^2 - \cancel{M_{A^0}^4} + M_W^4 + M_H^4 - 2 M_W^2 M_H^2}{4 M_{A^0}^2} = |\vec{P}_1|^2$$

$$|\vec{P}_1| = \frac{\lambda^{1/2}(M_{A^0}^2, M_W^2, M_H^2)}{(4 M_{A^0}^2)^{1/2}}$$

$$\boxed{|\vec{P}_1| = \frac{\lambda^{1/2}(M_{A^0}^2, M_W^2, M_H^2)}{2 M_{A^0}}}$$

$$\Gamma = \frac{g^2}{\cancel{4}}\left[M_W^2 - 2(M_H^2 + M_{A^0}^2) + \frac{(M_{A^0}^2 - M_H^2)^2}{M_W^2}\right] \frac{\lambda^{1/2}(M_{A^0}^2, M_W^2, M_H^2)}{\cancel{2} M_{A^0}} \frac{\cancel{4\pi}}{32\cancel{\pi^2} M_{A^0}^2}$$

$$\Rightarrow \boxed{\Gamma(A^0 \to W^+ H^-) = g^2\left[M_W^2 - 2(M_{H^-}^2 + M_{A^0}^2) + \frac{(M_{A^0}^2 - M_H^2)^2}{M_W^2}\right] \frac{\lambda^{1/2}(M_{A^0}^2, M_W^2, M_H^2)}{64\pi M_{A^0}^3}}$$

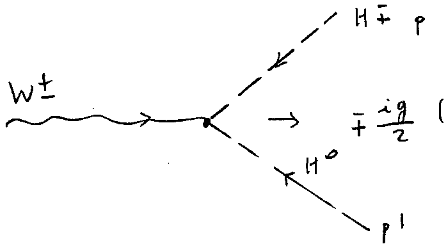
$$\boxed{\Gamma(A^0 \to W^+ H^-) = \frac{g^2 \lambda^{3/2}(M_{A^0}^2, M_W^2, M_{H^\pm}^2)}{64\pi M_{A^0}^3 M_W^2} = \frac{\sqrt{2}\, G_F\, \lambda^{3/2}(M_{A^0}^2, M_W^2, M_{H^\pm}^2)}{16\pi M_{A^0}^3}}$$

$$\Gamma(A^0 \to W^\pm H^\mp) = \frac{\sqrt{2}\, G_F\, \lambda^{3/2}\,(M_{A^0}^2, M_W^2, M_{H^\pm}^2)}{16\pi M_{A^0}^3}$$

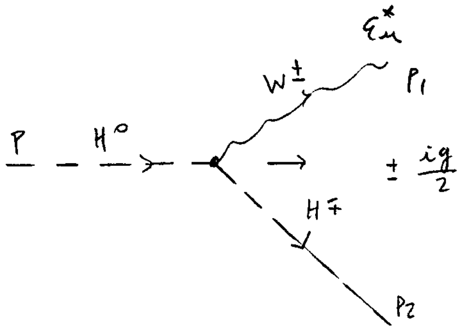
where $M_{A^0} > M_{W^+} + M_{H^-}$

and $\lambda(M_{A^0}^2, M_W^2, M_{H^\pm}^2) = M_{A^0}^4 + M_W^4 + M_{H^\pm}^4 - 2 M_{A^0}^2 M_W^2 - 2 M_{A^0}^2 M_{H^\pm}^2 - 2 M_W^2 M_{H^\pm}^2$

$\underline{H^0 \to W^\pm H^\mp \text{ decay}}$ $(m_{H^0} > M_{W^\pm} + M_{H^\pm})$ OK



$\rightarrow \mp \frac{ig}{2}(p'-p)^\mu \sin(\alpha-\beta)$



$\rightarrow \pm \frac{ig}{2}\sin(\alpha-\beta)(P+P_2)^\mu$

$$-iM = \pm \frac{ig}{2}\sin(\alpha-\beta)(P+P_2)^\mu \varepsilon_\mu^*$$

$$\overline{|M|^2} = \frac{g^2}{4}\sin^2(\alpha-\beta)(P+P_2)^\mu (P+P_2)^\nu \sum_\lambda \varepsilon_\mu^{\lambda *} \varepsilon_\nu^\lambda$$

$$\overline{|M|^2} = \frac{g^2}{4}\sin^2(\alpha-\beta)(P+P_2)^\mu (P+P_2)^\nu \left[-\eta_{\mu\nu} + \frac{P_{1\mu}P_{1\nu}}{M_W^2}\right]$$

$$\overline{|M|^2} = \frac{g^2}{4}\sin^2(\alpha-\beta)\left[-(P+P_2)^2 + \frac{(P_1\cdot(P+P_2))^2}{M_W^2}\right]$$

$$P = P_1 + P_2$$

$$P_1^2 = (P-P_2)^2$$

$$M_W^2 = m_{H^0}^2 + M_{H^\pm}^2 - 2P\cdot P_2$$

$$P\cdot P_2 = \frac{m_{H^0}^2 + M_{H^\pm}^2 - M_W^2}{2} \Rightarrow 2(P\cdot P_2) = m_{H^0}^2 + M_{H^\pm}^2 - M_W^2$$

$$(P+P_2)^2 = m_{H^0}^2 + M_{H^\pm}^2 + 2P\cdot P_2$$

$$= \cancel{m_{H^0}^2} + M_{H^\pm}^2 + m_{H^0}^2 + \cancel{M_{H^\pm}^2} - M_W^2$$

$$(P+P_2)^2 = 2m_{H^0}^2 + 2M_{H^\pm}^2 - M_W^2$$

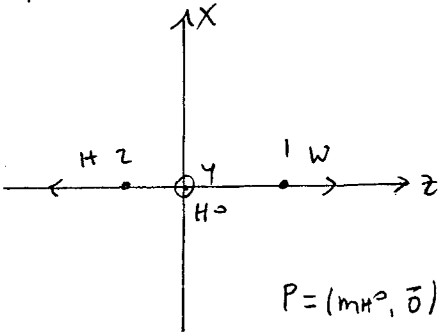
$$(P_1\cdot(P+P_2)) = (P-P_2)\cdot(P+P_2) = P^2 - P_2^2 = m_{H^0}^2 - M_{H^\pm}^2$$

$$\overline{|M|^2} = \frac{g^2}{4}\sin^2(\alpha-\beta)\left[-2m_{H^0}^2 - 2M_{H^\pm}^2 + M_W^2 + \frac{(m_{H^0}^2 - M_{H^\pm}^2)^2}{M_W^2}\right]$$

$$\overline{|M|^2} = \frac{g^2}{4M_W^2}\sin^2(\alpha-\beta)\left[-2m_{H^0}^2M_W^2 - 2M_{H^\pm}^2M_W^2 + M_W^4 + m_{H^0}^4 + M_{H^\pm}^4 - 2m_{H^0}^2M_{H^\pm}^2\right]$$

$$\boxed{\overline{|M|^2} = \frac{g^2}{4M_W^2}\sin^2(\alpha-\beta)\,\lambda(m_{H^0}^2, M_{W^\pm}^2, M_{H^\pm}^2)}$$

In the rest frame of H^0



$$P = (m_{H^0}, \vec{0})$$

$$P_1 = (E_1, \vec{P}_1)\ ; \quad P_2 = (E_2, -\vec{P}_1)$$

$$P_2^2 = (P-P_1)^2$$

$$M_{H^\pm}^2 = m_{H^0}^2 + M_W^2 - 2m_{H^0}E_1$$

$$E_1 = \frac{-M_{H^\pm}^2 + m_{H^0}^2 + M_W^2}{2m_{H^0}}$$

$$E_1^2 = M_W^2 + |\vec{P}_1|^2$$

$$|\vec{P}_1|^2 = \frac{(-M_{H^\pm}^2 + m_{H^0}^2 + M_W^2)^2}{4m_{H^0}^2} - M_W^2$$

$$|\vec{P}_1|^2 = \frac{M_{H^\pm}^4 + m_{H^0}^4 + M_W^4 - 2M_{H^\pm}^2m_{H^0}^2 - 2M_{H^\pm}^2M_W^2 + 2m_{H^0}^2M_W^2 - 4m_{H^0}^2M_W^2}{4m_{H^0}^2}$$

$$\boxed{|\vec{P}_1| = \frac{\lambda^{1/2}(m_{H^0}^2, M_W^2, M_{H^\pm}^2)}{2m_{H^0}}}$$

$$\Gamma(H^0\to W^\pm H^\mp) = \frac{g^2}{4M_W^2}\sin^2(\alpha-\beta)\,\frac{\lambda^{3/2}(m_{H^0}^2, M_W^2, M_{H^\pm}^2)}{2m_{H^0}}\,\frac{4\cancel{\pi}}{32\cancel{\pi}^2 m_{H^0}^2}$$

$$\Gamma = \frac{g^2}{4M_W^2} \frac{\sin^2(\alpha-\beta)\ \lambda^{3/2}}{16\pi\ m_{H^0}{}^3}$$

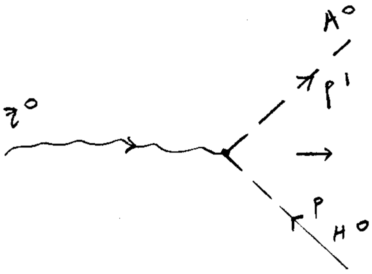
$$\Gamma = \frac{\sqrt{2}\ G_F\ [\sin\alpha\cos\beta - \sin\beta\cos\alpha]^2}{16\pi\ m_{H^0}{}^3}\ \lambda^{3/2}$$

$$\Gamma = \frac{\sqrt{2}\ G_F\ \cos^2\alpha\ \cos^2\beta\ [\tan\alpha - \tan\beta]^2\ \lambda^{3/2}}{16\pi\ m_{H^0}{}^3}$$

$$\boxed{\Gamma(H^0 \to W^\pm H^\mp) = \frac{\sqrt{2}\ G_F\ (\tan\alpha - \tan\beta)^2}{16\pi\ m_{H^0}{}^3\ (1+\tan^2\alpha)(1+\tan^2\beta)}\ \lambda^{3/2}(m_{H^0}{}^2, M_W{}^2, M_{H^\pm}{}^2)}$$

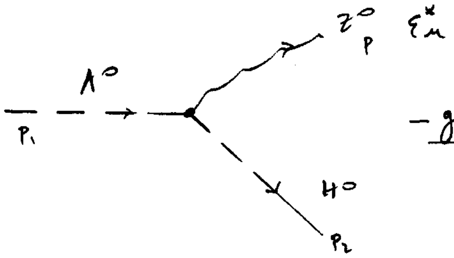
↗ ↑

$A^0 \to Z^0 H^0$ decay OK



$$\rightarrow\ -\frac{g\sin(\beta-\alpha)}{2\cos\theta_W}\ (p+p')^\mu$$

⇒



$$-\frac{g\sin(\beta-\alpha)}{2\cos\theta_W}(-P_2-P_1)^\mu = \frac{g\sin(\beta-\alpha)}{2\cos\theta_W}(P_1+P_2)^\mu$$

∴ Γ is obtained from the corresponding to $A^0 \to Z^0 h^0$ replacing

$$-\cos(\beta-\alpha) \rightarrow \sin(\beta-\alpha)$$

$$\Rightarrow\ \Gamma(A^0 \to Z^0 H^0) = \frac{g^2\sin^2(\beta-\alpha)\ \lambda^{3/2}(m_{A^0}{}^2, m_{H^0}{}^2, M_Z{}^2)}{64\pi\ m_{A^0}{}^3\ \underbrace{\cos^2\theta_W M_Z{}^2}_{M_W{}^2}}$$

$$\Gamma(A^0 \to Z^0 H^0) = \frac{\sqrt{2}\ G_F\ (\sin\beta\cos\alpha - \sin\alpha\cos\beta)^2\ \lambda^{3/2}(m_{A^0}{}^2, m_{H^0}{}^2, M_Z{}^2)}{16\pi\ m_{A^0}{}^3}$$

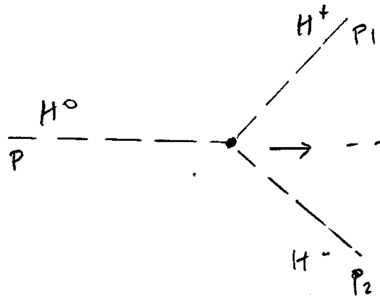
$$\Gamma(A^0 \to Z^0 H^0) = \frac{\sqrt{2}\, G_F \cos^2\alpha \cos^2\beta \, (\tan\beta - \tan\alpha)^2}{16\pi\, m_{A^0}^3}\, \lambda^{3/2}$$

$$\boxed{\Gamma(A^0 \to Z^0 H^0) = \frac{\sqrt{2}\, G_F \, (\tan\beta - \tan\alpha)^2 \, \lambda^{3/2}(m_{A^0}^2, m_{H^0}^2, M_Z^2)}{16\pi\, m_{A^0}^3 \, (1+\tan^2\alpha)(1+\tan^2\beta)}}$$

$$\nearrow \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \uparrow$$

$H^0 \to H^+H^-$ decay :

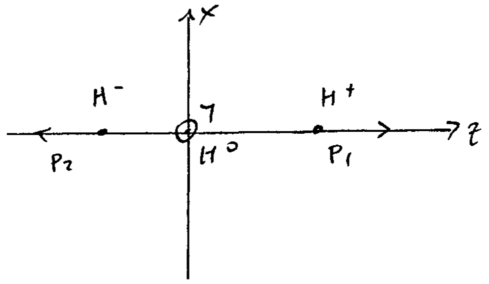
OK



$$\rightarrow \; -ig\left[M_W\cos(\beta-\alpha) - \frac{M_Z}{2\cos\theta_W}\cos 2\beta\,\cos(\beta+\alpha)\right]$$

$$-i\mathcal{M} = -ig\left[M_W\cos(\beta-\alpha) - \frac{M_Z}{2\cos\theta_W}\cos 2\beta\,\cos(\beta+\alpha)\right]$$

$$\overline{|\mathcal{M}|^2} = g^2\left[M_W\cos(\beta-\alpha) - \frac{M_Z}{2\cos\theta_W}\cos 2\beta\,\cos(\beta+\alpha)\right]^2$$



$$P = P_1 + P_2$$

$$P = (m_{H^0}, \vec{0})\,;\; P_1 = (E_1, \vec{P}_1)\,;\; P_2 = (E_1, -\vec{P}_1)$$

$$\Rightarrow \; E_1 = \frac{m_{H^0}}{2}$$

$$E_1^2 = m_{H^+}^2 + |\vec{P}_1|^2$$

$$\frac{m_{H^0}^2}{4} = m_{H^+}^2 + |\vec{P}_1|^2$$

$$|\vec{P}_1|^2 = \frac{m_{H^0}^2}{4} - m_{H^+}^2 = \frac{1}{4}m_{H^0}^2\left(1 - 4\frac{m_{H^+}^2}{m_{H^0}^2}\right)$$

$$|\vec{P}_1| = \frac{m_{H^0}}{2}\left(1 - \frac{4m_{H^+}^2}{m_{H^0}^2}\right)^{1/2}$$

$$\Gamma(H^0 \to H^+H^-) = g^2\left[M_W\cos(\beta-\alpha) - \frac{M_Z}{2\cos\theta_W}\cos 2\beta\,\cos(\beta+\alpha)\right]^2 \frac{\cancel{m_{H^0}}}{2}\left(1 - 4\frac{m_{H^+}^2}{m_{H^0}^2}\right)^{1/2}\frac{\cancel{4\pi}}{\cancel{32}\pi^2\,\cancel{m_{H^0}^2}}$$

$$= \frac{g^2 M_W^4}{M_W^2}\left[(\cos\beta\cos\alpha + \sin\beta\sin\alpha) - \frac{1}{2\cos^2\theta_W}\cos 2\beta\,(\cos\beta\cos\alpha - \sin\beta\sin\alpha)\right]^2 .$$

$$\cdot\; \frac{1}{16\pi m_{H^0}}\left(1 - \frac{4m_{H^\pm}^2}{m_{H^0}^2}\right)^{1/2}$$

$$\Gamma(H^0 \to H^+H^-) = \frac{\sqrt{2}\, G_F M_W^4}{4\pi m_{H^0}} \cos^2\beta \cos^2\alpha \left[(1+\tan\beta\tan\alpha) - \frac{1}{2\cos^2\theta_W}\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right) \cdot (1-\tan\beta\tan\alpha) \right]^2 \left(1 - 4\frac{m_{H^\pm}^2}{m_{H^0}^2}\right)^{1/2}$$

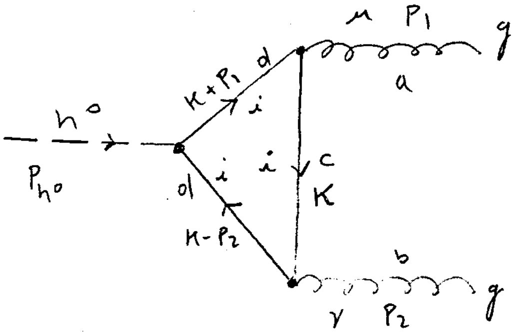
$$\downarrow \qquad \swarrow$$

$$\boxed{\Gamma(H^0 \to H^+H^-) = \frac{\sqrt{2}\, G_F M_W^4}{4\pi m_{H^0} (1+\tan^2\beta)(1+\tan^2\alpha)} \left[(1+\tan\beta\tan\alpha) - \frac{1}{2\cos^2\theta_W}\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right) \cdot (1-\tan\beta\tan\alpha) \right]^2 \left(1 - 4\frac{m_{H^\pm}^2}{m_{H^0}^2}\right)^{1/2}}$$

$h^\circ \to 2g$ decay :

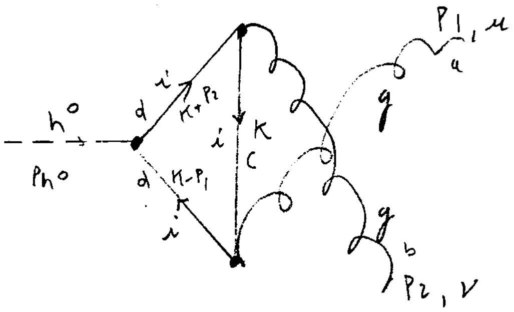
C -Martin

→ time



①

+



②

$$P_{h^\circ} = P_1 + P_2$$

For $i = d, s, b$

$$-iM_{1a} = \overset{\nearrow \text{Fermion loop}}{(-1)}\sum_{i=d,s,b}\int\frac{d^dK}{(2\pi)^d}\mathrm{Tr}\Big\{(-ig_s\gamma^\nu T^b_{dc})\left(\frac{i}{\not{K}-\not{P}_2-m_i}\right)\cdot\left(\frac{igm_i\sin\alpha}{2M_W\cos\beta}\right)\left(\frac{i}{\not{K}+\not{P}_1-m_i}\right)(-ig_s\gamma^\mu T^a_{cd})\cdot$$

$$\cdot\left(\frac{i}{\not{K}-m_i}\right)\Big\}\varepsilon^*_{1\mu}\varepsilon^*_{2\nu}\mu_1^{(4-d)}\mu_2^{\frac{(4-d)}{2}} \qquad (1)$$

$$-iM_{1a} = -\frac{g_s^2 g}{2M_W}\frac{\sin\alpha}{\cos\beta}\sum_{i=d,s,b}m_i\int\frac{d^dK}{(2\pi)^d}\mathrm{Tr}\Big\{\gamma^\nu(\not{K}-\not{P}_2+m_i)(\not{K}+\not{P}_1+m_i)\gamma^\mu\cdot$$

$$\cdot(\not{K}+m_i)\Big\}\cdot\frac{1}{[(K-P_2)^2-m_i^2][(K+P_1)^2-m_i^2][K^2-m_i^2]}\varepsilon^*_{1\mu}\varepsilon^*_{2\nu}$$

$$\mu_1^{(4-d)}\mu_2^{\frac{(4-d)}{2}}\mathrm{Tr}(T^aT^b)\cdot(-1) \qquad (2)$$

$$-iM_{2a} = (-1)\sum_{i=d,s,b}\int\frac{d^dK}{(2\pi)^d}\mathrm{Tr}\Big\{(-ig_s\gamma^\mu T^a_{dc})\left(\frac{i}{\not{K}-\not{P}_1-m_i}\right)\cdot\left(\frac{igm_i\sin\alpha}{2M_W\cos\beta}\right)\left(\frac{i}{\not{K}+\not{P}_2-m_i}\right)$$

$$(-ig_s\gamma^\nu T^b_{cd})\left(\frac{i}{\not{K}-m_i}\right)\Big\}\varepsilon^*_{1\mu}\varepsilon^*_{2\nu}\mu_1^{(4-d)}\mu_2^{\frac{(4-d)}{2}} \qquad (3)$$

$$-iM_{2a} = (-1)-\frac{g_s^2 g}{2M_W}\frac{\sin\alpha}{\cos\beta}\sum_{i=d,s,b}m_i\int\frac{d^dK}{(2\pi)^d}\mathrm{Tr}\Big\{\gamma^\mu(\not{K}-\not{P}_1+m_i)(\not{K}+\not{P}_2+m_i)\gamma^\nu(\not{K}+m_i)\Big\}\cdot$$

$$\cdot\frac{1}{[(K-P_1)^2-m_i^2][(K+P_2)^2-m_i^2][K^2-m_i^2]}\varepsilon^*_{1\mu}\varepsilon^*_{2\nu}\overset{T^q}{\mu_1^{(4-d)}}\mathrm{Tr}(T^aT^b)$$

$$\times\mu_2^{(4-d)/2}\underset{\to h^\circ}{} \qquad (4)$$

$$\mathrm{Tr}\{\gamma^\nu(\not{K}-\not{P}_2+m_i)(\not{K}+\not{P}_1+m_i)\overbrace{\gamma^\mu(\not{K}+m_i)}^{(\gamma^\mu\not{K}+m_i\gamma^\mu)}\}$$

$$=\mathrm{Tr}\{(\gamma^\nu\not{K}-\gamma^\nu\not{P}_2+m_i\gamma^\nu)(\not{K}\gamma^\mu\not{K}+m_i\not{K}\gamma^\mu+\not{P}_1\gamma^\mu\not{K}+m_i\not{P}_1\gamma^\mu+m_i\gamma^\mu\not{K}+m_i^2\gamma^\mu)\}$$

$$=\cancelto{0}{\mathrm{Tr}\{\gamma^\nu\not{K}\not{K}\gamma^\mu\not{K}\}}+m_i\mathrm{Tr}\{\gamma^\nu\not{K}\not{K}\gamma^\mu\}+\cancelto{0}{\mathrm{Tr}\{\gamma^\nu\not{K}\not{P}_1\gamma^\mu\not{K}\}}+m_i\mathrm{Tr}\{\gamma^\nu\not{K}\not{P}_1\gamma^\mu\}+m_i\mathrm{Tr}\{\gamma^\nu\not{K}\gamma^\mu\not{K}\}$$

$$+m_i^2\cancelto{0}{\mathrm{Tr}\{\gamma^\nu\not{K}\gamma^\mu\}}-\cancelto{0}{\mathrm{Tr}\{\gamma^\nu\not{P}_2\not{K}\gamma^\mu\not{K}\}}-m_i\mathrm{Tr}\{\gamma^\nu\not{P}_2\not{K}\gamma^\mu\}-\cancelto{0}{\mathrm{Tr}\{\gamma^\nu\not{P}_2\not{P}_1\gamma^\mu\not{K}\}}$$

$$-m_i\mathrm{Tr}\{\gamma^\nu\not{P}_2\not{P}_1\gamma^\mu\}-m_i\mathrm{Tr}\{\gamma^\nu\not{P}_2\gamma^\mu\not{K}\}-m_i^2\cancelto{0}{\mathrm{Tr}\{\gamma^\nu\not{P}_2\gamma^\mu\}}+m_i\mathrm{Tr}\{\gamma^\nu\not{K}\gamma^\mu\not{K}\}$$

$$+m_i^2\cancelto{0}{\mathrm{Tr}\{\gamma^\nu\not{K}\gamma^\mu\}}+m_i\mathrm{Tr}\{\gamma^\nu\not{P}_1\gamma^\mu\not{K}\}+m_i^2\cancelto{0}{\mathrm{Tr}\{\gamma^\nu\not{P}_1\gamma^\mu\}}+m_i^2\cancelto{0}{\mathrm{Tr}\{\gamma^\nu\gamma^\mu\not{K}\}}+m_i^3\mathrm{Tr}\{\gamma^\nu\gamma^\mu\}$$

$$=4m_iK^2\eta^{\mu\nu}+m_i\mathrm{Tr}\{\gamma^\nu\not{K}\gamma^\alpha\gamma^\mu P_{1\alpha}\}+2m_i\mathrm{Tr}\{\gamma^\mu\not{K}\gamma^\nu\not{K}\}-m_i\mathrm{Tr}\{\gamma^\nu\not{P}_2\gamma^\alpha\gamma^\mu K_\alpha\}$$

$$-m_i\mathrm{Tr}\{\gamma^\nu\not{P}_2\gamma^\alpha\gamma^\mu P_{1\alpha}\}-m_i\mathrm{Tr}\{\gamma^\mu\not{K}\gamma^\nu\not{P}_2\}+m_i\mathrm{Tr}\{\gamma^\mu\not{K}\gamma^\nu\not{P}_1\}+4m_i^3\eta^{\mu\nu}$$

$$=4m_iK^2\eta^{\mu\nu}+m_i\mathrm{Tr}\{\gamma^\nu\not{K}(2\eta^{\mu\alpha}-\gamma^\mu\gamma^\alpha)P_{1\alpha}\}+8m_i\{2K^\mu K^\nu-K^2\eta^{\mu\nu}\}$$

$$-m_i\mathrm{Tr}\{\gamma^\nu\not{P}_2(2\eta^{\mu\alpha}-\gamma^\mu\gamma^\alpha)K_\alpha\}-m_i\mathrm{Tr}\{\gamma^\nu\not{P}_2(2\eta^{\mu\alpha}-\gamma^\mu\gamma^\alpha)P_{1\alpha}\}$$

$$-4m_i\{K^\mu P_2^\nu+K^\nu P_2^\mu-(K\cdot P_2)\eta^{\mu\nu}\}+4m_i\{K^\mu P_1^\nu+K^\nu P_1^\mu-(K\cdot P_1)\eta^{\mu\nu}\}+4m_i^3\eta^{\mu\nu}$$

$$=4\cancel{m_i}K^2\eta^{\mu\nu}+2m_iP_1^\mu\mathrm{Tr}(\gamma^\nu\not{K})-m_i\mathrm{Tr}(\gamma^\nu\not{K}\gamma^\mu\not{P}_1)+16m_iK^\mu K^\nu-8m_i\cancel{K^2}\eta^{\mu\nu}$$

$$-2m_i\mathrm{Tr}(\gamma^\nu\not{P}_2)K^\mu+m_i\mathrm{Tr}(\gamma^\nu\not{P}_2\gamma^\mu\not{K})-2m_i\mathrm{Tr}(\gamma^\nu\not{P}_2)P_1^\mu+m_i\mathrm{Tr}(\gamma^\nu\not{P}_2\gamma^\mu\not{P}_1)$$

$$-4m_i\{K^\mu P_2^\nu+K^\nu P_2^\mu-(K\cdot P_2)\eta^{\mu\nu}\}+4m_i\{K^\mu P_1^\nu+K^\nu P_1^\mu-(K\cdot P_1)\eta^{\mu\nu}\}+4m_i^3\eta^{\mu\nu}$$

$$=-4m_iK^2\eta^{\mu\nu}+8m_iP_1^\mu K^\nu-4m_i\{\cancel{K^\nu P_1^\mu}+K^\mu P_1^\nu-(K\cdot P_1)\eta^{\mu\nu}\}+16m_iK^\mu K^\nu$$

$$-8m_iP_2^\nu K^\mu+4m_i\{P_2^\nu K^\mu\cancel{+P_2^\mu K^\nu}-(P_2\cdot K)\eta^{\mu\nu}\}-8m_iP_2^\nu P_1^\mu$$

$$+4m_i\{P_2^\nu P_1^\mu+P_2^\mu P_1^\nu-(P_1\cdot P_2)\eta^{\mu\nu}\}-4m_i\{\cancel{K^\mu P_2^\nu+K^\nu P_2^\mu}-(K\cdot P_2)\eta^{\mu\nu}\}$$

$$+4m_i\{K^\mu P_1^\nu\cancel{+K^\nu P_1^\mu}-(K\cdot P_1)\eta^{\mu\nu}\}+4m_i^3\eta^{\mu\nu}$$

$$=-4m_iK^2\eta^{\mu\nu}+8m_iP_1^\mu K^\nu-8m_iP_2^\nu K^\mu+16m_iK^\mu K^\nu-4m_iP_1^\mu P_2^\nu+4m_iP_2^\mu P_1^\nu$$

$$-4m_i(P_1\cdot P_2)\eta^{\mu\nu}+4m_i^3\eta^{\mu\nu}$$

$$\Rightarrow\mathrm{Tr}\{\ \}=4m_i\{-K^2\eta^{\mu\nu}+2P_1^\mu K^\nu-2P_2^\nu K^\mu+4K^\mu K^\nu-P_1^\mu P_2^\nu+P_2^\mu P_1^\nu-(P_1\cdot P_2)\eta^{\mu\nu}$$

$$+m_i^2\eta^{\mu\nu}\}$$

(The same as (230))

(5) Volume III

$$\frac{1}{abc} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[a(1-x-y) + bx + cy]^3}$$

$$\frac{1}{[(k+p_1)^2 - m_i^2]\,[(k-p_2)^2 - m_i^2]\,[k^2 - m_i^2]} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[k^2 + 2k\cdot(p_1 x - p_2 y) - m_i^2]^3} \qquad (6)$$

$$-iM_1a = (-1)\frac{-g_s^2 g}{2M_W}\frac{\sin\alpha}{\cos\beta}\sum_{i=d,s,b} m_i\,(I)\,\varepsilon^*_{1\mu}\varepsilon^*_{2\nu}\,\mu^{\frac{3}{2}(4-d)}\,\mathrm{Tr}(T^aT^b) \qquad (7)$$

$$-iM_1a = (-1)-\frac{g_s^2 g}{2M_W}\frac{\sin\alpha}{\cos\beta}\sum_i m_i^2\left(\frac{-4i\pi^2}{(2\pi)^4}\right)\cdot\int_0^1 dx\int_0^{1-x} dy\,\Big[4(p_1\cdot p_2)\,xy\,\eta^{\mu\nu} - 4xy\,p_2^{\mu}p_1^{\nu}$$
$$+ p_1^{\nu}p_2^{\mu} - (p_1\cdot p_2)\,\eta^{\mu\nu}\Big]\cdot\frac{1}{m_i^2 - 2xy(p_1\cdot p_2)}\,\mathrm{Tr}(T^aT^b)\,\varepsilon^*_{1\mu}\varepsilon^*_{2\nu} \qquad (8)$$

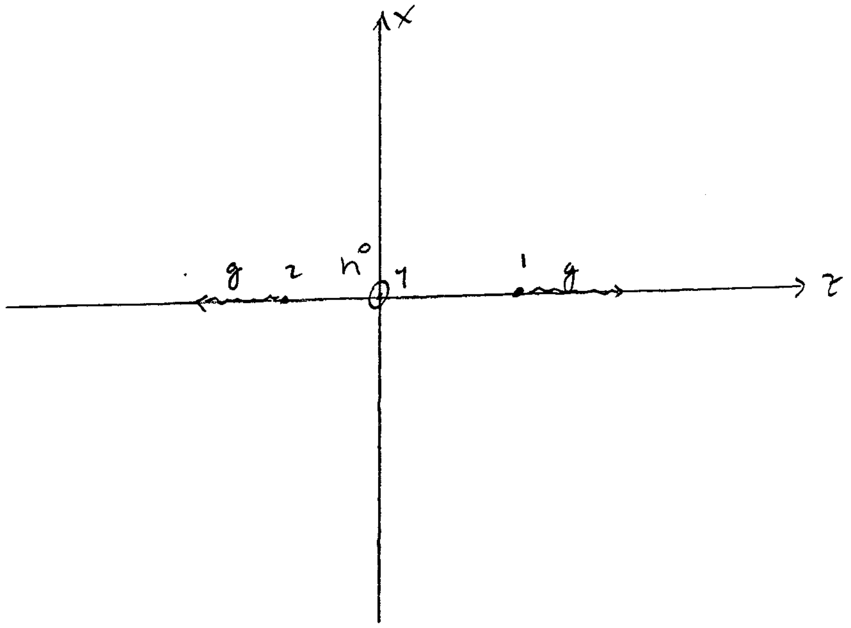
$$-iM_1a = (-1)-\frac{g_s^2 g}{2M_W}\frac{\sin\alpha}{\cos\beta}\left(\frac{-i}{4\pi^2}\right)\sum_i m_i^2\cdot\int_0^1 dx\,\Big\{(p_1\cdot p_2)\,\eta^{\mu\nu}\Big[-2\frac{(1-x)}{(p_1\cdot p_2)} - \frac{m_i^2}{x}\cdot\frac{1}{(p_1\cdot p_2)^2}\cdot$$
$$\cdot\ln\left|\frac{m_i^2 - 2x(1-x)(p_1\cdot p_2)}{m_i^2}\right| + \frac{1}{2x(p_1\cdot p_2)}\ln\left|\frac{m_i^2 - 2x(1-x)(p_1\cdot p_2)}{m_i^2}\right|\Big] + p_1^{\nu}p_2^{\mu}\Big[-\frac{1}{2x(p_1\cdot p_2)}\cdot$$
$$\cdot\ln\left|\frac{m_i^2 - 2x(1-x)(p_1\cdot p_2)}{m_i^2}\right| + \Big[\frac{2(1-x)}{(p_1\cdot p_2)} + \frac{m_i^2}{x(p_1\cdot p_2)^2}\ln\left|\frac{m_i^2 - 2x(1-x)(p_1\cdot p_2)}{m_i^2}\right|\Big]\Big]\Big\}\cdot$$
$$\mathrm{Tr}(T^aT^b)\,\varepsilon^*_{\mu_1}\varepsilon^*_{\nu_2} \qquad (9)$$

$$-iM_1a = (-1)-\frac{g_s^2 g}{2M_W}\frac{\sin\alpha}{\cos\beta}\left(\frac{-i}{4\pi^2}\right)\sum_i\Big\{\int_0^1 dx\,\left(p_1^{\nu}p_2^{\mu} - \frac{s}{2}\eta^{\mu\nu}\right)\Big[\ln\left|\frac{x^2 - x + \tau_i/4}{\tau_i/4}\right|\frac{\tau_i}{x}\cdot$$
$$\frac{(\tau_i - 1)}{4} + \tau_i(1-x)\Big]\Big\}\,\mathrm{Tr}(T^aT^b)\,\varepsilon^*_{1\mu}\varepsilon^*_{2\nu} \qquad (10)$$

$$-iM_1a = (-1)\frac{i g_s^2 g}{32 M_W \pi^2}\frac{\sin\alpha}{\cos\beta}\sum_{i=d,s,b}\Big\{\left(p_1^{\nu}p_2^{\mu} - \frac{s}{2}\eta^{\mu\nu}\right)\big[\tau_i(\tau_i - 1)f(\tau_i) + 2\tau_i\big]\Big\}$$
$$\cdot\,\mathrm{Tr}(T^aT^b)\,\varepsilon^*_{1\mu}\varepsilon^*_{2\nu} \qquad (11)$$

$$\boxed{-iM_1a = (-1)\frac{-i g_s^2 g}{64 M_W \pi^2}\frac{\sin\alpha}{\cos\beta}\,s\,\eta^{\mu\nu}\varepsilon^*_{1\mu}\varepsilon^*_{2\nu}\,\mathrm{Tr}(T^aT^b)\sum_{i=d,s,b}\tau_i\big[(\tau_i - 1)f(\tau_i) + 2\big]} \qquad (12)$$

This is because of:



$$\varepsilon_1^{\mu\,(\lambda=\pm 1)} = \mp (0,1,\pm i,0)\frac{1}{\sqrt{2}} \;;\quad \varepsilon_2^{\mu\,(\lambda=\pm 1)} = \mp\frac{1}{\sqrt{2}}(0,-1,\pm i,0)$$

$$P_1^\mu = (K^0,0,0,K^0) \;;\; P_2^\mu = (K^0,0,0,-K)$$

$$\Rightarrow P_{1\mu}\,\varepsilon^\mu_{1,2} = 0 \;;\quad P_{2\mu}\,\varepsilon^\mu_{1,2} = 0 \qquad (13)$$

in $-iM_{2a}$

$\mathrm{Tr}\{\gamma^\mu(\not{K}-\not{P}_1+m_i)(\not{K}+\not{P}_2+m_i)\gamma^\nu(\not{K}+m_i)\}$ is obtained from (5)

replacing $\nu \leftrightarrow \mu$, $P_1 \leftrightarrow P_2$

$$\Rightarrow \mathrm{Tr}\{\;\} = 4m_i\{-K^2\eta^{\mu\nu} + 2P_2^\nu K^\mu - 2P_1^\mu K^\nu + 4K^\mu K^\nu - P_2^\nu P_1^\mu + P_1^\nu P_2^\mu$$
$$-(P_1\cdot P_2)\eta^{\mu\nu} + m_i^2\eta^{\mu\nu}\} \qquad (14)$$

on the other hand:

$$\frac{1}{[(K+P_2)^2-m_i^2][(K-P_1)^2-m_i^2][K^2-m_i^2]} = 2\int_0^1 dx\int_0^{1-x}\frac{dy}{[K^2+2K\cdot(P_2x-P_1y)-m_i^2]^3} \qquad (15)$$

with $K' = K + (P_2x - P_1y)$

$K'^2 = K^2 + 2K\cdot(P_2x-P_1y) - 2xy(P_1\cdot P_2)$

$$\Rightarrow [k^2 + 2k\cdot(P_2 x - P_1 y) - m_i^2]^3 = [k'^2 + 2xy(P_1\cdot P_2) - m_i^2]^3 \qquad (16)$$

$$I' = \int \frac{d^d k}{(2\pi)^d}\, \mathrm{Tr}\{\ \}\cdot \frac{1}{[\ ][\ ][\ ]} \qquad (17)$$

$$I' = 4\int m_i \frac{d^d k'}{(2\pi)^d}\cdot \Big\{ -[k' - (P_2 x - P_1 y)]^2 \eta^{\mu\nu} + 2P_2^\nu [k' - (P_2 x - P_1 y)]^\mu$$

$$- 2P_1^\mu [k' - (P_2 x - P_1 y)]^\nu + 4[k' - (P_2 x - P_1 y)]^\mu [k' - (P_2 x - P_1 y)]^\nu - P_2^\nu P_1^\mu + P_1^\nu P_2^\mu$$

$$- (P_1\cdot P_2)\eta^{\mu\nu} + m_i^2 \eta^{\mu\nu} \Big\}\cdot 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[k'^2 + 2xy(P_1\cdot P_2) - m_i^2]^3}$$

$$I' = 8m_i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d k'}{(2\pi)^d} \Big\{ -k'^2 \eta^{\mu\nu} + 2k'\cdot(P_2 x - P_1 y)\eta^{\mu\nu} + 2xy(P_1\cdot P_2)\eta^{\mu\nu}$$

$$+ 2P_2^\nu k'^\mu - 2x P_2^\nu P_2^\mu + 2y P_2^\nu P_1^\mu - 2P_1^\mu k'^\nu + 2x P_1^\mu P_2^\nu - 2y P_1^\mu P_1^\nu$$

$$+ 4k'^\mu k'^\nu - 4k'^\mu (P_2 x - P_1 y)^\nu - 4k'^\nu (P_2 x - P_1 y)^\mu + 4(P_2 x - P_1 y)^\mu (P_2 x - P_1 y)^\nu$$

$$- P_2^\nu P_1^\mu + P_1^\nu P_2^\mu - (P_1\cdot P_2)\eta^{\mu\nu} + m_i^2 \eta^{\mu\nu} \Big\}\cdot [k'^2 + 2xy(P_1\cdot P_2) - m_i^2]^3$$

$$\int \frac{d^d k'\ k'^\mu}{[k'^2 - (m_i^2 - 2xy(P_1\cdot P_2))]^3} = 0 \qquad (18)$$

$$\int \frac{d^d k'}{(2\pi)^d} \frac{[-\eta^{\mu\nu} k'^2 + 4k'^\mu k'^\nu]}{[k'^2 - (m_i^2 - 2xy(P_1\cdot P_2))]^3} = -I_0\, \eta^{\mu\nu} [m_i^2 + (P_1 x - P_2 y)^2]$$

$$\Rightarrow I' = 8m_i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d k'}{(2\pi)^d} \Big\{ 4xy(P_1\cdot P_2)\eta^{\mu\nu} - 2x P_2^\mu P_2^\nu + 2y P_1^\mu P_2^\nu$$

$$4(x^2 P_2^\mu P_2^\nu - xy P_2^\mu P_1^\nu - xy P_1^\mu P_2^\nu + y^2 P_1^\mu P_1^\nu)$$

$$+ 2x P_1^\mu P_2^\nu - 2y P_1^\mu P_1^\nu + 4(x P_2^\mu - y P_1^\mu)(x P_2^\nu - y P_1^\nu) - P_2^\nu P_1^\mu + P_1^\nu P_2^\mu$$

$$- (P_1\cdot P_2)\eta^{\mu\nu} \Big\} \cdot [k'^2 + 2xy(P_1\cdot P_2) - m_i^2]^{-3} \qquad (20)$$

$$\int \frac{d^d k'}{[k'^2 - (m_i^2 - 2xy P_1\cdot P_2)]^3} = \frac{i\pi^2}{2[-m_i^2 + 2xy(P_1\cdot P_2)]} \qquad (21)$$

$$\Rightarrow I' = -\frac{8m_i}{(2\pi)^4} \frac{i\pi^2}{2} \int_0^1 dx \int_0^{1-x} dy \{\ \}\cdot [m_i^2 - 2xy(P_1\cdot P_2)]^{-1}. \qquad (22)$$

Now $P_1^\mu \varepsilon_{1\mu} = 0$; $P_2^\mu \varepsilon_{2\mu} = 0$ $\Rightarrow$

$$-iM_{2a} = (-1) - \frac{g_s^2 g}{2M_W} \frac{\sin\alpha}{\cos\beta} \sum_{i=d,s,b} m_i^2 \frac{(-4i\pi^2)}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \Big\{ 4xy(P_1\cdot P_2)\eta^{\mu\nu} - 4xy\, P_2^{\mu} P_1^{\nu} + P_1^{\nu} P_2^{\mu} - (P_1\cdot P_2)\eta^{\mu\nu} \Big\} \left[m_i^2 - 2xy(P_1\cdot P_2)\right]^{-1} \cdot \mathrm{Tr}(T^a T^b).$$
$$\varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \qquad (23)$$

Comparing (8) with (23) we can see that

$$\boxed{M_{1a} = M_{2a}} \qquad (24)$$

For $i = u,c,t$ we have to change $\frac{\sin\alpha}{\cos\beta} \rightarrow -\frac{\cos\alpha}{\sin\beta}$

$\Rightarrow$

$$\boxed{-iM_{1b} = -iM_{2b} = \frac{(-1)\, i g_s^2 g}{64 M_W \pi^2} \left(\frac{\cos\alpha}{\sin\beta}\right) S\, \eta^{\mu\nu} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \mathrm{Tr}(T^a T^b) \sum_{i=u,c,t} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right]}$$

$\Rightarrow$ (25)

$$M = M_{1a} + M_{2a} + M_{1b} + M_{2b} \qquad (26)$$

$$M = \frac{2 g_s^2 g}{64 M_W \pi^2} \left(\frac{\sin\alpha}{\cos\beta}\right) S\, \eta^{\mu\nu} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \mathrm{Tr}(T^a T^b) \sum_{i=d,s,b} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right] (-1)$$
$$- \frac{2 g_s^2 g}{64 M_W \pi^2} \left(\frac{\cos\alpha}{\sin\beta}\right) S\, \eta^{\mu\nu} \varepsilon_{1\mu} \varepsilon_{2\nu} \mathrm{Tr}(T^a T^b) \sum_{i=u,c,t} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right] (-1)$$

$$\boxed{M = \frac{g_s^2 g}{32 M_W \pi^2} S\, \eta^{\mu\nu} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \mathrm{Tr}(T^a T^b) \left\{ \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{i=d,s,b} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right] - \left(\frac{\cos\alpha}{\sin\beta}\right) \sum_{i=u,c,t} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right] \right\} \underset{\text{Fermion loop}}{\underset{\downarrow}{(-1)}}} \qquad (27)$$

$$\overline{|M|^2} = \frac{g_s^4 g^2}{1024 M_W^2 \pi^4} S^2 \eta^{\mu\nu} \eta^{\rho\sigma} \sum_{\lambda,\lambda'} \overbrace{\underbrace{\varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \varepsilon_{1\rho}}_{-\eta_{\mu\rho}} \varepsilon_{2\sigma}}^{-\eta_{\nu\sigma}} \frac{1}{4} \overset{\delta^{ab}\delta^{ab}}{\overset{\nearrow}{8}} \left| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{i=d,s,b} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right] - \left(\frac{\cos\alpha}{\sin\beta}\right) \sum_{i=u,c,t} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right] \right|^2$$

$$\overline{|M|^2} = \frac{g_s^4 g^2}{512 M_W^2 \pi^4} s^2 \delta_\rho^\nu \overset{4}{\widehat{\delta_\nu^\rho}} \quad | \quad |^2$$

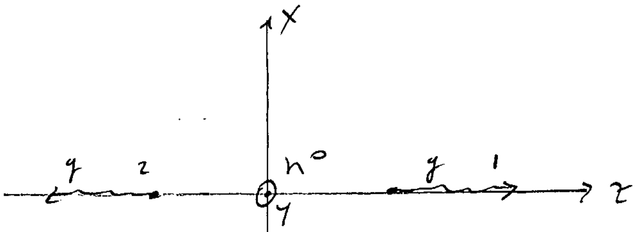
$$\overline{|M|^2} = \frac{g_s^4 g^2}{128 M_W^2 \pi^4} s^2 \left| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{i=d,s,b} \tau_i [(\tau_i-1) f(\tau_i)+2] - \left(\frac{\cos\alpha}{\sin\beta}\right) \sum_{i=u,c,t} \tau_i [(\tau_i-1) f(\tau_i)+2] \right|^2$$

---

(28)

$$d\Gamma = \frac{\overline{|M|^2} \, |\vec{P}_1| \, d\Omega}{32 \pi^2 m_{h^0}^2} \qquad (29)$$

In the rest frame of the h^0



$$P_1^\mu = (K^0, \overbrace{0, 0, K^0}^{\vec{P}_1}) ; \quad P_2^\mu = (K^0, \overbrace{0, 0, -K^0}^{-\vec{P}_1})$$

$$P_{h^0} = (m_{h^0}, 0, 0, 0)$$

$$P_{h^0} = P_1 + P_2 \Rightarrow m_{h^0} = 2K^0$$

$$s^2 = (P_1 + P_2)^4$$

$$s = (P_1 + P_2)^2 = 2 P_1 \cdot P_2 = 2 (K^{0^2} + K^{0^2}) = 4 K^{0^2}$$

$$\boxed{s = m_{h^0}^2} \qquad (30)$$

$$|\vec{P}_1|^2 = K^{0^2} = \frac{m_{h^0}^2}{4}$$

$$\Rightarrow \boxed{|\vec{P}_1| = \frac{m_{h^0}}{2}} \qquad (31)$$

$$\Gamma = \frac{g_s^4 g^2}{2 \times 128 M_{\cancel{W}}^2 \cancel{\pi^4}} \frac{m_{\cancel{h^0}}^{4} \; m_{\cancel{h^0}}}{2 \times 32 \cancel{\pi}^2 m_{\cancel{h^0}}^2} \cancel{4\pi} \Big| \qquad \Big|^2$$

↓
identical particles

$$\Gamma(h^0 \to 2g) = \frac{g_s^4 g^2 m_{h^0}^3}{2048 M_W^2 \pi^5 \times 2} \left| \quad \right|^2$$

$$\alpha_s = \frac{g_s^2}{4\pi} \; ; \; \alpha_s^2 = \frac{g_s^4}{16\pi^2}$$

$$\Gamma(h^0 \to 2g) = \frac{g^2 \alpha_s^2 m_{h^0}^3}{256 M_W^2 \pi^3} \left| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{i=d,s,b} T_i \left[(T_i - 1) f(T_i) + 2 \right] - \left(\frac{\cos\alpha}{\sin\beta}\right) \cdot \sum_{i=u,c,t} \right.$$

$$\left. T_i \left[(T_i - 1) f(T_i) + 2 \right] \right|^2$$

$$f(T_i) = \begin{cases} -2 \left(\arcsin\left(\frac{1}{T_i^{1/2}} \right) \right)^2 & T_i > 1 \\ \frac{1}{2} \left[\ln\left(\frac{1 + (1 - T_i)^{1/2}}{1 - (1 - T_i)^{1/2}} \right) - i\pi \right]^2 & T_i \leq 1 \end{cases}$$

$$T_i = \frac{4 m_i^2}{s} = \frac{4 m_i^2}{m_{h^0}^2}$$

$$\Gamma(h^0 \to 2g) = \frac{G_F \sqrt{2} \alpha_s^2 m_{h^0}^3}{64 \pi^3} \left| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{i=d,s,b} T_i \left[(T_i - 1) f(T_i) + 2 \right] \right.$$

$$\left. - \left(\frac{\cos\alpha}{\sin\beta}\right) \sum_{i=u,c,t} T_i \left[(T_i - 1) f(T_i) + 2 \right] \right|^2 \qquad (32)$$

$$\Gamma(h^0 \to 2g) \simeq \frac{G_F \sqrt{2} \alpha_s^2 m_{h^0}^3}{64 \pi^3} \left| \frac{\sin\alpha}{\cos\beta} T_b \left[(T_b - 1) f(T_b) + 2 \right] - \left(\frac{\cos\alpha}{\sin\beta}\right) \right.$$

$$\left. \cdot T_c \left[(T_c - 1) f(T_c) + 2 \right] - \left(\frac{\cos\alpha}{\sin\beta}\right) T_t \left[(T_t - 1) f(T_t) + 2 \right] \right|^2$$

$$\boxed{\Gamma(h^0 \to 2g) \simeq \frac{G_F \sqrt{2} \alpha_s^2 m_{h^0}^3}{64 \pi^3} \cdot \frac{(1 - \sin^2\alpha)}{\sin^2\beta} \left| \tan\beta \tan\alpha \, T_b \left[(T_b - 1) f(T_b) + 2 \right] - T_c \left[(T_c - 1) f(T_c) + 2 \right] - T_t \left[(T_t - 1) f(T_t) + 2 \right] \right|^2} \qquad (33)$$

where

$$\boxed{\tan\alpha = - \left\{ \frac{1 + \frac{(1 - \tan^2\beta)}{(1 + \tan^2\beta)} \left[\frac{1 - \frac{M_Z^2}{M_{HI}^2} - \frac{M_W^2}{M_{H3}^2}}{g^*(M_{HI}^2, M_Z^2, M_W^2, \tan^2\beta)} \right]}{1 - \frac{(1 - \tan^2\beta)}{(1 + \tan^2\beta)} \left[\frac{1 - \frac{M_Z^2}{M_{HI}^2} - \frac{M_W^2}{M_{HI}^2}}{g^*(M_{HI}^2, M_Z^2, M_W^2, \tan^2\beta)} \right]} \right\}^{1/2}} \qquad (34)$$

or

because

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

$$\boxed{\begin{aligned}\Gamma(h^0 \to 2g) \approx \frac{g^2 \alpha_s^2 m_{h^0}^3}{256\pi^3 M_W^2} \frac{(1-\sin^2\alpha)}{\sin^2\beta} \Big| & \tan\beta \tan\alpha\, \tau_b \left[(\tau_b - 1) f(\tau_b) + 2 \right] \\ & - \tau_c \left[(\tau_c - 1) f(\tau_c) + 2 \right] - \tau_t \left[(\tau_t - 1) f(\tau_t) + 2 \right] \Big|^2\end{aligned}} \quad (35)$$

because $\tau_b \ll 1$ $\left(\tau_b = 4 m_b^2 / m_{h^0}^2 \right)$

$$f(\tau_i) \approx \frac{1}{2} \left[\ln\left(\frac{1 + (1 - \frac{1}{2}\tau_i)}{1 - (1 - \frac{1}{2}\tau_i)} \right) - i\pi \right]^2$$

$$= \frac{1}{2} \left[\ln\left(\frac{2 - \frac{1}{2} T_i}{\frac{1}{2} T_i} \right) - i\pi \right]^2$$

$$= \frac{1}{2} \left[\ln\left(\frac{1 - \frac{1}{4} T_i}{\frac{1}{4} T_i} \right) - i\pi \right]^2$$

$$\approx \frac{1}{2} \left[-\frac{1}{4} T_i - \ln\left(\frac{1}{4} T_i \right) - i\pi \right]^2$$

$$\boxed{f(T_i) \approx \frac{1}{2} \left[\ln\left(\frac{1}{4} T_i \right) + i\pi \right]^2} \quad (36)$$

$$T_i \left[(T_i - 1) f(T_i) + 2 \right] \approx T_i \left[-f(T_i) + 2 \right]$$

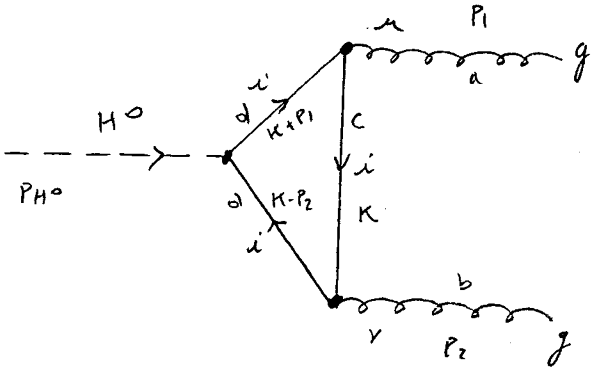
$$\approx T_i \left[-\frac{1}{2} \left[\ln\left(\frac{1}{4} T_i \right) + i\pi \right]^2 + 2 \right]$$

$$\Rightarrow \boxed{T_b \left[(T_b - 1) f(T_b) + 2 \right] \approx T_b \left[-\frac{1}{2} \left[\ln\left(\frac{1}{4} T_b \right) + i\pi \right]^2 + 2 \right]} \quad (37)$$

$$\boxed{T_c \left[(T_c - 1) f(T_c) + 2 \right] \approx T_c \left[-\frac{1}{2} \left[\ln\left(\frac{1}{4} T_c \right) + i\pi \right]^2 + 2 \right]} \quad (37b)$$

ok

$H^0 \to 2g$ decay



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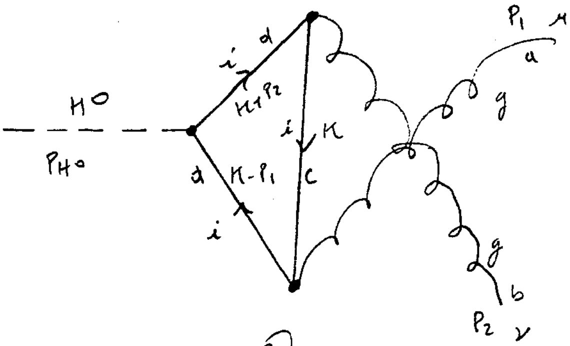


②

$$P_{H^0} = P_1 + P_2$$

For $i = d,s,b$ and $i = u,c,t$ the invariant amplitude is obtained from the corresponding to $h^0 \to 2g$ replacing in (27)

$$\underset{(d,s,b,e^-,\mu^-,\tau^-)}{\frac{\sin\alpha}{\cos\beta} \to -\frac{\cos\alpha}{\cos\beta}} \quad \text{and} \quad \underset{(u,c,t)}{\frac{\cos\alpha}{\sin\beta} \longrightarrow \frac{\sin\alpha}{\sin\beta}} \qquad (1)$$

$\Rightarrow$

$$\Gamma(H^0 \to 2g) = \frac{\sqrt{2}\, G_F \alpha_s^2 m_{H^0}^3}{64\pi^3} \left| \left(-\frac{\cos\alpha}{\cos\beta}\right) \sum_{i=d,s,b} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right] - \left(\frac{\sin\alpha}{\sin\beta}\right) \sum_{i=u,c,t} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right] \right|^2 \qquad (2)$$

$$= \frac{\sqrt{2}\, G_F \alpha_s^2 m_{H^0}^3}{64\pi^3} \left(\frac{\sin^2\alpha}{\sin^2\beta}\right) \left| \left(\frac{\tan\beta}{\tan\alpha}\right) \sum_{i=d,s,b} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right] + \sum_{i=u,c,t} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right] \right|^2$$

$$\boxed{\Gamma(H^0 \to 2g) = \frac{\sqrt{2}\, G_F \alpha_s^2 m_{H^0}^3}{64\pi^3} \cdot \sin^2\alpha \,(1 + \cot^2\beta) \left| \left(\frac{\tan\beta}{\tan\alpha}\right) \sum_{i=d,s,b} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right] + \sum_{i=u,c,t} \tau_i \left[(\tau_i - 1) f(\tau_i) + 2\right] \right|^2} \qquad (3)$$

$$\Rightarrow \boxed{\begin{aligned}\Gamma(H^{0} \rightarrow 2g) \approx \frac{\sqrt{2}\, G_F \alpha_s^2 m_{H^0}^3}{64\pi^3} \cdot \frac{(1+\cot^2\beta)}{(1+\cot^2\alpha)} \Bigg| \left(\frac{\tan\beta}{\tan\alpha}\right) \tau_b \left[(\tau_b - 1) f(\tau_b) + 2\right] \\ + \tau_t \left[(\tau_t - 1) f(\tau_t) + 2\right] \Bigg|^2\end{aligned}} \qquad (4)$$

Where $\tau_i = \dfrac{4 m_i^2}{m_{H^0}^2}$

$$f(x) = \begin{cases} -2\left(\arcsin\left(\dfrac{1}{x^{1/2}}\right)\right)^2 & x > 1 \\ \dfrac{1}{2}\left[\ln\left(\dfrac{1+(1-x)^{1/2}}{1-(1-x)^{1/2}}\right) - i\pi\right]^2 & x \leq 1 \end{cases}$$

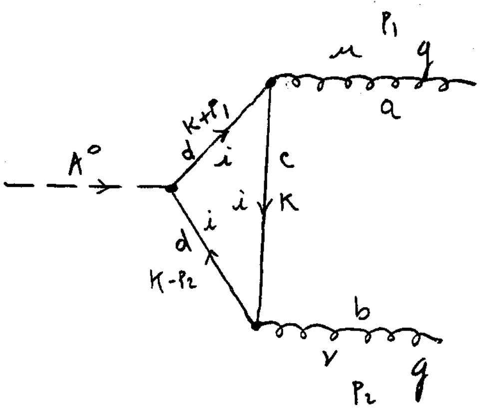
$$\tan\alpha = -\left\{\frac{1 + \dfrac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\dfrac{1 - \dfrac{M_Z^2}{M_{H^\pm}^2} - \dfrac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)}\right]}{1 - \dfrac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\dfrac{1 - \dfrac{M_Z^2}{M_{H^\pm}^2} - \dfrac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)}\right]}\right\}^{1/2}$$

$$g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta) = \left[\left(1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}\right)^2 - 4\left(\frac{M_Z^2}{M_{H^\pm}^2}\right)\left(1 - \frac{M_W^2}{M_{H^\pm}^2}\right)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2\right]^{1/2}$$

$A^0 \to 2g$ decay

$(M_{A^0} < M_{H^\pm})$

OK



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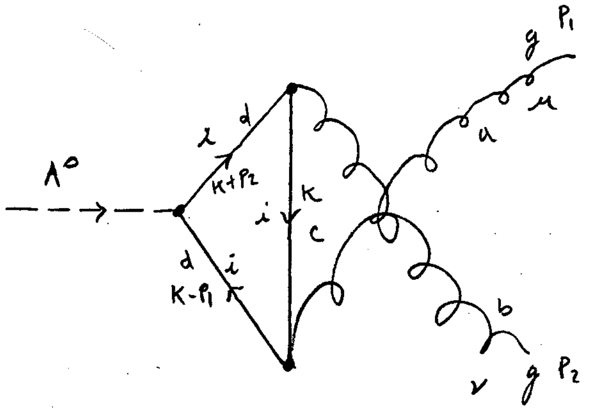


②

a) $P_{A^0} = p_1 + p_2 \quad \Rightarrow \quad m_{A^0}^2 = 2 p_1 \cdot p_2$

For $i = d, s, b$

$$-iM_{1a} = \overset{\nearrow \text{Fermion loop}}{(-1)} \sum_{i=d,s,b} \int \frac{d^dK}{(2\pi)^d} \mathrm{Tr}\left\{\left(-ig_s \gamma^\nu T^b_{dc}\right) \frac{i}{(\not K - \not p_2 - m_i)} \cdot \left(\frac{-g\, m_i \tan\beta}{2M_W}\gamma^5\right) \cdot \right.$$

$$\left. \cdot \frac{i}{(\not K + \not p_1 - m_i)} \left(-ig_s \gamma^\mu T^a_{cd}\right) \frac{i}{(\not K - m_i)}\right\} \varepsilon^*_{1\mu}\, \varepsilon^*_{2\nu}\, \mu_1^{(4-d)}\, \underset{\substack{\downarrow \\ A^0}}{\mu_2}^{\frac{(4-d)}{2}} \qquad (1)$$

$$-iM_{1a} = (-1)(-i) \frac{g_s^2 g}{2M_W} \tan\beta \sum_{i=d,s,b} m_i \int \frac{d^dK}{(2\pi)^d} \mathrm{Tr}\left\{\gamma^\nu (\not K - \not p_2 + m_i)\gamma^5 (\not K + \not p_1 + m_i)\gamma^\mu \right.$$

$$\left. (\not K + m_i)\right\} \frac{1}{\left((K-p_2)^2 - m_i^2\right)\left((K+p_1)^2 - m_i^2\right)(K^2 - m_i^2)}\, \varepsilon^*_{1\mu}\, \varepsilon^*_{2\nu}\, \mathrm{Tr}(T^a T^b) \cdot \mu^* \qquad (2)$$

where $\mu^* = \mu_1^{(4-d)} \mu_2^{\frac{(4-d)}{2}}$

$\mu^* \to 1$ when $d \to 4$

$$-iM_{2a} = (-1) \sum_{i=d,s,b} \int \frac{d^dK}{(2\pi)^d} \mathrm{Tr}\left\{\left(-ig_s \gamma^\mu T^a_{dc}\right) \frac{i}{(\not K - \not p_1 - m_i)} \cdot \left(-\frac{g\, m_i \tan\beta}{2M_W}\gamma^5\right) \cdot \right.$$

$$\left. \cdot \frac{i}{(\not K + \not p_2 - m_i)} \left(-ig_s \gamma^\nu T^b_{cd}\right) \frac{i}{(\not K - m_i)}\right\} \varepsilon^*_{1\mu}\, \varepsilon^*_{2\nu}\, \underbrace{\mu_1^{(4-d)}\, \underset{\substack{\downarrow \\ A^0}}{\mu_2}^{\frac{(4-d)}{2}}}_{\mu^*} \qquad (3)$$

$$-i M_{2a} = (-1)(-i)\frac{g_s^2\, g\, \tan\beta}{2M_W}\sum_{i=d,s,b} m_i \int \frac{d^dK}{(2\pi)^d}\,\mathrm{Tr}\left\{\gamma^\mu(\not K-\not P_1+m_i)\gamma^5(\not K+\not P_2+m_i)\gamma^\nu(\not K+m_i)\right\}\cdot$$

$$\cdot\frac{1}{[(K-P_1)^2-m_i^2]\,[(K+P_2)^2-m_i^2]\,[K^2-m_i^2]}\cdot \varepsilon^{*}_{1\mu}\,\varepsilon^{*}_{2\nu}\,\mathrm{Tr}(T^aT^b)\,\mu^{\epsilon} \qquad (4)$$

$$\mathrm{Tr}\left[\gamma^\nu(\not K-\not P_2+m_i)\gamma^5(\not K+\not P_1+m_i)\gamma^\mu(\not K+m_i)\right]$$

$$=\mathrm{Tr}\left[(\gamma^\nu\not K\gamma^5-\gamma^\nu\not P_2\gamma^5+m_i\gamma^\nu\gamma^5)(\not K\gamma^\mu\not K+m_i\not K\gamma^\mu+\not P_1\gamma^\mu\not K+m_i\not P_1\gamma^\mu\right.$$
$$\left.+\,m_i\gamma^\mu\not K+m_i^2\gamma^\mu)\right]$$

$$=\cancelto{0}{\mathrm{Tr}(\gamma^\nu\not K\gamma^5\not K\gamma^\mu\not K)}+m_i\cancel{\mathrm{Tr}(\gamma^\nu\not K\gamma^5\not K\gamma^\mu)}+\cancelto{0}{\mathrm{Tr}(\gamma^\nu\not K\gamma^5\not P_1\gamma^\mu\not K)}+m_i\cancel{\mathrm{Tr}(\gamma^\nu\not K\gamma^5\not P_1\gamma^\mu)}$$
$$+\,m_i\cancel{\mathrm{Tr}(\gamma^\nu\not K\gamma^5\gamma^\mu\not K)}+m_i^2\cancelto{0}{\mathrm{Tr}(\gamma^\nu\not K\gamma^5\gamma^\mu)}-\cancelto{0}{\mathrm{Tr}(\gamma^\nu\not P_2\gamma^5\not K\gamma^\mu\not K)}-m_i\cancel{\mathrm{Tr}(\gamma^\nu\not P_2\gamma^5\not K\gamma^\mu)}$$
$$-\cancelto{0}{\mathrm{Tr}(\gamma^\nu\not P_2\gamma^5\not P_1\gamma^\mu\not K)}-m_i\cancel{\mathrm{Tr}(\gamma^\nu\not P_2\gamma^5\not P_1\gamma^\mu)}-m_i\cancel{\mathrm{Tr}(\gamma^\nu\not P_2\gamma^5\gamma^\mu\not K)}-m_i^2\cancelto{0}{\mathrm{Tr}(\gamma^\nu\not P_2\gamma^5\gamma^\mu)}$$
$$+\,m_i\cancel{\mathrm{Tr}(\gamma^\nu\gamma^5\not K\gamma^\mu\not K)}+m_i^2\cancelto{0}{\mathrm{Tr}(\gamma^\nu\gamma^5\not K\gamma^\mu)}+m_i\cancel{\mathrm{Tr}(\gamma^\nu\gamma^5\not P_1\gamma^\mu\not K)}+m_i^2\cancelto{0}{\mathrm{Tr}(\gamma^\nu\gamma^5\not P_1\gamma^\mu)}$$
$$+\,m_i^2\cancelto{0}{\mathrm{Tr}(\gamma^\nu\gamma^5\gamma^\mu\not K)}+m_i^3\underset{\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu)=0}{\underset{\shortparallel}{\cancelto{0}{\mathrm{Tr}(\gamma^\nu\gamma^5\gamma^\mu)}}}$$

$$=m_iK^2\cancelto{0}{\mathrm{Tr}(\gamma^5\gamma^\nu\gamma^\mu)}+m_i\,\mathrm{Tr}(\gamma^5\gamma^\nu\not K\not P_1\gamma^\mu)+m_i\cancel{\mathrm{Tr}(\gamma^5\gamma^\nu\not K\gamma^\mu\not K)}$$
$$-\,m_i\,\mathrm{Tr}(\gamma^5\gamma^\nu\not P_2\not K\gamma^\mu)-m_i\,\mathrm{Tr}(\gamma^5\gamma^\nu\not P_2\not P_1\gamma^\mu)-m_i\,\mathrm{Tr}(\gamma^5\gamma^\nu\not P_2\gamma^\mu\not K)$$
$$-\,m_i\cancel{\mathrm{Tr}(\gamma^5\gamma^\nu\not K\gamma^\mu\not K)}-m_i\,\mathrm{Tr}(\gamma^5\gamma^\nu\not P_1\gamma^\mu\not K)$$

$$=\cancel{-4im_i\varepsilon^{\nu\alpha\beta\mu}K_\alpha P_{1\beta}}+\cancel{4im_i\varepsilon^{\nu\alpha\beta\mu}P_{2\alpha}K_\beta}+4im_i\varepsilon^{\nu\alpha\beta\mu}P_{2\alpha}P_{1\beta}$$
$$+\cancel{4im_i\varepsilon^{\nu\alpha\mu\beta}P_{2\alpha}K_\beta}+\cancel{4im_i\varepsilon^{\nu\alpha\mu\beta}P_{1\alpha}K_\beta}$$

but

$$-4im_i\varepsilon^{\nu\alpha\beta\mu}K_\alpha P_{1\beta}+4im_i\varepsilon^{\nu\alpha\mu\beta}P_{1\alpha}K_\beta = -4im_i\varepsilon^{\nu\beta\alpha\mu}K_\beta P_{1\alpha}$$
$$+4im_i\varepsilon^{\nu\alpha\mu\beta}K_\beta P_{1\alpha}$$

$$=-4im_i\varepsilon^{\nu\beta\alpha\mu}\cancel{K_\beta P_{1\alpha}}+4im_i\cancel{\varepsilon^{\nu\beta\alpha\mu}}K_\beta P_{1\alpha}=0$$

on the other hand $4im_i\varepsilon^{\nu\alpha\beta\mu}P_{2\alpha}K_\beta+4im_i\varepsilon^{\nu\alpha\mu\beta}P_{2\alpha}K_\beta$

$$=4im_i\varepsilon^{\nu\alpha\beta\mu}\cancel{P_{2\alpha}K_\beta}-4im_i\cancel{\varepsilon^{\nu\alpha\beta\mu}}P_{2\alpha}K_\beta=0$$

$$\Rightarrow\boxed{\mathrm{Tr}[\quad]=4im_i\varepsilon^{\nu\alpha\beta\mu}P_{2\alpha}P_{1\beta}}=+4im_i\varepsilon^{\nu\beta\alpha\mu}P_{2\beta}P_{1\alpha}$$

$$=-4im_i\varepsilon^{\mu\nu\beta\alpha}P_{1\alpha}P_{2\beta}=-4im_i\varepsilon^{\mu\alpha\nu\beta}P_{1\alpha}P_{2\beta}=4im_i\varepsilon^{\mu\alpha\beta\nu}P_{1\alpha}P_{2\beta}. \qquad (5)$$

$$\frac{1}{[(K+P_1)^2-m_i^2][(K-P_2)^2-m_i^2][K^2-m_i^2]} = 2\int_0^1 dX \int_0^{1-X} \frac{dY}{[K^2+2K\cdot(P_1X-P_2Y)-m_i^2]^3} \qquad (6)$$

$$\Rightarrow -iM_{1a} = (-1)(-i)\frac{g_s^2 g}{2M_W}\tan\beta \cdot 2\sum_{i=d,s,b} m_i \int_0^1 dX \int_0^{1-X} dY \int \frac{d^dK}{(2\pi)^d} \frac{4 i m_i \varepsilon^{\nu\alpha\beta\mu} P_{2\alpha} P_{1\beta}}{[K^2+2K\cdot(P_1X-P_2Y)-m_i^2]^3}$$

$$\varepsilon^*_{1\mu}\,\varepsilon^*_{2\nu}\,\frac{1}{2}\delta^{ab} \qquad (7)$$

$$K' = K + (P_1X - P_2Y)$$

$$K'^2 = K^2 + 2K\cdot(P_1X-P_2Y) - 2(P_1\cdot P_2)XY$$

$$d^dK = d^dK'$$

$$\Rightarrow -iM_{1a} = \frac{(-1)(-i)\,g_s^2 g \tan\beta}{M_W}\left(4i\,\varepsilon^{\nu\alpha\beta\mu} P_{2\alpha} P_{1\beta}\right)\sum_{i=d,s,b} m_i^2 \int_0^1 dX \int_0^{1-X} dY \int \frac{d^dK'}{(2\pi)^d}\,\cdot$$

$$\frac{1}{[K'^2+2(P_1\cdot P_2)XY-m_i^2]^3}\,\varepsilon^*_{1\mu}\,\varepsilon^*_{2\nu}\,\frac{1}{2}\delta^{ab}.$$

$$I_0(X,Y) = \int \frac{d^dK'}{[K'^2+2(P_1\cdot P_2)XY-m_i^2]^3} = \frac{i(-\pi)^{d/2}\,\Gamma(3-\frac{d}{2})}{\Gamma(3)\,[2(P_1\cdot P_2)XY-m_i^2]^{3-\frac{d}{2}}} = \frac{i\pi^2}{2[2(P_1\cdot P_2)XY-m_i^2]}$$

$$\therefore\quad -iM_{1a} = \frac{(-1)(-i)\,g_s^2 g \tan\beta}{M_W(2\pi)^4}\left(4i\,\varepsilon^{\mu\alpha\beta\nu} P_{1\alpha} P_{2\beta}\right)\sum_{i=d,s,b} m_i^2 \int_0^1 dX \int_0^{1-X} dY\; I_0(X,Y)\,\varepsilon^*_{1\mu}\,\varepsilon^*_{2\nu}\,\cdot$$

$$\cdot\,\frac{1}{2}\delta^{ab}\,. \qquad (8)$$

$$\mathrm{Tr}\{\gamma^\mu(\not{K}-\not{P}_1+m_i)\gamma^5(\not{K}+\not{P}_2+m_i)\gamma^\nu(\not{K}+m_i)\}$$

$$= \mathrm{Tr}\{(\gamma^\mu\not{K}\gamma^5 - \gamma^\mu\not{P}_1\gamma^5 + m_i\gamma^\mu\gamma^5)(\not{K}\gamma^\nu\not{K} + m_i\not{K}\gamma^\nu + \not{P}_2\gamma^\nu\not{K} + m_i\not{P}_2\gamma^\nu + m_i\gamma^\nu\not{K}$$
$$+ m_i^2\gamma^\nu)\}$$

$$= \cancelto{0}{\mathrm{Tr}(\gamma^\mu\not{K}\gamma^5\not{K}\gamma^\nu\not{K})} + m_i\mathrm{Tr}(\gamma^\mu|\not{K}\gamma^5\not{K}\gamma^\nu) + \cancelto{0}{\mathrm{Tr}(\gamma^\mu\not{K}\gamma^5\not{P}_2\gamma^\nu\not{K})} + m_i\mathrm{Tr}(|\gamma^\mu\not{K}\gamma^5\not{P}_2\gamma^\nu)$$
$$+ m_i\mathrm{Tr}(\gamma^\mu\not{K}|\gamma^5\gamma^\nu\not{K}) + m_i^2\cancelto{0}{\mathrm{Tr}(\gamma^\mu\not{K}\gamma^5\gamma^\nu)} - \cancelto{0}{\mathrm{Tr}(\gamma^\mu\not{P}_1\gamma^5\not{K}\gamma^\nu\not{K})} - m_i\mathrm{Tr}(\gamma^\mu\not{P}_1\gamma|^5\not{K}\gamma^\nu)$$
$$- \cancelto{0}{\mathrm{Tr}(\gamma^\mu\not{P}_1\gamma^5\not{P}_2\gamma^\nu\not{K})} - m_i\mathrm{Tr}(\gamma^\mu\not{P}_1\gamma|^5\not{P}_2\gamma^\nu) - m_i\mathrm{Tr}(\gamma^\mu|\not{P}_1\gamma^5\gamma^\nu\not{K}) - m_i^2\cancelto{0}{\mathrm{Tr}(\gamma^\mu\not{P}_1\gamma^5\gamma^\nu)}$$
$$+ m_i\mathrm{Tr}(\gamma^\mu\gamma^5\not{K}|\gamma^\nu\not{K}) + m_i^2\cancelto{0}{\mathrm{Tr}(\gamma^\mu\gamma^5\not{K}\gamma^\nu)} + m_i\mathrm{Tr}(\gamma^\mu|\gamma^5\not{P}_2\gamma^\nu\not{K}) + m_i^2\cancelto{0}{\mathrm{Tr}(\gamma^\mu\gamma^5\not{P}_2\gamma^\nu)}$$
$$+ m_i^2\cancelto{0}{\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\not{K})} + m_i^3\cancelto{0}{\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu)}$$
$$\downarrow$$
$$\mathrm{Tr}(\gamma^5\gamma^\nu\gamma^\mu) = 0$$

$$= m_i \operatorname{Tr}(\gamma^5 \overset{0}{\cancel{\gamma^\mu \gamma^\nu}}) K^2 + \cancel{m_i \operatorname{Tr}(\gamma^5 \gamma^\mu \not{K} \not{P}_2 \gamma^\nu)} + \cancel{m_i \operatorname{Tr}(\gamma^5 \gamma^\mu \not{K} \gamma^\nu \not{K})} - \cancel{m_i \operatorname{Tr}(\gamma^5 \gamma^\mu \not{P}_1 \not{K} \gamma^\nu)}$$

$$- m_i \operatorname{Tr}(\gamma^5 \gamma^\mu \not{P}_1 \not{P}_2 \gamma^\nu) - \cancel{m_i \operatorname{Tr}(\gamma^5 \gamma^\mu \not{P}_1 \gamma^\nu \not{K})} - \cancel{m_i \operatorname{Tr}(\gamma^5 \gamma^\mu \not{K} \gamma^\nu \not{K})} - \cancel{m_i \operatorname{Tr}(\gamma^5 \gamma^\mu \not{P}_2 \gamma^\nu \not{K})}$$

$$= \cancel{-4im_i \varepsilon^{\mu\alpha\beta\nu} K_\alpha P_{2\beta}} + \cancel{4im_i \varepsilon^{\mu\alpha\beta\nu} P_{1\alpha} K_\beta} + 4im_i \varepsilon^{\mu\alpha\beta\nu} P_{1\alpha} P_{2\beta}$$

$$+ \cancel{4im_i \varepsilon^{\mu\alpha\nu\beta} P_{1\alpha} K_\beta} + \cancel{4im_i \varepsilon^{\mu\alpha\nu\beta} P_{2\alpha} K_\beta}$$

but

$$-4im_i \varepsilon^{\mu\alpha\beta\nu} K_\alpha P_{2\beta} + 4im_i \varepsilon^{\mu\alpha\nu\beta} P_{2\alpha} K_\beta$$

$$= \cancel{-4im_i \varepsilon^{\mu\alpha\beta\nu}} K_\alpha P_{2\beta} + \cancel{4im_i} \underbrace{\varepsilon^{\mu\beta\nu\alpha}}_{\varepsilon^{\mu\alpha\beta\nu}} K_\alpha P_{2\beta} = 0$$

and $4im_i \varepsilon^{\mu\alpha\beta\nu} P_{1\alpha} K_\beta + 4im_i \varepsilon^{\mu\alpha\nu\beta} P_{1\alpha} K_\beta$

$$= \cancel{4im_i \varepsilon^{\mu\alpha\beta\nu} P_{1\alpha}} K_\beta - \cancel{4im_i \varepsilon^{\mu\alpha\beta\nu}} P_{1\alpha} K_\beta = 0$$

$$\Rightarrow \boxed{\operatorname{Tr}[\qquad] = 4im_i \varepsilon^{\mu\alpha\beta\nu} P_{1\alpha} P_{2\beta}.} \qquad (9)$$

$$-iM_{2a} = \frac{(-1)(-i)\, g_s^2 g \tan\beta}{2M_W} \sum_{i=d,s,b} m_i^2 \left(4i\varepsilon^{\mu\alpha\beta\nu} P_{1\alpha} P_{2\beta}\right) \int \frac{d^d K}{(2\pi)^d} \cdot \frac{1}{[(K-P_1)^2 - m_i^2][(K+P_2)^2 - m_i^2]} \cdot$$

$$\cdot \frac{1}{[K^2 - m_i^2]} \cdot \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \operatorname{Tr}(T^a T^b)$$

$$\frac{1}{[(K-P_1)^2 - m_i^2][(K+P_2)^2 - m_i^2][K^2 - m_i^2]} = 2\int_0^1 dX \int_0^{1-X} \frac{dY}{[K^2 - 2K\cdot(P_1 X - P_2 Y) - m_i^2]^3} \qquad (10)$$

$$K' = K - (P_1 X - P_2 Y)$$

$$K'^2 = K^2 - 2K\cdot(P_1 X - P_2 Y) - 2(P_1\cdot P_2) XY$$

$$d^d K' = d^d K$$

$$\Rightarrow -iM_{2a} = \frac{(-1)(-i)\, g_s^2 g \tan\beta}{M_W} \left(4i\varepsilon^{\mu\alpha\beta\nu} P_{1\alpha} P_{2\beta}\right) \sum_{i=d,s,b} m_i^2 \int_0^1 dX \int_0^{1-X} dY \int \frac{d^d K'}{(2\pi)^d} \cdot$$

$$\frac{1}{[K'^2 + 2(P_1\cdot P_2)XY - m_i^2]^3} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \frac{1}{2} \delta^{ab}.$$

$$\therefore -iM_{2a} = \frac{(-1)(-i)\, g_s^2 g \tan\beta}{M_W (2\pi)^4} \left(4i\varepsilon^{\mu\alpha\beta\nu} P_{1\alpha} P_{2\beta}\right) \sum_{i=d,s,b} m_i^2 \int_0^1 dX \int_0^{1-X} dY\, I_0(X,Y)\, \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \frac{1}{2} \delta^{ab}$$

(11)

$$-i(M_{1a}+M_{2a}) = \frac{\cancel{2}\,(-1)(-i)\,g_s^2\,g\,\tan\beta\,(i)\,\pi^2}{\cancel{2}\,M_W\,(2\pi)^4}\left(4i\,\varepsilon^{\mu\alpha\beta\nu}P_{1\alpha}P_{2\beta}\right)\sum_{i=d,s,b} m_i^2\int_0^1 dx\int_0^{1-x} dY\,\frac{1}{[2(P_1\cdot P_2)xY-m_i^2]}$$

$$\varepsilon_{1\mu}^{*}\,\varepsilon_{2\nu}^{*}\,\frac{1}{2}\,\delta^{ab}$$

$$-i(M_{1a}+M_{2a}) = -\frac{i\,g_s^2\,g\,\tan\beta}{4M_W\,\pi^2}\,\varepsilon^{\mu\alpha\beta\nu}P_{1\alpha}P_{2\beta}\sum_{i=d,s,b} m_i^2\int_0^1 dx\int_0^{1-x}\frac{dY}{[m_{A^0}^2\,xY-m_i^2]}\,\varepsilon_{1\mu}^{*}\,\varepsilon_{2\nu}^{*}\,\frac{1}{2}\delta^{ab}$$

$$\int_0^{1-x}\frac{dY}{[m_{A^0}^2\,xY-m_i^2]} = \int_0^{1-x}\frac{dY}{[aY+b]} = \frac{1}{a}\ln|aY+b|\Big|_0^{1-x}$$

$$= \frac{1}{m_{A^0}^2\,x}\ln|m_{A^0}^2\,x\,(1-x)-m_i^2| - \frac{1}{m_{A^0}^2\,x}\ln|-m_i^2|$$

$$= \frac{1}{m_{A^0}^2\,x}\ln\left|\frac{m_i^2-m_{A^0}^2\,x\,(1-x)}{m_i^2}\right|$$

$$= \frac{1}{m_{A^0}^2\,x}\ln\left|\frac{m_{A^0}^2x^2-m_{A^0}^2x+m_i^2}{m_i^2}\right| = \frac{1}{m_{A^0}^2\,x}\ln\left|\frac{x^2-x+\frac{T_i}{4}}{T_i/4}\right|$$

$$T_i = \frac{4m_i^2}{m_{A^0}^2}$$

$$-i(M_{1a}+M_{2a}) = -\frac{i\,g_s^2\,g\,\tan\beta}{4M_W\,\pi^2}\,\varepsilon^{\mu\alpha\beta\nu}P_{1\alpha}P_{2\beta}\sum_{i=d,s,b}\frac{T_i}{4}\int_0^1\frac{dx}{x}\ln\left|\frac{x^2-x+T_i/4}{T_i/4}\right|\varepsilon_{1\mu}^{*}\,\varepsilon_{2\nu}^{*}.$$

$$\frac{1}{2}\delta^{ab}.$$

$$-i(M_{1a}+M_{2a}) = -\frac{i\,g_s^2\,g\,\tan\beta}{32\,M_W\,\pi^2}\,\varepsilon^{\mu\alpha\beta\nu}P_{1\alpha}P_{2\beta}\,\delta^{ab}\Big(\sum_{i=d,s,b}T_i\,f(T_i)\Big)\varepsilon_{1\mu}^{*}\,\varepsilon_{2\nu}^{*} \qquad (12)$$

b)

For $i = u, c, t$ $\quad \tan\beta \rightarrow \cot\beta$.

$\therefore$

$$-i(M_{1b}+M_{2b}) = -\frac{i\,g_s^2\,g\,\cot\beta}{32\,M_W\,\pi^2}\,\varepsilon^{\mu\alpha\beta\nu}P_{1\alpha}P_{2\beta}\,\delta^{ab}\Big(\sum_{i=u,c,t}T_i\,f(T_i)\Big)\varepsilon_{1\mu}^{*}\,\varepsilon_{2\nu}^{*} \qquad (13)$$

Then:

$$-iM = -i(M_{1a}+M_{2a}+M_{1b}+M_{2b}) = -\frac{i\,g_s^2\,g}{32\,M_W\,\pi^2}\,\varepsilon^{\mu\alpha\beta\nu}P_{1\alpha}P_{2\beta}\,\delta^{ab}\,\varepsilon_{1\mu}^{*}\,\varepsilon_{2\nu}^{*}.$$

$$\cdot\Big[\tan\beta\sum_{i=d,s,b}T_i\,f(T_i) + \cot\beta\sum_{i=u,c,t}T_i\,f(T_i)\Big] \qquad (14)$$

$$\overline{|M|^2} = \frac{g_s^4 g^2}{32^2 M_W^2 \pi^4} \delta^{ab}\delta^{ab} \epsilon^{\mu\alpha\beta\nu} \epsilon^{\rho\sigma\delta\gamma} p_{1\alpha} p_{2\beta} p_{1\sigma} p_{2\delta} \sum_{\lambda,\lambda'} \epsilon^*_{1\mu} \epsilon^*_{2\nu} \epsilon_{1\rho} \epsilon_{2\gamma}$$

$$\left| \tan\beta \sum_{i=d,s,b} \tau_i f(\tau_i) + \cot\beta \sum_{i=u,c,t} \tau_i f(\tau_i) \right|^2$$

$$= \frac{g_s^4 g^2}{32^2 M_W^2 \pi^4}\, 8\, \epsilon^{\mu\alpha\beta\nu} \underset{\underset{\epsilon^{\rho\gamma\sigma\delta}}{\downarrow}}{\epsilon^{\rho\sigma\delta\gamma}} p_{1\alpha} p_{2\beta} p_{1\sigma} p_{2\delta} (-\eta_{\mu\rho})(-\eta_{\nu\gamma}) \,.$$

$$\cdot \left| \quad \right|^2$$

$$= \frac{g_s^4 g^2}{128 M_W^2 \pi^4} \epsilon^{\mu\alpha\beta\nu} \epsilon_{\mu\nu}{}^{\sigma\delta} p_{1\alpha} p_{2\beta} p_{1\sigma} p_{2\delta} \left| \quad \right|^2$$

$$= \frac{g_s^4 g^2}{128 M_W^2 \pi^4} \epsilon^{\mu\nu\alpha\beta} \epsilon_{\mu\nu}{}^{\sigma\delta} p_{1\alpha} p_{2\beta} p_{1\sigma} p_{2\delta} \cdot \left| \quad \right|^2$$

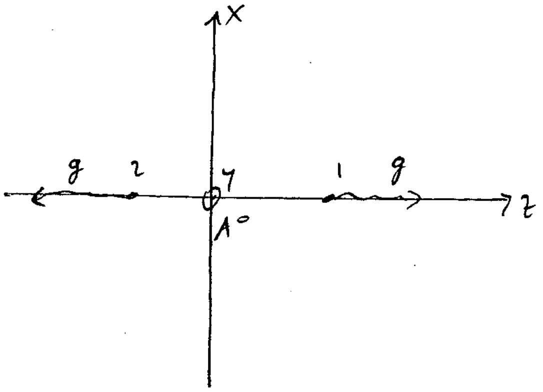
$$= \frac{g_s^4 g^2}{128 M_W^2 \pi^4} (-2)(\eta^{\alpha\sigma}\eta^{\beta\delta} - \eta^{\alpha\delta}\eta^{\beta\sigma}) p_{1\alpha} p_{2\beta} p_{1\sigma} p_{2\delta} \cdot \left| \quad \right|^2$$

$$= \frac{-g_s^4 g^2}{64 M_W^2 \pi^4} \left(p_{1\alpha} p_1^{\alpha} p_{2\beta} p_2^{\beta} - p_{1\alpha} p_{2\beta} p_1^{\beta} p_2^{\alpha} \right) \left| \quad \right|^2$$

$$= \frac{-g_s^4 g^2}{64 M_W^2 \pi^4} \left(\overset{0}{\cancel{p_1^2}} \overset{0}{\cancel{p_2^2}} - (p_1 \cdot p_2)(p_1 \cdot p_2) \right) \left| \quad \right|^2$$

$$\overline{|M|^2} = \frac{g_s^4 g^2}{64 M_W^2 \pi^4} (p_1 \cdot p_2)^2 \left| \tan\beta \sum_{i=d,s,b} \tau_i f(\tau_i) + \cot\beta \sum_{i=u,c,t} \tau_i f(\tau_i) \right|^2 \qquad (15)$$

In the rest frame of A^0



$$P_{A^0} = p_1 + p_2$$

$$M_{A^0}^2 = 2 p_1 \cdot p_2$$

$$\Rightarrow \boxed{p_1 \cdot p_2 = \frac{M_{A^0}^2}{2}} \qquad (16)$$

The decay rate is :

$$d\Gamma : \frac{\overline{|M|^2} \; |\vec{P}_1| \, d\Omega}{32\pi^2 \, M_{A^0}{}^2}$$

$$\Gamma = \frac{\overline{|M|^2} \; |\vec{P}_1|}{8\pi \, M_{A^0}{}^2} \qquad (17)$$

$$P_{A^0} = (M_{A^0}, 0, 0, 0)$$
$$P_1 = (E_1, 0, 0, E_1)$$
$$P_2 = (E_1, 0, 0, -E_1)$$

$$\Rightarrow \quad E_1 = \frac{M_{A^0}}{2} = |\vec{P}_1| \qquad (18)$$

$$\therefore \quad \Gamma = \frac{g_s^4 \, g^2}{64 M_W^2 \pi^4} \left(\frac{M_{A^0}^4}{4}\right) \left(\frac{M_{A^0}}{2}\right) \cdot \frac{1}{8\pi \, M_{A^0}{}^2} \Big| \qquad \Big|^2 \left(\frac{1}{2}\right) \rightarrow \text{identical particles}$$

$$\Gamma(A^0 \rightarrow 2g) = \frac{g_s^4 \, g^2}{4096 \, M_W^2 \pi^5} M_{A^0}^3 \left| \tan\beta \sum_{i=d,s,b} \tau_i \, f(\tau_i) + \cot\beta \sum_{i=u,c,t} \tau_i \, f(\tau_i) \right|^2 \frac{1}{2} \qquad (19)$$

$$\alpha_s = \frac{g_s^2}{4\pi} \quad \Rightarrow \quad g_s^4 = 16\pi^2 \alpha_s^2$$

$$\sqrt{2}\, G_F = \frac{g^2}{4 M_W^2}$$

$$\Gamma(A^0 \longrightarrow 2g) = \frac{16 \cancel{\pi^2} \alpha_s^2 \; \sqrt{2}\, G_F \, 4 \cancel{M_W^2}}{4096 \, \cancel{M_W^2} \cancel{\pi^5}} M_{A^0}^3 \Big| \qquad \Big|^2 \frac{1}{2}$$

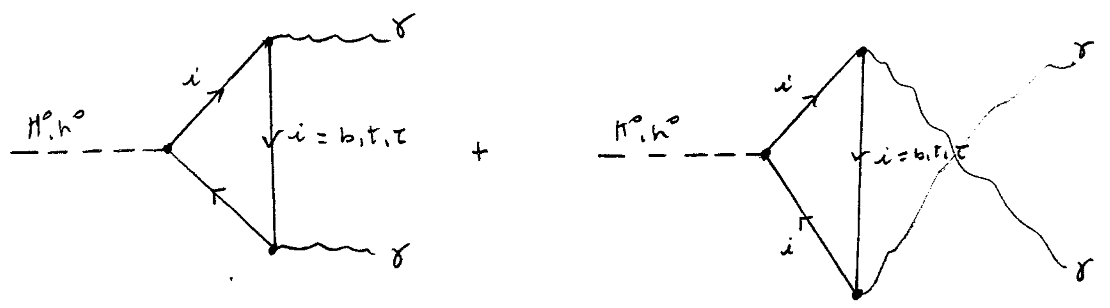
$$\boxed{\Gamma(A^0 \longrightarrow 2g) = \frac{\sqrt{2}\, G_F \, \alpha_s^2 \, M_{A^0}^3}{128 \; \pi^3} \left| \tan\beta \sum_{i=d,s,b} \tau_i \, f(\tau_i) + \cot\beta \sum_{i=u,c,t} \tau_i \, f(\tau_i) \right|^2} \qquad (20)$$

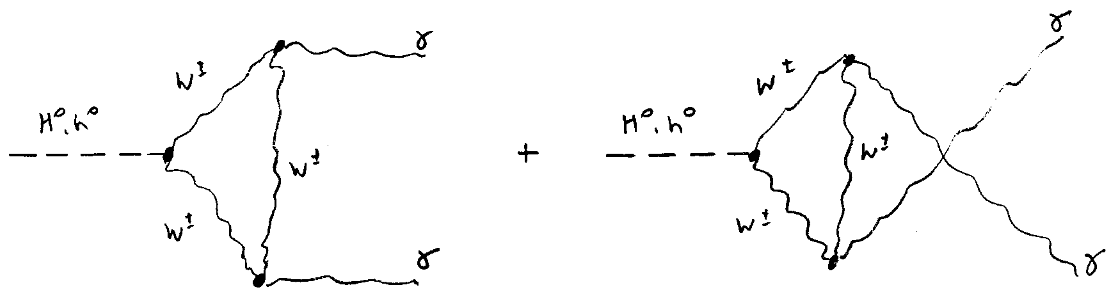
$$f(x) \equiv \begin{cases} -2 \left(\arcsin\left(\frac{1}{x^{1/2}}\right) \right)^2 & x > 1 \\ \frac{1}{2} \left[\ln\left(\frac{1 + (1-x)^{1/2}}{1 - (1-x)^{1/2}} \right) - i\pi \right]^2 & x \leq 1 \end{cases}$$

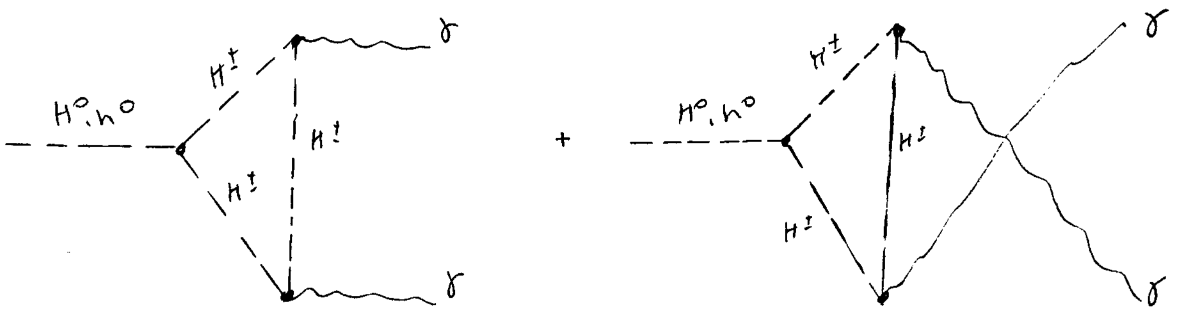
$$\tau_i \equiv \frac{4 m_i^2}{m_{A^0}{}^2}$$

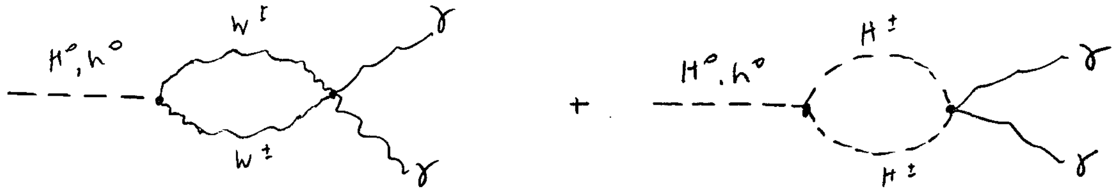
$$\boxed{\begin{array}{c} \Gamma(A^0 \longrightarrow 2g) = \dfrac{\sqrt{2}\, G_F \, \alpha_s^2 \, M_{A^0}^3}{128\pi^3} \left| \tan\beta \quad \tau_b \, f(\tau_b) + \cot\beta \; \tau_t \, f(\tau_t) \right|^2 \\ (\lim_{x \rightarrow 0} \; x \, f(x) = 0) \end{array}} \qquad (21)$$

$H^0, h^0 \to 2\gamma$ decay :





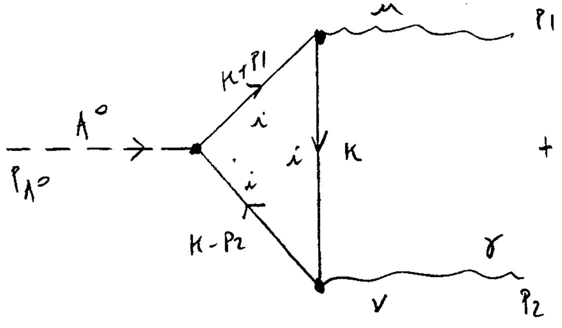




also are negligible compared with $H^0, h^0 \to 2g$.

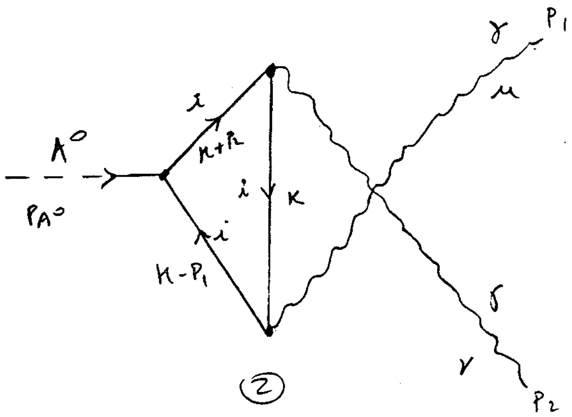
$A^0 \to 2\gamma$ decay :

OK



①

+



②

$$P_{A^0} = P_1 + P_2 \quad \Rightarrow \quad m_{A^0}^2 = 2P_1 \cdot P_2$$

a) For $i = d, s, b$

$$-iM_a = (-1) \sum_{\substack{i=d,s,b \\ e^-,\mu^-,\tau^-}} \int \frac{d^d K}{(2\pi)^d} \, Tr \left\{ (-ie\gamma^\nu Q_i) \frac{i}{(\not{K} - \not{P}_2 - m_i)} \left(\frac{-g m_i \tan\beta \, \gamma^5}{2M_W} \right) N_i \right.$$

$$\left. \frac{i}{(\not{K} + \not{P}_1 - m_i)} (-ie\gamma^\mu Q_i) \frac{i}{\not{K} - m_i} \right\} \varepsilon_{1\mu}^* \varepsilon_{2\nu}^* \underbrace{\underset{\substack{\downarrow \\ 2\gamma}}{\mu_1^{4-d}} \quad \underset{\substack{\downarrow \\ A^0}}{\mu_2^{\frac{(4-d)}{2}}}}_{\mu^* \, (\to 1 \text{ when } d \to 4)}$$

$$-iM_a = (-1)(-i) \frac{e^2 g \tan\beta}{2M_W} \sum_{\substack{i=d,s,b \\ e^-,\mu^-,\tau^-}} Q_i^2 m_i \int \frac{d^d k}{(2\pi)^d} \, Tr \left\{ \gamma^\nu (\not{K} - \not{P}_2 + m_i) \gamma^5 \cdot N_i \right.$$

$$\left. \cdot (\not{K} + \not{P}_1 + m_i) \gamma^\mu (\not{K} + m_i) \right\} \cdot \frac{1}{[(K-P_2)^2 - m_i^2][(K+P_1)^2 - m_i^2][K^2 - m_i^2]} \varepsilon_{1\mu}^* \varepsilon_{2\nu}^* \mu^*$$

$$-iM_{2a} = (-1) \sum_{\substack{i=d,s,b \\ e^-,\mu^-,\tau^-}} \int \frac{d^d k}{(2\pi)^d} Tr \left\{ (-ie\gamma^\mu Q_i) \frac{i}{(\not k - \not p_1 - m_i)} \left(\frac{-g m_i \tan\beta \gamma^5}{2M_W} \right) N_i \right.$$

$$\left. \frac{i}{(\not k + \not p_2 - m_i)} (-ie\gamma^\nu Q_i) \frac{i}{(\not k - m_i)} \right\} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \mu^\epsilon$$

$$-iM_{2a} = (-1)(-i) \frac{e^2 g \tan\beta}{2M_W} \sum_{\substack{i=d,s,b \\ e^-,\mu^-,\tau^-}} Q_i^2 m_i \int \frac{d^d k}{(2\pi)^d} Tr \left\{ \gamma^\mu (\not k - \not p_1 + m_i) \gamma^5 \cdot \right.$$

$$\left. \cdot (\not k + \not p_2 + m_i) \gamma^\nu (\not k + m_i) \right\} \cdot \frac{1}{[(k-p_1)^2 - m_i^2][(k+p_2)^2 - m_i^2][k^2 - m_i^2]} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \mu^\epsilon$$

Comparing with (12) in $A^0 \to gg$ whe have replacing: $g_s \to e$ adding a factor Q_i^2 and not considering the factor $Tr(T^a T^b) = \frac{1}{2}\delta^{ab}$

$$\Rightarrow -i(M_{1a} + M_{2a}) = -\frac{ie^2 g \tan\beta}{3^2 M_W \pi^2} \varepsilon^{\mu\alpha\beta\nu} p_{1\alpha} p_{2\beta} \cdot 2 \left(\sum_{\substack{i=d,s,b \\ e^-,\mu^-,\tau^-}} Q_i^{2 \nearrow \times N_i} T_i f(T_i) \right) \varepsilon^*_{1\mu} \varepsilon^*_{2\nu}$$

b) For $i = u, c, t$ $\quad \tan\beta \to \cot\beta$

$\therefore$

$$-i(M_{1b} + M_{2b}) = -\frac{ie^2 g \cot\beta}{32 M_W \pi^2} \varepsilon^{\mu\alpha\beta\nu} p_{1\alpha} p_{2\beta} \cdot 2 \left(\sum_{i=u,c,t} Q_i^{2 \nearrow \times N_i} T_i f(T_i) \right) \varepsilon^*_{1\mu} \varepsilon^*_{2\nu}$$

Then

$$-iM = -i(M_{1a} + M_{2a} + M_{1b} + M_{2b}) = -\frac{ie^2 g}{16 M_W \pi^2} \varepsilon^{\mu\alpha\beta\nu} p_{1\alpha} p_{2\beta} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \cdot$$

$$\left[\tan\beta \sum_{\substack{i=d,s,b \\ e^-,\mu^-,\tau^-}} N_i Q_i^2 T_i f(T_i) + \cot\beta \sum_{i=u,c,t} N_i Q_i^2 T_i f(T_i) \right]$$

(N_i : color factor)

$$\overline{|M|^2} = \frac{e^4 g^2}{16^2 M_W^2 \pi^4} \varepsilon^{\mu\alpha\beta\nu} \varepsilon^{\rho\sigma\delta\gamma} p_{1\alpha} p_{2\beta} p_{1\sigma} p_{2\delta} \sum_{\lambda,\lambda'} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \varepsilon_{1\rho} \varepsilon_{2\gamma}$$

$$\left| \tan \sum_i + \cot\beta \sum_i \right|^2$$

$$\overline{|M|^2} = \frac{e^4 g^2}{16^2 M_W^2 \pi^4} \varepsilon^{\mu\alpha\beta\nu} \varepsilon^{\rho\overset{\nearrow}{\sigma}\delta\sigma} (-\eta_{\mu\rho})(-\eta_{\nu\sigma}) P_{1\alpha} P_{2\beta} P_{1\sigma} P_{2\delta} . |\ |^2$$

$$\overline{|M|^2} = \frac{e^4 g^2}{16^2 M_W^2 \pi^4} \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{\mu\nu}{}^{\sigma\delta} P_{1\alpha} P_{2\beta} P_{1\sigma} P_{2\delta} . |\ |^2$$

$$\overline{|M|^2} = \frac{e^4 g^2}{16^2 M_W^2 \pi^4} (-2)(\eta^{\alpha\sigma}\eta^{\beta\delta} - \eta^{\alpha\delta}\eta^{\beta\sigma}) P_{1\alpha} P_{2\beta} P_{1\sigma} P_{2\delta} . |\ |^2$$

$$\overline{|M|^2} = \frac{e^4 g^2}{16^2 M_W^2 \pi^4} (-2) \left[\overset{0}{\cancel{P_1^2}} \overset{0}{\cancel{P_2^2}} - (P_1 . P_2)^2 \right] . |\ |^2$$

$$\overline{|M|^2} = \frac{2 e^4 g^2}{16^2 M_W^2 \pi^4} (P_1 . P_2)^2 \left| \tan\beta \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} N_i Q_i^2 \tau_i f(\tau_i) + \cot\beta \sum_{i=u,c,t} N_i Q_i^2 \tau_i f(\tau_i) \right|^2$$

In the rest frame of A^0

$$P_1 . P_2 = \frac{m_{A^0}^2}{2}$$

$$|\vec{P}_1| = \frac{m_{A^0}}{2}$$

$$\Gamma(A^0 \to 2\gamma) = \frac{\cancel{2} e^4 g^2}{16^2 M_W^2 \pi^4} \frac{m_{A^0}^4}{\cancel{4}} |\ |^2 \frac{m_{A^0}}{\cancel{2}} \frac{4\pi}{32\pi^2 m_{A^0}^2} \left(\frac{1}{2}\right) \to \text{identical particles} \quad N_c$$

$$\Gamma(A^0 \to 2\gamma) = \frac{\alpha^2 g^2}{16 \pi^2 M_W^2} m_{A^0}^3 |\ |^2 \frac{1}{64\pi}$$

$$= \frac{\sqrt{2} G_F \alpha^2}{4\pi^2} \frac{m_{A^0}^3}{64\pi} |\ |^2$$

$$\boxed{\begin{aligned} \Gamma(A^0 \to 2\gamma) = \frac{\sqrt{2} G_F \alpha^2}{256 \pi^3} m_{A^0}^3 \Bigg| & \tan\beta \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} N_i Q_i^2 \tau_i f(\tau_i) + \\ & + \cot\beta \sum_{i=u,c,t} N_i Q_i^2 \tau_i f(\tau_i) \Bigg|^2 \end{aligned}}$$

$$N_i = \begin{cases} 3 & \text{for quarks} \\ 1 & \text{for leptons} \end{cases}$$

$\Rightarrow$

$$\boxed{\Gamma(A^\circ \rightarrow 2\gamma) = \frac{\sqrt{2}\, G_F\, \alpha^2}{256\,\pi^3}\, m_{A^\circ}^3 \left| \tan\beta \left[\frac{1}{3} T_b f(T_b) + T_\tau f(T_\tau) \right] + \cot\beta \left[\frac{4}{3} T_t f(T_t) \right] \right|^2}$$

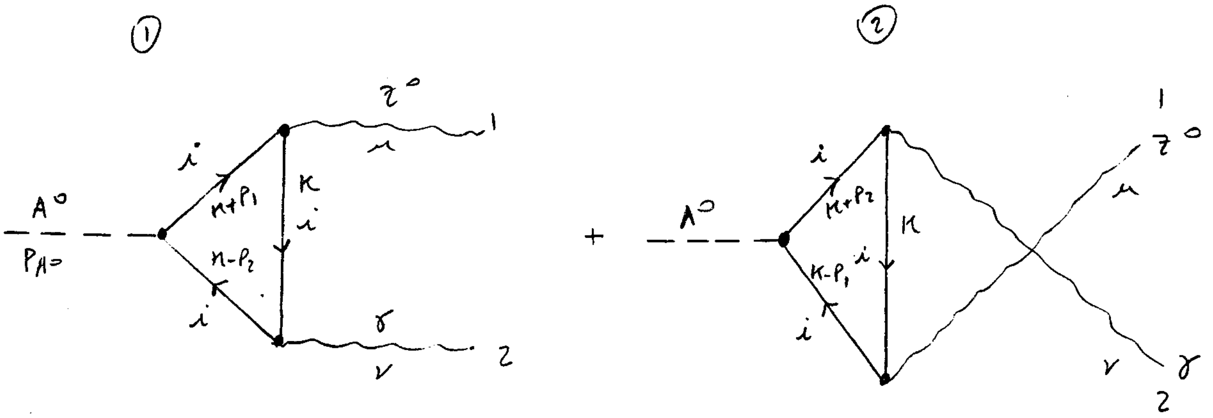
$$f(x) \equiv \begin{cases} -2\left(\arcsin\left(\frac{1}{x^{1/2}} \right) \right)^2 & x > 1 \\ \frac{1}{2}\left[\ln\left(\frac{1+(1-x)^{1/2}}{1-(1-x)^{1/2}} \right) - i\pi \right]^2 & x \leq 1 \end{cases}$$

$$T_i \equiv \frac{4 m_i^2}{m_{A^\circ}^2}$$

$$(\lim x \rightarrow 0 \;(x f(x)) = 0)$$

That is negligible compared with $\Gamma(A^\circ \rightarrow 2g)$ where we have α_s^2 instead of α^2.

$A^0 \to Z^0\gamma$ $\quad (m_{A^0} > M_{Z^0})$



$i = e^-, \mu^-, \tau^-, d, s, b$ ⓐ or u, c, t ⓑ

a) $i = e^-, \mu^-, \tau^-, d, s, b$

$$-iM_{1a} = \overset{\text{Fermion Loop}}{(-1)} \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} \overset{\text{color factor}}{N_i} \int \frac{d^dK}{(2\pi)^d}\, Tr\Big[\Big(-\frac{ig}{\cos\theta_W}\Big)\gamma^\mu \frac{1}{2}(C_V^i - C_A^i\gamma^5)\frac{i}{\not K - m_i}(-ie\gamma^\nu Q_i)$$

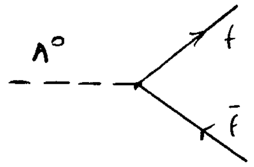
$$\frac{i}{(\not K - \not P_2 - m_i)}\Big(-\frac{g m_i \tan\beta}{2M_W}\gamma^5\Big)\frac{i}{(\not K + \not P_1) - m_i}\Big]\, \varepsilon^*_{1\mu}\, \varepsilon^*_{2\nu}\, \underbrace{\underset{\uparrow\, A^0}{\mu_1^{\frac{4-d}{2}}}\; \underset{\uparrow\, Z^0}{\mu_2^{\frac{4-d}{2}}}\; \underset{\uparrow\, \gamma^0}{\mu_3^{\frac{4-d}{2}}}}_{\mu^*} \qquad (1)$$

$$M_{1a} = \frac{-g^2 e \tan\beta}{4M_W \cos\theta_W} \sum_{i=d,s,b,\ldots} N_i Q_i m_i \int \frac{d^dK}{(2\pi)^d}\, Tr\big[\gamma^\mu (C_V^i - C_A^i\gamma^5)(\not K + m_i)\gamma^\nu \cdot$$

$$(\not K - \not P_2 + m_i)\gamma^5(\not K + \not P_1 + m_i)\big] \quad \frac{1}{(K^2-m_i^2)[(K-P_2)^2-m_i^2][(K+P_1)^2-m_i^2]}\; \varepsilon^*_{1\mu}\varepsilon^*_{2\nu}\,\mu^* \qquad (2)$$

$\mu^* \to 1$
when
$d \to 4$

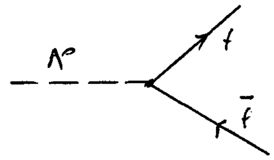
b)



$f = u, c, t, \nu_e, \nu_\mu, \nu_\tau$

$-\frac{g m_f}{2M_W}\cot\beta\,\gamma^5$

a)



$f = d, s, b, e^-, \mu^-, \tau^-$

$-\frac{g m_f}{2M_W}\tan\beta\,\gamma^5$

$$\mathrm{Tr}\left[\gamma^\mu(C_V^i - C_A^i\gamma^5)(\not{k}+m_i)\gamma^\nu(\not{k}-\not{p}_2+m_i)\gamma^5(\not{k}+\not{p}_1+m_i)\right]$$

$$= \mathrm{Tr}\left[(C_V^i\gamma^\mu - C_A^i\gamma^\mu\gamma^5)(\not{k}\gamma^\nu + m_i\gamma^\nu)(\not{k}\gamma^5\not{k} + \not{k}\gamma^5\not{p}_1 + m_i\not{k}\gamma^5 - \not{p}_2\gamma^5\not{k} - \not{p}_2\gamma^5\not{p}_1 - m_i\not{p}_2\gamma^5 + m_i\gamma^5\not{k} + m_i\gamma^5\not{p}_1 + m_i^2\gamma^5\right]$$

$$= \mathrm{Tr}\left[(C_V^i\gamma^\mu\not{k}\gamma^\nu \overset{\downarrow}{+} m_i C_V^i\overset{\downarrow}{\gamma^\mu}\gamma^\nu - C_A^i\overset{\downarrow}{\gamma^\mu}\gamma^5\not{k}\gamma^\nu - m_i \overset{\downarrow}{C_A^i}\gamma^\mu\gamma^5\gamma^\nu)(\not{k}\gamma^5\not{k} + \not{k}\gamma^5\not{p}_1 + m_i\not{k}\gamma^5 - \not{p}_2\gamma^5\not{k} - \not{p}_2\gamma^5\not{p}_1 - m_i\not{p}_2\gamma^5 + m_i\gamma^5\not{k} + m_i\gamma^5\not{p}_1 + m_i^2\gamma^5)\right]$$

$$= \overset{0}{\cancel{C_V^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{k}\gamma^5\not{k})}} + \overset{0}{\cancel{C_V^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{k}\gamma^5\not{p}_1)}} + m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{k}\gamma^5) - \overset{0}{\cancel{C_V^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{p}_2\gamma}}$$

$$- \overset{0}{\cancel{C_V^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{p}_2\gamma^5\not{p}_1)}} - m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{p}_2\gamma^5) + m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\gamma^5\not{k}) + m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\gamma^5\not{p}_1)$$

$$+ \overset{0}{\cancel{m_i^2 C_V^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\gamma^5)}} + \overset{0}{\cancel{m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{k}\gamma^5\not{k})}} + m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{k}\gamma^5\not{p}_1) + \overset{0}{\cancel{m_i^2 C_V^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{k}\gamma^5)}}$$

$$- m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{p}_2\gamma^5\not{k}) - m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{p}_2\gamma^5\not{p}_1) - \overset{0}{\cancel{m_i^2 C_V^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{p}_2\gamma^5)}} + \overset{0}{\cancel{m_i^2 C_V^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^5\not{k})}}$$

$$+ \overset{0}{\cancel{m_i^2 C_V^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^5\not{p}_1)}} + \overset{0}{\cancel{m_i^3 C_V^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^5)}} - \overset{0}{\cancel{C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\not{k}\gamma^\nu\not{k}\gamma^5\not{k})}} - \overset{0}{\cancel{C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\not{k}\gamma^\nu\not{k}\gamma^5\not{p}_1}}$$

$$- m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\not{k}\gamma^\nu\not{k}\gamma^5) + \overset{0}{\cancel{C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\not{k}\gamma^\nu\not{p}_2\gamma^5\not{k})}} + \overset{0}{\cancel{C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\not{k}\gamma^\nu\not{p}_2\gamma^5\not{p}_1)}}$$

$$+ m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\not{k}\gamma^\nu\not{p}_2\gamma^5) - m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\not{k}\gamma^\nu\gamma^5\not{k}) - m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\not{k}\gamma^\nu\gamma^5\not{p}_1)$$

$$- \overset{0}{\cancel{m_i^2 C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\not{k}\gamma^\nu\gamma^5)}} - m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\not{k}\gamma^5\not{k}) - m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\not{k}\gamma^5\not{p}_1)$$

$$- \overset{0}{\cancel{m_i^2 C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\not{k}\gamma^5)}} + m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\not{p}_2\gamma^5\not{k}) + m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\not{p}_2\gamma^5\not{p}_1)$$

$$+ \overset{0}{\cancel{m_i^2 C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\not{p}_2\gamma^5)}} - \overset{0}{\cancel{m_i^2 C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\gamma^5\not{k})}} - \overset{0}{\cancel{m_i^2 C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\gamma^5\not{p}_1)}} - m_i^3 C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\gamma^5$$

$$= - m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{p}_2\gamma^5) - m_i C_V^i\,\mathrm{Tr}(\not{k}\gamma^\mu\gamma^\nu\not{p}_2\gamma^5) + m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\gamma^5\not{p}_1)$$

$$+ m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{k}\gamma^5\not{p}_1) - m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{p}_2\gamma^5\not{p}_1) + m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{k})$$

$$- m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{p}_2) + m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{p}_2\not{k}) - m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{k}) - m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{p}_1)$$

$$- m_i C_A^i k^2\,\mathrm{Tr}(\gamma^\mu\gamma^\nu) - m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{k}\not{p}_1) + m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{p}_2 p_1) + m_i^3 C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu)$$

We have used: $\gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0$

$\mathrm{Trace}(\text{odd } \# \text{ of } \gamma\text{'s}) = 0$

$\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu) = 0$

$\not{k}\not{k} = k^2$

$$\gamma^\mu\not{k} + \not{k}\gamma^\mu = \gamma^\mu\gamma^\alpha k_\alpha + \gamma^\alpha\gamma^\mu k_\alpha = k_\alpha 2\eta^{\mu\alpha} = 2k^\mu$$

$$\Rightarrow \mathrm{Trace} = -2m_i C_V^i k^\mu\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^\nu\not{p}_2\gamma^5)}} + 2m_i C_V^i\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^\mu\gamma^5\not{p}_1)}}\,k^\nu - 2m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\not{p}_1)\,k^\nu$$

$$- m_i C_V^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{p}_2\gamma^5\not{p}_1) + m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{k}) - m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{p}_2)$$

$$+ m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{p}_2\not{k}) - m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{k}) - m_i C_A^i k^2\,\mathrm{Tr}(\gamma^\mu\gamma^\nu) + m_i C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu\not{p}_1$$

$$+ m_i^3 C_A^i\,\mathrm{Tr}(\gamma^\mu\gamma^\nu).$$

$$Tr = -2m_i C_A^i d P_1^\mu K^\nu - 4 i m_i C_V^i \varepsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{1\beta} + d m_i C_A^i (K^\mu K^\nu \cancel{+ K^\nu K^\mu} - K^2 \eta^{\mu\nu})$$
$$- d m_i C_A^i (K^\mu P_2^\nu + K^\nu P_2^\mu - (P_2 \cdot K)\eta^{\mu\nu}) + d m_i C_A^i P_{2\alpha} K_\beta (\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\nu\beta}$$
$$+ \eta^{\mu\beta}\eta^{\nu\alpha}) - d m_i C_A^i (K^\mu \cancel{K^\nu} + K^\nu K^\mu - K^2 \eta^{\mu\nu}) - d m_i C_A^i K^2 \eta^{\mu\nu} + d m_i C_A^i P_{2\alpha} P_{1\beta}$$
$$(\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha}) + d m_i^3 C_A^i \eta^{\mu\nu}.$$

$$Tr = d m_i \Big\{ -2 C_A^i P_1^\mu K^\nu - \frac{4i}{d} C_V^i \varepsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{1\beta} - C_A^i (K^\mu \cancel{P_2^\nu} + \cancel{K^\nu} P_2^\mu - \cancel{(P_2 \cdot K)}\eta^{\mu\nu}$$
$$+ C_A^i (\eta^{\mu\nu} \cancel{(P_2 \cdot K)} - \cancel{P_2^\mu K^\nu} + \cancel{P_2^\nu K^\mu}) - C_A^i K^2 \eta^{\mu\nu} + C_A^i (\eta^{\mu\nu}(P_1 \cdot P_2)$$
$$- P_2^\mu P_1^\nu + P_1^\mu P_2^\nu) + m_i^2 C_A^i \eta^{\mu\nu} \Big\}$$

$$Tr = d m_i \Big\{ -2 C_A^i P_1^\mu K^\nu - \frac{4i}{d} C_V^i \varepsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{1\beta} + C_A^i [-2K^\nu P_2^\mu + 2(P_2 \cdot K)\eta^{\mu\nu}]$$
$$- C_A^i K^2 \eta^{\mu\nu} + C_A^i (\eta^{\mu\nu}(P_1 \cdot P_2) - P_2^\mu P_1^\nu + P_1^\mu P_2^\nu) + m_i^2 C_A^i \eta^{\mu\nu} \Big\} \quad (3)$$

$$\Rightarrow M_{1a} = \frac{-g^2 e \tan\beta}{4 M_W \cos\theta_W} d \sum_{i=d,s,b,\dots} N_i Q_i m_i^2 \int \frac{d^d K}{(2\pi)^d} \Big\{ -2 C_A^i P_1^\mu K^\nu - \frac{4i}{d} C_V^i \varepsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{1\beta}$$
$$+ C_A^i [-2K^\nu P_2^\mu + 2(P_2 \cdot K)\eta^{\mu\nu}] - C_A^i K^2 \eta^{\mu\nu} + C_A^i (\eta^{\mu\nu}(P_1 \cdot P_2) - P_2^\mu P_1^\nu + P_1^\mu P_2^\nu) + m_i^2 C_A^i \eta^{\mu\nu} \Big\}$$
$$\cdot \frac{1}{(K^2 - m_i^2)[(K-P_2)^2 - m_i^2][(K+P_1)^2 - m_i^2]} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} u^\kappa . \quad (4)$$

$$\frac{1}{\underset{a}{(K^2 - m_i^2)} \underset{b}{[(K+P_1)^2 - m_i^2]} \underset{c}{[(K-P_2)^2 - m_i^2]}} = ?$$

$$\frac{1}{abc} = 2 \int_0^1 dx \int_0^{1-x} \frac{dy}{[a(1-x-y) + bx + cy]^3} \quad (5)$$

$$a[1-x-y] + bx + cy = (K^2 - m_i^2)(1-x-y) + [(K+P_1)^2 - m_i^2]x + [(K-P_2)^2 - m_i^2]y$$
$$= K^2 - \cancel{K^2 x} - \cancel{K^2 y} - m_i^2 + \cancel{m_i^2 x} + \cancel{m_i^2 y} + \cancel{K^2 x} + 2(K \cdot P_1)x + \cancel{P_1^2} x - \cancel{m_i^2 x} + \cancel{K^2 y} - 2(K \cdot P_2)y$$
$$+ \cancel{P_2^2}^{\,0} y - \cancel{m_i^2 y} = K^2 - m_i^2 + 2(P_1 \cdot K)x + M_Z^2 x - 2(P_2 \cdot K)y$$
$$= K^2 - m_i^2 + M_Z^2 x + 2((P_1 \cdot K)x - (P_2 \cdot K)y)$$

$$\frac{1}{(K^2 - m_i^2)[(K+P_1)^2 - m_i^2][(K-P_2)^2 - m_i^2]} = 2 \int_0^1 dx \int_0^{1-x} \frac{dy}{(K^2 - m_i^2 + M_Z^2 x + 2K \cdot (P_1 x - P_2 y))^3} \quad (6)$$

$$K' = K + (P_1 x - P_2 y) \quad (7)$$
$$K'^2 = K^2 + 2K \cdot (P_1 x - P_2 y) + (M_Z^2 x^2 - 2(P_1 \cdot P_2) xy)$$
$$\Rightarrow K^2 + 2K \cdot (P_1 x - P_2 y) = K'^2 + 2(P_1 \cdot P_2) xy - M_Z^2 x^2 .$$
$$d^d K' = d^d K$$

$$\Rightarrow$$

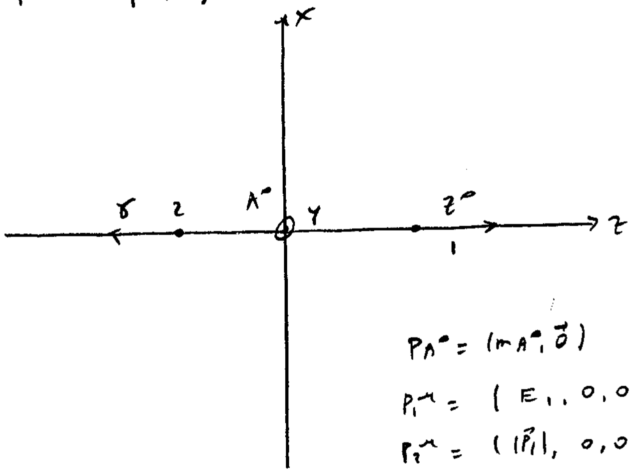
$$M_{1a} = \frac{-2g^2 e \tan\beta\, d}{4 M_W \cos\theta_W} \sum_{i=d,s,b,\cdots} N_i Q_i m_i^2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d K'}{(2\pi)^d} \Big\{ -2C_A^i P_1^\mu \left(K' - (P_1 x - P_2 y)\right)^\nu - \frac{4i}{d} C_V^i \epsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{1\beta}$$

$$-2C_A^i P_2^\mu \left(K' - (P_1 x - P_2 y)\right)^\nu + 2C_A^i n^{\mu\nu}\left(P_2 \cdot (K' - (P_1 x - P_2 y))\right) - C_A^i n^{\mu\nu}\left(K' - (P_1 x - P_2 y)\right)^2$$

$$+ C_A^i n^{\mu\nu} (P_1 \cdot P_2) - C_A^i P_2^\mu P_1^\nu + C_A^i P_1^\mu P_2^\nu + m_i^2 C_A^i n^{\mu\nu} \Big\} \cdot \left[K'^2 - m_i^2 + M_Z^2 x - M_Z^2 x^2 + 2(P_1 \cdot P_2) x y \right]^{-}$$

$$\cdot\ \epsilon_{1\mu}^* \epsilon_{2\nu}^* \mu^\nu . \qquad (8)$$

In the rest frame of A°:



$$P_{A^\circ} = (m_{A^\circ}, \vec{0})$$
$$P_1^\mu = (E_1, 0, 0, |\vec{P}_1|)$$
$$P_2^\mu = (|\vec{P}_1|, 0, 0, -|\vec{P}_1|)$$

$$\epsilon_2^{\nu\,(\lambda=\pm 1)} = \mp \frac{1}{\sqrt{2}} (0, -1, \pm i, 0)$$
$$\epsilon^{\lambda=0} = (|\vec{P}_1|, 0, 0, E_1) \frac{1}{M_{Z^\circ}}$$

$$\begin{cases} P_1^\mu \epsilon_{2\mu}^* = 0 \\ P_1^\mu \epsilon_{1\mu}^* = 0 \\ P_2^\nu \epsilon_{2\nu}^* = 0 \end{cases} \qquad (9)$$

$$M_{1a} = -\frac{2g^2 e \tan\beta\, d}{4 M_W \cos\theta_W} \sum_{i=d,s,b,\cdots} N_i Q_i m_i^2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d K'}{(2\pi)^d} \Big\{ -2C_A^i P_1^\mu K'^\nu - \frac{4i}{d} C_V^i \epsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{1\beta}$$

$$-2C_A^i P_2^\mu K'^\nu + 2C_A^i n^{\mu\nu}\left(P_2 \cdot (K' - (P_1 x - P_2 y))\right) - C_A^i n^{\mu\nu}\left(K' - (P_1 x - P_2 y)\right)^2 + C_A^i n^{\mu\nu} (P_1 \cdot P_2)$$

$$+ m_i^2 C_A^i n^{\mu\nu} \Big\} \left[K'^2 - m_i^2 + M_Z^2 x - M_Z^2 x^2 + 2(P_1 \cdot P_2) x y \right]^{-3} \cdot \epsilon_{1\mu}^* \epsilon_{2\nu}^* \mu^* . \qquad (10)$$

$$\int \frac{d^d K'\ K'^\mu}{\left[K'^2 - m_i^2 + M_Z^2 x(1-x) + 2(P_1 \cdot P_2) x y\right]^3} = 0 \qquad (11)$$

$$\int \frac{d^d K'}{\left[K'^2 - m_i^2 + M_Z^2 x(1-x) + 2(P_1 \cdot P_2) x y\right]^3} = I_0(x,y) = \frac{i(-\pi)^{d/2}\, \Gamma(3 - \frac{d}{2})}{2\left[-m_i^2 + M_Z^2 x(1-x) + 2(P_1 \cdot P_2) x y\right]^{3-\frac{d}{2}}} \qquad (12)$$

$$\int \frac{d^d K'\ K'^2}{\left[K'^2 - m_i^2 + M_Z^2 x(1-x) + 2(P_1 \cdot P_2) x y\right]^3} = I_0 \frac{d}{2} \frac{\left[-m_i^2 + M_Z^2 x(1-x) + 2(P_1 \cdot P_2) x y\right]}{(2 - \frac{d}{2})} \qquad (13)$$

$\Rightarrow$

$$M_{1a} = \frac{-2g^2 e \tan\beta\, d}{4M_W \cos\theta_W} \sum_{i=d,s,b,\dots} N_i Q_i m_i^2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{(2\pi)^d} I_0(x,y) \Big\{ -\frac{4i}{d} C_V^i \epsilon^{\mu\nu\alpha\beta} p_{2\alpha} p_{1\beta}$$

$$-2C_A^i \eta^{\mu\nu}(p_1\cdot p_2)x + C_A^i \eta^{\mu\nu}(p_1\cdot p_2) + m_i^2 C_A^i \eta^{\mu\nu} - C_A^i \eta^{\mu\nu} M_Z^2 x^2 + 2(p_1\cdot p_2)xy C_A^i \eta^{\mu\nu}$$

$$- C_A^i \eta^{\mu\nu} \frac{d}{2} \frac{[-m_i^2 + M_Z^2 x(1-x) + 2(p_1\cdot p_2)xy]}{(2-\frac{d}{2})} \Big\} \epsilon_{1\mu}^* \epsilon_{2\nu}^* \mu^\epsilon \qquad (14)$$

$$-iM_{2a} = (-1) \sum_{\substack{i=d,s,b,\\ e,\mu,\tau}} N_i \int \frac{d^dk}{(2\pi)^d} \mathrm{Tr}\Big[(-ie\gamma^\nu Q_i) \frac{i}{\not k - m_i} \Big(\frac{-ig}{\cos\theta_W}\Big) \gamma^\mu \frac{1}{2}(C_V^i - C_A^i\gamma^5) \cdot$$

$$\cdot \frac{i}{(\not k - \not p_1) - m_i} \Big(\frac{-g m_i \tan\beta}{2M_W} \gamma^5 \Big) \frac{i}{(\not k + \not p_2) - m_i} \Big] \epsilon_{1\mu}^* \epsilon_{2\nu}^* \mu^\epsilon \qquad (15)$$

$$-iM_{2a} = \frac{ig^2 e \tan\beta}{4M_W \cos\theta_W} \sum_{i=d,s,b,\dots} N_i Q_i m_i \int \frac{d^dk}{(2\pi)^d} \mathrm{Tr}\Big[\gamma^\nu (\not k + m_i) \gamma^\mu (C_V^i - C_A^i \gamma^5) [(\not k - \not p_1) + m_i] \gamma^5 \cdot$$

$$\cdot [(\not k + \not p_2) + m_i] \Big] \frac{1}{(k^2 - m_i^2)[(k-p_1)^2 - m_i^2][(k+p_2)^2 - m_i^2]} \epsilon_{1\mu}^* \epsilon_{2\nu}^* \mu^\epsilon \qquad (16)$$

$\mu^\epsilon \to 1$
when $d \to 4$

$$\mathrm{Tr}\big[\gamma^\nu (\not k + m_i) \gamma^\mu (C_V^i - C_A^i\gamma^5) [(\not k - \not p_1) + m_i] \gamma^5 [(\not k + \not p_2) + m_i] \big]$$

$$= \mathrm{Tr}\Big[(\gamma^\nu \not k \gamma^\mu + m_i \gamma^\nu \gamma^\mu)(C_V^i - C_A^i\gamma^5) [\not k \gamma^5 \not k + \not k \gamma^5 \not p_2 + m_i \not k \gamma^5 - \not p_1 \gamma^5 \not k - \not p_1 \gamma^5 \not p_2$$

$$- m_i \not p_1 \gamma^5 + m_i \gamma^5 \not k + m_i \gamma^5 \not p_2 + m_i^2 \gamma^5] \Big]$$

$$= \mathrm{Tr}\Big[(\overset{\downarrow}{C_V^i} \gamma^\nu \not k \gamma^\mu - \overset{\downarrow}{C_A^i} \gamma^\nu \not k \gamma^\mu \gamma^5 + \overset{\downarrow}{m_i C_V^i} \gamma^\nu \gamma^\mu - m_i \overset{\downarrow}{C_A^i} \gamma^\nu \gamma^\mu \gamma^5)(\not k \gamma^5 \not k + \not k \gamma^5 \not p_2 + m_i \not k \gamma^5$$

$$- \not p_1 \gamma^5 \not k - \not p_1 \gamma^5 \not p_2 - m_i \not p_1 \gamma^5 + m_i \gamma^5 \not k + m_i \gamma^5 \not p_2 + m_i^2 \gamma^5) \Big]$$

$$= C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \not k \gamma^5 \not k)} + C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \not k \gamma^5 \not p_2)} + m_i C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \not k \gamma^5)} - C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \not p_1 \gamma^5 \not k)}$$

$$- C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \not p_1 \gamma^5 \not p_2)} - m_i C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \not p_1 \gamma^5)} + m_i C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \gamma^5 \not k)} + m_i C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \gamma^5 \not p_2)}$$

$$+ m_i^2 C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \gamma^5)} - C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \gamma^5 \not k \gamma^5 \not k)} - C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \gamma^5 \not k \gamma^5 \not p_2)} - m_i C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \gamma^5 \not k \gamma^5)}$$

$$+ C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \gamma^5 \not p_1 \gamma^5 \not k)} + C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \gamma^5 \not p_1 \gamma^5 \not p_2)} + m_i C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \gamma^5 \not p_1 \gamma^5)}$$

$$- m_i C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \gamma^5 \gamma^5 \not k)} - m_i C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \gamma^5 \gamma^5 \not p_2)} - m_i^2 C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \not k \gamma^\mu \gamma^5 \gamma^5)}$$

$$+ m_i C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \not k \gamma^5 \not k)} + m_i C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \not k \gamma^5 \not p_2)} + m_i^2 C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \not k \gamma^5)} - m_i C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \not p_1 \gamma^5 \not k)}$$

$$- m_i C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \not p_1 \gamma^5 \not p_2)} - m_i^2 C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \not p_1 \gamma^5)} + m_i^2 C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^5 \not k)} + m_i^2 C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^5 \not p_2)}$$

$$+ m_i^3 C_V^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^5)} - m_i C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^5 \not k \gamma^5 \not k)} - m_i C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^5 \not k \gamma^5 \not p_2)} - m_i^2 C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^5 \not k \gamma^5)}$$

$$+ m_i C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^5 \not p_1 \gamma^5 \not k)} + m_i C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^5 \not p_1 \gamma^5 \not p_2)} + m_i^2 C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^5 \not p_1 \gamma^5)} - m_i^2 C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^5 \gamma^5 \not k)}$$

$$- m_i^2 C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^5 \gamma^5 \not p_2)} - m_i^3 C_A^i \cancel{\mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^5 \gamma^5)}$$

$$= - m_i C_V^i \mathrm{Tr}(\gamma^\nu \overset{\downarrow}{\not{K}} \gamma^\mu \not{p}_1 \gamma^5) + m_i C_V^i \mathrm{Tr}(\gamma^\nu \overset{\downarrow}{\not{K}} \gamma^\mu \gamma^5 \not{p}_2) - m_i C_A^i \mathrm{Tr}(\gamma^\nu \not{K} \gamma^\mu \gamma^5 \not{K} \gamma^5)$$

$$+ m_i C_A^i \mathrm{Tr}(\gamma^\nu \not{K} \gamma^\mu \gamma^5 \not{p}_1 \gamma^5) - m_i C_A^i \mathrm{Tr}(\gamma^\nu \not{K} \gamma^\mu \not{K}) - m_i C_A^i \mathrm{Tr}(\gamma^\nu \not{K} \gamma^\mu \not{p}_2)$$

$$+ m_i C_V^i \overset{0}{\mathrm{Tr}(\gamma^\nu \gamma^\mu \not{K} \gamma^5 \not{K})} + m_i C_V^i \overset{\downarrow}{\mathrm{Tr}}(\gamma^\nu \gamma^\mu \not{K} \gamma^5 \not{p}_2) - m_i C_V^i \mathrm{Tr}(\gamma^\nu \overset{\downarrow}{\gamma^\mu} \not{p}_1 \gamma^5 \not{K})$$

$$- m_i C_V^i \mathrm{Tr}(\gamma^\nu \gamma^\mu \not{p}_1 \gamma^5 \not{p}_2) + m_i C_A^i \mathrm{Tr}(\gamma^\nu \gamma^\mu) K^2 + m_i C_A^i \mathrm{Tr}(\gamma^\nu \gamma^\mu \not{K} \not{p}_2)$$

$$- m_i C_A^i \mathrm{Tr}(\gamma^\nu \gamma^\mu \not{p}_1 \not{K}) - m_i C_A^i \mathrm{Tr}(\gamma^\nu \gamma^\mu \not{p}_1 \not{p}_2) - m_i^3 C_A^i \mathrm{Tr}(\gamma^\mu \gamma^\nu)$$

$$= 2 m_i C_V^i K^\mu \overset{0}{\mathrm{Tr}(\gamma^\nu \gamma^5 \not{p}_2)} + 4 i m_i C_V^i \varepsilon^{\nu\alpha\mu\beta} K_\alpha p_{1\beta} - 4 i m_i C_V^i \varepsilon^{\nu\mu\alpha\beta} p_{1\alpha} K_\beta$$

$$- m_i C_A^i \mathrm{Tr}(\gamma^\nu \not{K} \gamma^\mu \not{p}_1) - m_i C_A^i \mathrm{Tr}(\gamma^\nu \not{K} \gamma^\mu \not{p}_2) - 4 i m_i C_V^i \varepsilon^{\nu\mu\alpha\beta} p_{1\alpha} p_{2\beta} + d m_i C_A^i K^2 \eta^{\mu\nu}$$

$$+ m_i C_A^i \mathrm{Tr}(\gamma^\nu \gamma^\mu \not{K} \not{p}_2) - m_i C_A^i \mathrm{Tr}(\gamma^\nu \gamma^\mu \not{p}_1 \not{K}) - m_i C_A^i \mathrm{Tr}(\gamma^\nu \gamma^\mu \not{p}_1 \not{p}_2) - d m_i^3 C_A^i \eta^{\mu\nu}$$

$$4 i m_i C_V^i \varepsilon^{\nu\alpha\mu\beta} K_\alpha p_{1\beta} - 4 i m_i C_V^i \varepsilon^{\nu\mu\alpha\beta} p_{1\alpha} K_\beta$$

$$= 4 i m_i C_V^i \varepsilon^{\nu\alpha\mu\beta} K_\alpha p_{1\beta} - 4 i m_i C_V^i \varepsilon^{\nu\mu\beta\alpha} p_{1\beta} K_\alpha$$

$$= 4 i m_i C_V^i \varepsilon^{\nu\alpha\mu\beta} K_\alpha p_{1\beta} - 4 i m_i C_V^i \varepsilon^{\nu\alpha\mu\beta} K_\alpha p_{1\beta} = 0$$

$$\Rightarrow \mathrm{Tr} = - d m_i C_A^i (K^\nu p_1^\mu + K^\mu p_1^\nu - (p_1 \cdot K) \eta^{\mu\nu}) - d m_i C_A^i (K^\nu p_2^\mu + K^\mu p_2^\nu - (p_2 \cdot K) \eta^{\mu\nu})$$

$$- 4 i m_i C_V^i \varepsilon^{\nu\mu\alpha\beta} p_{1\alpha} p_{2\beta} + d m_i C_A^i K^2 \eta^{\mu\nu} + m_i C_A^i K_\alpha p_{2\beta} \mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^\alpha \gamma^\beta)$$

$$- m_i C_A^i \mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^\alpha \gamma^\beta) p_{1\alpha} K_\beta - m_i C_A^i p_{1\alpha} p_{2\beta} \mathrm{Tr}(\gamma^\nu \gamma^\mu \gamma^\alpha \gamma^\beta) - d m_i^3 C_A^i \eta^{\mu\nu}$$

$$= - d m_i C_A^i (K^\nu p_1^\mu + K^\mu p_1^\nu - (p_1 \cdot K) \eta^{\mu\nu}) - d m_i C_A^i (K^\nu p_2^\mu + K^\mu p_2^\nu - (p_2 \cdot K) \eta^{\mu\nu})$$

$$- 4 i m_i C_V^i \varepsilon^{\nu\mu\alpha\beta} p_{1\alpha} p_{2\beta} + d m_i C_A^i K^2 \eta^{\mu\nu} + m_i C_A^i K_\alpha p_{2\beta}\, d\, [\eta^{\mu\nu} \eta^{\alpha\beta} - \eta^{\nu\alpha} \eta^{\mu\beta}$$

$$+ \eta^{\nu\beta} \eta^{\mu\alpha}] - d m_i C_A^i p_{1\alpha} K_\beta [\eta^{\mu\nu} \eta^{\alpha\beta} - \eta^{\nu\alpha} \eta^{\mu\beta} + \eta^{\nu\beta} \eta^{\mu\alpha}] - d m_i^3 C_A^i \eta^{\mu\nu}$$

$$- d m_i C_A^i p_{1\alpha} p_{2\beta} [\eta^{\mu\nu} \eta^{\alpha\beta} - \eta^{\nu\alpha} \eta^{\mu\beta} + \eta^{\nu\beta} \eta^{\mu\alpha}]$$

$$= - d m_i C_A^i (K^\nu p_1^\mu + K^\mu p_1^\nu - (p_1 \cdot K) \eta^{\mu\nu}) - d m_i C_A^i (K^\nu p_2^\mu + K^\mu p_2^\nu - (p_2 \cdot K) \eta^{\mu\nu})$$

$$- 4 i m_i C_V^i \varepsilon^{\nu\mu\alpha\beta} p_{1\alpha} p_{2\beta} + d m_i C_A^i K^2 \eta^{\mu\nu} + d m_i C_A^i \eta^{\mu\nu} (p_2 \cdot K) - d m_i C_A^i p_2^\mu K^\nu$$

$$+ d m_i C_A^i K^\mu p_2^\nu - d m_i C_A^i \eta^{\mu\nu} (p_1 \cdot K) + d m_i C_A^i p_1^\nu K^\mu - d m_i C_A^i p_1^\mu K^\nu - d m_i^3 C_A^i \eta^{\mu\nu}$$

$$- d m_i C_A^i \eta^{\mu\nu} (p_1 \cdot p_2) + d m_i C_A^i p_1^\nu p_2^\mu - d m_i C_A^i p_1^\mu p_2^\nu$$

$$= - 2 d m_i C_A^i K^\nu p_1^\mu - 2 d m_i C_A^i K^\nu p_2^\mu + 2 d m_i C_A^i \eta^{\mu\nu} (p_2 \cdot K) - 4 i m_i C_V^i \varepsilon^{\nu\mu\alpha\beta} p_{1\alpha} p_{2\beta}$$

$$+ d m_i C_A^i K^2 \eta^{\mu\nu} - d m_i^3 C_A^i \eta^{\mu\nu} - d m_i C_A^i \eta^{\mu\nu} (p_1 \cdot p_2) + d m_i C_A^i p_1^\nu p_2^\mu - d m_i C_A^i p_1^\mu p_2^\nu$$

$$\Rightarrow \mathrm{Tr} = d m_i \Big\{ - 2 C_A^i K^\nu p_1^\mu - 2 C_A^i K^\nu p_2^\mu + 2 C_A^i \eta^{\mu\nu} (p_2 \cdot K) - \frac{4i}{d} C_V^i \varepsilon^{\nu\mu\alpha\beta} p_{1\alpha} p_{2\beta}$$

$$+ C_A^i K^2 \eta^{\mu\nu} - m_i^2 C_A^i \eta^{\mu\nu} - C_A^i \eta^{\mu\nu} (p_1 \cdot p_2) + C_A^i p_1^\nu p_2^\mu - C_A^i p_1^\mu p_2^\nu \Big\} \qquad (17)$$

$$\frac{1}{\underset{a}{(K^2-m_i^2)}\,\underset{b}{[(K-P_1)^2-m_i^2]}\,\underset{c}{[(K+P_2)^2-m_i^2]}} = ?$$

$$a(1-X-Y)+bX+CY = (K^2-m_i^2)(1-X-Y) + [(K-P_1)^2-m_i^2]X + [(K+P_2)^2-m_i^2]Y$$

$$= \cancel{K^2} - \cancel{K^2X} - \cancel{K^2Y} - \cancel{m_i^2} + \cancel{m_i^2X} + \cancel{m_i^2Y} + \cancel{K^2X} - 2(K\cdot P_1)X + \cancel{P_1^2}X - \cancel{m_i^2X} + \cancel{K^2Y} + 2(P_2\cdot K)Y$$

$$- \cancel{m_i^2Y}$$

$$= K^2 - m_i^2 + M_Z^2X - 2K\cdot(P_1X-P_2Y)$$

$$\Rightarrow \frac{1}{(K^2-m_i^2)[(K-P_1)^2-m_i^2][(K+P_2)^2-m_i^2]} = 2\int_0^1 dX\int_0^{1-X}\frac{dY}{[K^2-m_i^2+M_Z^2X-2K\cdot(P_1X-P_2Y)]^3} \quad (18)$$

$$K' = K-(P_1X-P_2Y) \Rightarrow K = K' + (P_1X-P_2Y) \quad (19)$$

$$K'^2 = K^2 - 2K\cdot(P_1X-P_2Y) + M_Z^2X^2 - 2(P_1\cdot P_2)XY$$

$$d^dK' = d^dK$$

$$\therefore M_{2a} = -\frac{2g^2e\tan\beta\, d}{4M_W\cos\theta_W}\sum_{i=d,s,b,\dots} N_i a_i m_i^2\int_0^1 dX\int_0^{1-X}dY\int\frac{d^dK}{(2\pi)^d}\Big\{-2C_A^i K^\nu P_1^\mu - 2C_A^i K^\nu P_2^\mu +$$

$$+ 2C_A^i n^{\mu\nu}(P_2\cdot K) - \frac{4i}{d}C_V^i \varepsilon^{\nu\mu\alpha\beta}P_{1\alpha}P_{2\beta} + C_A^i K^2 n^{\mu\nu} - m_i^2 C_A^i n^{\mu\nu} - C_A^i n^{\mu\nu}(P_1\cdot P_2) + C_A^i P_1^\nu P_2^\mu$$

$$- C_A^i P_1^\mu P_2^\nu\Big\}[K^2-m_i^2+M_Z^2X-2K\cdot(P_1X-P_2Y)]^{-3}\,\varepsilon_{1\mu}^*\varepsilon_{2\nu}^*\,\mu^\alpha \quad (20)$$

$$M_{2a} = -\frac{2g^2e\tan\beta\, d}{4M_W\cos\theta_W}\sum_{i=d,s,b,\dots} N_i a_i m_i^2\int_0^1 dX\int_0^{1-X}dY\int\frac{d^dK'}{(2\pi)^d}\Big\{-2C_A^i \cancel{P_1^\mu}(K'+(P_1X-P_2Y))^\nu$$

$$-2C_A^i P_2^\mu(K'+(P_1X-P_2Y))^\nu + 2C_A^i n^{\mu\nu}(P_2\cdot(K'+(P_1X-P_2Y))) - \frac{4i}{d}C_V^i\varepsilon^{\nu\mu\alpha\beta}P_{1\alpha}P_{2\beta}$$

$$+ C_A^i n^{\mu\nu}(K'+(P_1X-P_2Y))^2 - m_i^2C_A^i n^{\mu\nu} - C_A^i n^{\mu\nu}(P_1\cdot P_2) + C_A^i P_1^\nu P_2^\mu - C_A^i P_1^\mu P_2^\nu\Big\}\cdot$$

$$\cdot[K'^2-m_i^2+M_Z^2X-M_Z^2X^2+2(P_1\cdot P_2)XY]^{-3}\,\varepsilon_{1\alpha}^*\varepsilon_{2\nu}^*\,\mu^\alpha \quad (21)$$

Using:

$$P_1^\mu\varepsilon_{2\mu}^* = 0$$

$$P_2^\mu\varepsilon_{2\mu}^* = 0$$

$$P_1^\mu\varepsilon_{1\mu}^* = 0$$

$$M_{2a} = \frac{-2g^2e\tan\beta\, d}{4M_W\cos\theta_W}\sum_{i=d,s,b,\dots} N_i a_i m_i^2\int_0^1 dX\int_0^{1-X}dY\int\frac{d^dK'}{(2\pi)^d}\Big\{-2C_A^i P_2^\mu\cancel{K'^\nu} + 2C_A^i n^{\mu\nu}(P_2\cdot(K'+(P_1X-P_2Y))$$

$$-\frac{4i}{d}C_V^i\varepsilon^{\nu\mu\alpha\beta}P_{1\alpha}P_{2\beta} + C_A^i n^{\mu\nu}(K'^2 + 2\cancel{K'}\cdot(P_1X-P_2Y) + M_Z^2X^2 - 2(P_1\cdot P_2)XY) - m_i^2C_A^i n^{\mu\nu}$$

$$- C_A^i n^{\mu\nu}(P_1\cdot P_2)\Big\}\cdot[K'^2-m_i^2+\underbrace{M_Z^2X-M_Z^2X^2}_{M_Z^2X(1-X)}+2(P_1\cdot P_2)XY]^{-3}\,\varepsilon_{1\mu}^*\varepsilon_{2\nu}^*\,\mu^\alpha \quad (22)$$

$$M_{2a} = \frac{-2g^2 e \tan\beta\, d}{4 M_W \cos\theta_W} \sum_{i=d,s,b,\ldots} N_i Q_i m_i^2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{(2\pi)^d} I_0(x,y) \Big\{ 2C_A^i \cancel{(P_1\cdot P_2)} x\, g^{\mu\nu}$$

$$- \frac{4i}{d} C_V^i \varepsilon^{\nu\mu\alpha\beta} P_{1\alpha} P_{2\beta} + C_A^i g^{\mu\nu} \frac{d}{2} \frac{[-m_i^2 + M_Z^2 x(1-x) + 2(P_1\cdot P_2)xy]}{(2-\frac{d}{2})} + C_A^i g^{\mu\nu} \cancel{M_Z^2} x^2$$

$$- 2C_A^i g^{\mu\nu} \cancel{(P_1\cdot P_2)} xy - m_i^2 C_A^i \cancel{g^{\mu\nu}} - C_A^i \cancel{g^{\mu\nu}} (P_1\cdot P_2) \Big\} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \mu^* \qquad (23)$$

$$\varepsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{1\beta}$$

$$\varepsilon^{\nu\mu\alpha\beta} P_{1\alpha} P_{2\beta} = -\varepsilon^{\mu\nu\beta\alpha} P_{1\beta} P_{2\alpha} = \varepsilon^{\mu\nu\alpha\beta} P_{1\beta} P_{2\alpha} \qquad (24)$$

$$M_{1a} + M_{2a} = \frac{-2g^2 e \tan\beta\, \cancel{d}}{4 M_W \cos\theta_W} \sum_{i=d,s,\ldots} N_i Q_i m_i^2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{(2\pi)^d} I_0(x,y) \Big\{ -\frac{8i}{\cancel{d}} C_V^i \varepsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{1\beta} \Big\}$$

$$\varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \mu^* . \qquad (25)$$

$$M_{1a} + M_{2a} = \frac{\cancel{4} i g^2 e \tan\beta\, \varepsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{1\beta}}{M_W \cos\theta_W \quad (2\pi)^d} \sum_{i=d,s,\ldots} N_i Q_i m_i^2 C_V^i \int_0^1 dx \int_0^{1-x} dy\, I_0(x,y)\, \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \mu^* \quad (26)$$

in the limit $d \to 4$

$$I_0(x,y) = \frac{i\pi^2}{2} \frac{1}{[-m_i^2 + M_Z^2 x(1-x) + 2(P_1\cdot P_2)xy]} \qquad (27)$$

Introducing:

$$I_i \equiv \int_0^1 dx \int_0^{1-x} \frac{dy}{[-m_i^2 + M_Z^2 x(1-x) + 2(P_1\cdot P_2)xy]} \qquad (28)$$

$$M_{1a} + M_{2a} = \frac{4 i g^2 e \tan\beta\, \varepsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{1\beta}}{(2\pi)^4 M_W \cos\theta_W} \frac{i\pi^2}{2} \sum_{i=d,s,\ldots} N_i Q_i m_i^2 C_V^i I_i\, \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \mu^*$$

$$\boxed{M_{1a} + M_{2a} = \frac{g^2 e \tan\beta\, \varepsilon^{\mu\nu\alpha\beta} P_{1\alpha} P_{2\beta}}{8\pi^2 M_W \cos\theta_W} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \sum_{i=d,s,b,\ldots} N_i Q_i m_i^2 C_V^i I_i} \qquad (29)$$

b) for $i = u, c, t$

we have to interchange $\tan\beta \longrightarrow \cot\beta$

$$\boxed{M_{1b} + M_{2b} = \frac{g^2 e \cot\beta\, \varepsilon^{\mu\nu\alpha\beta} P_{1\alpha} P_{2\beta}}{8\pi^2 M_W \cos\theta_W} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu} \sum_{i=u,c,t} N_i Q_i m_i^2 C_V^i I_i} \qquad (30)$$

$$\Rightarrow M = M_{1a} + M_{2a} + M_{1b} + M_{2b} = \frac{g^2 e\, \varepsilon^{\mu\nu\alpha\beta} P_{1\alpha} P_{2\beta} \varepsilon^*_{1\mu} \varepsilon^*_{2\nu}}{8\pi^2 M_W \cos\theta_W} \Big\{ \tan\beta \sum_{\substack{i=d,s,b\\ e,\mu,\tau}} N_i Q_i m_i^2 C_V^i I_i$$

$$+ \cot\beta \sum_{i=u,c,t} N_i Q_i m_i^2 C_V^i I_i \Big\} \qquad (31)$$

$$\overline{|M|^2} = \frac{g^4 e^2 \, \varepsilon^{\mu\nu\alpha\beta} \varepsilon^{\rho\sigma\gamma\delta} P_{1\alpha} P_{2\beta} P_{1\gamma} P_{2\delta}}{64\pi^4 M_W^2 \cos^2\theta_W} \Big(\sum_{\lambda} \varepsilon_{1\mu} \varepsilon_{1\rho}^*\Big)\Big(\sum_{\lambda'} \varepsilon_{2\nu} \varepsilon_{2\sigma}^*\Big) |\{ \}|^2$$

$$= \frac{g^4 e^2 \, \varepsilon^{\mu\nu\alpha\beta} \varepsilon^{\rho\sigma\gamma\delta} P_{1\alpha} P_{2\beta} P_{1\gamma} P_{2\delta}}{64\pi^4 M_W^2 \cos^2\theta_W} \left(-\eta_{\mu\rho} + \frac{P_{1\mu} P_{1\rho}}{M_Z^2}\right)\left(-\eta_{\nu\sigma}\right) |\{ \}|^2$$

$$= \frac{g^4 e^2 \, \varepsilon^{\mu\nu\alpha\beta} \varepsilon^{\rho\sigma\gamma\delta} P_{1\alpha} P_{2\beta} P_{1\gamma} P_{2\delta}}{64\pi^4 M_W^2 \cos^2\theta_W} \left(\eta_{\mu\rho}\eta_{\nu\sigma} - \frac{P_{1\mu} P_{1\rho} \eta_{\nu\sigma}}{M_Z^2}\right) |\{ \}|^2$$

$$= \frac{g^4 e^2}{64\pi^4 M_W^2 \cos^2\theta_W} \left\{ \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{\mu\nu}{}^{\gamma\delta} P_{1\alpha} P_{2\beta} P_{1\gamma} P_{2\delta} \right\} |\{ \}|^2$$

because $\varepsilon^{\mu\nu\alpha\beta} P_{1\alpha} P_{1\mu} = 0$

$$\overline{|M|^2} = \frac{g^4 e^2}{64\pi^4 M_W^2 \cos^2\theta_W} \left[-2\left(\eta^{\alpha\gamma}\eta^{\beta\delta} - \eta^{\alpha\delta}\eta^{\beta\gamma}\right) P_{1\alpha} P_{2\beta} P_{1\gamma} P_{2\delta} \right] |\{ \}|^2$$

$$\overline{|M|^2} = \frac{-2 g^4 e^2}{64\pi^4 M_W^2 \cos^2\theta_W} \left[P_1^2 \overset{0}{\cancel{P_2^2}} - (P_1 \cdot P_2)^2 \right] |\{ \}|^2$$

$$\overline{|M|^2} = \frac{g^4 e^2}{32\pi^4 M_W^2 \cos^2\theta_W} (P_1 \cdot P_2)^2 |\{ \}|^2$$

$$\boxed{\overline{|M|^2} = \frac{g^2 e^4 \quad (P_1 \cdot P_2)^2}{32\pi^4 M_W^2 \sin^2\theta_W \cos^2\theta_W} \left| \tan\beta \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} N_i Q_i m_i^2 C_V^i I_i + \cot\beta \sum_{i=u,c,t} N_i Q_i m_i^2 C_V^i I_i \right|^2} \quad (32$$

We have used: $\boxed{\varepsilon^{\mu\nu\alpha\beta} \varepsilon_{\mu\nu}{}^{\gamma\delta} = -2\left(\eta^{\alpha\gamma}\eta^{\beta\delta} - \eta^{\alpha\delta}\eta^{\beta\gamma}\right)}$ (33)

$$d\Gamma = \frac{\overline{|M|^2} \, |\vec{P}_1| \, d\Omega}{32\pi^2 m_{A^0}^2}$$

$$|\vec{P}_1|^2 = E_1^2 - M_Z^2$$

$$m_{A^0} = E_1 + |\vec{P}_1|$$

$$E_1^2 = (m_{A^0} - |\vec{P}_1|)^2 = m_{A^0}^2 - 2 m_{A^0} |\vec{P}_1| + |\vec{P}_1|^2$$

$$\Rightarrow \cancel{|\vec{P}_1|^2} = m_{A^0}^2 - 2 m_{A^0} |\vec{P}_1| + \cancel{|\vec{P}_1|^2} - M_Z^2$$

$$\boxed{|\vec{P}_1| = \frac{m_{A^0}^2 - M_Z^2}{2 m_{A^0}}} \qquad (34)$$

$$P_{A^0} = P_1 + P_2 \Rightarrow P_{A^0}^2 = P_1^2 + \overset{0}{\cancel{P_2^2}} + 2 P_1 \cdot P_2$$

$$m_{A^0}^2 = M_Z^2 + 2 P_1 \cdot P_2 \quad \therefore \quad \boxed{P_1 \cdot P_2 = \frac{1}{2}\left(m_{A^0}^2 - M_Z^2\right)} \qquad (35)$$

$$\Gamma = \frac{g^2 e^4 \frac{1}{4} (m_{A^0}^2 - M_Z^2)^2}{32\pi^4 M_W^2 \sin^2\theta_W \cos^2\theta_W} \frac{(m_{A^0}^2 - M_Z^2)}{2 m_{A^0}} \frac{1}{8\pi m_{A^0}^2} |\{ \quad \}|^2$$

$$\Gamma = \frac{g^2 e^4 m_{A^0}^3 \left(1 - \frac{M_Z^2}{m_{A^0}^2}\right)^3}{32 \times 4 \times 16 \pi^4 \pi M_W^2 \sin^2\theta_W \cos^2\theta_W} |\{ \quad \}|^2$$

but $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$

$$\alpha = \frac{e^2}{4\pi} \Rightarrow \alpha^2 = \frac{e^4}{16\pi^2}$$

$$\Gamma = \frac{G_F}{\sqrt{2}} \frac{\alpha^2 m_{A^0}^3 \left(1 - \frac{M_Z^2}{m_{A^0}^2}\right)^3}{16 \pi^3 \sin^2\theta_W \cos^2\theta_W} |\{ \quad \}|^2$$

$$\Gamma(A^0 \to Z^0\gamma) = \frac{\sqrt{2}\, G_F \alpha^2 m_{A^0}^3 \left(1 - \frac{M_Z^2}{m_{A^0}^2}\right)^3}{32 \pi^3 \sin^2\theta_W \cos^2\theta_W} \left| \tan\beta \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} N_i Q_i m_i^2 C_V^i I_i + \cot\beta \sum_{i=u,c,t} N_i Q_i m_i^2 C_V^i I_i \right| \quad (36)$$

$$\int_0^{1-X} \frac{dY}{[M_Z^2 X(1-X) - m_i^2 + 2(P_1 \cdot P_2) X Y]} = \int_0^{1-X} \frac{dY}{(a+bY)} = \int_a^{a+b(1-X)} \frac{dU}{bU} = \frac{1}{b} \ln \left| \frac{a+b(1-X)}{a} \right|$$

$U = a + bY$

$dU = b\,dY$

where $a = M_Z^2 X(1-X) - m_i^2$

$b = 2(P_1 \cdot P_2) X$

$$a + b(1-X) = M_Z^2 X(1-X) - m_i^2 + (m_{A^0}^2 - M_Z^2) X(1-X)$$

$$= -m_i^2 + m_{A^0}^2 X(1-X)$$

$$\Rightarrow \int_0^{1-X} \frac{dY}{[M_Z^2 X(1-X) - m_i^2 + 2(P_1 \cdot P_2) X Y]} = \frac{1}{2(P_1 \cdot P_2) X} \ln \left| \frac{m_{A^0}^2 X(1-X) - m_i^2}{M_Z^2 X(1-X) - m_i^2} \right|$$

$$= \frac{1}{2(P_1 \cdot P_2) X} \ln \left| \frac{m_{A^0}^2 X(X-1) + m_i^2}{M_Z^2 X(X-1) + m_i^2} \right|$$

$$= \frac{1}{2(P_1 \cdot P_2) X} \ln \left| \frac{X(X-1) + \frac{m_i^2}{m_{A^0}^2}}{X(X-1) + \frac{m_i^2}{M_Z^2}} \cdot \frac{m_A^2}{M_Z^2} \right| \quad (37)$$

Defining: $$\tau_i = \frac{4m_i^2}{m_{A^0}^2} \quad ; \quad \lambda_i = \frac{4m_i^2}{M_Z^2} \qquad (38)$$

$$\Rightarrow \quad I_i = \int_0^1 dx \, \frac{1}{(m_{A^0}^2 - M_Z^2)} \cdot \frac{1}{x} \ln \left| \frac{x(x-1) + \frac{m_i^2}{m_{A^0}^2}}{x(x-1) + \frac{m_i^2}{M_Z^2}} \cdot \frac{m_{A^0}^2}{M_Z^2} \right|$$

$$I_i = \frac{1}{(m_{A^0}^2 - M_Z^2)} \int_0^1 \frac{dx}{x} \ln \left| \left(\frac{x(x-1) + \tau_i/4}{x(x-1) + \lambda_i/4} \right) \cdot \frac{\lambda_i}{\tau_i} \right|$$

$$= \frac{1}{\left(\frac{4m_i^2}{\tau_i} - \frac{4m_i^2}{\lambda_i} \right)} \left\{ \int_0^1 \frac{dx}{x} \ln \left| \frac{x(x-1) + \tau_i/4}{\tau_i/4} \right| - \int_0^1 \frac{dx}{x} \ln \left| \frac{x(x-1) + \lambda_i/4}{\lambda_i/4} \right| \right\}$$

$$I_i = \frac{\tau_i \lambda_i}{4m_i^2 (\lambda_i - \tau_i)} \left\{ f(\tau_i) - f(\lambda_i) \right\} \equiv \frac{1}{4m_i^2} I(\tau_i, \lambda_i) \qquad (39)$$

where $f(z) = \int_0^1 \frac{dx}{x} \ln \left| \frac{x(x-1) + z/4}{z/4} \right|$. The value of this integral is:

$$f(z) = \begin{cases} -2 \left| \arcsin \left(\frac{1}{z^{1/2}} \right) \right|^2 & z > 1 \\ \frac{1}{2} \left[\ln \left(\frac{1 + (1-z)^{1/2}}{1 - (1-z)^{1/2}} \right) - i\pi \right]^2 & z \leq 1 \end{cases} \qquad (40)$$

$$I(\tau_i, \lambda_i) \equiv \frac{\tau_i \lambda_i}{(\lambda_i - \tau_i)} \left(f(\tau_i) - f(\lambda_i) \right) \qquad (41)$$

$$\therefore \boxed{\Gamma(A^0 \to Z^0 \gamma) = \frac{\sqrt{2} G_F \alpha^2 m_{A^0}^3 \left(1 - \frac{M_Z^2}{m_{A^0}^2} \right)^3}{512 \pi^3 \sin^2\theta_W \cos^2\theta_W} \left| \tan\beta \sum_{\substack{i = d,s,b \\ e^-,\mu^-,\tau^-}} N_i Q_i C_V^i I(\tau_i, \lambda_i) + \cot\beta \sum_{i=u,c,t} N_i Q_i C_V^i I(\tau_i, \lambda_i) \right|^2 \qquad (42)}$$

if $z \ll 1$

$$1 + (1-z)^{1/2} \approx 1 + 1 - \frac{1}{2}z = 2 - \frac{z}{2} \approx 2$$

$$1 - (1-z)^{1/2} \approx 1 - (1 - \frac{1}{2}z) = \frac{1}{2}z$$

$$\Rightarrow \quad f(z) = \frac{1}{2} \left[\ln \left(\frac{4}{z} \right) - i\pi \right]^2 = \frac{1}{2} \left[\ln \left(\frac{z}{4} \right) + i\pi \right]^2$$

So $$\boxed{f(z) \approx \frac{1}{2} \left[\ln \left(\frac{z}{4} \right) + i\pi \right]^2 \quad \text{for } z \ll 1} \qquad (43)$$

$$C_V^i = T_i^3 - 2\sin^2\theta_W\, Q_i$$

i	Q_i	T_i^3	C_V^i
u, c, t	$+2/3$	$1/2$	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_W$
d, s, b	$-1/3$	$-1/2$	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$
e^-, μ^-, τ^-	-1	$-1/2$	$-\frac{1}{2} + 2\sin^2\theta_W$

$$\Gamma(A^0 \to Z^0\gamma) \simeq \frac{\sqrt{2}\, G_F\, \alpha_{em}^2\, m_{A^0}^3 \left(1 - \frac{M_Z^2}{m_{A^0}^2}\right)^3}{512\pi^3 \sin^2\theta_W \cos^2\theta_W} \Big| \tan\beta \left[\cancel{3}\left(-\frac{1}{\cancel{3}}\right)\left(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W\right) I(\tau_b, \lambda_b) + \left(\frac{1}{2} - 2\sin^2\theta_W\right) I(\tau_\tau, \lambda_\tau) \right] + \cot\beta \left[\cancel{3}\left(\frac{2}{\cancel{3}}\right)\left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_W\right) I(\tau_t, \lambda_t) \right] \Big|^2$$

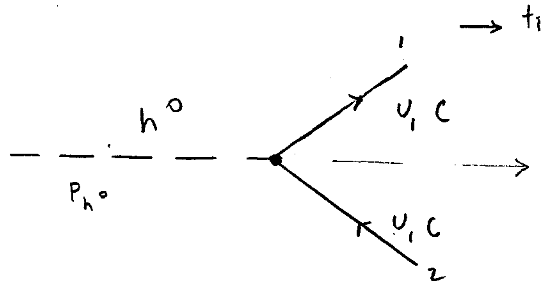
$$\boxed{\Gamma(A^0 \to Z^0\gamma) = \frac{\sqrt{2}\, G_F\, \alpha_{em}^2\, m_{A^0}^3 \left(1 - \frac{M_Z^2}{m_{A^0}^2}\right)^3}{512\pi^3 \sin^2\theta_W \cos^2\theta_W} \Big| \tan\beta \left[\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) I(\tau_b, \lambda_b) + \left(\frac{1}{2} - 2\sin^2\theta_W\right) I(\tau_\tau, \lambda_\tau) \right] + 2\cot\beta \left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_W\right) I(\tau_t, \lambda_t) \Big|^2}$$ (44,

$$\boxed{I(\tau_i, \lambda_i) = \frac{\tau_i \lambda_i}{(\lambda_i - \tau_i)} \left(f(\tau_i) - f(\lambda_i)\right)}$$

$$\boxed{\tau_i \equiv \frac{4m_i^2}{m_{A^0}^2} \; ; \quad \lambda_i \equiv \frac{4m_i^2}{M_Z^2}}$$

$h^0 \to t\bar{t}, b\bar{b}$ decays $\qquad \overset{\text{If}}{(m_{h^0} > 2m_t)}$ $\qquad$ OK C. Marin

To leading order in perturbation theory the Feynman diagram corresponding to this decay is:



$$\longrightarrow \quad \frac{-ig\, m_t \cos\alpha}{2 M_W \sin\beta}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

The invariant amplitude is:

$$-i\mathcal{M} = \tilde{U}_1 \left(\frac{-ig\, m_f \cos\alpha}{2 M_W \sin\beta} \right) V_2 \qquad (38)$$

$$\overline{|\mathcal{M}|^2} = \frac{g^2 m_f^2 \cos^2\alpha}{4 M_W^2 \sin^2\beta} \sum_s (U_1^+ \gamma^0 V_2)(U_1^+ \gamma^0 V_2)^+ \qquad f = u, c$$

$$= \frac{g^2 m_f^2 \cos^2\alpha}{4 M_W^2 \sin^2\beta} \sum_s (U_1^+ \gamma^0 V_2 V_2^+ \gamma^0 U_1)$$

$$= \frac{g^2 m_f^2 \cos^2\alpha}{4 M_W^2 \sin^2\beta} \sum_s (\bar{U}_1 V_2 \bar{V}_2 U_1)$$

$$= \frac{g^2 m_f^2 \cos^2\alpha}{4 M_W^2 \sin^2\beta} \left[Tr\left((\not{P}_1 + m_f)(\not{P}_2 - m_f) \right) \right]$$

$$= \frac{g^2 m_f^2 \cos^2\alpha}{\cancel{4} M_W^2 \sin^2\beta} \left[\cancel{4}(P_1 . P_2) - \cancel{4} m_f^2 \right]$$

$$\overline{|\mathcal{M}|^2} = \frac{g^2 m_f^2 \cos^2\alpha}{M_W^2 \sin^2\beta} \left[(P_1 . P_2) - m_f^2 \right] \qquad (39)$$

$$P_{h^0} = P_1 + P_2$$

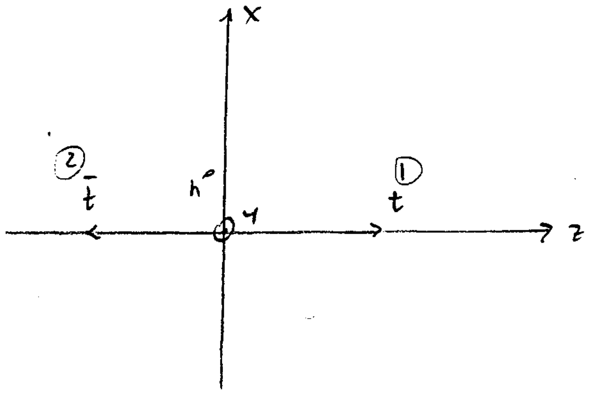
$$P_{h^0}^2 = 2m_f^2 + 2P_1 . P_2$$

$$\Rightarrow \quad P_1 . P_2 = \frac{m_{h^0}^2 - 2m_f^2}{2} = \frac{m_{h^0}^2}{2} - m_f^2$$

$$\boxed{\overline{|\mathcal{M}|^2} = \frac{g^2 m_f^2 \cos^2\alpha}{M_W^2 \sin^2\beta} \left[\frac{m_{h^0}^2}{2} - 2m_f^2 \right]} \qquad (40)$$

$$d\Gamma = \frac{\overline{|M|^2}\; |\vec{P}_1|\, d\Omega}{32\pi^2 m_{h^0}^2} \qquad (41)$$

In the rest frame of h°



$$P_{h^0} = (m_{h^0}, 0, 0, 0) \;;\; P_1 = (E, \vec{P}_1) \;;\; P_2 = (E, -\vec{P}_1)$$

$$P_1 . P_2 = E^2 + |\vec{P}_1|^2 \qquad (2E = m_{h^0})$$

$$|\vec{P}_1|^2 = \frac{m_{h^0}^2}{2} - m_f^2 - \frac{m_{h^0}^2}{4} = \frac{1}{4} m_{h^0}^2 - m_f^2$$

$$|\vec{P}_1| = \frac{(m_{h^0}^2 - 4m_f^2)^{1/2}}{2}$$

$$\Rightarrow \quad d\Gamma = \frac{g^2 m_f^2 \cos^2\alpha}{2 M_W^2 \sin^2\beta} \; \frac{(m_{h^0}^2 - 4m_f^2)^{3/2}}{2} \; \frac{d\Omega}{32\pi^2 m_{h^0}^2} \; N_c \qquad (N_c = 3)$$

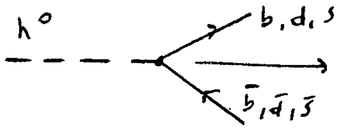
$$\Gamma = \frac{3 g^2 m_f^2 \cos^2\alpha \; (m_{h^0}^2 - 4m_f^2)^{3/2}}{32 M_W^2 \sin^2\beta \; \pi \; m_{h^0}^2}$$

for f = C

$$\therefore \quad \boxed{\Gamma(h^0 \to c\bar{c}) = \frac{3g^2 m_c^2 \cos^2\alpha \; m_{h^0}}{32\pi M_W^2 \sin^2\beta} \left(1 - \frac{4m_c^2}{m_{h^0}^2}\right)^{3/2}} \qquad (42)$$

$$\boxed{\Gamma(h^0 \to b\bar{b}) = \frac{3g^2 m_b^2 \sin^2\alpha \; m_{h^0}}{32\pi M_W^2 \cos^2\beta} \left(1 - \frac{4m_b^2}{m_{h^0}^2}\right)^{3/2}} \qquad (m_{h^0} > 2m_b)$$

(43)



$$\frac{i g m_b \sin\alpha}{2 M_W \cos\beta}$$

$$\Gamma(h^0 \to c\bar{c}) = \frac{3g^2 m_c^2 \cos^2\alpha \, m_{h^0}}{32\pi M_W^2 \sin^2\beta} \left(1 - \frac{4m_c^2}{m_{h^0}^2}\right)^{3/2} \qquad (44)$$

$$\Gamma(h^0 \to \tau^+\tau^-) = \frac{g^2 m_\tau^2 \sin^2\alpha \, m_{h^0}}{32\pi M_W^2 \cos^2\beta} \left(1 - \frac{4m_\tau^2}{m_{h^0}^2}\right)^{3/2} \qquad (45)$$

For $\tan\beta > 1$; $m_{h^0} \leq M_Z$

$$B(h^0 \to b\bar{b}) \approx \frac{\Gamma(h^0 \to b\bar{b})}{\Gamma(h^0 \to b\bar{b}) + \Gamma(h^0 \to c\bar{c}) + \Gamma(h^0 \to \tau^+\tau^-) + \Gamma(h^0 \to 2g)} \qquad (46)$$

$$= \frac{\dfrac{3\cancel{g^2} m_b^2 \sin^2\alpha \, \cancel{m_{h^0}}}{\cancel{32\pi M_W^2} \cos^2\beta}\left(1 - \dfrac{4m_b^2}{m_{h^0}^2}\right)^{3/2}}{\begin{aligned}&\dfrac{3\cancel{g^2} m_b^2 \sin^2\alpha \, \cancel{m_{h^0}}}{\cancel{32\pi} \cancel{M_W^2} \cos^2\beta}\left(1 - \dfrac{4m_b^2}{m_{h^0}^2}\right)^{3/2} + \dfrac{3\cancel{g^2} m_c^2 \cos^2\alpha \, \cancel{m_{h^0}}}{\cancel{32\pi} \cancel{M_W^2} \sin^2\beta}\left(1 - \dfrac{4m_c^2}{m_{h^0}^2}\right)^{3/2} + \\ &+ \dfrac{\cancel{g^2} m_\tau^2 \sin^2\alpha \, \cancel{m_{h^0}}}{\cancel{32\pi} \cancel{M_W^2} \cos^2\beta}\left(1 - \dfrac{4m_\tau^2}{m_{h^0}^2}\right)^{3/2} + \dfrac{\cancel{g^2} \alpha_s^2 m_{h^0}^{\cancel{3}}}{\underset{8\pi^2}{\cancel{256\pi^3}} \cancel{M_W^2}} \dfrac{(1-\sin^2\alpha)}{\sin^2\beta} \Big| \tan\beta \tan\alpha \\ &\cdot \Big(\tau_b[(\tau_b - 1) f(\tau_b) + 2] \Big) - \tau_t [(\tau_t - 1) f(\tau_t) + 2] \Big|^2\end{aligned}}$$

$$\boxed{B(h^0 \to b\bar{b}) \approx \frac{3 m_b^2 \sin^2\alpha \left(1 - \dfrac{4m_b^2}{m_{h^0}^2}\right)^{3/2}}{\begin{aligned}&3m_b^2 \sin^2\alpha \left(1 - \dfrac{4m_b^2}{m_{h^0}^2}\right)^{3/2} + \dfrac{3m_c^2(1-\sin^2\alpha)}{\tan^2\beta}\left(1 - \dfrac{4m_c^2}{m_{h^0}^2}\right)^{3/2} + \\ &+ m_\tau^2 \sin^2\alpha \left(1 - \dfrac{4m_\tau^2}{m_{h^0}^2}\right)^{3/2} + \dfrac{\alpha_s^2 m_{h^0}^2}{8\pi^2} \dfrac{(1-\sin^2\alpha)}{\tan^2\beta} \Big| \tan\beta \tan\alpha . \\ &\times \Big(\tau_b [(\tau_b - 1) f(\tau_b) + 2] \Big) - \tau_t [(\tau_t - 1) f(\tau_t) + 2] \Big|^2\end{aligned}} \qquad (47)}$$

I. $m_b, m_c, m_\tau \ll m_{h^0}$

$$\Rightarrow \boxed{B(h^0 \to b\bar{b}) \approx \frac{3m_b^2 \sin^2\alpha}{\begin{aligned}&3m_b^2 \sin^2\alpha + \dfrac{3m_c^2(1-\sin^2\alpha)}{\tan^2\beta} + m_\tau^2 \sin^2\alpha + \dfrac{\alpha_s^2 m_{h^0}^2}{8\pi^2} \cdot \\ &\cdot \dfrac{(1-\sin^2\alpha)}{\tan^2\beta} \Big| \tan\beta \tan\alpha \, \tau_b \left[-\frac{1}{2}\left(\ln\left(\frac{1}{4}\tau_b\right) + i\pi\right)^2 + 2 \right] \\ &- \tau_t \Big((\tau_t - 1) f(\tau_t) + 2 \Big) \Big|^2\end{aligned}} \qquad (48)}$$

where

$$\sin^2\alpha = \frac{1}{2}\left\{1 + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)}\right]\right\}$$

$$g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta) = \left[\left(1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}\right)^2 - 4\left(\frac{M_Z^2}{M_{H^\pm}^2}\right)\left(1 - \frac{M_W^2}{M_{H^\pm}^2}\right)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2\right]^{1/2}$$

$$f(\tau_i) = \begin{cases} -2\left(\arcsin\left(\frac{1}{\tau_i^{1/2}}\right)\right)^2 & \tau_i > 1 \\ \frac{1}{2}\left[\ln\left(\frac{1+(1-\tau_i)^{1/2}}{1-(1-\tau_i)^{1/2}}\right) - i\pi\right]^2 & \tau_i \leq 1 \end{cases}$$

$$\tau_i = \frac{4 m_i^2}{m_{h^0}^2}$$

$$\tan\alpha = -\left\{\frac{1 + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)}\right]}{1 - \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta}\right]}\right\}^{1/2}$$

$$B(h^0 \to \tau^+\tau^-) = \frac{m_\tau^2 \sin^2\alpha \left(1 - \frac{4m_\tau^2}{m_{h^0}^2}\right)^{3/2}}{\begin{gathered}3m_b^2\sin^2\alpha\left(1 - \frac{4m_b^2}{m_{h^0}^2}\right)^{3/2} + \frac{3m_c^2(1-\sin^2\alpha)}{\tan^2\beta}\left(1 - \frac{4m_c^2}{m_{h^0}^2}\right)^{3/2} + \\ + m_\tau^2\sin^2\alpha\left(1 - \frac{4m_\tau^2}{m_{h^0}^2}\right)^{3/2} + \frac{\alpha_s^2 m_{h^0}^2}{8\pi^2}\frac{(1-\sin^2\alpha)}{\tan^2\beta}\Big| \tan\beta\tan\alpha \cdot \\ \tau_b\left[(\tau_b - 1)f(\tau_b) + 2\right] - \tau_t\left[(\tau_t - 1)f(\tau_t) + 2\right]\Big|^2 .\end{gathered}} \tag{49}$$

If $m_b, m_c, m_\tau \ll m_{h^0}$

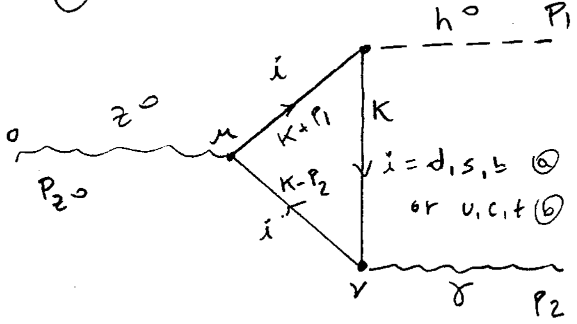
$$B(h^0 \to \tau^+\tau^-) \approx \frac{m_\tau^2\sin^2\alpha}{\begin{gathered}3m_b^2\sin^2\alpha + \frac{3m_c^2(1-\sin^2\alpha)}{\tan^2\beta} + m_\tau^2\sin^2\alpha + \frac{\alpha_s^2 m_{h^0}^2}{8\pi^2}\frac{(1-\sin^2\alpha)}{\tan^2\beta}\Big|\tan\beta\cdot\tan\alpha \\ \tau_b\left[-\frac{1}{2}\left(\ln\left(\frac{1}{4}\tau_b\right) + i\pi\right)^2 + 2\right] - \tau_t\left[(\tau_t - 1)f(\tau_t) + 2\right]\Big|^2\end{gathered}} \tag{50}$$

$Z^0 \to h^0\gamma$

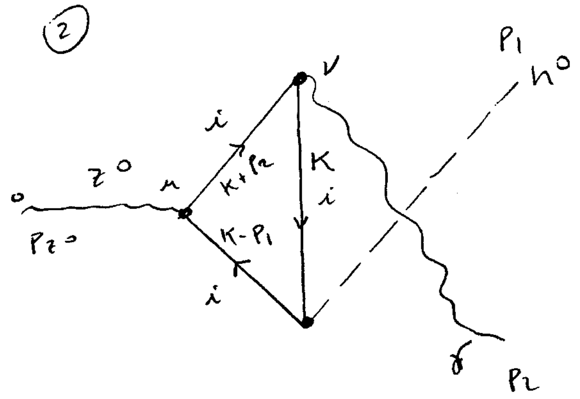
OK

$$P_{Z^0} = P_1 + P_2$$

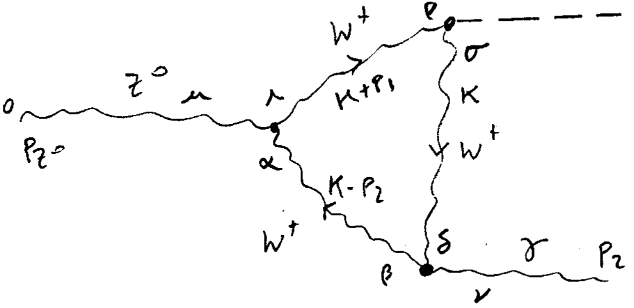
①



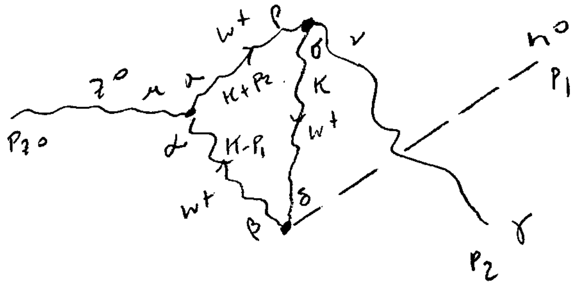
+ ②



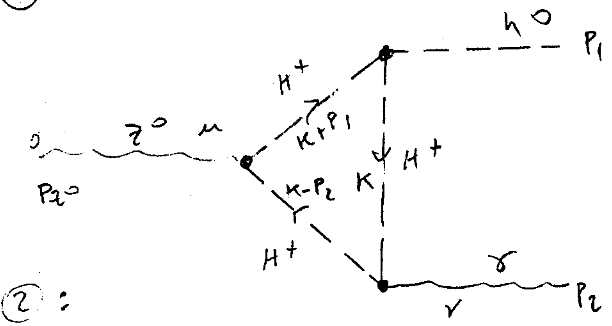
+ ③



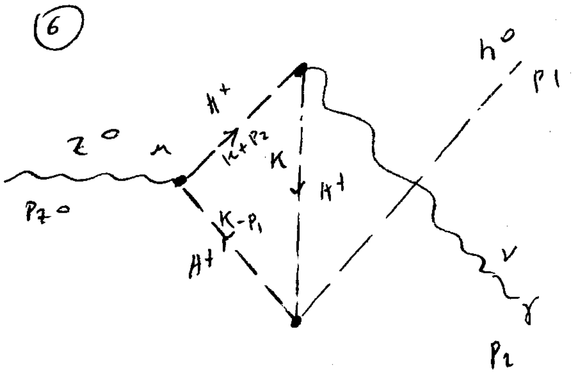
+ ④



+ ⑤



+ ⑥



① – ② :

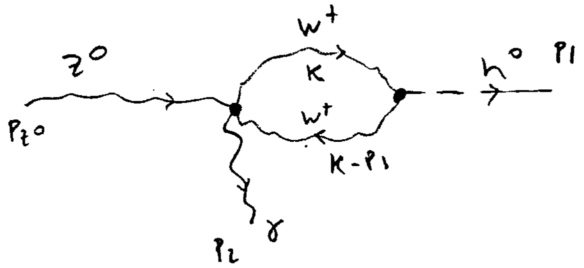
a) $i = d, s, b$

$$-iM_{1_a} = \overset{\text{Fermion loop}}{(-1)} \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} \overset{\text{color factor}}{N_i} \int \frac{d^d k}{(2\pi)^d} \left(\frac{i g m_i}{2M_W} \frac{\sin\alpha}{\cos\beta}\right) Tr \Big[\frac{i}{\not{k} - m_i} (-ie\gamma^\nu Q_i) \frac{i}{(\not{k} - \not{p}_2) - m_i}$$

$$\left(-\frac{ig}{\cos\theta_W}\right) \gamma^\mu \frac{1}{2} (C_V^i - C_A^i \gamma^5) \frac{i}{(\not{k} + \not{p}_1) - m_i} \Big] \varepsilon^*_{2\nu} \varepsilon_{0\mu} \underbrace{\underset{\substack{\downarrow\\ h^0}}{\mu_1}^{\left(\frac{4-d}{2}\right)} \underset{\substack{\gamma \\ \text{mass parameter}}}{\mu_2}^{\frac{(4-d)}{2}} \underset{Z^0}{\mu_3}^{\frac{(4-d)}{2}}}_{\mu^*} \qquad (1)$$

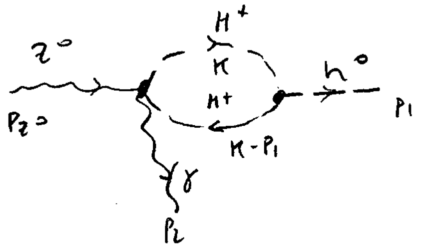
We also have the following diagrams

⑦



+

⑧



$$-iM_{1a} = (-1) - \frac{g^2 e}{4 M_W \cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_i Q_i N_i \int \frac{d^d k}{(2\pi)^d}\, \mathrm{Tr}\{[(\not{k}+m_i)\gamma^\nu[(\not{k}-\not{p}_2)+m_i]$$

$$\gamma^\mu (C_V^i - C_A^i\gamma^5)[(\not{k}+\not{p}_1)+m_i]\}\frac{1}{(k^2-m_i^2)((k-p_2)^2-m_i^2)((k+p_1)^2-m_i^2)} \,.$$

$\varepsilon^*_{2\nu}\,\varepsilon_{0\mu}\,\mu^\epsilon$

$\downarrow$

$\to 1$

when

$d\to 4$

(2)

$$\mathrm{Tr}\{(\not{k}+m_i)\gamma^\nu[(\not{k}-\not{p}_2)+m_i]\gamma^\mu(C_V^i - C_A^i\gamma^5)[(\not{k}+\not{p}_1)+m_i]\}$$

$$= \mathrm{Tr}\{(\not{k}+m_i)[\gamma^\nu\not{k} - \gamma^\nu\not{p}_2 + m_i\gamma^\nu]\gamma^\mu[C_V^i\not{k} + C_V^i\not{p}_1 + m_i C_V^i - C_A^i\gamma^5\not{k}$$

$$- C_A^i\gamma^5\not{p}_1 - m_i C_A^i\gamma^5]\}$$

$$= \mathrm{Tr}\{[\not{k}\gamma^\nu\overset{\downarrow}{\not{k}}\gamma^\mu - \not{k}\overset{\downarrow}{\gamma^\nu}\not{p}_2\gamma^\mu + m_i\overset{\downarrow}{\not{k}}\gamma^\nu\gamma^\mu + m_i\overset{\downarrow}{\gamma^\nu}\not{k}\gamma^\mu - m_i\gamma^\nu\overset{\downarrow}{\not{p}_2}\gamma^\mu + m_i^2\overset{\downarrow}{\gamma^\nu}\gamma^\mu]$$

$$[C_V^i\not{k} + C_V^i\not{p}_1 + m_i C_V^i - C_A^i\gamma^5\not{k} - C_A^i\gamma^5\not{p}_1 - m_i C_A^i\gamma^5]\}$$

$$= C_V^i\,\mathrm{Tr}[\not{k}\cancelto{0}{\gamma^\nu\not{k}\gamma^\mu}\not{k}] + C_V^i\,\mathrm{Tr}[\not{k}\cancelto{0}{\gamma^\nu\not{k}\gamma^\mu}\not{p}_1] + m_i C_V^i\,\mathrm{Tr}[\not{k}\bcancel{\gamma^\nu\not{k}}\gamma^\mu]$$

$$- C_A^i\,\mathrm{Tr}[\not{k}\gamma^\nu\cancelto{0}{\not{k}\gamma^\mu}\gamma^5\not{k}] - C_A^i\,\mathrm{Tr}[\not{k}\gamma^\nu\cancelto{0}{\not{k}\gamma^\mu}\gamma^5\not{p}_1] - m_i C_A^i\,\mathrm{Tr}[\gamma^5\not{k}\bcancel{\gamma^\nu}\not{k}\gamma^\mu]$$

$$- C_V^i\,\mathrm{Tr}[\not{k}\gamma^\nu\cancelto{0}{\not{p}_2\gamma^\mu}\not{k}] - C_V^i\,\mathrm{Tr}[\not{k}\gamma^\nu\cancelto{0}{\not{p}_2\gamma^\mu}\not{p}_1] - m_i C_V^i\,\mathrm{Tr}[\bcancel{\not{k}}\gamma^\nu\not{p}_2\gamma^\mu]$$

$$+ C_A^i\,\mathrm{Tr}[\not{k}\gamma^\nu\cancelto{0}{\not{p}_2\gamma^\mu}\gamma^5\not{k}] + C_A^i\,\mathrm{Tr}[\not{k}\gamma^\nu\cancelto{0}{\not{p}_2\gamma^\mu}\gamma^5\not{p}_1] + m_i C_A^i\,\mathrm{Tr}[\gamma^5\bcancel{\not{k}}\gamma^\nu\not{p}_2\gamma^\mu]$$

$$+ m_i C_V^i\,\mathrm{Tr}[\not{k}\bcancel{\gamma^\nu}\gamma^\mu\not{k}] + m_i C_V^i\,\mathrm{Tr}[\bcancel{\not{k}}\gamma^\nu\gamma^\mu\not{p}_1] + m_i^2 C_V^i\,\mathrm{Tr}[\not{k}\cancelto{0}{\gamma^\nu}\gamma^\mu]$$

$$- m_i C_A^i\,\mathrm{Tr}[\not{k}\gamma^\nu\bcancel{\gamma^\mu}\gamma^5\not{k}] - m_i C_A^i\,\mathrm{Tr}[\bcancel{\not{k}}\gamma^\nu\gamma^\mu\gamma^5\not{p}_1] - m_i^2 C_A^i\,\mathrm{Tr}[\not{k}\cancelto{0}{\gamma^\nu\gamma^\mu}\gamma^5]$$

$$+ m_i C_V^i\,\mathrm{Tr}[\gamma^\nu\bcancel{\not{k}}\gamma^\mu\not{k}] + m_i C_V^i\,\mathrm{Tr}[\bcancel{\gamma^\nu}\not{k}\gamma^\mu\not{p}_1] + m_i^2 C_V^i\,\mathrm{Tr}[\gamma^\nu\cancelto{0}{\not{k}}\gamma^\mu]$$

$$- m_i C_A^i\,\mathrm{Tr}[\gamma^\nu\bcancel{\not{k}}\gamma^\mu\gamma^5\not{k}] - m_i C_A^i\,\mathrm{Tr}[\gamma^\nu\bcancel{\not{k}}\gamma^\mu\gamma^5\not{p}_1] - m_i^2 C_A^i\,\mathrm{Tr}[\gamma^\nu\cancelto{0}{\not{k}}\gamma^\mu\gamma^5]$$

$$- m_i C_V^i\,\mathrm{Tr}[\gamma^\nu\bcancel{\not{p}_2}\gamma^\mu\not{k}] - m_i C_V^i\,\mathrm{Tr}[\bcancel{\gamma^\nu}\not{p}_2\gamma^\mu\not{p}_1] - m_i^2 C_V^i\,\mathrm{Tr}[\gamma^\nu\cancelto{0}{\not{p}_2}\gamma^\mu]$$

$$+ m_i C_A^i\,\mathrm{Tr}[\gamma^\nu\bcancel{\not{p}_2}\gamma^\mu\gamma^5\not{k}] + m_i C_A^i\,\mathrm{Tr}[\gamma^\nu\not{p}_2\bcancel{\gamma^\mu}\gamma^5\not{p}_1] + m_i^2 C_A^i\,\mathrm{Tr}[\gamma^\nu\cancelto{0}{\not{p}_2}\gamma^\mu\gamma^5]$$

$$+ m_i^2 C_V^i\,\mathrm{Tr}[\gamma^\nu\cancelto{0}{\gamma^\mu}\not{k}] + m_i^2 C_V^i\,\mathrm{Tr}[\gamma^\nu\cancelto{0}{\gamma^\mu}\not{p}_1] + m_i^3 C_V^i\,\mathrm{Tr}\bcancel{[}\gamma^\nu\gamma^\mu] - m_i^2 C_A^i\,\mathrm{Tr}[\gamma^5\cancelto{0}{\not{k}}\gamma^\nu\gamma^\mu]$$

$$- m_i^2 C_A^i\,\mathrm{Tr}[\gamma^5\cancelto{0}{\not{p}_1\gamma^\nu}\gamma^\mu] - m_i^3 C_A^i\,\mathrm{Tr}[\gamma^5\cancelto{0}{\gamma^\nu}\gamma^\mu]$$

$$\downarrow\ \mathrm{Tr}(\gamma^5\gamma^\nu\gamma^\mu) = 0$$

$$= 2 m_i C_V^i \, Tr\left[(2\eta^{\nu\alpha} - \gamma^\nu \gamma^\alpha) K_\alpha \not{K} \gamma^\mu \right] + 4 i m_i C_A^i \, \overset{0}{\varepsilon^{\alpha\nu\beta\mu} K_\alpha K_\beta}$$

$$- 8 m_i C_V^i \left[K^\mu P_2^\nu + K^\nu P_2^\mu - (P_2 \cdot K)\eta^{\mu\nu} \right] \underset{\downarrow}{-} 4 i m_i C_A^i \, \varepsilon^{\alpha\nu\beta\mu} K_\alpha P_{2\beta}$$

$$+ 4 m_i C_V^i K^2 \eta^{\mu\nu} + m_i C_V^i \, Tr\left[(2\eta^{\nu\alpha} - \gamma^\nu\gamma^\alpha)\gamma^\mu \not{P}_1 \right] K_\alpha - m_i C_A^i K^2 \, \overset{0}{Tr\left[\gamma^5 \gamma^\nu \gamma^\mu \right]}$$

$$+ 4 i m_i C_A^i \, \varepsilon^{\alpha\beta\nu\mu} P_{1\alpha} K_\beta + 4 m_i C_V^i \left[K^\nu P_1^\mu + K^\mu P_1^\nu - (P_1 \cdot K)\eta^{\mu\nu} \right]$$

$$+ 4 i m_i C_A^i \, \overset{0}{\varepsilon^{\alpha\nu\beta\mu} K_\alpha K_\beta} + 4 i m_i C_A^i \, \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} K_\beta - 4 m_i C_V^i \left[P_2^\nu P_1^\mu + P_2^\mu P_1^\nu - (P_1 \cdot P_2)\eta^{\mu\nu} \right]$$

$$- 4 i m_i C_A^i \, \varepsilon^{\alpha\nu\beta\mu} K_\alpha P_{2\beta} - 4 i m_i C_A^i \, \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} P_{2\beta} + 4 m_i^3 C_V^i \eta^{\mu\nu}$$

$$(\varepsilon^{\alpha\nu\beta\mu} K_\alpha K_\beta = 0)$$

$$= 16 m_i C_V^i K^\mu K^\nu - 2 m_i C_V^i \, Tr\left[\gamma^\nu \not{K} \not{K} \gamma^\mu \right] - 8 m_i C_V^i \left[K^\mu P_2^\nu + K^\nu P_2^\mu - (P_2 \cdot K)\eta^{\mu\nu} \right]$$

$$- 8 i m_i C_A^i \, \varepsilon^{\alpha\nu\beta\mu} K_\alpha P_{2\beta} + 4 m_i C_V^i K^2 \eta^{\mu\nu} + 8 m_i C_V^i P_1^\mu K^\nu - m_i C_V^i \, Tr\left[\gamma^\nu \not{K} \gamma^\mu \not{P}_1 \right]$$

$$+ 4 i m_i C_A^i \, \overset{0}{(- \varepsilon^{\alpha\nu\beta\mu} + \varepsilon^{\alpha\nu\beta\mu})} P_{1\alpha} K_\beta + 4 m_i C_V^i \left[K^\nu P_1^\mu + K^\mu P_1^\nu - (P_1 \cdot K)\eta^{\mu\nu} \right]$$

$$- 4 m_i C_V^i \left[P_2^\nu P_1^\mu + P_2^\mu P_1^\nu - (P_1 \cdot P_2)\eta^{\mu\nu} \right] - 4 i m_i C_A^i \, \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} P_{2\beta} + 4 m_i^3 C_V^i \eta^{\mu\nu}$$

$$= 16 m_i C_V^i K^\mu K^\nu - 8 m_i C_V^i K^2 \eta^{\mu\nu} - 8 m_i C_V^i K^\mu P_2^\nu - 8 m_i C_V^i K^\nu P_2^\mu + 8 m_i C_V^i (P_2 \cdot K)\eta^{\mu\nu}$$

$$- 8 i m_i C_A^i \, \varepsilon^{\alpha\nu\beta\mu} K_\alpha P_{2\beta} + 4 m_i C_V^i K^2 \eta^{\mu\nu} + 8 m_i C_V^i P_1^\mu K^\nu - 4 m_i C_V^i P_2^\nu P_1^\mu$$

$$- 4 m_i C_V^i P_2^\mu P_1^\nu + 4 m_i C_V^i (P_1 \cdot P_2)\eta^{\mu\nu} - 4 i m_i C_A^i \, \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} P_{2\beta} + 4 m_i^3 C_V^i \eta^{\mu\nu}$$

$\Rightarrow$

$$Tr = 16 m_i C_V^i K^\mu K^\nu - 4 m_i C_V^i K^2 \eta^{\mu\nu} - 8 m_i C_V^i K^\mu P_2^\nu - 8 m_i C_V^i K^\nu P_2^\mu + 8 m_i C_V^i (P_2 \cdot K)\eta^{\mu\nu}$$

$$- 8 i m_i C_A^i \, \varepsilon^{\alpha\nu\beta\mu} K_\alpha P_{2\beta} + 8 m_i C_V^i P_1^\mu K^\nu - 4 m_i C_V^i P_2^\nu P_1^\mu - 4 m_i C_V^i P_2^\mu P_1^\nu$$

$$+ 4 m_i C_V^i (P_1 \cdot P_2)\eta^{\mu\nu} - 4 i m_i C_A^i \, \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} P_{2\beta} + 4 m_i^3 C_V^i \eta^{\mu\nu} \qquad (3)$$

$$\frac{1}{[K^2 - m_i^2]\,[(K+P_1)^2 - m_i^2]\,[(K-P_2)^2 - m_i^2]} = 2 \int_0^1 dx \int_0^{1-x} \frac{dy}{[K^2 + 2K\cdot(P_1 x - P_2 y) - m_i^2 + m_{h0}^2 x]^3} \qquad (4)$$

$$-iM_1 a = \frac{-g^2 e}{4M_W \cos\theta_W} \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_i m_i Q_i \; 2\int_0^1 dx \int_0^{1-x} dy \int \frac{d^d K}{(2\pi)^d} \Big\{ 16 m_i C_V^i K^\mu K^\nu$$

$$-4m_i C_V^i K^2 n^{\mu\nu} - 8m_i C_V^i K^\mu P_2^\nu - 8 m_i C_V^i K^\nu P_2^\mu + 8 m_i C_V^i (P_2 \cdot K) n^{\mu\nu}$$

$$-8 i m_i C_A^i \varepsilon^{\alpha\nu\rho\mu} K_\alpha P_{2\rho} + 8 m_i C_V^i P_1^\mu K^\nu - 4 m_i C_V^i P_2^\nu P_1^\mu - 4 m_i C_V^i P_2^\mu P_1^\nu$$

$$+ 4 m_i C_V^i (P_1 \cdot P_2) n^{\mu\nu} - 4 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} P_{2\beta} + 4 m_i^3 C_V^i n^{\mu\nu} \Big\}.$$

$$\cdot \Big[K^2 + 2K \cdot (P_1 X - P_2 Y) \underset{+ m_{h^0}^2 X}{-} m_i^2 \Big]^{-3} \varepsilon_{2\nu}^* \varepsilon_{0\mu} \; (-1) \qquad (5)$$

$$K' = K + (P_1 X - P_2 Y) \qquad (6)$$

$$K'^2 = K^2 + 2K \cdot (P_1 X - P_2 Y) + (P_1 X - P_2 Y)^2$$

$$K'^2 = K^2 + 2K \cdot (P_1 X - P_2 Y) + m_{h^0}^2 X^2 - 2 (P_1 \cdot P_2) XY \qquad (7)$$

$$d^d K = d^d K'$$

$$-iM_1 a = \frac{-g^2 e}{4M_W \cos\theta_W} \left(\frac{\sin\alpha}{\cos\beta}\right) 2\sum_i m_i Q_i N_i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d K'}{(2\pi)^d} \Big\{ 16\, m_i C_V^i$$

$$(K' - (P_1 X - P_2 Y))^\mu (K' - (P_1 X - P_2 Y))^\nu - 4 m_i C_V^i [K' - (P_1 X - P_2 Y)]^2 n^{\mu\nu}$$

$$-8 m_i C_V^i (K' - (P_1 X - P_2 Y))^\mu P_2^\nu - 8 m_i C_V^i (K' - (P_1 X - P_2 Y))^\nu P_2^\mu$$

$$+ 8 m_i C_V^i n^{\mu\nu} (P_2 \cdot (K' - (P_1 X - P_2 Y))) - 8 i m_i C_A^i \varepsilon^{\alpha\nu\rho\mu} (K' - (P_1 X - P_2 Y))_\alpha P_{2\beta}$$

$$+ 8 m_i C_V^i P_1^\mu (K' - (P_1 X - P_2 Y))^\nu - 4 m_i C_V^i P_1^\mu P_2^\nu - 4 m_i C_V^i P_1^\nu P_2^\mu$$

$$+ 4 m_i C_V^i (P_1 \cdot P_2) n^{\mu\nu} - 4 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} P_{2\beta} + 4 m_i^3 C_V^i n^{\mu\nu} \Big\}.$$

$$\cdot \Big[K'^2 - m_{h^0}^2 X^2 + 2 (P_1 \cdot P_2) XY \underset{+ m_{h^0}^2 X}{-} m_i^2 \Big]^{-3} \varepsilon_{2\nu}^* \varepsilon_{0\mu} \qquad (8)$$

$$-iM_1 a = \frac{-g^2 e \; 2}{4M_W \cos\theta_W} \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_i m_i Q_i N_i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d K'}{(2\pi)^d} \Big\{ 16 m_i C_V^i K'^\mu K'^\nu$$

$$-16 m_i C_V^i K'^\mu (P_1 X - P_2 Y)^\nu - 16 m_i C_V^i K'^\nu (P_1 X - P_2 Y)^\mu + 16 m_i C_V^i (P_1 X - P_2 Y)^\mu \cdot$$

$$(P_1 X - P_2 Y)^\nu - 4 m_i C_V^i K'^2 n^{\mu\nu} + 8 m_i C_V^i K' \cdot (P_1 X - P_2 Y) n^{\mu\nu} - 4 m_i C_V^i (m_{h^0}^2 X^2$$

$$-2 (P_1 \cdot P_2) XY) n^{\mu\nu} - 8 m_i C_V^i K'^\mu P_2^\nu + 8 m_i C_V^i (P_1 X - P_2 Y)^\mu P_2^\nu - 8 m_i C_V^i K'^\nu P_2^\mu$$

$$+ 8 m_i C_V^i (P_1 X - P_2 Y)^\nu P_2^\mu + 8 m_i C_V^i n^{\mu\nu} \cancel{(P_2 \cdot K')} - 8 m_i C_V^i n^{\mu\nu} (P_1 \cdot P_2) X$$

$$-8 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} \cancel{K'_\alpha} P_{2\beta} + 8 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} x P_{1\alpha} P_{2\beta} - 8 i m_i C_A^i \underbrace{\varepsilon^{\alpha\nu\beta\mu} y P_{2\alpha} P_{2\beta}}_{=0}$$

$$+ 8 m_i C_V^i \cancel{P_1^\mu} K'^\nu - 8 m_i C_V^i P_1^\mu P_1^\nu x + 8 m_i C_V^i y P_1^\mu P_2^\nu - 4 m_i C_V^i P_1^\mu P_2^\nu - 4 m_i C_V^i P_1^\nu P_2^\mu$$

$$+ 4 m_i C_V^i (P_1\cdot P_2) \eta^{\mu\nu} - 4 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} P_{2\beta} + 4 m_i^3 C_V^i \eta^{\mu\nu} \Big\} . \varepsilon_{2\nu}^{\lambda} \varepsilon_{0\mu}$$

$$\cdot \Big[K'^2 - m_{h^0}^2 x^2 + 2(P_1\cdot P_2) x y \underset{+m_{h^0}^2 x}{-\, m_i^2} \Big]^{-3} \qquad (9)$$

$$\int \frac{d^d K'}{\big[K'^2 - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]^3} = \frac{i(-\pi)^{d/2}\,\Gamma(3-\frac{d}{2})}{\Gamma(\alpha)\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]^{3-\frac{d}{2}}}$$

$$= \frac{i\pi^2}{2\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]} = I_0 \qquad (10)$$

$$\int \frac{d^d K'\, K'^\mu}{\big[K'^2 - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]^3} = -\frac{i(-\pi)^{d/2}\,\Gamma(3-\frac{d}{2}) \times \overset{\nearrow q^\mu}{0}}{2\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]^{3-\frac{d}{2}}} = 0 \qquad (11)$$

$$\int \frac{d^d K'\, K'^\mu K'^\nu}{\big[K'^2 - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]^3} = \frac{+i(-\pi)^{d/2}\,\frac{\eta^{\mu\nu}}{2}\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]\Gamma(2-\frac{d}{2})}{2\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]^{3-d/2}\,\Gamma(3-\frac{d}{2})}$$

$$= \frac{I_0 \eta^{\mu\nu}}{2}\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]\frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})} \qquad (12)$$

$$\int \frac{d^d K'\, K'^2}{\big[K'^2 - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]^3} = \frac{I_0 d}{2}\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]\frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})} \qquad (13)$$

$\Rightarrow$

$$-iM_1 a = \frac{(-1) - g^2 e\ 2}{4 M_W \cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_i Q_i N_i \int_0^1 dx \int_0^{1-x} dy \frac{1}{(2\pi)^4}\Big\{ 16 m_i C_V^i \frac{I_0 \eta^{\mu\nu}}{2}\big[-m_{h^0}^2 x^2$$

$$+ 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]\frac{1}{2-d/2} + 16 m_i C_V^i (P_1 x - P_2 y)^\mu (P_1 x - P_2 y)^\nu I_0 - 4 m_i C_V^i \eta^{\mu\nu}\frac{I_0 d}{2} .$$

$$\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2 x}{-\, m_i^2}\big]\frac{1}{2-d/2} - 4 m_i C_V^i (m_{h^0}^2 x^2 - 2(P_1\cdot P_2)xy)\eta^{\mu\nu} I_0$$

$$+ 8 m_i C_V^i (P_1 x - P_2 y)^\mu P_2^\nu I_0 + 8 m_i C_V^i (P_1 x - P_2 y)^\nu P_2^\mu I_0 - 8 m_i C_V^i \eta^{\mu\nu}(P_1\cdot P_2) x I_0$$

$$+ 8 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} x P_{1\alpha} P_{2\beta} I_0 - 8 m_i C_V^i P_1^\mu P_1^\nu x I_0 + 8 m_i C_V^i y P_1^\mu P_2^\nu I_0$$

$$- 4 m_i C_V^i P_1^\mu P_2^\nu I_0 - 4 m_i C_V^i P_1^\nu P_2^\mu I_0 + 4 m_i C_V^i (P_1\cdot P_2)\eta^{\mu\nu} I_0 - 4 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} P_{2\beta} I_0 + 4 m_i^3 C_V^i \eta^{\mu\nu} I_0 \Big\} \times$$

$\times\ \varepsilon_{2\nu}^{*}\,\varepsilon_{0\mu}\,(-1)$ (14)

$$16 m_i C_V^i \frac{I_0}{2}\, \eta^{\mu\nu} \left[-m_{h^0}^2 X^2 + 2(P_1\cdot P_2) XY - m_i^2\right] \frac{1}{2 - d/2} - 4 m_i C_V^i \eta^{\mu\nu} \frac{I_0 d}{2}$$

$$\left[-m_{h^0}^2 X^2 + 2(P_1\cdot P_2) XY \underset{+m_{h^0}^2 x}{-\, m_i^2}\right] \frac{1}{2 - d/2}$$

$$= 2 m_i C_V^i I_0 \eta^{\mu\nu} \left[-m_{h^0}^2 X^2 + 2(P_1\cdot P_2) XY \underset{+m_{h^0}^2 X}{-\, m_i^2}\right] \lim_{d\to 4} \frac{[\cancel{4-d}]}{\frac{(\cancel{4-d})}{2}}$$

$$= 4 m_i C_V^i I_0 \eta^{\mu\nu} \left[-m_{h^0}^2 X^2 + 2(P_1\cdot P_2) XY \underset{+m_{h^0}^2 X}{-\, m_i^2}\right] \qquad (15)$$

$\Rightarrow$

$$-iM_1a = \frac{(-1)\ -g^2 e\, 2}{(2\pi)^4\, 4 M_W \cos\theta_W} \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_i m_i Q_i N_i \int_0^1 dx \int_0^{1-x} dY\ I_0 \Big\{ 4 m_i C_V^i \eta^{\mu\nu} \left[-m_{h^0}^2 X^2\right.$$

$$\left. + 2(P_1\cdot P_2) XY \underset{+m_{h^0}^2 X}{-\, m_i^2}\right] + 16 m_i C_V^i (P_1 X - P_2 Y)^\mu \cancel{(P_1 X - P_2 Y)^\nu} - 4 m_i C_V^i \eta^{\mu\nu} \left[m_{h^0}^2 X^2\right.$$

$$\left. - 2(P_1\cdot P_2) XY\right] + 8 m_i C_V^i (P_1 X - \cancel{P_2 Y})^\mu P_2^\nu + 8 m_i C_V^i (P_1 X - \cancel{P_2 Y})^\nu P_2^\mu - 8 m_i C_V^i \eta^{\mu\nu} (P_1\cdot P_2) X$$

$$+ 8 i\, m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} X P_{1\alpha} P_{2\beta} - 8 m_i C_V^i P_1^\mu P_1^\nu X + 8 m_i C_V^i Y \cancel{P_1^\mu} P_2^\nu - 4 m_i C_V^i \cancel{P_1^\mu} P_2^\nu$$

$$- 4 m_i C_V^i P_1^\nu P_2^\mu + 4 m_i C_V^i (P_1\cdot P_2) \eta^{\mu\nu} - 4 i\, m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} P_{2\beta} + 4 m_i^3 C_V^i \eta^{\mu\nu} \Big\} \varepsilon_{2\nu}^{*} \varepsilon_{0\mu}$$

$$P_{2\mu} \varepsilon_2^{\mu} = 0 \quad \text{or} \quad P_{2\nu} \varepsilon_2^{\nu *} = 0 \qquad (17) \qquad\qquad (16)$$

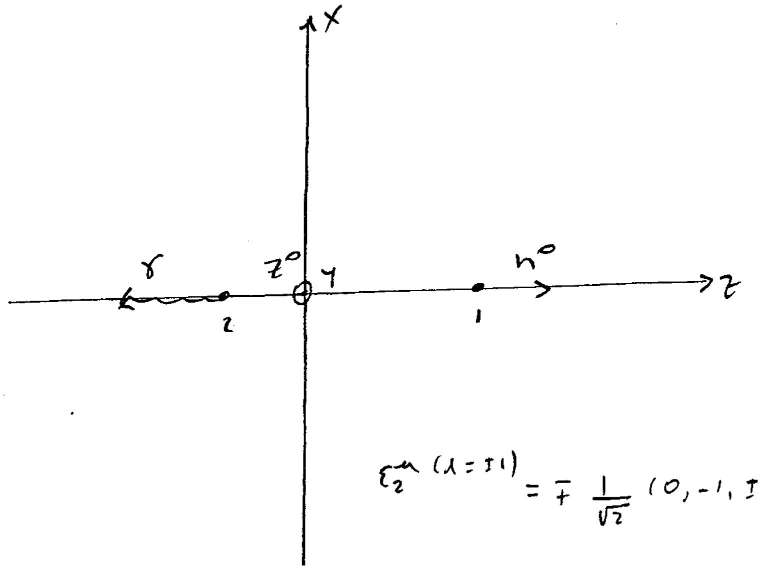
$$-iM_1a = \frac{(-1)\ -g^2 e\, 2}{(2\pi)^4\, 4 M_W \cos\theta_W} \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_i m_i Q_i N_i \int_0^1 dx \int_0^{1-x} dY\ I_0 \Big\{ 4 m_i C_V^i \eta^{\mu\nu} \left[-m_{h^0}^2 X^2\right.$$

$$\left. + 2(P_1\cdot P_2) XY \underset{+m_{h^0}^2 X}{-\, m_i^2}\right] + 16 m_i C_V^i X^2 \cancel{P_1^\mu} P_1^\nu - 16 m_i C_V^i X Y \cancel{P_2^\mu} P_1^\nu - 4 m_i C_V^i \eta^{\mu\nu} \left[m_{h^0}^2 X^2\right.$$

$$\left. - 2(P_1\cdot P_2) XY\right] + 8 m_i C_V^i \cancel{X} P_1^\nu P_2^\mu - 8 m_i C_V^i \eta^{\mu\nu} (P_1\cdot P_2) X + 8 i\, m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} X P_{1\alpha} P_{2\beta}$$

$$- 8 m_i C_V^i \cancel{P_1^\mu} P_1^\nu X - 4 m_i C_V^i \cancel{P_1^\nu} P_2^\mu + 4 m_i C_V^i \eta^{\mu\nu} (P_1\cdot P_2) - 4 i\, m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} P_{2\beta}$$

$$+ 4 m_i^3 C_V^i \eta^{\mu\nu} \Big\} \varepsilon_{2\nu}^{*} \varepsilon_{0\mu}. \qquad (18)$$



$$\varepsilon_2^{\mu}(\lambda=\pm1) = \mp\frac{1}{\sqrt{2}}(0,-1,\mp i,0)$$

$$P_1^{\mu} = (P_1^0, 0, 0, P)$$

$$P_2^{\mu} = (P_2^0, 0, 0, -P)$$

$$P_1^{\mu}\varepsilon_{2\mu} = 0 \quad \text{or} \quad P_1^{\nu}\varepsilon_{2\nu}^{x} = 0 \qquad (19)$$

$$-iM_1a = \frac{(-1)-g^2e\,2}{(2\pi)^4\,4M_W\cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_iQ_iN_i\int_0^1 dx\int_0^{1-x} dy\, I_0\Big\{4m_i\cancel{C_V^i}\,n^{\mu\nu}[-m_{h^0}^2x^2 + 2\cancel{(P_1\cdot P_2)}xy \underset{+m_{h^0}^2x}{-}\cancel{m_i}^2] - 4m_i\cancel{C_V^i}\,n^{\mu\nu}[m_{h^0}^2\cancel{x}^2 - 2\cancel{(P_1}\cdot P_2)xy] - 8m_iC_V^i\,\cancel{n^{\mu\nu}}(P_1\cdot P_2)x + 8i\,m_iC_A^i\,\varepsilon^{\alpha\nu\beta\mu}\,\cancel{x}\cancel{P_{1\alpha}}P_{2\beta} + 4m_iC_V^i\cancel{n^{\mu\nu}}(P_1\cdot P_2) - 4i\,m_iC_A^i\,\cancel{\varepsilon^{\alpha}}{}^{\nu\beta\mu}P_{1\alpha}P_{2\beta} + 4m_i^3\cancel{C_V^i}\,n^{\mu\nu}\Big\}\varepsilon_{2\nu}^{x}\,\varepsilon_{0\mu} \qquad (20)$$

$$-iM_1a = \frac{(-1)-g^2e\,2}{(2\pi)^4\,4M_W\cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_iQ_i\int_0^1 dx\int_0^{1-x} dy\, I_0\Big\{4m_iC_V^i\,n^{\mu\nu}[-2m_{h^0}^2x^2 - 2m_i^2 + 4(P_1\cdot P_2)xy - 2(P_1\cdot P_2)x + (P_1\cdot P_2) \underset{+m_{h^0}^2x}{+} 2m_i^2] + 8i\,m_iC_A^i\,\varepsilon^{\alpha\nu\beta\mu}\,x\,P_{1\alpha}P_{2\beta} - 4i\,m_iC_A^i\,\varepsilon^{\alpha\nu\beta\mu}P_{1\alpha}P_{2\beta}\Big\}\varepsilon_{2\nu}^{i}\,\varepsilon_{0\mu}N_i \qquad (21)$$

$$\int_0^{1-x}\frac{dy}{[-m_{h^0}^2x^2 + 2(P_1\cdot P_2)xy \underset{+m_{h^0}^2x}{-} m_i^2]} = \int_0^{1-x}\frac{dy}{(a+by)} = \frac{1}{b}\ln|a+by|\Big|_0^{1-x}$$

$$a = -m_{h^0}^2x^2 - m_i^2 + m_{h^0}^2x$$

$$b = 2(P_1\cdot P_2)x$$

$$= \frac{1}{b}\ln|a+b(1-x)| - \frac{1}{b}\ln|a|$$

$$= \frac{1}{b}\ln\left|\frac{a+b(1-X)}{a}\right|$$

$$\int_0^{1-X}\frac{dY}{\underset{+m_{h^0}^2X}{\left[-m_{h^0}^2X^2+2(P_1\cdot P_2)XY-m_i^2\right]}} = \frac{1}{2(P_1\cdot P_2)X}\ln\left|\frac{-m_{h^0}^2X^2-m_i^2+2(P_1\cdot P_2)X(1-X)+m_{h^0}^2X}{-m_{h^0}^2X^2-m_i^2+m_{h^0}^2X}\right|$$

$$\varepsilon^{\alpha\nu\beta\mu}\,P_{1\alpha}P_{2\beta}\,\varepsilon^{*}_{2\nu}\,\varepsilon_{0\mu}$$

$$\varepsilon_0^{(\lambda=\pm1)} = \mp(0,1,\pm i,0)\frac{1}{\sqrt{2}} \quad ; \quad P_{Z^0} = (M_{Z^0},0,0,0)$$

$$\varepsilon_0^{\lambda=0} = (0,0,0,1)$$

$$\varepsilon^{\alpha\nu\beta\mu}P_{1\alpha}P_{2\beta} = \varepsilon^{\alpha\nu\beta\mu}(P_{Z^0}-P_2)_\alpha P_{2\beta} = \varepsilon^{\alpha\nu\beta\mu}P_{Z^0\alpha}P_{2\beta}$$

$$M_{Z^0}^2 = m_{h^0}^2 + 2P_1\cdot P_2$$

$$\Rightarrow \int_0^{1-X}\frac{dY}{\underset{+m_{h^0}^2X}{\left[-m_{h^0}^2X^2+2(P_1\cdot P_2)XY-m_i^2\right]}} = \frac{1}{2(P_1\cdot P_2)X}\ln\left|\frac{-m_{h^0}^2X^2-m_i^2+(M_{Z^0}^2-m_{h^0}^2)(X-X^2)+m_{h^0}^2X}{-m_{h^0}^2X^2-m_i^2+m_{h^0}^2X}\right|$$

$$\int_0^{1-X}\frac{dY}{\underset{+m_{h^0}^2X}{\left[-m_{h^0}^2X^2+2(P_1\cdot P_2)XY-m_i^2\right]}} = \frac{1}{2(P_1\cdot P_2)X}\ln\left|\frac{-M_{Z^0}^2X^2+(M_{Z^0}^2-\cancel{m_{h^0}^2})X-m_i^2+\cancel{m_{h^0}^2X}}{-m_{h^0}^2X^2-m_i^2+m_{h^0}^2X}\right|$$

$$= \frac{1}{2(P_1\cdot P_2)X}\ln\left|\frac{M_{Z^0}^2X^2-M_{Z^0}^2X+m_i^2}{m_{h^0}^2X^2-m_{h^0}^2X+m_i^2}\right| \qquad (22)$$

$$-iM_{1a} = (-1)\frac{-g^2e\;i\pi^2}{(2\pi)^4\,4M_W\cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_iQ_i\int_0^1dX\int_0^{1-X}dY\left\{4m_iC_V^i\,n^{\mu\nu}+\right.$$

$$\frac{\left[-m_{h^0}^2X^2+2(P_1\cdot P_2)XY-2(P_1\cdot P_2)X+(P_1\cdot P_2)+m_i^2\right]}{\underset{+m_{h^0}^2X}{\left[-m_{h^0}^2X^2+2(P_1\cdot P_2)XY-m_i^2\right]}}4m_iC_V^i\,n^{\mu\nu} + \frac{4im_iC_A^i\,\varepsilon^{\alpha\nu\beta\mu}P_{1\alpha}P_{2\beta}}{\underset{+m_{h^0}^2X}{\left[-m_{h^0}^2X^2+2(P_1\cdot P_2)XY-m_i^2\right]}}\cdot$$

$$\left.\cdot(2X-1)\right\}\varepsilon^{*}_{2\nu}\,\varepsilon_{0\mu}\,N_i$$

$$\Rightarrow$$

$$-iM_{1a} = (-1)\frac{-g^2e\;i\pi^2}{(2\pi)^4\,4M_W\cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_iN_iQ_i\int_0^1dX\int_0^{1-X}dY\left\{8m_iC_V^i\,n^{\mu\nu}+4m_iC_V^i\,n^{\mu\nu}\cdot\right.$$

$$\left.\cdot\frac{\left[-m_{h^0}^2X+2m_i^2-2(P_1\cdot P_2)X+(P_1\cdot P_2)\right]}{\left[-m_{h^0}^2X^2+2(P_1\cdot P_2)XY+m_{h^0}^2X-m_i^2\right]} + \frac{4im_iC_A^i\,\varepsilon^{\alpha\nu\beta\mu}P_{1\alpha}P_{2\beta}(2X-1)}{\left[-m_{h^0}^2X^2+2(P_1\cdot P_2)XY+m_{h^0}^2X-m_i^2\right]}\right\}\varepsilon^{*}_{2\nu}\,\varepsilon_{0\mu} \qquad (23)$$

$$-iM_{2a} = \sum_{i=d,s,b} \int \frac{d^dk}{(2\pi)^d} \left(\frac{igm_i}{2M_W} \frac{\sin\alpha}{\cos\beta} \right) \mathrm{Tr}\left[\frac{i}{(\not{k}-\not{p}_1-m_i)} \left(\frac{-ig}{\cos\theta_W} \right) \gamma^\mu \frac{1}{2} ((C_V^i - C_A^i\gamma^5) \frac{i}{(\not{k}+\not{p}_2-m_i)} \right.$$

$$\left. (-ie\gamma^\nu Q_i) \frac{i}{\not{k}-m_i} \right] \varepsilon^*_{2\nu}\, \varepsilon_{0\mu}\, \underbrace{\mu_1^{\frac{(4-d)}{2}} \mu_2^{\frac{(4-d)}{2}} \mu_3^{\left(\frac{4-d}{2}\right)}}_{\mu^*} \underset{\substack{\downarrow \\ \text{Fermion loop}}}{(-1)} \quad (24)$$

$$-iM_{2a} = -\frac{g^2e}{4M_W\cos\theta_W} \left(\frac{\sin\alpha}{\cos\beta} \right) \sum_i m_i Q_i \int \frac{d^dk}{(2\pi)^d} \mathrm{Tr}\left\{ (\not{k}-\not{p}_1+m_i)\gamma^\mu (C_V^i - C_A^i\gamma^5)(\not{k}+\not{p}_2+m_i) \right.$$

$$\left. \gamma^\nu (\not{k}+m_i) \right\} \varepsilon^*_{2\nu}\, \varepsilon_{0\mu}\, \underset{\substack{\downarrow \\ 1}}{\mu^*}\, N_i \frac{1}{[(k-p_1)^2-m_i^2][(k+p_2)^2-m_i^2][k^2-m_i^2]} (-1) \quad (25)$$

$$\mathrm{Tr}\left\{ (\not{k}-\not{p}_1+m_i)\gamma^\mu (C_V^i - C_A^i\gamma^5)(\not{k}+\not{p}_2+m_i)\gamma^\nu(\not{k}+m_i) \right\}$$

$$= \mathrm{Tr}\left\{ \gamma^\nu(\not{k}+m_i)(\not{k}-\not{p}_1+m_i)\gamma^\mu (C_V^i - C_A^i\gamma^5)(\not{k}+\not{p}_2+m_i) \right\}$$

$$= \mathrm{Tr}\left\{ [k^2\gamma^\nu - \gamma^\nu\not{k}\not{p}_1 + m_i\gamma^\nu\not{k} + m_i\gamma^\nu\not{k} - m_i\gamma^\nu\not{p}_1 + m_i^2\gamma^\nu]\gamma^\mu [C_V^i\not{k} + C_V^i\not{p}_2 \right.$$

$$\left. + m_iC_V^i - C_A^i\gamma^5\not{k} - C_A^i\gamma^5\not{p}_2 - m_iC_A^i\gamma^5] \right\}$$

$$= \mathrm{Tr}\left\{ [k^2\gamma^\nu\gamma^\mu - \gamma^\nu\not{k}\not{p}_1\gamma^\mu + 2m_i\gamma^\nu\not{k}\gamma^\mu - m_i\gamma^\nu\not{p}_1\gamma^\mu + m_i^2\gamma^\nu\gamma^\mu][C_V^i\not{k} + C_V^i\not{p}_2 \right.$$

$$\left. + m_iC_V^i - C_A^i\overset{\downarrow}{\gamma^5}\not{k} - C_A^i\gamma^5\not{p}_2 - m_iC_A^i\gamma^5] \right\}$$

$$= C_V^ik^2\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^\nu\gamma^\mu\not{k}]}} + k^2C_V^i\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^\nu\gamma^\mu\not{p}_2]}} + m_iC_V^ik^2\,\bcancel{\mathrm{Tr}[\gamma^\nu\gamma^\mu]} - C_A^ik^2\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^\nu\gamma^\mu\gamma^5\not{k}]}}$$

$$- C_A^ik^2\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^\nu\gamma^\mu\gamma^5\not{p}_2]}} - m_iC_A^ik^2\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\nu\gamma^\mu]}} - C_V^i\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^\nu\not{k}\not{p}_1\gamma^\mu\not{k}]}}$$

$$- C_V^i\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^\nu\not{k}\not{p}_1\gamma^\mu\not{p}_2]}} - m_iC_V^i\,\bcancel{\mathrm{Tr}[\gamma^\nu\not{k}\not{p}_1\gamma^\mu]} + C_A^i\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\not{k}\gamma^\nu\not{k}\not{p}_1\gamma^\mu]}}$$

$$+ C_A^i\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\not{p}_2\gamma^\nu\not{k}\not{p}_1\gamma^\mu]}} + m_iC_A^i\,\bcancel{\mathrm{Tr}[\gamma^5\gamma^\nu\not{k}\not{p}_1\gamma^\mu]} + 2m_iC_V^i\,\bcancel{\mathrm{Tr}[\gamma^\nu\not{k}\gamma^\mu\not{k}]}$$

$$+ 2m_i\overset{\downarrow}{C_V^i}\,\bcancel{\mathrm{Tr}[\gamma^\nu\not{k}\gamma^\mu\not{p}_2]} + 2m_i^2C_V^i\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^\nu\not{k}\gamma^\mu]}} - 2m_i\overset{\downarrow}{C_A^i}\,\bcancel{\mathrm{Tr}[\gamma^5\not{k}\gamma^\nu\not{k}\gamma^\mu]}$$

$$- 2m_iC_A^i\,\bcancel{\mathrm{Tr}[\gamma^5\not{p}_2\gamma^\nu\not{k}\gamma^\mu]} - 2m_i^2C_A^i\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\nu\not{k}\gamma^\mu]}} - m_iC_V^i\,\cancel{\mathrm{Tr}[\gamma^\nu\not{p}_1\gamma^\mu\overset{\swarrow}{\not{k}}]}$$

$$- m_iC_V^i\,\bcancel{\mathrm{Tr}[\gamma^\nu\not{p}_1\gamma^\mu\not{p}_2]} - m_i^2C_V^i\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^\nu\not{p}_1\gamma^\mu]}} + m_iC_A^i\,\bcancel{\mathrm{Tr}[\gamma^5\not{k}\gamma^\nu\not{p}_1\gamma^\mu]}$$

$$+ m_iC_A^i\,\bcancel{\mathrm{Tr}[\gamma^5\not{p}_2\gamma^\nu\not{p}_1\gamma^\mu]} + m_i^2C_A^i\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\nu\not{p}_1\gamma^\mu]}} + m_i^2C_V^i\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^\nu\gamma^\mu\not{k}]}} + m_i^2C_V^i\,\cdot$$

$$\cdot\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^\nu\gamma^\mu\not{p}_2)}} + m_i^3C_V^i\,\bcancel{\mathrm{Tr}(\gamma^\nu\gamma^\mu)} - m_i^2C_A^i\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\not{k}\gamma^\nu\gamma^\mu]}} - m_i^2C_A^i\,\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\not{p}_2\gamma^\nu\gamma^\mu]}}$$

$$- m_i^3C_A^i\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^5\gamma^\nu\gamma^\mu)}}$$

$$= 4k^2 \eta^{\mu\nu} m_i C_V^i - m_i C_V^i \, Tr\left(\gamma^\nu \not{k} \left(2\eta^{\mu\alpha} - \gamma^\mu \gamma^\alpha\right)\right) p_{1\alpha} - 4 i m_i C_A^i \cancel{\varepsilon^{\nu\alpha\beta\mu}} k_\alpha p_{1\beta}$$

$$+ 8 m_i C_V^i \left[2k^\nu k^\mu - k^2 \eta^{\mu\nu}\right] + 8 m_i C_V^i \left[k^\nu p_2^\mu + k^\mu p_2^\nu - (p_2 \cdot k)\eta^{\mu\nu}\right]$$

$$+ 8 i m_i C_A^i \overset{0}{\cancel{\varepsilon^{\alpha\nu\beta\mu}}} k_\alpha k_\beta + 8 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} p_{2\alpha} k_\beta - 4 m_i C_V^i \left[p_1^\nu p_2^\mu + p_1^\mu p_2^\nu\right.$$

$$\left. - (p_1 \cdot p_2)\eta^{\mu\nu}\right] - 4 i m_i C_A^i \underset{\downarrow \atop (-\varepsilon^{\nu\alpha\beta\mu})}{\cancel{\varepsilon^{\alpha\nu\beta\mu}}} k_\alpha p_{1\beta} - 4 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} p_{2\alpha} p_{1\beta} + 4 m_i^3 C_V^i \eta^{\mu\nu}$$

$$- 4 m_i C_V^i \left(p_1^\nu k^\mu + p_1^\mu k^\nu - (p_1 \cdot k)\eta^{\mu\nu}\right)$$

$$= 4k^2 \eta^{\mu\nu} m_i C_V^i - 8 m_i C_V^i p_1^\mu k^\nu + m_i C_V^i \, Tr\left(\gamma^\nu \not{k} \gamma^\mu \not{p}_1\right) + 8 m_i C_V^i \left(2k^\mu k^\nu - k^2 \eta^{\mu\nu}\right)$$

$$+ 8 m_i C_V^i \left[k^\nu p_2^\mu + k^\mu p_2^\nu - (p_2 \cdot k)\eta^{\mu\nu}\right] - 4 m_i C_V^i \left[p_1^\nu p_2^\mu + p_1^\mu p_2^\nu - (p_1 \cdot p_2)\eta^{\mu\nu}\right]$$

$$+ 4 m_i^3 C_V^i \eta^{\mu\nu} + 8 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} p_{2\alpha} k_\beta - 4 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} p_{2\alpha} p_{1\beta} - 4 m_i C_V^i \left(p_1^\nu k^\mu + p_1^\mu k^\nu - (p_1 \cdot k)\eta^{\mu\nu}\right)$$

$$= 4k^2 \eta^{\mu\nu} m_i C_V^i - 8 m_i C_V^i p_1^\mu k^\nu + 4 m_i C_V^i \left[k^\nu p_1^\mu \cancel{+ k^\mu p_1^\nu} - (p_1 \cdot k)\eta^{\mu\nu}\right]$$

$$+ 8 m_i C_V^i \left(2k^\mu k^\nu - k^2 \eta^{\mu\nu}\right) + 8 m_i C_V^i \left[k^\nu p_2^\mu + k^\mu p_2^\nu - (p_2 \cdot k)\eta^{\mu\nu}\right]$$

$$- 4 m_i C_V^i \left[p_1^\nu p_2^\mu + p_1^\mu p_2^\nu - (p_1 \cdot p_2)\eta^{\mu\nu}\right] + 4 m_i^3 C_V^i \eta^{\mu\nu} + 8 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} p_{2\alpha} k_\beta$$

$$- 4 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} p_{2\alpha} p_{1\beta} - 4 m_i C_V^i \left(p_1^\nu k^\mu \cancel{+ p_1^\mu k^\nu} - (p_1 \cdot k)\eta^{\mu\nu}\right)$$

$\Rightarrow$

$$Tr(\,) = 16 m_i C_V^i k^\mu k^\nu - 4k^2 m_i C_V^i \eta^{\mu\nu} - 8 m_i C_V^i p_1^\mu k^\nu$$

$$+ 8 m_i C_V^i k^\nu p_2^\mu + 8 m_i C_V^i k^\mu p_2^\nu - 8 m_i C_V^i (p_2 \cdot k)\eta^{\mu\nu} - 4 m_i C_V^i p_1^\nu p_2^\mu - 4 m_i C_V^i p_1^\mu p_2^\nu$$

$$+ 4 m_i C_V^i (p_1 \cdot p_2)\eta^{\mu\nu} + 4 m_i^3 C_V^i \eta^{\mu\nu} + 8 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} p_{2\alpha} k_\beta - 4 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} p_{2\alpha} p_{1\beta} \qquad (26)$$

$$\frac{1}{\left[k^2 - m_i^2\right]\left[(k+p_2)^2 - m_i^2\right]\left[(k-p_1)^2 - m_i^2\right]} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{\left[k^2 - 2k\cdot(p_1 x - p_2 y) - m_i^2 + m_{h^0}^2 x\right]^3} \qquad (27)$$

$\Rightarrow$

$$-i\mathcal{M}_{2a} = \frac{(-1) - g^2 e}{4 M_W \cos\theta_W} \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_i m_i Q_i \, 2\int_0^1 dx \int_0^{1-x} dy \int \frac{d^d k}{(2\pi)^d} \left\{ 16 m_i C_V^i k^\mu k^\nu\right.$$

$$- 4k^2 m_i C_V^i \eta^{\mu\nu} - 8 m_i C_V^i p_1^\mu k^\nu + 8 m_i C_V^i k^\nu p_2^\mu + 8 m_i C_V^i k^\mu p_2^\nu$$

$$- 8 m_i C_V^i (p_2 \cdot k)\eta^{\mu\nu} - 4 m_i C_V^i p_1^\nu p_2^\mu - 4 m_i C_V^i p_1^\mu p_2^\nu + 4 m_i C_V^i (p_1 \cdot p_2)\eta^{\mu\nu}$$

$$\left. + 4 m_i^3 C_V^i \eta^{\mu\nu} + 8 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} p_{2\alpha} k_\beta - 4 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} p_{2\alpha} p_{1\beta} \right\} \cdot \left[k^2 - 2k\cdot(p_1 x - p_2 y) - m_i^2 + m_{h^0}^2 x\right]^{-3}$$

$$\varepsilon_{2\nu}^{*} \, \varepsilon_{0\mu} \, N_i \qquad (28)$$

$$K' = K - (P_1X - P_2Y) \Rightarrow K = K' + (P_1X - P_2Y) \qquad (29)$$

$$K'^2 = K^2 - 2K\cdot(P_1X - P_2Y) + m_{h^0}^2X^2 - 2P_1\cdot P_2\,XY \qquad (30)$$

$$d^dK = d^dK'$$

$$-iM_{2a} = \frac{(-1)-g^2e}{4M_W\cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_iQ_i\,2\int_0^1 dX\int_0^{1-X}dY\int\frac{d^dK'}{(2\pi)^d}\Big\{16m_iC_V^i\,(K'+(P_1X-P_2Y))^\mu$$

$$(K'+(P_1X-P_2Y))^\nu - 4\left[K'^2 + 2K'\cdot(P_1X-P_2Y) + m_{h^0}^2X^2 - 2(P_1\cdot P_2)XY\right]m_iC_V^i\eta^{\mu\nu}$$

$$-8m_iC_V^iP_1^\mu\,(K'+(P_1X-P_2Y))^\nu +$$

$$+8m_iC_V^iP_2^\mu\,(K'+(P_1X-P_2Y))^\nu + 8m_iC_V^iP_2^\nu\,(K'+(P_1X-P_2Y))^\mu$$

$$-8m_iC_V^i\eta^{\mu\nu}\left[P_2\cdot(K'+(P_1X-P_2Y))\right] - 4m_iC_V^iP_1^\nu P_2^\mu - 4m_iC_V^iP_1^\mu P_2^\nu + 4m_iC_V^i(P_1\cdot P_2)\eta^{\mu\nu}$$

$$+4m_i^3C_V^i\eta^{\mu\nu} + 8im_iC_A^i\,\varepsilon^{\alpha\nu\beta\mu}P_{2\alpha}(K'+(P_1X-P_2Y))_\beta - 4im_iC_A^i\,\varepsilon^{\alpha\nu\beta\mu}P_{2\alpha}P_{1\beta}\Big\}.$$

$$\cdot\big[K^2 - 2K\cdot(P_1X-P_2Y)\underset{+m_{h^0}^2X}{-m_i^2}\big]^{-3}\,\varepsilon_{\gamma\nu}^*\,\varepsilon_{0\mu}\,N_i \qquad (31)$$

$$-iM_{2a} = \frac{(-1)-g^2e\,2}{4M_W\cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_iQ_i\int_0^1 dX\int_0^{1-X}dY\int\frac{d^dK'}{(2\pi)^d}\Big\{16m_iC_V^iK'^\mu K'^\nu$$

$$+16m_iC_V^iK'^\mu(P_1X-P_2Y)^\nu + 16m_iC_V^i(P_1X-P_2Y)^\mu K'^\nu + 16m_iC_V^i(P_1X-P_2Y)^\mu(P_1X-P_2Y)^\nu$$

$$-4m_iC_V^iK'^2\eta^{\mu\nu} - 8m_iC_V^i\eta^{\mu\nu}K'\cdot(P_1X-P_2Y) - 4m_iC_V^im_{h^0}^2X^2\eta^{\mu\nu} + 8m_iC_V^i\eta^{\mu\nu}\times$$

$$(P_1\cdot P_2)XY - 8m_iC_V^iP_1^\mu K'^\nu - 8m_iC_V^iXP_1^\mu P_1^\nu + 8m_iC_V^iYP_1^\mu P_2^\nu + 8m_iC_V^iP_2^\mu K'^\nu$$

$$+8m_iC_V^iP_2^\mu(P_1X-P_2Y)^\nu + 8m_iC_V^iP_2^\nu K'^\mu + 8m_iC_V^iP_2^\nu(P_1X-P_2Y)^\mu - 8m_iC_V^i\eta^{\mu\nu}(P_2\cdot K')$$

$$-8m_iC_V^i\eta^{\mu\nu}P_2\cdot(P_1X-P_2Y) - 4m_iC_V^iP_1^\nu P_2^\mu - 4m_iC_V^iP_1^\mu P_2^\nu + 4m_iC_V^i(P_1\cdot P_2)\eta^{\mu\nu}$$

$$+4m_i^3C_V^i\eta^{\mu\nu} + 8im_iC_A^i\,\varepsilon^{\alpha\nu\beta\mu}P_{2\alpha}K'_\beta + 8im_iC_A^i\,\varepsilon^{\alpha\nu\beta\mu}XP_{2\alpha}P_{1\beta}$$

$$-8im_iC_A^i\,\varepsilon^{\alpha\nu\beta\mu}\overset{0}{Y P_{2\alpha}P_{2\beta}} - 4im_iC_A^i\,\varepsilon^{\alpha\nu\beta\mu}P_{2\alpha}P_{1\beta}\Big\}\,\varepsilon_{2\nu}^*\,\varepsilon_{0\mu}\cdot\big[K'^2 - m_{h^0}^2X^2$$

$$+2(P_1\cdot P_2)XY \underset{+m_{h^0}^2X}{-m_i^2}\big]^{-3}\,N_i \qquad (32)$$

$$\int \frac{d^d K'}{[K'^2 - m_{h^0}^2 X^2 + 2(P_1 \cdot P_2) X Y \underset{+m_{h^0}^2 X}{- m_i^2}]^3} = \frac{i(-\pi)^{d/2}\,\Gamma(3-\frac{d}{2})}{2\,[-m_{h^0}^2 X^2 + 2(P_1\cdot P_2) X Y \underset{+m_{h^0}^2 X}{- m_i^2}]^{3-\frac{d}{2}}} = I_0$$

$$\lim_{d \to 4} \int \frac{d^d K'}{[\quad]^3} = \frac{i\pi^2}{2\,[-m_{h^0}^2 X^2 + 2(P_1\cdot P_2) X Y \underset{+m_{h^0}^2 X}{- m_i^2}]} = I_0$$

$$\int \frac{d^d K'\; K'^{\mu}}{[K'^2 - m_{h^0}^2 X^2 + 2(P_1\cdot P_2) X Y \underset{+m_{h^0}^2 X}{- m_i^2}]^3} = 0$$

$$\int \frac{d^d K'\; K'^{\mu} K'^{\nu}}{[K'^2 - m_{h^0}^2 X^2 + 2(P_1\cdot P_2) X Y \underset{+m_{h^0}^2 X}{- m_i^2}]^3} = \frac{I_0\, n^{\mu\nu}}{2}[-m_{h^0}^2 X^2 + 2(P_1\cdot P_2) X Y \underset{+m_{h^0}^2 X}{- m_i^2}] \cdot \frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})}$$

$$\int \frac{d^d K'\; K'^2}{[K'^2 - m_{h^0}^2 X^2 + 2(P_1\cdot P_2) X Y \underset{+m_{h^0}^2 X}{- m_i^2}]^3} = \frac{I_0\, d}{2}[-m_{h^0}^2 X^2 + 2(P_1\cdot P_2) X Y \underset{+m_{h^0}^2 X}{- m_i^2}] \cdot \frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})}$$

$$-iM_{2a} = (-1)\frac{-g^2 e}{2 M_W \cos\theta_W (2\pi)^4}\left(\frac{\sin\alpha}{\cos\beta}\right) \sum_i m_i Q_i \int_0^1 dX \int_0^{1-X} dY \Big\{ 4 m_i C_V^i \underset{d\to 4}{\lim} \Big[2 I_0 n^{\mu\nu} \cdot$$

$$\cdot [-m_{h^0}^2 X^2 + 2(P_1\cdot P_2) X Y \underset{+m_{h^0}^2 X}{- m_i^2}] \frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})} - \frac{I_0 d}{2} n^{\mu\nu} [-m_{h^0}^2 X^2 + 2(P_1\cdot P_2) X Y \underset{+m_{h^0}^2 X}{- m_i^2}] \cdot$$

$$\cdot \frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})} \Big] + 16 m_i C_V^i X^2 P_1^{\mu} P_1^{\nu} I_0 - 16 m_i C_V^i X \cancel{Y} P_1^{\mu} P_2^{\nu} I_0 - 16 m_i C_V^i X Y P_2^{\mu} P_1^{\nu} I_0$$

$$+ 16 Y^2 m_i C_V^i P_2^{\mu} \cancel{P_2^{\nu}} I_0 - 4 m_i C_V^i m_{h^0}^2 X^2 n^{\mu\nu} I_0 + 8 m_i C_V^i n^{\mu\nu} (P_1\cdot P_2) X Y I_0$$

$$- 8 m_i C_V^i X P_1^{\mu} P_1^{\nu} I_0 + 8 m_i C_V^i Y P_1^{\mu} \cancel{P_2^{\nu}} I_0 + 8 m_i C_V^i X P_2^{\mu} P_1^{\nu} I_0$$

$$- 8 m_i C_V^i Y \cancel{P_2^{\mu}} P_2^{\nu} I_0 + 8 m_i C_V^i X \cancel{P_2^{\nu}} P_1^{\mu} I_0 - 8 m_i C_V^i \cancel{Y} P_2^{\nu} P_2^{\mu} I_0$$

$$- 8 m_i C_V^i X (P_1\cdot P_2) n^{\mu\nu} I_0 - 4 m_i C_V^i P_1^{\nu} \cancel{P_2^{\mu}} I_0 - 4 m_i C_V^i P_1^{\mu} \cancel{P_2^{\nu}} I_0$$

$$+ 4 m_i C_V^i (P_1\cdot P_2) n^{\mu\nu} I_0 + 4 m_i^3 C_V^i n^{\mu\nu} I_0 + 8 i m_i C_A^i \epsilon^{\alpha\nu\beta\mu} X P_{2\alpha} P_{1\beta} I_0$$

$$- 4 i m_i C_A^i \epsilon^{\alpha\nu\beta\mu} P_{2\alpha} P_{1\beta} I_0 \Big\} \epsilon_{2\nu}^* \epsilon_{0\mu} \cdot N_i \quad (33)$$

$$P_{2\nu}\,\varepsilon_2^{\nu *} = 0$$

$$-iM_{2a} = \frac{-g^2 e}{2M_W\cos\theta_W(2\pi)^4}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_i Q_i \int_0^1 dx \int_0^{1-x} dy \Big\{ 4m_i C_V^i\left[-m_{h^0}^2x^2 + 2(P_1\cdot P_2)xy - m_i^2 + m_{h^0}^2 x\right]$$

$$I_0 n^{\mu\nu} \lim_{d\to 4}\left[\frac{2\,\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})} - \frac{d}{2}\,\frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})}\right] + 16 m_i C_V^i x^2 P_1^\mu P_1^\nu I_0$$

$$-16 m_i C_V^i xy\,P_2^\mu P_1^\nu I_0 - 4m_i C_V^i m_{h^0}^2 x^2 n^{\mu\nu} I_0 + 8 m_i C_V^i n^{\mu\nu}(P_1\cdot P_2)xy\,I_0$$

$$-8m_i C_V^i x\,P_1^\mu P_1^\nu I_0 + 8m_i C_V^i x\,P_2^\mu P_1^\nu I_0 - 8 m_i C_V^i x(P_1\cdot P_2)n^{\mu\nu} I_0 - 4m_i C_V^i P_1^\nu P_2^\mu I_0$$

$$+4m_i C_V^i (P_1\cdot P_2) n^{\mu\nu} I_0 + 4m_i^3 C_V^i n^{\mu\nu} I_0 + 8 i m_i C_A^i \epsilon^{\alpha\nu\beta\mu} x\,P_{2\alpha}P_{1\beta} I_0$$

$$-4 i m_i C_A^i \epsilon^{\alpha\nu\beta\mu} P_{2\alpha}P_{1\beta} I_0\Big\}\,\varepsilon_{2\nu}^*\varepsilon_{0\mu} N_i\,(-1) \quad (34)$$

$$\Gamma(3-\tfrac{d}{2}) = (2-\tfrac{d}{2})\,\Gamma(2-\tfrac{d}{2})$$

$$\Rightarrow \quad \frac{\Gamma(2-d/2)}{\Gamma(3-\frac{d}{2})} = \frac{1}{(2-\frac{d}{2})}$$

$$\lim_{d\to 4}\left[\frac{2}{2-\frac{d}{2}} - \frac{d}{2}\cdot\frac{1}{2-\frac{d}{2}}\right] = \lim_{d\to 4}\frac{\cancel{(2-\frac{d}{2})}}{\cancel{(2-\frac{d}{2})}} = 1 \qquad (35)$$

$$-iM_{2a} = \frac{-g^2 e(-1)}{(2\pi)^4 2M_W\cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_i Q_i \int_0^1 dx\int_0^{1-x} dy\; I_0\Big\{4m_i C_V^i n^{\mu\nu}\left[-m_{h^0}^2x^2 + 2(P_1\cdot P_2)xy - m_i^2 + m_{h^0}^2 x\right]$$

$$+16 m_i C_V^i x^2 \cancel{P_1^\mu P_1^\nu} - 16 m_i C_V^i xy\,\cancel{P_2^\mu P_1^\nu} - 4m_i C_V^i m_{h^0}^2 x^2 n^{\mu\nu} + 8m_i C_V^i n^{\mu\nu}(P_1\cdot P_2)xy$$

$$-8m_i C_V^i x\,\cancel{P_1^\mu P_1^\nu} + 8 m_i C_V^i x\,\cancel{P_2^\mu P_1^\nu} - 8m_i C_V^i x(P_1\cdot P_2)n^{\mu\nu} - 4m_i C_V^i \cancel{P_1^\nu} P_2^\mu + 4m_i C_V^i (P_1\cdot P_2)n^{\mu\nu}$$

$$+4m_i^3 C_V^i n^{\mu\nu} + 8 i m_i C_A^i \epsilon^{\alpha\nu\beta\mu} x\,P_{2\alpha}P_{1\beta} - 4 i m_i C_A^i \epsilon^{\alpha\nu\beta\mu}P_{2\alpha}P_{1\beta}\Big\}\,\varepsilon_{2\nu}^*\varepsilon_{0\mu}N_i \qquad (36)$$

$$P_1^\nu\,\varepsilon_{2\nu}^* = 0$$

$$-iM_{2a} = \frac{-g^2e\,(-1)}{2M_W\cos\theta_W(2\pi)^4}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_i Q_i\int_0^1 dx\int_0^{1-x}dy\; I_0\Big\{4m_i C_V^i \eta^{\mu\nu}\left[-\cancel{m_{h^0}^2}x^2 + 2(\cancel{P_1}\cdot P_2)xy\right.$$

$$\left.\cancel{-m_i^2 + m_{h^0}^2 x}\right] - 4m_i C_V^i m_{h^0}^2\cancel{x^2}\eta^{\mu\nu} + 8m_i C_V^i \eta^{\mu\nu}(\cancel{P_1\cdot P_2})xy - 8m_i \cancel{C_V^i} x(P_1\cdot P_2)\eta^{\mu\nu}$$

$$+4m_i C_V^i(\cancel{P_1}\cdot P_2)\eta^{\mu\nu} + 4m_i^3\cancel{C_V^i}\eta^{\mu\nu} + 8 i m_i C_A^i \epsilon^{\alpha\nu\beta\mu} x\,P_{2\alpha}P_{1\beta} - 4 i m_i C_A^i\epsilon^{\alpha\nu\beta\mu}P_{2\alpha}P_{1\beta}\Big\}$$

$$\varepsilon_{2\nu}^*\varepsilon_{0\mu}N_i \qquad (37)$$

$$-iM_2 a = \frac{-g^2 e\,(-1)}{2(2\pi)^4 M_W \cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_i Q_i \int_0^1 dx \int_0^{1-x} dy\; I_0 \Big\{ 8 m_i C_V^i \eta^{\mu\nu}\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy - \underset{+m_{h^0}^2 x}{m_i^2}\big] + \cancel{8 C_V^i \eta^{\mu\nu} m_i^3} - \cancel{8 m_i C_V^i x (P_1\cdot P_2)\eta^{\mu\nu}} + \cancel{4 m_i C_V^i (P_1\cdot P_2)\eta^{\mu\nu}}$$

$$+ 8 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu}\, x\, P_{2\alpha}P_{1\beta} - 4 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} P_{2\alpha}P_{1\beta}\Big\}\varepsilon^*_{2\nu}\varepsilon_{0\mu} N_i \qquad (38)$$

$$- 4 m_i C_V^i \cancel{m_{h^0}}^2 x\, \eta^{\mu\nu}$$

$$-iM_2 a = \frac{-g^2 e\, i\pi^2 (-1)}{4(2\pi)^4 M_W \cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_i Q_i \int_0^1 dx \int_0^{1-x} dy \Big\{ 8 m_i C_V^i \eta^{\mu\nu} +$$

$$+ 4 m_i C_V^i \eta^{\mu\nu}\frac{\big[2m_i^2 - 2x(P_1\cdot P_2) + (P_1\cdot P_2) - m_{h^0}^2 x\big]}{\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy - \underset{+m_{h^0}^2 x}{m_i^2}\big]} + \frac{4 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu}(2x-1)P_{2\alpha}P_{1\beta}}{\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy - \underset{+m_{h^0}^2 x}{m_i^2}\big]}\Big\}\cdot \qquad (39)$$

$$\cdot\, \varepsilon^*_{2\nu}\varepsilon_{0\mu} N_i$$

$$\varepsilon^{\alpha\nu\beta\mu} P_{2\alpha}P_{1\beta} = \varepsilon^{\beta\nu\alpha\mu} P_{1\alpha}P_{2\beta} = -\varepsilon^{\beta\alpha\nu\mu} P_{1\alpha}P_{2\beta}$$

$$= -\varepsilon^{\alpha\nu\beta\mu} P_{1\alpha}P_{2\beta} \qquad (40)$$

$$-iM_2 a = \frac{-g^2 e\, i\pi^2 (-1)}{4(2\pi)^4 M_W \cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_i Q_i \int_0^1 dx \int_0^{1-x} dy \Big\{ 8 m_i C_V^i \eta^{\mu\nu} +$$

$$+ 4 m_i C_V^i \eta^{\mu\nu}\frac{\big[2m_i^2 - 2x(P_1\cdot P_2) + (P_1\cdot P_2) - m_{h^0}^2 x\big]}{\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy - \underset{+m_{h^0}^2 x}{m_i^2}\big]} - \frac{4 i m_i C_A^i \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha}P_{2\beta}(2x-1)}{\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy - \underset{+m_{h^0}^2 x}{m_i^2}\big]}\Big\}\times$$

$$\times\, \varepsilon^*_{2\nu}\varepsilon_{0\mu} N_i \qquad (41)$$

$$\Rightarrow\quad -i(M_1 + M_2)a = -\frac{g^2 e\, i\pi^2 (-1)}{2(2\pi)^4 M_W \cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_i m_i Q_i \int_0^1 dx \int_0^{1-x} dy \Big\{ 8 m_i C_V^i \eta^{\mu\nu}$$

$$+ \frac{\big[-2(P_1\cdot P_2)x + (P_1\cdot P_2) + \overset{-m_{h^0}^2 x}{2m_i^2}\big]}{\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy - \underset{+m_{h^0}^2 x}{m_i^2}\big]}\, 4 m_i C_V^i \eta^{\mu\nu}\Big\}\varepsilon^*_{2\nu}\varepsilon_{0\mu} N_i$$

$$\therefore$$

$$-i(M_1+M_2)a = \frac{-2g^2 e\, i\pi^2 \eta^{\mu\nu}}{(2\pi)^4 M_W \cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_{i=d,s,b,\ldots} m_i^2 Q_i C_V^i N_i \int_0^1 dx \int_0^{1-x} dy \Big\{ 2 +$$

$$+ \frac{\big[-2(P_1\cdot P_2)x + (P_1\cdot P_2) + 2m_i^2 - m_{h^0}^2 x\big]}{\big[-m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy - m_i^2 + m_{h^0}^2 x\big]}\Big\}\varepsilon^*_{2\nu}\varepsilon_{0\mu}\,(-1) \qquad (42)$$

b) $i = u, c, t$

$$\frac{igm_i}{2M_W}\left(\frac{\sin\alpha}{\cos\beta}\right) \longrightarrow -\frac{igm_i}{2MW}\left(\frac{\cos\alpha}{\sin\beta}\right)$$

$$-i(M_1+M_2)_b = \frac{2g^2 e\, i\pi^2 \eta^{\mu\nu}}{(2\pi)^4 M_w \cos\theta_W}\left(\frac{\cos\alpha}{\sin\beta}\right)\sum_{i=u,c,t} m_i^2 Q_i C_V^i \int_0^1 dx \int_0^{1-x} dy \left\{ 2 + \right.$$

$$\left. + \frac{[-2(P_1\cdot P_2)X + (P_1\cdot P_2) + 2m_i^2 \overset{-m_{h^0}^2 X}{]}}{[-m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY - m_i^2 \underset{+m_{h^0}^2 X}{]}} \right\} \varepsilon_{2\nu}^* \varepsilon_{0\mu} N_i \; (-1) \qquad (43)$$

$$\int_0^1 dx \int_0^{1-x} \frac{dY\, X}{[-m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY - m_i^2 \underset{+m_{h^0}^2 X}{]}}$$

$$= \int_0^1 dx \frac{1}{2(P_1\cdot P_2)} \ln\left|\frac{M_{Z^0}^2 X^2 - M_{Z^0}^2 X + m_i^2}{m_{h^0}^2 X^2 - m_{h^0}^2 X + m_i^2}\right|$$

$$= \int_0^1 dx \frac{1}{2(P_1\cdot P_2)} \ln\left|\frac{X^2 - X + \frac{m_i^2}{M_{Z^0}^2}}{X^2 - X + \frac{m_i^2}{m_{h^0}^2}}\left(\frac{M_{Z^0}^2}{m_{h^0}^2}\right)\right|$$

$$T_i \equiv \frac{4m_i^2}{m_{h^0}^2}; \quad \lambda_i \equiv \frac{4m_i^2}{M_{Z^0}^2}$$

$$= \frac{1}{2(P_1\cdot P_2)}\int_0^1 dx \ln\left|\frac{(X^2 - X + \frac{\lambda_i}{4})/(\lambda_i/4)}{(X^2 - X + T_i/4)/(T_i/4)}\right|$$

$$\int_0^1 dx \ln\left(\frac{X^2 - X + \frac{\lambda_i}{4}}{\frac{\lambda_i}{4}}\right) = \int_0^1 dx \ln\left(X^2 - X + \frac{\lambda_i}{4}\right) - \ln\left(\frac{\lambda_i}{4}\right)$$

$a = \frac{\lambda_i}{4}$; $b = -1$; $c = 1$; $b^2 - 4ac = 1 - 4\left(\frac{\lambda_i}{4}\right) = 1 - \lambda_i$

If $b^2 - 4ac = 1 - \lambda_i < 0 \qquad (\lambda_i > 1)$

$$\int = \left[\left(X - \frac{1}{2}\right)\ln\left(X^2 - X + \frac{\lambda_i}{4}\right) - 2X + (\lambda_i - 1)^{1/2}\tan^{-1}\left(\frac{2X-1}{(\lambda_i-1)^{1/2}}\right)\right]\Big|_0^1 - \ln\left(\frac{\lambda_i}{4}\right)$$

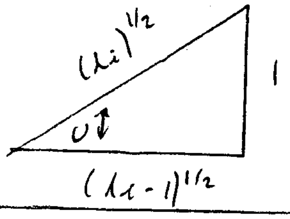
$$= \frac{1}{2}\cancel{\ln\left(\frac{\lambda_i}{4}\right)} - 2 + (\lambda_i - 1)^{1/2}\tan^{-1}\left(\frac{1}{(\lambda_i-1)^{1/2}}\right) + \frac{1}{\cancel{2}}\cancel{\ln}\left(\frac{\lambda_i}{4}\right) + (\lambda_i - 1)^{1/2}\tan^{-1}\left(\frac{1}{(\lambda_i-1)^{1/2}}\right)$$

$$- \cancel{\ln\left(\frac{\lambda_i}{4}\right)}$$

$$= -2 + 2(\lambda_i - 1)^{1/2} \tan^{-1}\left(\frac{1}{(\lambda_i - 1)^{1/2}}\right)$$

$$\tan^{-1}\left(\frac{1}{(\lambda_i - 1)^{1/2}}\right) = U$$

$$\Rightarrow \tan U = \frac{1}{(\lambda_i - 1)^{1/2}}$$



$$\Rightarrow U = \sin^{-1}\left(\frac{1}{\lambda_i^{1/2}}\right)$$

$$\Rightarrow \boxed{\int_0^1 dx \ln\left(\frac{x^2 - x + \frac{\lambda_i}{4}}{\frac{\lambda_i}{4}}\right) = -2 + 2(\lambda_i - 1)^{1/2} \sin^{-1}\left(\frac{1}{\lambda_i^{1/2}}\right) \text{ if } \lambda_i \geq 1}$$

(44)

If $b^2 - 4ac = 0$ $\lambda_i = 1$

$$\int_0^1 dx \ln(\quad) = \int_0^1 dx \ln\left(x^2 - x + \frac{1}{4}\right) - \ln\left(\frac{1}{4}\right)$$

$$= \int_0^1 dx \ln\left((x - \tfrac{1}{2})^2\right) - \ln\left(\frac{1}{4}\right)$$

$$= 2\int_0^1 dx \ln\left|x - \tfrac{1}{2}\right| - \ln\left(\frac{1}{4}\right)$$

$$= 2\left[(x - \tfrac{1}{2}) \ln\left|x - \tfrac{1}{2}\right| - (x - \tfrac{1}{2})\right]\Big|_0^1 - \ln\left(\frac{1}{4}\right)$$

$$= 2\left[\frac{1}{2}\ln\frac{1}{2} - \frac{1}{2}\right] - 2\left[-\frac{1}{2}\ln\frac{1}{2} + \frac{1}{2}\right] - \ln\left(\frac{1}{4}\right)$$

$$= \ln\left(\frac{1}{2}\right) - 1 + \ln\left(\frac{1}{2}\right) - 1 - \ln\left(\frac{1}{4}\right)$$

$$= 2\ln\left(\frac{1}{2}\right) - 2 + \ln 4$$

$$= \cancel{-2\ln 2} - 2 + \cancel{2\ln 2} = -2$$

$$\boxed{\int_0^1 dx \ln\left(\frac{x^2 - x + \frac{\lambda_i}{4}}{\frac{\lambda_i}{4}}\right) = -2 \text{ if } \lambda_i = 1}$$

(45)

If $b^2-4ac>0 \quad \lambda i<1$:

$$\int = \left[\left(x-\tfrac{1}{2}\right)\ln\left(x^2-x+\tfrac{\lambda i}{4}\right)-2x+(1-\lambda i)^{1/2}\tanh^{-1}\left(\frac{2x-1}{(1-\lambda i)^{1/2}}\right)\right]\Big|_0^1 - \ln\left(\frac{\lambda i}{4}\right)$$

$$= \cancel{\tfrac{1}{2}\ln\left(\tfrac{\lambda i}{4}\right)} - 2 + (1-\lambda i)^{1/2}\tanh^{-1}\left(\frac{1}{(1-\lambda i)^{1/2}}\right) + \cancel{\tfrac{1}{2}\ln\left(\tfrac{\lambda i}{4}\right)} - (1-\lambda i)^{1/2}\tanh^{-1}\left(\frac{-1}{(1-\lambda i)^{1/2}}\right)$$

$$\cancel{-\ln\left(\tfrac{\lambda i}{4}\right)}$$

$$= -2 + 2(1-\lambda i)^{1/2}\tanh^{-1}\left(\frac{1}{(1-\lambda i)^{1/2}}\right)$$

$$\tanh^{-1}\left(\frac{1}{(1-\lambda i)^{1/2}}\right) = V$$

$$\tanh V = \frac{1}{(1-\lambda i)^{1/2}} = \frac{\sinh V}{\cosh V} = \frac{e^V - e^{-V}}{e^V + e^{-V}}$$

$$\frac{1}{(1-\lambda i)^{1/2}} = a = \frac{e^{2V}-1}{e^{2V}+1}$$

$$e^{2V}-1 = a e^{2V} + a$$

$$e^{2V} = \frac{a+1}{1-a}$$

$$V = \tfrac{1}{2}\ln\left(\frac{\frac{1}{(1-\lambda i)^{1/2}}+1}{1-\frac{1}{(1-\lambda i)^{1/2}}}\right)$$

$$V = \tfrac{1}{2}\ln\left(\frac{1+(1-\lambda i)^{1/2}}{(1-\lambda i)^{1/2}-1}\right)$$

$$V = \tfrac{1}{2}\left[\ln\left(\frac{1+(1-\lambda i)^{1/2}}{1-(1-\lambda i)^{1/2}}\right) - \ln(-1)\right]$$

$$\ln(-1) = \ln(1e^{i\pi}) = i\pi$$

$$V = \tfrac{1}{2}\left[\ln\left(\frac{1+(1-\lambda i)^{1/2}}{1-(1-\lambda i)^{1/2}}\right) - i\pi\right]$$

$$\Rightarrow \boxed{\int_0^1 dx\,\ln\left(\frac{x^2-x+\frac{\lambda i}{4}}{\lambda i/4}\right) = -2 + (1-\lambda i)^{1/2}\left[\ln\left(\frac{1+(1-\lambda i)^{1/2}}{1-(1-\lambda i)^{1/2}}\right) - i\pi\right] \quad \text{if } \lambda i<1} \quad (46)$$

So:

$$\boxed{\int_0^1 dx \ln\left(\frac{x^2 - x + \frac{\lambda_i}{4}}{\frac{\lambda_i}{4}}\right) = \begin{cases} -2 + 2(\lambda_i - 1)^{1/2} \sin^{-1}\left(\frac{1}{\lambda_i^{1/2}}\right) & \text{if } \lambda_i \geq 1 \\ -2 + (1-\lambda_i)^{1/2}\left[\ln\left(\frac{1+(1-\lambda_i)^{1/2}}{1-(1-\lambda_i)^{1/2}}\right) - i\pi\right] & \text{if } \lambda_i < 1 \end{cases}}$$

(47)

Defining

$$g(\lambda_i) = \begin{cases} (\lambda_i - 1)^{1/2} \sin^{-1}\left(\frac{1}{\lambda_i^{1/2}}\right) & \text{if } \lambda_i \geq 1 \\ \frac{1}{2}(1-\lambda_i)^{1/2}\left[\ln\left(\frac{1+(1-\lambda_i)^{1/2}}{1-(1-\lambda_i)^{1/2}}\right) - i\pi\right] & \text{if } \lambda_i < 1 \end{cases} \qquad (48)$$

we have:

$$\int_0^1 dx \ln\left(\frac{x^2 - x + \frac{\lambda_i}{4}}{\frac{\lambda_i}{4}}\right) = -2 + 2g(\lambda_i)$$

$$\Rightarrow \boxed{\begin{aligned} &\int_0^1 dx \int_0^{1-x} \frac{x\,dy}{[-m_{h^0}^2 x^2 + 2(P_1 \cdot P_2)xy - m_i^2 + m_{h^0}^2 x]} \\ &= \frac{1}{\cancel{2}(P_1 \cdot P_2)}\left\{-\cancel{2} + \cancel{2}g(\lambda_i) + \cancel{2} - \cancel{2}g(T_i)\right\} \\ &= \frac{1}{(P_1 \cdot P_2)}\left(g(\lambda_i) - g(T_i)\right) \end{aligned}} \qquad (49)$$

$$\int_0^1 dx \int_0^{1-x} \frac{dy}{[-m_{h^0}^2 x^2 + 2(P_1 \cdot P_2)xy - m_i^2 + m_{h^0}^2 x]}$$

$$= \int_0^1 dx \frac{1}{2(P_1 \cdot P_2)x} \ln\left|\frac{M_{Z^0}^2 x^2 - M_{Z^0}^2 x + m_i^2}{m_{h^0}^2 x^2 - m_{h^0}^2 x + m_i^2}\right|$$

$$= \frac{1}{2(P_1 \cdot P_2)} \int_0^1 \frac{dx}{x} \ln\left|\frac{x^2 - x + \frac{m_i^2}{M_{Z^0}^2}}{x^2 - x + \frac{m_i^2}{m_{h^0}^2}}\left(\frac{M_{Z^0}^2}{m_{h^0}^2}\right)\right|$$

$$= \frac{1}{2(P_1 \cdot P_2)} \int_0^1 \frac{dx}{x} \ln \left| \frac{\left(x^2 - x + \frac{\lambda_i}{4}\right) / (\lambda_i/4)}{\left(x^2 - x + \frac{\tau_i}{4}\right) / (\tau_i/4)} \right|$$

but $$\int_0^1 \frac{dx}{x} \ln \left| \frac{x^2 - x + \frac{\tau_i}{4}}{(\tau_i/4)} \right| = f(\tau_i) = \begin{cases} -2\left(\arcsin\left(\frac{1}{\tau_i^{1/2}}\right)\right)^2 & \tau_i \geq 1 \\ \frac{1}{2}\left[\ln\left(\frac{1+(1-\tau_i)^{1/2}}{1-(1-\tau_i)^{1/2}}\right) - i\pi\right]^2 & \tau_i < 1 \end{cases}$$

$$\Rightarrow \boxed{\int_0^1 dx \int_0^{1-x} \frac{dy}{[-m_{h^0}^2 x^2 + 2(P_1 \cdot P_2) x y - m_i^2 + m_{h^0}^2 x]} = \frac{1}{2(P_1 \cdot P_2)} [f(\lambda_i) - f(\tau_i)]} \qquad (50)$$

$$\int_0^1 dx \int_0^{1-x} dy = \int_0^1 dx\, [1-x] = \left(x - \frac{x^2}{2}\right)\Big|_0^1 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \boxed{\int_0^1 dx \int_0^{1-x} dy = \frac{1}{2}} \qquad (51)$$

$$-i(M_1 + M_2)_a = \frac{-2g^2 e\, i\pi^2}{(2\pi)^4 M_W \cos\theta_W} \eta^{\mu\nu} \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{\substack{i=d,s,b \\ e^-,\mu^-,\tau^-}} m_i^2 Q_i C_V^i N_i \cdot \Big\{ 1 +$$

$$-2\cancel{(P_1 \cdot P_2)} \frac{1}{\cancel{(P_1 \cdot P_2)}} (g(\lambda_i) - g(\tau_i)) - \frac{m_{h^0}^2}{(P_1 \cdot P_2)} (g(\lambda_i) - g(\tau_i)) + [2m_i^2 + (P_1 \cdot P_2)] \cdot \frac{1}{2(P_1 \cdot P_2)} \cdot$$

$$\cdot [f(\lambda_i) - f(\tau_i)] \Big\} \varepsilon_{2\nu}^* \varepsilon_{0\mu} (-1)$$

$$= \frac{-2g^2 e\, i\pi^2}{(2\pi)^4 M_W \cos\theta_W} \eta^{\mu\nu} \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{i=d,s,b,\ldots} m_i^2 Q_i C_V^i N_i \cdot \Big\{ 1 +$$

$$+ (g(\lambda_i) - g(\tau_i)) \left[-2 \overset{①}{-} \frac{2m_{h^0}^2}{M_{Z^0}^2 - m_{h^0}^2}\right] + \left[2m_i^2 + \frac{1}{2}(M_{Z^0}^2 - m_{h^0}^2)\right] \frac{1}{(M_{Z^0}^2 - m_{h^0}^2)} \cdot$$

$$\cdot [f(\lambda_i) - f(\tau_i)] \Big\} \varepsilon_{2\nu}^* \varepsilon_{0\mu} \cdot (-1)$$

$$[\overset{①}{}] = \left[-2 - \frac{2\left(\frac{4m_i^2}{\tau_i}\right)}{\left(\frac{4m_i^2}{\lambda_i}\right) - \left(\frac{4m_i^2}{\tau_i}\right)}\right] = -2\left[1 + \frac{\frac{1}{\tau_i}}{\frac{1}{\lambda_i} - \frac{1}{\tau_i}}\right] = -2\left[1 + \frac{\lambda_i}{\tau_i - \lambda_i}\right]$$

$$= \frac{-2\tau_i}{(\tau_i - \lambda_i)}$$

$$\left[2m_i^2 + \frac{1}{2}(M_{Z^0}^2 - m_{h^0}^2)\right]\frac{1}{(M_{Z^0}^2 - m_{h^0}^2)} = \left[2\cancel{m_i^2} + \frac{1}{2}\left(\frac{4\cancel{m_i^2}}{\lambda_i} - \frac{4\cancel{m_i^2}}{\tau_i}\right)\right]\cdot\frac{1}{\left(\frac{4\cancel{m_i^2}}{\lambda_i} - \frac{4\cancel{m_i^2}}{\tau_i}\right)}$$

$$= \left[2 + 2\left(\frac{1}{\lambda_i} - \frac{1}{\tau_i}\right)\right]\cdot\frac{1}{4\left(\frac{1}{\lambda_i} - \frac{1}{\tau_i}\right)}$$

$$= \frac{1}{2}\left[\frac{1}{\left(\frac{1}{\lambda_i} - \frac{1}{\tau_i}\right)} + 1\right]$$

$$= \frac{1}{2}\left[\frac{\lambda_i\tau_i}{(\tau_i - \lambda_i)} + 1\right]$$

$$-i(M_1+M_2)_a = \frac{-2g^2 e\, i\pi^2\eta^{\mu\nu}}{(2\pi)^4 M_W\cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\cdot\sum_{i=d,s,b,\ldots} m_i^2 Q_i C_V^i N_i\cdot\Bigg\{1 +$$

$$-\frac{2\tau_i}{(\tau_i-\lambda_i)}(g(\lambda_i)-g(\tau_i)) + \frac{1}{2}\left[\frac{\lambda_i\tau_i}{(\tau_i-\lambda_i)} + 1\right](f(\lambda_i) - f(\tau_i))\Bigg\}\varepsilon_{2\nu}^*\varepsilon_{0\mu}\cdot(-1)$$

$$= \frac{-2g^2 e\, i\pi^2\eta^{\mu\nu}}{(2\pi)^4 M_W\cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\sum_{i=d,s,b,\ldots} Q_i C_V^i N_i (M_{Z^0}^2 - m_{h^0}^2)\frac{m_i^2}{(M_{Z^0}^2 - m_{h^0}^2)}$$

$$\Big\{\qquad\Big\}\varepsilon_{2\nu}^*\varepsilon_{0\mu}\ (-1)$$

$$\frac{m_i^2}{(M_{Z^0}^2 - m_{h^0}^2)} = \frac{\cancel{m_i^2}}{\frac{4\cancel{m_i^2}}{\lambda_i} - \frac{4\cancel{m_i^2}}{\tau_i}} = \frac{1}{4}\frac{\lambda_i\tau_i}{(\tau_i-\lambda_i)}$$

$$-i(M_1+M_2)a = \frac{-\cancel{2}g^2 e\, i\pi^2\eta^{\mu\nu}}{\cancel{2}(2\pi)^4 M_W\cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)\cdot\sum_i Q_i C_V^i N_i (M_{Z^0}^2 - m_{h^0}^2)\Bigg\{-\frac{1}{2}\frac{\lambda_i\tau_i}{(\tau_i-\lambda_i)}$$

$$+ \frac{\lambda_i\tau_i^2}{(\tau_i-\lambda_i)^2}(g(\lambda_i)-g(\tau_i)) - \frac{1}{4}\frac{\lambda_i\tau_i}{(\tau_i-\lambda_i)}\left[1 + \frac{\lambda_i\tau_i}{(\tau_i-\lambda_i)}\right](f(\lambda_i)-f(\tau_i))\Bigg\}\varepsilon_{2\nu}^*\varepsilon_{0\mu}$$

$$-i(M_1+M_2)a = -\frac{g^2 e\, i\,\eta^{\mu\nu}}{16\pi^2 M_W\cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)(M_{Z^0}^2 - m_{h^0}^2)\sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i\Bigg\{-\frac{1}{2}\frac{\tau_i\lambda_i}{(\tau_i-\lambda_i)}$$

$$-\frac{\tau_i^2\lambda_i}{(\tau_i-\lambda_i)^2}(g(\tau_i)-g(\lambda_i)) + \frac{1}{4}\frac{\tau_i\lambda_i}{(\tau_i-\lambda_i)}\left[1 + \frac{\tau_i\lambda_i}{(\tau_i-\lambda_i)}\right](f(\tau_i)-f(\lambda_i))\Bigg\}\varepsilon_{2\nu}^*\varepsilon_{0\mu}$$

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(52)

$$-i(M_1+M_2)_b = \frac{-ig^2 e\, \eta^{\mu\nu}}{16\pi^2 M_W \cos\theta_W}\left(-\frac{\cos\alpha}{\sin\beta}\right)(M_{Z^0}^2 - m_{h^0}^2)\sum_{i=u,c,t} Q_i C_V^i N_i \left\{-\frac{1}{2}\frac{\tau_i\lambda_i}{(\tau_i-\lambda_i)}\right.$$

$$\left. -\frac{\tau_i^2\lambda_i}{(\tau_i-\lambda_i)^2}\left(g(\tau_i)-g(\lambda_i)\right) + \frac{1}{4}\frac{\tau_i\lambda_i}{(\tau_i-\lambda_i)}\left[1+\frac{\tau_i\lambda_i}{(\tau_i-\lambda_i)}\right]\left(f(\tau_i)-f(\lambda_i)\right)\right\}\varepsilon_{2\nu}^{*}\varepsilon_{0\mu}$$

(53)

Defining:

$$F(\tau_i,\lambda_i) \equiv -\frac{1}{2}\frac{\tau_i\lambda_i}{(\tau_i-\lambda_i)} - \frac{\tau_i^2\lambda_i}{(\tau_i-\lambda_i)^2}\left(g(\tau_i)-g(\lambda_i)\right) + \frac{1}{4}\frac{\tau_i\lambda_i}{(\tau_i-\lambda_i)}\cdot$$

$$\cdot\left[1+\frac{\tau_i\lambda_i}{(\tau_i-\lambda_i)}\right]\left(f(\tau_i)-f(\lambda_i)\right) \qquad (54)$$

$\Longrightarrow$

$$(M_1+M_2)_a + (M_1+M_2)_b = \frac{g^2 e\,\eta^{\mu\nu}}{16\pi^2 M_W\cos\theta_W}\left(\frac{\sin\alpha}{\cos\beta}\right)(M_{Z^0}^2-m_{h^0}^2)\sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i F(\tau_i,\lambda_i)\cdot$$

$$\cdot\,\varepsilon_{2\nu}^{*}\varepsilon_{0\mu} + \frac{g^2 e\,\eta^{\mu\nu}}{16\pi^2 M_W\cos\theta_W}\left(-\frac{\cos\alpha}{\sin\beta}\right)(M_{Z^0}^2-m_{h^0}^2)\sum_{i=u,c,t} Q_i C_V^i N_i F(\tau_i,\lambda_i)\cdot$$

$$\cdot\,\varepsilon_{2\nu}^{*}\varepsilon_{0\mu}$$

$$\therefore\ \boxed{\begin{aligned}(M_1+M_2)_a + (M_1+M_2)_b &= \frac{g\, e^2\,\eta^{\mu\nu}\varepsilon_{0\mu}\varepsilon_{2\nu}^{*}}{16\pi^2 M_W\cos\theta_W\sin\theta_W}(M_{Z^0}^2-m_{h^0}^2)\left[\left(\frac{\sin\alpha}{\cos\beta}\right)\cdot\right.\\ &\left.\cdot\sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i F(\tau_i,\lambda_i) + \left(-\frac{\cos\alpha}{\sin\beta}\right)\sum_{i=u,c,t} Q_i C_V^i N_i F(\tau_i,\lambda_i)\right]\end{aligned}} \qquad (55)$$

Setting $M_{12ab} = (M_1+M_2)_a + (M_1+M_2)_b$

Considering only fermion contribution we have:

$$\overline{|M|^2} = \frac{g^2 e^4\,(M_{Z^0}^2-m_{h^0}^2)^2}{\underset{\downarrow \text{spin } Z^0 = 1}{3}\times 16^2\pi^4 M_W^2\sin^2\theta_W\cos^2\theta_W}\sum_{\lambda}\varepsilon_{0\mu}\varepsilon_{0\rho}^{*}\sum_{\lambda'}\varepsilon_{2\nu}^{*}\varepsilon_{2\sigma}\,\eta^{\mu\nu}\eta^{\rho\sigma}\left|\left(\frac{\sin\alpha}{\cos\beta}\right)\cdot\right.$$

$$\left.\cdot\sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i F(\tau_i,\lambda_i) + \left(-\frac{\cos\alpha}{\sin\beta}\right)\sum_{i=u,c,t} Q_i C_V^i N_i F(\tau_i,\lambda_i)\right|^2$$

$$\sum_{\lambda} \epsilon_{0\mu}\, \epsilon^{*}_{0\rho} \cdot \sum_{\lambda} \epsilon^{*}_{\gamma\nu}\, \epsilon_{\gamma\sigma}\ \eta^{\mu\nu}\eta^{\rho\sigma}$$

$$= \left(-\eta_{\mu\rho} + \frac{p_{0\mu}p_{0\rho}}{M_{Z^0}^2}\right)\left(-\eta_{\nu\sigma}\right)\eta^{\mu\nu}\eta^{\rho\sigma}$$

$$= \left(-\eta_{\mu\rho} + \frac{p_{0\mu}p_{0\rho}}{M_{Z^0}^2}\right)\left(-\delta^{\mu}_{\sigma}\right)\eta^{\rho\sigma}$$

$$= \left(-\eta_{\mu\rho} + \frac{p_{0\mu}p_{0\rho}}{M_{Z^0}^2}\right)\left(-\eta^{\mu\rho}\right)$$

$$= 4 - \frac{p_{0\mu}p_0^{\mu}}{M_{Z^0}^2} = 4 - \frac{M_{Z^0}^2}{M_{Z^0}^2} = 3 .$$

$$\Rightarrow \boxed{\begin{aligned} \overline{|M|^2} = \frac{g^2 e^4 (M_{Z^0}^2 - m_{h^0}^2)^2}{16^2\pi^4 M_W^2 \sin^2\theta_W \cos^2\theta_W} \Bigg| \left(\frac{\sin\alpha}{\cos\beta}\right) \cdot \sum_{\substack{i=d,s,b\\ e,\mu,\tau}} Q_i C_V^i N_i F(T_i,\lambda_i) + \left(-\frac{\cos\alpha}{\sin\beta}\right) \cdot \\ \cdot \sum_{i=u,c,t} Q_i C_V^i N_i F(T_i,\lambda_i) \Bigg|^2 \end{aligned}}$$

(56)

$$d\Gamma = \frac{\overline{|M|^2}\,|\vec{p}_1|\,d\Omega}{32\pi^2 M_{Z^0}^2} \qquad (57)$$

$$p_{Z^0} = (M_{Z^0}, \vec{0})\ ; \quad p_1 = (E_1, \vec{p}_1)\ ; \quad p_2 = (|\vec{p}_1|, -\vec{p}_1)$$

$$M_{Z^0} = E_1 + |\vec{p}_1| \quad \Rightarrow E_1 = M_{Z^0} - |\vec{p}_1|$$

$$E_1^2 = m_{h^0}^2 + |\vec{p}_1|^2$$

$$\Rightarrow (M_{Z^0} - |\vec{p}_1|)^2 = m_{h^0}^2 + |\vec{p}_1|^2$$

$$M_{Z^0}^2 - 2M_{Z^0}|\vec{p}_1| + \cancel{|\vec{p}_1|^2} = m_{h^0}^2 + \cancel{|\vec{p}_1|^2}$$

$$|\vec{p}_1| = \frac{M_{Z^0}^2 - m_{h^0}^2}{2M_{Z^0}} \qquad (58)$$

$$\therefore\ \Gamma = \frac{\cancel{g^2}\, \cancel{e^4} (M_{Z^0}^2 - m_{h^0}^2)^2}{\cancel{16^2}\cancel{\pi^4} M_{\cancel{W}}^2 \sin^2\theta_W \cos^2\theta_W}\ \frac{(M_{Z^0}^2 - m_{h^0}^2)}{2\cancel{M_{Z^0}}}\ \frac{\cancel{4\pi}}{\underset{8\pi}{\cancel{32\pi^2}} M_{Z^0}^2} \Big|\quad\Big|^2$$

$$\Gamma = \frac{G_F\, \alpha^2\, \overset{3}{\cancel{M_{Z^0}^4}} \left(1 - \frac{m_{h^0}^2}{M_{Z^0}^2}\right)^3 \cancel{M_{Z^0}^2}}{32\sqrt{2}\,\pi^3\, \cancel{M_{Z^0}} \sin^2\theta_W \cos^2\theta_W\, \cancel{M_{Z^0}^2}} \Big|\ \Big|^2$$

$$\boxed{\Gamma(Z^0 \xrightarrow[f]{} h^0\gamma) = \frac{\sqrt{2}\, G_F\, \alpha^2 M_{Z^0}^3 \left(1-\frac{m_{h^0}^2}{M_{Z^0}^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \left| \left(\frac{\sin\alpha}{\cos\beta}\right) \cdot \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i F(\tau_i,\lambda_i) + \left(-\frac{\cos\alpha}{\sin\beta}\right) \cdot \sum_{i=u,c,t} Q_i C_V^i N_i F(\tau_i,\lambda_i) \right|^2} \qquad (59)$$

where

$$\boxed{F(\tau_i,\lambda_i) \equiv -\frac{1}{2}\frac{\tau_i\lambda_i}{(\tau_i-\lambda_i)} - \frac{\tau_i^2\lambda_i}{(\tau_i-\lambda_i)^2}\left(g(\tau_i)-g(\lambda_i)\right) + \frac{1}{4}\frac{\tau_i\lambda_i}{(\tau_i-\lambda_i)}\left[1+\frac{\tau_i\lambda_i}{(\tau_i-\lambda_i)}\right] \cdot \left(f(\tau_i)-f(\lambda_i)\right)} \qquad (60)$$

$$\boxed{g(x) = \begin{cases} (x-1)^{1/2}\, \overset{\text{arc sin}}{\sin^{-1}}\left(\frac{1}{x^{1/2}}\right) & \text{if } x \geq 1 \\ \frac{1}{2}(1-x)^{1/2}\left[\ln\left(\frac{1+(1-x)^{1/2}}{1-(1-x)^{1/2}}\right) - i\pi\right] & \text{if } x < 1 \end{cases}} \qquad (60a)$$

$$\boxed{f(x) = \begin{cases} -2\left(\arcsin\left(\frac{1}{x^{1/2}}\right)\right)^2 & \text{if } x \geq 1 \\ \frac{1}{2}\left[\ln\left(\frac{1+(1-x)^{1/2}}{1-(1-x)^{1/2}}\right) - i\pi\right]^2 & \text{if } x < 1 \end{cases}} \qquad (60b)$$

$$\boxed{\tau_i \equiv \frac{4m_i^2}{m_{h^0}^2} \; ; \; \lambda_i \equiv \frac{4m_i^2}{M_{Z^0}^2}} \qquad (60c)$$

if $x \ll 1$

$$\ln\left(\frac{1+(1-x)^{1/2}}{1-(1-x)^{1/2}}\right) \simeq \ln\left(\frac{1+1-\frac{1}{2}x}{1-(1-\frac{1}{2}x)}\right) = \ln\left(\frac{2-\frac{1}{2}x}{\frac{1}{2}x}\right)$$

$$\approx \ln\left(\frac{4}{x}\right)$$

$$\Rightarrow f(x) \approx \frac{1}{2}\left[\ln 4 - \ln x - i\pi\right]^2$$

$$\boxed{f(x) \approx \frac{1}{2}\left[\ln\left(\frac{x}{4}\right) + i\pi\right]^2 \quad \text{if } x \ll 1.} \qquad (61)$$

$$\boxed{g(x) \approx \frac{1}{2}\left[\ln\left(\frac{4}{x}\right) - i\pi\right] = -\frac{1}{2}\left[\ln\left(\frac{x}{4}\right) + i\pi\right] \quad \text{if } x \ll 1} \qquad (62)$$

$$C_V^i = T_i^3 - 2\sin^2\theta_W Q_i ;$$

i	Q_i	T_i^3	C_V^i
u, c, t	$+2/3$	$+1/2$	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_W$
d, s, b	$-\frac{1}{3}$	$-1/2$	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$
e^-, μ^-, τ^-	-1	$-1/2$	$-\frac{1}{2} + 2\sin^2\theta_W$

$$\Gamma(Z^0 \xrightarrow[F]{} h^0\gamma) = \frac{\sqrt{2}\, G_F \alpha^2 M_{Z^0}^3 \left(1 - \frac{m_{h^0}^2}{M_{Z^0}^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \frac{(1-\sin^2\alpha)}{\sin^2\beta} \Big| \tan\alpha \tan\beta \cdot \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i F(\tau_i,\lambda_i)$$

$$- \sum_{i=u,c,t} Q_i C_V^i N_i F(\tau_i,\lambda_i) \Big|^2 \qquad (63)$$

$$= \frac{\sqrt{2}\, G_F \alpha^2 M_{Z^0}^3 \left(1 - \frac{m_{h^0}^2}{M_{Z^0}^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \frac{(1-\sin^2\alpha)(1+\tan^2\beta)}{\tan^2\beta} \Big| \tan\alpha \tan\beta \,\cdot$$

$$\cdot \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i F(\tau_i,\lambda_i) - \sum_{i=u,c,t} Q_i C_V^i N_i F(\tau_i,\lambda_i) \Big|^2$$

$$\Rightarrow \boxed{\begin{aligned} \Gamma(Z^0 \xrightarrow[F]{} h^0\gamma) = {} & \frac{\sqrt{2}\, G_F \alpha^2 M_{Z^0}^3 \left(1 - \frac{m_{h^0}^2}{M_{Z^0}^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \frac{(1-\sin^2\alpha)(1+\tan^2\beta)}{\tan^2\beta} \Big| \tan\alpha \cdot \tan\beta \,\cdot \\ & \left(\left(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W\right) F(\tau_b,\lambda_b) + \left(-\frac{1}{2} + 2\sin^2\theta_W\right) F(\tau_\tau,\lambda_\tau) \right) \\ & + 2\left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_W\right) F(\tau_t,\lambda_t) \Big|^2 \end{aligned}} \quad (64)$$

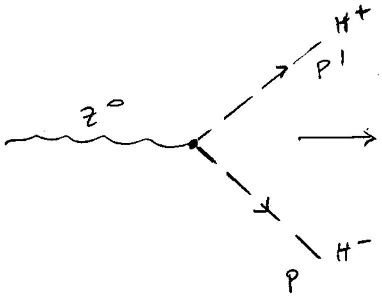
where : $F(\tau_i,\lambda_i)$ is defined in (60), (60 a), (60 b), (60 c) and :

$$\sin^2\alpha = \frac{1}{2}\left\{ 1 + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)} \right] \right\}$$

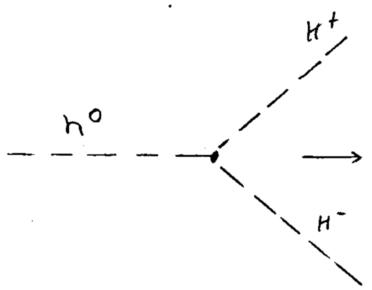
$$g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta) = \left[\left(1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2} \right)^2 - 4\left(\frac{M_Z^2}{M_{H^\pm}^2}\right)\left(1 - \frac{M_W^2}{M_{H^\pm}^2}\right)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2 \right]^{1/2}$$

$$\tan\alpha = \left\{ \frac{1 + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)} \right]}{1 - \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)} \right]} \right\}^{1/2}$$

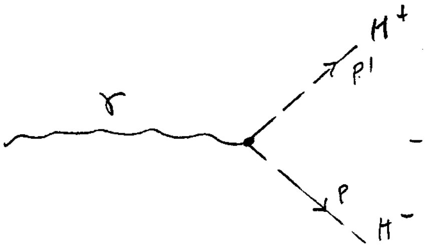
⑤ and ⑥ ($H^\pm$ contribution)



$$\longrightarrow \; \frac{-ig\cos 2\theta_W}{2\cos\theta_W}\,(-p+p')^\mu \qquad (65)$$



$$\longrightarrow \; -ig\left[M_W \sin(\beta-\alpha) + \frac{M_Z}{2\cos\theta_W}\cos 2\beta \sin(\beta+\alpha)\right] \qquad (66)$$



$$-ie\,(-p+p')^\mu \qquad (67)$$

$Z^0 H^+ H^-$:

$$3\left(\frac{d}{2}-1\right) + x + 1 = d$$

$$x = d - 1 - \frac{3d}{2} + 3 = 2 - \frac{d}{2}$$

$h^0 H^+ H^-$:

$$3\left(\frac{d}{2}-1\right) + x + 1 = d \;\Rightarrow\; x = 2 - \frac{d}{2}$$

$\gamma H^+ H^-$: $\quad x = 2 - \frac{d}{2}$

$$-iM_5 = -ig\left[M_W \sin(\beta-\alpha) + \frac{M_Z}{2\cos\theta_W}\cos 2\beta \sin(\beta+\alpha)\right]\int \frac{d^dk}{(2\pi)^d}\,\frac{i}{(k^2 - m_{H^\pm}^2)}\,(-ie)(2k-p_2)^\nu$$

$$\frac{i}{((k-p_2)^2 - m_{H^\pm}^2)}\left(-\frac{ig\cos 2\theta_W}{2\cos\theta_W}\right)(2k+p_1-p_2)^\mu\,\frac{i}{((k+p_1)^2 - m_{H^\pm}^2)}\,\varepsilon^*_{Z\nu}\,\varepsilon_{0\mu}\,\underbrace{\underset{\substack{\downarrow\\ h^0}}{\mu_1}^{\frac{4-d}{2}}\cdot\underset{\substack{\downarrow\\ \gamma}}{\mu_2}^{\frac{4-d}{2}}}_{\mu^*}\cdot$$

$$\cdot\;\underset{\substack{\downarrow\\ Z^0\\ \checkmark}}{\mu_3}^{\frac{4-d}{2}}$$

(68)

$$-iM_5 = g^2 e\left[M_W \sin(\beta-\alpha) + \frac{M_Z}{2\cos\theta_W}\cos 2\beta \sin(\beta+\alpha)\right]\frac{\cos 2\theta_W}{2\cos\theta_W}\int\frac{d^d\kappa}{(2\pi)^d}(2\kappa-P_2)^\nu(2\kappa+P_1-P_2)^\mu\cdot$$

$$\cdot\frac{1}{(\kappa^2-m_{H^\pm}^2)((\kappa-P_2)^2-m_{H^\pm}^2)((\kappa+P_1)^2-m_{H^\pm}^2)}\ \varepsilon^*_{2\nu}\,\varepsilon_{0\mu}\ \mu^\epsilon$$

$$\downarrow$$

$\mu^\epsilon \to 1$ when $d\to 4$

$$-iM_5 = g^2 e M_W\left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W}\right]\frac{(1-2\sin^2\theta_W)}{2\cos\theta_W}\cdot\int\frac{d^d\kappa}{(2\pi)^d}\frac{(4\kappa^\mu\kappa^\nu + 2\kappa^\nu P_1^\mu - 2\kappa^\nu P_2^\mu - 2\kappa^\mu P_2^\nu - P_1^\mu P_2^\nu + P_2^\mu P_2^\nu)\ \varepsilon^*_{2\nu}\varepsilon_{0\mu}}{(\kappa^2-m_{H^\pm}^2)[(\kappa-P_2)^2-m_{H^\pm}^2][(\kappa+P_1)^2-m_{H^\pm}^2]} \quad (69)$$

$$-iM_6 = \int\frac{d^d\kappa}{(2\pi)^d}(-ie)(2\kappa+P_2)^\nu\frac{i}{[\kappa^2-m_{H^\pm}^2]}(-ig)\left[M_W\sin(\beta-\alpha)+\frac{M_Z}{2\cos\theta_W}\cos 2\beta\sin(\beta+\alpha)\right]\cdot$$

$$\cdot\frac{i}{[(\kappa-P_1)^2-m_{H^\pm}^2]}\left(\frac{-ig\cos 2\theta_W}{2\cos\theta_W}\right)(2\kappa+P_2-P_1)^\mu\frac{i}{[(\kappa+P_2)^2-m_{H^\pm}^2]}\ \varepsilon^*_{2\nu}\varepsilon_{0\mu}\ \mu^\epsilon \quad (70)$$

$$-iM_6 = g^2 e\left[M_W\sin(\beta-\alpha)+\frac{M_Z}{2\cos\theta_W}\cos 2\beta\sin(\beta+\alpha)\right]\frac{\cos 2\theta_W}{2\cos\theta_W}\int\frac{d^d\kappa}{(2\pi)^d}(2\kappa+P_2)^\nu(2\kappa+P_2-P_1)^\mu\cdot$$

$$\frac{1}{[\kappa^2-m_{H^\pm}^2][(\kappa-P_1)^2-m_{H^\pm}^2][(\kappa+P_2)^2-m_{H^\pm}^2]}\ \varepsilon^*_{2\nu}\varepsilon_{0\mu}$$

$$-iM_6 = g^2 e M_W\left[\sin(\beta-\alpha)+\frac{\cos 2\beta\sin(\beta+\alpha)}{2\cos^2\theta_W}\right]\frac{(1-2\sin^2\theta_W)}{2\cos\theta_W}\int\frac{d^d\kappa}{(2\pi)^d}[4\kappa^\mu\kappa^\nu + 2\kappa^\nu P_2^\mu$$

$$-2\kappa^\nu P_1^\mu + 2\kappa^\mu P_2^\nu + P_2^\mu P_2^\nu - P_1^\mu P_2^\nu]\cdot\frac{1}{[\kappa^2-m_{H^\pm}^2][(\kappa-P_1)^2-m_{H^\pm}^2][(\kappa+P_2)^2-m_{H^\pm}^2]}\ \varepsilon^*_{2\nu}\varepsilon_{0\mu} \quad (71)$$

For iM_5:

$$\frac{1}{abc} = 2\int_0^1 dx\int_0^{1-x}\frac{dy}{[a(1-x-y)+bx+cy]^3}$$

$$a = \kappa^2 - m_{H^\pm}^2\ ;\ b = (\kappa+P_1)^2 - m_{H^\pm}^2\ ;\ c = (\kappa-P_2)^2 - m_{H^\pm}^2$$

$$a(1-x-y)+bx+cy = (\kappa^2-m_{H^\pm}^2)(1-x-y) + [(\kappa+P_1)^2-m_{H^\pm}^2]x + [(\kappa-P_2)^2-m_{H^\pm}^2]y$$

$$= \cancel{\kappa^2} - \cancel{\kappa^2 x} - \cancel{\kappa^2 y} - \cancel{m_{H^\pm}^2} + \cancel{x m_{H^\pm}^2} + \cancel{y m_{H^\pm}^2} + \cancel{\kappa^2 x} + 2\cancel{(\kappa\cdot P_1)x} + \cancel{m_{h^0}^2 x} - \cancel{m_{H^\pm}^2 x}$$

$$+ \cancel{\kappa^2 y} - 2\cancel{(\kappa\cdot P_2)y} - \cancel{m_{H^\pm}^2 y} = \kappa^2 - m_{H^\pm}^2 + m_{h^0}^2 x + 2\kappa\cdot(P_1 x - P_2 y)$$

$$\frac{1}{[\kappa^2-m_{H^\pm}^2][(\kappa+P_1)^2-m_{H^\pm}^2][(\kappa-P_2)^2-m_{H^\pm}^2]} = 2\int_0^1 dx\int_0^{1-x}\frac{dy}{[\kappa^2-m_{H^\pm}^2+m_{h^0}^2 x+2\kappa\cdot(P_1x-P_2y)]^3} \quad (72)$$

$$-iM_S = g^2 e M_W \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W} \right] \frac{(1-2\sin^2\theta_W)}{\cancel{2}\cos\theta_W} \cancel{2} \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d K}{(2\pi)^d} \cdot$$

$$\frac{(4K^\mu K^\nu + 2K^\nu P_1^\mu - 2K^\nu P_2^\mu - 2K^\mu P_2^\nu - P_1^\mu P_2^\nu + P_2^\mu P_2^\nu)}{[K^2 - m_{H^\pm}^2 + m_{h^0}^2 x + 2K\cdot(P_1 x - P_2 y)]^3} \varepsilon_{2\nu}^* \varepsilon_{0\mu} \quad (73)$$

$$P_{2\nu} \varepsilon_2^{\nu *} = 0 \quad \Rightarrow$$

$$-iM_S = g^2 e M_W \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W} \right] \frac{(1-2\sin^2\theta_W)}{\cos\theta_W} \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d K}{(2\pi)^d} \cdot$$

$$\frac{(4K^\mu K^\nu + 2K^\nu P_1^\mu - 2K^\nu P_2^\mu)\, \varepsilon_{2\nu}^* \varepsilon_{0\mu}}{[K^2 - m_{H^\pm}^2 + m_{h^0}^2 x + 2K\cdot(P_1 x - P_2 y)]^3} \quad (74)$$

$$K' = K + (P_1 x - P_2 y)$$

$$K'^2 = K^2 + 2K\cdot(P_1 x - P_2 y) + m_{h^0}^2 x^2 - 2(P_1\cdot P_2) x y$$

$$\Rightarrow K^2 + 2K\cdot(P_1 x - P_2 y) = K'^2 - m_{h^0}^2 x^2 + 2(P_1\cdot P_2) x y$$

$$d^d K = d^d K'$$

$$-iM_S = g^2 e M_W \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W} \right] \frac{(1-2\sin^2\theta_W)}{\cos\theta_W} \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d K'}{(2\pi)^d} \cdot$$

$$\left[4(K' - (P_1 x - P_2 y))^\mu (K' - (P_1 x - P_2 y))^\nu + 2(K' - (P_1 x - P_2 y))^\nu P_1^\mu - 2(K' - (P_1 x - P_2 y))^\nu P_2^\mu \right]$$

$$\cdot \frac{1}{[K'^2 - m_{H^\pm}^2 + m_{h^0}^2 x - m_{h^0}^2 x^2 + 2(P_1\cdot P_2) x y]^3} \cdot \varepsilon_{2\nu}^* \varepsilon_{0\mu} \quad (75)$$

$$-iM_S = 2g^2 e M_W \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W} \right] \frac{(1-2\sin^2\theta_W)}{\cos\theta_W} \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d K'}{(2\pi)^d} \cdot$$

$$\left[\cancel{2K'^\mu K'^\nu} - 2K'^\mu (\cancel{P_1 x} - P_2 y)^\nu - 2K'^\nu (\cancel{P_1 x} - P_2 y)^\mu + 2(P_1 x - P_2 y)^\mu (\cancel{P_1 x} - P_2 y)^\nu + \right.$$

$$\left. + K'^\nu \cancel{P_1^\mu} - (P_1 x - \cancel{P_2 y})^\nu P_1^\mu - K'^\nu \cancel{P_2^\mu} + (P_1 x - \cancel{P_2 y})^\nu P_2^\mu \right] \cdot \varepsilon_{2\nu}^* \varepsilon_{0\mu} \cdot$$

$$\cdot \frac{1}{[K'^2 - m_{H^\pm}^2 + m_{h^0}^2 x - m_{h^0}^2 x^2 + 2(P_1\cdot P_2) x y]^3}$$

$$P_1^\nu \varepsilon_{2\nu}^* = 0 \quad \text{and} \quad P_2^\nu \varepsilon_{2\nu}^* = 0 \quad \Rightarrow$$

$$-iM_5 = 2g^2 e M_W \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W} \right] \left(\frac{1-2\sin^2\theta_W}{\cos\theta_W} \right) \int_0^1 dX \int_0^{1-X} dY \int \frac{d^d K'}{(2\pi)^d} \cdot$$

$$\left[2K'^\mu K'^\nu - 2K'^\nu (P_1 X - P_2 Y)^\mu + K'^\nu P_1^\mu - K'^\nu P_2^\mu \right] \varepsilon_{2\nu}^* \varepsilon_{0\mu} \cdot$$

$$\cdot \frac{1}{[K'^2 - m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1 \cdot P_2) X Y]^3} \qquad (76)$$

$$\int \frac{d^d K' \; K'^\nu}{[K'^2 - m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1 \cdot P_2) X Y]^3} = 0$$

$$\int \frac{d^d K' \; K'^\mu K'^\nu}{[K'^2 - m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1 \cdot P_2) X Y]^3} = \frac{i(-\pi)^{d/2}}{2} \frac{\eta^{\mu\nu}}{2} \left(-m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + \right.$$

$$\left. + 2(P_1 \cdot P_2) X Y \right)^{-2+\frac{d}{2}} \Gamma\left(2 - \frac{d}{2}\right) = I_0 \frac{\eta^{\mu\nu}}{2} (-m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1 \cdot P_2) X Y) \frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})}$$

$$\boxed{\begin{aligned} -iM_5 = {} & \frac{4g^2 e M_W}{(2\pi)^4} \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W} \right] \frac{(1-2\sin^2\theta_W)}{\cos\theta_W} \int_0^1 dX \int_0^{1-X} dY \\ & \frac{i(-\pi)^{d/2}}{4} \eta^{\mu\nu} (-m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1 \cdot P_2) X Y)^{-2+\frac{d}{2}} \Gamma\left(2 - \frac{d}{2}\right) \varepsilon_{2\nu}^* \varepsilon_{0\mu} \end{aligned}}$$

$$(77)$$

$$\frac{1}{[K^2 - m_{H^\pm}^2][(K-P_1)^2 - m_{H^\pm}^2][(K+P_2)^2 - m_{H^\pm}^2]} = 2\int_0^1 dX \int_0^{1-X} \frac{dY}{[K^2 - m_{H^\pm}^2 + m_{h^0}^2 X - 2K \cdot (P_1 X - P_2 Y)]^3}$$

$$(78)$$

$$-iM_6 = g^2 e M_W \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W} \right] \frac{(1-2\sin^2\theta_W)}{\cancel{2}\cos\theta_W} \cancel{2} \int_0^1 dX \int_0^{1-X} dY \int \frac{d^d K}{(2\pi)^d} \cdot$$

$$\cdot \left[4K^\mu K^\nu + 2K^\nu P_2^\mu - 2K^\nu P_1^\mu \right] \frac{1}{[K^2 - m_{H^\pm}^2 + m_{h^0}^2 X - 2K \cdot (P_1 X - P_2 Y)]^3} \varepsilon_{2\nu}^* \varepsilon_{0\mu} \qquad (79)$$

$$K' = K - (P_1 X - P_2 Y)$$

$$K'^2 = K^2 - 2K \cdot (P_1 X - P_2 Y) + m_{h^0}^2 X^2 - 2(P_1 \cdot P_2) X Y$$

$$d^d K = d^d K'$$

$$-iM_6 = 2g^2 e M_W \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W} \right] \frac{(1-2\sin^2\theta_W)}{\cos\theta_W} \int_0^1 dx \int_0^{1-x} dY \int \frac{d^dK'}{(2\pi)^d} \cdot$$

$$\left[2\left(K' + (P_1X - P_2Y)\right)^\mu \left(K' + (P_1X - P_2Y)\right)^\nu + \left(K' + (P_1X-P_2Y)\right)^\nu P_2^\mu - \left(K' + (P_1X-P_2Y)\right)^\nu P_1^\mu \right].$$

$$\cdot \frac{1}{\left[K'^2 - m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right]^3} \varepsilon^*_{2\nu} \varepsilon_{0\mu} \qquad (80)$$

$$-iM_6 = 2g^2 e M_W \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W} \right] \frac{(1-2\sin^2\theta_W)}{\cos\theta_W} \int_0^1 dx \int_0^{1-x} dY \int \frac{d^dK'}{(2\pi)^d} \cdot$$

$$\cdot \left[2K'^\mu \cancel{K'^\nu} + 2K'^\mu (P_1X - \cancel{P_2Y)^\nu} + 2K'^\nu (P_1\cancel{X} - P_2Y)^\mu + 2(P_1X - P_2Y)^\mu (P_1X - \cancel{P_2Y)^\nu} \right.$$

$$\left. + K'^\nu P_2^\mu + (P_1X - \cancel{P_2Y)^\nu} P_2^\mu - K'^\nu P_1^\mu - (P_1X - \cancel{P_2Y)^\nu} P_1^\mu \right] \cdot \frac{1}{\left[K'^2 - m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right]^3} \cdot$$

$$\cdot\, \varepsilon^*_{2\nu} \varepsilon_{0\mu} \qquad (81)$$

$$-iM_6 = 2g^2 e M_W \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W} \right] \frac{(1-2\sin^2\theta_W)}{\cos\theta_W} \int_0^1 dx \int_0^{1-x} dY \int \frac{d^dK'}{(2\pi)^d} \cdot$$

$$\left[2K'^\mu K'^\nu + 2K'^\nu (P_1X - P_2Y)^\mu + K'^\nu P_2^\mu - K'^\nu P_1^\mu \right] \cdot \frac{1}{\left[K'^2 - m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right]^3}$$

$$\cdot\, \varepsilon^*_{2\nu} \varepsilon_{0\mu}$$

$$\boxed{\begin{aligned} -iM_6 = {} & \frac{4g^2 e M_W}{(2\pi)^4} \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W} \right] \frac{(1-2\sin^2\theta_W)}{\cos\theta_W} \int_0^1 dx \int_0^{1-x} dY \,. \\ & \cdot \frac{i(-\pi)^{d/2} \eta^{\mu\nu}}{4} \left(-m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right)^{-2+\frac{d}{2}} \Gamma\left(2 - \frac{d}{2}\right) \varepsilon^*_{2\nu} \varepsilon_{0\mu} \end{aligned}}$$

(82)

$$\int \frac{d^dK'}{\left[K'^2 - m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right]^3} = I_0(X,Y) = \frac{i(-\pi)^{d/2}\,\Gamma\left(3 - \frac{d}{2}\right)}{2\left[-m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right]^{3-\frac{d}{2}}}$$

$$\boxed{\begin{aligned} -i(M_5 + M_6) = {} & \frac{4g^2 e M_W}{(2\pi)^4} \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W} \right] \frac{(1-2\sin^2\theta_W)}{\cos\theta_W} \int_0^1 dx \int_0^{1-x} dY \; \eta^{\mu\nu} \\ & \left(-m_{H^\pm}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right) I_0(X,Y) \frac{\Gamma(2 - d/2)}{\Gamma(3-d/2)} \varepsilon^*_{2\nu} \varepsilon_{0\mu} \end{aligned}} \qquad (83)$$

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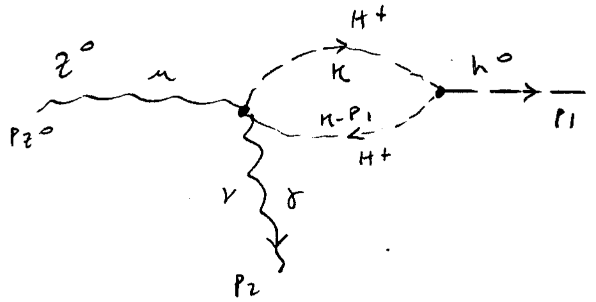
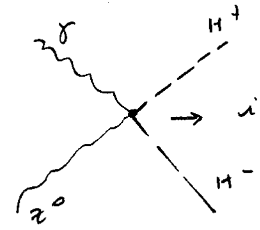




$\rightarrow\ ieg \frac{\cos 2\theta_W}{\cos\theta_W} \eta^{\mu\nu}$

$$-iM_8 = -ig\left[M_W \sin(\beta-\alpha) + \frac{M_Z}{2\cos\theta_W}\cos 2\beta \sin(\beta+\alpha)\right]\int \frac{d^dk}{(2\pi)^d}\frac{i}{[(k-p_1)^2 - m_{H^\pm}^2]}\cdot$$

$$ieg\frac{\cos 2\theta_W}{\cos\theta_W}\eta^{\mu\nu}\frac{i}{(k^2-m_{H^\pm}^2)}\varepsilon^*_{2\nu}\varepsilon_{0\mu}\ \underset{\substack{\downarrow \\ h^0H^+H^-}}{\mu_1^{\frac{4-d}{2}}}\ \underset{\substack{\downarrow \\ Z^0\gamma H^+H^-}}{\mu_2^{(4-d)}} \qquad (84)$$

μ'^{ε}

$(\mu'^{\varepsilon} \rightarrow 1$ when $d \rightarrow 4)$

$Z^0\gamma H^+H^-$

$$4\left(\frac{d}{2}-1\right) + X = d$$

$$X = d - 2d + 4$$

$$X = 4 - d$$

$$-iM_8 = -g^2 e M_W\left[\sin(\beta-\alpha) + \frac{M_W}{2\cos^2\theta_W}\cos 2\beta \sin(\beta+\alpha)\right]\frac{(1-2\sin^2\theta_W)}{\cos\theta_W}\cdot$$

$$\int\frac{d^dk}{(2\pi)^d}\eta^{\mu\nu}\frac{[(k+p_2)^2 - m_{H^\pm}^2]}{[k^2-m_{H^\pm}^2][(k-p_1)^2-m_{H^\pm}^2][(k+p_2)^2-m_{H^\pm}^2]}\varepsilon^*_{2\nu}\varepsilon_{0\mu} \qquad (85)$$

$$= -2g^2 e M_W\left[\sin(\beta-\alpha) + \frac{1}{2\cos^2\theta_W}\cos 2\beta \sin(\beta+\alpha)\right]\frac{(1-2\sin^2\theta_W)}{\cos\theta_W}\cdot$$

$$\cdot\int_0^1 dx\int_0^{1-x} dy\int\frac{d^dk}{(2\pi)^d}\eta^{\mu\nu}\frac{[(k+p_2)^2-m_{H^\pm}^2]}{[k^2-m_{H^\pm}^2+m_{h^0}^2x-2k\cdot(p_1x-p_2y)]^3}\varepsilon^*_{2\nu}\varepsilon_{0\mu} \qquad (86)$$

$d^dk = d^dk'$; $k' = k - (p_1x - p_2y)$

$$k^2 - 2k\cdot(p_1x-p_2y) = k'^2 - m_{h^0}^2x^2 + 2(p_1\cdot p_2)xy$$

$$(k+p_2)^2 - m_{H^\pm}^2 = (k' + p_1x - p_2y + p_2)^2 - m_{H^\pm}^2 = (k'+p_1x+p_2(1-y))^2 - m_{H^\pm}^2$$

$$= K'^2 + m_{h^0}^2 x^2 + 2x\,(K'\cdot P_1) + 2(1-y)(K'\cdot P_2) + 2x(1-y)(P_1\cdot P_2) - m_{H^\pm}^2$$

$$\Rightarrow\ -iM_8 = -2g^2 e M_W \left[\operatorname{Sin}(\beta-\alpha) + \frac{1}{2\cos^2\theta_W}\cos 2\beta\, \operatorname{Sin}(\beta+\alpha)\right] \frac{(1-2\operatorname{Sin}^2\theta_W)}{\cos\theta_W}\,\cdot$$

$$\cdot \int_0^1 dx \int_0^{1-x} dy \int \frac{d^dK'}{(2\pi)^d}\ \eta^{\mu\nu} \left[K'^2 + m_{h^0}^2 x^2 + 2x\,(K'\cdot P_1) + 2(1-y)(K'\cdot P_2) + \right.$$

$$\left. + 2x(1-y)\, P_1\cdot P_2 - m_{H^\pm}^2 \right] \cdot \frac{1}{[K'^2 - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy - m_{H^\pm}^2 + m_{h^0}^2 x]^3} \cdot \varepsilon_{2\nu}^{*} \varepsilon_{0\mu} \qquad (87)$$

$$\int \frac{d^dK'\ K'^2}{[K'^2 - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy - m_{H^\pm}^2 + m_{h^0}^2 x]^3} = I_0\, \frac{d}{2}\,(-m_{H^\pm}^2 + m_{h^0}^2 x - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy)\,\cdot$$

$$\cdot\ \frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})} \qquad (88)$$

$$\Rightarrow\ -iM_8 = -\frac{2g^2 e M_W}{(2\pi)^4} \left[\operatorname{Sin}(\beta-\alpha) + \frac{1}{2\cos^2\theta_W}\cos 2\beta\, \operatorname{Sin}(\beta+\alpha)\right] \frac{(1-2\operatorname{Sin}^2\theta_W)}{\cos\theta_W}\,\cdot$$

$$\cdot \int_0^1 dx \int_0^{1-x} dy\ \eta^{\mu\nu}\, I_0(x,y) \left[\frac{d}{2}\,(-m_{H^\pm}^2 + m_{h^0}^2 x - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy)\, \frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})} \right.$$

$$\left. + (m_{h^0}^2 x^2 + 2x(1-y)(P_1\cdot P_2) - m_{H^\pm}^2) \right]\ \varepsilon_{2\nu}^{*} \varepsilon_{0\mu} \qquad (89)$$

$$-i(M_5 + M_6 + M_8) = \frac{2g^2 e M_W}{(2\pi)^4} \left[\operatorname{Sin}(\beta-\alpha) + \frac{1}{2\cos^2\theta_W}\cos 2\beta\, \operatorname{Sin}(\beta+\alpha)\right] \frac{(1-2\operatorname{Sin}^2\theta_W)}{\cos\theta_W} \cdot \varepsilon_{2\nu}^{*} \varepsilon_{0\mu}$$

$$\int_0^1 dx \int_0^{1-x} dy\ \eta^{\mu\nu} \left\{ 2\,(-m_{H^\pm}^2 + m_{h^0}^2 x - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy)\, \frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})} \right.$$

$$- \frac{d}{2}\,(-m_{H^\pm}^2 + m_{h^0}^2 x - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy)\, \frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})} - m_{h^0}^2 x^2 - 2x(1-y)(P_1\cdot P_2)$$

$$\left. + m_{H^\pm}^2 \right\} I_0(x,y) \qquad (90)$$

Now:

$$\lim_{d\to 4} \left\{ 2\,(-m_{H^\pm}^2 + m_{h^0}^2 x - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy)\, \frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-d/2)} - \frac{d}{2}\,(-m_{H^\pm}^2 + m_{h^0}^2 x - m_{h^0}^2 x^2 \right.$$

$$\left. + 2(P_1\cdot P_2)xy)\, \frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})} \right\} = (-m_{H^\pm}^2 + m_{h^0}^2 x - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy) \lim_{d\to 4} \left\{ 2\,\Gamma(2-\frac{d}{2}) \right.$$

$$\left. - \frac{d}{2}\,\Gamma(2-d/2) \right\} \cdot \frac{1}{\Gamma(3-\frac{d}{2})}$$

$$= \left(-m_{HI}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right) \lim_{d\to 4} \frac{\cancel{\Gamma(2-\frac{d}{2})}\cdot\cancel{(2-\frac{d}{2})}}{\cancel{\Gamma(2-\frac{d}{2})(2-\frac{d}{2})}}$$

$$= \left(-m_{HI}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right) \qquad (91)$$

$\Rightarrow$

$$-i(M_5 + M_6 + M_8) = \frac{\cancel{2}g^2 e M_W}{(2\pi)^4}\left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W}\right]\frac{(1-2\sin^2\theta_W)}{\cos\theta_W}\,\varepsilon^{x}_{2\nu}\,\varepsilon_{0\mu}\,\eta^{\mu\nu}$$

$$\int_0^1 dX \int_0^{1-X} dY \Big\{ -m_{HI}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY - m_{h^0}^2 X^2 - 2X(P_1\cdot P_2)$$
$$+ 2XY(P_1\cdot P_2) + m_{HI}^2\Big\} \frac{i\pi^2}{\cancel{2}}\cdot\frac{1}{\left[-m_{HI}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right]}$$

$$= \frac{i g^2 e M_W}{16\pi^2}\left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W}\right]\frac{(1-2\sin^2\theta_W)}{\cos\theta_W}\,\varepsilon^{x}_{2\nu}\,\varepsilon_{0\mu}\,\eta^{\mu\nu}$$

$$\int_0^1 dX \int_0^{1-X} dY \left\{ 2 + \frac{(2m_{HI}^2 - m_{h^0}^2 X - 2X(P_1\cdot P_2)}{\left[-m_{HI}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right]}\right\} \qquad (92)$$

$$\int_0^{1-X} \frac{dY}{\left[-m_{HI}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right]} = \frac{1}{b}\ln\left|\frac{a + b(1-X)}{a}\right|$$

$$a = -m_{HI}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2$$

$$b = 2(P_1\cdot P_2)X$$

$$= \frac{1}{2(P_1\cdot P_2)X}\ln\left|\frac{-m_{HI}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)X(1-X)}{-m_{HI}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2}\right|$$

$$P_{Z^0} = P_1 + P_2 \Rightarrow M_Z^2 = m_{h^0}^2 + 2(P_1\cdot P_2)$$

$$2(P_1\cdot P_2) = M_Z^2 - m_{h^0}^2$$

$$\Rightarrow \int_0^{1-X} \frac{dY}{\left[-m_{HI}^2 + m_{h^0}^2 X - m_{h^0}^2 X^2 + 2(P_1\cdot P_2)XY\right]} = \frac{1}{2(P_1\cdot P_2)X}\ln\left|\frac{M_Z^2 X - M_Z^2 X^2 - m_{HI}^2}{m_{h^0}^2 X - m_{h^0}^2 X^2 - m_{HI}^2}\right|$$

$$= \frac{1}{2(P_1\cdot P_2)X}\ln\left|\left(\frac{X - X^2 - \frac{m_{HI}^2}{M_Z^2}}{X - X^2 - \frac{m_{HI}^2}{m_{h^0}^2}}\right)\left(\frac{M_Z^2}{m_{h^0}^2}\right)\right|$$

$$= \frac{1}{2(P_1 \cdot P_2)x} \ln \left| \frac{-x + x^2 + \frac{\lambda_H}{4}}{-x + x^2 + \frac{\tau_H}{4}} \left(\frac{\tau_H}{\lambda_H}\right) \right| \qquad (93)$$

$$\lambda_H \equiv 4 \frac{m_{H^\pm}^2}{M_Z^2} \; ; \quad \tau_H = \frac{4 m_{H^\pm}^2}{m_{h^0}^2}$$

$$\Rightarrow \boxed{\begin{aligned} &\int_0^1 dx \int_0^{1-x} \frac{dy \quad x}{[-m_{H^\pm}^2 + m_{h^0}^2 x - m_{h^0}^2 x^2 + 2(P_1 \cdot P_2) x y]} \\ &= \frac{1}{2(P_1 \cdot P_2)} \int_0^1 dx \ln \left| \frac{(x^2 - x + \frac{\lambda_H}{4})}{(x^2 - x + \frac{\tau_H}{4})} \left(\frac{\tau_H}{\lambda_H}\right) \right| \\ &= \frac{1}{(P_1 \cdot P_2)} (g(\lambda_H) - g(\tau_H)) \end{aligned}}$$

(see 49)

(94)

$$\Rightarrow \boxed{\begin{aligned} &\int_0^1 dx \int_0^{1-x} \frac{dy}{[-m_{H^\pm}^2 + m_{h^0}^2 x - m_{h^0}^2 x^2 + 2(P_1 \cdot P_2) x y]} \\ &= \frac{1}{2(P_1 \cdot P_2)} [f(\lambda_H) - f(\tau_H)] \end{aligned}}$$

(see 50)

(95)

$$\Rightarrow \boxed{\int_0^1 dx \int_0^{1-x} dy = \frac{1}{2}}$$

(see 51)

$$-i(M_5 + M_6 + M_8) = \frac{i g^2 e M_W}{16\pi^2} \left[\sin(\beta - \alpha) + \frac{\cos 2\beta \sin(\beta + \alpha)}{2\cos^2\theta_W}\right] \frac{(1 - 2\sin^2\theta_W)}{\cos\theta_W} \epsilon^x_{z\nu} \epsilon_{o\mu} n^{\mu\nu}_x$$

$$\times \left\{ 1 + \frac{2 m_{H^\pm}^2}{2(P_1 \cdot P_2)} [f(\lambda_H) - f(\tau_H)] - \frac{(m_{h^0}^2 + 2(P_1 \cdot P_2))}{(P_1 \cdot P_2)} [g(\lambda_H) - g(\tau_H)] \right\}$$

$$= \frac{i g^2 e M_W}{16\pi^2} \left[\sin(\beta - \alpha) + \frac{\cos 2\beta \sin(\beta + \alpha)}{2\cos^2\theta_W}\right] \frac{(1 - 2\sin^2\theta_W)}{\cos\theta_W} \epsilon^\alpha_{z\nu} \epsilon_{o\mu} n^{\mu\nu}$$

$$\times \left\{ 1 + \frac{2 m_{H^\pm}^2}{(M_Z^2 - m_{h^0}^2)} [f(\lambda_H) - f(\tau_H)] - \frac{2 M_Z^2}{(M_Z^2 - m_{h^0}^2)} [g(\lambda_H) - g(\tau_H)] \right\} \qquad (96)$$

$$\frac{m_{H^\pm}^2}{M_Z^2 - m_{h^0}^2} = \frac{\cancel{m_{H^\pm}^2}}{\frac{4\cancel{m_{H^\pm}^2}}{\lambda_H} - \frac{4\cancel{m_{H^\pm}^2}}{\tau_H}} = \frac{1}{4}\,\frac{\lambda_H \tau_H}{(\tau_H - \lambda_H)} \qquad (97)$$

$$\frac{M_Z^2}{M_Z^2 - m_{h^0}^2} = \frac{1}{1 - \frac{m_{h^0}^2}{M_Z^2}} = \frac{1}{1 - \frac{\lambda_H}{\tau_H}} = \frac{\tau_H}{(\tau_H - \lambda_H)} \qquad (98)$$

$$-i\,(M_5 + M_6 + M_8) = \frac{i g^2 e M_W}{16\pi^2}\left[\sin(\beta-\alpha) + \frac{\cos 2\beta \,\sin(\beta+\alpha)}{2\cos^2\theta_W}\right]\frac{(1-2\sin^2\theta_W)}{\cos\theta_W}\,\varepsilon^*_{2\nu}\,\varepsilon_{0\mu}\,\eta^{\mu\nu}\,\times$$

$$\times\left\{1 + \frac{1}{2}\frac{\lambda_H\tau_H}{(\tau_H-\lambda_H)}\big(f(\lambda_H) - f(\tau_H)\big) - \frac{2\tau_H}{(\tau_H-\lambda_H)}\big(g(\lambda_H)-g(\tau_H)\big)\right\}\frac{(M_Z^2-m_{h^0}^2)}{(M_Z^2-m_{h^0}^2)}$$

$$= \frac{i g^2 e M_W}{32\pi^2}\left[\sin(\beta-\alpha) + \frac{\cos 2\beta\,\sin(\beta+\alpha)}{2\cos^2\theta_W}\right]\frac{(1-2\sin^2\theta_W)}{\cos\theta_W}\,\frac{(M_Z^2-m_{h^0}^2)}{m_{H^\pm}^2}\,\varepsilon^*_{2\nu}\,\varepsilon_{0\mu}\,\eta^{\mu\nu}\,\times$$

$$\times\left\{\frac{1}{2}\frac{\lambda_H\tau_H}{(\tau_H-\lambda_H)} + \frac{1}{4}\frac{\lambda_H^2\tau_H^2}{(\tau_H-\lambda_H)^2}\big(f(\lambda_H)-f(\tau_H)\big) - \frac{\lambda_H\tau_H^2}{(\tau_H-\lambda_H)^2}\big(g(\lambda_H)-g(\tau_H)\big)\right\}$$

$$= -\frac{i g^2 e M_W}{32\pi^2}\left[\sin(\beta-\alpha) + \frac{\cos 2\beta\,\sin(\beta+\alpha)}{2\cos^2\theta_W}\right]\frac{(1-2\sin^2\theta_W)}{\cos\theta_W}\,\frac{(M_Z^2-m_{h^0}^2)}{m_{H^\pm}^2}\,\varepsilon^*_{2\nu}\,\varepsilon_{0\mu}\,\eta^{\mu\nu}\,\times$$

$$\times\left\{-\frac{1}{2}\frac{\lambda_H\tau_H}{(\tau_H-\lambda_H)} + \frac{1}{4}\frac{\lambda_H^2\tau_H^2}{(\tau_H-\lambda_H)^2}\big(f(\tau_H)-f(\lambda_H)\big) - \frac{\lambda_H\tau_H^2}{(\tau_H-\lambda_H)^2}\big(g(\tau_H)-g(\lambda_H)\big)\right\} \qquad (99)$$

Defining :

$$\boxed{I(\tau_H,\lambda_H) \equiv -\frac{1}{2}\frac{\tau_H\lambda_H}{(\tau_H-\lambda_H)} - \frac{\tau_H^2\lambda_H}{(\tau_H-\lambda_H)^2}\big(g(\tau_H)-g(\lambda_H)\big) + \frac{1}{4}\frac{\tau_H^2\lambda_H^2}{(\tau_H-\lambda_H)^2}\big(f(\tau_H)-f(\lambda_H)\big)} \qquad (100)$$

$$\Rightarrow\quad (M_5 + M_6 + M_8) = \frac{g^2 e M_W}{32\pi^2}\left[\sin(\beta-\alpha) + \frac{\cos 2\beta\,\sin(\beta+\alpha)}{2\cos^2\theta_W}\right]\frac{(1-2\sin^2\theta_W)}{\cos\theta_W}\,\times$$

$$\frac{(M_Z^2-m_{h^0}^2)}{m_{H^\pm}^2}\,\varepsilon^*_{2\nu}\,\varepsilon_{0\mu}\,\eta^{\mu\nu}\,I(\tau_H,\lambda_H) \qquad (101)$$

$$\Rightarrow \boxed{(M_5 + M_6 + M_8) = \frac{g e^2}{32\pi^2 M_W}\left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W}\right]\frac{(1-2\sin^2\theta_W)}{\sin\theta_W \cos\theta_W} \times \left(\frac{M_W^2}{M_{H^\pm}^2}\right)(M_Z^2 - m_{h^0}^2)\, \epsilon^*_{2\nu}\, \epsilon_{0\mu}\, \eta^{\mu\nu}\, I(\tau_H, \lambda_H)} \qquad (102)$$

The Fermion - charged Higgs contribution to M is:

$$M = M_f + M_{H^\pm} = (M_1 + M_2)_a + (M_1 + M_2)_b + (M_5 + M_6 + M_8) \qquad (103)$$

$$M = \frac{g e^2 \eta^{\mu\nu} \epsilon_{0\mu} \epsilon^*_{2\nu} (M_Z^2 - m_{h^0}^2)}{16\pi^2 M_W \sin\theta_W \cos\theta_W} \Bigg\{ \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i F(\tau_i, \lambda_i)$$

$$+ \left(-\frac{\cos\alpha}{\sin\beta}\right) \sum_{i=u,c,t} Q_i C_V^i N_i F(\tau_i, \lambda_i) + \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W}\right] \frac{(1-2\sin^2\theta_W)}{2} \cdot$$

$$\cdot \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) \Bigg\} \qquad (104)$$

$$\overline{|M|^2} = \frac{1}{3} \frac{g^2 e^4 (M_Z^2 - m_{h^0}^2)^2}{16^2 \pi^4 M_W^2 \sin^2\theta_W \cos^2\theta_W}\, \eta^{\mu\nu} \eta^{\rho\sigma} \sum_{\lambda} \epsilon^{\lambda}_{0\mu} \epsilon^{\lambda *}_{0\rho} \sum_{\lambda'} \epsilon^{\lambda' *}_{2\nu} \epsilon^{\lambda'}_{2\sigma}$$

$$\Bigg| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i F(\tau_i, \lambda_i) - \left(\frac{\cos\alpha}{\sin\beta}\right) \sum_{i=u,c,t} Q_i C_V^i N_i F(\tau_i, \lambda_i)$$

$$+ \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W}\right] \frac{1}{2}(1-2\sin^2\theta_W)\left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) \Bigg|^2$$

$$= \frac{1}{3} \frac{g^2 e^4 (M_Z^2 - m_{h^0}^2)^2}{16^2 \pi^4 M_W^2 \sin^2\theta_W \cos^2\theta_W} \left(-\eta_{\mu\rho} + \frac{P_{0\mu} P_{0\rho}}{M_Z^2}\right)\left(-\eta_{\nu\sigma}\right) \eta^{\mu\nu} \eta^{\rho\sigma} \,|\,|^2$$

$$= \frac{1}{3} \frac{g^2 e^4 (M_Z^2 - m_{h^0}^2)^2}{16^2 \pi^4 M_W^2 \sin^2\theta_W \cos^2\theta_W} \left(-\delta^{\nu}_{\rho} + \frac{P_0^{\nu} P_{0\rho}}{M_Z^2}\right)\left(-\delta^{\rho}_{\nu}\right) |\,|^2$$

$$= \cancel{\frac{1}{3}} \frac{g^2 e^4 (M_Z^2 - m_{h^0}^2)^2}{16^2 \pi^4 M_W^2 \sin^2\theta_W \cos^2\theta_W} \left[\cancel{4} - 1\right] |\,|^2$$

$$\overline{|M|^2} = \frac{g^2 e^4 (M_Z^2 - m_{h^0}^2)^2}{16^2 \pi^4 M_W^2 \sin^2\theta_W \cos^2\theta_W} \Bigg| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i F(\tau_i, \lambda_i) - \left(\frac{\cos\alpha}{\sin\beta}\right) \sum_{i=u,c,t} Q_i C_V^i N_i F(\tau_i, \lambda_i)$$

$$+ \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W}\right] \frac{1}{2}(1-2\sin^2\theta_W)\left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) \Bigg|^2 \qquad (105)$$

$$\Rightarrow \quad \Gamma = \frac{g^2 e^4 (M_Z^2 - m_{h^0}^2)^2}{16^2 \pi^4 M_W^2 \sin^2\theta_W \cos^2\theta_W} \left| \; \right|^2 \frac{(M_{Z^0}^2 - m_{h^0}^2)}{2 M_{Z^0}} \frac{1}{8\pi M_{Z^0}^2} \qquad (\text{see } 57, 58)$$

$$\Gamma = \frac{\alpha^2 \sqrt{2}\, G_F}{64\pi^3} \frac{\cancel{M_Z^6} \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3}{\cancel{M_Z^3} \quad \sin^2\theta_W \cos^2\theta_W} \left| \; \right|^2$$

$$\boxed{\begin{aligned} \Gamma(Z^0 \xrightarrow[f+H]{} h^0\gamma) = {} & \frac{\sqrt{2}\, G_F\, \alpha^2 M_{Z^0}^3 \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \left| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{\substack{i = d,s,b \\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i F(\tau_i, \lambda_i) \right. \\ & - \left(\frac{\cos\alpha}{\sin\beta}\right) \sum_{i = u,c,t} Q_i C_V^i N_i F(\tau_i, \lambda_i) + \left[\sin(\beta - \alpha) + \frac{\cos 2\beta \sin(\beta + \alpha)}{2\cos^2\theta_W} \right] \cdot \frac{1}{2} \cdot \\ & \left. \cdot (1 - 2\sin^2\theta_W) \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) \right|^2 \qquad (106) \end{aligned}}$$

$$\Gamma(Z^0 \xrightarrow[f+H]{} h^0\gamma) \approx \frac{\sqrt{2}\, G_F\, \alpha^2 M_{Z^0}^3 \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \left| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{i = b,\tau^-} Q_i C_V^i N_i F(\tau_i, \lambda_i) \right.$$

$$- 2\left(\frac{\cos\alpha}{\sin\beta}\right) C_V^t F(\tau_t, \lambda_t) + \left[\sin(\beta - \alpha) + \frac{\cos 2\beta \sin(\beta + \alpha)}{2\cos^2\theta_W} \right] \frac{1}{2} (1 - 2\sin^2\theta_W) \left(\frac{M_W^2}{M_{H^\pm}^2}\right) \cdot$$

$$\left. \cdot I(\tau_H, \lambda_H) \right|^2$$

$$\Gamma(Z^0 \xrightarrow[f+H]{} h^0\gamma) = \frac{\sqrt{2}\, G_F\, \alpha^2 M_{Z^0}^3 \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \left| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{i = b,\tau^-} Q_i C_V^i N_i F(\tau_i, \lambda_i) \right.$$

$$- 2\left(\frac{\cos\alpha}{\sin\beta}\right) C_V^t F(\tau_t, \lambda_t) + \left[(\sin\beta\cos\alpha - \sin\alpha\cos\beta) + \frac{(1 - \tan^2\beta)}{(1 + \tan^2\beta)} \frac{(\sin\beta\cos\alpha + \sin\alpha\cos\beta)}{2\cos^2\theta_W} \right] \cdot$$

$$\left. \cdot \frac{1}{2} (1 - 2\sin^2\theta_W) \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) \right|^2$$

$$= \frac{\sqrt{2}\, G_F\, \alpha^2 M_{Z^0}^3 \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \left| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{i = b,\tau^-} Q_i C_V^i N_i F(\tau_i, \lambda_i) \right.$$

$$- 2\left(\frac{\cos\alpha}{\sin\beta}\right) C_V^t F(\tau_t, \lambda_t) + \cos\alpha\cos\beta \left[(\tan\beta - \tan\alpha) + \frac{(1 - \tan^2\beta)}{(1 + \tan^2\beta)} \frac{(\tan\beta + \tan\alpha)}{2\cos^2\theta_W} \right] \cdot$$

$$\left. \cdot \frac{1}{2} (1 - 2\sin^2\theta_W) \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) \right|^2$$

$$= \frac{\sqrt{2}\, G_F\, \alpha^2 M_{Z^0}^3 \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \left| \left(\frac{\cos\alpha}{\sin\beta}\right) \left[\tan\alpha \tan\beta \sum_{i=b,\tau^-} Q_i C_V^{i} N_i F(\tau_i,\lambda_i) \right.\right.$$

$$\left. - 2 C_V^{t} F(\tau_t,\lambda_t) \right] + \cos\alpha \cos\beta \left[(\tan\beta - \tan\alpha) + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \frac{(\tan\beta + \tan\alpha)}{2\cos^2\theta_W} \right] \cdot$$

$$\left. \cdot \frac{1}{2} (1 - 2\sin^2\theta_W) \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) \right|^2$$

$$= \frac{\sqrt{2}\, G_F\, \alpha^2 M_{Z^0}^3 \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \cos^2\alpha \cos^2\beta \left| \frac{1}{\sin\beta \cos\beta} \left[\tan\alpha \tan\beta \sum_{i=b,\tau^-} Q_i C_V^{i} N_i F(\tau_i,\lambda_i) \right.\right.$$

$$\left. - 2 C_V^{t} F(\tau_t,\lambda_t) \right] + \left[(\tan\beta - \tan\alpha) + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \frac{(\tan\beta + \tan\alpha)}{2\cos^2\theta_W} \right] \frac{1}{2} (1 - 2\sin^2\theta_W) \left(\frac{M_W^2}{M_{H^\pm}^2}\right) \cdot$$

$$\left. \cdot I(\tau_H, \lambda_H) \right|^2$$

$$\sin\beta \cos\beta = (\sin^2\beta \cos^2\beta)^{1/2} = \left(\frac{1}{\csc^2\beta} \cdot \frac{1}{\sec^2\beta}\right)^{1/2} = \left(\frac{1}{1+\cot^2\beta} \cdot \frac{1}{1+\tan^2\beta}\right)^{1/2}$$

$$\sin\beta \cos\beta = \left(\frac{\tan^2\beta}{(1+\tan^2\beta)^2}\right)^{1/2}$$

$$\sin\beta \cos\beta = \frac{\tan\beta}{1+\tan^2\beta}$$

$$\Rightarrow \Gamma(Z^0 \xrightarrow[f+H]{} h^0\gamma) \approx \frac{\sqrt{2}\, G_F\, \alpha^2 M_{Z^0}^3 \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \cdot \frac{1}{(1+\tan^2\alpha)(1+\tan^2\beta)} \left| \frac{(1+\tan^2\beta)}{\tan\beta} \cdot \left[\right.\right.$$

$$\left. \tan\alpha \tan\beta \sum_{i=b,\tau^-} Q_i C_V^{i} N_i F(\tau_i,\lambda_i) - 2 C_V^{t} F(\tau_t,\lambda_t) \right] + \left[(\tan\beta - \tan\alpha) + \right.$$

$$\left.\left. + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \frac{(\tan\beta + \tan\alpha)}{2\cos^2\theta_W} \right] \frac{1}{2} (1 - 2\sin^2\theta_W) \left(\frac{M_W^2}{M_{H^\pm}^2}\right) \cdot I(\tau_H, \lambda_H) \right|^2$$

$$\therefore \Gamma(Z^0 \xrightarrow[f+H^\pm]{} h^0\gamma) = \frac{\sqrt{2}\, G_F\, \alpha^2 M_{Z^0}^3 \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \cdot \frac{1}{(1+\tan^2\alpha)(1+\tan^2\beta)} \left| \frac{(1+\tan^2\beta)}{\tan\beta} \cdot \left[\tan\alpha \tan\beta \cdot \right.\right.$$

$$\cdot \Big(- (-\tfrac{1}{2} + \tfrac{2}{3}\sin^2\theta_W) F(\tau_b,\lambda_b) - (-\tfrac{1}{2} + 2\sin^2\theta_W) F(\tau_\tau,\lambda_\tau) \Big) - 2 (\tfrac{1}{2} - \tfrac{4}{3}\sin^2\theta_W) \cdot$$

$$\left. F(\tau_t,\lambda_t) \right] + \left[(\tan\beta - \tan\alpha) + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \frac{(\tan\beta + \tan\alpha)}{2\cos^2\theta_W} \right] \frac{1}{2} (1 - 2\sin^2\theta_W) \cdot$$

$$\left. \cdot \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) \right|^2 \qquad (107)$$

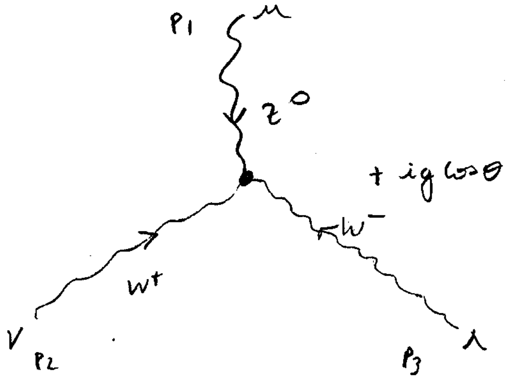
Where

$$I\,(\tau_H,\lambda_H) = -\frac{1}{2}\frac{\tau_H\lambda_H}{(\tau_H-\lambda_H)} - \frac{\tau_H^2\lambda_H}{(\tau_H-\lambda_H)^2}\left[g(\tau_H)-g(\lambda_H)\right] + \frac{1}{4}\frac{\tau_H^2\lambda_H^2}{(\tau_H-\lambda_H)^2}\left[f(\tau_H)-f(\lambda_H)\right]$$

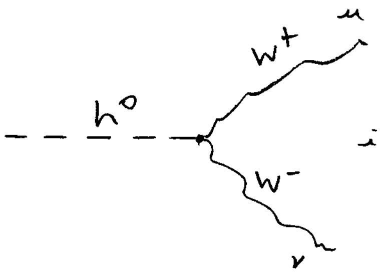
$$\tau_H = \frac{4\,m_{H^\pm}^2}{m_{h^0}^2}\;;\quad \lambda_H = \frac{4\,m_{H^\pm}^2}{M_Z^2}$$

$$\tau_i = \frac{4\,m_i^2}{m_{h^0}^2}\;;\quad \lambda_i = \frac{4\,m_i^2}{M_Z^2}$$

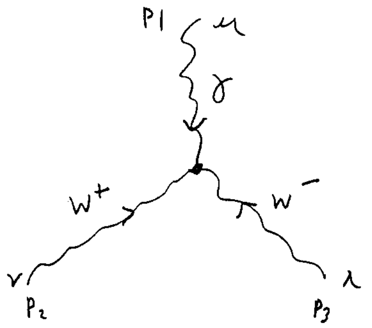
③ and ④ :



$$+ig\cos\theta_W\left[(P_1-P_2)^\lambda\eta^{\mu\nu}+(P_2-P_3)^\mu\eta^{\nu\lambda}+(P_3-P_1)^\nu\eta^{\mu\lambda}\right] \qquad (108)$$



$$ig M_W \sin(\beta-\alpha)\eta^{\mu\nu} \qquad (109)$$



$$+ig\sin\theta_W\left[(P_1-P_2)^\lambda\eta^{\lambda\nu}+(P_2-P_3)^\mu\eta^{\nu\lambda}+(P_3-P_1)^\nu\eta^{\mu\lambda}\right] \qquad (110)$$

$$-i\left[\eta_{\mu\nu}-\frac{K_\mu K_\nu}{M_W^2}\right]\frac{1}{K^2-M_W^2} \qquad (111)$$

$$-iM_3=\int\frac{d^dK}{(2\pi)^d}\left[igM_W\sin(\beta-\alpha)\eta^{\rho\sigma}\right](-i)\left[\eta_{\sigma\delta}-\frac{K_\sigma K_\delta}{M_W^2}\right]\frac{1}{K^2-M_W^2}\cdot$$

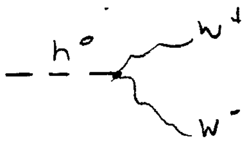
$$\left[+ig\sin\theta_W\right]\left[(-P_2-K)^\beta\eta^{\nu\delta}+(2K-P_2)^\nu\eta^{\beta\delta}+(2P_2-K)^\delta\eta^{\beta\nu}\right]\cdot$$

$$(-i)\left[\eta_{\beta\alpha}-\frac{(K-P_2)_\beta(K-P_2)_\alpha}{M_W^2}\right]\frac{1}{(K-P_2)^2-M_W^2}\left[ig\cos\theta_W\right]\cdot$$

$$\cdot\left[(P_{Z^0}-K+P_2)^\lambda\eta^{\mu\alpha}+(2K+P_1-P_2)^\mu\eta^{\alpha\lambda}+(-K-P_1-P_{Z^0})^\alpha\eta^{\mu\lambda}\right]$$

$$(-i)\left[\eta_{\lambda\rho}-\frac{(K+P_1)_\lambda(K+P_1)_\rho}{M_W^2}\right]\frac{1}{(K+P_1)^2-M_W^2}\,\varepsilon^\alpha_{Z\nu}\,\varepsilon_{0\mu}\,\underset{\substack{\downarrow\\ h^0}}{\mu_1^{\left(\frac{4-d}{2}\right)}}\;\underset{\substack{\downarrow\\ \gamma}}{\mu_2^{\left(\frac{4-d}{2}\right)}}\;\underset{\substack{\downarrow\\ Z^0}}{\mu_3^{\left(\frac{4-d}{2}\right)}}$$

(112)

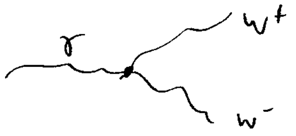


dimension of the mass parameter.

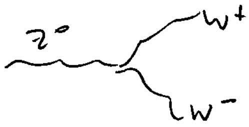
$$3\left(\frac{d}{2}-1\right) + x + 1 = d$$

$$X = d - \frac{3d}{2} + 3 - 1$$

$$X = -\frac{d}{2} + 2$$



or



$$3\left(\frac{d}{2}-1\right) + x + 1 = d$$

$$\Rightarrow \quad x = 2 - \frac{d}{2}$$

$P_{Z^0} = P_1 + P_2$

$$-iM_3 = g^3 M_W \sin(\beta-\alpha) \sin\theta_W \cos\theta_W \int \frac{d^dK}{(2\pi)^d} \left\{ \eta^{\rho\sigma} \left[\eta_{\sigma\delta} - \frac{K_\sigma K_\delta}{M_W^2}\right] \right.$$

$$\cdot \left[(-P_2-K)^\beta \eta^{\nu\delta} + (2K-P_2)^\nu \eta^{\beta\delta} + (2P_2-K)^\delta \eta^{\beta\nu}\right] \cdot \left[\eta_{\beta\alpha} - \frac{(K-P_2)_\beta (K-P_2)_\alpha}{M_W^2}\right] \cdot$$

$$\cdot \left[(P_{Z^0}-K+P_2)^\lambda \eta^{\mu\alpha} + (2K+P_1-P_2)^\mu \eta^{\alpha\lambda} + (-K-P_1-P_{Z^0})^\alpha \eta^{\mu\lambda}\right] \cdot \left[\eta_{\lambda\rho} - \frac{(K+P_1)_\lambda (K+P_1)_\rho}{M_W^2}\right]$$

$$\cdot \frac{1}{(K^2-M_W^2)\left[(K-P_1)^2-M_W^2\right]\left[(K+P_1)^2-M_W^2\right]} \varepsilon^*_{Z\nu} \varepsilon_{0\mu} \underset{\substack{\downarrow \\ 1 \text{ when } d\to 4}}{\mu'^x} \qquad (113)$$

$$\mu'^x = \mu_1^{\frac{(4-d)}{2}} \mu_2^{\frac{(4-d)}{2}} \mu_3^{\frac{(4-d)}{2}}$$ (mass parameter)

$\mu'^x \to 1$ when $d \to 4$

↓ begin

$$\eta^{\rho\sigma}\left[\eta_{\sigma\delta}-\frac{K_\sigma K_\delta}{M_W^2}\right]\left[\eta_{\lambda\rho}-\frac{(K+P_1)_\lambda(K+P_1)_\rho}{M_W^2}\right]$$

$$=\left[\delta^{\rho}_{\delta}-\frac{K^\rho K_\delta}{M_W^2}\right]\left[\eta_{\lambda\rho}-\frac{(K+P_1)_\lambda\,(K+P_1)_\rho}{M_W^2}\right]$$

$$=\eta_{\lambda\delta}-\frac{(K+P_1)_\lambda(K+P_1)_\delta}{M_W^2}-\frac{K_\lambda K_\delta}{M_W^2}+\frac{(K\cdot(K+P_1))\,(K+P_1)_\lambda K_\delta}{M_W^4}\qquad(114)$$

$$-iM_3=g^3M_W\,\mathrm{Sin}(\beta-\alpha)\,\mathrm{Sin}\theta_W\cos\theta_W\int\frac{d^dK}{(2\pi)^d}\left[\overset{\downarrow}{\eta_{\lambda\delta}}-\frac{\overset{\downarrow}{(K+P_1)_\lambda}(K+P_1)_\delta}{M_W^2}-\frac{\overset{\downarrow}{K_\lambda K_\delta}}{M_W^2}\right.$$
$$\left.+\frac{\overset{\downarrow}{(K\cdot(K+P_1))}(K+P_1)_\lambda K_\delta}{M_W^4}\right]\left[(P_{Z^0}-K+P_2)^\lambda\eta^{\mu\alpha}+(2K+P_1-P_2)^\mu\eta^{\alpha\lambda}+\right.$$
$$\left.+(-K-P_1-P_{Z^0})^\alpha\eta^{\mu\lambda}\right]\left[-(P_2+K)_\alpha\eta^{\nu\delta}+\frac{K^2}{M_W^2}(K-P_2)_\alpha\eta^{\nu\delta}+(2K-P_2)^\nu\delta^{\delta}_{\alpha}\right.$$
$$\left.-\frac{(2K-P_2)^\nu(K-P_2)^\delta(K-P_2)_\alpha}{M_W^2}+(2P_2-K)^\delta\delta^{\nu}_{\alpha}-\frac{(2P_2-K)^\delta(K-P_2)^\nu(K-P_2)_\alpha}{M_W^2}\right]\cdot$$
$$\cdot\frac{1}{(K^2-M_W^2)\left[(K-P_2)^2-M_W^2\right]\left[(K+P_1)^2-M_W^2\right]}\,\varepsilon^{\alpha}_{2\nu}\,\varepsilon_{0\mu}\qquad(115)$$

$$-iM_3=g^3M_W\,\mathrm{Sin}(\beta-\alpha)\,\mathrm{Sin}\theta_W\cos\theta_W\int\frac{d^dK}{(2\pi)^d}\left[\overset{\downarrow}{(P_{Z^0}-K+P_2)_\delta\eta^{\mu\alpha}}+\overset{\downarrow}{2(K-P_2)^\mu\delta^{\alpha}_{\delta}}\right.$$
$$\overset{\downarrow}{-(K+P_1+P_{Z^0})^\alpha\delta^{\mu}_{\delta}}-\frac{\overset{\downarrow}{((K+P_1)\cdot(P_{Z^0}-K+P_2))}}{M_W^2}(K+P_1)_\delta\eta^{\mu\alpha}-\frac{\overset{\downarrow}{2(K+P_1)^\alpha}(K+P_1)_\delta\cdot(K-P_2)^\mu}{M_W^2}$$
$$+\frac{\overset{\downarrow}{(K+P_1)^\mu}(K+P_1)_\delta(K+P_1+P_{Z^0})^\alpha}{M_W^2}-\frac{\overset{\downarrow}{(K\cdot(P_{Z^0}-K+P_2))}}{M_W^2}K_\delta\eta^{\mu\alpha}$$
$$-\frac{\overset{\downarrow}{2(K-P_2)^\mu}}{M_W^2}K^\alpha K_\delta+\frac{\overset{\downarrow}{(K+P_1+P_{Z^0})^\alpha}K^\mu K_\delta}{M_W^2}+\frac{\overset{\downarrow}{(K\cdot(K+P_1))}((K+P_1)\cdot(P_{Z^0}-K+P_2))K_\delta\eta^{\mu\alpha}}{M_W^4}$$
$$\overset{\downarrow\ \downarrow}{+2}\frac{(K\cdot(K+P_1))(K+P_1)^\alpha K_\delta(K-P_2)^\mu}{M_W^4}-\frac{\overset{\downarrow}{(K\cdot(K+P_1))}(K+P_1)^\mu K_\delta(K+P_1+P_{Z^0})^\alpha}{M_W^4}\Bigg]\cdot\Bigg[$$
$$-(P_2+K)_\alpha\eta^{\nu\delta}+\frac{K^2}{M_W^2}(K-P_2)_\alpha\eta^{\nu\delta}+2K^\nu\delta^{\delta}_{\alpha}-\frac{2K^\nu(K-P_2)^\delta(K-P_2)_\alpha}{M_W^2}+(2P_2-K)^\delta\delta^{\nu}_{\alpha}-$$
$$-\frac{(2P_2-K)^\delta K^\nu(K-P_2)_\alpha}{M_W^2}\Bigg]\cdot\frac{1}{(K^2-M_W^2)\left[(K-P_2)^2-M_W^2\right]\left[(K+P_1)^2-M_W^2\right]}\,\varepsilon^{\mu}_{2\nu}\,\varepsilon_{0\mu}\qquad(116)$$

because $(\overset{P_{Z^0}^{\mu}}{P_1}+P_2)^\mu\,\varepsilon_{0\mu}=0$

$$
\begin{aligned}
-iM_3 = {} & g^3 M_W \sin(\beta-\alpha)\sin\theta_W\cos\theta_W \int \frac{d^dK}{(2\pi)^d}\Big\{\Big[10K^\mu K^\nu - 9P_2^\mu K^\nu + [4(P_1\cdot P_2) + 2K^2 + (P_1\cdot K) \\
& -2(P_2\cdot K)]\eta^{\mu\nu} - \frac{\eta^{\mu\nu}}{M_W^2}\Big[3K^4 + 2(P_1\cdot K)K^2 - 6(P_1\cdot P_2)K^2 + 8(P_2\cdot K)^2 - 8(P_2\cdot K)K^2 + 8(P_2\cdot K)(P_1\cdot P_2) \\
& + 2(P_2\cdot K)m_{h^0}^2 - K^2 m_{h^0}^2 + 2(P_1\cdot P_2)m_{h^0}^2 - (P_1\cdot K)m_{h^0}^2 + 4(P_1\cdot P_2)^2 - 2(P_1\cdot P_2)(P_1\cdot K)\Big] \\
& + \frac{K^\mu K^\nu}{M_W^2}\Big[3K^2 + (P_1\cdot K) - 4(P_2\cdot K) - 6(P_1\cdot P_2) - m_{h^0}^2\Big] + \frac{P_2^\mu K^\nu}{M_W^2}\Big[-2K^2 + (P_1\cdot K) + 2(P_1\cdot P_2) \\
& + 4(P_2\cdot K) + m_{h^0}^2\Big] + \frac{K^\nu P_1^\mu}{M_W^2}\Big[2K^2 - 6(P_2\cdot K) - 2(P_1\cdot K) - 4(P_1\cdot P_2) - 2m_{h^0}^2\Big] + \frac{K^\mu K^\nu}{M_W^4}\Big[-K^2(P_1\cdot K) \\
& + 2m_{h^0}^2(P_1\cdot K) + 2K^2(P_2\cdot K) + 2K^2(P_1\cdot P_2) + 2(P_1\cdot K)(P_2\cdot K) + 4(P_1\cdot P_2)(P_1\cdot K) - K^4 + K^2 m_{h^0}^2\Big] \\
& + \frac{P_1^\mu K^\nu}{M_W^4}\Big[2(P_1\cdot P_2)(P_1\cdot K) + (P_1\cdot K)m_{h^0}^2\Big] + \frac{P_2^\mu K^\nu}{M_W^4}\Big[K^2(P_1\cdot K) - 2K^2(P_2\cdot K) \\
& - m_{h^0}^2(P_1\cdot K) - 2(P_1\cdot K)(P_2\cdot K) + K^4 - m_{h^0}^2K^2 - 2(P_1\cdot P_2)(P_1\cdot K) - 2K^2(P_1\cdot P_2)\Big] + \frac{\eta^{\mu\nu}}{M_W^4}\Big[\\
& 4K^2(P_2\cdot K)^2 - 4K^4(P_2\cdot K) + K^6 + 2m_{h^0}^2K^2(P_2\cdot K) - m_{h^0}^2K^4 - 2(P_1\cdot P_2)K^4 + 4(P_1\cdot K)(P_2\cdot K)^2 \\
& - 4K^2(P_1\cdot K)(P_2\cdot K) + K^4(P_1\cdot K) + 2m_{h^0}^2(P_1\cdot K)(P_2\cdot K) - m_{h^0}^2K^2(P_1\cdot K) + 4(P_1\cdot K)(P_2\cdot K)(P_1\cdot P_2) \\
& - 2K^2(P_1\cdot K)(P_1\cdot P_2) + 4(P_1\cdot P_2)(P_2\cdot K)K^2\Big]\Big\} \times \frac{1}{[K^2-M_W^2][(K-P_2)^2-M_W^2][(K+P_1)^2-M_W^2]}\,\epsilon_{2\nu}^*\epsilon_{0\mu}
\end{aligned}
\qquad (126)
$$

$$a = K^2 - M_W^2;\quad b = (K+P_1)^2 - M_W^2;\quad c = (K-P_2)^2 - M_W^2$$

$$
\begin{aligned}
a(1-x-y) + bx + cy & = (K^2 - M_W^2)(1-x-y) + [(K+P_1)^2 - M_W^2]x + [(K-P_2)^2 - M_W^2]y \\
& = \cancel{K^2} - \cancel{K^2x} - \cancel{K^2y} - \cancel{M_W^2} + \cancel{M_W^2x} + \cancel{M_W^2y} + \cancel{K^2x} + \cancel{2(P_1\cdot K)}x + \cancel{m_{h^0}^2}x - \cancel{M_W^2}x + \cancel{K^2}y - 2\cancel{(P_2\cdot K)}y \\
& \quad - \cancel{M_W^2}y \\
& = K^2 - M_W^2 + m_{h^0}^2x + 2K\cdot(P_1x - P_2y)
\end{aligned}
$$

$$\frac{1}{[K^2-M_W^2][(K+P_1)^2-M_W^2][(K-P_2)^2-M_W^2]} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[K^2 - M_W^2 + m_{h^0}^2x + 2K\cdot(P_1x - P_2y)]^3}$$

$$\boxed{K' = K + (P_1x - P_2y)} = K + h \qquad (h = P_1x - P_2y) \qquad (127)$$

$$K'^2 = K^2 + 2K\cdot(P_1x - P_2y) + m_{h^0}^2x^2 - 2(P_1\cdot P_2)xy$$

$$d^dK' = d^dK$$

$$\Rightarrow \frac{1}{[K^2-M_W^2][(K+P_1)^2-M_W^2][(K-P_2)^2-M_W^2]} = 2\int_0^1 dx\int_0^{1-x}\frac{dy}{[K'^2 - m_{h^0}^2x^2 + 2(P_1\cdot P_2)xy - M_W^2 + m_{h^0}^2x]^3}$$

(128)

$$-iM_4 = \int \frac{d^dK}{(2\pi)^d}\left[(-2P_2-K)^\sigma \eta^{\nu\rho} + (2K+P_2)^\nu \eta^{\rho\sigma} + (-K+P_2)^\rho \eta^{\sigma\nu}\right] ig\sin\theta_W .$$

$$(-i)\left[\eta_{\sigma\delta} - \frac{K_\sigma K_\delta}{M_W^2}\right]\frac{1}{K^2-M_W^2}\left[igM_W\sin(\beta-\alpha)\eta^{\beta\delta}\right].$$

$$\cdot\ (-i)\left[\eta_{\beta\alpha} - \frac{(K-P_1)_\beta(K-P_1)_\alpha}{M_W^2}\right]\frac{1}{(K-P_1)^2-M_W^2}\cdot ig\cos\theta_W\left[(P_{Z^0}-K+P_1)^\lambda\eta^{\mu\alpha}\right.$$

$$\left. + (2K+P_2-P_1)^\mu \eta^{\alpha\lambda} + (-K-P_2-P_{Z^0})^\alpha \eta^{\mu\lambda}\right](-i)\left[\eta_{\lambda\rho} - \frac{(K+P_2)_\lambda(K+P_2)_\rho}{M_W^2}\right]$$

$$\frac{1}{(K+P_2)^2-M_W^2}\ \varepsilon^{*}_{2\nu}\ \varepsilon_{0\mu}\ \underbrace{\underset{h^0}{\underset{\uparrow}{\mu_1^{\frac{(4-d)}{2}}}}\ \ \underset{\gamma}{\underset{\uparrow}{\mu_2^{\frac{(4-d)}{2}}}}\ \ \underset{Z^0\text{ (vertices)}}{\underset{\uparrow}{\mu_3^{\frac{(4-d)}{2}}}}}_{\mu^x} \qquad (129)$$

$$-iM_4 = g^3 M_W \sin(\beta-\alpha)\sin\theta_W\cos\theta_W \int\frac{d^dK}{(2\pi)^d}\left\{\left[(-2P_2-K)^\sigma\eta^{\nu\rho} + (2K+P_2)^\nu\eta^{\rho\sigma}\right.\right.$$

$$\left. + (-K+P_2)^\rho\eta^{\sigma\nu}\right]\left[\eta_{\sigma\delta} - \frac{K_\sigma K_\delta}{M_W^2}\right]\eta^{\beta\delta}\left[\eta_{\beta\alpha} - \frac{(K-P_1)_\beta(K-P_1)_\alpha}{M_W^2}\right].\left[(2P_1+P_2-K)^\lambda.\right.$$

$$\left.\eta^{\mu\alpha} + (2K+P_2-P_1)^\mu\eta^{\alpha\lambda} + (-K-P_1-2P_2)^\alpha\eta^{\mu\lambda}\right]\left[\eta_{\lambda\rho} - \frac{(K+P_2)_\lambda(K+P_2)_\rho}{M_W^2}\right]$$

$$\left.\cdot\ \frac{1}{[K^2-M_W^2][(K-P_1)^2-M_W^2][(K+P_2)^2-M_W^2]}\right\}\varepsilon^{*}_{2\nu}\ \varepsilon_{0\mu}\ \underset{\substack{\mu'^x\to 1\\ \text{when } d\to 4}}{\underset{\downarrow}{\mu'^x}} \qquad (130)$$

$$\eta^{\beta\delta}\left[\eta_{\beta\alpha} - \frac{(K-P_1)_\beta(K-P_1)_\alpha}{M_W^2}\right] = \delta^\delta_\alpha - \frac{(K-P_1)^\delta(K-P_1)_\alpha}{M_W^2}$$

$$\Rightarrow \left[\eta_{\sigma\delta} - \frac{K_\sigma K_\delta}{M_W^2}\right]\eta^{\beta\delta}\left[\eta_{\beta\alpha} - \frac{(K-P_1)_\beta(K-P_1)_\alpha}{M_W^2}\right] = \left[\eta_{\sigma\delta} - \frac{K_\sigma K_\delta}{M_W^2}\right].$$

$$\cdot\left(\delta^\delta_\alpha - \frac{(K-P_1)^\delta(K-P_1)_\alpha}{M_W^2}\right)$$

$$= \left(\eta_{\sigma\alpha} - \frac{(K-P_1)_\sigma(K-P_1)_\alpha}{M_W^2} - \frac{K_\sigma K_\alpha}{M_W^2} + \frac{(K\cdot(K-P_1))\,K_\sigma(K-P_1)_\alpha}{M_W^4}\right)$$

(131)

$$\Rightarrow -iM_4 = g^3 M_W \sin(\beta-\alpha) \sin\theta_W \cos\theta_W \int \frac{d^dK}{(2\pi)^d} \Big\{ \Big[(-2P_2-K)_\alpha \eta^{\nu\rho} + \delta^\rho_\alpha (2K+P_2)^\nu$$

$$+ (-K+P_2)^\rho \delta^\nu_\alpha + \frac{((2P_2+K)\cdot(K-P_1))(K-P_1)_\alpha \eta^{\nu\rho}}{M_W^2} - \frac{(2K+P_2)^\nu (K-P_1)^\rho (K-P_1)_\alpha}{M_W^2}$$

$$+ \frac{(K-P_2)^\rho (K-P_1)^\nu (K-P_1)_\alpha}{M_W^2} + \frac{((2P_2+K)\cdot K) K_\alpha \eta^{\nu\rho}}{M_W^2} - \frac{(2K+P_2)^\nu K^\rho K_\alpha}{M_W^2}$$

$$+ \frac{(K-P_2)^\rho K^\nu K_\alpha}{M_W^2} - \frac{((2P_2+K)\cdot K)(K\cdot(K-P_1))\eta^{\nu\rho}(K-P_1)_\alpha}{M_W^4}$$

$$+ \frac{((K\cdot(K-P_1))(2K+P_2)^\nu K^\rho (K-P_1)_\alpha}{M_W^4} - \frac{((K\cdot(K-P_1))(K-P_2)^\rho K^\nu (K-P_1)_\alpha}{M_W^4} \Big] \cdot$$

$$\cdot \Big[(2P_1+P_2-K)_\rho \eta^{\mu\alpha} + \delta^\alpha_\rho (2K+P_2-P_1)^\mu - \delta^\mu_\rho (K+P_1+2P_2)^\alpha$$

$$- \frac{((K+P_2)\cdot(2P_1+P_2-K))(K+P_2)_\rho \eta^{\mu\alpha}}{M_W^2} - \frac{(2K+P_2-P_1)^\mu (K+P_2)^\alpha (K+P_2)_\rho}{M_W^2}$$

$$+ (K+P_1+2P_2)^\alpha \frac{(K+P_2)^\mu (K+P_2)_\rho}{M_W^2} \Big] \cdot \frac{1}{[K^2-M_W^2][(K-P_1)^2-M_W^2][(K+P_2)^2-M_W^2]} \Big\} \varepsilon^*_{2\nu} \varepsilon_{0\mu}$$

(132)

$$P_1^\nu \varepsilon^*_{2\nu} = 0 \quad ; \quad P_2^\nu \varepsilon^*_{2\nu} = 0$$

$$-iM_4 = g^3 M_W \sin(\beta-\alpha) \sin\theta_W \cos\theta_W \int \frac{d^dK}{(2\pi)^d} \Big\{ \Big[-(2P_2+K)_\alpha \eta^{\nu\rho} + 2\delta^\rho_\alpha K^\nu$$

$$+ (-K+P_2)^\rho \delta^\nu_\alpha + \frac{((2P_2+K)\cdot(K-P_1))(K-P_1)_\alpha \eta^{\nu\rho}}{M_W^2} - \frac{2K^\nu (K-P_1)^\rho (K-P_1)_\alpha}{M_W^2}$$

$$+ \frac{K^\nu (K-P_2)^\rho (K-P_1)_\alpha}{M_W^2} + \frac{((2P_2+K)\cdot K) K_\alpha \eta^{\nu\rho}}{M_W^2} - \frac{2K^\nu K^\rho K_\alpha}{M_W^2}$$

$$+ \frac{(K-P_2)^\rho K^\nu K_\alpha}{M_W^2} - \frac{((2P_2+K)\cdot K)(K\cdot(K-P_1))\eta^{\nu\rho}(K-P_1)_\alpha}{M_W^4} + \frac{((K\cdot(K-P_1))2K^\nu K^\rho \cdot}{M_W^4}$$

$$\frac{\cdot (K-P_1)_\alpha}{} - \frac{((K\cdot(K-P_1))(K-P_2)^\rho K^\nu (K-P_1)_\alpha}{M_W^4} \Big] \cdot \Big[(2P_1+P_2-K)_\rho \eta^{\mu\alpha} + \delta^\alpha_\rho (2K+P_2-P_1)^\mu$$

$$- \delta^\mu_\rho (K+P_1+2P_2)^\alpha - \frac{((K+P_2)\cdot(2P_1+P_2-K))(K+P_2)_\rho \eta^{\mu\alpha}}{M_W^2} - \frac{(2K+P_2-P_1)^\mu (K+P_2)^\alpha (K+P_2)_\rho}{M_W^2}$$

$$+ \frac{(K+P_1+2P_2)^\alpha (K+P_2)^\mu (K+P_2)_\rho}{M_W^2} \Big] \cdot \frac{1}{[K^2-M_W^2][(K-P_1)^2-M_W^2][(K+P_2)^2-M_W^2]} \Big\} \varepsilon^*_{2\nu} \varepsilon_{0\mu}$$

(133)

OK

$$- \frac{1}{M_W^4} K^\mu K^\nu \left[2(\cancel{P_1 \cdot K})^2 - K^2 \cancel{(P_1 \cdot K)} + 2 \cancel{(P_1 \cdot P_2)} (P_1 \cdot K) \right] + \frac{1}{M_W^4} P_1^\mu K^\nu \left[\cancel{2(P_1 \cdot K)^2} - \cancel{K^2 (P_1 \cdot K)} \right.$$

$$\left. + 2 \cancel{(P_1 \cdot P_2)} (P_1 \cdot K) \right] + \frac{K^\mu K^\nu}{M_W^4} \left[- \cancel{K^2} (P_1 \cdot K) + 2 (\cancel{P_1 \cdot K})^2 - (\cancel{P_1 \cdot K}) m_{h^0}^2 \right] \text{ OK}$$

$$+ \frac{P_1^\mu K^\nu}{M_W^4} \left[K^2 (\cancel{P_1 \cdot K}) - 2 \cancel{(P_1 \cdot K)^2} + \cancel{(P_1 \cdot K) m_{h^0}^2} \right] - 2 \cancel{\frac{K^\mu K^\nu}{M_W^6}} [0] - \cancel{\frac{P_1^\mu K^2}{M_W^6}} [0]$$

$$+ \frac{K^\mu K^\nu}{M_W^6} \cancel{(K^2 - P_1 \cdot K)} [0] + \frac{P_1^\mu K^\nu}{M_W^6} \cancel{(K^2 - P_1 \cdot K)} [0] \Big\} .$$

$$\cdot \frac{1}{[K^2 - M_W^2][(K-P_1)^2 - M_W^2][(K+P_2)^2 - M_W^2]} \cdot \varepsilon_{2\nu}^* \varepsilon_{0\mu} \qquad (140)$$

$\Rightarrow$

$$+9 P_2^\mu K^\nu \quad \left(8K^\nu (P_1^\mu + P_2^\mu) - 8 P_1^\mu K^\nu + P_2^\mu K^\nu \right)$$

$$-i M_4 = g^3 M_W \sin(\beta - \alpha) \sin\theta_W \cos\theta_W \int \frac{d^d K}{(2\pi)^d} \Big\{ \underbrace{P_2^\mu K^\nu - 8 P_1^\mu K^\nu} + 10 K^\mu K^\nu + \eta^{\mu\nu} \left[4 (P_1 \cdot P_2) \right.$$

$$\left. + 2 (P_2 \cdot K) - (P_1 \cdot K) + 2K^2 \right] + \frac{K^\mu K^\nu}{M_W^2} \left[4 (P_2 \cdot K) + 3K^2 - 6 (P_1 \cdot P_2) - (P_1 \cdot K) - m_{h^0}^2 \right]$$

$$+ \frac{P_1^\mu K^\nu}{M_W^2} \left[-6 (P_2 \cdot K) - 2K^2 + 4 (P_1 \cdot P_2) - 2 (P_1 \cdot K) + 2 m_{h^0}^2 \right] + \frac{P_2^\mu K^\nu}{M_W^2} \left[4 (P_2 \cdot K) + 2K^2 \right.$$

$$\left. - 2 (P_1 \cdot P_2) + (P_1 \cdot K) - m_{h^0}^2 \right] + \frac{\eta^{\mu\nu}}{M_W^2} \left[-8 (P_2 \cdot K) K^2 - 8 (P_2 \cdot K)^2 - 3K^4 + 2 m_{h^0}^2 (P_2 \cdot K) \right.$$

$$+ 8 (P_1 \cdot P_2)(P_2 \cdot K) + m_{h^0}^2 K^2 + 6 (P_1 \cdot P_2) K^2 - 2 (P_1 \cdot P_2)(P_1 \cdot K) - 2 m_{h^0}^2 (P_1 \cdot P_2) - 4 (P_1 \cdot P_2)^2 - m_{h^0}^2 (P_1 \cdot K)$$

$$\left. + 2K^2 (P_1 \cdot K) \right] + \frac{\eta^{\mu\nu}}{M_W^4} \left[4K^4 (P_2 \cdot K) + 4K^2 (P_2 \cdot K)^2 - 2 m_{h^0}^2 K^2 (P_2 \cdot K) - 4K^2 (P_1 \cdot P_2)(P_2 \cdot K) \right.$$

$$- m_{h^0}^2 K^4 - 2K^4 (P_1 \cdot P_2) + K^6 - 4K^2 (P_1 \cdot K)(P_2 \cdot K) - 4 (P_1 \cdot K)(P_2 \cdot K)^2 + 2 m_{h^0}^2 (P_1 \cdot K)(P_2 \cdot K)$$

$$\left. + 4 (P_1 \cdot K)(P_2 \cdot K)(P_1 \cdot P_2) - K^4 (P_1 \cdot K) + K^2 m_{h^0}^2 (P_1 \cdot K) + 2K^2 (P_1 \cdot P_2)(P_1 \cdot K) \right] + \frac{K^\mu K^\nu}{M_W^4} \left[K^4 + 2K^2 (P_2 \cdot K) \right.$$

$$\left. - K^2 m_{h^0}^2 - 2K^2 (P_1 \cdot P_2) - K^2 (P_1 \cdot K) - 2 (P_1 \cdot K)(P_2 \cdot K) \right] + \frac{P_1^\mu K^\nu}{M_W^4} \left[2 (P_1 \cdot P_2)(P_1 \cdot K) + (P_1 \cdot K) m_{h^0}^2 \right]$$

$$- \frac{P_2^\mu K^\nu}{M_W^4} \left[K^4 + 2K^2 (P_2 \cdot K) - K^2 m_{h^0}^2 - 2K^2 (P_1 \cdot P_2) - K^2 (P_1 \cdot K) - 2 (P_1 \cdot K)(P_2 \cdot K) + m_{h^0}^2 (P_1 \cdot K) \right.$$

$$\left. + 2 (P_1 \cdot P_2)(P_1 \cdot K) \right] \Big\} \cdot \frac{1}{[K^2 - M_W^2][(K-P_1)^2 - M_W^2][(K+P_2)^2 - M_W^2]} \cdot \varepsilon_{2\nu}^* \varepsilon_{0\mu} \qquad (141)$$

$$\frac{1}{abc} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[a(1-x-y) + bx + cy]^3}$$

$$a = K^2 - M_W^2 \, ; \quad b = (K-P_1)^2 - M_W^2 \, ; \quad c = (K+P_2)^2 - M_W^2$$

$$a(1-x-y) + bx + cy = (K^2 - M_W^2)(1-x-y) + [(K-P_1)^2 - M_W^2]x + [(K+P_2)^2 - M_W^2]y$$

$$= \cancel{K^2} - \cancel{K^2 x} - \cancel{K^2 y} - \cancel{M_W^2} + \cancel{M_W^2 x} + \cancel{M_W^2 y} + \cancel{K^2 x} - 2(\cancel{P_1}\cdot K)x + m_{\cancel{h^0}}^2 x - \cancel{M_W^2 x}$$
$$+ \cancel{K^2 y} + 2(P_2\cdot \cancel{K})y - \cancel{M_W^2 y}$$

$$= K^2 - 2(P_1 x - P_2 y)\cdot K + m_{h^0}^2 x - M_W^2$$

$$\frac{1}{[K^2 - M_W^2][(K-P_1)^2 - M_W^2][(K+P_2)^2 - M_W^2]} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[K^2 - M_W^2 - 2K\cdot(P_1 x - P_2 y) + m_{h^0}^2 x]^3}$$

$$\boxed{K' = K - \overset{h}{\widehat{(P_1 x - P_2 y)}}} = K - h \qquad (142)$$

$$K'^2 = K^2 - 2K\cdot(P_1 x - P_2 y) + (m_{h^0}^2 x^2 - 2 P_1\cdot P_2\, xy)$$

$$d^d K' = d^d K$$

$$\Rightarrow \frac{1}{[K^2 - M_W^2][(K-P_1)^2 - M_W^2][(K+P_2)^2 - M_W^2]} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[K'^2 - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy - M_W^2 + m_{h^0}^2 x]^3}$$

(143)

The terms without $\frac{1}{M_W^2}$, $\frac{1}{M_W^4}$ in (126) + (141) are:

$$10(K'-h)^\mu (K'-h)^\nu - 9P_2^\mu(\cancel{K'}-h)^\nu + [4\cancel{(P_1\cdot P_2)} + 2(\cancel{K'}-h)^2 + P_1\cdot(\cancel{K'}-\cancel{h})$$
$$-2P_2\cdot(\cancel{K'}-h)]\eta^{\mu\nu} + 10(\cancel{K'}+h)^\mu (K'+h)^\nu + 9P_2^\mu(\cancel{K'}+h)^\nu + [4\cancel{(P_1\cdot P_2)}$$
$$+2P_2\cdot(\cancel{K'}+h) - P_1\cdot(\cancel{K'}+\cancel{h}) + 2(\cancel{K'}+h)^2]\eta^{\mu\nu}$$

$$= 10\cancel{K'^\mu} K'^\nu - 10\cancel{K'^\mu} h^\nu - 10\cancel{h^\mu} K'^\nu + 10\cancel{h^\mu} h^\nu + 18\cancel{P_2^\mu} h^\nu + 10\cancel{K'^\mu} K'^\nu$$
$$+ 10\cancel{K'^\mu} h^\nu + 10\cancel{h^\mu} K'^\nu + 10\cancel{h^\mu} h^\nu + \eta^{\mu\nu}[8(P_1\cdot P_2) + 4K'^2 + 4h^2$$
$$-2P_1\cdot h + 4P_2\cdot h]$$

no contribution because $P_1^\nu, P_2^\nu \varepsilon_{2\nu}^{*} = 0$

$$= 20K'^\mu K'^\nu + 20h^\mu \cancel{h^\nu} + 18P_2^\mu \cancel{h^\nu} + \eta^{\mu\nu}[8(P_1\cdot P_2) + 4K'^2 + 4h^2 - 2(P_1\cdot h)$$
$$+ 4(P_2\cdot h)] = \underbrace{20K'^\mu K'^\nu + \eta^{\mu\nu}[8(P_1\cdot P_2) + 4K'^2 + 4h^2 - 2(P_1\cdot h) + 4(P_2\cdot h)]}$$

(144) OK

$$-i(M_3+M_4) = g^3 M_W \sin(\beta-\alpha)\sin\theta_W\cos\theta_W\, 2\int_0^1 dx\int_0^{1-x} dy\int\frac{d^dk'}{(2\pi)^d}\Big\{ 20\, k'^\mu k'^\nu$$

$$+\eta^{\mu\nu}\left[8(p_1\cdot p_2)+4k'^2+4h^2-2(p_1\cdot h)+4(p_2\cdot h)\right]+\frac{k'^\mu k'^\nu}{M_W^2}\left[6k'^2+6h^2-2(p_1\cdot h)+8(p_2\cdot h)\right.$$

$$\left.-12(p_1\cdot p_2)-2m_{h^0}^2\right]+\frac{h^\mu k'^\nu}{M_W^2}\left[12(k'\cdot h)-2(p_1\cdot k')+8(p_2\cdot k')\right]+\frac{p_1^\mu k'^\nu}{M_W^2}\left[-8(k'\cdot h)\right.$$

$$\left.-12(p_2\cdot k')-4(p_1\cdot k')\right]+\frac{p_2^\mu k'^\nu}{M_W^2}\left[8(k'\cdot h)+2(p_1\cdot k')+8(p_2\cdot k')\right]+\frac{\eta^{\mu\nu}}{M_W^2}\left[-6k'^4\right.$$

$$-12k'^2h^2-24(k'\cdot h)^2-6h^4+8(p_1\cdot k')(k'\cdot h)+4(p_1\cdot h)k'^2+4(p_1\cdot h)h^2+12(p_1\cdot p_2)k'^2+12(p_1\cdot p_2)h^2$$

$$-16(p_2\cdot k')^2-16(p_2\cdot h)^2-32(p_2\cdot k')(k'\cdot h)-16(p_2\cdot h)k'^2-16(p_2\cdot h)h^2+16(p_1\cdot p_2)(p_2\cdot h)$$

$$+4m_{h^0}^2(p_2\cdot h)+2k'^2m_{h^0}^2+2h^2m_{h^0}^2-4(p_1\cdot p_2)m_{h^0}^2-2(p_1\cdot h)m_{h^0}^2-8(p_1\cdot p_2)^2$$

$$\left.-4(p_1\cdot p_2)(p_1\cdot h)\right]+\frac{k'^\mu k'^\nu}{M_W^4}\Big\{-2k'^2(p_1\cdot k')-4(k'\cdot h)(p_1\cdot h)-2h^2(p_1\cdot k')+2m_{h^0}^2(p_1\cdot k')$$

$$-2m_{h^0}^2(p_1\cdot h)+4k'^2(p_2\cdot k')+8(k'\cdot h)(p_2\cdot h)+4h^2(p_2\cdot k')-8(k'\cdot h)(p_1\cdot p_2)-4(p_1\cdot k')(p_2\cdot h)$$

$$-4(p_1\cdot h)(p_2\cdot k')+4(p_1\cdot p_2)(p_1\cdot k')-4(p_1\cdot p_2)(p_1\cdot h)+8k'^2(k'\cdot h)+8h^2(k'\cdot h)-4(k'\cdot h)m_{h^0}^2\Big\}$$

$$+\frac{h^\mu k'^\nu}{M_W^4}\left[-2k'^2(p_1\cdot h)-4(k'\cdot h)(p_1\cdot k')-2h^2(p_1\cdot h)-2m_{h^0}^2(p_1\cdot k')+2m_{h^0}^2(p_1\cdot h)\right.$$

$$+4k'^2(p_2\cdot h)+8(k'\cdot h)(p_2\cdot k')+4h^2(p_2\cdot h)-4k'^2(p_1\cdot p_2)-4h^2(p_1\cdot p_2)-4(p_1\cdot k')(p_2\cdot k')$$

$$-4(p_1\cdot h)(p_2\cdot h)-4(p_1\cdot p_2)(p_1\cdot k')+4(p_1\cdot p_2)(p_1\cdot h)+2k'^4+8(k'\cdot h)^2+4k'^2h^2+2h^4-2k'^2m_{h^0}^2$$

$$\left.-2h^2m_{h^0}^2\right]+\frac{p_1^\mu k'^\nu}{M_W^4}\left[4(p_1\cdot p_2)(p_1\cdot k')+2m_{h^0}^2(p_1\cdot k')\right]+\frac{p_2^\mu k'^\nu}{M_W^4}\left[2k'^2(p_1\cdot k')+\right.$$

$$+4(k'\cdot h)(p_1\cdot h)+2h^2(p_1\cdot k')-4k'^2(p_2\cdot k')-8(k'\cdot h)(p_2\cdot h)-4h^2(p_2\cdot k')+4m_{h^0}^2(k'\cdot h)$$

$$+8(p_1\cdot p_2)(k'\cdot h)+4(p_1\cdot k')(p_2\cdot h)+4(p_1\cdot h)(p_2\cdot k')-8k'^2(k'\cdot h)-8h^2(k'\cdot h)-2m_{h^0}^2(p_1\cdot k')$$

$$\left.-4(p_1\cdot p_2)(p_1\cdot k')\right]+\frac{\eta^{\mu\nu}}{M_W^4}\left[8k'^2(p_2\cdot k')^2+8h^2(p_2\cdot k')^2+32(k'\cdot h)(p_2\cdot k')(p_2\cdot h)+8k'^2(p_2\cdot h)^2\right.$$

$$+8h^2(p_2\cdot h)^2+8k'^4(p_2\cdot h)+32k'^2(k'\cdot h)(p_2\cdot k')+16k'^2h^2(p_2\cdot h)+32(k'\cdot h)^2(p_2\cdot h)+32h^2(k'\cdot h)(p_2\cdot k')+8h^4(p_2\cdot h)$$

$$+2k'^6+6k'^4h^2+24k'^2(k'\cdot h)^2+24h^2(k'\cdot h)^2+6k'^2h^4+2h^6-4m_{h^0}^2k'^2(p_2\cdot h)-8m_{h^0}^2(k'\cdot h)(p_2\cdot k')-4m_{h^0}^2h^2(p_2\cdot h)$$

$$-2m_{h^0}^2k'^4-8m_{h^0}^2(k'\cdot h)^2-4m_{h^0}^2k'^2h^2-2m_{h^0}^2h^4-4(p_1\cdot p_2)k'^4-8(p_1\cdot p_2)k'^2h^2-4(p_1\cdot p_2)h^4-16(p_1\cdot p_2)(k'\cdot h)^2$$

$$-8(p_1\cdot h)(p_2\cdot k')^2-16(p_1\cdot k')(p_2\cdot k')(p_2\cdot h)-8(p_1\cdot h)(p_2\cdot h)^2-8k'^2(p_1\cdot k')(p_2\cdot k')-8k'^2(p_1\cdot h)(p_2\cdot h)$$

$$-16(k'\cdot h)(p_1\cdot k')(p_2\cdot h)-16(k'\cdot h)(p_1\cdot h)(p_2\cdot k')-8(k'\cdot h)^2(p_1\cdot h)-8h^2(p_1\cdot k')(p_2\cdot k')-8h^2(p_1\cdot h)(p_2\cdot h)-2k'^4(p_1\cdot h)$$

$$-8k'^2(k'\cdot h)(p_1\cdot k')-4k'^2h^2(p_1\cdot h)-8h^2(k'\cdot h)(p_1\cdot k')-2h^4(p_1\cdot h)+4m_{h^0}^2(p_1\cdot k')(p_2\cdot k')+4m_{h^0}^2\times$$

$$(p_1\cdot h)(p_2\cdot h)+2m_{h^0}^2k'^2(p_1\cdot h)+2m_{h^0}^2h^2(p_1\cdot h)+4m_{h^0}^2(p_1\cdot k')(k'\cdot h)+8(p_1\cdot p_2)(p_1\cdot k')(p_2\cdot k')$$

$$+8(p_1\cdot p_2)(p_1\cdot h)(p_2\cdot h)+4(p_1\cdot p_2)k'^2(p_1\cdot h)+8(p_1\cdot p_2)(p_1\cdot k')(k'\cdot h)+4(p_1\cdot p_2)h^2(p_1\cdot h)$$

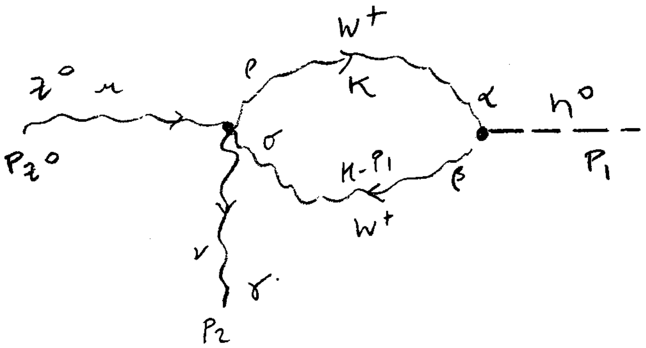
$$\left.-8(p_1\cdot p_2)k'^2(p_2\cdot h)-16(p_1\cdot p_2)(k'\cdot h)(p_2\cdot k')-8h^2(p_1\cdot p_2)(p_2\cdot h)\right]\Big\}\times$$

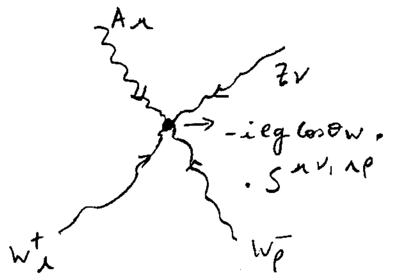
$$\times\frac{1}{\left[k'^2-m_{h^0}^2x^2+2(p_1\cdot p_2)xy-M_W^2+m_{h^0}^2x\right]^3}\times\varepsilon^*_{2\nu}\,\varepsilon_{0\mu}\qquad(153)$$

$$h\equiv p_1x-p_2y$$

7

$P_{Z^0} = P_1 + P_2$





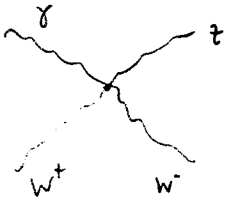
where: $S^{\mu\nu,\lambda\rho} = 2\eta^{\mu\nu}\eta^{\lambda\rho} - \eta^{\mu\lambda}\eta^{\nu\rho} - \eta^{\mu\rho}\eta^{\nu\lambda}$

$$-iM_7 = \int \frac{d^d K}{(2\pi)^d}\left[ig M_W \sin(\beta-\alpha)\,\eta^{\alpha\beta}\right](-i)\left(\eta_{\beta\sigma} - \frac{(K-P_1)_\beta (K-P_1)_\sigma}{M_W^2}\right)\frac{1}{(K-P_1)^2 - M_W^2}$$

$$\cdot(-ieg\cos\theta_W)\left[2\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\rho\nu} - \eta^{\mu\rho}\eta^{\nu\sigma}\right](-i)\Big(\eta_{\rho\alpha} -$$

$$-\frac{K_\rho K_\alpha}{M_W^2}\Big)\frac{1}{K^2 - M_W^2}\,\varepsilon^*_{Z\nu}\,\varepsilon_{0\mu}\;\underbrace{\mu_1^{\frac{(4-d)}{2}}\;\mu_{2'}^{(4-d)}}$$

$\mu^{\prime\prime *} \to 1$ when $d \to 4$.



$$4\left(\frac{d}{2}-1\right) + X = d$$

$$X = d - 2d + 4$$

$$X = 4 - d$$

$$-iM_7 = -eg^2 M_W \sin(\beta-\alpha)\cos\theta_W \int \frac{d^d K}{(2\pi)^d}\,\eta^{\alpha\beta}\left(\eta_{\beta\sigma} - \frac{(K-P_1)_\beta (K-P_1)_\sigma}{M_W^2}\right)$$

$$\left[2\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\rho\nu} - \eta^{\mu\rho}\eta^{\nu\sigma}\right]\left(\eta_{\rho\alpha} - \frac{K_\rho K_\alpha}{M_W^2}\right)\cdot$$

$$\cdot \frac{1}{[K^2 - M_W^2][(K-P_1)^2 - M_W^2]}\,\varepsilon^*_{Z\nu}\,\varepsilon_{0\mu}$$

$$= -eg^2 M_W \sin(\beta-\alpha)\cos\theta_W \int \frac{d^d K}{(2\pi)^d}\left(\delta^\alpha_\sigma - \frac{(K-P_1)^\alpha (K-P_1)_\sigma}{M_W^2}\right)$$

$$\left[2\eta^{\mu\nu}\delta^\sigma_\alpha - \eta^{\mu\sigma}\delta^\nu_\alpha - \eta^{\nu\sigma}\delta^\mu_\alpha - \frac{2\eta^{\mu\nu}K^\sigma K_\alpha}{M_W^2} + \frac{\eta^{\mu\sigma}K^\nu K_\alpha}{M_W^2} + \frac{\eta^{\nu\sigma}K^\mu K_\alpha}{M_W^2}\right]$$

$$\cdot \frac{1}{[k^2 - M_W^2][(k-P_1)^2 - M_W^2]} \varepsilon^{*}_{2\nu} \varepsilon_{0\mu}$$

$$-iM_7 = -eg^2 M_W \cos\theta_W \sin(\beta-\alpha) \int \frac{d^dk}{(2\pi)^d} \Big[8\cancel{\eta}^{\mu\nu} - \cancel{\eta}^{\mu\nu} - \cancel{\eta}^{\mu\nu} - \frac{2\eta^{\mu\nu}}{M_W^2} k^2 + 2\frac{k^\mu k^\nu}{M_W^2}$$

$$- \frac{2\eta^{\mu\nu}}{M_W^2} (k-P_1)^2 + 2\frac{(k-P_1)^\mu (k-P_1)^\nu}{M_W^2} + \frac{2\eta^{\mu\nu}}{M_W^4} (k\cdot(k-P_1))^2 - \frac{(k-P_1)^\mu k^\nu (k\cdot(k-P_1))}{M_W^4}$$

$$- \frac{k^\mu (k-P_1)^\nu (k\cdot(k-P_1))}{M_W^4} \Big] \cdot \frac{1}{[k^2 - M_W^2][(k-P_1)^2 - M_W^2]} \varepsilon^{*}_{2\nu} \varepsilon_{0\mu}$$

$$= -eg^2 M_W \cos\theta_W \int \frac{d^dk}{(2\pi)^d} \Big[6\eta^{\mu\nu} - 2\frac{k^2}{M_W^2}\eta^{\mu\nu} + 2\frac{k^\mu k^\nu}{M_W^2} - \frac{2\eta^{\mu\nu}}{M_W^2} (k^2 - 2k\cdot P_1$$

$$+ m_{h0}^2) + \frac{2}{M_W^2} (k^\mu k^\nu - k^\mu \cancel{P_1^\nu} - \underbrace{P_1^\mu k^\nu + P_1^\mu \cancel{P_1^\nu}}_{P_1^\nu \varepsilon^{*}_{2\nu} = 0}) + \frac{2\eta^{\mu\nu}}{M_W^4} (k^2 - P_1\cdot k)^2$$

$$- \frac{(k^\mu k^\nu - P_1^\mu k^\nu)(k^2 - P_1\cdot k)}{M_W^4} - \frac{(k^\mu k^\nu - \cancel{k^\mu P_1^\nu})(k^2 - P_1\cdot k)}{M_W^4} \Big] \sin(\beta-\alpha)$$

$$\cdot \frac{1}{[k^2 - M_W^2][(k-P_1)^2 - M_W^2]} \varepsilon^{*}_{2\nu} \varepsilon_{0\mu}$$

$$-iM_7 = -g^3 M_W \sin\theta_W \cos\theta_W \sin(\beta-\alpha) \int \frac{d^dk}{(2\pi)^d} \Big[6\eta^{\mu\nu} + \frac{4k^\mu k^\nu}{M_W^2} + \frac{\eta^{\mu\nu}}{M_W^2}\big(-4k^2$$

$$+ 4(P_1\cdot k) - 2m_{h0}^2\big) - 2\frac{P_1^\mu k^\nu}{M_W^2} + \frac{2\eta^{\mu\nu}}{M_W^4}\big(k^4 - 2k^2(P_1\cdot k) + (P_1\cdot k)^2\big)$$

$$- \frac{2k^\mu k^\nu}{M_W^4}(k^2 - P_1\cdot k) + \frac{P_1^\mu k^\nu}{M_W^4}(k^2 - P_1\cdot k) \Big] \big[(k+P_2)^2 - M_W^2\big] \cdot$$

$$\cdot \frac{1}{[k^2 - M_W^2][(k-P_1)^2 - M_W^2][(k+P_2)^2 - M_W^2]} \varepsilon^{*}_{2\nu} \varepsilon_{0\mu}$$

$$-iM_7 = -g^3 M_W \sin\theta_W \cos\theta_W \sin(\beta-\alpha) \int \frac{d^d K}{(2\pi)^d} \Big[6\eta^{\mu\nu}\left(K^2 + 2(P_2\cdot K) - M_W^2\right)$$

$$+ \frac{4K^\mu K^\nu}{M_W^2}\left(K^2 + 2(P_2\cdot K) - M_W^2\right) + \frac{\eta^{\mu\nu}}{M_W^2}\Big(-4K^4 - 8K^2(P_2\cdot K) + 4K^2 M_W^2 + 4K^2(P_1\cdot K)$$

$$+ 8(P_1\cdot K)(P_2\cdot K) - 4(P_1\cdot K)M_W^2 - 2m_{h^0}^2 K^2 - 4m_{h^0}^2(P_2\cdot K) + 2m_{h^0}^2 M_W^2\Big) - \frac{2P_1^\mu K^\nu}{M_W^2}\left(K^2 + 2P_2\cdot K\right.$$

$$\left.- M_W^2\right) + \frac{2\eta^{\mu\nu}}{M_W^4}\Big(K^6 + 2K^4(P_2\cdot K) - K^4 M_W^2 - 2K^4(P_1\cdot K) - 4K^2(P_1\cdot K)(P_2\cdot K) + 2K^2 M_W^2(P_1\cdot K)$$

$$+ K^2(P_1\cdot K)^2 + 2(P_1\cdot K)^2(P_2\cdot K) - M_W^2(P_1\cdot K)^2\Big) - \frac{2K^\mu K^\nu}{M_W^4}\Big(K^4 + 2K^2(P_2\cdot K) - K^2 M_W^2 - K^2(P_1\cdot K)$$

$$- 2(P_1\cdot K)(P_2\cdot K) + M_W^2(P_1\cdot K)\Big) + \frac{P_1^\mu K^\nu}{M_W^4}\Big(K^4 + 2K^2(P_2\cdot K) - K^2 M_W^2 - K^2(P_1\cdot K) - 2(P_1\cdot K)(P_2\cdot K)$$

$$+ M_W^2(P_1\cdot K)\Big)\Big] \cdot \frac{1}{[K^2 - M_W^2][(K-P_1)^2 - M_W^2][(K+P_2)^2 - M_W^2]}\, \varepsilon^*_{2\nu}\varepsilon_{0\mu}$$

$$= -g^3 M_W \sin\theta_W \cos\theta_W \sin(\beta-\alpha) \int \frac{d^d K}{(2\pi)^d} \Big\{ -4K^\mu K^\nu + 2P_1^\mu K^\nu + \eta^{\mu\nu}\big(10K^2$$

$$+ 12(P_2\cdot K) - 4(P_1\cdot K) - 6M_W^2 + 2m_{h^0}^2\big) + \frac{K^\mu K^\nu}{M_W^2}\left[6K^2 + 8(P_2\cdot K) - 2(P_1\cdot K)\right]$$

$$+ \frac{P_1^\mu K^\nu}{M_W^2}\left[-3K^2 - 4(P_2\cdot K) + (P_1\cdot K)\right] + \frac{\eta^{\mu\nu}}{M_W^2}\Big[-6K^4 - 8K^2(P_2\cdot K) + 8K^2(P_1\cdot K)$$

$$+ 8(P_1\cdot K)(P_2\cdot K) - 2m_{h^0}^2 K^2 - 4m_{h^0}^2(P_2\cdot K) - 2(P_1\cdot K)^2\Big] + \frac{\eta^{\mu\nu}}{M_W^4}\Big[2K^6 + 4K^4(P_2\cdot K)$$

$$-4K^4(P_1\cdot K) - 8K^2(P_1\cdot K)(P_2\cdot K) + 2K^2(P_1\cdot K)^2 + 4(P_1\cdot K)^2(P_2\cdot K)\Big] + \frac{K^\mu K^\nu}{M_W^4}\Big[-2K^4$$

$$-4K^2(P_2\cdot K) + 2K^2(P_1\cdot K) + 4(P_1\cdot K)(P_2\cdot K)\Big] + \frac{P_1^\mu K^\nu}{M_W^4}\Big[K^4 + 2K^2(P_2\cdot K) - K^2(P_1\cdot K)$$

$$-2(P_1\cdot K)(P_2\cdot K)\Big]\Big\} \cdot \frac{1}{[K^2 - M_W^2][(K-P_1)^2 - M_W^2][(K+P_2)^2 - M_W^2]}\, \varepsilon^*_{2\nu}\varepsilon_{0\mu}$$

$$\frac{1}{[K^2 - M_W^2][(K-P_1)^2 - M_W^2][(K+P_2)^2 - M_W^2]} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[K'^2 - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy - M_W^2 + m_{h^0}^2 x]^3}$$

Where $K' = K - (P_1 x - P_2 y) = K - h$

$d^d K' = d^d K$

$$-iM_7 = -g^3 M_W \sin\theta_W \cos\theta_W \sin(\beta-\alpha)\; 2\int_0^1 dx \int_0^{1-x} dy \int \frac{d^dK'}{(2\pi)^d}\Big\{-4K'^\mu K'^\nu - 4k^\mu K'^\nu$$

$$+2P_1^\mu K'^\nu + n^{\mu\nu}\big(10K'^2 + 20(K'\cdot k) + 10k^2 + 12(P_2\cdot K') + 12(P_2\cdot k) - 4(P_1\cdot K')$$

$$-4(P_1\cdot k) - 6M_W^2 + 2m_{h^0}^2\big) + \frac{K'^\mu K'^\nu}{M_W^2}\big(6K'^2 + 12(K'\cdot k) + 6k^2 + 8(P_2\cdot K') + 8(P_2\cdot k)$$

$$-2(P_1\cdot K') - 2(P_1\cdot k)\big) + \frac{k^\mu K'^\nu}{M_W^2}\big(6K'^2 + 12(K'\cdot k) + 6k^2 + 8(P_2\cdot K') + 8(P_2\cdot k) - 2(P_1\cdot K')$$

$$-2(P_1\cdot k)\big) + \frac{P_1^\mu K'^\nu}{M_W^2}\big(-3K'^2 - 6(K'\cdot k) - 3k^2 - 4(P_2\cdot K') - 4(P_2\cdot k) + (P_1\cdot K') + (P_1\cdot k)\big)$$

$$+\frac{n^{\mu\nu}}{M_W^2}\big(-6K'^4 - 24K'^2(K'\cdot k) - 12k^2K'^2 - 24(K'\cdot k)^2 - 24k^2(K'\cdot k) - 6k^4 - 8K'^2(P_2\cdot K')$$

$$-8K'^2(P_2\cdot k) - 16(K'\cdot k)(P_2\cdot K') - 16(K'\cdot k)(P_2\cdot k) - 8k^2(P_2\cdot K') - 8k^2(P_2\cdot k) + 8K'^2(P_1\cdot K')$$

$$+8K'^2(P_1\cdot k) + 16(K'\cdot K)(P_1\cdot K') + 16(K'\cdot k)(P_1\cdot k) + 8k^2(P_1\cdot K') + 8k^2(P_1\cdot k) + 8(P_1\cdot K')(P_2\cdot K')$$

$$+8(P_1\cdot K')(P_2\cdot k) + 8(P_1\cdot k)(P_2\cdot K') + 8(P_1\cdot k)(P_2\cdot k) - 2m_{h^0}^2K'^2 - 4m_{h^0}^2(K'\cdot k) - 2m_{h^0}^2k^2$$

$$-4m_{h^0}^2(P_2\cdot K') - 4m_{h^0}^2(P_2\cdot k) - 2(P_1\cdot K')^2 - 4(P_1\cdot K')(P_1\cdot k) - 2(P_1\cdot k)^2\big)$$

$$+\frac{n^{\mu\nu}}{M_W^4}\Big[2K'^6 + 12K'^4(K'\cdot k) + 6K'^4k^2 + 24K'^2(K'\cdot k)^2 + 24K'^2k^2(K'\cdot k) + 16(K'\cdot k)^3$$

$$+24k^2(K'\cdot k)^2 + 6k^4K'^2 + 12k^4(K'\cdot k) + 2k^6 + 4K'^4(P_2\cdot K') + 4K'^4(P_2\cdot k)$$

$$+16K'^2(K'\cdot k)(P_2\cdot K') + 16K'^2(K'\cdot k)(P_2\cdot k) + 8k^2K'^2(P_2\cdot K') + 8k^2K'^2(P_2\cdot k) + 16(K'\cdot k)^2(P_2\cdot K')$$

$$+16(K'\cdot k)^2(P_2\cdot k) + 16k^2(K'\cdot k)(P_2\cdot K') + 16k^2(K'\cdot k)(P_2\cdot k) + 4k^4(P_2\cdot K') + 4k^4(P_2\cdot k) - 4K'^4(P_1\cdot K')$$

$$-4K'^4(P_1\cdot k) - 16K'^2(K'\cdot k)(P_1\cdot K') - 16K'^2(K'\cdot k)(P_1\cdot k) - 8k^2K'^2(P_1\cdot K') - 8k^2K'^2(P_1\cdot k) - 16(K'\cdot k)^2(P_1\cdot K')$$

$$-16(K'\cdot k)^2(P_1\cdot k) - 16k^2(K'\cdot k)(P_1\cdot K') - 16k^2(K'\cdot k)(P_1\cdot k) - 4k^4(P_1\cdot K') - 4k^4(P_1\cdot k)$$

$$-8K'^2(P_1\cdot K')(P_2\cdot K') - 8K'^2(P_1\cdot K')(P_2\cdot k) - 8K'^2(P_1\cdot k)(P_2\cdot K') - 8K'^2(P_1\cdot k)(P_2\cdot k) - 16(K'\cdot k)\times$$

$$(P_1\cdot K')(P_2\cdot K') - 16(K'\cdot k)(P_1\cdot K')(P_2\cdot k) - 16(K'\cdot k)(P_1\cdot k)(P_2\cdot K') - 16(K'\cdot k)(P_1\cdot k)(P_2\cdot k)$$

$$-8k^2(P_1\cdot K')(P_2\cdot K') - 8k^2(P_1\cdot K')(P_2\cdot k) - 8k^2(P_1\cdot k)(P_2\cdot K') - 8k^2(P_1\cdot k)(P_2\cdot k) + 2K'^2(P_1\cdot K')^2$$

$$+4K'^2(P_1\cdot K')(P_1\cdot k) + 2K'^2(P_1\cdot k)^2 + 4(K'\cdot k)(P_1\cdot K')^2 + 8(K'\cdot k)(P_1\cdot K')(P_1\cdot k) + 4(K'\cdot k)(P_1\cdot k)^2$$

$$+2k^2(P_1\cdot K')^2 + 4k^2(P_1\cdot K')(P_1\cdot k) + 2k^2(P_1\cdot k)^2 + 4(P_1\cdot K')^2(P_2\cdot K') + 4(P_1\cdot K')^2(P_2\cdot k)$$

$$+8(P_1\cdot K')(P_1\cdot k)(P_2\cdot K') + 8(P_1\cdot K')(P_1\cdot k)(P_2\cdot k) + 4(P_1\cdot k)^2(P_2\cdot K') + 4(P_1\cdot k)^2(P_2\cdot k)\Big]$$

$$+\frac{K'^\mu K'^\nu}{M_W^4}\Big[-2K'^4 - 8K'^2(K'\cdot k) - 4k^2K'^2 - 8(K'\cdot k)^2 - 8k^2(K'\cdot k) - 2k^4 - 4K'^2(P_2\cdot K') - 4K'^2(P_2\cdot k)$$

$$-8(K'\cdot K)(P_2\cdot K') - 8(K'\cdot k)(P_2\cdot k) - 4k^2(P_2\cdot K') - 4k^2(P_2\cdot k) + 2K'^2(P_1\cdot K') + 2K'^2(P_1\cdot k) + 4(K'\cdot k)(P_1\cdot K')$$

$$+4(K'\cdot k)(P_1\cdot k) + 2k^2(P_1\cdot K') + 2k^2(P_1\cdot k) + 4(P_1\cdot K')(P_2\cdot K') + 4(P_1\cdot K')(P_2\cdot k) + 4(P_1\cdot k)(P_2\cdot K') + 4(P_1\cdot k)(P_2\cdot k)\Big]$$

$$+\frac{k^\mu K'^\nu}{M_W^4}\Big[-2K'^4 - 8K'^2(K'\cdot k) - 4k^2K'^2 - 8(K'\cdot k)^2 - 8k^2(K'\cdot k) - 2k^4 - 4K'^2(P_2\cdot K') - 4K'^2(P_2\cdot k) - 8(K'\cdot k)(P_2\cdot K')$$

$$-8(K'\cdot k)(P_2\cdot k) - 4k^2(P_2\cdot K') - 4k^2(P_2\cdot k) + 2K'^2(P_1\cdot K') + 2K'^2(P_1\cdot k) + 4(K'\cdot k)(P_1\cdot K') + 4(K'\cdot k)(P_1\cdot k)$$

$$+2k^2(P_1\cdot K') + 2k^2(P_1\cdot k) + 4(P_1\cdot K')(P_2\cdot K') + 4(P_1\cdot K')(P_2\cdot k) + 4(P_1\cdot k)(P_2\cdot K') + 4(P_1\cdot k)(P_2\cdot k)\Big]$$

$$+ \frac{P_1^{\mu} k'^{\nu}}{M_W^4} \Big[k'^4 + 4k'^2 (k'\cdot k) + 2k^2 k'^2 + 4(k'\cdot k)^2 + 4k^2 (k'\cdot k) + k^4 + 2k'^2 (P_2\cdot k')$$

$$+ 2k'^2 (P_2\cdot k) + 4(k'\cdot k)(P_2\cdot k') + 4(k'\cdot k)(P_2\cdot k) + 2k^2 (P_2\cdot k') + 2k^2 (P_2\cdot k) - k'^2 (P_1\cdot k')$$

$$- k'^2 (P_1\cdot k) - 2(k'\cdot k)(P_1\cdot k') - 2(k'\cdot k)(P_1\cdot k) - k^2 (P_1\cdot k') - k^2 (P_1\cdot k) - 2(P_1\cdot k')(P_2\cdot k') - 2(P_1\cdot k') \times$$

$$(P_2\cdot k) - 2(P_1\cdot k)(P_2\cdot k') - 2(P_1\cdot k)(P_2\cdot k) \Big] \Big\} \times \frac{1 \times \epsilon^{*}_{2\nu}\, \epsilon_{0\mu}}{\left[k'^2 - m_{h^0}^2 x^2 + 2(P_1\cdot P_2)xy - M_W^2 + m_{h^0}^2 x\right]^3}$$

$$-i(M_3+M_4+M_7) = g^3 M_W \sin\theta_W \cos\theta_W \sin(\beta-\alpha)\, 2\int_0^1 dx\int_0^{1-x} dy \int \frac{d^dk'}{(2\pi)^d}\Big\{ \overset{OK}{24\, k'^\mu k'^\nu}$$

$$+\,\eta^{\mu\nu}\,\overset{OK}{[}-6k'^2 - 6h^2 + 8(P_1\cdot P_2) + 2(P_1\cdot h) - 8(P_2\cdot h) + 6M_W^2 - 2m_{h^0}^2\,] + \frac{k'^\mu k'^\nu}{M_W^2}[-12\overset{\downarrow}{(P_1\cdot P_2)}$$

$$-2m_{h^0}^2\,] + \frac{\eta^{\mu\nu}}{M_W^2}[-8(P_1\cdot k')(k'\cdot h) - 4k'^2(P_1\cdot h) - 4h^2(P_1\cdot h) + 12\overset{\downarrow}{(P_1\cdot P_2)}k'^2 + 12(P_1\cdot P_2)h^2$$

$$-16\cancel{(P_2\cdot k')}^2 - 16(P_2\cdot h)^2 - 16(P_2\cdot k')(k'\cdot h) - 8k'^2(P_2\cdot h) - 8h^2(P_2\cdot h) + 16(P_1\cdot P_2)(P_2\cdot h) + 8m_{h^0}^2(P_2\cdot h)$$

$$+4k'^2m_{h^0}^2 + 4h^2m_{h^0}^2 - 4(P_1\cdot P_2)m_{h^0}^2 - 2m_{h^0}^2(P_1\cdot h) - 8(P_1\cdot P_2)^2 - 4(P_1\cdot P_2)(P_1\cdot h)$$

$$-8(P_1\cdot k')\underset{\uparrow}{(P_2\cdot k')} - 8(P_1\cdot h)(P_2\cdot h) + 2(P_1\cdot k')^2 + 2(P_1\cdot h)^2\,] + \frac{\eta^{\mu\nu}}{M_W^4}[\,8k'^2\cancel{(P_2\cdot k')}^2 + 8h^2\cancel{(P_2\cdot k')}^2$$

$$+32(k'\cdot h)(P_2\cdot k')(P_2\cdot h) + 8k'^2(P_2\cdot h)^2 + 8h^2(P_2\cdot h)^2 + 4k'^4(P_2\cdot h) + 16k'^2(k'\cdot h)(P_2\cdot k')$$

$$+8k'^2h^2(P_2\cdot h) + 16(k'\cdot h)^2(P_2\cdot h) + 16h^2(k'\cdot h)(P_2\cdot k') + 4h^4(P_2\cdot h) - 4m_{h^0}^2k'^2(P_2\cdot h)$$

$$-8m_{h^0}^2\underset{\downarrow}{(k'\cdot h)}(P_1\cdot k') - 4m_{h^0}^2h^2(P_2\cdot h) - 2m_{h^0}^2k'^4 - 8m_{h^0}^2(k'\cdot h)^2 - 4m_{h^0}^2k'^2h^2 - 2m_{h^0}^2h^4$$

$$-4(P_1\cdot P_2)k'^4 - 8(P_1\cdot P_2)k'^2h^2 - 4(P_1\cdot P_2)h^4 - 16(P_1\cdot P_2)(k'\cdot h)^2 - 8(P_1\cdot h)\cancel{(P_2\cdot k')}^2 - 16(P_1\cdot k')(P_2\cdot k')\times$$

$$\times(P_2\cdot h) - 8(P_1\cdot h)(P_2\cdot h)^2 - 2k'^4(P_1\cdot h) + 8k'^2(k'\cdot h)(P_1\cdot k') + 4k'^2h^2(P_1\cdot h) + 8(k'\cdot h)^2(P_1\cdot h)$$

$$+8h^2(k'\cdot h)(P_1\cdot k') + 2h^4(P_1\cdot h) + 4m_{h^0}^2(P_1\cdot k')(P_2\cdot k') + 4m_{h^0}^2(P_1\cdot h)(P_2\cdot h) + 2m_{h^0}^2k'^2(P_1\cdot h)$$

$$+2m_{h^0}^2h^2(P_1\cdot h) + 4m_{h^0}^2(P_1\cdot k')(k'\cdot h) + 8(P_1\cdot P_2)(P_1\cdot k')(P_2\cdot k') + 8(P_1\cdot P_2)(P_1\cdot h)(P_2\cdot h)$$

$$+4(P_1\cdot P_2)k'^2(P_1\cdot h) + 8(P_1\cdot P_2)(P_1\cdot k')(k'\cdot h) + 4(P_1\cdot P_2)h^2(P_1\cdot h) - 8(P_1\cdot P_2)k'^2(P_2\cdot h)$$

$$-16(P_1\cdot P_2)(k'\cdot h)(P_2\cdot k') - 8h^2(P_1\cdot P_2)(P_2\cdot h) - 2k'^2(P_1\cdot k')^2 - 2k'^2(P_1\cdot h)^2 - 8(k'\cdot h)(P_1\cdot k')(P_1\cdot h)$$

$$-2h^2(P_1\cdot k')^2 - 2h^2(P_1\cdot h)^2 - 4(P_1\cdot k')^2(P_2\cdot h) - 8(P_1\cdot k')(P_2\cdot k')(P_1\cdot h) - 4(P_1\cdot h)^2(P_2\cdot h)\,]$$

$$+\frac{k'^\mu k'^\nu}{M_W^4}[-2m_{h^0}^2(P_1\cdot h) - 4(P_1\cdot P_2)(P_1\cdot h) + 4h^2k'^2 + 8(k'\cdot h)^2 + 2h^4 + 4k'^2(P_2\cdot h)$$

$$+8(k'\cdot h)(P_2\cdot k') + 4h^2(P_2\cdot h) - 2k'^2(P_1\cdot h) - 4(k'\cdot h)(P_1\cdot k') - 2h^2(P_1\cdot h) - 4\overset{\downarrow}{(P_1\cdot k')}(P_2\cdot k')$$

$$-4(P_1\cdot h)(P_2\cdot h)\,]\Big\}\times\frac{1}{[k'^2 - m_{h^0}^2x^2 + 2(P_1\cdot P_2)xy - M_W^2 + m_{h^0}^2x\,]^3}\,\varepsilon^*_{2\nu}\,\varepsilon_{0\mu}\,.$$

$$\int\frac{d^dk'\,(P_2\cdot k')^2}{[k'^2 - m_{h^0}^2x^2 + 2(P_1\cdot P_2)xy - M_W^2 + m_{h^0}^2x\,]^3} = \int d^dk'\,\frac{k'^\alpha k'^\beta P_{2\alpha}P_{2\beta}}{[\qquad]^3}$$

$$= \frac{I_0}{2}\,\underbrace{\eta^{\alpha\beta}P_{2\alpha}P_{2\beta}}_{P_2^2\,=\,0}\,[\qquad]\,\frac{\Gamma(2-\frac{d}{2})}{\Gamma(3-\frac{d}{2})} = 0.$$

Similarly $$\int\frac{d^dk'\,k'^2(P_2\cdot k')^2}{[k'^2 - m_{h^0}^2x^2 + 2(P_1\cdot P_2)xy - M_W^2 + m_{h^0}^2x\,]^3} = 0$$

The fermion - charged Higgs contribution - W contribution to M is:

$$M = M_f + M_{H^\pm} + M_{W^\pm} = (M_1+M_2)_a + (M_1+M_2)_b + (M_5+M_6+M_8) + (M_3+M_4+M_7)$$

$$M = \frac{g e^2 \eta^{\mu\nu} \varepsilon_{0\mu} \varepsilon^*_{2\nu} (M_Z^2 - m_{h^0}^2)}{16\pi^2 M_W} \Bigg\{ \left(\frac{\sin\alpha}{\cos\beta}\right) \frac{1}{\sin\theta_W \cos\theta_W} \sum_{\substack{i=d,s,b\\ e,\mu,\tau}} Q_i C_V^i N_i F(\tau_i,\lambda_i)$$

$$+ \left(-\frac{\cos\alpha}{\sin\beta}\right) \frac{1}{\sin\theta_W \cos\theta_W} \sum_{i=u,c,t} Q_i C_V^i N_i F(\tau_i,\lambda_i) + \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W}\right] \times$$

$$\times \frac{(1-2\sin^2\theta_W)}{2\sin\theta_W\cos\theta_W} \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H,\lambda_H) - \frac{\sin(\beta-\alpha)}{2} \cot\theta_W \Bigg[4(3-\tan^2\theta_W) K(\tau_W,\lambda_W)$$

$$+ \left[\left(1+\frac{2}{\tau_W}\right)\tan^2\theta_W - \left(5+\frac{2}{\tau_W}\right)\right] I(\tau_W,\lambda_W)\Bigg]\Bigg\}$$

$$\tau_W = \frac{4M_W^2}{m_{h^0}^2} ; \quad \lambda_W = \frac{4M_W^2}{M_Z^2}$$

$$K(\tau_W,\lambda_W) = -\frac{\tau_W\lambda_W}{4(\tau_W-\lambda_W)} [f(\tau_W) - f(\lambda_W)]$$

$$I(\tau_W,\lambda_W) = -\frac{1}{2}\frac{\tau_W\lambda_W}{(\tau_W-\lambda_W)} - \frac{\tau_W^2\lambda_W}{(\tau_W-\lambda_W)^2}(g(\tau_W)-g(\lambda_W)) +$$

$$+ \frac{1}{4}\frac{\tau_W^2\lambda_W^2}{(\tau_W-\lambda_W)^2}(f(\tau_W)-f(\lambda_W))$$

$$\overline{|M|^2} = \frac{g^2 e^4 \eta^{\mu\nu}\eta^{\rho\sigma}}{3\ 16^2\pi^4 M_W^2} (M_Z^2-m_{h^0}^2)^2 \left(\sum_\lambda \varepsilon_{0\mu}\varepsilon^*_{0\rho}\right)\left(\sum_{\lambda'} \varepsilon^*_{2\nu}\varepsilon_{2\sigma}\right) |\{\ \}|^2$$

$$= \frac{g^2 e^4 \eta^{\mu\nu}\eta^{\rho\sigma}}{16^2\pi^4 M_W^2} (M_Z^2-m_{h^0}^2)^2 \left(-\eta_{\mu\rho} + \frac{P_{0\mu}P_{0\rho}}{M_Z^2}\right)\left(-\eta_{\nu\sigma}\right) |\{\ \}|^2$$

$$= \frac{g^2 e^4 \eta^{\mu\nu}\eta^{\rho\sigma}}{16^2\pi^4 M_W^2} (M_Z^2-m_{h^0}^2)^2 \left[\eta_{\mu\rho}\eta_{\nu\sigma} - \frac{\eta_{\nu\sigma}P_{0\mu}P_{0\rho}}{M_Z^2}\right] |\{\ \}|^2$$

$$= \frac{g^2 e^4 (M_Z^2-m_{h^0}^2)^2}{16^2\pi^4 M_W^2} \left[\delta^\nu_\rho\,\delta^\rho_\nu - \frac{\delta^\mu_\sigma \eta^{\rho\sigma} P_{0\mu}P_{0\rho}}{M_Z^2}\right] |\{\ \}|^2$$

$$= \frac{g^2 e^4 (M_Z^2-m_{h^0}^2)^2}{16^2\pi^4 M_W^2} \left[4 - \frac{P_0^2}{M_Z^2}\right] |\{\ \}|^2$$

$$= \frac{g^2 e^4 (M_Z^2-m_{h^0}^2)^2}{\cancel{3}\ 16^2\pi^4 M_W^2} [\cancel{4-1}] |\{\ \}|^2$$

$$\overline{|M|^2} = \frac{g^2 e^4 (M_Z^2-m_{h^0}^2)^2}{16^2\pi^4 M_W^2} |\{\ \}|^2$$

$$d\Gamma = \frac{|M|^2\, |\vec{P}_1|\, d\Omega}{32\pi^2 M_Z^2}$$

$$|\vec{P}_1| = \frac{M_Z^2 - m_{h^0}^2}{2 M_{Z^0}}$$

$$\therefore\ \Gamma = \frac{g^2 e^4 (M_Z^2 - m_{h^0}^2)^2}{16^2 \pi^4 M_W^2} |H J|^2 \frac{(M_Z^2 - m_{h^0}^2)}{2 M_{Z^0}} \frac{\overset{1}{\cancel{4\pi}}}{\underset{8\pi}{\cancel{32\pi^2}} M_{Z^0}^2}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} \quad ; \quad \alpha = \frac{e^2}{4\pi} \Rightarrow \quad \alpha^2 = \frac{e^4}{16\pi^2}$$

$$\Gamma_{\cdot} = \frac{G_F}{\sqrt{2}} \frac{\alpha^2}{32\pi^3} \frac{(M_Z^2 - m_{h^0}^2)^3}{M_Z^3} |d\,J|^2$$

$$\Gamma = \frac{\sqrt{2}\, G_F\, \alpha^2}{64\pi^3} M_Z^3 \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3 \Bigg| \left(\frac{\sin\alpha}{\cos\beta}\right) \frac{1}{\sin\theta_W \cos\theta_W} \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i F(\tau_i, \lambda_i)$$

$$+ \left(-\frac{\cos\alpha}{\sin\beta}\right) \frac{1}{\sin\theta_W \cos\theta_W} \sum_{i=u,c,t} Q_i C_V^i N_i F(\tau_i, \lambda_i) + \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W}\right].$$

$$\cdot \frac{(1 - 2\sin^2\theta_W)}{2\sin\theta_W \cos\theta_W} \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) - \frac{\sin(\beta-\alpha)}{2} \frac{\cos^2\theta_W}{\sin\theta_W \cos\theta_W} \left[4(3 - \tan^2\theta_W) K(\tau_W, \lambda_W)\right.$$

$$\left. + \left[\left(1 + \frac{2}{\tau_W}\right)\tan^2\theta_W - \left(5 + \frac{2}{\tau_W}\right)\right] I(\tau_W, \lambda_W)\right] \Bigg|^2$$

$$\boxed{\begin{aligned} \Gamma = {} & \frac{\sqrt{2}\, G_F\, \alpha^2}{64\pi^3} M_Z^3 \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3 \frac{1}{\sin^2\theta_W \cos^2\theta_W} \Bigg| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{\substack{i=d,s,b\\ e^-,\mu^-,\tau^-}} Q_i C_V^i N_i F(\tau_i, \lambda_i) \\ & - \left(\frac{\cos\alpha}{\sin\beta}\right) \sum_{i=u,c,t} Q_i C_V^i N_i F(\tau_i, \lambda_i) + \left[\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W}\right] \frac{(1 - 2\sin^2\theta_W)}{2} \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) \\ & - \frac{\sin(\beta-\alpha)}{2} \cos^2\theta_W \left[4(3 - \tan^2\theta_W) K(\tau_W, \lambda_W) + \left[\left(1 + \frac{2}{\tau_W}\right)\tan^2\theta_W - \left(5 + \frac{2}{\tau_W}\right)\right] I(\tau_W, \lambda_W)\right] \Bigg|^2 \end{aligned}}$$

$$\Gamma \approx \frac{\sqrt{2}\, G_F\, \alpha^2 M_Z^3}{64\pi^3} \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3 \frac{1}{\sin^2\theta_W \cos^2\theta_W} \Bigg| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{i=b,\tau^-} Q_i C_V^i N_i F(\tau_i, \lambda_i)$$

$$- 2\left(\frac{\cos\alpha}{\sin\beta}\right) C_V^t F(\tau_t, \lambda_t) + \left[(\sin\beta\cos\alpha - \sin\alpha\cos\beta) + \frac{(1 - \tan^2\beta)}{(1 + \tan^2\beta)} \frac{(\sin\beta\cos\alpha + \sin\alpha\cos\beta)}{2\cos^2\theta_W}\right].$$

$$\cdot \frac{(1 - 2\sin^2\theta_W)}{2} \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) - \frac{(\sin\beta\cos\alpha - \sin\alpha\cos\beta)}{2} \cos^2\theta_W \left[4(3 - \tan^2\theta_W) K(\tau_W, \lambda_W) + \right.$$

$$+ \left[\left(1 + \frac{2}{T_W}\right) \tan^2\theta_W - \left(5 + \frac{2}{T_W}\right)\right] I(T_W, \lambda_W) \Big] \Big|^2$$

$$\Gamma = \frac{\sqrt{2}\, G_F\, \alpha^2 M_Z^3}{64\pi^3} \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3 \frac{1}{\sin^2\theta_W \cos^2\theta_W} \Big| \left(\frac{\sin\alpha}{\cos\beta}\right) \sum_{i=b,\tau^-} a_i\, C_V^i N_i\, F(\tau_i, \lambda_i)$$

$$- 2\left(\frac{\cos\alpha}{\sin\beta}\right) C_V^t\, F(\tau_t, \lambda_t) + \cos\alpha \cos\beta \left[(\tan\beta - \tan\alpha) + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \frac{(\tan\beta + \tan\alpha)}{2\cos^2\theta_W} \right] \cdot$$

$$\cdot \frac{(1 - 2\sin^2\theta_W)}{2} \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) - \frac{\cos\alpha \cos\beta\, (\tan\beta - \tan\alpha) \cos^2\theta_W}{2} \Big[4(3 - \tan^2\theta_W) K(T_W, \lambda_W)$$

$$+ \left[\left(1 + \frac{2}{T_W}\right) \tan^2\theta_W - \left(5 + \frac{2}{T_W}\right)\right] I(T_W, \lambda_W) \Big] \Big|^2$$

$$= \frac{\sqrt{2}\, G_F\, \alpha^2 M_Z^3}{64\pi^3} \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3 \frac{\cos^2\alpha \cos^2\beta}{\sin^2\theta_W \cos^2\theta_W} \Big| \frac{1}{\sin\beta \cos\beta} \Big[\tan\alpha \tan\beta \sum_{i=b,\tau^-} a_i\, C_V^i N_i\, F(\tau_i, \lambda_i)$$

$$- 2 C_V^t\, F(\tau_t, \lambda_t) \Big] + \left[(\tan\beta - \tan\alpha) + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \frac{(\tan\alpha + \tan\beta)}{2\cos^2\theta_W} \right] \frac{(1 - 2\sin^2\theta_W)}{2} \cdot$$

$$\cdot \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) - \frac{(\tan\beta - \tan\alpha)\cos^2\theta_W}{2} \Big[4(3 - \tan^2\theta_W) K(T_W, \lambda_W) + \Big[\left(1 + \frac{2}{T_W}\right)\tan^2\theta_W$$

$$- \left(5 + \frac{2}{T_W}\right)\Big] I(T_W, \lambda_W) \Big] \Big|^2$$

$$\sin\beta \cos\beta = \frac{\tan\beta}{1 + \tan^2\beta}$$

$$= \frac{\sqrt{2}\, G_F\, \alpha^2 M_Z^3}{64\pi^3} \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3 \frac{1}{\sin^2\theta_W \cos^2\theta_W\, (1 + \tan^2\alpha)(1 + \tan^2\beta)} \Big| \frac{(1 + \tan^2\beta)}{\tan\beta} \Big[\tan\alpha \tan\beta \Big($$

$$- \left(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W\right) F(\tau_b, \lambda_b) - \left(-\frac{1}{2} + 2\sin^2\theta_W\right) F(\tau_\tau, \lambda_\tau)\Big) - 2\left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_W\right) F(\tau_t, \lambda_t)\Big]$$

$$+ \left[(\tan\beta - \tan\alpha) + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \frac{(\tan\alpha + \tan\beta)}{2\cos^2\theta_W} \right] \frac{(1 - 2\sin^2\theta_W)}{2} \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H)$$

$$- \frac{(\tan\beta - \tan\alpha)\cos^2\theta_W}{2} \Big[4(3 - \tan^2\theta_W) K(T_W, \lambda_W) + \Big[\left(1 + \frac{2}{T_W}\right)\tan^2\theta_W -$$

$$- \left(5 + \frac{2}{T_W}\right)\Big] I(T_W, \lambda_W) \Big] \Big|^2$$

$$\Gamma\left(\underset{f+H^\pm+W^\pm}{Z^0 \to h^0\gamma}\right) = \frac{\sqrt{2}\, G_F\, \alpha^2 M_Z^3 \left(1 - \frac{m_{h^0}^2}{M_Z^2}\right)^3}{64\pi^3 \sin^2\theta_W \cos^2\theta_W} \cos^2\alpha \cos^2\beta \left| \frac{1}{\sin\beta\cos\beta} \left[\tan\alpha \tan\beta \cdot \right.\right.$$

$$\left. \cdot \left(\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) F(\tau_b, \lambda_b) + \left(\frac{1}{2} - 2\sin^2\theta_W\right) F(\tau_\tau, \lambda_\tau) \right) - 2\left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_W\right) F(\tau_t, \lambda_t) \right]$$

$$+ \left[(\tan\beta - \tan\alpha) + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \frac{(\tan\alpha + \tan\beta)}{2\cos^2\theta_W} \frac{(1 - 2\sin^2\theta_W)}{2} \left(\frac{M_W^2}{M_{H^\pm}^2}\right) I(\tau_H, \lambda_H) \right.$$

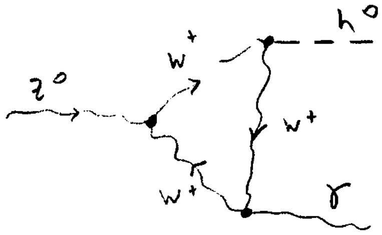
$$- \frac{(\tan\beta - \tan\alpha)\cos^2\theta_W}{2} \cdot \left[4(3 - \tan^2\theta_W) K(\tau_W, \lambda_W) + \left[\left(1 + \frac{2}{\tau_W}\right)\tan^2\theta_W - \right.\right.$$

$$\left.\left.\left. - \left(5 + \frac{2}{\tau_W}\right)\right] I(\tau_W, \lambda_W) \right] \right|^2$$

where $\tau_W \equiv \frac{4M_W^2}{m_{h^0}^2}$; $\lambda_W \equiv \frac{4M_W^2}{M_Z^2}$

$$K(\tau_W, \lambda_W) \equiv \frac{-\tau_W \lambda_W}{4(\tau_W - \lambda_W)} \left[f(\tau_W) - f(\lambda_W) \right]$$

Including the $W^\pm$ diagrams:



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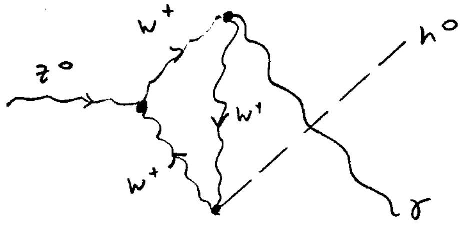

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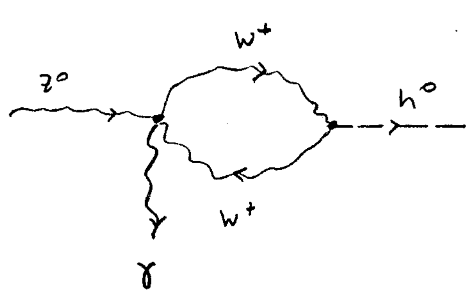
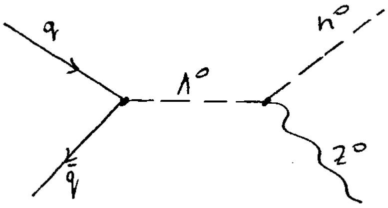


Two Higgs doublet Model

$q\bar{q} \to \underset{H^0}{h^0} Z^0$



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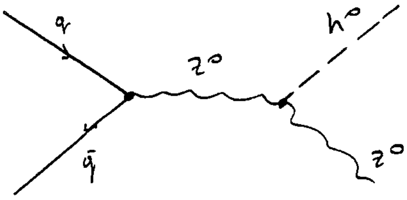

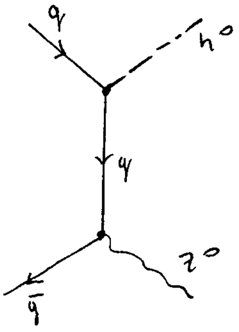

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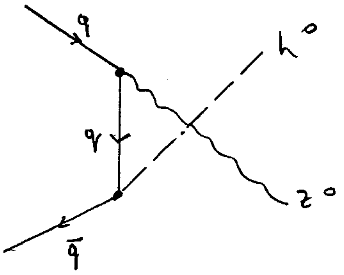
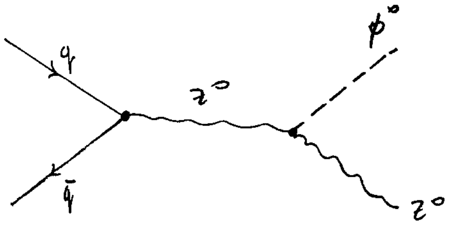


$q\bar{q} \to \underset{\substack{\downarrow \\ \text{higgs}}}{\phi^0} Z^0$ standard model:



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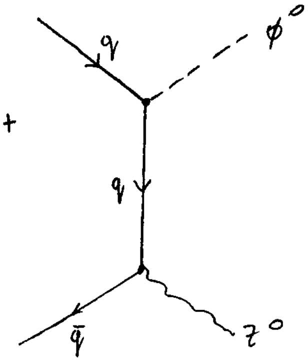

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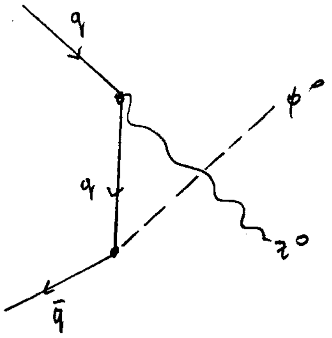
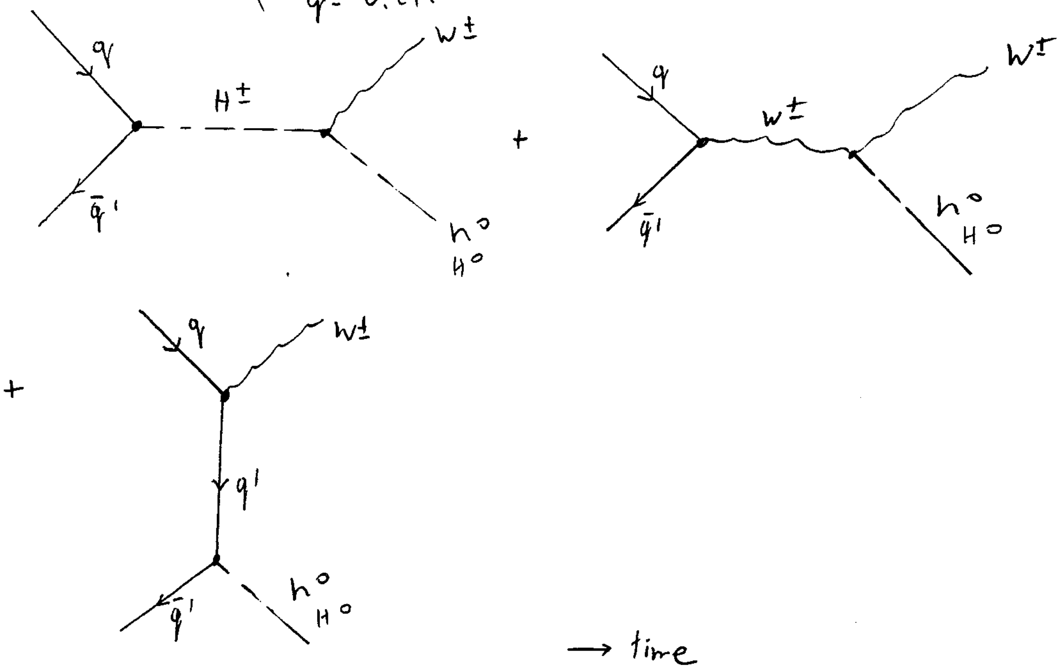


Two Higgs doublet Model :

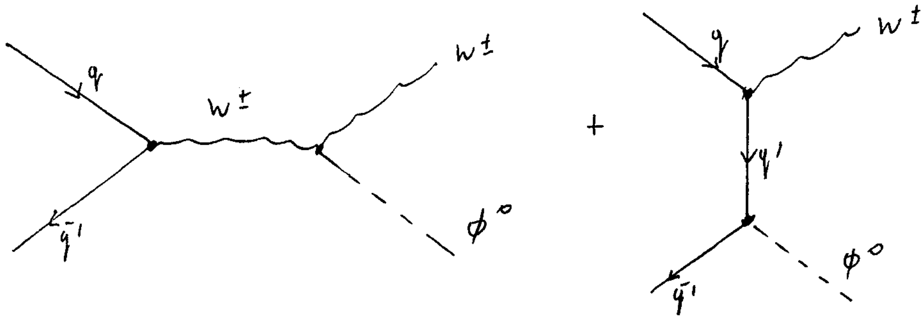
$q\bar{q}' \to W^+ \underset{H^0}{h^0}$

($q' = d, s, b$ $q = u, c, t$ for H^+) ($q' = u, c, t$ $q = d, s, b$ for H^-)



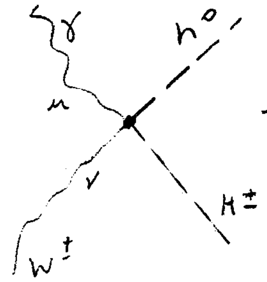
Standard Model :

$q\bar{q}' \to W^+ \phi^0$ ↳ higgs

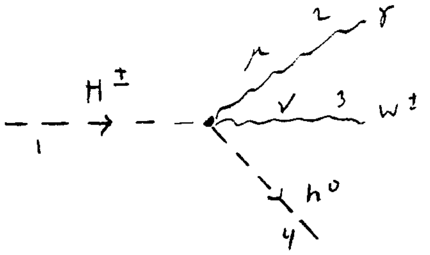


$H^\pm \to W^\pm \gamma h^0$ $\quad (m_{H^\pm} > M_{W^\pm} + m_{h^0})$

C. Marin



$$\frac{ige}{2} \cos(\beta - \alpha) \eta^{\mu\nu}$$



$$P_1 = P_2 + P_3 + P_4$$

$$-iM = \frac{ige}{2} \cos(\beta-\alpha) \eta^{\mu\nu} \varepsilon^*_{2\mu} \varepsilon^*_{3\nu} \qquad (1)$$

$$\overline{|M|^2} = \frac{g^2e^2}{4} \cos^2(\beta-\alpha) \eta^{\mu\nu} \eta^{\rho\sigma} \Big(\sum_\lambda \varepsilon^*_{2\mu} \varepsilon_{2\rho}\Big)\Big(\sum_{\lambda'} \varepsilon^*_{3\nu} \varepsilon_{3\sigma}\Big)$$

$$\overline{|M|^2} = \frac{g^2e^2}{4} \cos^2(\beta-\alpha) \eta^{\mu\nu} \eta^{\rho\sigma} (-\eta_{\mu\rho}) \Big(-\eta_{\nu\sigma} + \frac{P_{3\nu} P_{3\sigma}}{M_W^2}\Big)$$

$$= -\frac{g^2e^2}{4} \cos^2(\beta-\alpha) \eta^{\nu\sigma} \Big(-\eta_{\nu\sigma} + \frac{P_{3\nu} P_{3\sigma}}{M_W^2}\Big)$$

$$= -\frac{g^2e^2}{4} \cos^2(\beta-\alpha) (-4 + 1) = \frac{3}{4} g^2 e^2 \cos^2(\beta-\alpha)$$

$$\Rightarrow \boxed{\overline{|M|^2} = \frac{3}{4} g^2 e^2 \cos^2(\beta-\alpha)} \qquad (2)$$

$$d\Gamma = \frac{(2\pi)^{-3} \overline{|M|^2} \, d m_{23}^2 \, d m_{34}^2}{32 m_1^3} \qquad (3)$$

For m_{34}^2 fixed the range of m_{23}^2 is determined by its values when $\vec{P_2}$ is parallel or antiparallel to $\vec{P_3}$

$$m_{23}^2 = (p_2+p_3)^2 = (E_2^* + E_3^*,\ \vec{p}_2^{\,*} + \vec{p}_3^{\,*})^2$$

$$= (E_2^* + E_3^*)^2 - (\vec{p}_2^{\,*} + \vec{p}_3^{\,*})^2$$

$$= (E_2^* + E_3^*)^2 - (E_2^{*2} - m_2^2) - (E_3^{*2} - m_3^2) - 2\vec{p}_2^{\,*}\cdot\vec{p}_3^{\,*}$$

$$m_{23\,\substack{\min\\ \max}}^2 = 2E_2^* E_3^* + \underset{\substack{\downarrow\\ \gamma}}{m_2^2} + \underset{\substack{\downarrow\\ W^\pm}}{m_3^2} \mp 2\,(E_2^{*2} - m_2^2)^{1/2}(E_3^{*2} - m_3^2)^{1/2}$$

$$\boxed{m_{23\,\substack{\min\\ \max}}^2 = 2E_2^* E_3^* + M_{W^\pm}^2 \mp 2\,(E_2^*)\,(E_3^{*2} - M_{W^\pm}^2)^{1/2}} \qquad (4)$$

In the rest frame of $p_3 + p_4$: $(m_{34}, 0, 0, 0) = p_3 + p_4$

$$m_{34}^2 = (p_3+p_4)^2 = M_{W^\pm}^2 + m_{h^0}^2 + 2(p_3.p_4) = M_{W^\pm}^2 + m_{h^0}^2 + 2\,p_3.(p_3+p_4-p_3)$$

$$m_{34}^2 = \cancel{M_W^2} + m_{h^0}^2 + 2E_3^* m_{34} - \cancel{2}M_W^2 = m_{h^0}^2 - M_W^2 + 2E_3^* m_{34}$$

$$\boxed{E_3^* = \frac{m_{34}^2 + M_W^2 - m_{h^0}^2}{2m_{34}}} \qquad (5)$$

$$m_{34}^2 = (p_1 - p_2)^2 = m_{H^\pm}^2 - 2(p_1.p_2) = m_{H^\pm}^2 - 2\big((p_2+p_3+p_4).p_2\big)$$

$$m_{34}^2 = m_{H^\pm}^2 - 2E_2^* m_{34}$$

$$\boxed{E_2^* = \frac{m_{H^\pm}^2 - m_{34}^2}{2m_{34}}} \qquad (6)$$

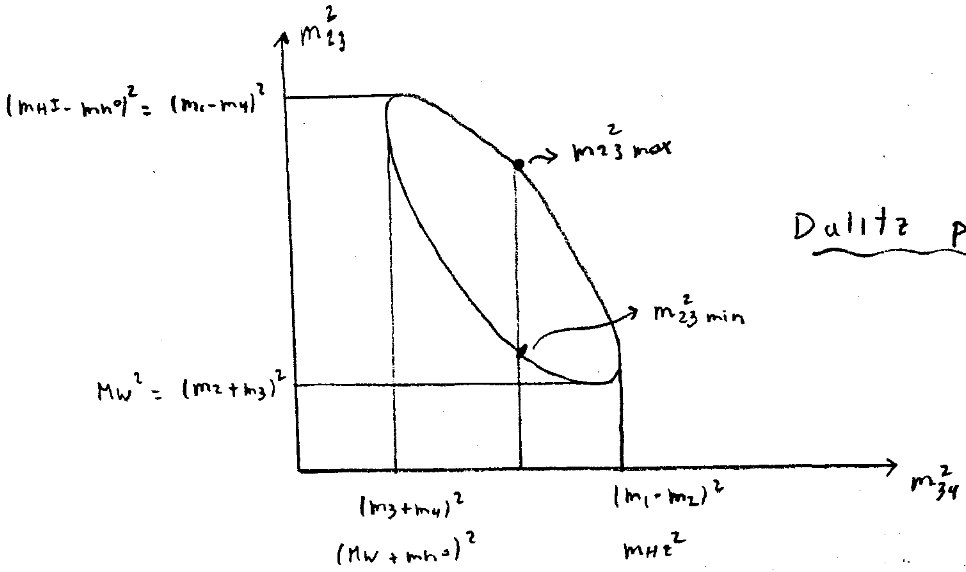
$\Rightarrow$

$$m_{23\,\substack{\min\\ \max}}^2 = \frac{1}{2m_{34}^2}(m_{34}^2 + M_W^2 - m_{h^0}^2)(m_{H^\pm}^2 - m_{34}^2) + M_W^2 \mp \frac{1}{m_{34}}(m_{H^\pm}^2 - m_{34}^2)\,.$$

$$\cdot\left[\frac{(m_{34}^2 + M_W^2 - m_{h^0}^2)^2}{4m_{34}^2} - M_W^2\right]^{1/2}$$

$$\boxed{\begin{aligned} m_{23\,\substack{\min\\ \max}}^2 = {} & \frac{1}{2m_{34}^2}(m_{34}^2 + M_W^2 - m_{h^0}^2)(m_{H^\pm}^2 - m_{34}^2) + M_W^2 \mp \frac{1}{2m_{34}^2}(m_{H^\pm}^2 - m_{34}^2)\,. \\ & \cdot\left[(m_{34}^2 + M_W^2 - m_{h^0}^2)^2 - 4M_W^2 m_{34}^2\right]^{1/2} \end{aligned}} \qquad (7)$$

Dalitz plot



$$m_{34}^2 = (P_3 + P_4)^2 = m_3^2 + m_4^2 + 2(P_3 \cdot P_4) = m_3^2 + m_4^2 + 2(E_3 E_4 - \vec{P}_3 \cdot \vec{P}_4)$$

if $\vec{P}_3 = \vec{P}_4 = \vec{0} \Rightarrow E_3 = m_3,\ E_4 = m_4.$

$$\therefore\ m_{34\,min}^2 = (m_3 + m_4)^2 \qquad (8)$$

$$m_{34}^2 = (P_1 - P_2)^2 = m_1^2 + m_2^2 - 2(P_1 \cdot P_2) = m_1^2 + m_2^2 - 2(E_1 E_2 - \vec{P}_1 \cdot \vec{P}_2)$$

if $\vec{P}_1 = \vec{P}_2 = \vec{0} \Rightarrow E_1 = m_1,\ E_2 = m_2$

$$\therefore\ m_{34\,max}^2 = (m_1 - m_2)^2 \qquad (9)$$

$$m_{23}^2 = (P_2 + P_3)^2 = m_2^2 + m_3^2 + 2(P_2 \cdot P_3) = m_2^2 + m_3^2 + 2(E_2 E_3 - \vec{P}_2 \cdot \vec{P}_3)$$

if $\vec{P}_2 = \vec{P}_3 = \vec{0} \Rightarrow E_2 = m_2,\ E_3 = m_3$

$$\therefore\ m_{23\,min}^2 = (m_2 + m_3)^2 \qquad (10)$$

$$m_{23}^2 = (P_1 - P_4)^2 = m_1^2 + m_4^2 - 2(P_1 \cdot P_4) = m_1^2 + m_4^2 - 2(E_1 E_4 - \vec{P}_1 \cdot \vec{P}_4)$$

if $\vec{P}_1 = \vec{P}_4 = \vec{0} \Rightarrow E_1 = m_1,\ E_4 = m_4$

$$\therefore\ m_{23\,max}^2 = (m_1 - m_4)^2 \qquad (11)$$

$$d\Gamma = (2\pi)^{-3} \frac{3}{4} g^2 e^2 \frac{\cos^2(\beta - \alpha)\ dm_{23}^2\, dm_{34}^2}{32\, m_{H^\pm}^3} \qquad (12)$$

$$d\Gamma = \frac{(2\pi)^{-3} \frac{3}{4} g^2 e^2 \cos^2(\beta - \alpha)}{32\, m_{H^\pm}^3} \frac{1}{m_{34}^2} (m_{H^\pm}^2 - m_{34}^2) \cdot \left[(m_{34}^2 + M_W^2 - m_{h^0}^2)^2 - 4 M_W^2 m_{34}^2\right]^{1/2} dm_{34}^2 \qquad (13)$$

setting $x \equiv m_{34}^2$

$$d\Gamma = \frac{(2\pi)^{-3}\ \frac{3}{4}\, g^2 e^2 \cos^2(\beta-\alpha)}{32\, m_{H^\pm}^3} \left(\frac{m_{H^\pm}^2}{X} - 1\right)\left[X^2 - 2X(M_W^2 + m_{h^0}^2) + M_W^4 + m_{h^0}^4 - 2M_W^2 m_{h^0}^2\right]$$

$$\cdot\, dX \qquad (14)$$

$$a = M_W^4 + m_{h^0}^4 - 2M_W^2 m_{h^0}^2 = (M_W^2 - m_{h^0}^2)^2$$

$$b = -2\,(M_W^2 + m_{h^0}^2)$$

$$c = 1$$

$$\mathbb{X} = a + bX + cX^2$$

$$q = 4ac - b^2 = 4\,(M_W^2 - m_{h^0}^2)^2 - 4\,(M_W^2 + m_{h^0}^2)^2$$

$$q = 4\,[\cancel{M_W^4} - 2M_W^2 m_{h^0}^2 + \cancel{m_{h^0}^4} - \cancel{M_W^4} - \cancel{m_{h^0}^4} - 2M_W^2 m_{h^0}^2]$$

$$q = -16\, M_W^2 m_{h^0}^2$$

$$k = \frac{4c}{q} = \frac{4}{-16\, M_W^2 m_{h^0}^2} = \frac{-1}{4M_W^2 m_{h^0}^2}$$

$$\int \sqrt{\mathbb{X}}\, dX = \frac{(2cX + b)}{4c}\sqrt{\mathbb{X}} + \frac{1}{2k}\int \frac{dX}{\sqrt{\mathbb{X}}} \qquad (15)$$

$$\int \frac{dX}{\sqrt{\mathbb{X}}} = \frac{1}{\sqrt{c}} \ln\left(2\sqrt{c\mathbb{X}} + 2cX + b\right) \qquad (c > 0) \qquad (16)$$

in fact:

$$\frac{d}{dX}\frac{1}{\sqrt{c}} \ln\left(2\sqrt{c}\sqrt{a+bX+cX^2} + 2cX + b\right)$$

$$= \frac{1}{\sqrt{c}}\, \frac{1}{(2\sqrt{c}\sqrt{a+bX+cX^2} + 2cX + b)} \left[\cancel{2}\sqrt{c}\,\frac{1}{\cancel{2}}\,(a+bX+cX^2)^{-1/2}(b+2cX) + 2c\right]$$

$$= \frac{1}{\sqrt{c}\,(2\sqrt{c}\sqrt{a+bX+cX^2} + 2cX + b)} \left[\frac{(b+2cX)\sqrt{c}}{(a+bX+cX^2)^{1/2}} + 2c\right]$$

$$= \frac{1}{(2\sqrt{c}\sqrt{a+bX+cX^2} + 2cX + b)} \left[\frac{2\sqrt{c}\,(\cancel{a+bX+cX^2})^{1/2} + 2cX + b}{(a+bX+cX^2)^{1/2}}\right]$$

$$= \frac{1}{(a+bX+cX^2)^{1/2}}$$

$$\int \frac{dx}{\sqrt{\mathcal{X}}} = \left(\ln\left[2(\sqrt{a+bx+cx^2}|\sqrt{c} + 2(x+b\right]\right)\frac{1}{\sqrt{c}}$$

$$= \ln\left[2\left((M_W^2 - m_{h^0}^2)^2 - 2(M_W^2 + m_{h^0}^2)x + x^2\right)^{1/2} + 2x - 2(M_W^2 + m_{h^0}^2)\right]$$

$$\int \sqrt{\mathcal{X}}\, dx = \frac{(\cancel{2}x - \cancel{2}(M_W^2 + m_{h^0}^2))}{\cancel{4}_2}\left[(M_W^2 - m_{h^0}^2)^2 - 2(M_W^2 + m_{h^0}^2)x + x^2\right]^{1/2}$$

$$- 2M_W^2 m_{h^0}^2 \ln\left[2\left[(M_W^2 - m_{h^0}^2)^2 - 2(M_W^2 + m_{h^0}^2)x + x^2\right]^{1/2} + 2x\right.$$

$$\left. - 2(M_W^2 + m_{h^0}^2)\right]$$

$$\int_{(M_W+m_{h^0})^2}^{m_{H^\pm}^2} \sqrt{\mathcal{X}}\, dx = \frac{1}{2}\left[m_{H^\pm}^2 - (M_W^2 + m_{h^0}^2)\right]\left[(M_W^2 - m_{h^0}^2)^2 - 2(M_W^2 + m_{h^0}^2)m_{H^\pm}^2 + m_{H^\pm}^4\right]^{1/2}$$

$$- 2M_W^2 m_{h^0}^2 \ln\left[2\left[(M_W^2 - m_{h^0}^2)^2 - 2(M_W^2 + m_{h^0}^2)m_{H^\pm}^2 + m_{H^\pm}^4\right]^{1/2} + 2m_{H^\pm}^2 - 2(M_W^2 + m_{h^0}^2)\right]$$

$$+ 2M_W^2 m_{h^0}^2 \ln\left[4M_W m_{h^0}\right]$$

$$\boxed{\begin{aligned}\int_{(M_W+m_{h^0})^2}^{m_{H^\pm}^2} \sqrt{\mathcal{X}}\, dx = {}& \frac{1}{2}\left[m_{H^\pm}^2 - (M_W^2 + m_{h^0}^2)\right]\left[(M_W^2 - m_{h^0}^2)^2 - 2(M_W^2 + m_{h^0}^2)m_{H^\pm}^2 + m_{H^\pm}^4\right]^{1/2} \\ & - 2M_W^2 m_{h^0}^2 \ln\left\{\frac{\left[(M_W^2 - m_{h^0}^2)^2 - 2(M_W^2 + m_{h^0}^2)m_{H^\pm}^2 + m_{H^\pm}^4\right]^{1/2} + m_{H^\pm}^2 - M_W^2 - m_{h^0}^2}{2M_W m_{h^0}}\right\}\end{aligned}}$$

(17)

$$\int \frac{\sqrt{\mathcal{X}}\, dx}{x} = \sqrt{\mathcal{X}} + \frac{b}{2}\int \frac{dx}{\sqrt{\mathcal{X}}} + a\int \frac{dx}{x\sqrt{\mathcal{X}}}$$

$$= \sqrt{\mathcal{X}} + \frac{b}{2}\left[\frac{1}{\sqrt{c}}\ln\left(2\sqrt{c\mathcal{X}} + 2cx + b\right)\right] - \sqrt{a}\ln\left(\frac{2\sqrt{a\mathcal{X}} + bx + 2a}{x}\right)$$

$(a > 0)$ (18)

$$= \left[(M_W^2 - m_{h^0}^2)^2 - 2(M_W^2 + m_{h^0}^2)x + x^2\right]^{1/2} - (M_W^2 + m_{h^0}^2)\left[\ln\left|2\left[(M_W^2 - m_{h^0}^2)^2 -\right.\right.\right.$$

$$\left.\left.\left. - 2(M_W^2 + m_{h^0}^2)x + x^2\right]^{1/2} + 2x - 2(M_W^2 + m_{h^0}^2)\right|\right] - |m_{h^0}^2 - M_W^2| \times$$

$$\times \ln\left|\frac{2|m_{h^0}^2 - M_W^2|\left[(M_W^2 - m_{h^0}^2)^2 - 2(M_W^2 + m_{h^0}^2)x + x^2\right]^{1/2} - 2(M_W^2 + m_{h^0}^2)x + 2(M_W^2 - m_{h^0}^2)^2}{x}\right|$$

$$\int_{(M_W+m_{h^0})^2}^{m_{H^\pm}^2} \frac{\sqrt{X}}{X}\,dX = \left[(M_W^2-m_{h^0}^2)^2-2(M_W^2+m_{h^0}^2)m_{H^\pm}^2+m_{H^\pm}^4\right]^{1/2}-(M_W^2+m_{h^0}^2)\cdot$$

$$\cdot\left[\ln\left|2\left[(M_W^2-m_{h^0}^2)^2-2(M_W^2+m_{h^0}^2)m_{H^\pm}^2+m_{H^\pm}^4\right]^{1/2}+2m_{H^\pm}^2-2(M_W^2+m_{h^0}^2)\right|\right]-|m_{h^0}^2-M_W^2|\ln\left|\frac{2|m_{h^0}^2-M_W^2|\left[(M_W^2-m_{h^0}^2)^2-2(M_W^2+m_{h^0}^2)m_{H^\pm}^2+m_{H^\pm}^4\right]^{1/2}-2(M_W^2+m_{h^0}^2)m_{H^\pm}^2+2(M_W^2-m_{h^0}^2)^2}{m_{H^\pm}^2}\right|+(M_W^2+m_{h^0}^2)\left[\ln(4M_W m_{h^0})\right]+$$

$$+|m_{h^0}^2-M_W^2|\left[\ln(4M_W m_{h^0})\right].$$

(19)

because :

When $X=(M_W+m_{h^0})^2 = M_W^2+2M_W m_{h^0}+m_{h^0}^2$

$$\left[(M_W^2-m_{h^0}^2)^2-2(M_W^2+m_{h^0}^2)(M_W+m_{h^0})^2+(M_W+m_{h^0})^4\right]=$$

$$=\cancel{M_W^4}-2M_W^2\cancel{m_{h^0}^2}+\cancel{m_{h^0}^4}-2\cancel{M_W^4}-4\cancel{M_W^3}m_{h^0}-2\cancel{M_W^2}m_{h^0}^2-2M_W^2\cancel{m_{h^0}^2}-4M_W\cancel{m_{h^0}^3}-2\cancel{m_{h^0}^4}$$

$$+\cancel{M_W^4}+4\cancel{M_W^3}m_{h^0}+6M_W^2\cancel{m_{h^0}^2}+4\cancel{M_W}m_{h^0}^3+\cancel{m_{h^0}^4}$$

$$=0 \qquad (20)$$

$$X_H^W \equiv \frac{m_W^2}{m_{H^\pm}^2} \quad ; \quad X_H^{h^0} \equiv \frac{m_{h^0}^2}{m_{H^\pm}^2} \qquad (21)$$

$$\boxed{\begin{aligned}\int_{(m_W+m_{h^0})^2}^{m_{H^\pm}^2} \sqrt{X}\, dX = m_{H^\pm}^4 \Big\{ & \frac{1}{2}\left[1-(X_H^W+X_H^{h^0})\right]\left[(X_H^W-X_H^{h^0})^2-2(X_H^W+X_H^{h^0})+1\right]^{1/2} \\ & -2X_H^W X_H^{h^0} \ln\left|\frac{[(X_H^W-X_H^{h^0})^2-2(X_H^W+X_H^{h^0})+1]^{1/2}+1-X_H^W-X_H^{h^0}}{2(X_H^W X_H^{h^0})^{1/2}}\right|\Big\} \end{aligned}} \qquad (22)$$

$$\boxed{\begin{aligned}\int_{(m_W+m_{h^0})^2}^{m_{H^\pm}^2} \frac{\sqrt{X}}{X}\, dX = m_{H^\pm}^2 \Big\{ & \left[(X_H^W-X_H^{h^0})^2-2(X_H^W+X_H^{h^0})+1\right]^{1/2} \\ & -(X_H^W+X_H^{h^0})\cdot\ln\left|\frac{[(X_H^W-X_H^{h^0})^2-2(X_H^W+X_H^{h^0})+1]^{1/2}+1-(X_H^W+X_H^{h^0})}{2(X_H^W X_H^{h^0})^{1/2}}\right| \\ & -|X_H^{h^0}-X_H^W| \ln\left|\frac{|X_H^{h^0}-X_H^W|\,[(X_H^W-X_H^{h^0})^2-2(X_H^W+X_H^{h^0})+1]^{1/2}-(X_H^W+X_H^{h^0})+(X_H^W-X_H^{h^0})^2}{2(X_H^W X_H^{h^0})^{1/2}}\right|\Big\} \end{aligned}} \qquad (23)$$

$$\begin{aligned}\left[(X_H^W-X_H^{h^0})^2-2(X_H^W+X_H^{h^0})+1\right]^{1/2} &= \Big[(X_H^W)^2+(X_H^{h^0})^2-2(X_H^W)(X_H^{h^0}) \\ &\quad -2X_H^W-2X_H^{h^0}+1\Big]^{1/2} = \left[\lambda(1,X_H^W,X_H^{h^0})\right]^{1/2}\end{aligned}$$

$$\Rightarrow \boxed{\begin{aligned}\int_{(m_W+m_{h^0})^2}^{m_{H^\pm}^2} \sqrt{X}\, dX = m_{H^\pm}^4 \Big\{ & \frac{1}{2}\left[1-X_H^W-X_H^{h^0}\right]\lambda^{1/2}(1,X_H^W,X_H^{h^0}) \\ & -2X_H^W X_H^{h^0} \ln\left|\frac{\lambda^{1/2}(1,X_H^W,X_H^{h^0})+1-X_H^W-X_H^{h^0}}{2(X_H^W X_H^{h^0})^{1/2}}\right|\Big\} \end{aligned}} \qquad (24)$$

(136)

$$\int_{(M_W+m_{h^0})^2}^{m_{H^\pm}^2} \frac{\sqrt{X}}{X}\,dX = m_{H^\pm}^2 \left\{ \lambda^{1/2}(1,X_H^W,X_H^{h^0}) - (X_H^W + X_H^{h^0})\cdot \ln\left| \frac{\lambda^{1/2}(1,X_H^W,X_H^{h^0}) + 1 - X_H^W - X_H^{h^0}}{2(X_H^W X_H^{h^0})^{1/2}} \right| - |X_H^{h^0} - X_H^W| \ln \left| \frac{|X_H^{h^0}-X_H^W|\lambda^{1/2}(1,X_H^W,X_H^{h^0}) - (X_H^W + X_H^{h^0}) + (X_H^W - X_H^{h^0})^2}{2(X_H^W X_H^{h^0})^{1/2}} \right| \right\} \quad (25)$$

$$\Rightarrow \Gamma = \frac{(2\pi)^{-3}\frac{3}{4} g^2 e^2 \cos^2(\beta-\alpha)}{32}\left(\frac{m_{H^\pm}}{M_W}\right) M_W \left\{ \lambda^{1/2}(1,X_H^W,X_H^{h^0}) - (X_H^W + X_H^{h^0})\cdot \ln\left| \frac{\lambda^{1/2}(1,X_H^W,X_H^{h^0}) + 1 - X_H^W - X_H^{h^0}}{2(X_H^W X_H^{h^0})^{1/2}} \right| - |X_H^{h^0}-X_H^W| \ln \left| \frac{|X_H^{h^0}-X_H^W|\lambda^{1/2} - (X_H^W + X_H^{h^0}) + (X_H^W - X_H^{h^0}}{2(X_H^W X_H^{h^0})^{1/2}} \right| - \frac{1}{2}(1 - X_H^W - X_H^{h^0})\lambda^{1/2} + 2X_H^W X_H^{h^0} \ln \left| \frac{\lambda^{1/2} + 1 - X_H^W - X_H^{h^0}}{2(X_H^W X_H^{h^0})^{1/2}} \right| \right\} \quad (26)$$

but $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$; $g = \frac{e}{\sin\theta_W} \Rightarrow e^2 = g^2\sin^2\theta_W$

$$\Gamma = \frac{(2\pi)^{-3}\frac{3}{4} g^4 \sin^2\theta_W \cos^2(\beta-\alpha)}{32\sqrt{X_H^W}\, M_W^4} M_W^5 \left\{ \quad \right\}$$

$$\Gamma = \frac{(2\pi)^{-3}\frac{3}{4} G_F^2 \sin^2\theta_W \cos^2(\beta-\alpha) M_W^5}{(X_H^W)^{1/2}} \left\{ \quad \right\}$$

$$\Rightarrow \Gamma = \frac{3 G_F^2 \sin^2\theta_W \cos^2(\beta-\alpha) M_W^5}{32\pi^3 (X_H^W)^{1/2}} \left\{ \quad \right\}$$

$$\cos^2(\beta-\alpha) = [\cos\beta\cos\alpha + \sin\beta\sin\alpha]^2 = \cos^2\beta\cos^2\alpha\,[1 + \tan\beta\tan\alpha]^2$$

$$= \frac{1}{(1+\tan^2\beta)(1+\tan^2\alpha)}[1 + \tan\beta\tan\alpha]^2$$

$$\Gamma (H^{\pm} \to W^{\pm} \gamma h^{o}) = \frac{3 G_F^2 \sin^2\theta_W \, [1 + \tan\beta \tan\alpha]^2 M_W^5}{32 (1+\tan^2\beta)(1+\tan^2\alpha)\pi^3 (X_H^W)^{1/2}} \left\{ \frac{\lambda^{1/2}(1, X_H^W, X_H^{h^o})}{2} \cdot \right.$$

$$\cdot \left(1 + X_H^W + X_H^{h^o}\right) + \left(2 X_H^W X_H^{h^o} - X_H^W - X_H^{h^o}\right) \cdot \ln \left| \frac{\lambda^{1/2}(1, X_H^W, X_H^{h^o}) + 1 - X_H^W - X_H^{h^o}}{2 (X_H^W X_H^{h^o})^{1/2}} \right|$$

$$\left. - |X_H^{h^o} - X_H^W| \ln \left| \frac{|X_H^{h^o} - X_H^W| \lambda^{1/2}(1, X_H^W, X_H^{h^o}) - (X_H^W + X_H^{h^o}) + (X_H^W - X_H^{h^o})^2}{2 (X_H^W X_H^{h^o})^{1/2}} \right| \right\} \qquad (27)$$

where

$$X_H^W \equiv \frac{M_W^2}{m_{H^\pm}^2} \; ; \quad X_H^{h^o} \equiv \frac{m_{h^o}^2}{m_{H^\pm}^2} \; ; \quad X_H^Z \equiv \frac{M_Z^2}{m_{H^\pm}^2} \; ;$$

$$\lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2ab - 2ac - 2bc \qquad (28)$$

$$\tan\alpha = \left\{ \frac{1 + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \left[\frac{1 - X_H^Z - X_H^W}{g^*} \right]}{1 - \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \left[\frac{1 - X_H^Z - X_H^W}{g^*} \right]} \right\}^{1/2} \qquad (29)$$

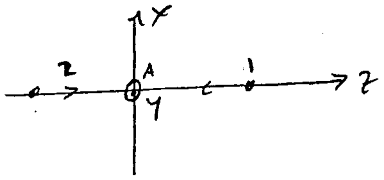
$$g^* \equiv \left[(1 + X_H^Z - X_H^W)^2 - 4(X_H^Z)(1 - X_H^W) \left(\frac{1-\tan^2\beta}{1+\tan^2\beta} \right)^2 \right]^{1/2} \qquad (30)$$

$$m_{h^0}^2 = \frac{1}{2} m_{H^\pm}^2 \left\{ 1 - X_H^W + X_H^Z - g^* \right\} \qquad (31)$$

$$\Rightarrow \boxed{X_H^{h^o} = \frac{1}{2} (1 - X_H^W + X_H^Z - g^*)} \qquad (32)$$

Production of h^0, H^0 and A^0

1+2 → A Cross section:



$$d\hat{\sigma} = \frac{\overline{|M|^2}\, dQ}{F}$$

$$dQ = (2\pi)^4\, \delta^4(P_A - P_1 - P_2)\, \frac{d^3P_A}{(2\pi)^3\, 2E_A}$$ (charge A = 0)

$$F = 4\left((P_1\cdot P_2)^2 - m_1^2 m_2^2\right)^{1/2}$$

$$d\hat{\sigma} = \frac{\overline{|M|^2}\,(2\pi)^4\,\delta^4(P_A - P_1 - P_2)}{4\left((P_1\cdot P_2)^2 - m_1^2 m_2^2\right)^{1/2}}\,\frac{d^3P_A}{(2\pi)^3\, 2E_A}$$

$$\hat{\sigma} = \frac{\overline{|M|^2}\,\cancel{(2\pi)^4}\,\delta(E_A - E_1 - E_2)}{4\left((P_1\cdot P_2)^2 - m_1^2 m_2^2\right)^{1/2}\,\cancel{(2\pi)^3}\,2E_A}$$; $\hat{s} = (P_1+P_2)^2 = (E_1+E_2)^2$

$\hat{s} = P_1^2 + P_2^2 + 2\,P_1\cdot P_2$

$\hat{s} = m_1^2 + m_2^2 + 2\,P_1\cdot P_2$

$$\hat{\sigma} = \frac{\overline{|M|^2}\,(2\pi)\,\delta(M_A - \sqrt{\hat{s}})}{4\left[\frac{(\hat{s} - m_1^2 - m_2^2)^2}{4} - m_1^2 m_2^2\right]^{1/2} 2M_A}$$

but: $$\delta\left(1 - \frac{M_A^2}{\hat{s}}\right) = \hat{s}\,\delta(\hat{s} - M_A^2)$$
$$= \hat{s}\,\delta\left[(\sqrt{\hat{s}} - M_A)(\sqrt{\hat{s}} + M_A)\right]$$
$$= \hat{s}\left[\frac{\delta(\sqrt{\hat{s}} - M_A) + \delta(\sqrt{\hat{s}}\cancel{+M_A})}{2M_A}\right]$$

because $$\delta\left((X - X_1)(X - X_2)\right) = \frac{\delta(X - X_1) + \delta(X - X_2)}{|X_1 - X_2|}$$

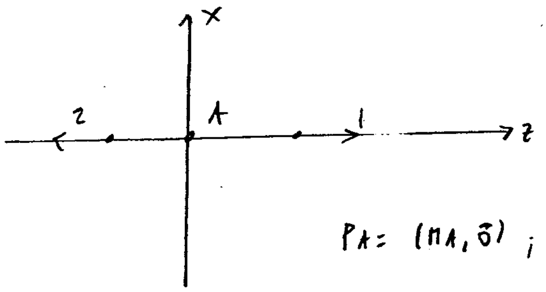
$$\delta\left(1 - \frac{M_A^2}{\hat{s}}\right) = \frac{\hat{s}}{2M_A}\,\delta(M_A - \sqrt{\hat{s}})$$; $z \equiv \frac{M_A^2}{\hat{s}}$

$$\Rightarrow \hat{\sigma} = \frac{\overline{|M|^2}\,(2\pi)\,\cancel{2M_A}\,\delta(1-z)}{4\hat{s}\left[\frac{(\hat{s} - m_1^2 - m_2^2)^2}{4} - m_1^2 m_2^2\right]^{1/2}\cancel{2M_A}} = \frac{2\pi\,\overline{|M|^2}\,\delta(1-z)}{4\hat{s}\left[\frac{(\hat{s} - m_1^2 - m_2^2)^2}{4} - m_1^2 m_2^2\right]^{1/2}}$$

$$d\Gamma = \frac{\overline{|M'|^2}\,|\vec{P}_1|\,d\Omega}{32\pi^2 M_A^2}$$

For identical particles: $$\Gamma = \frac{\overline{|M'|^2}\,|\vec{P}'_1|}{8\pi M_A^2}\,\frac{1}{2}$$ → identical particles

If $m_1 = m_2 = 0$ ✓ $$\hat{\sigma} = \frac{\cancel{2}\pi\,\overline{|M|^2}\,\delta(1-z)}{\cancel{2}\,M_A^4} = \frac{\pi\,\overline{|M|^2}\,\delta(1-z)}{M_A^4}$$



$p_A = (M_A, \vec{0})$; $p_1 = (E_1, \vec{p})$; $p_2 = (E_2, -\vec{p})$

$$p_A = p_1 + p_2$$

$$M_A = E_1 + E_2$$

$$M_A = 2E \quad \text{if} \quad m_1 = m_2$$

$$E_1^2 = m_1^2 + |\vec{p}_1|^2$$

if $m_1 = 0$

$$|\vec{p}_1| = E_1 = E = \frac{M_A}{2}$$

$$\Gamma = \frac{\overline{|M'|^2}\, M_A}{2\;8\pi\, M_A^2\; 2} = \frac{\overline{|M'|^2}}{16\pi M_A}\,\frac{1}{2}$$

$$\overline{|M|^2} = \underset{\text{polarization}}{\frac{1}{4}} \times \overbrace{\frac{1}{8} \times \frac{1}{8}}^{\text{color factors}}\, \overline{|M'|^2}$$

$$\overline{|M'|^2} = 64 \times 4\, \overline{|M|^2}$$

$$\Gamma = \frac{\cancel{64} \times 4\, \overline{|M|^2}}{\cancel{32}\,\pi M_A} = \frac{8\, \overline{|M|^2}}{\pi M_A}$$

$$\Rightarrow \overline{|M|^2} = \frac{\pi M_A\, \Gamma(A \to 2g)}{8}$$

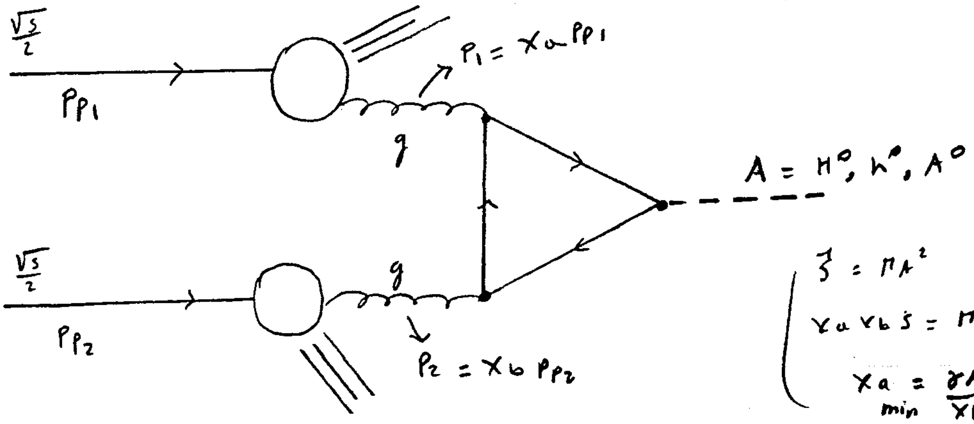
$$\Rightarrow \hat{\sigma}(gg \to A) = \frac{\pi^2 M_A\, \Gamma(A \to 2g)}{8 M_A^4}\, \delta(1-z)$$

$$\therefore \boxed{\hat{\sigma}(gg \to A) = \frac{\pi^2\, \Gamma(A \to 2g)}{8 M_A^3}\, \delta(1-z)}$$

where $z \equiv \frac{M_A^2}{\hat{s}}$

$\hat{s}$: partonic c.m. energy squared.

PP
or $p\bar{p} \to AX$ $\quad (A = h^\circ, H^\circ, A^\circ)$



$$\left(\begin{aligned} &\hat{s} = M_A^2 \\ &x_a x_b s = M_A^2 \\ &x_{a\,\text{min}} = \frac{\tau_A}{x_b \to 1} = \tau_A \end{aligned} \right)$$

$$\hat{s} = (p_1 + p_2)^2 = 2\, p_1 p_2 = 2\, x_a x_b P_{P1} P_{P2}$$

$$s = (P_{P1} + P_{P2})^2 = 2\, P_{P1} P_{P2}$$

$$\Rightarrow \quad \boxed{\hat{s} = x_a x_b s}$$

$$\sigma(p\bar{p} \xrightarrow{g} AX) = \int dx_a\, dx_b\, g(x_a, \overset{\text{factorization scale}}{M^2})\, g(x_b, M^2)\, \hat{\sigma}(gg \to A)$$

defining $\sigma_0(gg \to A) = \dfrac{\pi^2\, \Gamma(A \to 2g)}{8 M_A^3}$

$$\sigma(p\bar{p} \to AX) = \int dx_a\, dx_b\, g(x_a, M^2)\, g(x_b, M^2)\, \sigma_0\, \delta\left(1 - \frac{M_A^2}{\hat{s}}\right)$$

$$\sigma(p\bar{p} \to AX) = \sigma_0 \int dx_a\, dx_b\, g(x_a, M^2)\, g(x_b, M^2)\, \delta\left(1 - \frac{M_A^2}{x_a x_b s}\right)$$

$$= \sigma_0 \int dx_a\, dx_b\, g(x_a, M^2)\, g(x_b, M^2)\, x_a x_b\, \delta\left(x_a x_b - \frac{M_A^2}{s}\right)$$

$$= \sigma_0 \int dx_a\, dx_b\, g(x_a, M^2)\, g(x_b, M^2)\, x_b\, \delta\left(x_b - \frac{M_A^2}{x_a s}\right)$$

$$\sigma(p\bar{p} \to AX) = \sigma_0 \left(\int \frac{dx_a}{x_a}\, g(x_a, M^2)\, g\left(\frac{M_a^2}{s x_a}, M^2\right) \right) \frac{M_A^2}{s}$$

$$\tau_A \equiv \frac{M_A^2}{s}$$

$$\boxed{\sigma(p\bar{p} \xrightarrow{g} AX) = \frac{\pi^2\, \Gamma(A \to 2g)}{8 M_A^3} \left(\int_{\tau_A}^{1} \frac{dx_a}{x_a}\, g(x_a, M^2)\, g\left(\frac{\tau_A}{x_a}, M^2\right) \right) \tau_A}$$

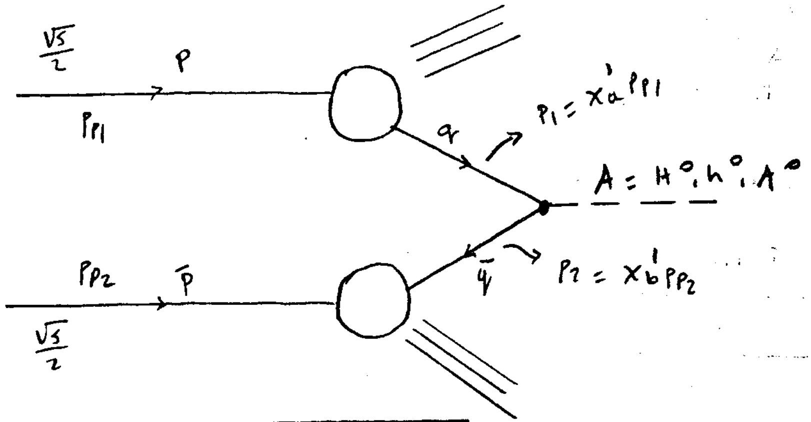
$A = h^0, H^0, A^0$

$\Gamma(h^0 \to 2g)$ is given in (10)

$\Gamma(H^0 \to 2g)$ is " in (29)

$\Gamma(A^0 \to 2g)$ " " " (44)

We also have:



$$\boxed{\hat{s} = x_a' x_b' s}$$

$$\sigma(p\bar{p} \xrightarrow{q} AX) = \sum_q \int dx_a' \, dx_b' \, f_q(x_a', M^2) \, f_{\bar{q}}(x_b', M^2) \, \hat{\sigma}(q\bar{q} \to A)$$

$$\hat{\sigma}(q\bar{q} \to A) = \frac{\pi \overline{|M|^2} \delta(1-z)}{M_A^4}$$

$$d\Gamma = \frac{\overline{|M'|^2} \, |\vec{p}_1| \, d\Omega}{32\pi^2 M_A^2}$$

$$\Gamma = \frac{\overline{|M'|^2} \, |\vec{p}_1|}{8\pi M_A^2} = \frac{\overline{|M'|^2}}{16\pi M_A}$$

$$\overline{|M|^2} = \frac{1}{4} \overline{|M'|^2} \frac{1}{3} \Rightarrow \overset{\text{decay}}{\overline{|M'|^2}} = 4 \overline{|M|^2} \times 3$$

$$\Gamma = \frac{3 \overline{|M|^2}}{4\pi M_A}$$

$$\therefore \overline{|M|^2} = 4\pi M_A \Gamma(A \to q\bar{q}) / 3$$

$$\boxed{\hat{\sigma}(q\bar{q} \to A) = \frac{4\pi^2 \Gamma(A \to q\bar{q}) \delta(1-z)}{M_A^3 \times 3}}$$ where $z \equiv \frac{M_A^2}{\hat{s}}$

defining: $\sigma_0(q\bar{q} \to A) \equiv \frac{4\pi^2 \Gamma(A \to q\bar{q})}{3 m_A^3}$

$$\sigma(p\bar{p} \xrightarrow{q} AX) = \sum_q \int dx_a' \, dx_b' \; f_q(x_a', m^2) \, f_{\bar{q}}(x_b', m^2) \, \sigma_0(q\bar{q} \to A) \, \delta(1 - \frac{m_A^2}{\hat{s}})$$

$$= \sum_q \sigma_0(q\bar{q} \to A) \int dx_a' \, dx_b' \; f_q(x_a', m^2) \, f_{\bar{q}}(x_b', m^2) \, \delta\left(1 - \frac{m_A^2}{x_a' x_b' s}\right)$$

$$= \sum_q \sigma_0(q\bar{q} \to A) \int dx_a' \, dx_b' \; f_q(x_a', m^2) \, f_{\bar{q}}(x_b', m^2) \, x_a' x_b' \, \delta\left(x_a' x_b' - \frac{m_A^2}{s}\right)$$

$$= \sum_q \sigma_0(q\bar{q} \to A) \int dx_a' \, dx_b' \; f_q(x_a', m^2) \, f_{\bar{q}}(x_b', m^2) \, x_b' \, \delta(x_b' - \frac{m_A^2}{x_a' s})$$

$$\sigma(p\bar{p} \xrightarrow{q} AX) = \sum_q \sigma_0(q\bar{q} \to A) \int \frac{dx_a'}{x_a'} \, f_q(x_a', m^2) \, f_{\bar{q}}(\frac{m_A^2}{s x_a'}, m^2) \, \frac{m_A^2}{s}$$

$$\gamma_A = \frac{m_A^2}{s}$$

$$\boxed{\sigma(p\bar{p} \xrightarrow{q} AX) = \frac{4\pi^2}{3 m_A^3} \left(\sum_q \Gamma(A \to q\bar{q}) \right) \left(\int_{\gamma_A}^{1} \frac{dx_a'}{x_a'} \, f_q(x_a', m^2) \, f_{\bar{q}}\left(\frac{\gamma_A}{x_a'}, m^2\right) \right) \gamma_A} \qquad q = u, d, s, c, b$$

f_q : parton density function

m^2 : factorization scale $(m = m_A)$

$$\boxed{\begin{aligned} \Rightarrow \sigma(p\bar{p} \to AX) = {} & \frac{\pi^2 \Gamma(A \to 2g)}{8 m_A^3} \left(\int_{\gamma_A}^{1} \frac{dx_a}{x_a} \, g(x_a, m^2) \, g\left(\frac{\gamma_A}{x_a}, m^2\right) \right) \gamma_A \\ & + \frac{4\pi^2}{\underset{\uparrow}{3} m_A^3} \left[\sum_{q = u,d,s,c,b} \Gamma(A \to q\bar{q}) \left(\int_{\gamma_A}^{1} \frac{dx_a}{x_a} \, f_q^{p}(x_a, m^2) \, f_{\bar{q}}^{\bar{p}}\left(\frac{\gamma_A}{x_a}, m^2\right) \right) \right] \gamma_A \end{aligned}}$$

$A = h^0, H^0, A^0$; $\gamma_A = \frac{m_A^2}{s}$.

$\Gamma(h^0 \to 2g)$ is given in (10)

$\Gamma(H^0 \to 2g)$ " " in (29)

$\Gamma(A^0 \to 2g)$ " " in (44)

$\Gamma(h^0 \to q\bar{q})$ " " " (16, 17)

$\Gamma(H^0 \to q\bar{q})$ " " " (28)

$\Gamma(A^0 \to q\bar{q})$ " " " (40)

Production of $h^0W^{\pm}X$

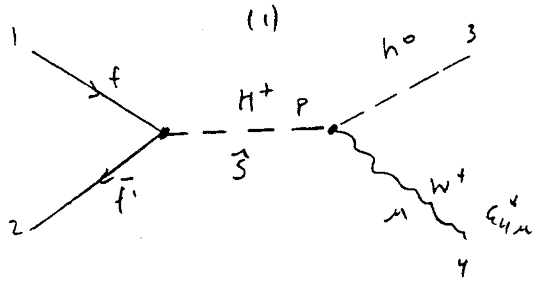
$q\bar{q}' \to h^0 W^+$; $q'\bar{q} \to h^0 W^-$

$q = u, c$; $q' = d, s, b$

a) $f\bar{f}' \to h^0 W^+$; b) $f'\bar{f} \to h^0 W^-$ ($f = u, c$; $f' = d, s, b$)

Let's consider a) : $f\bar{f}' \to h^0 W^+$:

(1)



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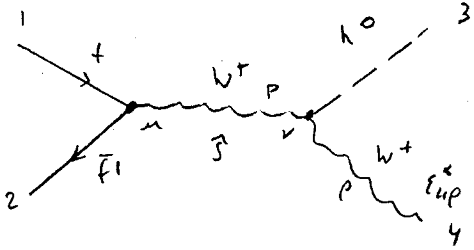
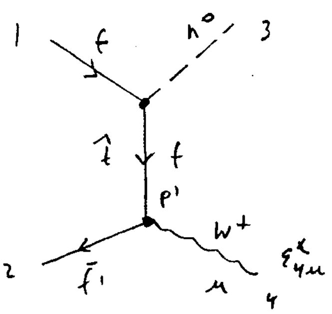
(2)



$p = p_1 + p_2$

(3)



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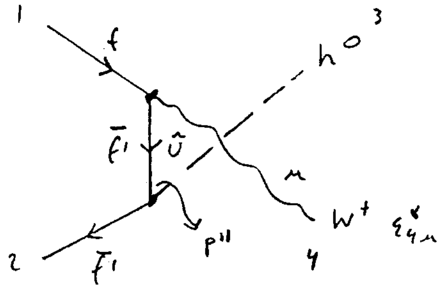
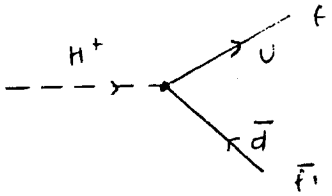
(4)



$p' = p_1 - p_3$

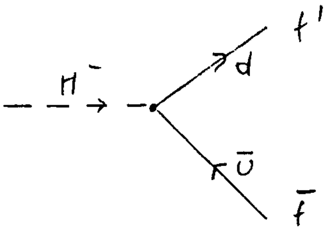
$p'' = p_1 - p_4$



$$\frac{ig}{2\sqrt{2}M_W}\left[m_d \tan\beta(1+\gamma^5) + m_u \cot\beta(1-\gamma^5)\right] V_{ud}$$

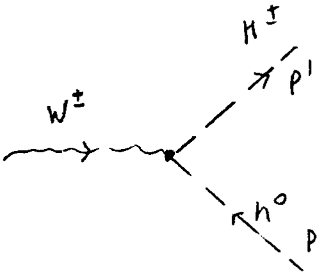
$$\frac{ig}{2\sqrt{2}M_W}(A + B\gamma^5)V_{ff'}$$

$A = m_{f'} \tan\beta + m_f \cot\beta$; $B = m_{f'} \tan\beta - m_f \cot\beta$

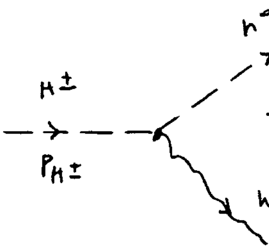


$$\frac{ig}{2\sqrt{2}M_W}\left[m_d \tan\beta(1-\gamma^5) + m_u \cot\beta(1+\gamma^5)\right] V^*_{ud}$$

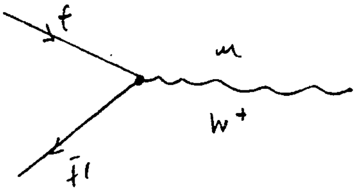
$$\frac{ig}{2\sqrt{2}M_W}(A - B\gamma^5)V^*_{ff'}$$



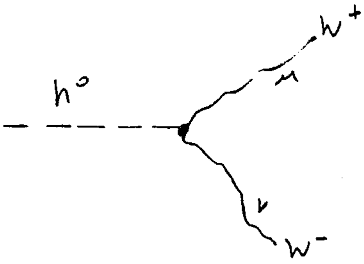
$$\mp \frac{ig}{2}\cos(\beta-\alpha)(p+p')^\mu$$



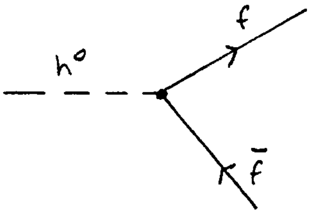
$$\mp \frac{ig}{2}\cos(\beta-\alpha)(P_{h^0}+P_{H^\pm})^\mu$$



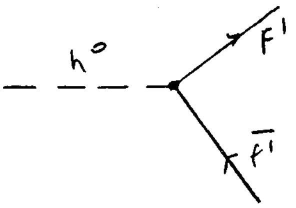
$$-\frac{ig}{\sqrt{2}}\gamma^\mu \frac{1}{2}(1-\gamma^5)V^*_{ff'}$$



$$ig M_W \sin(\beta-\alpha)\,\eta^{\mu\nu}.$$



$$\frac{igm_f}{2M_W}C_f \quad ; \quad C_f = -\frac{\cos\alpha}{\sin\beta}$$



$$\frac{igm_{f'}}{2M_W}C_{f'} \quad ; \quad C_{f'} = \frac{\sin\alpha}{\cos\beta}$$

$$-iM_1 = \varepsilon^*_{4\mu}\left(-\frac{ig}{2}\cos(\beta-\alpha)(P_1+P_2+P_3)^\mu\right)\frac{i}{s-M_{H^+}^2+iM_{H^+}\Gamma_{H^+}}\bar{V}_2\left(\frac{ig}{2\sqrt{2}M_W}(A-B\gamma^5)\right)U_1 V^*_{ff'}$$

$$M_1 = \frac{-g^2 V^*_{ff'}\cos(\beta-\alpha)\varepsilon^*_{4\mu}(P_1+P_2+P_3)^\mu}{4\sqrt{2}M_W(s-M_{H^+}^2+iM_{H^+}\Gamma_{H^+})}\bar{V}_2(A-B\gamma^5)U_1$$

$$-iM_2 = \varepsilon^*_{4\rho}\left(igM_W\sin(\beta-\alpha)\,n^{\nu\rho}\right)(-i)\left(n_{\mu\nu}-\frac{P_\mu P_\nu}{M_W^2}\right)\frac{1}{\hat{s}-M_W^2+iM_W\Gamma_W}\,\cdot$$

$$\cdot\,\bar{V}_2\left(-\frac{ig}{\sqrt{2}}\gamma^\mu\frac{1}{2}(1-\gamma^5)\right)V^*_{ff'}\,U_1$$

$$M_2 = \frac{g^2M_W\sin(\beta-\alpha)\,V^*_{ff'}}{2\sqrt{2}}\,\varepsilon^{*\nu}_4\left(n_{\mu\nu}-\frac{P_\mu P_\nu}{M_W^2}\right)\frac{1}{\hat{s}-M_W^2+iM_W\Gamma_W}\cdot\bar{V}_2\gamma^\mu(1-\gamma^5)U_1$$

Defining:

$$C_W \equiv \frac{\sin(\beta-\alpha)}{\hat{s}-M_W^2+iM_W\Gamma_W} \qquad (1)$$

$$C_{H^+} \equiv \frac{\cos(\beta-\alpha)}{\hat{s}-M_{H^+}^2+iM_{H^+}\Gamma_{H^+}} \qquad (2)$$

$$\Rightarrow \boxed{M_1 = \frac{-g^2V^*_{ff'}C_{H^+}}{4\sqrt{2}M_W}\,\varepsilon^*_{4\mu}(P_1+P_2+P_3)^\mu\,\bar{V}_2(A-B\gamma^5)U_1} \qquad (3)$$

$$\Rightarrow \boxed{M_2 = \frac{g^2M_WC_WV^*_{ff'}\varepsilon^*_{4\rho}n^{\nu\rho}}{2\sqrt{2}}\left(n_{\mu\nu}-\frac{P_\mu P_\nu}{M_W^2}\right)\bar{V}_2\gamma^\mu(1-\gamma^5)U_1} \qquad (4)$$

$$-iM_3 = \varepsilon^*_{4\mu}\,\bar{V}_1\left(-\frac{ig}{\sqrt{2}}\gamma^\mu\frac{1}{2}(1-\gamma^5)\right)V^*_{ff'}\,\frac{i(\not{P}'+m_f)}{\hat{t}-m_f^2}\left(\frac{igm_f}{2M_W}c_f\right)U_1$$

$$M_3 = \frac{-g^2m_fC_fV^*_{ff'}}{4\sqrt{2}M_W(\hat{t}-m_f^2)}\,\varepsilon^*_{4\mu}\,\bar{V}_2\gamma^\mu(1-\gamma^5)(\not{P}'+m_f)U_1$$

defining:

$$C_{tf} \equiv \frac{C_f}{\hat{t}-m_f^2} \qquad (5)$$

$$\Rightarrow \boxed{M_3 = \frac{-g^2m_fC_{tf}V^*_{ff'}}{4\sqrt{2}M_W}\,\varepsilon^*_{4\mu}\,\bar{V}_2\gamma^\mu(1-\gamma^5)(\not{P}'+m_f)U_1} \qquad (6)$$

$$-iM_4 = \bar{V}_2 \left(\frac{i g m_{f'}}{2 M_W} C_{f'} \right) \frac{i(\not{P}'' + m_{f'})}{\hat{U} - m_{f'}^2} \left(-\frac{ig}{\sqrt{2}} \gamma^\mu \frac{1}{2}(1-\gamma^5) \right) V^*_{ff'} \, U_1 \; \varepsilon^*_{4\mu}$$

$$M_4 = - \frac{g^2 m_{f'} C_{f'} V^*_{ff'} \varepsilon^*_{4\mu}}{4\sqrt{2} M_W (\hat{U} - m_{f'}^2)} \; \bar{V}_2 (\not{P}'' + m_{f'}) \gamma^\mu (1-\gamma^5) U_1$$

defining:

$$C_{Uf'} \equiv \frac{C_{f'}}{(\hat{U} - m_{f'}^2)} \qquad (7)$$

$$\Rightarrow \boxed{M_4 = - \frac{g^2 m_{f'} C_{Uf'} V^*_{ff'} \varepsilon^*_{4\mu}}{4\sqrt{2} M_W} \; \bar{V}_2 (\not{P}'' + m_{f'}) \gamma^\mu (1-\gamma^5) U_1} \qquad (8)$$

$$-iM = -i(M_1 + M_2 + M_3 + M_4) \qquad (9)$$

$$|M|^2 = (M_1 + M_2 + M_3 + M_4)^* (M_1 + M_2 + M_3 + M_4)$$

$$|M|^2 = |M_1|^2 + |M_2|^2 + |M_3|^2 + |M_4|^2 + M_1^* M_2 + M_2^* M_1 + M_1^* M_3 + M_3^* M_1 + M_1^* M_4$$
$$+ M_4^* M_1 + M_2^* M_3 + M_3^* M_2 + M_2^* M_4 + M_4^* M_2 + M_3^* M_4 + M_4^* M_3 \qquad (10).$$

$$|M_1|^2 = \frac{g^4 |V_{ff'}|^2 |C_{H^+}|^2}{32 M_W^2} (P_1+P_2+P_3)^\mu (P_1+P_1+P_3)^\nu \Big(\sum_\lambda \varepsilon_{4\mu} \varepsilon^*_{4\nu}\Big) \sum_s (\bar V_2 (A-B\gamma^5) U_1)^+ (\bar V_2 (A-B\gamma^5) U_1) \qquad (11)$$

$$(\underset{\underset{V_2^+\gamma^0}{\downarrow}}{\bar V_2} (A-B\gamma^5) U_1)^+ = U_1^+ (A-B\gamma^5) \gamma^0 V_2 = \bar U_1 (A+B\gamma^5) V_2 \qquad (12)$$

$$|M_1|^2 = \frac{g^4 |V_{ff'}|^2 |C_{H^+}|^2}{32 M_W^2} (P_1+P_2+P_3)^\mu (P_1+P_2+P_3)^\nu \Big(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\Big) \sum_s \bar U_1 (A+B\gamma^5) V_2 . \bar V_2 (A-B\gamma^5) U_1$$

$$= \frac{g^4 |V_{ff'}|^2 |C_{H^+}|^2}{32 M_W^2} (P_1+P_2+P_3)^\mu (P_1+P_2+P_3)^\nu \Big(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\Big) \mathrm{Tr}\,[(\not P_1 + m_f)(A+B\gamma^5) (\not P_2 - m_{f'})(A-B\gamma^5)] \qquad (13)$$

$$\mathrm{Tr}\,[(\not P_1 + m_f)(A+B\gamma^5)(\not P_2 - m_{f'})(A-B\gamma^5)] = \mathrm{Tr}\,[(\overset{\downarrow}{A \not P_1} + \overset{\downarrow}{B \not P_1 \gamma^5} + \overset{\downarrow}{m_f A} + \overset{\downarrow}{m_f B\gamma^5}) . (A\not P_2 - B\not P_2 \gamma^5 - m_{f'} A + m_{f'} B\gamma^5)]$$

$$= A^2 \mathrm{Tr}(\not P_1 \not P_2) - AB\,\mathrm{Tr}\cancelto{0}{(\not P_1 \not P_2 \gamma^5)} - m_{f'} A^2 \cancelto{0}{\mathrm{Tr}(\not P_1)} + m_{f'} AB\,\cancelto{0}{\mathrm{Tr}(\not P_1 \gamma^5)} + AB\,\mathrm{Tr}\cancelto{0}{(\not P_1 \gamma^5 \not P_2)}$$
$$- B^2 \mathrm{Tr}(\not P_1 \gamma^5 \not P_2 \gamma^5) - m_{f'} AB\,\mathrm{Tr}\cancelto{0}{(\not P_1 \gamma^5)} + m_{f'} B^2 \cancelto{0}{\mathrm{Tr}(\not P_1)} + m_f A^2 \cancelto{0}{\mathrm{Tr}(\not P_2)} - m_f AB\,\cancelto{0}{\mathrm{Tr}(\not P_2 \gamma^5)}$$
$$-4 m_f m_{f'} A^2 + m_f m_{f'} AB\,\cancelto{0}{\mathrm{Tr}(\gamma^5)} + m_f AB\,\mathrm{Tr}\cancelto{0}{(\gamma^5 \not P_2)} - m_f B^2 \mathrm{Tr}\cancelto{0}{(\gamma^5 \not P_2 \gamma^5)} - m_f m_{f'} AB\,\cancelto{0}{\mathrm{Tr}(\gamma^5)}$$
$$+ 4 m_f m_{f'} B^2$$

$$= 4A^2 (P_1 . P_2) + 4B^2 (P_1 . P_2) + 4 m_f m_{f'} (B^2 - A^2)$$

$$\Rightarrow$$

$$\mathrm{Tr} = 4 [(A^2+B^2)(P_1 . P_2) - m_f m_{f'} (A^2 - B^2)] \qquad (14)$$

$$|M_1|^2 = 4 \frac{g^4 |V_{ff'}|^2 |C_{H^+}|^2}{32 M_W^2} \Big[-(P_1+P_2+P_3)^2 + \frac{((P_1+P_2+P_3).P_4)^2}{M_W^2}\Big] [(A^2+B^2)(P_1 . P_2) - m_f m_{f'} (A^2-B^2)] \qquad (15)$$

$$\hat S = (P_1+P_2)^2 = m_f^2 + m_{f'}^2 + 2(P_1 . P_2)$$

$$\boxed{(P_1 . P_2) = \frac{\hat S - m_f^2 - m_{f'}^2}{2}} \qquad (16)$$

$$\hat S = (P_3+P_4)^2 = m_{h^0}^2 + M_W^2 + 2(P_3 . P_4)$$

$$\boxed{(P_3 . P_4) = \frac{\hat S - m_{h^0}^2 - M_W^2}{2}} \qquad (17)$$

$$(p_1 + p_2 + p_3) \cdot p_4 = (2p_3 + p_4) \cdot p_4 = 2(p_3 \cdot p_4) + M_W^2$$

$$(p_1 + p_2 + p_3) \cdot p_4 = \hat{s} - m_{h^0}^2 - \cancel{M_W^2} + \cancel{M_W^2}$$

$$\boxed{(p_1 + p_2 + p_3) \cdot p_4 = \hat{s} - m_{h^0}^2} \qquad (18)$$

$$(p_1 + p_2 + p_3)^2 = (2p_3 + p_4) \cdot (2p_3 + p_4) = 4 m_{h^0}^2 + 4(p_3 \cdot p_4) + M_W^2$$

$$(p_1 + p_2 + p_3)^2 = 4 \cancel{m_{h^0}}^2 + 2(\hat{s} - \cancel{m_{h^0}}^2 - M_W^2) + M_W^2$$

$$\boxed{(p_1 + p_2 + p_3)^2 = 2\hat{s} + 2 m_{h^0}^2 - M_W^2} \qquad (19)$$

$$|M_1|^2 = \frac{g^4 |V_{ff'}|^2 |C_{H^+}|^2}{8 M_W^2} \left[-2\hat{s} - 2 m_{h^0}^2 + M_W^2 + \frac{1}{M_W^2} (\hat{s} - m_{h^0}^2)^2 \right] \left[(A^2 + B^2) \frac{(\hat{s} - m_f^2 - m_{f'}^2)}{2} \right.$$
$$\left. - m_f m_{f'} (A^2 - B^2) \right]$$

$$|M_1|^2 = \frac{g^4 |V_{ff'}|^2 |C_{H^+}|^2}{16 M_W^4} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \left[(A^2 + B^2)(\hat{s} - m_f^2 - m_{f'}^2) - 2 m_f m_{f'} (A^2 - B^2) \right]$$

$$\boxed{|M_1|^2 = \frac{g^4 |V_{ff'}|^2 |C_{H^+}|^2}{16 M_W^4} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \left[A^2 [\hat{s} - (m_f + m_{f'})^2] + B^2 [\hat{s} - (m_f - m_{f'})^2] \right]}$$

(20)

$$|M_2|^2 = \frac{g^4 M_W^2 |C_W|^2 |V_{ff'}|^2}{8} \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}\right) \left(\eta_{\rho\sigma} - \frac{p_\rho p_\sigma}{M_W^2}\right) \left(\sum_\lambda \varepsilon_4^\nu \varepsilon_4^{*\sigma}\right) \sum_s (\bar{v}_2 \gamma^\mu (1 - \gamma^5) u_1)^\dagger$$
$$(\bar{v}_2 \gamma^\rho (1 - \gamma^5) u_1) \qquad (21)$$

$$(\bar{v}_2 \gamma^\mu (1 - \gamma^5) u_1)^\dagger = (v_2^\dagger \gamma^0 \gamma^\mu (1 - \gamma^5) u_1)^\dagger = u_1^\dagger (1 - \gamma^5) \overset{\gamma^0 \gamma^\mu}{\widehat{\gamma^{\mu\dagger} \gamma^0}} v_2$$

$$= u_1^\dagger \gamma^0 (1 + \gamma^5) \gamma^\mu v_2 = \bar{u}_1 (1 + \gamma^5) \gamma^\mu v_2 = \bar{u}_1 \gamma^\mu (1 - \gamma^5) v_2$$

$$\Rightarrow |M_2|^2 = \frac{g^4 M_W^2 |C_W|^2 |V_{ff'}|^2}{8} \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}\right) \left(\eta_{\rho\sigma} - \frac{p_\rho p_\sigma}{M_W^2}\right) \left(-\eta^{\nu\sigma} + \frac{p_4^\nu p_4^\sigma}{M_W^2}\right) \sum_s \bar{u}_1 (1 + \gamma^5) \gamma^\mu v_2 .$$
$$. \bar{v}_2 \gamma^\rho (1 - \gamma^5) u_1$$

$$|M_2|^2 = \frac{g^4 M_W^2 |C_W|^2 |V_{ff'}|^2}{8} \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}\right) \left(\eta_{\rho\sigma} - \frac{p_\rho p_\sigma}{M_W^2}\right) \left(-\eta^{\nu\sigma} + \frac{p_4^\nu p_4^\sigma}{M_W^2}\right) \mathrm{Tr} \left[(\not{p}_1 + m_f)(1 + \gamma^5) \right.$$
$$\left. \gamma^\mu (\not{p}_2 - m_{f'}) \gamma^\rho (1 - \gamma^5) \right] \qquad (23)$$

$$\mathrm{Tr}\,[\,(\not p_1 + m_f)(1+\gamma^5)\gamma^\mu(\not p_2 - m_{f'})\gamma^\rho(1-\gamma^5)\,]$$

$$= \mathrm{Tr}\,[\,(\overset{\downarrow}{\not p_1\gamma^\mu} + \overset{\downarrow}{\not p_1\gamma^5\gamma^\mu} + \overset{\downarrow}{m_f\gamma^\mu} + \overset{\downarrow}{m_f\gamma^5\gamma^\mu})(\not p_2\gamma^\rho - \not p_2\gamma^\rho\gamma^5 - m_{f'}\gamma^\rho + m_{f'}\gamma^\rho\gamma^5)\,]$$

$$= \mathrm{Tr}(\not p_1\gamma^\mu\not p_2\gamma^\rho) - \mathrm{Tr}(\not p_1\gamma^\mu\not p_2\gamma^\rho\gamma^5) - m_{f'}\cancelto{0}{\mathrm{Tr}(\not p_1\gamma^\mu\gamma^\rho)} + m_{f'}\cancelto{0}{\mathrm{Tr}(\not p_1\gamma^\mu\gamma^\rho\gamma^5)}$$
$$+ \mathrm{Tr}(\not p_1\gamma^5\gamma^\mu\not p_2\gamma^\rho) - \mathrm{Tr}(\not p_1\gamma^5\gamma^\mu\not p_2\gamma^\rho\gamma^5) - m_{f'}\cancelto{0}{\mathrm{Tr}(\not p_1\gamma^5\gamma^\mu\gamma^\rho)} + m_{f'}\cancelto{0}{\mathrm{Tr}(\not p_1\gamma^5\gamma^\mu\gamma^\rho\gamma^5)}$$
$$+ m_f\cancelto{0}{\mathrm{Tr}(\gamma^\mu\not p_2\gamma^\rho)} - m_f\cancelto{0}{\mathrm{Tr}(\gamma^\mu\not p_2\gamma^\rho\gamma^5)} - m_f m_{f'}\mathrm{Tr}(\gamma^\mu\gamma^\rho) + m_f m_{f'}\cancelto{0}{\mathrm{Tr}(\gamma^\mu\gamma^\rho\gamma^5)}$$
$$+ m_f\cancelto{0}{\mathrm{Tr}(\gamma^5\gamma^\mu\not p_2\gamma^\rho)} - m_f\cancelto{0}{\mathrm{Tr}(\gamma^5\gamma^\mu\not p_2\gamma^\rho\gamma^5)} - m_f m_{f'}\cancelto{0}{\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\rho)} + m_f m_{f'}\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\rho\gamma^5)$$

$$= \mathrm{Tr}(\gamma^\mu\not p_2\gamma^\rho\not p_1) - \mathrm{Tr}(\gamma^5\not p_1\gamma^\mu\not p_2\gamma^\rho) - \mathrm{Tr}(\gamma^5\not p_1\gamma^\mu\not p_2\gamma^\rho) + \mathrm{Tr}(\not p_1\gamma^\mu\not p_2\gamma^\rho)$$
$$-4m_f\cancel{m_{f'}}\eta^{\mu\rho} + 4m_f\cancel{m_{f'}}\eta^{\mu\rho}$$

$$= 2\,\mathrm{Tr}(\gamma^\mu\not p_2\gamma^\rho\not p_1) - 2\,\mathrm{Tr}(\gamma^5\not p_1\gamma^\mu\not p_2\gamma^\rho) = 2\,\mathrm{Tr}(\gamma^\mu\not p_2\gamma^\rho\not p_1) - 2\,\mathrm{Tr}(\gamma^5\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\rho)\,p_{1\alpha}p_{2\beta}$$

$$\Rightarrow \mathrm{Tr} = 8\,[\,p_2^\mu p_1^\rho + p_2^\rho p_1^\mu - (p_1\cdot p_2)\eta^{\mu\rho}\,] + 8i\,\varepsilon^{\alpha\mu\beta\rho}p_{1\alpha}p_{2\beta} \qquad (24)$$

$$\Rightarrow |M_2|^2 = \frac{\cancel{8}g^4 M_W^2|C_W|^2|V_{ff'}|^2}{\cancel{8}}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}\right)\left(\eta_{\rho\sigma} - \frac{p_\rho p_\sigma}{M_W^2}\right)\left(-\eta^{\nu\sigma} + \frac{p_4^\nu p_4^\sigma}{M_W^2}\right)\{p_2^\mu p_1^\rho + p_2^\rho p_1^\mu$$
$$- (p_1\cdot p_2)\eta^{\mu\rho} + i\,\varepsilon^{\alpha\mu\beta\rho}p_{1\alpha}p_{2\beta}\} \qquad (25)$$

$$|M_2|^2 = g^4 M_W^2|C_W|^2|V_{ff'}|^2\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}\right)\left(-\delta^\nu_\rho + \frac{p_4^\nu p_{4\rho}}{M_W^2} + \frac{p_\rho p^\nu}{M_W^2} - \frac{p_\rho p_4^\nu (p_4\cdot p)}{M_W^4}\right)\{p_2^\mu p_1^\rho + p_2^\rho p_1^\mu$$
$$- (p_1\cdot p_1)\eta^{\mu\rho} + i\,\varepsilon^{\alpha\mu\beta\rho}p_{1\alpha}p_{2\beta}\} \qquad (26)$$

$$= g^4 M_W^2|C_W|^2|V_{ff'}|^2\left[-\eta_{\mu\rho} + \frac{p_{4\mu}p_{4\rho}}{M_W^2} + \frac{p_\mu p_\rho}{M_W^2} - \frac{p_{4\mu}p_\rho(p_4\cdot p)}{M_W^4} + \frac{p_\mu p_\rho}{M_W^2} - \frac{p_\mu p_{4\rho}(p\cdot p_4)}{M_W^4}\right.$$
$$\left. - \frac{p_\mu p^2 p_\rho}{M_W^4} + \frac{p_\mu p_\rho (p_4\cdot p)^2}{M_W^6}\right][\,p_2^\mu p_1^\rho + p_2^\rho p_1^\mu - (p_1\cdot p_2)\eta^{\mu\rho} + i\,\varepsilon^{\alpha\mu\beta\rho}p_{1\alpha}p_{2\beta}\,] \qquad (27)$$

$$\eta_{\mu\rho}\,\varepsilon^{\alpha\mu\beta\rho} = 0$$

$$p_{4\mu}p_{4\rho}\,\varepsilon^{\alpha\mu\beta\rho} = 0$$

$$p_\mu p_\rho\,\varepsilon^{\alpha\mu\beta\rho} = 0$$

$$p_\rho\,\varepsilon^{\alpha\mu\beta\rho}p_{1\alpha}p_{2\beta} = \cancelto{0}{p_{1\rho}p_{1\alpha}p_{2\beta}\,\varepsilon^{\alpha\mu\beta\rho}} + \cancelto{0}{p_{2\rho}p_{1\alpha}p_{2\beta}\,\varepsilon^{\alpha\mu\beta\rho}} = 0$$

$$p_\mu\,\varepsilon^{\alpha\mu\beta\rho}p_{1\alpha}p_{2\beta} = \cancelto{0}{p_{1\mu}p_{1\alpha}p_{2\beta}\,\varepsilon^{\alpha\mu\beta\rho}} + \cancelto{0}{p_{2\mu}p_{1\alpha}p_{2\beta}\,\varepsilon^{\alpha\mu\beta\rho}} = 0$$

$$\Rightarrow |M_2|^2 = g^4 M_W^2|C_W|^2|V_{ff'}|^2\left[\overset{\downarrow}{-\eta_{\mu\rho}} + \overset{\downarrow}{\frac{p_{4\mu}p_{4\rho}}{M_W^2}} + \overset{\downarrow}{\frac{2p_\mu p_\rho}{M_W^2}} - \overset{\downarrow}{\frac{(p\cdot p_4)}{M_W^4}}(p_{4\mu}p_\rho + p_\mu p_{4\rho})\right.$$
$$\left.\overset{\downarrow}{-\frac{p_\mu p_\rho\,\hat{s}}{M_W^4}} + \overset{\downarrow}{\frac{p_\mu p_\rho (p\cdot p_4)^2}{M_W^6}}\right][\,p_2^\mu p_1^\rho + p_2^\rho p_1^\mu - (p_1\cdot p_2)\eta^{\mu\rho}\,] \qquad (28)$$

$$|M_2|^2 = g^4 M_W^2 |C_W|^2 |V_{ff'}|^2 \Big\{ \cancel{2(P_1 \cdot P_2)} + \cancel{2\frac{(P_2 \cdot P_4)(P_1 \cdot P_4)}{M_W^2}} - \cancel{(P_1 \cdot P_2)} + \cancel{\frac{4(P \cdot P_2)(P \cdot P_1)}{M_W^2}} - \cancel{\frac{2\hat{s}(P_1 \cdot P_2)}{M_W^2}}$$

$$- 2\frac{(P \cdot P_4)}{M_W^4}\Big((P_2 \cdot P_4)(P \cdot P_1) + (P \cdot P_2)(P_1 \cdot P_4) - (P \cdot P_4)(P_1 \cdot P_2)\Big) - \frac{\hat{s}}{M_W^4}\Big(2(P \cdot P_2)(P \cdot P_1) - \hat{s}(P_1 \cdot P_2)\Big)$$

$$+ \frac{(P \cdot P_4)^2}{M_W^6}\Big[2(P \cdot P_2)(P \cdot P_1) - \hat{s}(P_1 \cdot P_2)\Big]\Big\} \qquad (29)$$

$$\hat{t} = (P_1 - P_3)^2 = (P_4 - P_2)^2 = M_W^2 + m_{f'}^2 - 2(P_2 \cdot P_4)$$

$$\boxed{(P_2 \cdot P_4) = \frac{M_W^2 + m_{f'}^2 - \hat{t}}{2}} \qquad (30)$$

$$\hat{U} = (P_1 - P_4)^2 = m_f^2 + M_W^2 - 2(P_1 \cdot P_4)$$

$$\boxed{(P_1 \cdot P_4) = \frac{M_W^2 + m_f^2 - \hat{U}}{2}} \qquad (31)$$

$$P \cdot P_1 = (P_1 + P_2) \cdot P_1 = m_f^2 + (P_1 \cdot P_2) = m_f^2 + \frac{\hat{s} - m_f^2 - m_{f'}^2}{2}$$

$$\boxed{(P \cdot P_1) = \frac{\hat{s} + m_f^2 - m_{f'}^2}{2}} \qquad (32)$$

$$P \cdot P_2 = (P_1 + P_2) \cdot P_2 = m_{f'}^2 + (P_1 \cdot P_2) = m_{f'}^2 + \frac{\hat{s} - m_f^2 - m_{f'}^2}{2}$$

$$\boxed{(P \cdot P_2) = \frac{\hat{s} - m_f^2 + m_{f'}^2}{2}} \qquad (33)$$

$$(P \cdot P_4) = (P_1 \cdot P_4) + (P_2 \cdot P_4) = \frac{2M_W^2 + m_f^2 + m_{f'}^2 - \hat{U} - \hat{t}}{2}$$

$$\hat{s} + \hat{t} + \hat{U} = m_f^2 + m_{f'}^2 + m_{h^0}^2 + M_W^2$$

$$\boxed{(P \cdot P_4) = \frac{M_W^2 + \hat{s} - m_{h^0}^2}{2}} \qquad (34)$$

$$|M_2|^2 = \frac{g^4 |C_W|^2 |V_{ff'}|^2}{M_W^4}\Big\{ \frac{M_W^6}{2}(\hat{s} - m_f^2 - m_{f'}^2) + \frac{1}{2}M_W^4 (M_W^2 + m_{f'}^2 - \hat{t})(M_W^2 + m_f^2 - \hat{U})$$

$$+ M_W^4 (\hat{s} - m_f^2 + m_{f'}^2)(\hat{s} + m_f^2 - m_{f'}^2) - \hat{s} M_W^4 (\hat{s} - m_f^2 - m_{f'}^2)$$

$$- M_W^2 (M_W^2 + \hat{s} - m_{h^0}^2)\Big[\frac{1}{4}(M_W^2 + m_{f'}^2 - \hat{t})(\hat{s} + m_f^2 - m_{f'}^2) + \frac{1}{4}(\hat{s} - m_f^2 + m_{f'}^2)(M_W^2 + m_f^2$$

$$- \hat{U}) - \frac{1}{4}(M_W^2 + \hat{s} - m_{h^0}^2)(\hat{s} - m_f^2 - m_{f'}^2)\Big] - \hat{s} M_W^2 \Big[\frac{1}{2}(\hat{s} - m_f^2 + m_{f'}^2)(\hat{s} + m_f^2 - m_{f'}^2)$$

$$- \frac{\hat{s}}{2}(\hat{s} - m_f^2 - m_{f'}^2)\Big] + \frac{1}{4}(M_W^2 + \hat{s} - m_{h^0}^2)^2 \Big[\frac{1}{2}(\hat{s} - m_f^2 + m_{f'}^2)(\hat{s} + m_f^2 - m_{f'}^2)$$

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neglecting m_f^2, $m_{f'}^2$ compared with $\hat{s}$:

$$|M_2|^2 \simeq \frac{g^4 |C_W|^2 |V_{ff'}|^2}{M_W^4} \Big\{ \frac{\hat{s}}{2} \cancel{M_W^6} + \frac{1}{2} M_W^4 (M_W^2 - \hat{t})(M_W^2 - \hat{u}) + \cancel{M_W^4 \hat{s}^2} - \cancel{M_W^4 \hat{s}^2}$$

$$- M_W^2 (M_W^2 + \hat{s} - m_{h^0}^2) \Big[\frac{1}{4} (M_W^2 - \hat{t}) \hat{s} + \frac{1}{4} \hat{s} (M_W^2 - \hat{u}) - \frac{1}{4} (M_W^2 + \hat{s} - m_{h^0}^2) \hat{s} \Big]$$

$$- \hat{s} M_W^2 \Big[\cancel{\frac{1}{2} \hat{s}^2} - \frac{1}{2} \hat{s}^2 \Big] + \frac{1}{4} (M_W^2 + \hat{s} - m_{h^0}^2)^2 \Big[\cancel{\frac{1}{2} \hat{s}^2 - \frac{1}{2} \hat{s}^2} \Big] \Big\}$$

$$|M_2|^2 = \frac{g^4 |C_W|^2 |V_{ff'}|^2}{M_W^4} \Big\{ \frac{\hat{s}}{2} M_W^6 + \frac{1}{2} M_W^4 (M_W^2 - \hat{t})(M_W^2 - \hat{u}) - \frac{M_W^2}{4} (M_W^2 + \hat{s} - m_{h^0}^2) \hat{s} \Big[M_W^2 - \hat{t}$$

$$+ \cancel{M_W^2} - \hat{u} - \cancel{M_W^2} - \hat{s} + m_{h^0}^2 \Big] \Big\}$$

$$|M_2|^2 = \frac{g^4 |C_W|^2 |V_{ff'}|^2}{M_W^4} \Big\{ \frac{\hat{s}}{2} M_W^6 + \frac{1}{2} M_W^4 (M_W^4 - \hat{u} M_W^2 - \hat{t} M_W^2 + \hat{u}\hat{t}) - \frac{M_W^2}{4} (M_W^2 + \hat{s} - m_{h^0}^2) \hat{s} .$$

$$\cdot [0] \Big\}$$

$$\hat{s} + \hat{t} + \hat{u} \approx m_{h^0}^2 + M_W^2$$

$$\Rightarrow |M_2|^2 \approx \frac{g^4 |C_W|^2 |V_{ff'}|^2}{2 \cancel{M_W^4}} \cancel{M_W^4} \Big\{ \hat{s} M_W^2 + M_W^4 - \hat{u} M_W^2 - \hat{t} M_W^2 + \hat{u}\hat{t} \Big\}$$

$$= \frac{g^4 |C_W|^2 |V_{ff'}|^2}{2} \Big\{ \hat{s} M_W^2 + M_W^2 (\hat{s} - m_{h^0}^2) + \hat{u}\hat{t} \Big\}$$

$$\boxed{|M_2|^2 = \frac{g^4 |C_W|^2 |V_{ff'}|^2}{2} \Big\{ 2 \hat{s} M_W^2 - m_W^2 m_{h^0}^2 + \hat{u}\hat{t} \Big\}} \qquad (36)$$

$$\hat{u}\hat{t} = m_{h^0}^2 M_W^2 + \frac{1}{4} \lambda (\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \qquad (37)$$

(see (55) in $\mu^- \mu^+ \to H^\mp W^\pm$) and (58) here

$$\Rightarrow \boxed{|M_2|^2 = \frac{g^4 |C_W|^2 |V_{ff'}|^2}{2} \Big\{ 2 \hat{s} M_W^2 + \frac{1}{4} \lambda (\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \Big\}}$$

$$\boxed{|M_2|^2 = \frac{g^4 |C_W|^2 |V_{ff'}|^2}{8} \Big\{ 8 \hat{s} M_W^2 + \lambda (\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \Big\}} \qquad (38)$$

Let's return to (20)

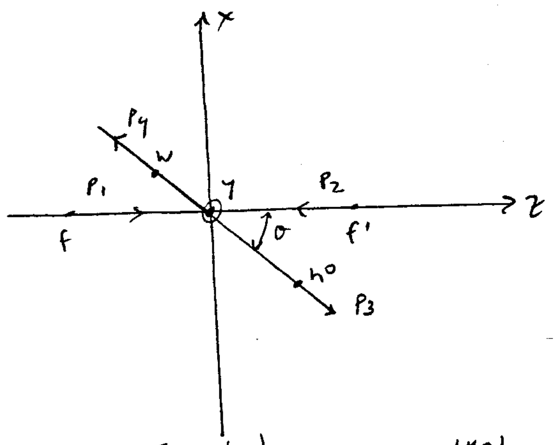
neglecting m_f^2, $m_{f'}^2$, $m_f m_{f'}$ compared with $\hat{s}$:

$$|M_1|^2 = \frac{g^4 |V_{ff'}|^2 |C_{H^+}|^2}{16 M_W^4} \lambda (\hat{s}, m_{h^0}^2, M_W^2) \hat{s} (A^2 + B^2)$$

$$= \frac{g^4 |V_{ff'}|^2 |C_{H^+}|^2}{16 M_W^4} \lambda (\hat{s}, m_{h^0}^2, M_W^2) \hat{s} \left[2 m_{f'}^2 \tan^2\beta + 2 m_f^2 \cot^2\beta \right]$$

$$\boxed{|M_1|^2 = \frac{g^4 |V_{ff'}|^2 |C_{H^+}|^2 \hat{s} \lambda (\hat{s}, m_{h^0}^2, M_W^2)}{8 M_W^4} \left[m_{f'}^2 \tan^2\beta + m_f^2 \cot^2\beta \right]} \qquad (20a)$$

$$p^\mu = (m\cosh\phi,\ -m\sin\theta\sinh\phi,\ 0,\ m\cos\theta\sinh\phi) \qquad (39)$$



$$P_1 = (m_f\cosh\phi_1,\ 0,\ 0,\ m_f\sinh\phi_1) \qquad (40)$$

$$P_2 = (m_{f'}\cosh\phi_2,\ 0,\ 0,\ -m_{f'}\sinh\phi_2) \qquad (41)$$

$$P_3 = (m_{h^0}\cosh\phi_3,\ -m_{h^0}\sin\theta\sinh\phi_3,\ 0,\ m_{h^0}\cos\theta\sinh\phi_3) \qquad (42)$$

$$P_4 = (M_W\cosh\phi_4,\ M_W\sin\theta\sinh\phi_4,\ 0,\ -M_W\cos\theta\sinh\phi_4) \qquad (43)$$

$$P_1 + P_2 = P_3 + P_4$$

$$\Rightarrow \boxed{m_{h^0}\sinh\phi_3 = M_W\sinh\phi_4} \qquad (44)$$

$$\boxed{m_f\sinh\phi_1 = m_{f'}\sinh\phi_2} \qquad (45)$$

$$\hat{s} = (P_1+P_2)^2 = (m_f\cosh\phi_1 + m_{f'}\cosh\phi_2)^2$$

$$\hat{s} = m_f^2\cosh^2\phi_1 + m_{f'}^2\cosh^2\phi_2 + 2m_f m_{f'}\cosh\phi_1\cosh\phi_2$$

$$\hat{s} = m_f^2(1 + \sinh^2\phi_1) + m_{f'}^2(1+\sinh^2\phi_2) + 2m_f m_{f'}(1+\sinh^2\phi_1)^{1/2}(1+\sinh^2\phi_2)^{1/2}$$

$$\hat{s} = m_f^2 + m_{f'}^2 + 2m_f^2\sinh^2\phi_1 + 2(m_f^2 + m_f^2\sinh^2\phi_1)^{1/2}(m_{f'}^2 + m_f^2\sinh^2\phi_1)^{1/2}$$

$$(\hat{s} - m_f^2 - m_{f'}^2 - 2m_f^2\sinh^2\phi_1)^2 = 4m_f^2(1+\sinh^2\phi_1)(m_{f'}^2 + m_f^2\sinh^2\phi_1)$$

$$\hat{s}^2 + m_f^4 + m_{f'}^4 + 4\cancel{m_f^4\sinh^4\phi_1} - 2\hat{s}m_f^2 - 2\hat{s}m_{f'}^2 - 4\hat{s}m_f^2\sinh^2\phi_1 + 2\cancel{m_f^2}m_{f'}^2$$
$$+ 4m_f^4\cancel{\sinh^2\phi_1} + 4m_f^2\cancel{m_{f'}^2\sinh^2\phi_1} = 4\cancel{m_f^2}m_{f'}^2 + \cancel{4m_f^4\sinh^2\phi_1} + \cancel{4m_f^2m_{f'}^2\sinh^2\phi_1}$$
$$+ 4m_f^4\cancel{\sinh^4\phi_1}$$

$$\hat{s}^2 + m_f^4 + m_{f'}^4 - 2\hat{s}m_f^2 - 2\hat{s}m_{f'}^2 - 4\hat{s}m_f^2\sinh^2\phi_1 - 2m_f^2m_{f'}^2 = 0$$

$$\sinh^2\phi_1 = \frac{\lambda(\hat{s}, m_f^2, m_{f'}^2)}{4\hat{s}m_f^2}$$

$$\boxed{\sinh\phi_1 = \frac{\lambda^{1/2}(\hat{s}, m_f^2, m_{f'}^2)}{2\hat{s}^{1/2}m_f}} \qquad (46)$$

$$\hat{t} = (P_1 - P_3)^2 = (m_f \cosh\phi_1 - m_{h^0}\cosh\phi_3 ,\ m_{h^0}\sin\theta \sinh\phi_3 ,\ 0 ,\ m_f \sinh\phi_1 - m_{h^0}\cos\theta \sinh\phi_3)^2$$

$$\hat{t} = (m_f \cosh\phi_1 - m_{h^0}\cosh\phi_3)^2 - m_{h^0}^2 \sin^2\theta \sinh^2\phi_3 - m_f^2 \sinh^2\phi_1 - m_{h^0}^2 \cos^2\theta \sinh^2\phi_3$$
$$+ 2 m_f m_{h^0} \sinh\phi_1 \cos\theta \sinh\phi_3$$

$$\hat{t} = m_f^2 \cancel{\cosh^2}\phi_1 + m_{h^0}^2 \cancel{\cosh^2}\phi_3 - 2m_f m_{h^0} \cosh\phi_1 \cosh\phi_3 - m_{h^0}^2 \sin^2\theta \Big) \sinh^2\phi_3 - m_f^2 \cancel{\sinh^2}\phi_1$$
$$- m_{h^0}^2 \cos^2\theta \cancel{\sinh^2}\phi_3 + 2m_f m_{h^0} \sinh\phi_1 \cos\theta \sinh\phi_3$$

$$\hat{t} = m_f^2 + m_{h^0}^2 - 2m_f m_{h^0} \cosh\phi_1 \cosh\phi_3 + 2m_f m_{h^0} \sinh\phi_1 \cos\theta \sinh\phi_3 \qquad (47)$$

$$\cosh^2\phi_1 - \sinh^2\phi_1 = 1$$

$$\cosh^2\phi_1 = 1 + \frac{\lambda(\hat{s}, m_f^2, m_{f'}^2)}{4\hat{s} m_f^2}$$

$$\boxed{\cosh\phi_1 = \frac{(4\hat{s} m_f^2 + \lambda(\hat{s}, m_f^2, m_{f'}^2))^{1/2}}{2\sqrt{\hat{s}}\, m_f}} \qquad (48)$$

$$\hat{t} = m_f^2 + m_{h^0}^2 - \frac{m_{h^0} (4\hat{s} m_f^2 + \lambda(\hat{s}, m_f^2, m_{f'}^2))^{1/2}}{(\hat{s})^{1/2}} \cosh\phi_3 + \frac{m_{h^0} \lambda^{1/2}(\hat{s}, m_f^2, m_{f'}^2)}{\hat{s}^{1/2}} \cos\theta \sinh\phi_3 \qquad (49)$$

on the other hand:

$$\hat{s} = (P_3 + P_4)^2 = (m_{h^0}\cosh\phi_3 + M_W \cosh\phi_4 ,\ 0 ,\ 0 ,\ 0)^2$$

$$\hat{s} = m_{h^0}^2 \cosh^2\phi_3 + M_W^2 \cosh^2\phi_4 + 2m_{h^0} M_W \cosh\phi_3 \cosh\phi_4$$

$$m_{h^0}^2 \sinh^2\phi_3 = M_W^2 \sinh^2\phi_4$$

$$m_{h^0}^2 (\cosh^2\phi_3 - 1) = M_W^2 (\cosh^2\phi_4 - 1)$$

$$\cosh^2\phi_4 = \frac{m_{h^0}^2}{M_W^2} (\cosh^2\phi_3 - 1) + 1$$

$$\cosh\phi_4 = \left(\frac{m_{h^0}^2}{M_W^2} (\cosh^2\phi_3 - 1) + 1 \right)^{1/2} \qquad (50)$$

$$\hat{s} = m_{h^0}^2 \cosh^2\phi_3 + M_W^2 \left[\frac{m_{h^0}^2}{M_W^2} (\cosh^2\phi_3 - 1) + 1 \right] + 2m_{h^0} M_W \cosh\phi_3 \left[\frac{m_{h^0}^2}{M_W^2} (\cosh^2\phi_3 - 1) + 1 \right]^{1/2}$$

$$\hat{s} = 2m_{h^0}^2 \cosh^2\phi_3 - m_{h^0}^2 + M_W^2 + 2m_{h^0} M_W \cosh\phi_3 \left[\frac{m_{h^0}^2}{M_W^2} (\cosh^2\phi_3 - 1) + 1 \right]^{1/2}$$

$$(\hat{s} - 2m_{h^0}^2 \cosh^2\phi_3 + m_{h^0}^2 - M_W^2)^2 = 4m_{h^0}^2 M_W^2 \cosh^2\phi_3 \left[\frac{m_{h^0}^2}{M_W^2} (\cosh^2\phi_3 - 1) + 1 \right]$$

$$\hat{s}^2 + 4m_{h^0}^4 \cancel{\cosh^4\phi_3} + m_{h^0}^4 + M_W^4 - 4\hat{s} m_{h^0}^2 \cosh^2\phi_3 + 2\hat{s} m_{h^0}^2 - 2\hat{s} M_W^2 - 4\cancel{m_{h^0}^4} \cosh^2\phi_3$$
$$+ 4m_{h^0}^2 M_W^2 \cancel{\cosh^2\phi_3} - 2m_{h^0}^2 M_W^2 = 4m_{h^0}^4 \cancel{\cosh^4}\phi_3 - 4\cancel{m_{h^0}^4 \cosh^2\phi_3} + 4\cancel{m_{h^0}^2 M_W^2} \cosh^2\phi_3$$

$$\cosh^2\phi_3 = \frac{-2m_{h^0}^2 M_W^2 + m_{h^0}^4 + M_W^4 + 2\hat{s} m_{h^0}^2 - 2\hat{s} M_W^2 + \hat{s}^2}{4\hat{s} m_{h^0}^2} = \frac{(\hat{s} + m_{h^0}^2 - M_W^2)^2}{4\hat{s} m_{h^0}^2}$$

$$\Rightarrow \boxed{\cosh\phi_3 = \frac{\hat{s} + m_{h^0}^2 - M_W^2}{2\sqrt{\hat{s}}\, m_{h^0}}} \qquad (51)$$

$$\cosh^2\phi_3 - \sinh^2\phi_3 = 1$$

$$\Rightarrow \sinh^2\phi_3 = \frac{(\hat{s} + m_{h^0}^2 - M_W^2)^2}{4\hat{s}\, m_{h^0}^2} - 1$$

$$\sinh^2\phi_3 = \frac{\lambda(\hat{s}, m_{h^0}^2, M_W^2)}{4\hat{s}\, m_{h^0}^2}$$

$$\boxed{\sinh\phi_3 = \frac{\lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)}{2\sqrt{\hat{s}}\, m_{h^0}}} \qquad (52)$$

$$\cosh\phi_4 = \left[\frac{\cancel{m_{h^0}^2}}{M_W^2}\, \frac{\lambda(\hat{s}, m_{h^0}^2, M_W^2)}{4\hat{s}\, \cancel{m_{h^0}^2}} + 1\right]^{1/2}$$

$$\boxed{\cosh\phi_4 = \left(\frac{\lambda(\hat{s}, m_{h^0}^2, M_W^2)}{4\hat{s}\, M_W^2} + 1\right)^{1/2}} \qquad (53)$$

$$\hat{t} = m_f^2 + m_{h^0}^2 - \frac{\cancel{m_{h^0}}\,(4\hat{s}\, m_f^2 + \lambda(\hat{s}, m_f^2, m_{f'}^2))^{1/2}}{(\hat{s})^{1/2}}\, \frac{(\hat{s} + m_{h^0}^2 - M_W^2)}{2(\hat{s})^{1/2}\, \cancel{m_{h^0}}} + \frac{\cancel{m_{h^0}}\,\lambda^{1/2}(\hat{s}, m_f^2, m_{f'}^2)}{\sqrt{\hat{s}}}\cos\theta \,.$$

$$\cdot\, \frac{\lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)}{2\sqrt{\hat{s}}\, \cancel{m_{h^0}}}$$

$$\boxed{\hat{t} = \frac{1}{2}\left[2m_f^2 + 2m_{h^0}^2 - \frac{(4\hat{s}\, m_f^2 + \lambda(\hat{s}, m_f^2, m_{f'}^2))^{1/2}\,(\hat{s} + m_{h^0}^2 - M_W^2)}{\hat{s}}\right] + \frac{1}{2}\cos\theta\, \frac{\lambda^{1/2}(\hat{s}, m_f^2, m_{f'}^2)\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)}{\hat{s}}} \qquad (54)$$

$$\hat{s} + \hat{t} + \hat{u} = m_f^2 + m_{f'}^2 + m_{h^0}^2 + M_W^2$$

$$\hat{u} = \cancel{m_f^2} + m_{f'}^2 + \cancel{m_{h^0}^2} + M_W^2 - \hat{s} - \cancel{m_f^2} - \cancel{m_{h^0}^2} + \frac{1}{2}\, \frac{(4\hat{s}\, m_f^2 + \lambda(\hat{s}, m_f^2, m_{f'}^2))^{1/2}\,(\hat{s} + m_{h^0}^2 - M_W^2)}{\hat{s}}$$

$$- \frac{1}{2}\cos\theta\, \frac{\lambda^{1/2}(\hat{s}, m_f^2, m_{f'}^2)\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)}{\hat{s}}$$

$$\boxed{\hat{u} = \frac{1}{2}\left[2m_{f'}^2 + 2M_W^2 - 2\hat{s} + \frac{(4\hat{s}\, m_f^2 + \lambda(\hat{s}, m_f^2, m_{f'}^2))^{1/2}\,(\hat{s} + m_{h^0}^2 - M_W^2)}{\hat{s}}\right] - \frac{1}{2}\cos\theta\, \frac{\lambda^{1/2}(\hat{s}, m_f^2, m_{f'}^2)\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)}{\hat{s}}} \qquad (55)$$

If $m_f = m_{f'} = 0$

$$\hat{t} = \frac{1}{2}\left[2\cancel{m_{h^0}^2} - \hat{s} - \cancel{m_{h^0}^2} + M_W^2 \right] + \frac{1}{2}\cos\theta\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)$$

$$\boxed{\hat{t} = \frac{1}{2}\left[m_{h^0}^2 + M_W^2 - \hat{s} \right] + \frac{1}{2}\cos\theta\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)} \qquad (56)$$

$$\hat{u} = \frac{1}{2}\left[2\cancel{M_W^2} - 2\cancel{\hat{s}} + \cancel{\hat{s}} + m_{h^0}^2 - \cancel{M_W^2} \right] - \frac{1}{2}\cos\theta\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)$$

$$\boxed{\hat{u} = \frac{1}{2}\left[m_{h^0}^2 + M_W^2 - \hat{s} \right] - \frac{1}{2}\cos\theta\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)} \qquad (57)$$

$$\hat{u}\hat{t} = \frac{1}{4}(m_{h^0}^2 + M_W^2 - \hat{s})^2 - \frac{1}{4}\cos^2\theta\, \lambda(\hat{s}, m_{h^0}^2, M_W^2)$$

$$\hat{u}\hat{t} = \frac{1}{4}\left[\cancel{m_{h^0}^4} + \cancel{M_W^4} + \cancel{\hat{s}^2} + 2 m_{h^0}^2 M_W^2 - \cancel{2 m_{h^0}^2 \hat{s}} - \cancel{2 M_W^2 \hat{s}} - \cancel{\hat{s}^2} - \cancel{m_{h^0}^4} - \cancel{M_W^4} + \cancel{2\hat{s} m_{h^0}^2} + \cancel{2\hat{s} M_W^2} \right.$$
$$\left. + 2 m_{h^0}^2 M_W^2 \right] + \frac{1}{4}\sin^2\theta\, \lambda(\hat{s}, m_{h^0}^2, M_W^2)$$

$$\hat{u}\hat{t} = \frac{1}{4}\left[4 m_{h^0}^2 M_W^2 \right] + \frac{1}{4}\sin^2\theta\, \lambda(\hat{s}, m_{h^0}^2, M_W^2)$$

$$\boxed{\hat{u}\hat{t} = m_{h^0}^2 M_W^2 + \frac{1}{4}\lambda(\hat{s}, m_{h^0}^2, M_W^2)\sin^2\theta} \qquad (58)$$

$$M_1^* M_2 = -\frac{g^4 |V_{ff'}|^2 C_{H^+}^* C_W}{16}\left(\sum_\lambda \varepsilon_{4\mu}\varepsilon_{4\rho}^*\right)\eta^{\nu\rho}(p_1+p_2+p_3)^\mu \left(\eta_{\sigma\nu} - \frac{p_\sigma p_\nu}{M_W^2}\right)\sum_s \bar{v}_1 (A + B\gamma^5) v_2 \,.$$
$$. \bar{v}_2 \gamma^\sigma (1-\gamma^5) v_1 \qquad (59)$$

$$M_1^* M_2 = -\frac{g^4 |V_{ff'}|^2 C_{H^+}^* C_W}{16}\left(-\eta_{\mu\rho} + \frac{p_{4\mu} p_{4\rho}}{M_W^2}\right)\eta^{\nu\rho}(p_1+p_2+p_3)^\mu \left(\eta_{\sigma\nu} - \frac{p_\sigma p_\nu}{M_W^2}\right).$$
$$. \mathrm{Tr}\left[(\not{p}_1 + m_f)(A + B\gamma^5)(\not{p}_2 - m_{f'})\gamma^\sigma (1-\gamma^5) \right] \qquad (60)$$

$$\mathrm{Tr}\left[(\not{p}_1 + m_f)(A + B\gamma^5)(\not{p}_2 - m_{f'})\gamma^\sigma (1-\gamma^5) \right]$$
$$= \mathrm{Tr}\left[(A\overset{\downarrow}{\not{p}_1} + B\overset{\downarrow}{\not{p}_1}\gamma^5 + m_f \overset{\downarrow}{A} + m_f \overset{\downarrow}{B}\gamma^5)(\not{p}_2\gamma^\sigma - \not{p}_2\gamma^\sigma\gamma^5 - m_{f'}\gamma^\sigma + m_{f'}\gamma^\sigma\gamma^5) \right]$$
$$= A\,\mathrm{Tr}(\not{p}_1\not{p}_2\gamma^\sigma) - A\,\overset{0}{\mathrm{Tr}(\not{p}_1\not{p}_2\gamma^\sigma\gamma^5)} - A m_{f'}\mathrm{Tr}(\not{p}_1\gamma^\sigma) + m_{f'} A\, \overset{0}{\mathrm{Tr}(\not{p}_1\gamma^\sigma\gamma^5)}$$
$$+ B\,\overset{0}{\mathrm{Tr}(\not{p}_1\gamma^5\not{p}_2\gamma^\sigma)} - B\,\overset{0}{\mathrm{Tr}(\not{p}_1\gamma^5\not{p}_2\gamma^\sigma\gamma^5)} - m_{f'}B\,\overset{0}{\mathrm{Tr}(\not{p}_1\gamma^5\gamma^\sigma)} + m_{f'}B\,\mathrm{Tr}(\not{p}_1\gamma^5\gamma^\sigma\gamma^5)$$
$$+ m_f A\,\mathrm{Tr}(\not{p}_2\gamma^\sigma) - m_f A\,\overset{0}{\mathrm{Tr}(\not{p}_2\gamma^\sigma\gamma^5)} - m_f m_{f'} A\,\overset{0}{\mathrm{Tr}(\gamma^\sigma)} + m_f m_{f'} A\,\overset{0}{\mathrm{Tr}(\gamma^\sigma\gamma^5)} + m_f B\,\overset{0}{\mathrm{Tr}(\gamma^5\not{p}_2\gamma^\sigma)}$$
$$- m_f B\,\mathrm{Tr}(\gamma^5\not{p}_2\gamma^\sigma\gamma^5) - m_f m_{f'} B\,\overset{0}{\mathrm{Tr}(\gamma^5\gamma^\sigma)} + m_f m_{f'} B\,\overset{0}{\mathrm{Tr}(\gamma^5\gamma^\sigma\gamma^5)}$$

$$= -4 A m_{f'} p_1^\sigma - 4 m_{f'} B p_1^\sigma + 4 m_f A p_2^\sigma - 4 m_f B p_2^\sigma$$

$$\Rightarrow \mathrm{Tr} = 4\left[-(A+B) m_{f'} p_1^\sigma + (A-B) m_f p_2^\sigma \right] \qquad (61)$$

$$M_1^* M_2 = -\frac{g^4 |V_{ff'}|^2 C_H^* + C_W}{16} \left(-\eta_{\mu\rho} + \frac{P_{4\mu} P_{4\rho}}{M_W^2}\right) \eta^{\nu\rho} (P_1 + P_2 + P_3)^\mu \left(\eta_{\sigma\nu} - \frac{P_\sigma P_\nu}{M_W^2}\right) 4 \times$$

$$\times \left(-(A+B) m_{f'} P_1^\sigma + (A-B) m_f P_2^\sigma\right) \qquad (62)$$

$$M_1^* M_2 = \frac{-g^4 |V_{ff'}|^2 C_H^* + C_W}{4} \left(-\delta_\mu^\nu + \frac{P_{4\mu} P_4^\nu}{M_W^2}\right) (P_1 + P_2 + P_3)^\mu \Big[-(A+B) m_{f'} P_{1\nu} + (A-B) m_f P_{2\nu}$$

$$+ \frac{(A+B) m_{f'} (P\cdot P_1) P_\nu}{M_W^2} - \frac{(A-B) m_f (P\cdot P_2) P_\nu}{M_W^2} \Big] \qquad (63)$$

$$= -\frac{g^4 |V_{ff'}|^2 C_H^* + C_W}{4} \left[-(P_1+P_2+P_3)^\nu + \frac{((P_1+P_2+P_3)\cdot P_4) P_4^\nu}{M_W^2}\right] \Big[-(A+B) m_{f'} P_{1\nu} + (A-B) m_f P_{2\nu}$$

$$+ \frac{(A+B) m_{f'} (P\cdot P_1) P_\nu}{M_W^2} - \frac{(A-B) m_f (P\cdot P_2) P_\nu}{M_W^2} \Big] \qquad (64)$$

$$= -\frac{g^4 |V_{ff'}|^2 C_H^* + C_W}{4} \Big[(A+B) m_{f'} (P_1\cdot(P_1+P_2+P_3)) - (A-B) m_f ((P_1+P_2+P_3)\cdot P_2)$$

$$- \frac{(A+B) m_{f'} (P\cdot P_1)(P\cdot(P_1+P_2+P_3))}{M_W^2} + \frac{(A-B) m_f (P\cdot P_2)(P\cdot(P_1+P_2+P_3))}{M_W^2}$$

$$- \frac{(A+B) m_{f'}}{M_W^2} (P_1\cdot P_4)((P_1+P_2+P_3)\cdot P_4) + \frac{(A-B) m_f}{M_W^2} (P_2\cdot P_4)((P_1+P_2+P_3)\cdot P_4)$$

$$+ \frac{(A+B) m_{f'} (P\cdot P_1)(P\cdot P_4)((P_1+P_2+P_3)\cdot P_4)}{M_W^4} - \frac{(A-B) m_f (P\cdot P_2)(P\cdot P_4)((P_1+P_2+P_3)\cdot P_4)}{M_W^4} \Big] \qquad (65)$$

$$P_1\cdot(P_1+P_2+P_3) = m_f^2 + \left(\frac{\hat{s} - m_f^2 - m_{f'}^2}{2}\right) + (P_1\cdot P_3)$$

$$\hat{t} = (P_1 - P_3)^2 = m_f^2 + m_{h^0}^2 - 2(P_1\cdot P_3)$$

$$\boxed{(P_1\cdot P_3) = \frac{m_f^2 + m_{h^0}^2 - \hat{t}}{2}} \qquad (66)$$

$$P_1\cdot(P_1+P_2+P_3) = m_f^2 + \frac{\hat{s} - \cancel{m_f^2} - m_{f'}^2}{\cancel{2}} + \frac{\cancel{m_f^2} + m_{h^0}^2 - \hat{t}}{\cancel{2}}$$

$$\boxed{P_1\cdot(P_1+P_2+P_3) = \frac{\hat{s} + 2m_f^2 - m_{f'}^2 + m_{h^0}^2 - \hat{t}}{2} \approx \frac{\hat{s} + m_{h^0}^2 - \hat{t}}{2}} \qquad (67)$$

$$P_2\cdot(P_1+P_2+P_3) = m_{f'}^2 + \frac{\hat{s} - m_f^2 - m_{f'}^2}{2} + P_2\cdot P_3$$

$$\hat{u} = (P_1 - P_4)^2 = (P_3 - P_2)^2 = m_{f'}^2 + m_{h^0}^2 - 2(P_2\cdot P_3)$$

$$\boxed{(P_2\cdot P_3) = \frac{m_{f'}^2 + m_{h^0}^2 - \hat{u}}{2}} \qquad (68)$$

$$P_2\cdot(P_1+P_2+P_3) = m_{f'}^2 + \frac{\hat{s}-m_f^2-m_{f'}^2}{2} + \frac{m_{f'}^2+m_{h^0}^2-\hat{u}}{2}$$

$$\boxed{P_2\cdot(P_1+P_2+P_3) = \frac{\hat{s}+2m_{f'}^2-m_f^2+m_{h^0}^2-\hat{u}}{2} \approx \frac{\hat{s}+m_{h^0}^2-\hat{u}}{2}} \qquad (69)$$

$$P\cdot(P_1+P_2+P_3) = \frac{\cancel{\hat{s}}+2m_f^2-\cancel{m_{f'}^2}+\cancel{m_{h^0}^2}-\cancel{\hat{t}}+\cancel{\hat{s}}+2\cancel{m_{f'}^2}-\cancel{m_f^2}+\cancel{m_{h^0}^2}-\cancel{\hat{u}}}{2}$$

$$\boxed{P\cdot(P_1+P_2+P_3) = \frac{2\hat{s}-\hat{t}-\hat{u}+m_f^2+m_{f'}^2+2m_{h^0}^2}{2} \approx \frac{2\hat{s}-\hat{t}-\hat{u}+2m_{h^0}^2}{2}} \qquad (70)$$

$$M_1^* M_2 = -\frac{g^4|V_{ff'}|^2 C_{H^+}^* C_W}{4}\Big[(A+B)m_{f'}\frac{(\hat{s}+m_{h^0}^2-\hat{t})}{2} - (A-B)m_f\frac{(\hat{s}+m_{h^0}^2-\hat{u})}{2}$$

$$-\frac{(A+B)m_{f'}}{M_W^2}\frac{\hat{s}}{2}\frac{(2\hat{s}-\hat{t}-\hat{u}+2m_{h^0}^2)}{2} + \frac{(A-B)m_f}{M_W^2}\frac{\hat{s}}{2}\frac{(2\hat{s}-\hat{t}-\hat{u}+2m_{h^0}^2)}{2}$$

$$-\frac{(A+B)m_{f'}}{M_W^2}\frac{(M_W^2-\hat{u})}{2}(\hat{s}-m_{h^0}^2) + \frac{(A-B)m_f}{M_W^2}\frac{(M_W^2-\hat{t})}{2}(\hat{s}-m_{h^0}^2)$$

$$+(A+B)m_{f'}\frac{\hat{s}}{2}\frac{(\hat{s}+M_W^2-m_{h^0}^2)}{2}\frac{(\hat{s}-m_{h^0}^2)}{M_W^4} - \frac{(A-B)m_f}{M_W^4}\frac{\hat{s}}{2}\frac{(\hat{s}+M_W^2-m_{h^0}^2)}{2}(\hat{s}-m_{h^0}^2)\Big] \qquad (71)$$

$$(3\hat{s}-M_W^2+m_{h^0}^2)$$

$$= -\frac{g^4|V_{ff'}|^2 C_{H^+}^* C_W}{16 M_W^4}\Big\{(A+B)m_{f'}\Big[2M_W^4(\hat{s}+m_{h^0}^2-\hat{t}) - M_W^2\hat{s}(2\hat{s}-\hat{t}-\hat{u}+2m_{h^0}^2)$$

$$-2M_W^2(M_W^2-\hat{u})(\hat{s}-m_{h^0}^2) + \hat{s}(\hat{s}+M_W^2-m_{h^0}^2)(\hat{s}-m_{h^0}^2)\Big]$$

$$(3\hat{s}-M_W^2+m_{h^0}^2)$$

$$+(A-B)m_f\Big[-2M_W^4(\hat{s}+m_{h^0}^2-\hat{u}) + M_W^2\hat{s}(2\hat{s}-\hat{t}-\hat{u}+2m_{h^0}^2)$$

$$+2M_W^2(M_W^2-\hat{t})(\hat{s}-m_{h^0}^2) - \hat{s}(\hat{s}+M_W^2-m_{h^0}^2)(\hat{s}-m_{h^0}^2)\Big]\Big\}$$

$$\hat{s}+\hat{t}+\hat{u} \approx m_{h^0}^2 + M_W^2$$

$$= \frac{-g^4|V_{ff'}|^2 C_{H^+}^* C_W}{16 M_W^4}\Big\{(A+B)m_{f'}\Big[\cancel{2M_W^4\hat{s}} + \cancel{2M_W^4 m_{h^0}^2} - \cancel{2M_W^4\hat{t}} - \cancel{3M_W^2\hat{s}^2} + \cancel{\hat{s}M_W^4} - \cancel{\hat{s}M_W^2 m_{h^0}^2}$$

$$-\cancel{2M_W^4\hat{s}} + \cancel{2M_W^4 m_{h^0}^2} + \cancel{2M_W^2\hat{u}\hat{s}} - \cancel{2M_W^2 m_{h^0}^2\hat{u}} + \cancel{\hat{s}^3} - \cancel{\hat{s}^2 m_{h^0}^2} + \cancel{\hat{s}^2 M_W^2} - \cancel{\hat{s}M_W^2 m_{h^0}^2}$$

$$-\cancel{m_{h^0}^2\hat{s}^2} + \cancel{\hat{s}m_{h^0}^4}\Big] + (A-B)m_f\Big[-\cancel{2M_W^4\hat{s}} - \cancel{2M_W^4 m_{h^0}^2} + \cancel{2M_W^4\hat{u}} + \cancel{3\hat{s}^2 M_W^2} - \cancel{M_W^4\hat{s}} + \cancel{\hat{s}m_{h^0}^2 M_W^2}$$

$$+\cancel{2M_W^4\hat{s}} - \cancel{2M_W^4 m_{h^0}^2} - \cancel{2M_W^2\hat{s}\hat{t}} + \cancel{2M_W^2 m_{h^0}^2\hat{t}} - \cancel{\hat{s}^3} + \cancel{\hat{s}^2 m_{h^0}^2} - \cancel{\hat{s}^2 M_W^2} + \cancel{\hat{s}M_W^2 m_{h^0}^2}$$

$$+\cancel{m_{h^0}^2\hat{s}^2} - \cancel{m_{h^0}^4\hat{s}}\Big]\Big\}$$

$$= -\frac{g^4 |V_{ff'}|^2 C_{H^+}^* C_W}{16 M_W^4} \Big\{ (A+B) m_{f'} \Big[4M_W^4 m_{h^0}^2 - \cancel{2\hat{s} M_W^2 m_{h^0}^2} - 2M_W^4 \hat{t} - \cancel{2M_W^2 \hat{s}^2} + \cancel{\hat{s} M_W^4}$$

$$+ \cancel{2M_W^2 \hat{u} \hat{s}} - \cancel{2M_W^2 m_{h^0}^2 \hat{u}} + \cancel{\hat{s}^3} - \cancel{2m_{h^0}^2 \hat{s}^2} + \cancel{\hat{s} m_{h^0}^4} \Big] + (A-B) m_f \Big[-4M_W^4 m_{h^0}^2 + \cancel{2\hat{s} M_W^2 m_{h^0}^2}$$

$$+ 2M_W^4 \hat{u} + \cancel{2M_W^2 \hat{s}^2} - \cancel{\hat{s} M_W^4} - \cancel{2M_W^2 \hat{s} \hat{t}} + \cancel{2M_W^2 m_{h^0}^2 \hat{t}} - \cancel{\hat{s}^3} + \cancel{2m_{h^0}^2 \hat{s}^2} - \cancel{\hat{s} m_{h^0}^4} \Big] \Big\}$$

$$= -\frac{g^4 |V_{ff'}|^2 C_{H^+}^* C_W}{16 M_W^4} \Big\{ (A+B) m_{f'} \Big[\hat{s}\, \lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{u} (\hat{s} - m_{h^0}^2) + 2M_W^4 (2m_{h^0}^2 - \hat{t}) \Big]$$

$$+ (A-B) m_f \Big[-\hat{s}\, \lambda(\hat{s}, m_{h^0}^2, M_W^2) - 2M_W^2 \hat{t} (\hat{s} - m_{h^0}^2) + 2M_W^4 (2m_{h^0}^2 - \hat{u}) \Big] \Big\}$$

$$\Rightarrow \boxed{\begin{aligned} M_1^* M_2 = {} & -\frac{g^4 |V_{ff'}|^2 C_{H^+}^* C_W}{8 M_W^4} \Big\{ m_{f'}^2 \tan\beta \Big[\hat{s}\, \lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{u} (\hat{s} - m_{h^0}^2) \\ & + 2M_W^4 (2m_{h^0}^2 - \hat{t}) \Big] - m_f^2 \cot\beta \Big[\hat{s}\, \lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{t} (\hat{s} - m_{h^0}^2) \\ & + 2M_W^4 (2m_{h^0}^2 - \hat{u}) \Big] \Big\} \end{aligned}} \quad (72)$$

$$\boxed{\begin{aligned} M_2^* M_1 = (M_1^* M_2)^* = {} & -\frac{g^4 |V_{ff'}|^2 C_{H^+} C_W^*}{8 M_W^4} \Big\{ m_{f'}^2 \tan\beta \Big[\hat{s}\, \lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{u} (\hat{s} - m_{h^0}^2) \\ & + 2M_W^4 (2m_{h^0}^2 - \hat{t}) \Big] - m_f^2 \cot\beta \Big[\hat{s}\, \lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{t} (\hat{s} - m_{h^0}^2) + 2M_W^4 (2m_{h^0}^2 - \hat{u}) \Big] \Big\} \end{aligned}} \quad (73)$$

$$\boxed{\begin{aligned} M_1^* M_2 + M_2^* M_1 = {} & -\frac{g^4 |V_{ff'}|^2}{8 M_W^4} 2\,\mathrm{Re}(C_{H^+}^* C_W) \Big\{ m_{f'}^2 \tan\beta \Big[\hat{s}\, \lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{u} (\hat{s} - m_{h^0}^2) \\ & + 2M_W^4 (2m_{h^0}^2 - \hat{t}) \Big] - m_f^2 \cot\beta \Big[\hat{s}\, \lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{t} (\hat{s} - m_{h^0}^2) + 2M_W^4 (2m_{h^0}^2 - \hat{u}) \Big] \Big\} \end{aligned}} \quad (74)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{1}{64\pi^2 \hat{s}} \left(\frac{P_f}{P_i}\right) \overline{|M|^2} \qquad (75)$$

$$d\hat{t} = \frac{1}{2} \frac{\lambda^{1/2}(\hat{s}, m_f^2, m_{f'}^2)\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)}{\hat{s}} \, d\cos\theta \qquad (76)$$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\cos\theta} \frac{1}{2\pi} = \frac{1}{2\pi} \frac{d\sigma}{d\hat{t}} \frac{d\hat{t}}{d\cos\theta} \qquad (77)$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2\pi} \left(\frac{d\sigma}{d\hat{t}}\right) \frac{1}{2} \frac{\lambda^{1/2}(\hat{s}, m_f^2, m_{f'}^2)\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)}{\hat{s}^{1/2}\, \hat{s}^{1/2}} \qquad (78)$$

$$\boxed{\begin{array}{l} P_i = |\vec{P}_1| = |\vec{P}_2| = m_f \sinh\phi_1 = \dfrac{\lambda^{1/2}(\hat{s}, m_f^2, m_{f'}^2)}{2\hat{s}^{1/2}} \\ \hline P_f = |\vec{P}_3| = |\vec{P}_4| = m_{h^0} \sinh\phi_3 = \dfrac{\lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)}{2\hat{s}^{1/2}} \end{array}} \qquad \begin{array}{l} (79) \\ (80) \end{array}$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{1}{\pi} \left(\frac{d\sigma}{d\hat{t}}\right) P_i P_f \qquad (81)$$

$$\Rightarrow \left(\frac{d\sigma}{d\hat{t}}\right) = \frac{\pi}{P_i P_f} \left(\frac{d\sigma}{d\Omega}\right) = \frac{\cancel{\pi}}{P_i \cancel{P_f}} \frac{1}{64\pi^2 \hat{s}} \left(\frac{\cancel{P_f}}{P_i}\right) \overline{|M|^2}$$

$$\boxed{\left(\frac{d\sigma}{d\hat{t}}\right) = \frac{1}{64\pi \hat{s} P_i^2} \overline{|M|^2}} \qquad (82)$$

$P_i^2 \approx \dfrac{\hat{s}}{4}$ (83) neglecting $m_f, m_{f'}$ (compared with $\sqrt{\hat{s}}$

$$\Rightarrow \boxed{\left(\frac{d\sigma}{d\hat{t}}\right) = \frac{1}{16\pi \hat{s}^2} \overline{|M|^2}} \qquad (84)$$

$$|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2 |V_{tf'}|^2}{32 M_W^2} \left(\sum_\lambda \varepsilon_{4\mu} \varepsilon^*_{4\nu}\right) \sum_s [\bar{V}_2 \gamma^\mu (1-\gamma^5)(\not{p}'+m_f)U_1]^+ [\bar{V}_2 \gamma^\nu (1-\gamma^5)(\not{p}'+m_f)U_1] \quad (85)$$

$$[\bar{V}_2 \gamma^\mu (1-\gamma^5)(\not{p}'+m_f)U_1]^+ = [V_2^+ \gamma^0 \gamma^\mu (1-\gamma^5)(\gamma^\alpha p'_\alpha + m_f)U_1]^+$$

$$= U_1^+ (\gamma^{\alpha +} p'_\alpha + m_f)(1-\gamma^5) \underset{\gamma^0\gamma^\mu}{\underset{\vee}{\gamma^{\mu +} \gamma^0}} V_2$$

$$= U_1^+ (\gamma^{\alpha +} p'_\alpha + m_f) \gamma^0 (1+\gamma^5) \gamma^\mu V_2$$

$$= U_1^+ (\underset{\gamma^0\gamma^\alpha}{\underset{\vee}{\gamma^{\alpha +}\gamma^0}} p'_\alpha + m_f \gamma^0)(1+\gamma^5)\gamma^\mu V_2$$

$$= \bar{U}_1 (\not{p}' + m_f)(1+\gamma^5)\gamma^\mu V_2 \quad (86)$$

$\Rightarrow$

$$|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2 |V_{tf'}|^2}{32 M_W^2} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) \sum_s (\bar{U}_1 (\not{p}'+m_f)(1+\gamma^5)\gamma^\mu V_2)(\bar{V}_2 \gamma^\nu (1-\gamma^5)(\not{p}'+m_f)U_1)$$

$$|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2 |V_{tf'}|^2}{32 M_W^2} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) \mathrm{Tr}\Big[(\not{p}_1 + m_f)(\not{p}' + m_f)(1+\gamma^5)\gamma^\mu (\not{p}_2 - m_{f'})\gamma^\nu (1-\gamma^5)(\not{p}' + m_f)\Big] \quad (87)$$

neglecting $m_f, m_{f'}$ in the trace :

$$\mathrm{Tr} = \mathrm{Tr}[\not{p}_1 \not{p}' (1+\gamma^5)\gamma^\mu \not{p}_2 \gamma^\nu (1-\gamma^5)\not{p}']$$

$$= \mathrm{Tr}[(\not{p}_1 \not{p}' \gamma^\mu \not{p}_2 + \not{p}_1 \not{p}' \gamma^5 \gamma^\mu \not{p}_2)(\gamma^\nu \not{p}' - \gamma^\nu \gamma^5 \not{p}')]$$

$$= \mathrm{Tr}(\not{p}_1 \not{p}' \gamma^\mu \not{p}_2 \cancel{\gamma^\nu} \not{p}') - \mathrm{Tr}(\not{p}_1 \not{p}' \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5 \not{p}') + \mathrm{Tr}(\not{p}_1 \not{p}' \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu \not{p}')$$
$$- \mathrm{Tr}(\not{p}_1 \not{p}' \gamma^5 \gamma^\mu \cancel{\not{p}_2} \gamma^\nu \gamma^5 \not{p}')$$

$$= 2\,\mathrm{Tr}(\not{p}_1 \not{p}' \gamma^\mu \not{p}_2 \gamma^\nu \not{p}') + 2\,\mathrm{Tr}(\not{p}_1 \not{p}' \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu \not{p}')$$

$$= 2\,\mathrm{Tr}[(-\not{p}_1 \not{p}' + 2(p_1 \cdot p'))\not{p}' \gamma^\mu \not{p}_2 \gamma^\nu] + 2\,\mathrm{Tr}[(-\not{p}_1 \not{p}' + 2(p_1 \cdot p'))\not{p}' \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu]$$

$$= -2\hat{t}\,\mathrm{Tr}(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) + 4(p_1 \cdot p')\mathrm{Tr}(\not{p}' \gamma^\mu \not{p}_2 \gamma^\nu) + 2\hat{t}\,\mathrm{Tr}(\gamma^5 \not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu)$$
$$- 4(p_1 \cdot p')\mathrm{Tr}(\gamma^5 \not{p}' \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$= -2\hat{t}\,\mathrm{Tr}(\gamma^\mu \not{p}_2 \gamma^\nu \not{p}_1) + 4(p_1 \cdot p')\mathrm{Tr}(\gamma^\mu \not{p}_2 \gamma^\nu \not{p}') + 2\hat{t}\,\mathrm{Tr}(\gamma^5 \gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu) p_{1\alpha} p_{2\beta}$$
$$- 4(p_1 \cdot p')\mathrm{Tr}(\gamma^5 \gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu) p'_\alpha p_{2\beta}$$

$$\underset{\mathrm{Tr}}{\Rightarrow} = -8\hat{t}[p_2^\mu p_1^\nu + p_2^\nu p_1^\mu - (p_1 \cdot p_2)\eta^{\mu\nu}] + 16(p_1 \cdot p')[p_2^\mu p'^\nu + p_2^\nu p'^\mu - (p_2 \cdot p')\eta^{\mu\nu}]$$
$$-8i\hat{t}\,\varepsilon^{\alpha\mu\beta\nu} p_{1\alpha} p_{2\beta} + 16i(p_1 \cdot p')\varepsilon^{\alpha\mu\beta\nu} p'_\alpha p_{2\beta} \quad (88)$$

$$n_{\mu\nu}\,\varepsilon^{\alpha\mu\beta\nu}=0$$

$$P_{4\mu}P_{4\nu}\,\varepsilon^{\alpha\mu\beta\nu}=0$$

$$\Rightarrow\ |M_3|^2=\frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{32M_W^2}\left(-n_{\mu\nu}+\frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)8\Big\{-\hat t\,[P_2^{\mu}P_1^{\nu}+P_2^{\nu}P_1^{\mu}-(P_1\cdot P_2)n^{\mu\nu}]$$
$$+2(P_1\cdot P')[P_2^{\mu}P'^{\nu}+P_2^{\nu}P'^{\mu}-(P_2\cdot P')n^{\mu\nu}]\Big\}\qquad(89)$$

$$|M_3|^2=\frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{4M_W^2}\Big\{-\hat t\,\Big[2\cancel{(P_1\cdot P_2)}+\frac{2(P_2\cdot P_4)(P_1\cdot P_4)}{M_W^2}-\cancel{(P_1\cdot P_2)}\Big]$$
$$+2(P_1\cdot P')\Big[2\cancel{(P_2\cdot P')}+2\frac{(P_2\cdot P_4)(P'\cdot P_4)}{M_W^2}-\cancel{(P_2\cdot P')}\Big]\Big\}$$

$P_1 - P_3$

$$|M_3|^2=\frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{4M_W^2}\Big\{-\hat t\,\Big[(P_1\cdot P_2)+2\frac{(P_2\cdot P_4)(P_1\cdot P_4)}{M_W^2}\Big]+2(P_1\cdot \overset{\nearrow}{P'})\Big[(P_2\cdot \overset{\nearrow}{P'})+$$
$$+2\frac{(P_2\cdot P_4)(P'\cdot P_4)}{M_W^2}\Big]\Big\}$$

$$|M_3|^2=\frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{4M_W^2}\Big\{-\hat t\,\Big[\frac{\hat s}{2}+\cancel{2}\frac{(M_W^2-\hat t)}{\cancel{2}M_W^2}\frac{(M_W^2-\hat u)}{2}\Big]-2(P_1\cdot P_3)\Big[\cancel{(P_2\cdot P_4)}$$
$$+2\cancel{(P_2\cdot P_4)}-2\frac{(P_2\cdot P_4)^2}{M_W^2}\Big]\Big\}$$

$$|M_3|^2=\frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{4M_W^2}\Big\{-\frac{\hat t}{2}\Big[\hat s+\frac{1}{M_W^2}(M_W^2-\hat t)(M_W^2-\hat u)\Big]-(m_{h^0}^2-\hat t)\Big[\frac{3}{2}(M_W^2-\hat t)$$
$$-\frac{2(M_W^2-\hat t)^2}{4M_W^2}\Big]\Big\}\qquad(90)$$

because:

$$\hat t=(P_1-P_3)^2=m_{h^0}^2-2(P_1\cdot P_3)$$

$$\boxed{(P_1\cdot P_3)=\frac{m_{h^0}^2-\hat t}{2}}\qquad(91)$$

$$|M_3|^2=\frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{4M_W^2}\Big\{\frac{-\hat t}{2M_W^2}\Big[\hat s\cancel{M_W^2}+\cancel{M_W^4}-\cancel{\hat u M_W^2}-\cancel{\hat t M_W^2}+\cancel{\hat u\hat t}\Big]-\frac{(m_{h^0}^2-\hat t)}{4M_W^2}$$
$$\cdot\Big[6M_W^2(M_W^2-\hat t)-2(M_W^2-\hat t)^2\Big]\Big\}$$

$$\hat s+\hat t+\hat u\approx m_{h^0}^2+M_W^2$$

$$M_W^4-\hat u M_W^2-\hat t M_W^2=M_W^2(M_W^2-\hat u-\hat t)=M_W^2(\hat s-m_{h^0}^2)$$

$$|M_3|^2=\frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{4M_W^2}\Big\{-\frac{\hat t}{2M_W^2}\Big[\hat s M_W^2+M_W^2(\hat s-m_{h^0}^2)+\hat u\hat t\Big]-\frac{(m_{h^0}^2-\hat t)}{\underset{2}{\cancel{4}}M_W^2}\cdot$$
$$\cdot\cancel{2}[M_W^2-\hat t][2M_W^2+\hat t]\Big\}$$

$$|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ -\hat t \left[2\hat s M_W^2 - m_{h^0}^2 M_W^2 + \hat u \hat t \right] - (m_{h^0}^2 - \hat t)(M_W^2 - \hat t)(2M_W^2 + \hat t) \right\}$$

$$= \frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ -\hat t \left[2\cancel{\hat s M_W^2} - \cancel{m_{h^0}^2} M_W^2 + \cancel{\hat u \hat t} \right] - 2M_W^4 m_{h^0}^2 + \hat t M_W^2 m_{h^0}^2 \right.$$

$$\left. + m_{h^0}^2 \hat t^2 + 2M_W^4 \hat t - \hat t^2 M_W^2 - \hat t^3 \right\}$$

$$= \frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ -2\hat s \cancel{\hat t} M_W^2 + \hat t \cancel{m_{h^0}^2} M_W^2 - \hat t^2 (\cancel{m_{h^0}^2} + \cancel{M_W^2} - \hat s) - 2M_W^4 m_{h^0}^2 \right.$$

$$\left. + \hat t \cancel{M_W^2} m_{h^0}^2 + m_{h^0}^2 \cancel{\hat t^2} + 2M_W^4 \hat t - \cancel{\hat t^2} M_W^2 \right\}$$

$$= \frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ -2\hat s \hat t \cancel{M_W^2} + 2\hat t \cancel{M_W^2} m_{h^0}^2 - 2\cancel{\hat t^2} M_W^2 + \hat s \hat t^2 - 2\cancel{M_W^4} m_{h^0}^2 \right.$$

$$\left. + 2\cancel{M_W^4} \hat t \right\}$$

$$2\hat t M_W^2 (M_W^2 - \hat s + m_{h^0}^2) = 2\hat t M_W^2 (\hat u + \hat t)$$

$$= \frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ 2\hat u \hat t M_W^2 + 2\cancel{\hat t^2} M_W^2 - 2\cancel{\hat t^2} M_W^2 + \hat s \hat t^2 - 2M_W^4 m_{h^0}^2 \right\}$$

$$= \frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ 2M_W^2 (\hat u \hat t - m_{h^0}^2 M_W^2) + \hat s \hat t^2 \right\}$$

$$|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ \frac{1}{2} M_W^2 \lambda(\hat s, m_{h^0}^2, M_W^2) \sin^2\theta + \hat s \hat t^2 \right\}$$

$$\boxed{|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2 |V_{ff'}|^2}{8 M_W^4} \hat s \left\{ \hat t^2 + \frac{M_W^2}{2\hat s} \lambda(\hat s, m_{h^0}^2, M_W^2) \sin^2\theta \right\}} \qquad (92)$$

$$|M_4|^2 = \frac{g^4 m_{f'}^2 C_{uf'}^2 |V_{ff'}|^2}{32 M_W^2} \left(\sum_\lambda \varepsilon_{4\mu} \varepsilon_{4\nu}^* \right) \sum_s \left(\bar V_2 (\not{P}'' + m_{f'}) \gamma^\mu (1-\gamma^5) U_1 \right)^\dagger \left(\bar V_2 (\not{P}'' + m_{f'}) \right.$$

$$\left. \gamma^\nu (1-\gamma^5) U_1 \right) \qquad (93)$$

$$\left(\bar V_2 (\not{P}'' + m_{f'}) \gamma^\mu (1-\gamma^5) U_1 \right)^\dagger = \left(V_2^\dagger \gamma^0 (\not{P}'' + m_{f'}) \gamma^\mu (1-\gamma^5) U_1 \right)^\dagger$$

$$= U_1^\dagger (1-\gamma^5) \gamma^{\mu\dagger} (\gamma^{\alpha\dagger} P''_\alpha + m_{f'}) \gamma^{0\dagger} V_2$$

$$= U_1^\dagger (1-\gamma^5) \gamma^{\mu\dagger} (\gamma^0 \gamma^\alpha P''_\alpha + m_{f'} \gamma^0) V_2$$

$$= U_1^\dagger (1-\gamma^5) \gamma^0 \gamma^\mu (\not{P}'' + m_{f'}) V_2$$

$$= \bar U_1 (1+\gamma^5) \gamma^\mu (\not{P}'' + m_{f'}) V_2 = \bar U_1 \gamma^\mu (1-\gamma^5) (\not{P}'' + m_{f'}) V_2 \qquad (94)$$

$$\Rightarrow |M_4|^2 = \frac{g^4 m_{f'}^2 C_{uf'}^2 |V_{ff'}|^2}{32 M_W^2} \left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2} \right) \sum_s \bar U_1 \gamma^\mu (1-\gamma^5) (\not{P}'' + m_{f'}) V_2 \bar V_2 (\not{P}'' + m_{f'})$$

$$\gamma^\nu (1-\gamma^5) U_1$$

$$|M_4|^2 = \frac{g^4 m_{f'}^2 C_{U_{ff'}}^2 |V_{ff'}|^2}{32 M_W^2} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) Tr\Big((\not{p}_1 + m_f)\gamma^\mu (1-\gamma^5)(\not{p}'' + m_{f'})(\not{p}_2 - m_f^{'})$$
$$(\not{p}'' + m_{f'})\gamma^\nu (1-\gamma^5)\Big) \qquad (95)$$

neglecting $m_f, m_{f'}$ in the trace we have:

$$Tr(\not{p}_1 \gamma^\mu (1-\gamma^5)\not{p}'' \not{p}_2 \not{p}'' \gamma^\nu (1-\gamma^5))$$

$$\not{p}_2 \not{p}'' = -\not{p}'' \not{p}_2 + 2(P_2 . P'')$$

$$\Rightarrow Tr = Tr(\not{p}_1 \gamma^\mu (1-\gamma^5)\not{p}'' (-\not{p}'' \not{p}_2 + 2(P_2 . P''))\gamma^\nu (1-\gamma^5))$$
$$= Tr\left[(\not{p}_1 \gamma^\mu \not{p}'' - \not{p}_1 \gamma^\mu \gamma^5 \not{p}'')(-\not{p}'' \not{p}_2 \gamma^\nu + \not{p}'' \not{p}_2 \gamma^\nu \gamma^5 + 2(P_2 . P'')\gamma^\nu - 2(P_2 . P'')\gamma^\nu\gamma^5)\right]$$
$$= -Tr(\not{p}_1 \gamma^\mu \not{p}'' \not{p}'' \not{p}_2 \gamma^\nu) + Tr(\not{p}_1 \gamma^\mu \not{p}'' \not{p}'' \not{p}_2 \gamma^\nu \gamma^5) + 2(P_2 . P'') Tr(\not{p}_1 \gamma^\mu \not{p}'' \gamma^\nu)$$
$$-2(P_2 . P'') Tr(\not{p}_1 \gamma^\mu \not{p}'' \gamma^\nu \gamma^5) + Tr(\not{p}_1 \gamma^\mu \gamma^5 \not{p}'' \not{p}'' \not{p}_2 \gamma^\nu) - Tr(\not{p}_1 \gamma^\mu \gamma^5 \not{p}'' \not{p}'' \not{p}_2 \gamma^\nu \gamma^5)$$
$$-2(P_2 . P'') Tr(\not{p}_1 \gamma^\mu \gamma^5 \not{p}'' \gamma^\nu) + 2(P_2 . P'') Tr(\not{p}_1 \gamma^\mu \gamma^5 \not{p}'' \gamma^\nu \gamma^5)$$
$$= -\hat{U}\, Tr(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) + \hat{U}\, Tr(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5) + 2(P_2 . P'') Tr(\not{p}_1 \gamma^\mu \not{p}'' \gamma^\nu)$$
$$-2(P_2 . P'') Tr(\gamma^5 \not{p}_1 \gamma^\mu \not{p}'' \gamma^\nu) + \hat{U}\, Tr(\not{p}_1 \gamma^\mu \gamma^5 \not{p}_2 \gamma^\nu) - Tr(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu)\hat{U}$$
$$-2(P_2 . P'') Tr(\gamma^5 \not{p}_1 \gamma^\mu \not{p}'' \gamma^\nu) + 2(P_2 . P'') Tr(\not{p}_1 \gamma^\mu \not{p}'' \gamma^\nu)$$

$$= -2\hat{U}\, Tr(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) + 2\hat{U}\, Tr(\gamma^5 \not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) + 4(P_2 . P'') Tr(\not{p}_1 \gamma^\mu \not{p}'' \gamma^\nu)$$
$$-4(P_2 . P'') Tr(\gamma^5 \not{p}_1 \gamma^\mu \not{p}'' \gamma^\nu)$$

$$\Rightarrow Tr = -8\hat{U}(P_2^\mu P_1^\nu + P_2^\nu P_1^\mu - (P_1 . P_2)\eta^{\mu\nu}) - 8i\hat{U}\,\varepsilon^{\alpha\mu\beta\nu} P_{1\alpha} P_{2\beta} + 16(P_2 . P'')(P''^\mu P_1^\nu$$
$$+ P''^\nu P_1^\mu - (P'' . P_1)\eta^{\mu\nu}) + 16i(P_2 . P'')\varepsilon^{\alpha\mu\beta\nu} P_{1\alpha} P''_\beta \qquad (96)$$

$$\eta_{\mu\nu}\varepsilon^{\alpha\mu\beta\nu} = 0$$

$$P_{4\mu} P_{4\nu}\varepsilon^{\alpha\mu\beta\nu} = 0$$

$$\Rightarrow |M_4|^2 = \frac{g^4 m_{f'}^2 C_{U_{ff'}}^2 |V_{ff'}|^2}{32 M_W^2}\left(-\eta_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)\Big\{-8\hat{U}(P_2^\mu P_1^\nu + P_2^\nu P_1^\mu - (P_1 . P_2)\eta^{\mu\nu})$$
$$+ 16(P_2 . P'')(P''^\mu P_1^\nu + P''^\nu P_1^\mu - (P'' . P_1)\eta^{\mu\nu})\Big\} \qquad (97)$$

$$= \frac{g^4 m_{f'}^2 C_{U_{ff'}}^2 |V_{ff'}|^2}{32 M_W^2}\Big\{-8\hat{U}\left(2(P_1 . P_2) + \frac{2(P_2 . P_4)(P_1 . P_4)}{M_W^2} - (P_1 . P_2)\right)$$
$$+ 16(P_2 . P'')\left(2(P'' . P_1) + \frac{2(P_4 . P'')(P_4 . P_1)}{M_W^2} - (P'' . P_1)\right)\Big\}$$

$$= \frac{g^4 m_{f'}^2 C_{uf'}^2 |V_{ff'}|^2}{32 M_W^2} \left\{ -8\hat{u} \left((P_1 \cdot P_2) + \frac{2 (P_2 \cdot P_4)(P_1 \cdot P_4)}{M_W^2} \right) + 16 (P_2 \cdot P'') \left((P'' \cdot P_1) + \frac{2 (P_4 \cdot P'')(P_4 \cdot P_1)}{M_W^2} \right) \right\}$$

$$P_2 \cdot P'' = P_2 \cdot (P_1 - P_4) = P_2 \cdot (P_3 - P_2) = P_2 \cdot P_3$$

$$P'' \cdot P_1 = (P_1 - P_4) \cdot P_1 = - P_1 \cdot P_4$$

$$P'' \cdot P_4 = (P_1 - P_4) \cdot P_4 = P_1 \cdot P_4 - M_W^2$$

$$\Rightarrow |M_4|^2 = \frac{g^4 m_{f'}^2 C_{uf'}^2 |V_{ff'}|^2}{32 M_W^2} \left\{ -8\hat{u} \left(\frac{\hat{s}}{2} + \frac{1}{2} \frac{(M_W^2 - \hat{t})(M_W^2 - \hat{u})}{M_W^2} \right) + \frac{16}{2} (m_{h^0}^2 - \hat{u}) \left(\frac{\hat{u} - M_W^2}{2} + \right.\right.$$

$$\left.\left. + \frac{2}{M_W^2} \left(\frac{1}{2} (M_W^2 - \hat{u}) - M_W^2 \right) \frac{(M_W^2 - \hat{u})}{2} \right) \right\}$$

$$|M_4|^2 = \frac{g^4 m_{f'}^2 C_{uf'}^2 |V_{ff'}|^2}{4 M_W^2} \left\{ - \frac{\hat{u}}{2} \left(\hat{s} + \frac{(M_W^2 - \hat{t})(M_W^2 - \hat{u})}{M_W^2} \right) + (m_{h^0}^2 - \hat{u}) \left[\frac{1}{2} (\hat{u} - M_W^2) \right.\right.$$

$$\left.\left. + \frac{1}{2 M_W^2} (M_W^2 - \hat{u})^2 + \hat{u} - M_W^2 \right] \right\}$$

$$|M_4|^2 = \frac{g^4 m_{f'}^2 C_{uf'}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ - \hat{u} \left(\hat{s} M_W^2 + (M_W^2 - \hat{t})(M_W^2 - \hat{u}) \right) + (m_{h^0}^2 - \hat{u}) \left[3 M_W^2 (\hat{u} - M_W^2) \right.\right.$$

$$\left.\left. + (M_W^2 - \hat{u})^2 \right] \right\}$$

$$= \frac{g^4 m_{f'}^2 C_{uf'}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ - \hat{u} \hat{s} M_W^2 - \hat{u} M_W^4 + \hat{u}^2 M_W^2 + \hat{u} \hat{t} M_W^2 - \hat{u}^2 \hat{t} + (m_{h^0}^2 - \hat{u}) (\right.$$

$$\left. M_W^2 \hat{u} - 2 M_W^4 + \hat{u}^2) \right\}$$

$$= \frac{g^4 m_{f'}^2 C_{uf'}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ - \hat{u} \cancel{\hat{s}} M_W^2 - \cancel{\hat{u} M_W^4} + \cancel{\hat{u}^2 M_W^2} + \hat{u} \cancel{\hat{t}} M_W^2 - \cancel{\hat{u}^2} \hat{t} + m_{h^0}^2 \cancel{M_W^2} \hat{u} - 2 m_{h^0}^2 M_W^4 \right.$$

$$\left. + \cancel{m_{h^0}^2} \hat{u}^2 - \cancel{M_W^2} \hat{u}^2 + 2 \cancel{M_W^4} \hat{u} - \cancel{\hat{u}^3} \right\}$$

$$= \frac{g^4 m_{f'}^2 C_{uf'}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ M_W^4 \hat{u} - \hat{u}^2 (\hat{u} - m_{h^0}^2 + \hat{t}) + \hat{u} M_W^2 (\overbrace{-\hat{s} + \hat{t} + m_{h^0}^2}^{2\hat{t} + \hat{u} - M_W^2}) \right.$$

$$\left. - 2 m_{h^0}^2 M_W^4 \right\}$$

$$\overset{M_W^2 - \hat{s}}{\frown}$$

$$\hat{s} + \hat{t} + \hat{u} = m_{h^0}^2 + M_W^2$$

$$= \frac{g^4 m_{f'}^2 C_{uf'}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ \cancel{M_W^4 \hat{u}} - \cancel{\hat{u}^2 M_W^2} + \hat{u}^2 \hat{s} + 2 \hat{u} \hat{t} M_W^2 + \cancel{\hat{u}^2 M_W^2} - \cancel{\hat{u} M_W^4} - 2 m_{h^0}^2 M_W^4 \right\}$$

$$= \frac{g^4 m_{f'}^2 C_{uf'}^2 |V_{ff'}|^2}{8 M_W^4} \left\{ \hat{s} \hat{u}^2 + 2 M_W^2 (\hat{u} \hat{t} - m_{h^0}^2 M_W^2) \right\}$$

$$\boxed{|M_4|^2 = \frac{g^4 m_{f'}^2 C_{u_{f'}}^2 |V_{ff'}|^2 \hat{s}}{8 M_W^4} \left\{ \hat{v}^2 + \frac{M_W^2}{2\hat{s}} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\sigma \right\}} \qquad (98)$$

$$M_1^* M_3 = \frac{g^4 |V_{ff'}|^2 C_{H^+}^* m_f C_{tf}}{32 M_W^2} \left(\sum_\lambda \varepsilon_{4\mu} \varepsilon_{4\nu}^*\right) (P_1 + P_2 + P_3)^\mu \sum_s \bar{U}_1 (A + B\gamma^5) V_2 \,.\, \bar{V}_2 \gamma^\nu (1-\gamma^5)$$

$$(\not{P}' + m_f) U_1 \qquad (99)$$

$$M_1^* M_3 = \frac{g^4 |V_{ff'}|^2 C_{H^+}^* m_f C_{tf}}{32 M_W^2} \left(-n_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) (P_1 + P_2 + P_3)^\mu \, Tr[(\not{P}_1 + m_f)(A + B\gamma^5)$$

$$(\not{P}_2 - m_{f'}) \gamma^\nu (1-\gamma^5)(\not{P}' + m_f)] \qquad (100)$$

$$Tr = Tr[(A\not{P}_1 + B\not{P}_1\gamma^5 + m_f A + m_f B\gamma^5)(\not{P}_2 - m_{f'})(\gamma^\nu \not{P}' + \gamma^\nu m_f - \gamma^\nu\gamma^5\not{P}' - m_f\gamma^\nu\gamma^5)]$$

$$= Tr[(A\not{P}_1\not{P}_2 - A\overset{\downarrow}{m_{f'}}\not{P}_1 + B\not{P}_1\gamma^5\not{P}_2 - m_{f'}\overset{\downarrow}{B}\not{P}_1\gamma^5 + m_f\overset{\downarrow}{A}\not{P}_2 - m_f\overset{\downarrow}{m_{f'}}A + m_f\overset{\downarrow}{B}\gamma^5\not{P}_2$$

$$- m_f \overset{\downarrow}{m_{f'}} B\gamma^5)(\gamma^\nu\not{P}' + \underset{\uparrow}{m_f}\gamma^\nu - \gamma^\nu\gamma^5\not{P}' - \underset{\uparrow}{m_f}\gamma^\nu\gamma^5)]$$

Neglecting terms of order m_f^2, $m_{f'}^2$ inside the trace

$\Rightarrow$

$$Tr = Tr[(A\not{P}_1\not{P}_2 + B\not{P}_1\gamma^5\not{P}_2)(\gamma^\nu\not{P}' - \gamma^\nu\gamma^5\not{P}')]$$

$$= A\,Tr(\not{P}_1\not{P}_2\gamma^\nu\not{P}') - A\,Tr(\not{P}_1\not{P}_2\gamma^\nu\gamma^5\not{P}') + B\,Tr(\not{P}_1\gamma^5\not{P}_2\gamma^\nu\not{P}') - B\,Tr(\not{P}_1\gamma^5\not{P}_2\gamma^\nu\gamma^5\not{P}')$$

$$= (A-B)\,Tr(\not{P}_1\not{P}_2\gamma^\nu\not{P}') + (A-B)\,Tr(\gamma^5\not{P}_1\not{P}_2\gamma^\nu\not{P}')$$

$$= (A-B)\,Tr(\gamma^\alpha\not{P}_2\gamma^\nu\not{P}')P_{1\alpha} + (A-B)\,Tr(\gamma^5\gamma^\alpha\gamma^\beta\gamma^\nu\gamma^\sigma)P_{1\alpha}P_{2\beta}P'_\sigma$$

$$= (A-B)\,Tr(\gamma^\alpha\not{P}_2\gamma^\nu\not{P}')P_{1\alpha} - 4i(A-B)\,\varepsilon^{\alpha\beta\nu\sigma}P_{1\alpha}P_{2\beta}P'_\sigma$$

$$= (A-B)\,Tr(\gamma^\alpha\not{P}_2\gamma^\nu\not{P}')P_{1\alpha} - 4i(A-B)\,\varepsilon^{\alpha\beta\nu\sigma}P_{1\alpha}P_{2\beta}(P_{1\sigma} - P_{3\sigma})$$

$\Rightarrow$

$$Tr = (A-B)\,Tr(\gamma^\alpha\not{P}_2\gamma^\nu\not{P}')P_{1\alpha} - 4i(A-B)\,\varepsilon^{\alpha\beta\nu\sigma}P_{1\alpha}P_{2\beta}P_{3\sigma} \qquad (101)$$

$$(\varepsilon^{\alpha\beta\nu\sigma}P_{1\alpha}P_{1\sigma} = 0)$$

$$n_{\mu\nu}(P_1 + P_2 + P_3)^\mu \varepsilon^{\alpha\beta\nu\sigma}P_{1\alpha}P_{2\beta}P_{3\sigma} = (P_1 + P_2 + P_3)_\nu\,\varepsilon^{\lambda\beta\nu\sigma}P_{1\alpha}P_{2\beta}P_{3\sigma} = 0.$$

$$P_{4\mu}P_{4\nu}(P_1 + P_2 + P_3)^\mu \varepsilon^{\alpha\beta\nu\sigma}P_{1\alpha}P_{2\beta}P_{3\sigma} = (P_4.(P_1 + P_2 + P_3))(P_1 + P_2 - P_3)_\nu\,\varepsilon^{\alpha\beta\nu\sigma}P_{1\alpha}P_{2\beta}P_{3\sigma}$$

$$= 0.$$

$$\Rightarrow M_1^* M_3 = \frac{g^4 |V_{ff'}|^2 C_{H^+}^* m_f C_{tf}}{32 M_W^2} \left(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)(P_1 + P_2 + P_3)^\mu (A-B) P_{1\alpha}\, 4 [P_2^\alpha P'^\nu$$

$$+ P_2^\nu P'^\alpha - (P_2.P')n^{\alpha\nu}] \qquad (102)$$

$$M_1^* M_3 = \frac{g^4 |V_{ff'}|^2 C_{H^+}^* m_f C_{tf}}{32 M_W^2}\left(-(P_1+P_2+P_3)_\nu + \frac{((P_1+P_2+P_3)\cdot P_4)P_{4\nu}}{M_W^2}\right) 4(A-B)\left[(P_1\cdot P_2)P'^{\nu}\right.$$

$$\left. + (P_1\cdot P')P_2^{\nu} - (P_2\cdot P')P_1^{\nu}\right]$$

$$= \frac{g^4 |V_{ff'}|^2 C_{H^+}^* m_f C_{tf}}{8 M_W^2}(A-B)\Big\{-(P_1\cdot P_2)((P_1+P_2+P_3)\cdot P') - (P_1\cdot P')((P_1+P_2+P_3)\cdot P_2)$$

$$+ (P_2\cdot P')((P_1+P_2+P_3)\cdot P_1) + \frac{(P_1\cdot P_2)(P_4\cdot P')((P_1+P_2+P_3)\cdot P_4)}{M_W^2} + \frac{(P_1\cdot P')(P_2\cdot P_4)((P_1+P_2+P_3)\cdot P_4)}{M_W^2}$$

$$- \frac{(P_2\cdot P')(P_1\cdot P_4)((P_1+P_2+P_3)\cdot P_4)}{M_W^2}\Big\} \qquad (103)$$

$$(P_1+P_2+P_3)\cdot P' = (P_1+P_2+P_3)\cdot(P_1-P_3) = -\cancel{P_1\cdot P_3} + P_1\cdot P_2 - P_2\cdot P_3 + \cancel{P_1\cdot P_3} - m_{h^0}^2$$

$$= \frac{\hat s}{2} - \frac{(m_{h^0}^2-\hat u)}{2} - m_{h^0}^2$$

$$\boxed{(P_1+P_2+P_3)\cdot P' = \frac{\hat s - 3m_{h^0}^2 + \hat u}{2}} \qquad (104)$$

$$P_1\cdot P' = P_1\cdot(P_1-P_3) = -P_1\cdot P_3$$

$$P_2\cdot P' = P_2\cdot(P_4-P_2) = P_2\cdot P_4$$

$$P_4\cdot P' = P_4\cdot(P_4-P_2) = M_W^2 - P_2\cdot P_4 = M_W^2 - \frac{(M_W^2-\hat t)}{2} = \frac{M_W^2+\hat t}{2} \qquad (105)$$

$$M_1^* M_3 = \frac{g^4 |V_{ff'}|^2 C_{H^+}^* m_f^2 C_{tf}}{8 M_W^2} 2\cot\beta\Big\{-\frac{\hat s}{2}\frac{(\hat s - 3m_{h^0}^2 + \hat u)}{2} + \frac{(m_{h^0}^2-\hat t)}{2}\frac{(\hat s + m_{h^0}^2 - \hat u)}{2}$$

$$+ \frac{(M_W^2-\hat t)}{2}\frac{(\hat s + m_{h^0}^2 - \hat t)}{2} + \frac{(\hat s - m_{h^0}^2)}{M_W^2}\left[\frac{\hat s}{2}\left(\frac{M_W^2+\hat t}{2}\right) - \frac{(m_{h^0}^2-\hat t)}{2}\frac{(M_W^2-\hat t)}{2} - \frac{(M_W^2-\hat t)}{2}\frac{(M_W^2-\hat u)}{2}\right]\Big\}$$

$$M_1^* M_3 = \frac{g^4 |V_{ff'}|^2 C_{H^+}^* m_f^2 \cot\beta\, C_{tf}}{16 M_W^2}\Big\{-\cancel{\hat s^2} + 3\hat s m_{h^0}^2 - \cancel{\hat s \hat u} + \cancel{m_{h^0}^2 \hat s} + \cancel{m_{h^0}^4} - \cancel{m_{h^0}^2 \hat u} - \cancel{\hat s \hat t} - \hat t m_{h^0}^2$$

$$+ \hat u \hat t + \cancel{M_W^2 \hat s} + \cancel{M_W^2 m_{h^0}^2} - M_W^2 \hat t - \hat s \hat t - \cancel{m_{h^0}^2 \hat t} + \hat t^2 + \frac{(\hat s - m_{h^0}^2)}{M_W^2}\Big[\cancel{\hat s M_W^2} + \hat s \hat t - \cancel{m_{h^0}^2 M_W^2}$$

$$+ m_{h^0}^2 \hat t + \cancel{M_W^2 \hat t} - \hat t^2 - \cancel{M_W^4} + \cancel{M_W^2 \hat u} + M_W^2 \hat t - \hat u \hat t\Big]\Big\}$$

$$= \frac{g^4 |V_{ff'}|^2 C_{H^+}^* m_f^2 \cot\beta\, C_{tf}}{16 M_W^2}\Big\{-\hat s(\overset{0}{\cancel{\hat s + \hat u + \hat t - M_W^2 - m_{h^0}^2}}) + \cancel{m_{h^0}^2 \overbrace{(m_{h^0}^2 + M_W^2 - \hat t - \hat u)}^{\hat s}}$$

$$+ 3\cancel{\hat s} m_{h^0}^2 - \hat t m_{h^0}^2 + \cancel{\hat u \hat t} - \hat t M_W^2 - \hat s \hat t + \cancel{\hat t^2} + \frac{(\hat s - m_{h^0}^2)}{M_W^2}\Big[M_W^2(\overset{0}{\cancel{\hat s - m_{h^0}^2 + \hat t - M_W^2}}$$

$$\cancel{+\hat u}) + \hat s \hat t + \cancel{m_{h^0}^2 \hat t} - \cancel{\hat t^2} + \cancel{M_W^2 \hat t} - \cancel{\hat u \hat t}\Big]\Big\}$$

$$= \frac{g^4 |V_{ff'}|^2 C_{H^+}^* m_f^2 \cot\beta\, C_{tf}}{16 M_W^2}\Big\{4m_{h^0}^2 \hat s - \hat t(\overbrace{m_{h^0}^2 + M_W^2 - \hat u - \hat t}^{\hat s}) - \hat s \hat t + \frac{(\hat s - m_{h^0}^2)}{M_W^2}\cdot$$

$$\cdot\Big[\hat t(m_{h^0}^2 \underbrace{+ M_W^2}_{} - \hat t - \hat u) + \hat s \hat t\Big]\Big\}$$

$$M_1^* M_3 = \frac{g^4 |V_{ff'}|^2 C_{H+}^* m_f^2 \cot\beta \, C_{tf}}{16 M_W^2} \left\{ 4 m_{h^0}^2 \hat{s} - 2\hat{s}\hat{t} + \frac{(\hat{s} - m_{h^0}^2)}{M_W^2} (2\hat{s}\hat{t}) \right\}$$

$$= 2 g^4 \frac{|V_{ff'}|^2 C_{H+}^* m_f^2 \cot\beta \, C_{tf} \hat{s}}{16 M_W^2} \left\{ 2 m_{h^0}^2 - \hat{t} + \frac{\hat{t}(\hat{s} - m_{h^0}^2)}{M_W^2} \right\}$$

$$= \frac{g^4 |V_{ff'}|^2 C_{H+}^* m_f^2 \cot\beta \, C_{tf} \hat{s}}{8 M_W^4} \left\{ 2 m_{h^0}^2 M_W^2 - \hat{t} M_W^2 + \hat{t}\hat{s} - \hat{t} m_{h^0}^2 \right\}$$

$$= \frac{g^4 |V_{ff'}|^2 C_{H+}^* m_f^2 \cot\beta \, C_{tf} \hat{s}}{8 M_W^4} \left\{ 2 m_{h^0}^2 M_W^2 + \hat{t} (\overbrace{\hat{s} - m_{h^0}^2 - M_W^2}^{-\hat{u} - \hat{t}}) \right\}$$

$$= \frac{g^4 |V_{ff'}|^2 C_{H+}^* m_f^2 \cot\beta \, C_{tf} \hat{s}}{8 M_W^4} \left\{ 2 m_{h^0}^2 M_W^2 - \hat{u}\hat{t} - \hat{t}^2 \right\}$$

$$\boxed{M_1^* M_3 = \frac{g^4 |V_{ff'}|^2 C_{H+}^* m_f^2 \cot\beta \, C_{tf} \hat{s}}{8 M_W^4} \left\{ m_{h^0}^2 M_W^2 - \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta - \hat{t}^2 \right\}} \quad (106)$$

$$\boxed{M_3^* M_1 = \frac{g^4 |V_{ff'}|^2 C_{H+} m_f^2 \cot\beta \, C_{tf} \hat{s}}{8 M_W^4} \left\{ m_{h^0}^2 M_W^2 - \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta - \hat{t}^2 \right\}} \quad (107)$$

$$\Rightarrow \boxed{M_1^* M_3 + M_3^* M_1 = \frac{g^4 |V_{ff'}|^2 \, \mathrm{Re}(C_{H+}) m_f^2 \cot\beta \, C_{tf} \hat{s}}{4 M_W^4} \left\{ m_{h^0}^2 M_W^2 - \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta - \hat{t}^2 \right\}} \quad (108)$$

$$M_1^* M_4 = \frac{g^4 |V_{ff'}|^2 C_{H+}^* m_{f'} C_{uf'}}{32 M_W^2} \left(\sum_\lambda \varepsilon_{4\mu} \varepsilon_{4\nu}^* \right) (P_1 + P_2 + P_3)^\mu \sum_s \bar{U}_1 (A + B\gamma^5) V_2 \bar{V}_2 (\not{P}'' + m_{f'}) \gamma^\nu$$
$$(1 - \gamma^5) U_1 \qquad (109)$$

$$M_1^* M_4 = \frac{g^4 |V_{ff'}|^2 C_{H+}^* m_{f'} C_{uf'}}{32 M_W^2} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2} \right) (P_1 + P_2 + P_3)^\mu \, \mathrm{Tr} \left[(\not{P}_1 + m_f)(A + B\gamma^5)(\not{P}_2 - m_{f'}) \right.$$
$$\left. (\not{P}'' + m_{f'}) \gamma^\nu (1 - \gamma^5) \right] \qquad (110)$$

neglecting $m_f, m_{f'}$ inside the trace (but not in A, B)

$$M_1^* M_4 = \frac{g^4 |V_{ff'}|^2 C_{H+}^* m_{f'} C_{uf'}}{32 M_W^2} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2} \right) (P_1 + P_2 + P_3)^\mu \, \mathrm{Tr} \left[\not{P}_1 (A + B\gamma^5) \not{P}_2 \not{P}'' \gamma^\nu (1 - \gamma^5) \right] \quad (111)$$

$$\mathrm{Tr}\left[\not{P}_1 (A + B\gamma^5) \not{P}_2 \not{P}'' \gamma^\nu (1 - \gamma^5) \right] = \mathrm{Tr}\left[(A \not{P}_1 \not{P}_2 \not{P}'' \gamma^\nu + B \not{P}_1 \gamma^5 \not{P}_2 \not{P}'' \gamma^\nu)(1 - \gamma^5) \right]$$

$$= A\,\mathrm{Tr}(\not{P}_1 \not{P}_2 \not{P}'' \gamma^\nu) - A\,\mathrm{Tr}(\not{P}_1 \not{P}_2 \not{P}'' \gamma^\nu \gamma^5) + B\,\mathrm{Tr}(\not{P}_1 \gamma^5 \not{P}_2 \not{P}'' \gamma^\nu) - B\,\mathrm{Tr}(\not{P}_1 \gamma^5 \not{P}_2 \not{P}'' \gamma^\nu \gamma^5)$$

$$= A\,\mathrm{Tr}(\not{P}_2 \not{P}'' \gamma^\nu \not{P}_1) - A\,\mathrm{Tr}(\gamma^5 \not{P}_1 \not{P}_2 \not{P}'' \gamma^\nu) - B\,\mathrm{Tr}(\gamma^5 \not{P}_1 \not{P}_2 \not{P}'' \gamma^\nu) + B\,\mathrm{Tr}(\not{P}_1 \not{P}_2 \not{P}'' \gamma^\nu)$$

$$= A\,\mathrm{Tr}(\gamma^\alpha \not{P}'' \gamma^\nu \not{P}_1) P_{2\alpha} - A\,\mathrm{Tr}(\gamma^5 \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\nu) P_{1\alpha} P_{2\beta} P''_\sigma - B\,\mathrm{Tr}(\gamma^5 \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\nu).$$

$$T_r = 4A\left[P''^{\alpha} P_1^{\nu} + P''^{\nu} P_1^{\alpha} - (P_1 . P'')\,\eta^{\nu\alpha}\right] P_{2\alpha} + 4iA\,\varepsilon^{\alpha\beta\sigma\nu} P_{1\alpha} P_{2\beta} P''_{\sigma} + 4iB\,\varepsilon^{\alpha\beta\sigma\nu} P_{1\alpha} P_{2\beta}$$

$$P''_{\sigma} + 4B\left[P''^{\alpha} P_1^{\nu} + P''^{\nu} P_1^{\alpha} - (P_1 . P'')\,\eta^{\nu\alpha}\right] P_{2\alpha} \qquad (112)$$

$$\eta_{\mu\nu}(P_1+P_2+P_3)^{\mu}\,\varepsilon^{\alpha\beta\sigma\nu} P_{1\alpha} P_{2\beta}(P_{1\sigma} - P_{4\sigma}) = -(P_1+P_2+P_3)_{\nu}\,\varepsilon^{\alpha\beta\sigma\nu} P_{1\alpha} P_{2\beta} P_{4\sigma}$$

$$= -\varepsilon^{\alpha\beta\sigma\nu} P_{1\alpha} P_{2\beta} P_{3\nu} P_{4\sigma} = -\varepsilon^{\alpha\beta\sigma\nu} P_{1\alpha} P_{2\beta} P_{3\nu}(P_1+P_2-P_3)_{\sigma} = 0 .$$

also $\quad P_{4\nu}\,\varepsilon^{\alpha\beta\sigma\nu} P_{1\alpha} P_{2\beta}(P_{1\sigma} - P_{4\sigma}) = 0$

$$\Rightarrow M_1^{*} M_4 = \frac{g^4 |V_{ff'}|^2 C_{H^+}^{*} m_{f'} C_{uf'}}{32 M_W^2}\left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right)(P_1+P_2+P_3)^{\mu}\, 4(A+B)\left[P''^{\alpha} P_1^{\nu} + P''^{\nu} P_1^{\alpha}\right.$$

$$\left. - (P_1 . P'')\,\eta^{\nu\alpha}\right] P_{2\alpha} \qquad (113)$$

$$M_1^{*} M_4 = \frac{g^4 |V_{ff'}|^2 C_{H^+}^{*} m_{f'}^2 C_{uf'}}{32 M_W^2}\left(-(P_1+P_2+P_3)_{\nu} + \frac{P_{4\nu}\,((P_1+P_2+P_3) . P_4)}{M_W^2}\right) 8\tan\beta\left[(P'' . P_2) P_1^{\nu}\right.$$

$$\left. + P''^{\nu}(P_1 . P_2) - (P_1 . P'') P_2^{\nu}\right]$$

$$M_1^{*} M_4 = \frac{g^4 |V_{ff'}|^2 C_{H^+}^{*} m_{f'}^2 C_{uf'}}{4 M_W^2}\tan\beta\left\{-(P_2 . P'')((P_1+P_2+P_3) . P_1) - (P_1 . P_2)((P_1+P_2+P_3) . P'')\right.$$

$$+ (P_1 . P'')((P_1+P_2+P_3) . P_2) + \frac{((P_1+P_2+P_3) . P_4)}{M_W^2}\left[(P'' . P_2)(P_1 . P_4) + (P_1 . P_2)(P'' . P_4)\right.$$

$$\left.\left. - (P_1 . P'')(P_2 . P_4)\right]\right\} \qquad (114)$$

$$(P_2 . P'') = P_2 . (P_1 - P_4) = P_2 . (P_3 - P_2) = P_2 . P_3$$

$$(P_1 . P'') = P_1 . (P_1 - P_4) = -P_1 . P_4$$

$$(P'' . P_4) = P_4 . (P_1 - P_4) = P_1 . P_4 - M_W^2 = \frac{M_W^2 - \hat{u}}{2} - M_W^2 = -\frac{M_W^2 - \hat{u}}{2} \qquad (115)$$

$$(P_1+P_2+P_3) . P'' = (P_1+P_2+P_3) . (P_1 - P_4) = (P_1+P_2+P_3) . (P_3 - P_2) = P_1 . P_3 - P_1 . P_2 + \cancel{P_2 . P_3} + m_{h^0}^2$$

$$-\cancel{P_2 . P_3} = \frac{1}{2}(m_{h^0}^2 - \hat{t}) - \frac{\hat{s}}{2} + m_{h^0}^2 = \frac{m_{h^0}^2 - \hat{t} - \hat{s} + 2m_{h^0}^2}{2} = \frac{3m_{h^0}^2 - \hat{t} - \hat{s}}{2}$$

$$\Rightarrow \boxed{(P_1+P_2+P_3) . P'' = \frac{3m_{h^0}^2 - \hat{t} - \hat{s}}{2}} \qquad (116)$$

$$M_1^{*} M_4 = \frac{g^4 |V_{ff'}|^2 C_{H^+}^{*} m_{f'}^2 C_{uf'}}{4 M_W^2}\tan\beta\left\{\frac{1}{2}(\hat{u} - m_{h^0}^2)\,\frac{1}{2}(\hat{s} + m_{h^0}^2 - \hat{t}) - \frac{\hat{s}}{2}\,\frac{1}{2}(3m_{h^0}^2 - \hat{t} - \hat{s})\right.$$

$$+ \frac{1}{2}(\hat{u} - M_W^2)\,\frac{1}{2}(\hat{s} + m_{h^0}^2 - \hat{u}) + \frac{(\hat{s} - m_{h^0}^2)}{M_W^2}\left[\frac{1}{2}(m_{h^0}^2 - \hat{u})\,\frac{1}{2}(M_W^2 - \hat{u}) - \frac{\hat{s}}{2}\,\frac{1}{2}(M_W^2 + \hat{u})\right.$$

$$\left.\left. + \frac{1}{2}(M_W^2 - \hat{u})\,\frac{1}{2}(M_W^2 - \hat{t})\right]\right\} \qquad (117)$$

$$M_1^* M_4 = g^4 \frac{|V_{tt'}|^2 C_{H^+}^* m_{t'}^2 C_{ut'} \tan\beta}{16 M_W^2} \Big\{ \hat u \hat s + \cancel{m_{h^0}^2 \hat u} - \cancel{\hat u \hat t} - m_{h^0}^2 \hat s - \cancel{m_{h^0}^4} + \cancel{m_{h^0}^2 \hat t} - 3\hat s m$$

$$+ \hat s \hat t + \hat s^2 + \cancel{\hat u \hat s} + \hat u m_{h^0}^2 - \cancel{\hat u^2} - M_W^2 \hat s - M_W^2 m_{h^0}^2 + \cancel{M_W^2 \hat u} + \frac{(\hat s - m_{h^0}^2)}{M_W^2} \Big[\cancel{m_{h^0}^2 M_W^2}$$

$$- \hat u m_{h^0}^2 - \hat u M_W^2 + \hat u^2 - \cancel{\hat s M_W^2} - \hat u \hat s + \cancel{M_W^4} - \cancel{M_W^2 \hat t} - \cancel{M_W^2 \hat u} + \hat u \hat t \Big] \Big\}$$

$$= g^4 \frac{|V_{tt'}|^2 C_{H^+}^* m_{t'}^2 C_{ut'} \tan\beta}{16 M_W^2} \Big\{ 2\hat u \hat s - \cancel{m_{h^0}^4} + \cancel{m_{h^0}^2 \hat t} - 3\hat s m_{h^0}^2 + \cancel{\hat u m_{h^0}^2} - M_W^2 m_{h^0}^2$$

$$+ \frac{(\hat s - m_{h^0}^2)}{M_W^2} \Big[-2\hat u \hat s + M_W^2 (m_{h^0}^2 \overset{0}{\cancel{- \hat s + M_W^2}} - \hat t - \hat u) \Big] \Big\}$$

$$= g^4 \frac{|V_{tt'}|^2 C_{H^+}^* m_{t'}^2 C_{ut'} \tan\beta}{16 M_W^4} \Big\{ M_W^2 \Big[2\hat u \hat s - m_{h^0}^2 (\overbrace{m_{h^0}^2 - \hat t - \hat u + M_W^2}^{\hat s}) - 3\hat s m_{h^0}^2 \Big]$$

$$- 2\hat u \hat s (\hat s - m_{h^0}^2) \Big\}$$

$$= g^4 \frac{|V_{tt'}|^2 C_{H^+}^* m_{t'}^2 C_{ut'} \tan\beta}{16 M_W^4} \Big\{ 2\hat u \hat s M_W^2 - 4\hat s m_{h^0}^2 M_W^2 - 2\hat u \hat s^2 + 2\hat u \hat s m_{h^0}^2 \Big\}$$

$$= g^4 \frac{|V_{tt'}|^2 C_{H^+}^* m_{t'}^2 C_{ut'} \tan\beta\, \hat s}{8 M_W^4} \Big\{ \cancel{\hat u M_W^2} - 2 m_{h^0}^2 M_W^2 - \cancel{\hat u \hat s} + \cancel{\hat u m_{h^0}^2} \Big\}$$

$$= g^4 \frac{|V_{tt'}|^2 C_{H^+}^* m_{t'}^2 C_{ut'} \tan\beta\, \hat s}{8 M_W^4} \Big\{ \hat u (\overbrace{M_W^2 + m_{h^0}^2 - \hat s}^{\hat u + \hat t}) - 2 m_{h^0}^2 M_W^2 \Big\}$$

$$= g^4 \frac{|V_{tt'}|^2 C_{H^+}^* m_{t'}^2 C_{ut'} \tan\beta\, \hat s}{8 M_W^4} \Big\{ \hat u^2 + \hat u \hat t - 2 m_{h^0}^2 M_W^2 \Big\}$$

$$M_1^* M_4 = g^4 \frac{|V_{tt'}|^2 C_{H^+}^* m_{t'}^2 C_{ut'} \tan\beta\, \hat s}{8 M_W^4} \Big\{ \hat u^2 - m_{h^0}^2 M_W^2 + \frac{1}{4} \lambda (\hat s, m_{h^0}^2, M_W^2) \sin^2\theta \Big\}$$

$$M_4^* M_1 = g^4 \frac{|V_{tt'}|^2 C_{H^+} m_{t'}^2 C_{ut'} \tan\beta\, \hat s}{8 M_W^4} \Big\{ \hat u^2 - m_{h^0}^2 M_W^2 + \frac{1}{4} \lambda (\hat s, m_{h^0}^2, M_W^2) \sin^2\theta \Big\}$$

$\Rightarrow$

$$\boxed{M_1^* M_4 + M_4^* M_1 = g^4 \frac{|V_{tt'}|^2 (\mathrm{Re}\, C_{H^+}) m_{t'}^2 C_{ut'} \tan\beta\, \hat s}{4 M_W^4} \Big\{ \hat u^2 - m_{h^0}^2 M_W^2 + \frac{1}{4} \lambda (\hat s, m_{h^0}^2, M_W^2) \sin^2\theta \Big\}}$$

(118)

$$M_2^* M_3 = -\frac{g^4 m_f C_w^* C_{tf} |V_{tf'}|^2}{16} \eta^{\nu\rho} \Big|\sum_\lambda \varepsilon_{4\rho} \varepsilon^*_{4\sigma}\Big| \left(\eta_{\mu\nu} - \frac{P_\mu P_\nu}{M_W^2}\right) \sum_s [\bar{U}_1 \gamma^\mu (1-\gamma^5) V_2)(\bar{V}_2 \gamma^\sigma (1-\gamma^5) (\not{p}' + m_f)$$

$$U_1 . \qquad (119)$$

$$M_2^* M_3 = \frac{-g^4 m_f C_w^* C_{tf} |V_{tf'}|^2}{16} \eta^{\nu\rho} \left(-\eta_{\rho\sigma} + \frac{P_{4\rho} P_{4\sigma}}{M_W^2}\right) \left(\eta_{\mu\nu} - \frac{P_\mu P_\nu}{M_W^2}\right) \mathrm{Tr}\, [(\not{p}_1 + m_f) \gamma^\mu (1-\gamma^5) (\not{p}_2 - m_{f'})$$

$$\gamma^\sigma (1-\gamma^5) (\not{p}' + m_f)] \qquad (120)$$

$$\mathrm{Tr}\, [(\not{p}_1 + m_f) \gamma^\mu (1-\gamma^5) (\not{p}_2 - m_{f'}) \gamma^\sigma (1-\gamma^5) (\not{p}' + m_f)]$$

$$= \mathrm{Tr}[(\not{p}_1 \gamma^\mu - \not{p}_1 \gamma^\mu \gamma^5 + m_f \gamma^\mu - m_f \gamma^\mu \gamma^5)(\not{p}_2 - m_{f'}) (\gamma^\sigma \not{p}' + m_f \gamma^\sigma - \gamma^\sigma \gamma^5 \not{p}' - m_f \gamma^\sigma \gamma^5)]$$

$$= \mathrm{Tr}\, [(\not{p}_1 \gamma^\mu - \not{p}_1 \gamma^\mu \gamma^5 + m_f \gamma^\mu - m_f \gamma^\mu \gamma^5) (\not{p}_2 \gamma^\sigma \not{p}' + m_f \not{p}_2 \gamma^\sigma - \not{p}_2 \gamma^\sigma \gamma^5 \not{p}' - m_f \not{p}_2 \gamma^\sigma \gamma^5$$

$$- m_{f'} \gamma^\sigma \not{p}' - m_f \cancel{m_{f'}} \underbrace{\gamma^\sigma + m_{f'} \gamma^\sigma \gamma^5 \not{p}' + m_f}_{\text{neglecting this terms}} \cancel{m_{f'}} \gamma^\sigma \gamma^5)]$$

$$= \mathrm{Tr}\, (\not{p}_1 \gamma^\mu \cancel{\not{p}_2 \overset{0}{\gamma^\sigma}} \not{p}') + m_f \mathrm{Tr} (\not{p}_1 \cancel{\gamma^\mu} \not{p}_2 \gamma^\sigma) - \mathrm{Tr}\, (\not{p}_1 \gamma^\mu \cancel{\not{p}_2 \overset{0}{\gamma^\sigma}} \gamma^5 \not{p}') - m_f \mathrm{Tr} (\not{p}_1 \cancel{\gamma^\mu} \not{p}_2 \gamma^\sigma \gamma^5)$$

$$- m_{f'} \mathrm{Tr} (\not{p}_1 \gamma^\mu \cancel{\gamma^\sigma} \not{p}') + m_{f'} \mathrm{Tr} (\cancel{\not{p}_1} \gamma^\mu \gamma^\sigma \gamma^5 \not{p}') - \mathrm{Tr} (\not{p}_1 \gamma^\mu \cancel{\gamma^5 \not{p}_2} \overset{0}{\gamma^\sigma} \not{p}') - m_f \mathrm{Tr} (\not{p}_1 \gamma^\mu \cancel{\gamma^5} \not{p}_2 \gamma^\sigma)$$

$$+ \mathrm{Tr}\, (\not{p}_1 \gamma^\mu \gamma^5 \cancel{\not{p}_2 \overset{0}{\gamma^\sigma}} \gamma^5 \not{p}') + m_f \mathrm{Tr} (\not{p}_1 \gamma^\mu \cancel{\gamma^5} \not{p}_2 \gamma^\sigma \gamma^5) + m_{f'} \mathrm{Tr} (\cancel{\not{p}_1} \gamma^\mu \gamma^5 \gamma^\sigma \not{p}')$$

$$- m_{f'} \mathrm{Tr} (\not{p}_1 \gamma^\mu \gamma^5 \gamma^\sigma \cancel{\gamma^5} \not{p}') + m_f \mathrm{Tr} (\gamma^\mu \not{p}_2 \gamma^\sigma \not{p}') - m_f \mathrm{Tr} (\gamma^\mu \not{p}_2 \gamma^\sigma \gamma^5 \not{p}')$$

$$- m_f \mathrm{Tr} (\gamma^\mu \gamma^5 \not{p}_2 \gamma^\sigma \not{p}') + m_f \mathrm{Tr}\, (\gamma^\mu \gamma^5 \not{p}_2 \gamma^\sigma \gamma^5 \not{p}')$$

(neglecting terms of order $m_f^2,\ m_{f'}^2,\ m_f m_{f'}$)

$$= 2 m_f \mathrm{Tr}\, (\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\sigma) - 2 m_f \mathrm{Tr} (\gamma^5 \not{p}_1 \gamma^\mu \not{p}_2 \gamma^\sigma) - m_{f'} \mathrm{Tr} \cancel{(\not{p}_1 \gamma^\mu} \gamma^\sigma \not{p}')$$

$$- m_{f'} \mathrm{Tr} (\gamma^5 \cancel{\not{p}_1 \gamma^\mu} \gamma^\sigma \not{p}') + m_{f'} \cancel{\mathrm{Tr} (\gamma^5} \not{p}_1 \gamma^\mu \gamma^\sigma \not{p}') + m_{f'} \cancel{\mathrm{Tr} (\not{p}_1} \gamma^\mu \gamma^\sigma \not{p}')$$

$$+ 2 m_f \mathrm{Tr} (\gamma^\mu \not{p}_2 \gamma^\sigma \not{p}') + 2 m_f \mathrm{Tr} (\gamma^5 \gamma^\mu \not{p}_2 \gamma^\sigma \not{p}')$$

$\Rightarrow$

$$\mathrm{Tr} = 8 m_f [P_2^\mu P_1^\sigma + P_2^\sigma P_1^\mu - (P_1 \cdot P_2) \eta^{\mu\sigma}] + 8 i m_f \varepsilon^{\alpha\mu\beta\sigma} P_{1\alpha} P_{2\beta}$$

$$+ 8 m_f [P_2^\mu P'^\sigma + P_2^\sigma P'^\mu - (P' \cdot P_2) \eta^{\mu\sigma}] - 8 i m_f \varepsilon^{\mu\alpha\sigma\beta} P_{2\alpha} P'_\beta \qquad (121)$$

$$M_2^* M_3 = \frac{-g^4 m_f^2 C_w^* C_{tf} |V_{tf'}|^2}{16} \left(-\delta^\nu_\sigma + \frac{P_4^\nu P_{4\sigma}}{M_W^2}\right) \left(\eta_{\mu\nu} - \frac{P_\mu P_\nu}{M_W^2}\right) 8 \{ P_2^\mu P_1^\sigma + P_2^\sigma P_1^\mu - (P_1 \cdot P_2) \eta^{\mu\sigma}$$

$$+ P_2^\mu P'^\sigma + P_2^\sigma P'^\mu - (P' \cdot P_2) \eta^{\mu\sigma} + i \varepsilon^{\alpha\mu\beta\sigma} P_{1\alpha} P_{2\beta} - i \varepsilon^{\mu\alpha\sigma\beta} P_{2\alpha} P'_\beta \} \qquad (122)$$

$$= \frac{-g^4 m_f^2 C_w^* C_{tf} |V_{tf'}|^2}{2} \left(-\eta_{\mu\sigma} + \frac{P_\mu P_\sigma}{M_W^2} + \frac{P_{4\mu} P_{4\sigma}}{M_W^2} - \frac{(P \cdot P_4)\, P_\mu P_{4\sigma}}{M_W^4}\right) \{ P_2^\mu P_1^\sigma + P_2^\sigma P_1^\mu$$

$$- (P_1 \cdot P_2) \eta^{\mu\sigma} + P_2^\mu P'^\sigma + P_2^\sigma P'^\mu - (P' \cdot P_2) \eta^{\mu\sigma} + i \varepsilon^{\alpha\mu\beta\sigma} P_{1\alpha} P_{2\beta} - i \underset{\downarrow\ \alpha\mu\beta\sigma}{\varepsilon^{\mu\alpha\sigma\beta}} P_{2\alpha} P'_\beta \}$$

$$\eta_{\mu\sigma}\,\varepsilon^{\alpha\mu\rho\sigma} = 0$$

$$P_\mu P_\sigma\,\varepsilon^{\alpha\mu\rho\sigma} = 0$$

$$P_{4\mu} P_{4\sigma}\,\varepsilon^{\alpha\mu\rho\sigma} = 0$$

$$P_\mu P_{4\sigma}\,\varepsilon^{\alpha\mu\rho\sigma} P_{1\alpha} P_{2\beta} = (P_{1\mu} + P_{2\mu}) P_{4\sigma}\,\varepsilon^{\alpha\mu\rho\sigma} P_{1\alpha} P_{2\beta} = 0$$

$$P_{4\sigma} P_\mu\,\varepsilon^{\alpha\mu\rho\sigma} P_{2\alpha} P'_\rho = (P_{1\mu} + P_{2\mu})\,\varepsilon^{\alpha\mu\rho\sigma} P_{2\alpha} (P_{1\rho} - P_{3\rho})(P_1 + P_2 - P_3)_\sigma = 0$$

$$\Rightarrow M_2^* M_3 = \frac{-g^4 m_f^2 C_W^* C_{tf} |V_{tf'}|^2}{2}\left(\overset{\downarrow}{-\eta_{\mu\sigma}} + \overset{\downarrow}{\frac{P_\mu P_\sigma}{M_W^2}} + \overset{\downarrow}{\frac{P_{4\mu} P_{4\sigma}}{M_W^2}} - \overset{\downarrow}{\frac{(P.P_4) P_\mu P_{4\sigma}}{M_W^4}}\right)\Big\{ P_2^\mu P_1^\sigma + P_2^\sigma P_1^\mu$$

$$- (P_1.P_2)\eta^{\mu\sigma} + P_2^\mu P'^\sigma + P_2^\sigma P'^\mu - (P'.P_2)\eta^{\mu\sigma}\Big\} \qquad (123)$$

$$= \frac{-g^4 m_f^2 C_W^* C_{tf} |V_{tf'}|^2}{2}\Big\{ (P_1.P_2) + (P'.P_2) + \frac{2(P_2.P)(P_1.P)}{M_W^2} - \frac{\hat{s}(P_1.P_2)}{M_W^2} + \frac{2(P_2.P)(P'.P)}{M_W^2}$$

$$- \frac{\hat{s}(P'.P_2)}{M_W^2} + \frac{2(P_2.P_4)(P_1.P_4)}{M_W^2} + \frac{2(P_2.P_4)(P'.P_4)}{M_W^2} - \frac{(P.P_4)}{M_W^4}\Big[(P_2.P)(P_1.P_4) + (P_2.P_4)(P_1.P)$$

$$- (P_1.P_2)(P.P_4) + (P_2.P)(P'.P_4) + (P_2.P_4)(P.P') - (P'.P_2)(P.P_4)\Big]\Big\}$$

$$\rightarrow P'.P_2 = (P_1 - P_3).P_2 = (P_4 - P_2).P_2 = P_2.P_4$$

$$\rightarrow P.P_1 = (P_1 + P_2).P_1 = P_1.P_2 \quad ; \quad P_2.P = P_2.(P_1 + P_2) = P_1.P_2$$

$$P'.P = (P_1 - P_3).(P_1 + P_2) = P_1.P_2 - P_1.P_3 - P_2.P_3 = \frac{\hat{s}}{2} - \frac{(m_{h^0}^2 - \hat{t})}{2} - \frac{(m_{h^0}^2 - \hat{u})}{2}$$

$$\rightarrow \boxed{P'.P = \frac{\hat{s} - 2m_{h^0}^2 + \hat{u} + \hat{t}}{2} = \frac{M_W^2 - m_{h^0}^2}{2}} \qquad (124)$$

$$P'.P_4 = (P_4 - P_2).P_4 = M_W^2 - P_2.P_4 = M_W^2 - \frac{(M_W^2 - \hat{t})}{2} = \frac{M_W^2 + \hat{t}}{2}$$

$$\rightarrow \boxed{P'.P_4 = \frac{M_W^2 + \hat{t}}{2}} \qquad (125)$$

$$M_2^* M_3 = \frac{-g^4 m_f^2 C_W^* C_{tf} |V_{tf'}|^2}{2}\Big\{ \frac{\hat{s}}{2} + \frac{1}{2}(M_W^2 - \hat{t}) + \frac{1}{M_W^2}\Big[\cancel{\frac{\hat{s}^2}{2}} - \cancel{\frac{\hat{s}^2}{2}} + \frac{\hat{s}}{2}(\cancel{M_W^2} - m_{h^0}^2)$$

$$- \frac{\hat{s}}{2}(\cancel{M_W^2} - \hat{t}) + \frac{(M_W^2 - \hat{t})(M_W^2 - \hat{u})}{2} + \frac{(M_W^2 - \hat{t})}{2}(M_W^2 + \hat{t})\Big] - \frac{1}{2}\frac{(M_W^2 + \hat{s} - m_{h^0}^2)}{M_W^4}\Big[$$

$$\frac{\hat{s}}{2}\frac{1}{2}(M_W^2 - \hat{u}) + \frac{1}{2}(M_W^2 - \hat{t})\frac{\hat{s}}{2} - \frac{\hat{s}}{2}\frac{(M_W^2 + \hat{s} - m_{h^0}^2)}{2} + \frac{\hat{s}}{2}\frac{(M_W^2 + \hat{t})}{2} + \frac{1}{2}(M_W^2 - \hat{t})\frac{(M_W^2 - m_{h^0}^2)}{2}$$

$$- \frac{(M_W^2 - \hat{t})}{2}\frac{(M_W^2 + \hat{s} - m_{h^0}^2)}{2}\Big]\Big\}$$

$$M_2^* M_3 = \frac{-g^4 m_f^2 C_W^* C_{tf} |V_{tf'}|^2}{2} \Big\{ \frac{\hat s}{2} + \frac{1}{2}(M_W^2 - \hat t) + \frac{1}{2M_W^2}\Big[-\hat s m_{h^0}^2 + \hat s \hat t + \cancel{M_W^4} - \hat u M_W^2 - \hat t M_W^2$$

$$+ \hat u \hat t + \cancel{M_W^4} - \hat t^2 \Big] - \frac{1}{8}\frac{(M_W^2 + \hat s - m_{h^0}^2)}{M_W^4}\Big[2\hat s \cancel{M_W^2} - \cancel{\hat u}\hat s - \cancel{\hat s \hat t} - \hat s \cancel{M_W^2} - \cancel{\hat s^2} + \hat s \cancel{m_{h^0}^2} + \cancel{\hat s M_W^2}$$

$$+ \cancel{\hat s \hat t} + \cancel{M_W^4} - m_{h^0}^2 \cancel{M_W^2} - \hat t \cancel{M_W^2} + \hat t \cancel{m_{h^0}^2} - \cancel{M_W^4} - \cancel{M_W^2}\hat s + M_W^2 \cancel{m_{h^0}^2} + \hat t \cancel{M_W^2} + \cancel{\hat s \hat t} - \hat t \cancel{m_{h^0}^2} \Big] \Big\}$$

$$M_2^* M_3 = \frac{-g^4 m_f^2 C_W^* C_{tf} |V_{tf'}|^2}{2} \Big\{ \frac{1}{2}(\hat s + M_W^2 - \hat t) + \frac{1}{2M_W^2}\Big[-\hat s m_{h^0}^2 + \hat s \hat t + 2M_W^4 - \hat u M_W^2 - \hat t M_W^2$$

$$+ \hat u \hat t - \hat t^2 \Big] - \frac{1}{8}\frac{(M_W^2 + \hat s - m_{h^0}^2)}{M_W^4} \cdot \Big[\underbrace{\hat s M_W^2 - \hat u \hat s - \hat s^2 + \hat s m_{h^0}^2}_{\hat s (M_W^2 + m_{h^0}^2 - \hat u - \hat s) = \hat s \hat t} + \hat s \hat t \Big] \Big\}$$

$$= \frac{-g^4 m_f^2 C_W^* C_{tf} |V_{tf'}|^2}{2} \Big\{ \frac{1}{2}(\hat s + M_W^2 - \hat t) + \frac{1}{2M_W^2}\Big[-\hat s m_{h^0}^2 + \hat s \hat t + 2M_W^4 - \hat u M_W^2 - \hat t M_W^2$$

$$+ \hat u \hat t - \hat t^2 \Big] - \frac{1}{4M_W^4} \hat s \hat t (M_W^2 + \hat s - m_{h^0}^2) \Big\}$$

$$= \frac{-g^4 m_f^2 C_W^* C_{tf} |V_{tf'}|^2}{8M_W^4} \Big\{ 2\hat s \xcancel{M_W^4} + 2\xcancel{M_W^6} - 2\cancel{M_W^4}\hat t - 2\hat s \cancel{m_{h^0}^2} M_W^2 + \hat s \hat t \cancel{M_W^2} + 4\cancel{M_W^6}$$

$$- 2\hat u \xcancel{M_W^4} - 2\hat t \xcancel{M_W^4} + 2M_W^{\cancel{2}} \hat u \hat t - 2\hat t^2 \cancel{M_W^2} - \hat s^2 \cancel{\hat t} + m_{h^0}^{\cancel{2}} \hat s \hat t \Big\}$$

$$-2M_W^4 (\hat u + \hat t - M_W^2) = -2M_W^4 (m_{h^0}^2 - \hat s)$$

$$= \frac{-g^4 m_f^2 C_W^* C_{tf} |V_{tf'}|^2}{8M_W^4} \Big\{ -2M_W^4 m_{h^0}^2 + 4\hat s M_W^4 - 2M_W^4 \hat t - 2\hat s m_{h^0}^2 M_W^2 + \hat s \cancel{\hat t} M_W^2 + 4M_W^6$$

$$+ 2M_W^2 \hat u \hat t - 2\hat t^2 M_W^2 - \cancel{\hat s^2} \hat t + m_{h^0}^2 \cancel{\hat s} \hat t \Big\}$$

$$= \frac{-g^4 m_f^2 C_W^* C_{tf} |V_{tf'}|^2}{8M_W^4} \Big\{ -2\cancel{M_W^4} m_{h^0}^2 + 4\hat s \cancel{M_W^4} - 2\cancel{M_W^4}\hat t - 2\hat s \cancel{m_{h^0}^2} M_W^2 + \hat s \cancel{\hat t} \hat u + \cancel{\hat s} \hat t^2$$

$$+ 4\cancel{M_W^6} + 2M_W^{\cancel{2}} \hat u \hat t - 2\hat t^2 \cancel{M_W^2} \Big\}$$

$$M_2^* M_3 = \frac{-g^4 m_f^2 C_W^* C_{tf} |V_{tf'}|^2}{8M_W^4} \Big\{ 2M_W^2 (\hat u \hat t - \cancel{M_W^2} m_{h^0}^2) + \hat s (\hat u \cancel{\hat t} - m_{h^0}^2 M_W^2) - \hat s m_{h^0}^2 M_W^2$$

$$+ 4\hat s M_W^4 - 2\hat t M_W^4 + \hat t^2 (\hat s - 2M_W^2) + 4M_W^6 \Big\}$$

$$M_2^* M_3 = \frac{-g^4 m_f^2 C_W^* C_{tf} |V_{tf'}|^2}{8M_W^4} \Big\{ \frac{1}{2}\lambda(\hat s, m_{h^0}^2, M_W^2) \sin^2\theta \Big(M_W^2 + \frac{\hat s}{2}\Big) - \hat s m_{h^0}^2 M_W^2 + 4\hat s M_W^4 - 2\hat t M_W^4$$

$$+ \hat t^2 (\hat s - 2M_W^2) + 4M_W^6 \Big\}$$

(126)

$\Rightarrow$

$$M_2^* M_3 + M_3^* M_2 = \frac{-g^4 m_f^2 (\mathrm{Re}(C_W)) C_{tf} |V_{tf'}|^2}{4M_W^4} \Big\{ \frac{1}{2}\lambda(\hat s, m_{h^0}^2, M_W^2) \sin^2\theta \Big[M_W^2 + \frac{\hat s}{2} \Big] - \hat s m_{h^0}^2 M_W^2 + 4\hat s M_W^4 - 2\hat t M_W^4$$

$$+ \hat t^2 (\hat s - 2M_W^2) + 4M_W^6 \Big\}$$

(127)

$$M_2^* M_4 = -\frac{g^4 m_{f'} C_W^* C_{Uf'} |V_{ff'}|^2}{16} n^{\nu\rho} \left(\sum_\lambda \varepsilon_{4\rho} \varepsilon^*_{4\sigma}\right) \sum_s \bar{U}_1 \gamma^\mu (1-\gamma^5) V_2 \bar{V}_2 (\not{p}'' + m_{f'}) \gamma^\sigma (1-\gamma^5) U_1 \quad 17$$

$$\cdot \left(\eta_{\mu\nu} - \frac{P_\mu P_\nu}{M_W^2}\right) \qquad (128)$$

$$M_2^* M_4 = -\frac{g^4 m_{f'} C_W^* C_{Uf'} |V_{ff'}|^2}{16} n^{\nu\rho} \left(-\eta_{\rho\sigma} + \frac{P_{4\rho} P_{4\sigma}}{M_W^2}\right)\left(\eta_{\mu\nu} - \frac{P_\mu P_\nu}{M_W^2}\right) \mathrm{Tr}\left[(\not{p}_1 + m_f)\gamma^\mu\right.$$

$$\left.(1-\gamma^5)(\not{p}_2 - m_{f'})(\not{p}'' + m_{f'})\gamma^\sigma(1-\gamma^5)\right] \qquad (129)$$

$$\mathrm{Tr} = \mathrm{Tr}\Big[\left(\not{p}_1\gamma^\mu - \not{p}_1\gamma^\mu\gamma^5 + m_f\gamma^\mu - m_f\gamma^\mu\gamma^5\right)(\not{p}_2 - m_{f'})\big(\not{p}''\gamma^\sigma - \not{p}''\gamma^\sigma\gamma^5$$

$$+ \underset{\downarrow}{m_{f'}\gamma^\sigma} - \underset{\downarrow}{m_{f'}\gamma^\sigma\gamma^5}\big)\Big]$$

$$= \mathrm{Tr}\Big[\left(\not{p}_1\gamma^\mu - \not{p}_1\gamma^\mu\gamma^5 + m_f\gamma^\mu - m_f\gamma^\mu\gamma^5\right)\big(\not{p}_2\not{p}''\gamma^\sigma - \not{p}_2\not{p}''\gamma^\sigma\gamma^5 + m_{f'}\not{p}_2\gamma^\sigma$$

$$- m_{f'}\not{p}_2\gamma^\sigma\gamma^5 - m_{f'}\not{p}''\gamma^\sigma + m_{f'}\not{p}''\gamma^\sigma\gamma^5\big)\Big]$$

$$= \overset{0}{\mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_2\not{p}''\gamma^\sigma)} - \overset{0}{\cancel{\mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_2\not{p}''\gamma^\sigma\gamma^5)}} + m_{f'}\mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_2\gamma^\sigma) - m_{f'}\mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_2$$

$$\gamma^\sigma\gamma^5) - m_{f'}\mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}''\gamma^\sigma) + m_{f'}\mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}''\gamma^\sigma\gamma^5) - \overset{0}{\cancel{\mathrm{Tr}(\not{p}_1\gamma^\mu\gamma^5\not{p}_2\not{p}''\gamma^\sigma)}}$$

$$+ \overset{0}{\cancel{\mathrm{Tr}(\not{p}_1\gamma^\mu\gamma^5\not{p}_2\not{p}''\gamma^\sigma\gamma^5)}} - m_{f'}\mathrm{Tr}(\not{p}_1\gamma^\mu\gamma^5\not{p}_2\gamma^\sigma) + m_{f'}\mathrm{Tr}(\not{p}_1\gamma^\mu\gamma^5\not{p}_2\gamma^\sigma\gamma^5)$$

$$+ m_{f'}\mathrm{Tr}(\not{p}_1\gamma^\mu\gamma^5\not{p}''\gamma^\sigma) - m_{f'}\mathrm{Tr}(\not{p}_1\gamma^\mu\gamma^5\not{p}''\gamma^\sigma\gamma^5) + m_f\,\mathrm{Tr}(\cancel{\gamma^\mu}\not{p}_2\not{p}''\gamma^\sigma)$$

$$- m_f\,\mathrm{Tr}(\cancel{\gamma^\mu}\not{p}_2\not{p}''\gamma^\sigma\gamma^5) - m_f\,\mathrm{Tr}(\gamma^\mu\cancel{\gamma^5}\not{p}_2\not{p}''\gamma^\sigma) + m_f\,\mathrm{Tr}(\gamma^\mu\gamma^5\cancel{\not{p}_2}\not{p}''\gamma^\sigma\gamma^5)$$

$$\Rightarrow \mathrm{Tr}\left[\qquad\right] = +2m_{f'}\mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_2\gamma^\sigma) - 2m_{f'}\mathrm{Tr}(\gamma^5\not{p}_1\gamma^\mu\not{p}_2\gamma^\sigma)$$

$$-2m_{f'}\mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}''\gamma^\sigma) + 2m_{f'}\mathrm{Tr}(\gamma^5\not{p}_1\gamma^\mu\not{p}''\gamma^\sigma)$$

$$\Rightarrow \mathrm{Tr} = 8m_{f'}\left[P_2^\mu P_1^\sigma + P_2^\sigma P_1^\mu - (P_1\cdot P_2)\eta^{\mu\sigma}\right] + 8im_{f'}\varepsilon^{\alpha\mu\beta\sigma}P_{1\alpha}P_{2\beta}$$

$$-8m_{f'}\left[P''^\mu P_1^\sigma + P''^\sigma P_1^\mu - (P_1\cdot P'')\eta^{\mu\sigma}\right] - 8im_{f'}\varepsilon^{\alpha\mu\beta\sigma}P_{1\alpha}P''_\beta$$

$$(130)$$

$$M_2^* M_4 = -\frac{g^4 m_{f'}^2 C_W^* C_{Uf'} |V_{ff'}|^2}{2}\left(-\delta^\nu_\sigma + \frac{P_4^\nu P_{4\sigma}}{M_W^2}\right)\left(\eta_{\mu\nu} - \frac{P_\mu P_\nu}{M_W^2}\right)\Big\{P_2^\mu P_1^\sigma + P_2^\sigma P_1^\mu - (P_1\cdot P_2)\eta^{\mu\sigma}$$

$$- P''^\mu P_1^\sigma - P''^\sigma P_1^\mu + (P_1\cdot P'')\eta^{\mu\sigma} + i\varepsilon^{\alpha\mu\beta\sigma}P_{1\alpha}P_{2\beta} - i\varepsilon^{\alpha\mu\beta\sigma}P_{1\alpha}P''_\beta\Big\}$$

$$= \frac{-g^4 m_{f'}^2 C_W^* C_{Uf'} |V_{ff'}|^2}{2}\left(-\eta_{\mu\sigma} + \frac{P_\mu P_\sigma}{M_W^2} + \frac{P_{4\mu}P_{4\sigma}}{M_W^2} - \frac{(P\cdot P_4)}{M_W^4}P_\mu P_{4\sigma}\right)\Big\{P_2^\mu P_1^\sigma$$

$$+ P_2^\sigma P_1^\mu - (P_1\cdot P_2)\eta^{\mu\sigma} - P''^\mu P_1^\sigma - P''^\sigma P_1^\mu + (P_1\cdot P'')\eta^{\mu\sigma} + i\varepsilon^{\alpha\mu\beta\sigma}P_{1\alpha}P_{2\beta}$$

$$- i\varepsilon^{\alpha\mu\beta\sigma}P_{1\alpha}P''_\beta\Big\} \qquad (131)$$

$$\eta_{\mu\sigma}\,\varepsilon^{\alpha\mu\beta\sigma} = 0$$

$$p_\mu p_\sigma\,\varepsilon^{\alpha\mu\beta\sigma} = 0$$

$$p_{4\mu} p_{4\sigma}\,\varepsilon^{\alpha\mu\beta\sigma} = 0$$

$$p_\mu p_{4\sigma}\,\varepsilon^{\alpha\mu\beta\sigma} p_{1\alpha} p_{2\beta} = (p_1+p_2)_\mu\, p_{4\sigma}\,\varepsilon^{\alpha\mu\beta\sigma} p_{1\alpha} p_{2\beta} = 0$$

$$p_\mu p_{4\sigma}\,\varepsilon^{\alpha\mu\beta\sigma} p_{1\alpha} p''_\beta = p_\mu p_{4\sigma}\,\varepsilon^{\alpha\mu\beta\sigma} p_{1\alpha}(p_1-p_4)_\beta = 0 .$$

$$\Rightarrow M_2^* M_4 = \frac{-g^4 m_{f'}^2 C_W^* C_{uf'} |V_{tf'}|^2}{2}\Big(\overset{\downarrow}{-\eta_{\mu\sigma}} + \overset{\downarrow}{\frac{p_\mu p_\sigma}{M_W^2}} + \overset{\downarrow}{\frac{p_{4\mu} p_{4\sigma}}{M_W^2}} - \overset{\downarrow}{\frac{(p\cdot p_4) p_\mu p_{4\sigma}}{M_W^4}}\Big)\Big\{p_2^\mu p_1^\sigma + p_2^\sigma p_1^\mu$$
$$-(p_1\cdot p_2)\eta^{\mu\sigma} - p''^\mu p_1^\sigma - p''^\sigma p_1^\mu + (p_1\cdot p'')\eta^{\mu\sigma}\Big\}$$

$$= -\frac{g^4 m_{f'}^2 C_W^* C_{uf'} |V_{tf'}|^2}{2}\Big\{(p_1\cdot p_2) - (p_1\cdot p'') + \frac{2(p_2\cdot p)(p_1\cdot p)}{M_W^2} - \frac{\hat{s}(p_1\cdot p_2)}{M_W^2}$$
$$-2\frac{(p''\cdot p)(p_1\cdot p)}{M_W^2} + \frac{\hat{s}(p_1\cdot p'')}{M_W^2} + \frac{2(p_2\cdot p_4)(p_1\cdot p_4)}{M_W^2} - \frac{2(p''\cdot p_4)(p_1\cdot p_4)}{M_W^2} - \frac{(p\cdot p_4)}{M_W^4}\Big[(p\cdot p_2)(p_1\cdot p_4)$$
$$+ (p_1\cdot p)(p_2\cdot p_4) - (p_1\cdot p_2)(p\cdot p_4) - (p\cdot p'')(p_1\cdot p_4) - (p\cdot p_1)(p_4\cdot p'') + (p_1\cdot p'')(p\cdot p_4)\Big]\Big\}$$

$$p_1\cdot p'' = p_1\cdot(p_1-p_4) = -p_1\cdot p_4 = \frac{1}{2}(\hat{u}-M_W^2)$$

$$p_2\cdot p = p_1\cdot p_2 = \frac{\hat{s}}{2}$$

$$p_1\cdot p = p_1\cdot p_2 = \frac{\hat{s}}{2}$$

$$p\cdot p'' = (p_1+p_2)\cdot(p_1-p_4) = -p_1\cdot p_4 + p_1\cdot p_2 - p_2\cdot p_4$$

$$p\cdot p'' = \frac{1}{2}(\hat{u}-M_W^2) + \frac{\hat{s}}{2} - \frac{1}{2}(M_W^2-\hat{t}) = \frac{1}{2}(\hat{s}+\hat{u}+\hat{t}-M_W^2-M_W^2)$$

$$\boxed{p\cdot p'' = \frac{1}{2}(m_{h^0}^2 - M_W^2)} \qquad (132)$$

$$p_4\cdot p'' = p_4\cdot(p_1-p_4) = p_1\cdot p_4 - M_W^2 = \frac{1}{2}(M_W^2-\hat{u}) - M_W^2$$

$$\boxed{p_4\cdot p'' = -\frac{1}{2}(M_W^2+\hat{u})} \qquad (133)$$

$$M_2^* M_4 = \frac{-g^4 m_{f'}^2 C_W^* C_{uf'} |V_{tf'}|^2}{2}\Big\{\frac{\hat{s}}{2} + \frac{1}{2}(M_W^2-\hat{u}) + \frac{1}{M_W^2}\Big[\cancel{\frac{\hat{s}^2}{2}} - \cancel{\frac{\hat{s}^2}{2}} - (m_{h^0}^2-M_W^2)\frac{\hat{s}}{2}$$
$$+\frac{\hat{s}}{2}(\hat{u}-M_W^2) + (M_W^2-\hat{t})\frac{1}{2}(M_W^2-\hat{u}) + (M_W^2+\hat{u})\frac{1}{2}(M_W^2-\hat{u})\Big] - \frac{1}{2M_W^4}(M_W^2+\hat{s}-m_{h^0}^2)\cdot$$
$$\cdot\Big[\frac{\hat{s}}{2}\frac{1}{2}(M_W^2-\hat{u}) + \frac{\hat{s}}{2}\frac{1}{2}(M_W^2-\hat{t}) - \frac{\hat{s}}{2}\frac{1}{2}(M_W^2+\hat{s}-m_{h^0}^2) - \frac{1}{2}(m_{h^0}^2-M_W^2)\frac{1}{2}(M_W^2-\hat{u})$$
$$+\frac{\hat{s}}{2}\frac{1}{2}(M_W^2+\hat{u}) + \frac{1}{2}(\hat{u}-M_W^2)\frac{1}{2}(M_W^2+\hat{s}-m_{h^0}^2)\Big]\Big\}$$

$$M_2^* M_4 = \frac{-g^4 m_{f'}|^2 C_W^* C_{uf'} |V_{ff'}|^2}{2} \Big\{ \frac{1}{2}(\hat s + M_W^2 - \hat u) + \frac{1}{2M_W^2}\Big[-\cancel{m_{h^0}^2} \hat s + \cancel{\hat u \hat s} + \cancel{M_W^4}$$

$$-\cancel{\hat u M_W^2} - \cancel{\hat t M_W^2} + \cancel{\hat u \hat t} + \cancel{M_W^4} - \cancel{\hat u^2}\Big] - \frac{1}{8M_W^4}(M_W^2 + \hat s - m_{h^0}^2)\Big[\cancel{\hat s M_W^2} - \cancel{\hat u \hat s}$$

$$+ \cancel{\hat s M_W^2} - \hat s \hat t - \cancel{\hat s M_W^2} - \hat s^2 + \hat s m_{h^0}^2 - \cancel{m_{h^0}^2 M_W^2} + \cancel{m_{h^0}^2 \hat u} + \cancel{M_W^4} - \cancel{M_W^2 \hat u} + \hat s M_W^2 + \cancel{\hat u \hat s}$$

$$+ \cancel{M_W^2 \hat u} + \hat u \hat s - \cancel{\hat u m_{h^0}^2} - \cancel{M_W^4} - \cancel{M_W^2 \hat s} + \cancel{M_W^2 m_{h^0}^2}\Big]\Big\}$$

$$= \frac{-g^4 m_{f'}|^2 C_W^* C_{uf'} |V_{ff'}|^2}{2} \Big\{ \frac{1}{2}(\hat s + M_W^2 - \hat u) + \frac{1}{2M_W^2}\Big[-\hat s m_{h^0}^2 + \hat u \hat s + 2M_W^4 - \hat u M_W^2$$

$$- \hat t M_W^2 + \hat u \hat t - \hat u^2\Big] - \frac{1}{8M_W^4}(M_W^2 + \hat s - m_{h^0}^2)(-\hat s \hat t - \hat s^2 + \hat s m_{h^0}^2 + \hat s M_W^2 + \hat u \hat s)\Big\}$$

$$\hat s(m_{h^0}^2 + M_W^2 - \hat s - \hat t) = \hat u \hat s$$

$$M_2^* M_4 = -\frac{g^4 m_{f'}|^2 C_W^* C_{uf'} |V_{ff'}|^2}{2} \Big\{ \frac{1}{2}(\hat s + M_W^2 - \hat u) + \frac{1}{2M_W^2}(-\cancel{\hat s m_{h^0}^2} + \cancel{\hat u \hat s} + 2\cancel{M_W^4} - \cancel{\hat u M_W^2}$$

$$-\cancel{\hat t M_W^2} + \hat u \hat t - \hat u^2) - \frac{1}{4M_W^4}(M_W^2 + \hat s - m_{h^0}^2)\hat u \hat s\Big\}$$

$$= \frac{-g^4 m_{f'}|^2 C_W^* C_{uf'} |V_{ff'}|^2}{2} \Big\{ \frac{1}{2}(\hat s + M_W^2 - \hat u) + \frac{1}{2M_W^2}(-\hat s m_{h^0}^2 + \hat u \hat s + M_W^4 + \hat s M_W^2$$

$$- M_W^2 m_{h^0}^2 + \hat u \hat t - \hat u^2) - \frac{1}{4M_W^4}\hat u \hat s(M_W^2 + \hat s - m_{h^0}^2)\Big\}$$

$$M_2^* M_4 = \frac{-g^4 m_{f'}|^2 C_W^* C_{uf'} |V_{ff'}|^2}{8M_W^4} \Big\{ 2\cancel{\hat s M_W^4} + 2\cancel{M_W^6} - 2\cancel{\hat u M_W^4} - 2\cancel{\hat s m_{h^0}^2 M_W^2} + 2\cancel{\hat u \hat s} M_W^2 + 2\cancel{M_W^6}$$

$$+ 2\cancel{\hat s M_W^4} - 2\cancel{M_W^4 m_{h^0}^2} + 2\cancel{M_W^2 \hat u \hat t} - 2\cancel{M_W^2 \hat u^2} - \cancel{\hat u \hat s M_W^2} - \cancel{\hat u \hat s^2} + \cancel{\hat u \hat s m_{h^0}^2}\Big\}$$

$$= \frac{-g^4 m_{f'}|^2 C_W^* C_{uf'} |V_{ff'}|^2}{8M_W^4} \Big\{ 2M_W^2(\hat u \hat t - m_{h^0}^2 M_W^2) + 4\cancel{M_W^6} + 4\cancel{\hat s M_W^4} - 2\cancel{\hat u} M_W^4$$

$$-2\hat u M_W^2 + \hat u \hat s M_W^2 - \hat s m_{h^0}^2 M_W^2 - \hat s m_{h^0}^2 M_W^2 - \hat u \hat s^2 + \hat u \hat s m_{h^0}^2\Big\}$$

$$\hat u \hat s m_{h^0}^2 + \hat u \hat s M_W^2 - \hat u \hat s^2 = \hat u \hat s(m_{h^0}^2 + M_W^2 - \hat s) = \hat u \hat s(\hat u + \hat t)$$

$$= \hat u^2 \hat s + \hat u \hat s \hat t$$

$$M_2^* M_4 = \frac{-g^4 m_{f'}|^2 C_W^* C_{uf'} |V_{ff'}|^2}{8M_W^4} \Big\{ 2M_W^2(\hat u \hat t - m_{h^0}^2 M_W^2) + 4M_W^6 + 4\cancel{\hat s M_W^4} - 2\cancel{\hat u M_W^4}$$

$$-2\hat u^2 M_W^2 + \cancel{\hat u^2 \hat s} + \cancel{\hat u \hat s \hat t} - \cancel{\hat s m_{h^0}^2 M_W^2} - \cancel{\hat s m_{h^0}^2 M_W^2}\Big\}$$

$$M_2^* M_4 = \frac{-g^4 m_{f'}^2 C_W^* C_{uf'} |V_{ff'}|^2}{8 M_W^4} \Big\{ 2M_W^2 (\hat{u}\hat{t} - m_{h^0}^2 M_W^2) + \hat{s}(\hat{u}\hat{t} - m_{h^0}^2 M_W^2) - \hat{s} m_{h^0}^2 M_W^2$$

$$+ 4\hat{s} M_W^4 - 2\hat{u} M_W^4 + \hat{u}^2 (\hat{s} - 2M_W^2) + 4M_W^6 \Big\}$$

$$M_2^* M_4 = -\frac{g^4 m_{f'}^2 C_W^* C_{uf'} |V_{ff'}|^2}{8 M_W^4} \Big\{ \frac{1}{2} M_W^2 \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta + \frac{1}{4} \hat{s} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta$$

$$- \hat{s} m_{h^0}^2 M_W^2 + 4\hat{s} M_W^4 - 2\hat{u} M_W^4 + \hat{u}^2 (\hat{s} - 2M_W^2) + 4M_W^6 \Big\}$$

$$M_2^* M_4 = -\frac{g^4 m_{f'}^2 C_W^* C_{uf'} |V_{ff'}|^2}{8 M_W^4} \Big\{ \frac{\lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta}{2} \Big[M_W^2 + \frac{1}{2}\hat{s} \Big] - \hat{s} m_{h^0}^2 M_W^2 + 4\hat{s} M_W^4$$

$$- 2\hat{u} M_W^4 + \hat{u}^2 (\hat{s} - 2M_W^2) + 4M_W^6 \Big\}$$

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(134)

$$M_4^* M_2 = -\frac{g^4 m_{f'}^2 C_W C_{uf'} |V_{ff'}|^2}{8 M_W^4} \underset{\downarrow \text{ the same}}{\{ \quad \}} \qquad (135)$$

$$\Rightarrow \boxed{\begin{aligned} M_2^* M_4 + M_4^* M_2 = & -\frac{g^4 m_{f'}^2 (\mathrm{Re}\, C_W) C_{uf'} |V_{ff'}|^2}{4 M_W^4} \Big\{ \frac{\lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta}{2} \Big[M_W^2 + \frac{\hat{s}}{2} \Big] - \hat{s} m_{h^0}^2 M_W^2 \\ & + 4\hat{s} M_W^4 - 2\hat{u} M_W^4 + \hat{u}^2 (\hat{s} - 2M_W^2) + 4M_W^6 \Big\} \qquad (136) \end{aligned}}$$

$$M_3^* M_4 = \frac{g^4 m_f m_{f'} C_{tf} C_{uf'} |V_{ff'}|^2}{32 M_W^2} \Big(\sum_\lambda \epsilon_{4\mu} \epsilon_{4\nu}^* \Big) \sum_s \bar{u}_1 (\not{p}' + m_f) \gamma^\mu (1-\gamma^5) v_2 \bar{v}_2 (\not{p}'' + m_{f'}) \gamma^\nu (1-\gamma^5) u_1$$

$$= \frac{g^4 m_f m_{f'} C_{tf} C_{uf'} |V_{ff'}|^2}{32 M_W^2} \Big(-g_{\mu\nu} + \frac{p_{4\mu} p_{4\nu}}{M_W^2} \Big) \mathrm{Tr} \Big[(\not{p}_1 + m_f)(\not{p}' + m_f) \gamma^\mu (1-\gamma^5)(\not{p}_2 - m_{f'})$$

$$(\not{p}'' + m_{f'}) \gamma^\nu (1-\gamma^5) \Big] \qquad (137)$$

$$\mathrm{Tr} \approx \mathrm{Tr} \Big\{ (\not{p}_1 \not{p}' + m_f \not{p}_1 + m_f \not{p}')(\gamma^\mu - \gamma^\mu\gamma^5)(\not{p}_2 \not{p}'' + m_{f'} \not{p}_2 - m_{f'} \not{p}'')(\gamma^\nu - \gamma^\nu\gamma^5) \Big\}$$

$$= \mathrm{Tr} \Big\{ \Big[\not{p}_1 \overset{\downarrow}{\not{p}'} \gamma^\mu - \not{p}_1 \overset{\downarrow}{\not{p}'} \gamma^\mu \gamma^5 + \overset{\downarrow}{m_f} \not{p}_1 \gamma^\mu - m_f \overset{\downarrow}{\not{p}_1} \gamma^\mu \gamma^5 + m_f \not{p}' \gamma^\mu - m_f \not{p}' \gamma^\mu \gamma^5 \Big] \Big[\not{p}_2 \not{p}'' \gamma^\nu$$

$$- \not{p}_2 \not{p}'' \gamma^\nu \gamma^5 + m_{f'} \not{p}_2 \gamma^\nu - m_{f'} \not{p}_2 \gamma^\nu \gamma^5 - m_{f'} \not{p}'' \gamma^\nu + m_{f'} \not{p}'' \gamma^\nu \gamma^5 \Big] \Big\}$$

$$= \mathrm{Tr}(\not{p}_1 \not{p}' \cancel{\gamma^\mu \not{p}_2} \not{p}'' \gamma^\nu) - \mathrm{Tr}(\gamma^5 \not{p}_1 \not{p}' \cancel{\gamma^\mu} \not{p}_2 \not{p}'' \gamma^\nu) + \mathrm{Tr}(\gamma^5 \not{p}_1 \not{p}' \cancel{\gamma^\mu} \not{p}_2 \not{p}'' \gamma^\nu)$$

$$- \mathrm{Tr}(\not{p}_1 \not{p}' \cancel{\gamma^\mu} \not{p}_2 \not{p}'' \gamma^\nu) + \underset{m_{f'}^2}{O(m_f^2)} \approx 0.$$

$$\Rightarrow \boxed{M_3^* M_4 + M_4^* M_3 \approx 0} \qquad (138)$$

$$\overline{|M|^2} = |V_{ff'}|^2 G_F^2 \Big\{ |C_{H^+}|^2 \hat{s}\, \lambda(\hat{s}, m_{h^0}^2, M_W^2) \left[m_{f'}^2 \tan^2\beta + m_f^2 \cot^2\beta \right] + M_W^4 |C_W|^2 \cdot$$

$$\cdot \left[8\hat{s} M_W^2 + \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \right] - 2\,\mathrm{Re}(C_{H^+}^* C_W) \Big[m_{f'}^2 \tan\beta \big(\hat{s}\lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{u} (\hat{s} - m_{h^0}^2)$$

$$+ 2M_W^4 (2m_{h^0}^2 - \hat{t}) \big) - m_f^2 \cot\beta \big(\hat{s}\lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{t} (\hat{s} - m_{h^0}^2) + 2M_W^4 (2m_{h^0}^2 - \hat{u}) \big) \Big] +$$

$$+ m_f^2 C_{tf}^2 \hat{s} \left[\hat{t}^2 + \frac{M_W^2}{2\hat{s}} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \right] + m_{f'}^2 C_{uf'}^2 \hat{s} \left[\hat{u}^2 + \frac{M_W^2}{2\hat{s}} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \right]$$

$$+ 2\,\mathrm{Re}(C_{H^+})\, m_f^2 \cot\beta\, C_{tf}\, \hat{s} \left[m_{h^0}^2 M_W^2 - \frac{1}{4}\lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta - \hat{t}^2 \right]$$

$$+ 2\,\mathrm{Re}(C_{H^+})\, m_{f'}^2 \tan\beta\, C_{uf'}\, \hat{s} \left[\hat{u}^2 - m_{h^0}^2 M_W^2 + \frac{1}{4}\lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \right]$$

$$- 2\,\mathrm{Re}(C_W)\, m_f^2 C_{tf} \Big[\frac{1}{2}\lambda(\hat{s}, m_{h^0}^2, M_W^2) \cancel{\sin^2\theta} \left(M_W^2 + \frac{\hat{s}}{2} \right) - \hat{s}\, m_{h^0}^2 \cancel{M_W}^2 + 4\cancel{\hat{s}} M_W^4 - 2\hat{t} M_W^4$$

$$+ \hat{t}^2 (\hat{s} - 2M_W^2) + 4\cancel{M_W}^6 \Big] - 2\,\mathrm{Re}(C_W)\, m_{f'}^2 C_{uf'} \Big[\frac{1}{2}\lambda(\hat{s}, m_{h^0}^2, M_W^2) \cancel{\sin^2\theta} \left(M_W^2 + \frac{\hat{s}}{2} \right) - \hat{s}\, \cancel{m_{h^0}^2} M_W^2 +$$

$$+ 4\cancel{\hat{s}} M_W^4 - 2\hat{u} M_W^4 + \hat{u}^2 (\hat{s} - 2M_W^2) + 4\cancel{M_W}^6 \Big] \Big\}$$

$$\overline{|M|^2} = |V_{ff'}|^2 G_F^2 \Big\{ |C_{H^+}|^2 \hat{s}\, \lambda(\hat{s}, m_{h^0}^2, M_W^2) \left[m_{f'}^2 \tan^2\beta + m_f^2 \cot^2\beta \right] + M_W^4 |C_W|^2 \cdot$$

$$\cdot \left[8\hat{s} M_W^2 + \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \right] - 2\,\mathrm{Re}(C_{H^+}^* C_W) \Big[m_{f'}^2 \tan\beta \big(\hat{s}\lambda(\hat{s}, m_{h^0}^2, M_W^2) +$$

$$+ 2M_W^2 \hat{u} (\hat{s} - m_{h^0}^2) + 2M_W^4 (2m_{h^0}^2 - \hat{t}) \big) - m_f^2 \cot\beta \big(\hat{s}\lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{t} (\hat{s} - m_{h^0}^2) +$$

$$+ 2M_W^4 (2m_{h^0}^2 - \hat{u}) \big) \Big] + \frac{M_W^2}{2} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \left[m_f^2 C_{tf}^2 + m_{f'}^2 C_{uf'}^2 \right] + \hat{s} \Big[m_f^2 C_{tf}^2 \hat{t}^2$$

$$+ m_{f'}^2 C_{uf'}^2 \hat{u}^2 \Big] + 2\,\mathrm{Re}(C_{H^+})\, \hat{s} \left[m_{h^0}^2 M_W^2 - \frac{1}{4}\lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \right] \left[m_f^2 \cot\beta\, C_{tf} - m_{f'}^2 \tan\beta\, C_{uf'} \right]$$

$$+ 2\,\mathrm{Re}(C_{H^+})\, \hat{s} \left[-m_f^2 \cot\beta\, C_{tf}\, \hat{t}^2 + m_{f'}^2 \tan\beta\, C_{uf'}\, \hat{u}^2 \right] - 2\,\mathrm{Re}(C_W) \Big[\frac{1}{2}\lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \left(M_W^2 + \frac{\hat{s}}{2} \right)$$

$$- \hat{s}\, m_{h^0}^2 M_W^2 + 4\hat{s} M_W^4 + 4M_W^6 \Big] \left[m_f^2 C_{tf} + m_{f'}^2 C_{uf'} \right] - 2\,\mathrm{Re}(C_W)\, m_f^2 C_{tf} \Big[-2\hat{t} M_W^4 +$$

$$+ \hat{t}^2 (\hat{s} - 2M_W^2) \Big] - 2\,\mathrm{Re}(C_W)\, m_{f'}^2 C_{uf'} \left[-2\hat{u} M_W^4 + \hat{u}^2 (\hat{s} - 2M_W^2) \right] \Big\}$$

$$\overline{|M|^2} = |V_{ff'}|^2 G_F^2 \Big\{ |C_{H^+}|^2 \hat{s}\, \lambda(\hat{s}, m_{h^0}^2, M_W^2) \left[m_{f'}^2 \tan^2\beta + m_f^2 \cot^2\beta \right] + M_W^4 |C_W|^2 \cdot \Big[8\hat{s} M_W^2 +$$

$$+ \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \Big] - 2\,\mathrm{Re}(C_{H^+}^* C_W) \Big[m_{f'}^2 \tan\beta \big(\hat{s}\lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{u} (\hat{s} - m_{h^0}^2) +$$

$$+ 2M_W^4 (2m_{h^0}^2 - \hat{t}) \big) - m_f^2 \cot\beta \big(\hat{s}\lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{t} (\hat{s} - m_{h^0}^2) + 2M_W^4 (2m_{h^0}^2 - \hat{u}) \big) \Big] +$$

$$+ \frac{M_W^2}{2} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \left[\frac{m_f^2 C_f^2}{\hat{t}^2} + \frac{m_{f'}^2 C_{f'}^2}{\hat{u}^2} \right] + \hat{s} \left[m_f^2 C_f^2 + m_{f'}^2 C_{f'}^2 \right] + 2\,\mathrm{Re}(C_{H^+}) \cdot$$

$$\cdot \hat{s} \left[m_{h^0}^2 M_W^2 - \frac{1}{4}\lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \right] \left[\frac{m_f^2 \cot\beta\, C_f}{\hat{t}} - \frac{m_{f'}^2 \tan\beta\, C_{f'}}{\hat{u}} \right] + 2\,\mathrm{Re}(C_{H^+})\, \hat{s} \cdot$$

$$\cdot \left[m_{f'}^2 \tan\beta\, C_{f'}\, \hat{u} - m_f^2 \cot\beta\, C_f\, \hat{t} \right] - 2\,\mathrm{Re}(C_W) \Big[\frac{1}{2}\lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \left(M_W^2 + \frac{\hat{s}}{2} \right) - \hat{s}\, m_{h^0}^2 M_W^2$$

$$+ 4\hat{s} M_W^4 + 4M_W^6 \Big] \left[\frac{m_f^2 C_f}{\hat{t}} + \frac{m_{f'}^2 C_{f'}}{\hat{u}} \right] - 2\,\mathrm{Re}(C_W)\, m_f^2 C_f \left[-2M_W^4 + \hat{t} (\hat{s} - 2M_W^2) \right]$$

$$- 2\,\mathrm{Re}(C_W)\, m_{f'}^2 C_{f'} \left[-2M_W^4 + \hat{u} (\hat{s} - 2M_W^2) \right] \Big\} \qquad (139)$$

$$\left(\frac{d\sigma}{d\hat{t}}\right)(f\bar{f}' \to h^0 W^+) = \frac{1}{16\pi\hat{s}^2}|V_{ff'}|^2 G_F^2 \bigg\{ |C_{H^+}|^2 \hat{s}\,\lambda(\hat{s}, m_{h^0}^2, M_W^2)[m_{f'}^2\tan^2\beta + m_f^2\cot^2\beta] +$$

$$+ M_W^4 |C_W|^2 . [8\hat{s}M_W^2 + \lambda(\hat{s},m_{h^0}^2,M_W^2)\sin^2\theta] - 2C_{H^+}\,\mathrm{Re}(C_W)[m_{f'}^2\tan\beta(\hat{s}\lambda(\hat{s},m_{h^0}^2,M_W^2) +$$

$$+2M_W^2\hat{u}(\hat{s}-m_{h^0}^2) + 2M_W^4(2m_{h^0}^2-\hat{t})) - m_f^2\cot\beta(\hat{s}\lambda(\hat{s},m_{h^0}^2,M_W^2) + 2M_W^2\hat{t}(\hat{s}-m_{h^0}^2) +$$

$$+2M_W^4(2m_{h^0}^2-\hat{u}))] + \frac{M_W^2}{2}\lambda(\hat{s},m_{h^0}^2,M_W^2)\sin^2\theta\left[\frac{m_f^2C_f^2}{\hat{t}^2} + \frac{m_{f'}^2C_{f'}^2}{\hat{u}^2}\right] + \hat{s}[m_f^2C_f^2 + m_{f'}^2C_{f'}^2]$$

$$+ 2C_{H^+}\hat{s}\left[m_{h^0}^2M_W^2 - \frac{1}{4}\lambda(\hat{s},m_{h^0}^2,M_W^2)\sin^2\theta\right]\left[\frac{m_f^2\cot\beta\, C_f}{\hat{t}} - \frac{m_{f'}^2\tan\beta\, C_{f'}}{\hat{u}}\right] + 2C_{H^+}\hat{s}.$$

$$.[m_{f'}^2\tan\beta\, C_{f'}\hat{u} - m_f^2\cot\beta\, C_f\hat{t}] - 2\,\mathrm{Re}(C_W)\left[\frac{1}{2}\lambda(\hat{s},m_{h^0}^2,M_W^2)\sin^2\theta\left(M_W^2+\frac{\hat{s}}{2}\right) - \hat{s}m_{h^0}^2M_W^2 +\right.$$

$$\left.+ 4\hat{s}M_W^4 + 4M_W^6\right]\left[\frac{m_f^2C_f}{\hat{t}} + \frac{m_{f'}^2C_{f'}}{\hat{u}}\right] - 2\,\mathrm{Re}(C_W)m_f^2C_f[-2M_W^4+\hat{t}(\hat{s}-2M_W^2)]$$

$$-2\,\mathrm{Re}(C_W)m_{f'}^2C_{f'}[-2M_W^4+\hat{u}(\hat{s}-2M_W^2)]\bigg\} \qquad (140)$$

where (if we neglect Γ_{H^+})

$$\boxed{C_{H^+} \equiv \frac{\cos(\beta-\alpha)}{\hat{s}-M_{H^+}^2}} \qquad (141)$$

$$\boxed{C_W = \frac{\sin(\beta-\alpha)\;(\hat{s}-M_W^2-i M_W\Gamma_W)}{(\hat{s}-M_W^2)^2 + M_W^2\Gamma_W^2}} \;;\; \boxed{\mathrm{Re}(C_W) = \frac{\sin(\beta-\alpha)\;(\hat{s}-M_W^2)}{(\hat{s}-M_W^2)^2 + M_W^2\Gamma_W^2}} \qquad (142,43)$$

$$\boxed{|C_W|^2 = \frac{\sin^2(\beta-\alpha)}{(\hat{s}-M_W^2)^2 + M_W^2\Gamma_W^2}} \qquad (144)$$

$$\boxed{\mathrm{Re}(C_W) = \frac{\sin(\beta-\alpha)\,(\hat{s}-M_W^2)}{(\hat{s}-M_W^2)^2 + M_W^2\Gamma_W^2}} \;;\; \boxed{C_f = -\frac{\cos\alpha}{\sin\beta} \;;\; C_{f'} = \frac{\sin\alpha}{\cos\beta}} \qquad (145,46)$$

$M_W = 80.423 \pm 0.039$ GeV

$\Gamma_W = 2.118 \pm 0.042$ GeV

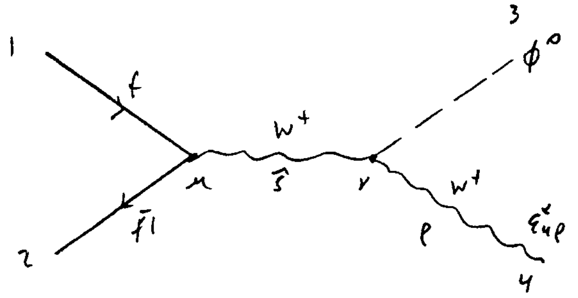
$f = u, c$; $f' = d, s, b$.

For light quarks $f = u$; $f' = d, s$

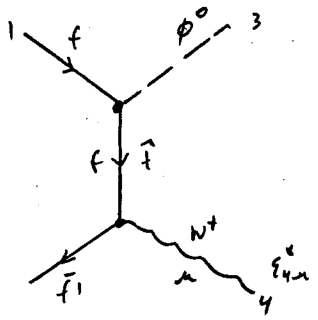
$$\boxed{\left(\frac{d\sigma}{d\hat{t}}\right)(f\bar{f}' \to h^0 W^+) = \frac{1}{16\pi\hat{s}^2}|V_{ff'}|^2 G_F^2 M_W^4 |C_W|^2 [8\hat{s}M_W^2 + \lambda(\hat{s},m_{h^0}^2,M_W^2)\sin^2\theta]}$$

(147)

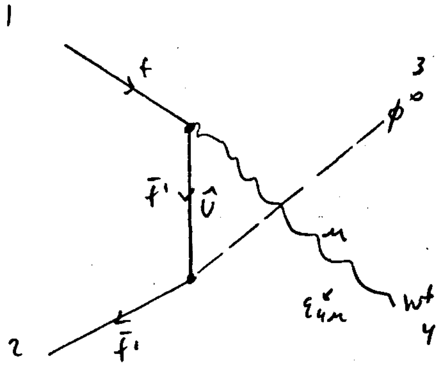
In the Standard Model we only have three diagrams:



+



+



$\phi^0 \equiv$ Standard Model Higgs Particle.

and then

$$\left(\frac{d\sigma}{d\hat{t}}\right)(f\bar{f}' \to \phi^0 W^+) = \frac{1}{16\pi\hat{s}^2}|V_{ff'}|^2 G_F^2 \Big\{ M_W^4 |C_W'|^2 [8\hat{s}M_W^2 + \lambda(\hat{s}, m_{\phi^0}^2, M_W^2)\sin^2\theta]$$

$$+ \frac{M_W^2}{2}\lambda(\hat{s}, m_{\phi^0}^2, M_W^2)\sin^2\theta\left[\frac{m_f^2}{\hat{t}^2} + \frac{m_{f'}^2}{\hat{u}^2}\right] + \hat{s}[m_f^2 + m_{f'}^2] + 2\,\mathrm{Re}(C_W')\Big[\frac{1}{2}\lambda(\hat{s}, m_{\phi^0}^2, M_W^2).$$

$$\cdot\sin^2\theta\left(M_W^2 + \frac{\hat{s}}{2}\right) - \hat{s}\,m_{\phi^0}^2 M_W^2 + 4\hat{s}M_W^4 + 4M_W^6\Big]\left[\frac{m_f^2}{\hat{t}} + \frac{m_{f'}^2}{\hat{u}}\right] + 2\,\mathrm{Re}(C_W')m_f^2[-2M_W^4 +$$

$$+ \hat{t}(\hat{s} - 2M_W^2)] + 2\,\mathrm{Re}(C_W')m_{f'}^2[-2M_W^4 + \hat{u}(\hat{s} - 2M_W^2)]\Big\} \qquad (148)$$

---

where $\boxed{C_W' = \frac{\hat{s} - M_W^2 - i M_W \Gamma_W}{(\hat{s} - M_W^2)^2 + M_W^2\Gamma_W^2}}$; $\boxed{\mathrm{Re}(C_W') = \frac{\hat{s} - M_W^2}{(\hat{s} - M_W^2)^2 + M_W^2\Gamma_W^2}}$ (149,50)

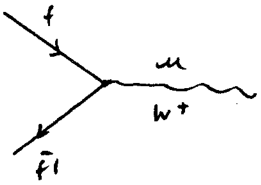
$$\boxed{|C_W'|^2 = \frac{1}{(\hat{s} - M_W^2)^2 + M_W^2\Gamma_W^2}} \qquad (151)$$

For light quarks $f = u$; $f' = d, s$.

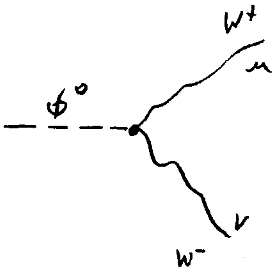
$$\boxed{\left(\frac{d\sigma}{d\hat{t}}\right)(f\bar{f}' \to \phi^0 W^+) = \frac{1}{16\pi\hat{s}^2}|V_{ff'}|^2 G_F^2 M_W^4 |C_W'|^2 [8\hat{s}M_W^2 + \lambda(\hat{s}, m_{\phi^0}^2, M_W^2)\sin^2\theta]} \qquad (152)$$

that is the same as $\left(\frac{d\sigma}{d\hat{t}}\right)(f\bar{f}' \to h^0 W^+)$ if we don't consider the factor $\sin^2(\beta-\alpha)$

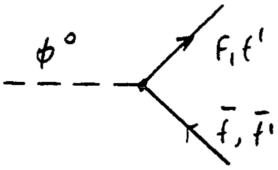
In the standard model:



$-\frac{ig}{\sqrt{2}}\gamma^\mu \frac{1}{2}(1-\gamma^5)V^*_{ff'}$

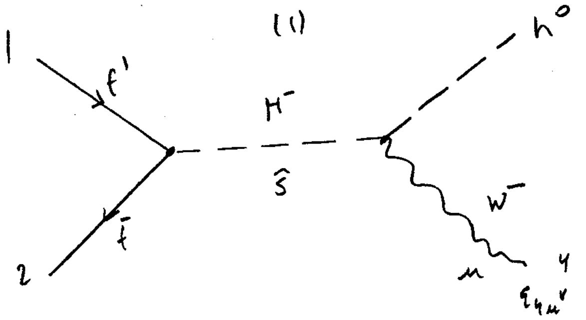


$igM_W\eta^{\mu\nu}$

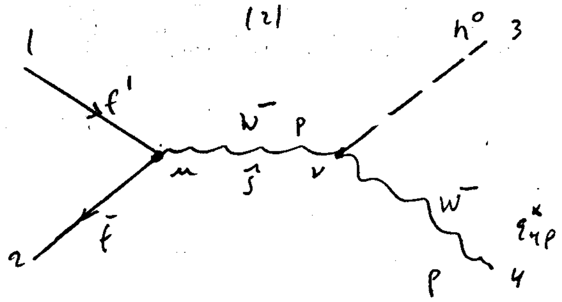


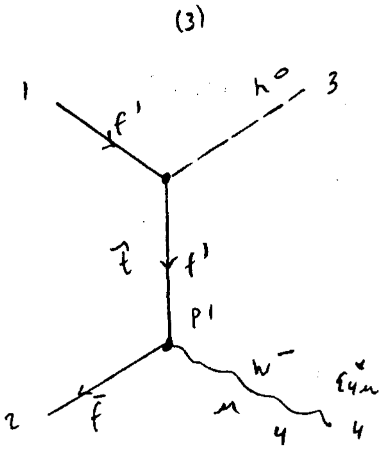
$-\frac{igm_f}{2M_W}$, $-\frac{igm_{f'}}{2M_W}$ so $c_f = c_{f'} = -1$

To evaluate $\left(\frac{d\sigma}{d\hat{t}}\right)_{f'\bar{f}\to h^0 W^-}$, the Feynman diagrams are:

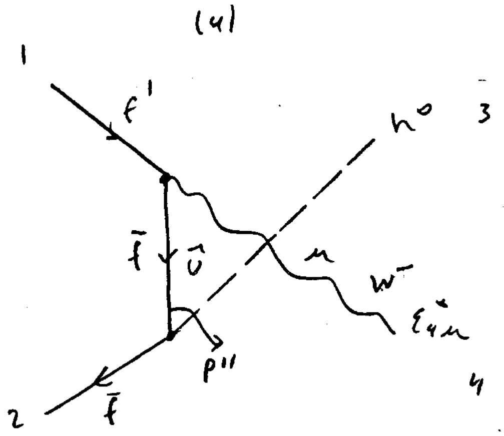


+





+



$f = u, c \ ; \ f' = d, s, b.$

$$-i\mathcal{M}_3 = \bar{V}_2\left(-\frac{ig}{\sqrt{2}}\gamma^\mu \frac{1}{2}(1-\gamma^5)\right)V_{ff'}\,\varepsilon^*_{4\mu}\,\frac{i(\not{p}'+m_{f'})}{\hat{t}-m_{f'}^2}\left(\frac{igm_{f'}}{2M_W}C_{f'}\right)U_1 \qquad (153)$$

$$C_{tf'} \equiv \frac{C_{f'}}{\hat{t}-m_{f'}^2} \qquad (154)$$

$$\Rightarrow \boxed{\mathcal{M}_3 = -\frac{g^2 m_{f'} C_{tf'}}{4\sqrt{2}M_W}V_{ff'}\,\varepsilon^*_{4\mu}\,\bar{V}_2\gamma^\mu(1-\gamma^5)(\not{p}'+m_{f'})U_1} \qquad (155)$$

$$-i\mathcal{M}_4 = \bar{V}_2\left(\frac{igm_f}{2M_W}C_f\right)\frac{i(\not{p}''+m_f)}{\hat{u}-m_f^2}\left(-\frac{ig}{\sqrt{2}}\gamma^\mu\frac{1}{2}(1-\gamma^5)\right)V_{ff'}\,\varepsilon^*_{4\mu}\,U_1$$

$$C_{uf} \equiv \frac{C_f}{\hat{u}-m_f^2} \qquad (156)$$

$$\Rightarrow \boxed{\mathcal{M}_4 = -\frac{g^2 m_f C_{uf} V_{ff'}\,\varepsilon^*_{4\mu}}{4\sqrt{2}M_W}\bar{V}_2(\not{p}''+m_f)\gamma^\mu(1-\gamma^5)U_1} \qquad (157)$$

$$-iM_1 = \varepsilon^{*}_{4\mu}\left(\frac{ig}{2}\cos(\beta-\alpha)\,(p_1+p_2+p_3)^{\mu}\right)\frac{i}{\hat{s}-M_{H^-}^{2}+iM_{H^-}\Gamma_{H^-}}\,\overline{V_2}\left(\frac{ig}{2\sqrt{2}M_W}(A+B\gamma^5)\right)V_{ff'}\,U_1$$

$$\boxed{M_1 = \frac{g^2V_{ff'}}{4\sqrt{2}M_W}\,C_{H^+}\,\varepsilon^{*}_{4\mu}(p_1+p_2+p_3)^{\mu}\,\overline{V_2}\,(A+B\gamma^5)U_1 \qquad (C_{H^+}=C_{H^-})} \qquad (158)$$

$$-iM_2 = \varepsilon^{*}_{4\rho}\,(igM_W\sin(\beta-\alpha)n^{\nu\rho}\,(-i)\left[n_{\mu\nu}-\frac{p_\mu p_\nu}{M_W^2}\right]\frac{1}{\hat{s}-M_W^2+iM_W\Gamma_W}\,\overline{V}_2\left(-\frac{ig}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right)$$

$$\times V_{ff'}U_1$$

$$\boxed{M_2 = \frac{g^2M_W\,C_W}{2\sqrt{2}}\,V_{ff'}\,\varepsilon^{*}_{4\rho}\,n^{\nu\rho}\left(n_{\mu\nu}-\frac{p_\mu p_\nu}{M_W^2}\right)\overline{V}_2\gamma^{\mu}(1-\gamma^5)U_1} \qquad (159)$$

(91) is like (2) changing $i \to -i$; $V^{*}_{ff'} \to V_{ff'}$ ~~and~~ $-B \to B$

$$\Rightarrow \boxed{|M_1|^2 = \frac{g^4\,|V_{ff'}|^2\,|C_{H^+}|^2\,\hat{s}\,\lambda(\hat{s},m_{h^0}^2,M_W^2)}{8M_W^4}\left[m_{f'}^2\tan^2\beta+m_f^2\cot^2\beta\right]} \qquad (160)$$

(92) is identical to (4) changing $V^{*}_{ff'} \to V_{ff'}$

$$\Rightarrow \boxed{|M_2|^2 = \frac{g^4\,|C_W|^2\,|V_{ff'}|^2}{8}\left[8\hat{s}M_W^2+\lambda(\hat{s},m_{h^0}^2,M_W^2)\sin^2\theta\right]} \qquad (161)$$

$$\begin{aligned}M_1^{*}M_2 = \frac{g^4\,|V_{ff'}|^2\,C^{*}_{H^+}C_W}{16M_W^4}\Big\{&(A-B)m_f\left[\hat{s}\,\lambda(\hat{s},m_{h^0}^2,M_W^2)+2M_W^2\,\hat{u}\,(\hat{s}-m_{h^0}^2)+2M_W^4(2m_{h^0}^2-\hat{t})\right]\\&-(A+B)m_{f'}\left[+\hat{s}\,\lambda(\hat{s},m_{h^0}^2,M_W^2)+2M_W^2\,\hat{t}\,(\hat{s}-m_{h^0}^2)+2M_W^4(2m_{h^0}^2-\hat{u})\right]\Big\}\end{aligned}$$

(162)

(in (61) $m_{f'} \leftrightarrow m_f$)

$$\begin{aligned}M_1^{*}M_2 = \frac{g^4\,|V_{ff'}|^2\,C^{*}_{H^+}C_W}{16M_W^4}\Big\{&2m_f^2\cot\beta\left[\hat{s}\,\lambda(\hat{s},m_{h^0}^2,M_W^2)+2M_W^2\,\hat{u}\,(\hat{s}-m_{h^0}^2)+2M_W^4(2m_{h^0}^2-\hat{t})\right]\\&-2m_{f'}^2\tan\beta\left[\hat{s}\,\lambda(\hat{s},m_{h^0}^2,M_W^2)+2M_W^2\,\hat{t}\,(\hat{s}-m_{h^0}^2)+2M_W^4(2m_{h^0}^2-\hat{u})\right]\Big\}\end{aligned} \qquad (163)$$

$$\boxed{\begin{aligned}M_1^{*}M_2+M_2^{*}M_1 = -\frac{g^4\,|V_{ff'}|^2}{8M_W^4}\,2\,\mathrm{Re}\,(C^{*}_{H^+}C_W)\Big\{&-m_f^2\cot\beta\left[\hat{s}\,\lambda(\hat{s},m_{h^0}^2,M_W^2)+2M_W^2\,\hat{u}\,(\hat{s}-m_{h^0}^2)+\right.\\&\left.+2M_W^4(2m_{h^0}^2-\hat{t})\right]+m_{f'}^2\tan\beta\left[\hat{s}\,\lambda(\hat{s},m_{h^0}^2,M_W^2)+2M_W^2\,\hat{t}\,(\hat{s}-m_{h^0}^2)+2M_W^4(2m_{h^0}^2-\hat{u})\right]\Big\}\end{aligned}}$$

(164)

$$\boxed{|M_3|^2 = \frac{g^4 m_{f'}^2 C_{tf'}^2 |V_{tf'}|^2 \hat{s}}{8M_W^4} \left\{ \hat{t}^2 + \frac{M_W^2}{2\hat{s}} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \right\}} \quad (165)$$

$$\boxed{|M_4|^2 = \frac{g^4 m_f^2 C_{uf}^2 |V_{tf'}|^2}{8M_W^4} \hat{s} \left\{ \hat{u}^2 + \frac{M_W^2}{2\hat{s}} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \right\}} \quad (166)$$

To get $M_1^* M_3$ we change $B \to -B$; $m_f \to m_{f'}$; $C_{tf} \to C_{tf'}$; $\cot\beta \to \tan\beta$

$$\boxed{M_1^* M_3 + M_3^* M_1 = -g \frac{^4 |V_{tf'}|^2 \, Re\,(C_{H^+}) m_{f'}^2 \tan\beta \, C_{tf'}}{4M_W^4} \hat{s} \left\{ m_{h^0}^2 M_W^2 - \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta - \hat{t}^2 \right\}} \quad (167)$$

To get $M_1^* M_4$ we change $B \to -B$; $m_{f'} \to m_f$; $C_{uf'} \to C_{uf}$; $\tan\beta \to \cot\beta$

$$\boxed{M_1^* M_4 + M_4^* M_1 = -\frac{g^4 |V_{tf'}|^2 \, Re\,(C_{H^+}) m_f^2 C_{uf} \cot\beta \, \hat{s}}{4M_W^4} \left\{ \hat{u}^2 - m_{h^0}^2 M_W^2 + \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \right\}} \quad (168)$$

$$\boxed{\begin{aligned} M_2^* M_3 + M_3^* M_2 = & -\frac{g^4 m_{f'}^2 \, Re\,(C_W) C_{tf'} |V_{tf'}|^2}{4M_W^4} \left\{ \frac{1}{2} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \left(M_W^2 + \frac{\hat{s}}{2}\right) - \hat{s} m_{h^0}^2 M_W^2 \right. \\ & \left. + 4\hat{s} M_W^4 - 2\hat{t} M_W^4 + \hat{t}^2 (\hat{s} - 2M_W^2) + 4M_W^6 \right\} \end{aligned}} \quad (169)$$

$$\boxed{\begin{aligned} M_2^* M_4 + M_4^* M_2 = & -\frac{g^4 m_f^2 \, Re\,(C_W) C_{uf} |V_{tf'}|^2}{4M_W^4} \left\{ \frac{1}{2} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta \left(M_W^2 + \frac{\hat{s}}{2}\right) - \hat{s} m_{h^0}^2 M_W^2 \right. \\ & \left. + 4\hat{s} M_W^4 - 2\hat{u} M_W^4 + \hat{u}^2 (\hat{s} - 2M_W^2) + 4M_W^6 \right\} \end{aligned}} \quad (170)$$

$$\boxed{M_3^* M_4 + M_4^* M_3 = 0} \quad (171)$$

$$\begin{aligned} \overline{|M|^2} = & |V_{tf'}|^2 G_F^2 \Big\{ |C_{H^+}|^2 \hat{s} \lambda(\hat{s}, m_{h^0}^2, M_W^2) [m_{f'}^2 \tan^2\beta + m_f^2 \cot^2\beta] + M_W^4 |C_W|^2 [8\hat{s} M_W^2 + \lambda(\hat{s}, m_{h^0}^2, M_W^2) \cdot \\ & \sin^2\theta] - 2\, Re\,(C_{H^+}^* C_W) \Big[-m_f^2 \cot\beta [\hat{s} \lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{u} (\hat{s} - m_{h^0}^2) + 2M_W^4 (2m_{h^0}^2 - \hat{t})] \\ & + m_{f'}^2 \tan\beta [\hat{s} \lambda(\hat{s}, m_{h^0}^2, M_W^2) + 2M_W^2 \hat{t} (\hat{s} - m_{h^0}^2) + 2M_W^4 (2m_{h^0}^2 - \hat{u})] \Big] + m_{f'}^2 C_{tf'}^2 \hat{s} [\hat{t}^2 + \\ & + \frac{M_W^2}{2\hat{s}} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta] + m_f^2 C_{uf}^2 \hat{s} [\hat{u}^2 + \frac{M_W^2}{2\hat{s}} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta] \\ & - 2 Re\,(C_{H^+}) m_{f'}^2 \tan\beta \, C_{tf'} \hat{s} [m_{h^0}^2 M_W^2 - \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta - \hat{t}^2] + 2 Re\,(C_{H^+}) m_f^2 C_{uf} \cot\beta \, \hat{s} \cdot \\ & \cdot [m_{h^0}^2 M_W^2 - \hat{u}^2 - \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta] - 2 Re\,(C_W) m_{f'}^2 C_{tf'} [\frac{1}{2} \lambda \sin^2\theta (M_W^2 + \frac{\hat{s}}{2}) - \hat{s} m_{h^0}^2 M_W^2 \\ & + 4\hat{s} M_W^4 - 2\hat{t} M_W^4 + \hat{t}^2 (\hat{s} - 2M_W^2) + 4M_W^6] - 2 Re\,(C_W) m_f^2 C_{uf} [\frac{1}{2} \lambda \sin^2\theta (M_W^2 + \frac{\hat{s}}{2}) - \hat{s} m_{h^0}^2 M_W^2 \\ & + 4\hat{s} M_W^4 - 2\hat{u} M_W^4 + \hat{u}^2 (\hat{s} - 2M_W^2) + 4M_W^6] \Big\} \end{aligned}$$

$$\overline{|M|^2} = |V_{ff'}|^2 G_F^2 \Big\{ |C_{H^+}|^2 \hat{s}\lambda \left[m_{f'}^2 \tan^2\beta + m_f^2\cot^2\beta\right] + M_W^4 |C_W|^2 \left[8\hat{s}M_W^2 + \lambda\sin^2\theta\right]$$
$$-2\mathrm{Re}(C_{H^+}^* C_W)\Big[-m_f^2\cot\beta\left[\hat{s}\lambda + 2M_W^2\hat{u}(\hat{s}-m_{h^0}^2)+2M_W^4(2m_{h^0}^2-\hat{t})\right] + m_{f'}^2\tan\beta\big[\hat{s}\lambda +$$
$$+2M_W^2\hat{t}(\hat{s}-m_{h^0}^2)+2M_W^4(2m_{h^0}^2-\hat{u})\big]\Big] + \frac{M_W^2}{2}\lambda\sin^2\theta\left[m_{f'}^2 C_{tf'}^2 + m_f^2 C_{uf}^2\right] + \hat{s}\big[m_{f'}^2 C_{tf'}^2\hat{t}^2 +$$
$$+m_f^2 C_{uf}^2\hat{u}^2\big] - 2\mathrm{Re}(C_{H^+})\hat{s}\left[m_{h^0}^2M_W^2 - \frac{1}{4}\lambda\sin^2\theta\right]\left[m_{f'}^2\tan\beta\, C_{tf'} - m_f^2\cot\beta\, C_{uf}\right] +$$
$$+2\mathrm{Re}(C_{H^+})\hat{s}\left[m_{f'}^2\tan\beta\, C_{tf'}\hat{t}^2 - m_f^2\cot\beta\, C_{uf}\hat{u}^2\right] - 2\mathrm{Re}(C_W)\Big[\frac{1}{2}\lambda\sin^2\theta\left(M_W^2+\frac{\hat{s}}{2}\right) - \hat{s}m_{h^0}^2M_W^2$$
$$+4\hat{s}M_W^4+4M_W^6\Big]\left[m_{f'}^2C_{tf'} + m_f^2C_{uf}\right] - 2\mathrm{Re}(C_W)m_{f'}^2C_{tf'}\left[-2\hat{t}M_W^4+\hat{t}^2(\hat{s}-2M_W^2)\right]$$
$$-2\mathrm{Re}(C_W)m_f^2C_{uf}\left[-2\hat{u}M_W^4+\hat{u}^2(\hat{s}-2M_W^2)\right]\Big\}$$

$$\overline{|M|^2} = |V_{ff'}|^2 G_F^2 \Big\{ |C_{H^+}|^2 \hat{s}\lambda \left[m_{f'}^2 \tan^2\beta + m_f^2\cot^2\beta\right] + M_W^4 |C_W|^2 \left[8\hat{s}M_W^2 + \lambda\sin^2\theta\right]$$
$$-2\mathrm{Re}(C_{H^+}^* C_W)\Big[-m_f^2\cot\beta\left[\hat{s}\lambda + 2M_W^2\hat{u}(\hat{s}-m_{h^0}^2)+2M_W^4(2m_{h^0}^2-\hat{t})\right] + m_{f'}^2\tan\beta\big[\hat{s}\lambda +$$
$$+2M_W^2\hat{t}(\hat{s}-m_{h^0}^2)+2M_W^4(2m_{h^0}^2-\hat{u})\big]\Big] + \frac{M_W^2}{2}\lambda\sin^2\theta\left[\frac{m_{f'}^2 C_{f'}^2}{\hat{t}^2} + \frac{m_f^2 C_f^2}{\hat{u}^2}\right] +$$
$$+\hat{s}\left[m_{f'}^2 C_{f'}^2 + m_f^2 C_f^2\right] + 2\mathrm{Re}(C_{H^+})\hat{s}\left[m_{h^0}^2M_W^2 - \frac{1}{4}\lambda\sin^2\theta\right]\left[-\frac{m_{f'}^2\tan\beta\, C_{f'}}{\hat{t}} + \frac{m_f^2\cot\beta\, C_f}{\hat{u}}\right]$$
$$+2\mathrm{Re}(C_{H^+})\hat{s}\left[m_{f'}^2\tan\beta\, C_{f'}\hat{t} - m_f^2\cot\beta\, C_f\hat{u}\right] - 2\mathrm{Re}(C_W)\Big[\frac{1}{2}\lambda\sin^2\theta\left(M_W^2+\frac{\hat{s}}{2}\right) - \hat{s}m_{h^0}^2M_W^2$$
$$+4\hat{s}M_W^4+4M_W^6\Big]\left[\frac{m_{f'}^2C_{f'}}{\hat{t}} + \frac{m_f^2C_f}{\hat{u}}\right] - 2\mathrm{Re}(C_W)m_{f'}^2C_{f'}\left[-2M_W^4+\hat{t}(\hat{s}-2M_W^2)\right]$$
$$-2\mathrm{Re}(C_W)m_f^2C_f\left[-2M_W^4+\hat{u}(\hat{s}-2M_W^2)\right]\Big\} \qquad (172)$$

$\Rightarrow$

$$\left(\frac{d\sigma}{d\hat{t}}\right)(f'\bar{f}\to h^0W^-) = \frac{1}{16\pi\hat{s}^2}|V_{ff'}|^2 G_F^2 \Big\{ |C_{H^+}|^2 \hat{s}\lambda(\hat{s},m_{h^0}^2,M_W^2)\left[m_{f'}^2\tan^2\beta + m_f^2\cot^2\beta\right]$$
$$+M_W^4|C_W|^2\left[8\hat{s}M_W^2 + \lambda(\hat{s},m_{h^0}^2,M_W^2)\sin^2\theta\right] - 2C_{H^+}\mathrm{Re}(C_W)\Big[m_{f'}^2\tan\beta\big[\hat{s}\lambda(\hat{s},m_{h^0}^2,M_W^2)+$$
$$+2M_W^2\hat{t}(\hat{s}-m_{h^0}^2)+2M_W^4(2m_{h^0}^2-\hat{u})\big] - m_f^2\cot\beta\big[\hat{s}\lambda(\hat{s},m_{h^0}^2,M_W^2)+2M_W^2\hat{u}(\hat{s}-m_{h^0}^2)$$
$$+2M_W^4(2m_{h^0}^2-\hat{t})\big]\Big] + \frac{M_W^2}{2}\lambda(\hat{s},m_{h^0}^2,M_W^2)\sin^2\theta\left[\frac{m_{f'}^2C_{f'}^2}{\hat{t}^2} + \frac{m_f^2C_f^2}{\hat{u}^2}\right] +$$
$$+\hat{s}\left[m_{f'}^2C_{f'}^2+m_f^2C_f^2\right] + 2C_{H^+}\hat{s}\left[m_{h^0}^2M_W^2 - \frac{1}{4}\lambda(\hat{s},m_{h^0}^2,M_W^2)\sin^2\theta\right]\Big[\frac{m_f^2\cot\beta\, C_f}{\hat{u}} -$$
$$-\frac{m_{f'}^2\tan\beta\, C_{f'}}{\hat{t}}\Big] + 2C_{H^+}\hat{s}\left[m_{f'}^2\tan\beta\, C_{f'}\hat{t} - m_f^2\cot\beta\, C_f\hat{u}\right] - 2\mathrm{Re}(C_W)\Big[\frac{1}{2}\lambda(\hat{s},m_{h^0}^2,M_W^2)\cdot$$
$$\sin^2\theta\left(M_W^2+\frac{\hat{s}}{2}\right) - \hat{s}m_{h^0}^2M_W^2 + 4\hat{s}M_W^4 + 4M_W^6\Big]\left[\frac{m_{f'}^2C_{f'}}{\hat{t}} + \frac{m_f^2C_f}{\hat{u}}\right] - 2\mathrm{Re}(C_W)m_{f'}^2C_{f'}\cdot\big[-2M_W^4+$$
$$+\hat{t}(\hat{s}-2M_W^2)\big] - 2\mathrm{Re}(C_W)m_f^2C_f\left[-2M_W^4+\hat{u}(\hat{s}-2M_W^2)\right]\Big\} \qquad (173)$$

---

that is $\left(\frac{d\sigma}{d\hat{t}}\right)(f\bar{f'}\to h^0W^+)$ changing $\hat{t}\leftrightarrow\hat{u}$

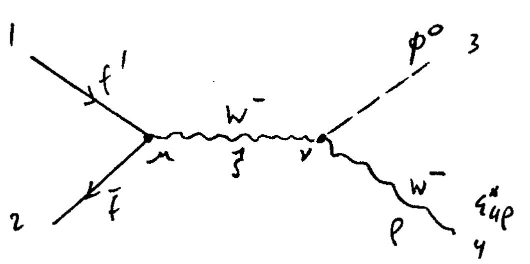
For light quarks $f = u ; \quad f' = d , s$

$$\boxed{\left(\frac{d\sigma}{d\hat{t}}\right)(f'\bar{f} \rightarrow h^0 W^-) = \frac{1}{16\pi\hat{s}^2} |V_{ff'}|^2 G_F^2 M_W^4 |C_W|^2 [8\hat{s}M_W^2 + \lambda(\hat{s}, m_{h^0}^2, M_W^2)\sin^2\theta]} \tag{174}$$

Then for light quarks:

$$\boxed{\left(\frac{d\sigma}{d\hat{t}}\right)(f\bar{f}' \rightarrow h^0 W^+) = \left(\frac{d\sigma}{d\hat{t}}\right)(f'\bar{f} \rightarrow h^0 W^-)} \tag{175}$$

In the standard Model we have 3 diagrams



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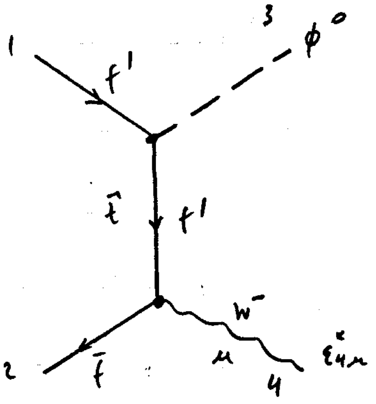

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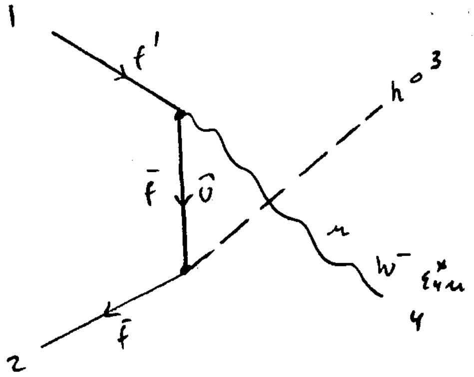


$\phi^0 \equiv$ standard Model Higgs particle

and:

$$\left(\frac{d\sigma}{d\hat{t}}\right)(f'\bar{f} \rightarrow \phi^0 W^-) = \frac{1}{16\pi\hat{s}^2} |V_{ff'}|^2 G_F^2 \Big\{ M_W^4 |C_{W'}|^2 [8\hat{s}M_W^2 + \lambda(\hat{s}, m_{\phi^0}^2, M_W^2)\sin^2\theta] +$$
$$+ \frac{M_W^2}{2} \lambda(\hat{s}, m_{\phi^0}^2, M_W^2)\sin^2\theta \left[\frac{m_{f'}^2}{\hat{t}^2} + \frac{m_f^2}{\hat{u}^2}\right] + \hat{s}[m_{f'}^2 + m_f^2] + 2\mathrm{Re}\,C(_{W'})\Big[\frac{1}{2}\lambda(\hat{s}, m_{\phi^0}^2, M_W^2).$$
$$\sin^2\theta\left(M_W^2 + \frac{\hat{s}}{2}\right) - \hat{s}m_{\phi^0}^2 M_W^2 + 4\hat{s}M_W^4 + 4M_W^6\Big]\left[\frac{m_{f'}^2}{\hat{t}} + \frac{m_f^2}{\hat{u}}\right] + 2\mathrm{Re}\,C(_{W'})m_{f'}^2[-2M_W^4 +$$
$$+ \hat{t}(\hat{s} - 2M_W^2)] + 2\mathrm{Re}(C_{W'})m_f^2[-2M_W^4 + \hat{u}(\hat{s} - 2M_W^2)]\Big\} \tag{176}$$

For light quarks $f = u ; \quad f' = d , s$

$$\boxed{\left(\frac{d\sigma}{d\hat{t}}\right)(f'\bar{f} \rightarrow \phi^0 W^-) = \frac{1}{16\pi\hat{s}^2} |V_{ff'}|^2 G_F^2 M_W^4 |C_{W'}|^2 [8\hat{s}M_W^2 + \lambda(\hat{s}, m_{\phi^0}^2, M_W^2)\sin^2\theta]} \tag{177}$$

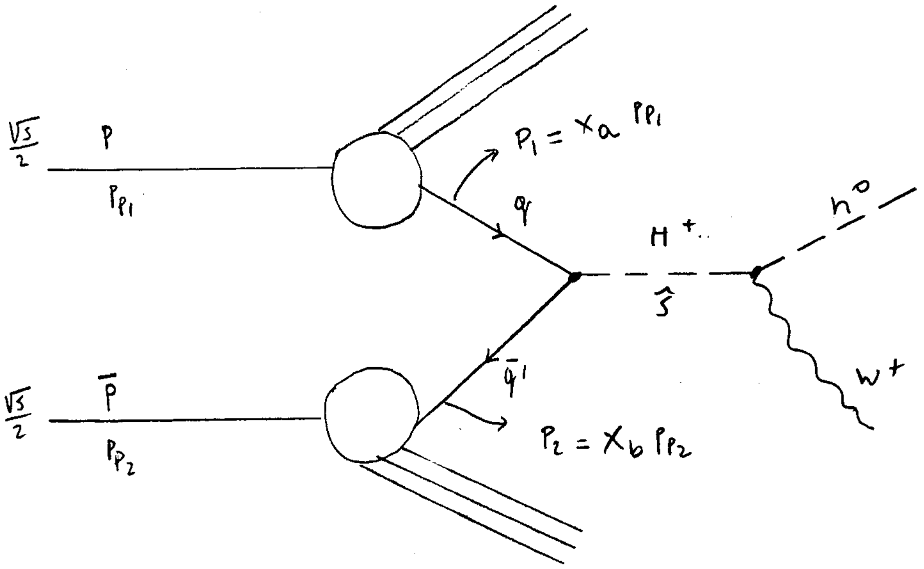
and :

$\left(\frac{d\sigma}{d\hat{t}}\right)(f'\bar{f} \to \phi^0 W^-)$ is the same as $\left(\frac{d\sigma}{d\hat{t}}\right)(f'\bar{f} \to h^0 W^-)$ if we don't consider the factor $\sin^2(\beta-\alpha)$.

Also :

$$\boxed{\left(\frac{d\sigma}{d\hat{t}}\right)(f\bar{f}' \to \phi^0 W^+) = \left(\frac{d\sigma}{d\hat{t}}\right)(f'\bar{f} \to \phi^0 W^-)} \qquad (178)$$

$\sigma(p\bar{p} \to h^0 W^+ X)$



$+ \ldots W^+ , \; q, \bar{q}'$
other diagrams

$$\hat{s} = x_a x_b s \qquad (179)$$

$$\frac{d^2\sigma}{dY\, dP_T^2}(p\bar{p} \to h^0 W^+ X) = \sum_{q,q'} \int_{X_{a\,min}}^{1} dX_a \Big[f_q^{p}(X_a, M_a^2)\, f_{\bar{q}'}^{\bar{p}}(X_b, M_b^2) +$$

$$+ f_{\bar{q}'}^{p}(X_a, M_a^2)\, f_q^{\bar{p}}(X_b, M_b^2) \Big] \frac{X_b \hat{s}}{m_{h^0}^2 - \hat{u}} \cdot \frac{d\sigma}{d\hat{t}}(q\bar{q}' \to h^0 W^+) \qquad (180)$$

where $$X_{a\,min} = \frac{\sqrt{s}\, m_T\, e^{Y} + m_{h^0}^2 - M_W^2}{s - \sqrt{s}\, m_T\, e^{-Y}} \qquad (181)$$

$$m_T = (M_W^2 + P_T^2)^{1/2} \qquad (182)$$

$$X_b = \frac{X_a \sqrt{s}\, m_T\, e^{-Y} + m_{h^0}^2 - M_W^2}{X_a s - \sqrt{s}\, m_T\, e^{Y}} \qquad (183)$$

$$P_T^2 = \frac{\lambda(\hat{s}, m_{h^0}^2, M_W^2) \sin^2\theta}{4\hat{s}} \qquad (184)$$

$$\hat{u}\hat{t} = m_{h^0}^2 M_W^2 + \hat{s} P_T^2 \qquad (185)$$

Y is the rapidity of W^+ and P_T is the transverse momentum of W^+.

$f_{\substack{q \\ \bar{q}'}}(x, M^2)$ are the parton density functions. M^2 is the factorization scale

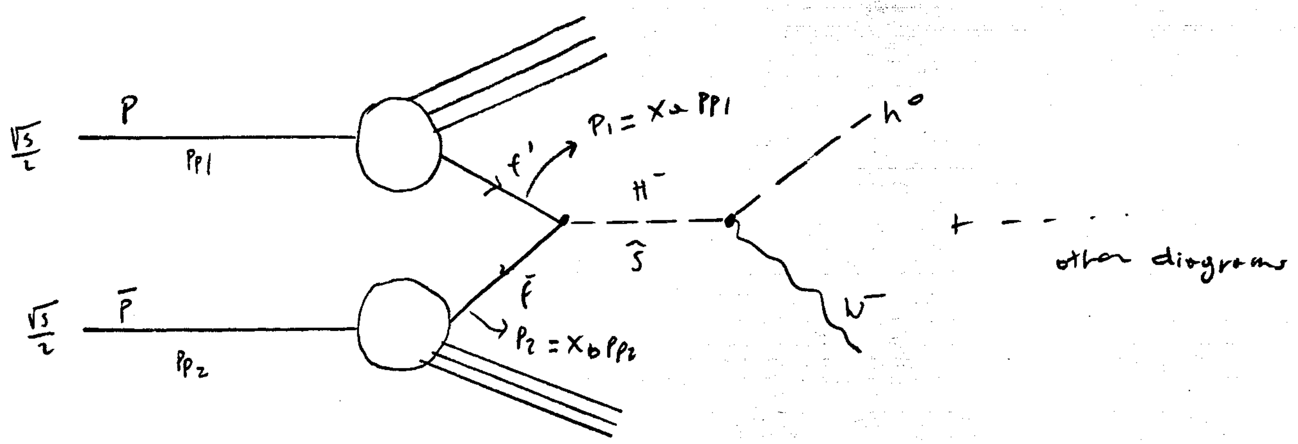
$M_a^2 = M_b^2 = \hat{s}$ (invariant mass of the pair $h^0 W^+$)

$$\hat{t} = \frac{m_{h^0}^2 + M_W^2 - \hat{s}}{2} + \frac{1}{2} \cos\theta \, \lambda^{1/2} (\hat{s}, m_{h^0}^2, M_W^2) \qquad (186)$$

$$\hat{u} = \frac{m_{h^0}^2 + M_W^2 - \hat{s}}{2} - \frac{1}{2} \cos\theta \, \lambda^{1/2} (\hat{s}, m_{h^0}^2, M_W^2) \qquad (187)$$

$$\cos\theta = \left(1 - \frac{4\hat{s}\, P_T^2}{\lambda(\hat{s}, m_{h^0}^2, M_W^2)} \right)^{1/2} \qquad (188)$$

$\sigma\,(p\bar{p} \to h^0 W^- X)$



$$\hat{s} = x_a x_b s$$

$$\frac{d^2\sigma}{dy\, dp_T^2}(p\bar{p} \to h^0 W^- X) = \sum_{q,q'} \int_{x_{a\,min}}^{1} dx_a \Big[f_{q'}^{p}(x_a, M_a^2)\, f_{\bar{q}}^{\bar{p}}(x_b, M_b^2) +$$

$$+ f_{\bar{q}}^{p}(x_a, M_a^2)\, f_{q'}^{\bar{p}}(x_b, M_b^2) \Big] \frac{x_b \hat{s}}{m_{h^0}^2 - \hat{u}} \; \frac{d\sigma}{d\hat{t}}(q'\bar{q} \to h^0 W^-) \qquad (189)$$

$$f_{q'}^{p} = f_{\bar{q}'}^{\bar{p}} \;;\; f_{\bar{q}}^{\bar{p}} = f_{q}^{p} \;;\; f_{\bar{q}}^{p} = f_{q}^{\bar{p}} \;;\; f_{q'}^{\bar{p}} = f_{\bar{q}'}^{p} \qquad (190)$$

where

$$x_{a\,min} = \frac{\sqrt{s}\, m_T\, e^{y} + m_{h^0}^2 - M_W^2}{s - \sqrt{s}\, m_T\, e^{-y}}$$

$$m_T = (M_W^2 + p_T^2)^{1/2}$$

$$x_b = \frac{x_a \sqrt{s}\, m_T\, e^{-y} + m_{h^0}^2 - M_W^2}{x_a s - \sqrt{s}\, m_T\, e^{y}}$$

$$p_T^2 = \frac{\lambda(\hat{s}, m_{h^0}^2, M_W^2)\, \sin^2\theta}{4\hat{s}}$$

$$\hat{u}\hat{t} = m_{h^0}^2 M_W^2 + \hat{s}\, p_T^2$$

$$\hat{t} = \frac{m_{h^0}^2 + M_W^2 - \hat{s}}{2} + \frac{1}{2}\cos\theta\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)$$

$$\hat{u} = \frac{m_{h^0}^2 + M_W^2 - \hat{s}}{2} - \frac{1}{2}\cos\theta\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_W^2)$$

$$\cos\theta = \left(1 - \frac{4\hat{s}\, p_T^2}{\lambda(\hat{s}, m_{h^0}^2, M_W^2)} \right)^{1/2}$$

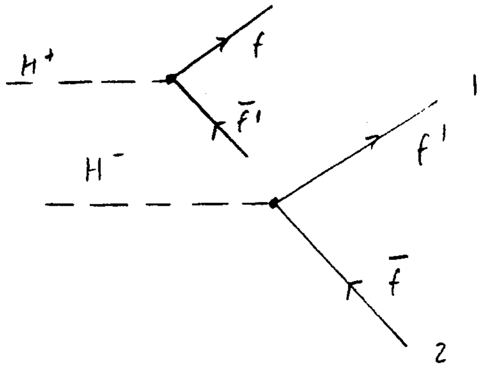
y is the rapidity of W^- and p_T is the transverse momentum of W^-

$f_{q}(x, \mu^2)$ are the parton density functions. μ^2 is the factorization scale
$\bar{q}'$

$\mu_a^2 = \mu_b^2 = \hat{s}$ (invariant mass of the pair $h^0 W^-$)

Production of H^+

$H^- \to f'\bar{f}$ ($H^+ \to f\bar{f}'$ is done).



$f = u, c, t, \nu_e, \nu_\mu, \nu_\tau$

$f' = d, s, b, e^-, \mu^-, \tau^-$

$$\frac{ig}{2\sqrt{2}M_W}\left[m_{f'} \tan\beta\,(1-\gamma^5) + m_f \cot\beta\,(1+\gamma^5)\right]$$

$$= \frac{ig}{2\sqrt{2}M_W}\left[A - B\gamma^5 \right]$$

$$A \equiv m_{f'} \tan\beta + m_f \cot\beta$$

$$B = m_{f'} \tan\beta - m_f \cot\beta$$

$$-iM = \bar{U}_1 \frac{ig}{2\sqrt{2}M_W}(A - B\gamma^5) V_2 \; V^*_{ff'}$$

$$\overline{|M|^2} = \frac{g^2}{8M_W^2}\sum_s [\bar{U}_1(A - B\gamma^5)V_2][\bar{U}_1(A-B\gamma^5)V_2]^+ \; |V_{ff'}|^2$$

$$[U_1^+\gamma^0(A-B\gamma^5)V_2]^+ = V_2^+(A-B\gamma^5)\gamma^0 U_1 = \bar{V}_2(A+B\gamma^5)U_1$$

$$\overline{|M|^2} = \frac{g^2}{8M_W^2}\sum_s \bar{U}_1(A-B\gamma^5)V_2 \, . \, \bar{V}_2(A+B\gamma^5)U_1 \; |V_{ff'}|^2$$

$$= \frac{g^2}{8M_W^2}\,\text{Tr}\left[(\not{p}_2 - m_f)(A+B\gamma^5)(\not{p}_1 + m_{f'})(A-B\gamma^5)\right] |V_{ff'}|^2$$

$$\text{Tr} = \text{Tr}\left[(A\not{p}_2 + B\not{p}_2\gamma^5 - m_f A - m_f B\gamma^5)(A\not{p}_1 - B\not{p}_1\gamma^5 + Am_{f'} - Bm_{f'}\gamma^5)\right]$$

$$= A^2\,\text{Tr}(\not{p}_2\not{p}_1) - AB\,\overset{0}{\cancel{\text{Tr}(\not{p}_2\not{p}_1\gamma^5)}} + A^2 m_{f'}\,\overset{0}{\cancel{\text{Tr}(\not{p}_2)}} - ABm_{f'}\,\overset{0}{\cancel{\text{Tr}(\not{p}_2\gamma^5)}}$$

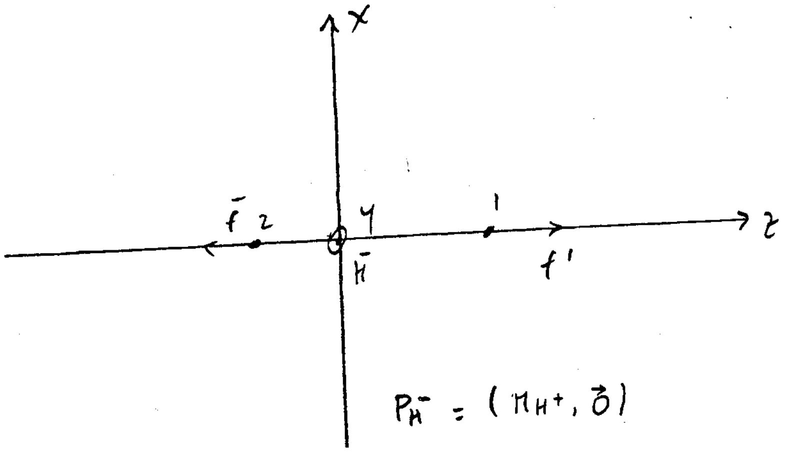
$$+ AB\,\overset{0}{\cancel{\text{Tr}(\not{p}_2\gamma^5\not{p}_1)}} - B^2\,\text{Tr}(\not{p}_2\gamma^5\not{p}_1\gamma^5) + Am_{f'}B\,\overset{0}{\cancel{\text{Tr}(\not{p}_2\gamma^5)}} - \overset{2}{B}m_{f'}\,\overset{0}{\cancel{\text{Tr}(\not{p}_2)}}$$

$$- m_f A^2\,\overset{0}{\cancel{\text{Tr}(\not{p}_1)}} + m_f AB\,\overset{0}{\cancel{\text{Tr}(\not{p}_1\gamma^5)}} - m_f m_{f'} A^2\,\text{Tr}(I) + ABm_f m_{f'}\,\overset{0}{\cancel{\text{Tr}(\gamma^5)}}$$

$$- m_f AB\,\overset{0}{\cancel{\text{Tr}(\gamma^5\not{p}_1)}} + m_f B^2\,\overset{0}{\cancel{\text{Tr}(\gamma^5\not{p}_1\gamma^5)}} - ABm_f m_{f'}\,\overset{0}{\cancel{\text{Tr}(\gamma^5)}} + B^2 m_f m_{f'}\,\text{Tr}(I)$$

$$\text{Tr} = 4A^2(P_1\cdot P_2) + 4B^2(P_1\cdot P_2) - 4m_f m_{f'} A^2 + 4m_f m_{f'} B^2$$

$$\Rightarrow \overline{|M|^2} = \frac{g^2}{2M_W^2}|V_{ff'}|^2\left[(A^2+B^2)(P_1\cdot P_2) - m_f m_{f'}(A^2 - B^2)\right]$$



$$P_{\bar{H}} = (M_{H^+}, \vec{0})$$

$$P_1 = P_{f'} = (E_1, \vec{P}_1) \; ; \; P_{\bar{f}} = (E_2, -\vec{P}_1) = P_2$$

$$P_{H^+} = P_{f'} + P_{\bar{f}} = P_1 + P_2$$

$$M_{H^+}^2 = m_f^2 + m_{f'}^2 + 2(P_1 \cdot P_2)$$

$$\boxed{(P_1 \cdot P_2) = \frac{M_{H^+}^2 - m_f^2 - m_{f'}^2}{2}}$$

$$P_1 = P_{H^+} - P_2$$

$$m_{f'}^2 = M_{H^+}^2 + m_f^2 - 2 M_{H^+} E_2 \Rightarrow E_2 = \frac{-m_{f'}^2 + M_{H^+}^2 + m_f^2}{2 M_{H^+}}$$

but $E_2^2 = m_f^2 + |\vec{P}_1|^2$

$$|\vec{P}_1|^2 = \frac{(m_f^2 + M_{H^+}^2 - m_{f'}^2)^2}{4 M_{H^+}^2} - m_f^2$$

$$|\vec{P}_1| = M_{H^+} \left[\frac{m_f^4 + M_{H^+}^4 + m_{f'}^4 + 2 m_f^2 M_{H^+}^2 - 2 m_f^2 m_{f'}^2 - 2 M_{H^+}^2 m_{f'}^2 - 4 m_f^2 M_{H^+}^2}{4 M_{H^+}^4} \right]^{1/2}$$

$$|\vec{P}_1| = \frac{M_{H^+}}{2} \left[1 + \left(\frac{m_f^2}{M_{H^+}^2}\right)^2 + \left(\frac{m_{f'}^2}{M_{H^+}^2}\right)^2 - 2\left(\frac{m_f^2}{M_{H^+}^2}\right) - 2\left(\frac{m_f^2}{M_{H^+}^2}\right)\left(\frac{m_{f'}^2}{M_{H^+}^2}\right) - 2\left(\frac{m_{f'}^2}{M_{H^+}^2}\right)\right]^{1/2}$$

$$|\vec{P}_1| = \frac{1}{2} M_{H^+} \lambda^{1/2}\left(\frac{m_f^2}{M_{H^+}^2}, \frac{m_{f'}^2}{M_{H^+}^2}, 1\right)$$

$$d\Gamma = \frac{\overline{|M|^2} \, |\vec{P}_1| \, d\Omega}{32 \pi^2 M_{H^+}^2}$$

$$\Gamma = \frac{\cancel{g}^2}{\cancel{2} \cancel{M_W}^2} |V_{ff'}|^2 \left[\frac{(A^2+B^2)}{2} (M_{H^+}^2 - m_f^2 - m_{f'}^2) - m_f m_{f'} (A^2-B^2) \right].$$

$$\cdot \frac{1}{\cancel{2}} \cancel{M_{H^+}} \lambda^{1/2} \cdot \frac{1}{8\pi \cancel{M_{H^+}}^2} \cdot N_C$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

$$\Gamma = \frac{G_F}{\sqrt{2}} \frac{|V_{ff'}|^2}{4\pi M_{H^+}} \frac{N_C \lambda^{1/2}}{2} \left\{ A^2 [M_{H^+}^2 - (m_f + m_{f'})^2] + B^2 [M_{H^+}^2 - (m_f - m_{f'})^2] \right\}$$

$\Rightarrow$

$$\boxed{\Gamma(H^+ \to f\bar{f}') = \Gamma(H^- \to f'\bar{f}) = \frac{\sqrt{2}\, G_F |V_{ff'}|^2 N_C}{16\pi M_{H^\pm}} \cdot \lambda^{1/2}\left(\frac{m_f^2}{M_{H^\pm}^2}, \frac{m_{f'}^2}{M_{H^\pm}^2}, 1\right) \cdot \left\{ A^2 [M_{H^+}^2 - (m_f + m_{f'})^2] + B^2 [M_{H^+}^2 - (m_f - m_{f'})^2] \right\}}$$

$$N_C = \begin{cases} 3 & \text{for quarks} \\ 1 & \text{for leptons} \end{cases}$$

$A \equiv m_{f'} \tan\beta + m_f \cot\beta$

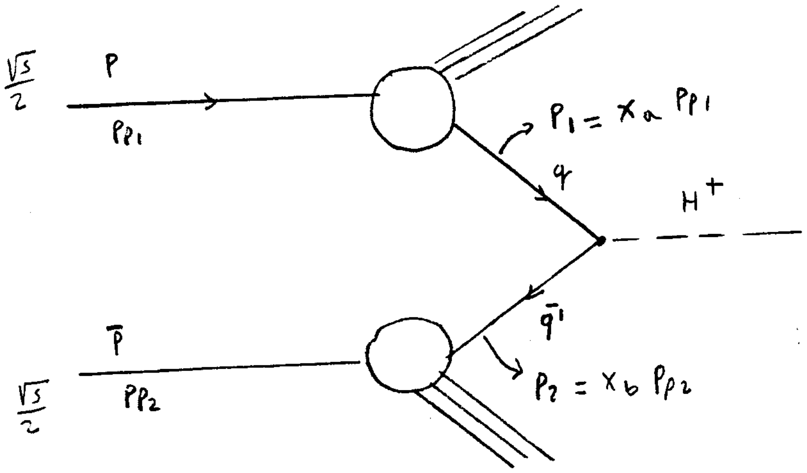
$B \equiv m_{f'} \tan\beta - m_f \cot\beta$

$V_{ff'}$ is an element of the CKM matrix.

$H^+ \to u\bar{d}, u\bar{s}, u\bar{b}, c\bar{d}, c\bar{s}, c\bar{b}, t\bar{d}, t\bar{s}, t\bar{b}, e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau$

$H^- \to d\bar{u}, s\bar{u}, b\bar{u}, d\bar{c}, s\bar{c}, b\bar{c}, d\bar{t}, s\bar{t}, b\bar{t}, e^-\bar{\nu}_e, \mu^-\bar{\nu}_\mu, \tau^-\bar{\nu}_\tau$

$\sigma(p\bar{p} \to H^{\pm} X)$



$(m_q \approx m_q' \approx 0)$

$\hat{s} = x_a x_b s$

$$\sigma(p\bar{p} \to H^+X) = \frac{4\pi^2}{\underset{\overset{\downarrow}{C_f}}{3} M_{H^\pm}^3} \left\{ \sum \Gamma(H^+ \to q\bar{q}') \left\{ \int_{\tau_A}^{1} \frac{dx_a}{x_a} \left[f_q^p(x_a, M^2) \cdot f_{\bar{q}'}^{\bar{p}}\left(\frac{\tau_H}{x_a}, M^2\right) + f_{\bar{q}'}^{p}(x_a, M^2) \cdot f_q^{\bar{p}}\left(\frac{\tau_H}{x_a}, M^2\right) \right] \right\} \right\} \tau_H$$

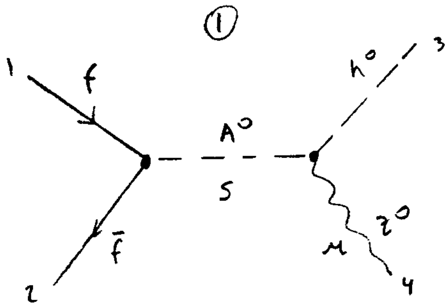
$$\tau_H = \frac{M_{H^+}^2}{s}$$

M^2 = factorization scale $(M = m_{H^+})$

Production of $h^0 Z^0 X$

$f\bar{f} \to h^0 Z^0$:

(Unitary gauge)



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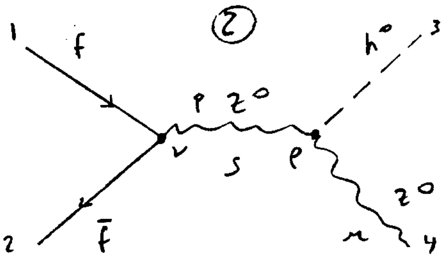

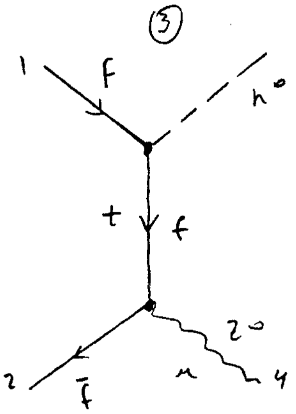

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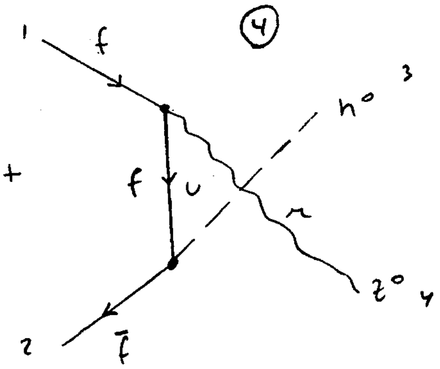
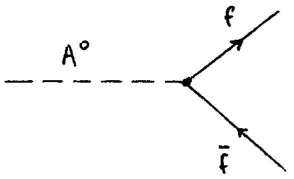
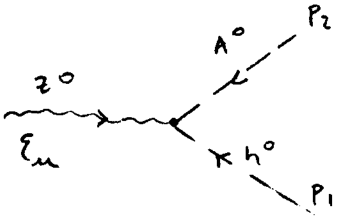


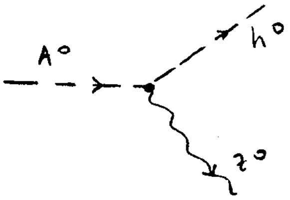


$$-\frac{g m_f}{2M_W} A_f \gamma^5 \qquad A_f = \begin{cases} \cot\beta & f = u, c \\ \tan\beta & f = d, s, b, e^-, \mu^-, \tau^- \end{cases}$$



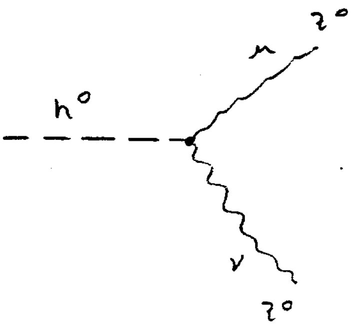
$$\frac{g}{2\cos\theta_W} (P_1 - P_2)^\mu \cos(\beta-\alpha)$$

(

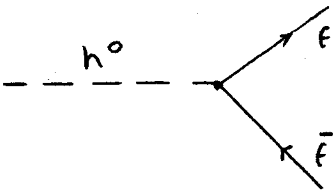


$$\frac{g}{2\cos\theta_W} (-P_{h^0} - P_{A^0})^\mu \cos(\beta-\alpha)$$

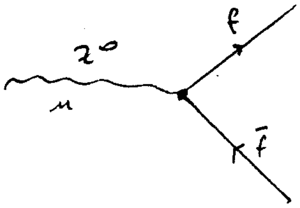
)



$$\frac{i g M_Z}{\cancel{2}\cos\theta_W} \sin(\beta-\alpha)\eta^{\mu\nu} \cancel{.2!}$$ (identical particles in the final state)



$$\frac{i g m_f}{2 M_W} C_f \quad ; \quad C_f = \begin{cases} \dfrac{\sin\alpha}{\cos\beta} & f = d, s, b, e^-, \mu^-, \tau^- \\ -\dfrac{\cos\alpha}{\sin\beta} & f = u, c \end{cases}$$

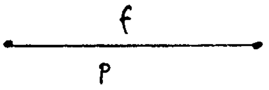


$$-\frac{ig}{\cos\theta_W}\gamma^\mu \frac{1}{2}\left(C_V^f - C_A^f \gamma^5\right)$$

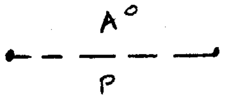
$$C_A^f = T_f^3$$

$$C_V^f = T_f^3 - 2\sin^2\theta_W Q_f$$

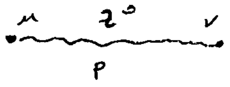
	T_f^3	Q_f
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	0
e^-, μ^-, τ^-	$-\frac{1}{2}$	-1
u c t	$\frac{1}{2}$	$+\frac{2}{3}$
d s b	$-\frac{1}{2}$	$-\frac{1}{3}$



$$\frac{i}{\not p - m_f} = \frac{i(\not p_f + m_f)}{p_f^2 - m_f^2}$$



$$\frac{i}{p^2 - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0}}$$



$$-i\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_Z^2}\right) \cdot \frac{1}{p^2 - M_Z^2 + i M_Z \Gamma_Z}$$

$$-iM_1 = -\varepsilon^*_{4\mu} \frac{g}{2\cos\theta_W} (P_{A^0} + P_3)^\mu \cos(\beta-\alpha) \frac{i}{s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0}} \bar{V}_2 \left(\frac{-g m_f}{2 M_W} A_f \gamma^5\right) U_1$$

$$\boxed{-iM_1 = \frac{i g^2 m_f}{4 M_W \cos\theta_W} \cos(\beta-\alpha) A_f \frac{(P_{A^0} + P_3)^\mu}{(s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0})} \varepsilon^*_{4\mu} (\bar{V}_2 \gamma^5 U_1)} \qquad (1)$$

$$-iM_2 = \varepsilon^*_{4\mu} \left(\frac{i g M_Z}{\cos\theta_W} \sin(\beta-\alpha) \eta^{\mu\rho}\right) (-i) \left(\eta_{\nu\rho} - \frac{P_\nu P_\rho}{M_Z^2}\right) \frac{1}{s - M_Z^2 + i M_Z \Gamma_Z} \bar{V}_2 \left(-\frac{ig}{\cos\theta_W}\right) \gamma^\nu \cdot$$

$$\cdot \frac{1}{2} (C_V^f - C_A^f \gamma^5) U_1 \qquad (2)$$

$$-iM_2 = -\frac{i g^2 M_Z}{2\cos^2\theta_W} \sin(\beta-\alpha) \cdot \frac{1}{(s - M_Z^2 + i M_Z \Gamma_Z)} \varepsilon^*_{4\mu} \left(\delta^\mu_\nu - \frac{(P_1+P_2)_\nu (P_1+P_2)^\mu}{M_Z^2}\right)$$

$$\bar{V}_2 \gamma^\nu (C_V^f - C_A^f \gamma^5) U_1 \qquad (3)$$

$$(\not{P}_1 - m_f) U_1 = 0 \quad \Rightarrow \quad \not{P}_1 U_1 = m_f U_1$$

$$\bar{V}_2 (\not{P}_2 + m_f) = 0 \quad \Rightarrow \quad \bar{V}_2 \not{P}_2 = -m_f \bar{V}_2$$

$$(P_1 + P_2)_\nu \bar{V}_2 \gamma^\nu (C_V^f - C_A^f \gamma^5) U_1 = \bar{V}_2 \not{P}_1 (C_V^f - C_A^f \gamma^5) U_1 + \bar{V}_2 \not{P}_2 (C_V^f - C_A^f \gamma^5) U_1$$

$$= \bar{V}_2 (C_V^f + C_A^f \gamma^5) m_f U_1 - m_f \bar{V}_2 (C_V^f - C_A^f \gamma^5) U_1$$

$$= m_f C_V^f \cancel{\bar{V}_2 U_1} + m_f C_A^f \bar{V}_2 \gamma^5 U_1 - m_f C_V^f \cancel{\bar{V}_2 U_1} + m_f C_A^f \bar{V}_2 \gamma^5 U_1$$

$$= 2 m_f C_A^f \bar{V}_2 \gamma^5 U_1 \qquad (4)$$

$$-iM_2 = -\frac{i g^2 M_Z}{2\cos^2\theta_W} \sin(\beta-\alpha) \frac{1}{(s - M_Z^2 + i M_Z \Gamma_Z)} \varepsilon^*_{4\mu} \left\{ \bar{V}_2 \gamma^\mu (C_V^f - C_A^f \gamma^5) U_1 \right.$$

$$\left. - \frac{2 m_f C_A^f}{M_Z^2} (P_1 + P_2)^\mu \bar{V}_2 \gamma^5 U_1 \right\} \qquad (5)$$

defining $C_Z \equiv \dfrac{\sin(\beta-\alpha)}{(s - M_Z^2 + i M_Z \Gamma_Z)}$ (6)

$$\boxed{-iM_2 = -i(M_{2a} + M_{2b})} \qquad (7)$$

$$\boxed{\begin{aligned} -iM_{2a} &= -i\frac{g^2 M_Z C_Z}{2\cos^2\theta_W}\,\varepsilon^*_{4\mu}\,\bar{V}_2\,\gamma^\mu\,(C_V^f - C_A^f\gamma^5)\,U_1 && (8)\\ -iM_{2b} &= \frac{i\,g^2 C_Z\, m_f\, C_A^f}{\cos^2\theta_W\, M_Z}\,(P_1+P_2)^\mu\,\varepsilon^*_{4\mu}\,\bar{V}_2\,\gamma^5 U_1 && (9)\end{aligned}}$$

$$-iM_3 = \bar{V}_2\left(-\frac{ig}{\cos\theta_W}\gamma^\mu\frac{1}{2}(C_V^f - C_A^f\gamma^5)\right)\varepsilon^*_{4\mu}\; i\,\frac{(\not{p}_1 - \not{p}_3 + m_f)}{t-m_f^2}\left(\frac{igm_f}{2M_W}C_f\right)U_1$$

$$\boxed{-iM_3 = \frac{i g^2 m_f C_f}{4M_W\cos\theta_W}\,\frac{1}{(t-m_f^2)}\,\varepsilon^*_{4\mu}\,\bar{V}_2\,\gamma^\mu(C_V^f - C_A^f\gamma^5)(\not{p}_1 - \not{p}_3 + m_f)\,U_1} \qquad (10)$$

$$-iM_4 = \bar{V}_2\left(\frac{igm_f}{2M_W}C_f\right)\frac{i(\not{p}_1 - \not{p}_4 + m_f)}{U-m_f^2}\left(\frac{-ig}{\cos\theta_W}\gamma^\mu\frac{1}{2}(C_V^f - C_A^f\gamma^5)\right)U_1\,\varepsilon^*_{4\mu}$$

$$\boxed{-iM_4 = \frac{i g^2 m_f C_f}{4M_W\cos\theta_W}\,\frac{1}{(U-m_f^2)}\,\varepsilon^*_{4\mu}\,\bar{V}_2\,(\not{p}_1 - \not{p}_4 + m_f)\gamma^\mu(C_V^f - C_A^f\gamma^5)\,U_1} \qquad (11)$$

M_1, M_3, M_4 are proportional to m_f (also M_{2b})

By analogy with $\mu^-\mu^+ \to h^0 Z^0$. the only important term is:

M_{2a} for $f = e^-, \mu^-, d, s, U$.

$\therefore\quad M \approx M_{2a}$

$$\overline{|M|^2} = \frac{g^4 |C_Z|^2\,[\,|C_A^f|^2 + |C_V^f|^2\,]}{32\cos^4\theta_W\, M_Z^4}\,[\,8 s M_Z^2 + \lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta\,]\,M_Z^4$$

$$\overline{|M|^2} = \frac{g^4 M_Z^4 |C_Z|^2\,[\,|C_A^f|^2 + |C_V^f|^2\,]}{32\, M_W^4}\,[\,8 s M_Z^2 + \lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta\,] \qquad (12)$$

$$\Rightarrow \left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{64\pi^2 s^2}\,G_F^2 M_Z^4 |C_Z|^2\,\lambda^{1/2}(s, m_{h^0}^2, M_Z^2)\,[\,|C_A^f|^2 + |C_V^f|^2\,]\,[\,8 s M_Z^2 + \lambda\sin^2\theta\,] \qquad (13)$$

if $f = e^-, \mu^-, d, s, U$.

if $f = c, b, \tau^-$

$$M = M_1 + M_2 + M_3 + M_4 = M_1 + M_{2a} + M_{2b} + M_3 + M_4 . \qquad (14)$$

Defining :
$$C_A^1 = \frac{\cos(\beta - \alpha)}{(s - m_{A^0}^2 + i m_{A^0} \Gamma_{A^0})} \qquad (15)$$

$$\Rightarrow \boxed{M_1 = -\frac{g^2 m_f C_A^1}{4 M_W \cos\theta_W} A_f (P_1 + P_2 + P_3)^\mu \, \varepsilon_{4\mu}^* \, (\bar{V}_2 \gamma^5 U_1)} \qquad (16)$$

$\Rightarrow$

$$\boxed{\begin{aligned} M_{2a} &= \frac{g^2 M_Z C_Z}{2\cos^2\theta_W} \varepsilon_{4\mu}^* \bar{V}_2 \gamma^\mu (C_V^f - C_A^f \gamma^5) U_1 & (17) \\ M_{2b} &= -\frac{g^2 m_f C_Z C_A^f}{M_W \cos\theta_W} (P_1 + P_2)^\mu \varepsilon_{4\mu}^* (\bar{V}_2 \gamma^5 U_1) & (18) \end{aligned}}$$

Defining
$$C_{U_f} = \frac{C_f}{U - m_f^2} \; ; \quad C_{t_f} = \frac{C_f}{t - m_f^2} \qquad (19)$$

$(2m_f - \not{p}_3)U_1$

$$\Rightarrow \boxed{M_3 = -\frac{g^2 m_f C_{t_f}}{4 M_W \cos\theta_W} \varepsilon_{4\mu}^* \bar{V}_2 \gamma^\mu (C_V^f - C_A^f \gamma^5)(\not{p}_1 - \not{p}_3 + m_f) U_1} \qquad (20)$$

$\bar{V}_2(\not{p}_3 - \not{p}_2 + m_f)\gamma^\mu = \bar{V}_2 (2m_f + \not{p}_3)\gamma^\mu$

$$\Rightarrow \boxed{M_4 = \frac{-g^2 m_f C_{U_f}}{4 M_W \cos\theta_W} \varepsilon_{4\mu}^* \bar{V}_2 (\not{p}_1 - \not{p}_4 + m_f)\gamma^\mu (C_V^f - C_A^f \gamma^5) U_1} \qquad (21)$$

$$\boxed{M_1 + M_{2b} \equiv M_{12b} = -\frac{g^2 m_f}{M_W \cos\theta_W} \left\{ \frac{A_f C_A^1}{4} (P_1 + P_2 + P_3)^\mu + C_Z C_A^f (P_1 + P_2)^\mu \right\} \varepsilon_{4\mu}^* (\bar{V}_2 \gamma^5 U_1)} \qquad (22)$$

$$\therefore \quad M = M_{12b} + M_{2a} + M_3 + M_4$$

$$|M|^2 = (M_{12b} + M_{2a} + M_3 + M_4)(M_{12b}^* + M_{2a}^* + M_3^* + M_4^*)$$

$$\begin{aligned} |M|^2 = {} & |M_{12b}|^2 + |M_{2a}|^2 + |M_3|^2 + |M_4|^2 + M_{12b} M_{2a}^* + M_{12b} M_3^* + M_{12b} M_4^* \\ & + M_{2a} M_{12b}^* + M_{2a} M_3^* + M_{2a} M_4^* + M_3 M_{12b}^* + M_3 M_{2a}^* + M_3 M_4^* + M_4 M_{12b}^* \\ & + M_4 M_{2a}^* + M_4 M_3^* . \qquad (23) \end{aligned}$$

$$(\bar{V}_2 \gamma^5 U_1)^\dagger = U_1^\dagger \gamma^5 \gamma^0 V_2 = -U_1^\dagger \gamma^0 \gamma^5 V_2 = -\bar{U}_1 \gamma^5 V_2 \qquad (24)$$

$$|M_{12b}|^2 = \frac{-g^4 m_f^2}{M_W^2 \cos^2\theta_W} \left\{ \frac{A_f C_A'^{*}}{4} (p_1 + p_2 + p_3)^\mu + C_Z^* C_A^f (p_1 + p_2)^\mu \right\} \left\{ \frac{A_f C_A'}{4} (p_1 + p_2 + p_3)^\nu + \right.$$

$$\left. + C_Z C_A^f (p_1 + p_2)^\nu \right\} \left(\sum_\lambda \varepsilon_{4\mu} \varepsilon_{4\nu}^* \right) \sum_s (\bar{U}_1 \gamma^5 V_2)(\bar{V}_2 \gamma^5 U_1)$$

$$= \frac{-g^4 m_f^2}{M_W^2 \cos^2\theta_W} \left\{ \frac{A_f C_A'^{*}}{4} (p_1 + p_2 + p_3)^\mu + C_Z^* C_A^f (p_1 + p_2)^\mu \right\} \left\{ \frac{A_f C_A'}{4} (p_1 + p_2 + p_3)^\nu + \right.$$

$$\left. + C_Z C_A^f (p_1 + p_2)^\nu \right\} \left(-\eta_{\mu\nu} + \frac{p_{4\mu} p_{4\nu}}{M_Z^2} \right) Tr\left((\not p_1 + m_f) \gamma^5 (\not p_2 - m_f) \gamma^5 \right) \quad (25)$$

$$Tr\left((\not p_1 + m_f) \gamma^5 (\not p_2 - m_f) \gamma^5 \right) = -Tr\left((\not p_1 + m_f)(\not p_2 + m_f) \right)$$

$$= -Tr(\not p_1 \not p_2) - 4 m_f^2 = -4 (p_1 \cdot p_2) - 4 m_f^2 \quad (26)$$

$$\Rightarrow |M_{12b}|^2 = \frac{4 g^4 m_f^2}{M_W^2 \cos^2\theta_W} \left\{ \frac{A_f C_A'^{*}}{4} (p_1 + p_2 + p_3)^\mu + C_Z^* C_A^f (p_1 + p_2)^\mu \right\} \left\{ -\frac{A_f C_A'}{4} (p_1 + p_2 + p_3)_\mu \right.$$

$$\left. - C_Z C_A^f (p_1 + p_2)_\mu + \frac{A_f C_A'}{4} ((p_1 + p_2 + p_3) \cdot p_4) \frac{p_{4\mu}}{M_Z^2} + C_Z C_A^f ((p_1 + p_2) \cdot p_4) \frac{p_{4\mu}}{M_Z^2} \right\}$$

$$[(p_1 \cdot p_2) + m_f^2]$$

$$= \frac{4 g^4 m_f^2}{M_W^2 \cos^2\theta_W} \left\{ - \frac{A_f^2}{16} |C_A'|^2 \cancel{(p_1 + p_2 + p_3)^2} - \frac{A_f C_A'^{*} C_Z C_A^f}{4} \cancel{((p_1 + p_2 + p_3) \cdot (p_1 + p_2))} \right.$$

$$+ \frac{A_f^2 |C_A'|^2}{16} \frac{((p_1 + p_2 + p_3) \cdot p_4)^2}{M_Z^2} + \frac{A_f C_A'^{*} C_Z C_A^f}{4 M_Z^2} \cancel{((p_1 + p_2) \cdot p_4)} ((p_1 + p_2 + p_3) \cdot p_4)$$

$$- \frac{C_Z^* C_A^f A_f C_A'}{4} ((p_1 + \cancel{p_2}) \cdot (p_1 + p_2 + p_3)) - |C_Z|^2 C_A^{f\,2} \cancel{(p_1 + p_2)^2} + \frac{C_Z^* C_A^f \cancel{A_f} C_A'}{4 M_Z^2} \cdot$$

$$\left. \cdot ((p_1 + p_2) \cdot p_4) ((p_1 + p_2 \cancel{+} p_3) \cdot p_4) + |C_Z|^2 C_A^{f\,2} \frac{((p_1 + p_2) \cdot p_4)^2}{M_Z^2} \right\} [p_1 \cdot p_2 + m_f^2]$$

$$S = (p_1 + p_2)^2 = 2 m_f^2 + 2 (p_1 \cdot p_2)$$

$$\Rightarrow \boxed{(p_1 \cdot p_2) = \frac{S}{2} - m_f^2} \quad (27) \qquad \boxed{(p_2 \cdot p_3) = \frac{m_f^2 + m_{h^0}^2 - U}{2}} \quad (27a)$$

$$(p_1 + p_2) \cdot p_3 = (p_3 + p_4) \cdot p_3 = m_{h^0}^2 + (p_3 \cdot p_4)$$

$$S = (p_3 + p_4)^2 = m_{h^0}^2 + M_Z^2 + 2 (p_3 \cdot p_4)$$

$$\Rightarrow \boxed{(p_3 \cdot p_4) = \frac{S - m_{h^0}^2 - M_Z^2}{2}} \quad (28)$$

$$\boxed{(p_1+p_2)\cdot p_3 = m_{h^0}^2 + \frac{s-m_{h^0}^2-M_Z^2}{2} = \frac{s+m_{h^0}^2-M_Z^2}{2}} \qquad (29)$$

$$(p_1+p_2)\cdot p_4 = (p_3+p_4)\cdot p_4 = \frac{s-m_{h^0}^2-M_Z^2}{2} + M_Z^2$$

$$\boxed{(p_1+p_2)\cdot p_4 = \frac{s-m_{h^0}^2+M_Z^2}{2}} \qquad (30)$$

$$(p_1+p_2+p_3)\cdot p_4 = (2p_3+p_4)\cdot p_4 = s-m_{h^0}^2-\cancel{M_Z^2}+\cancel{M_Z}^2$$

$$\boxed{(p_1+p_2+p_3)\cdot p_4 = s-m_{h^0}^2} \qquad (31)$$

$$(p_1+p_2+p_3)^2 = (2p_3+p_4)^2 = 4\cancel{m_{h^0}^2}+M_Z^2+2\,(s-\cancel{m_{h^0}^2}-M_Z^2)$$

$$\boxed{(p_1+p_2+p_3)^2 = 2m_{h^0}^2+2s-M_Z^2} \qquad (32)$$

$$(p_1+p_2+p_3)\cdot(p_1+p_2) = s + p_3\cdot(p_1+p_2) = s + \frac{s+m_{h^0}^2-M_Z^2}{2}$$

$$\boxed{(p_1+p_2+p_3)\cdot(p_1+p_2) = \frac{3s+m_{h^0}^2-M_Z^2}{2}} \qquad (33)$$

$$|M_{12b}|^2 = \frac{2s\,g^4 m_f^2}{M_W^2\cos^2\theta_W}\left\{\frac{A_f^2}{16}|C_A'|^2\left[-2m_{h^0}^2-2s+M_Z^2+\frac{(s-m_{h^0}^2)^2}{M_Z^2}\right]\right.$$

$$+\frac{A_f\,C_A'^{*}\,C_Z\,C_A^f}{4}\left[\frac{-3s-m_{h^0}^2+M_Z^2}{2}+\frac{(s-m_{h^0}^2+M_Z^2)}{2M_Z^2}(s-m_{h^0}^2)\right]$$

$$+\frac{A_f\,C_A'\,C_Z^{*}\,C_A^f}{4}\left[\frac{-3s-m_{h^0}^2+M_Z^2}{2}+\frac{(s-m_{h^0}^2+M_Z^2)}{2M_Z^2}(s-m_{h^0}^2)\right]$$

$$\left.+|C_Z|^2(C_A^f)^2\left[-s+\frac{(s-m_{h^0}^2+M_Z^2)^2}{4M_Z^2}\right]\right\}$$

$$|M_{12b}|^2 = \frac{2s\,g^4 m_f^2}{M_W^2\cos^2\theta_W}\left\{\frac{A_f^2}{16}|C_A'|^2\,\frac{\lambda(s,\cancel{m}_{h^0}^2,M_Z^2)}{M_Z^2}+\frac{1}{4M_Z^2}A_f\,C_A^f\,\mathrm{Re}(C_A'^{*}C_Z)\,\lambda(s,m_{h^0}^2,M_Z^2)\right.$$

$$\left.+\frac{|C_Z|^2(C_A^f)^2}{4M_Z^2}\lambda(s,m_{h^0}^2,M_Z^2)\right\}$$

$$\boxed{|M_{12W}|^2 = \frac{5 g^4 m_f^2}{2 M_W^4}\, \lambda(s, m_{h^0}^2, M_Z^2) \left[\frac{A_f^2}{4} |C_A'|^2 + A_f (C_A^f)\, \mathrm{Re}(C_A'^{*} C_Z) + |C_Z|^2 (C_A^f)^2 \right]}$$

(34)

$$|M_{Za}|^2 = \frac{g^4 M_Z^2 |C_Z|^2}{4 \cos^4\theta_W} \left(\sum_\lambda \varepsilon_{4\mu} \varepsilon^*_{4\nu} \right) \sum_s \left(\bar{V}_2 \gamma^\mu (C_V^f - C_A^f \gamma^5) U_1 \right)^\dagger \left(\bar{V}_2 \gamma^\nu (C_V^f - C_A^f \gamma^5) U_1 \right) \quad (35)$$

$$\left(\underset{\overset{\wedge}{V_2^\dagger \gamma^0}}{\bar{V}_2} \gamma^\mu (C_V^f - C_A^f \gamma^5) U_1 \right)^\dagger = U_1^\dagger (C_V^f - C_A^f \gamma^5)\, \overset{\gamma^0 \gamma^\mu}{\overset{\frown}{\gamma^{\mu\,\dagger}}}\, \gamma^0 V_2$$

$$= U_1^\dagger \gamma^0 (C_V^f + C_A^f \gamma^5) \gamma^\mu V_2 = \bar{U}_1 (C_V^f + C_A^f \gamma^5) \gamma^\mu V_2 \quad (36)$$

$$\Rightarrow |M_{Za}|^2 = \frac{g^4 M_Z^2 |C_Z|^2}{4 \cos^4\theta_W} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_Z^2} \right) \sum_s \left(\bar{U}_1 (C_V^f + C_A^f \gamma^5) \gamma^\mu V_2 \right) \left(\bar{V}_2 \gamma^\nu (C_V^f - C_A^f \gamma^5) U_1 \right.$$

$$\overbrace{C_V^f \gamma^\mu + C_A^f \gamma^5 \gamma^\mu}$$

$$= \frac{g^4 M_Z^2 |C_Z|^2}{4 \cos^4\theta_W} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_Z^2} \right) \mathrm{Tr}\left[(\not{p}_1 + m_f)(C_V^f + C_A^f \gamma^5) \gamma^\mu (\not{p}_2 - m_f) \gamma^\nu \right.$$

$$\left. (C_V^f - C_A^f \gamma^5) \right] \quad (37)$$

$$\mathrm{Tr} = \mathrm{Tr}\left[(C_V^f \not{p}_1 \gamma^\mu + C_A^f \not{p}_1 \gamma^5 \gamma^\mu + m_f C_V^f \gamma^\mu + m_f C_A^f \gamma^5 \gamma^\mu)(C_V^f \not{p}_2 \gamma^\nu - m_f C_V^f \gamma^\nu \right.$$

$$\left. - C_A^f \not{p}_2 \gamma^\nu \gamma^5 + m_f C_A^f \gamma^\nu \gamma^5) \right]$$

$$= (C_V^f)^2 \mathrm{Tr}(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) - m_f (C_V^f)^2 \cancelto{0}{\mathrm{Tr}(\not{p}_1 \gamma^\mu \gamma^\nu)} - C_A^f C_V^f \mathrm{Tr}(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5)$$

$$+ m_f C_A^f C_V^f \cancelto{0}{\mathrm{Tr}(\not{p}_1 \gamma^\mu \gamma^\nu \gamma^5)} + C_A^f C_V^f \mathrm{Tr}(\not{p}_1 \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu) - m_f C_A^f C_V^f \cancelto{0}{\mathrm{Tr}(\not{p}_1 \gamma^5 \gamma^\mu \gamma^\nu)}$$

$$- (C_A^f)^2 \mathrm{Tr}(\not{p}_1 \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5) + m_f (C_A^f)^2 \cancelto{0}{\mathrm{Tr}(\not{p}_1 \gamma^5 \gamma^\mu \gamma^\nu \gamma^5)} + m_f (C_V^f)^2 \cancelto{0}{\mathrm{Tr}(\gamma^\mu \not{p}_2 \gamma^\nu)}$$

$$- m_f^2 (C_V^f)^2 \mathrm{Tr}(\gamma^\mu \gamma^\nu) - m_f C_V^f C_A^f \cancelto{0}{\mathrm{Tr}(\gamma^\mu \not{p}_2 \gamma^\nu \gamma^5)} + m_f^2 C_A^f C_V^f \cancelto{0}{\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^5)}$$

$$+ m_f C_A^f C_V^f \cancelto{0}{\mathrm{Tr}(\gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu)} - m_f^2 C_A^f C_V^f \cancelto{0}{\mathrm{Tr}(\gamma^5 \gamma^\mu \gamma^\nu)} - m_f (C_A^f)^2 \cancelto{0}{\mathrm{Tr}(\gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5)}$$

$$+ m_f^2 (C_A^f)^2 \mathrm{Tr}(\gamma^5 \gamma^\mu \gamma^\nu \gamma^5)$$

$$= (C_V^f)^2 \mathrm{Tr}(\gamma^\nu \not{p}_1 \gamma^\mu \not{p}_2) - C_A^f C_V^f \mathrm{Tr}(\gamma^5 \not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) - C_A^f C_V^f \mathrm{Tr}(\gamma^5 \not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$+ (C_A^f)^2 \mathrm{Tr}(\gamma^\nu \not{p}_1 \gamma^\mu \not{p}_2) - m_f^2 (C_V^f)^2 \mathrm{Tr}(\gamma^\mu \gamma^\nu) + m_f^2 (C_A^f)^2 \mathrm{Tr}(\gamma^\mu \gamma^\nu)$$

$$= \left[(C_V^f)^2 + (C_A^f)^2 \right] 4 \left(P_1^\nu P_2^\mu + P_1^\mu P_2^\nu - (P_1 \cdot P_2) \eta^{\mu\nu} \right] + 8 i C_A^f C_V^f \varepsilon^{\alpha\mu\beta\nu} P_{1\alpha} P_{2\beta}$$

$$- 4 m_f^2 (C_V^f)^2 \eta^{\mu\nu} + 4 m_f^2 (C_A^f)^2 \eta^{\mu\nu}$$

$$\Rightarrow \mathrm{Tr} = 4 \{ \left[(C_V^f)^2 + (C_A^f)^2 \right] (P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - (P_1 \cdot P_2) \eta^{\mu\nu}) + 2 i C_A^f C_V^f \varepsilon^{\alpha\mu\beta\nu} P_{1\alpha} P_{2\beta}$$

$$+ m_f^2 \eta^{\mu\nu} \left[(C_A^f)^2 - (C_V^f)^2 \right] \quad (38)$$

$$\eta_{\mu\nu}\,\epsilon^{\alpha\mu\rho\nu} = 0$$

and $$P_{4\mu}P_{4\nu}\,\epsilon^{\alpha\mu\rho\nu} = 0$$

Then :

$$|M_{Za}|^2 = \frac{\cancel{4}g^4 M_Z^2\,|C_Z|^2}{\cancel{4}\cos^4\theta_W}\left(-\eta_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\Big\{\left[|C_V^f|^2 + |C_A^f|^2\right]\left(P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - (P_1\cdot P_2)\eta^{\mu\nu}\right)$$
$$+ m_f^2\eta^{\mu\nu}\left[|C_A^f|^2 - |C_V^f|^2\right]\Big\} \qquad (39)$$

$$|M_{Za}|^2 = \frac{g^4 M_Z^2\,|C_Z|^2}{\cos^4\theta_W}\Big\{-\left[|C_V^f|^2 + |C_A^f|^2\right]\left(2(P_1\cdot P_2) - 4(P_1\cdot P_2)\right) - 4m_f^2\left[|C_A^f|^2 - |C_V^f|^2\right]$$
$$+ \frac{\left[|C_V^f|^2 + |C_A^f|^2\right]}{M_Z^2}\left[2(P_1\cdot P_4)(P_2\cdot P_4) - (P_1\cdot P_2)M_Z^2\right] + m_f^2\left[|C_A^f|^2 - |C_V^f|^2\right]\Big\}$$

$$|M_{Za}|^2 = \frac{g^4 M_Z^2\,|C_Z|^2}{\cos^4\theta_W}\Big\{(P_1\cdot P_2)\left[|C_V^f|^2 + |C_A^f|^2\right] - 3m_f^2\left[|C_A^f|^2 - |C_V^f|^2\right]$$
$$+ \frac{2(P_1\cdot P_4)(P_2\cdot P_4)}{M_Z^2}\left[|C_V^f|^2 + |C_A^f|^2\right]\Big\} \qquad (40)$$

$$s = (P_1 + P_2)^2 = 2m_f^2 + 2(P_1\cdot P_2)$$

$$\Rightarrow \quad (P_1\cdot P_2) = \frac{s}{2} - m_f^2$$

$$(P_1\cdot P_4) = P_1\cdot(P_1 + P_2 - P_3) = \cancel{m_f^2} + \frac{s}{2} - \cancel{m_f^2} - (P_1\cdot P_3)$$

$$(P_1\cdot P_4) = \frac{s}{2} - (P_1\cdot P_3)$$

$$t = (P_1 - P_3)^2 = m_f^2 + m_{h^0}^2 - 2(P_1\cdot P_3)$$

$$\boxed{(P_1\cdot P_3) = \frac{m_f^2 + m_{h^0}^2 - t}{2}} \qquad (41)$$

$$(P_1\cdot P_4) = \frac{s}{2} - \frac{(m_f^2 + m_{h^0}^2 - t)}{2} = \frac{s + t - m_f^2 - m_{h^0}^2}{2}$$

$$s + t + u = 2m_f^2 + m_{h^0}^2 + M_Z^2$$

$$s + t - m_f^2 - m_{h^0}^2 = m_f^2 + M_Z^2 - u$$

$$\Rightarrow \quad \boxed{(P_1\cdot P_4) = \frac{m_f^2 + M_Z^2 - u}{2}} \qquad (42)$$

$$(P_2 \cdot P_4) = (P_3 + P_4 - P_1)\cdot P_4 = \frac{s - m_{h^0}^2 - M_Z^2}{2} + M_Z^2 - \frac{(m_f^2 + M_Z^2 - u)}{2}$$

$$(P_2 \cdot P_4) = \frac{s - m_{h^0}^2 - \cancel{M_Z^2} + 2\cancel{M_Z^2} - m_f^2 - \cancel{M_Z^2} + u}{2}$$

$$\boxed{(P_2 \cdot P_4) = \frac{s - m_{h^0}^2 - m_f^2 + u}{2}} \qquad (43)$$

$$|M_{Za}|^2 = \frac{g^4 M_Z^2 |C_Z|^2}{\cos^4\theta_W}\Big\{ \left(\frac{s}{2} - m_f^2\right)\left[|C_V^f|^2 + |C_A^f|^2\right] - 3m_f^2\left[|C_A^f|^2 - |C_V^f|^2\right]$$

$$+ \frac{\cancel{2}}{M_Z^2}\frac{(m_f^2 + M_Z^2 - u)}{\cancel{2}}\,\frac{1}{2}\,(s - \overset{m_f^2+M_Z^2-t}{\widehat{m_{h^0}^2}} - m_f^2 + u)\left[|C_V^f|^2 + |C_A^f|^2\right]\Big\}$$

$$|M_{Za}|^2 = \frac{g^4 M_Z^2 |C_Z|^2}{\cos^4\theta_W}\Big\{ \left[|C_V^f|^2 + |C_A^f|^2\right]\Big[\frac{s}{2} - m_f^2 + \frac{1}{2M_Z^2}\big((m_f^2 + M_Z^2)^2 - t(m_f^2 + M_Z^2)$$

$$- u(m_f^2 + M_Z^2) + ut\big)\Big] - 3m_f^2\left[|C_A^f|^2 - |C_V^f|^2\right]\Big\}$$

$$|M_{Za}|^2 = \frac{g^4 M_Z^2 |C_Z|^2}{\cos^4\theta_W}\Big\{ \left[|C_V^f|^2 + |C_A^f|^2\right]\Big[\frac{s}{2} - m_f^2 + \frac{1}{2M_Z^2}\big((m_f^2 + M_Z^2)(s - m_f^2 - m_{h^0}^2)$$

$$+ ut\big)\Big] - 3m_f^2\left[|C_A^f|^2 - |C_V^f|^2\right]\Big\}$$

$$|M_{Za}|^2 = \frac{g^4 M_Z^2 |C_Z|^2}{\cos^4\theta_W}\Big\{ \left[|C_V^f|^2 + |C_A^f|^2\right]\Big[\frac{\cancel{s}}{2} - m_f^2 + \frac{1}{2}\frac{m_f^2}{M_Z^2}(s - m_f^2 - m_{h^0}^2)$$

$$+ \frac{1}{2}(\cancel{s} - m_f^2 - m_{h^0}^2) + \frac{ut}{2M_Z^2}\Big] - 3m_f^2\left[|C_A^f|^2 - |C_V^f|^2\right]\Big\}$$

$$|M_{Za}|^2 = \frac{g^4 M_Z^2 |C_Z|^2}{\cos^4\theta_W}\Big\{ \left[|C_V^f|^2 + |C_A^f|^2\right]\Big[s - \frac{3}{2}m_f^2 - \frac{m_{h^0}^2}{2} + \frac{ut}{2M_Z^2}$$

$$+ \frac{1}{2}\frac{m_f^2}{M_Z^2}(s - m_f^2 - m_{h^0}^2)\Big] - 3m_f^2\left[|C_A^f|^2 - |C_V^f|^2\right]\Big\} \qquad (44)$$

neglecting m_f^2 $(s >> m_f^2)$

$$|M_{Za}|^2 = \frac{g^4 M_Z^2 |C_Z|^2}{\cos^4\theta_W}\Big\{ \left[|C_V^f|^2 + |C_A^f|^2\right]\Big[s - \frac{m_{h^0}^2}{2} + \frac{ut}{2M_Z^2}\Big]\Big\}$$

$$|M_{Za}|^2 = \frac{g^4 |C_Z|^2}{2\cos^4\theta_W}\left[|C_V^f|^2 + |C_A^f|^2\right]\left[2sM_Z^2 - m_{h^0}^2 M_Z^2 + ut\right]$$

but $ut = m_{h^0}^2 M_Z^2 + \frac{1}{4}\lambda \sin^2\theta$ (45)

$$\Rightarrow |M_{Za}|^2 = \frac{g^4 |C_Z|^2}{2\cos^4\theta_W}\left[|C_V^f|^2 + |C_A^f|^2\right]\left[2sM_Z^2 + \frac{1}{4}\lambda \sin^2\theta\right]$$

$$\boxed{|M_{Za}|^2 = \frac{g^4 |C_Z|^2}{8\cos^4\theta_W} \left[|C_V^f|^2 + |C_A^f|^2 \right] \left(8 s M_Z^2 + \lambda(s, m_{h^0}^2, M_Z^2) \sin^2\theta \right)} \qquad (46)$$

$$|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2}{16 M_W^2 \cos^2\theta_W} \left(\sum_\lambda \varepsilon_{4\mu} \varepsilon^*_{4\nu} \right) \left[\bar{V}_2 \gamma^\mu (C_V^f - C_A^f \gamma^5)(2m_f - \not{p}_3) U_1 \right]^\dagger \cdot$$
$$\cdot \left[\bar{V}_2 \gamma^\nu (C_V^f - C_A^f \gamma^5)(2m_f - \not{p}_3) U_1 \right] \qquad (47)$$

$$\left[V_2^\dagger \gamma^0 \gamma^\mu (C_V^f - C_A^f \gamma^5)(2m_f - \gamma^\alpha p_{3\alpha}) U_1 \right]^\dagger$$
$$= U_1^\dagger (2m_f - \gamma^{\alpha\dagger} p_{3\alpha})(C_V^f - C_A^f\gamma^5) \overset{\gamma^0\gamma^\mu}{\gamma^{\mu\dagger}\gamma^0} V_2$$
$$= U_1^\dagger (2m_f - \gamma^{\alpha\dagger} p_{3\alpha}) \gamma^0 (C_V^f + C_A^f \gamma^5) \gamma^\mu V_2$$
$$= U_1^\dagger (2m_f \gamma^0 - \gamma^0 \not{p}_3)(C_V^f + C_A^f \gamma^5) \gamma^\mu V_2$$
$$= \bar{U}_1 (2m_f - \not{p}_3)(C_V^f + C_A^f \gamma^5) \gamma^\mu V_2 \qquad (48)$$

$$\Rightarrow |M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2}{16 M_W^2 \cos^2\theta_W} \left(-\eta_{\mu\nu} + \frac{p_{4\mu} p_{4\nu}}{M_Z^2} \right) \sum_s \left(\bar{U}_1 (2m_f - \not{p}_3)(C_V^f + C_A^f\gamma^5)\gamma^\mu V_2 \right) \cdot$$
$$\cdot \left(\bar{V}_2 \gamma^\nu (C_V^f - C_A^f \gamma^5)(2m_f - \not{p}_3) U_1 \right)$$

$$|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2}{16 M_W^2 \cos^2\theta_W} \left(-\eta_{\mu\nu} + \frac{p_{4\mu} p_{4\nu}}{M_Z^2} \right) \mathrm{Tr} \left[(\not{p}_1 + m_f)(2m_f - \not{p}_3)(C_V^f + C_A^f \gamma^5) \gamma^\mu (\not{p}_2 - m_f) \gamma^\nu \right.$$
$$\left. (C_V^f - C_A^f \gamma^5)(2m_f - \not{p}_3) \right] \qquad (49)$$

$$\mathrm{Tr} = \mathrm{Tr}\left[(2m_f \not{p}_1 - \not{p}_1\not{p}_3 + 2m_f^2 - m_f \not{p}_3)(C_V^f + C_A^f\gamma^5)(\gamma^\mu \not{p}_2 \gamma^\nu - m_f \gamma^\mu\gamma^\nu) \right.$$
$$\left. (2m_f C_V^f - C_V^f \not{p}_3 - 2m_f C_A^f \gamma^5 + C_A^f \gamma^5 \not{p}_3) \right]$$

$$= \mathrm{Tr}\left[(\overset{\text{ok}}{\downarrow} 2m_f C_V^f \not{p}_1 + \overset{\downarrow\,\text{ok}}{2m_f C_A^f \not{p}_1 \gamma^5} - \overset{\downarrow\,\text{OK}}{C_V^f \not{p}_1 \not{p}_3} - \overset{\downarrow\,\text{OK}}{C_A^f \not{p}_1\not{p}_3\gamma^5} + \overset{\downarrow\,\text{OK}}{2m_f^2 C_V^f} + \overset{\downarrow\,\text{OK}}{2m_f^2 C_A^f \gamma^5} \right.$$
$$- \overset{\downarrow\,\text{OK}}{m_f C_V^f \not{p}_3} - \overset{\downarrow}{m_f C_A^f \not{p}_3 \gamma^5})(2m_f C_V^f \gamma^\mu \not{p}_2 \gamma^\nu - C_V^f \gamma^\mu \not{p}_2 \gamma^\nu \not{p}_3 - 2m_f C_A^f \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5$$
$$+ C_A^f \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5 \not{p}_3 - 2m_f^2 C_V^f \gamma^\mu\gamma^\nu + m_f C_V^f \gamma^\mu \gamma^\nu \not{p}_3 + 2m_f^2 C_A^f \gamma^\mu\gamma^\nu\gamma^5 - m_f C_A^f \gamma^\mu\gamma^\nu \cdot$$
$$\left. \gamma^5 \not{p}_3) \right]$$

$$= 4m_f^2 |C_V^f|^2 \mathrm{Tr}(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) - 2m_f |C_V^f|^2 \mathrm{Tr}(\not{p}_1 \overset{0}{\cancel{\gamma^\mu}} \not{p}_2 \gamma^\nu \not{p}_3) - 4m_f^2 C_V^f C_A^f \mathrm{Tr}(\not{p}_1 \gamma^\mu$$
$$\not{p}_2 \gamma^\nu \gamma^5) + 2m_f C_V^f C_A^f \mathrm{Tr}(\not{p}_1 \gamma^\mu \overset{0}{\cancel{\not{p}_2}} \gamma^\nu \gamma^5 \not{p}_3) - 4m_f^3 |C_V^f|^2 \mathrm{Tr}(\overset{0}{\cancel{\not{p}_1}} \gamma^\mu \gamma^\nu)$$
$$+ 2m_f^2 |C_V^f|^2 \mathrm{Tr}(\not{p}_1 \gamma^\mu \gamma^\nu \not{p}_3) + 4m_f^3 C_V^f C_A^f \mathrm{Tr}(\not{p}_1 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu \gamma^5) - 2m_f^2 C_V^f C_A^f \mathrm{Tr}(\not{p}_1 \gamma^\mu \gamma^\nu$$
$$\gamma^5 \not{p}_3) + 4m_f^2 C_V^f C_A^f \mathrm{Tr}(\not{p}_1 \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu) - 2m_f C_V^f C_A^f \mathrm{Tr}(\not{p}_1 \gamma^5 \overset{0}{\cancel{\gamma^\mu}} \not{p}_2 \gamma^\nu \not{p}_3)$$
$$- 4m_f^2 |C_A^f|^2 \mathrm{Tr}(\not{p}_1 \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5) + 2m_f |C_A^f|^2 \mathrm{Tr}(\not{p}_1 \gamma^5 \gamma^\mu \overset{0}{\cancel{\not{p}_2}} \gamma^\nu \gamma^5 \not{p}_3) - 4m_f^3 C_V^f C_A^f \mathrm{Tr}(\not{p}_1 \gamma^5$$
$$\gamma^\mu \gamma^\nu) + 2m_f^2 C_A^f C_V^f \mathrm{Tr}(\not{p}_1 \gamma^5 \gamma^\mu \gamma^\nu \not{p}_3) + 4m_f^3 |C_A^f|^2 \mathrm{Tr}(\not{p}_1 \gamma^5 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu \gamma^5) - 2m_f^2 |C_A^f|^2 \mathrm{Tr}(\not{p}_1 \gamma^5 \gamma^\mu \gamma^\nu \gamma^5 \not{p}_3$$

$$\downarrow -2m_f (C_V^f)^2 \mathrm{Tr}(\not{p}_1\not{p}_3 \overset{0}{\cancel{\gamma^\mu}} \not{p}_2\gamma^\nu) + (C_V^f)^2 \mathrm{Tr}(\not{p}_1\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu\not{p}_3) + 2m_f C_V^f C_A^f \mathrm{Tr}(\not{p}_1\not{p}_3\cancel{\gamma^\mu}\not{p}_2\gamma^\nu\gamma^5)(2$$

$$- C_V^f C_A^f \mathrm{Tr}(\not{p}_1\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu\gamma^5\not{p}_3) + 2m_f^2 (C_V^f)^2 \mathrm{Tr}(\not{p}_1\not{p}_3\gamma^\mu\gamma^\nu) - m_f (C_V^f)^2\mathrm{Tr}(\not{p}_1\not{p}_3\overset{0}{\cancel{\gamma^\mu}}\gamma^\nu\not{p}_3)$$

$$-2m_f^2 C_V^f C_A^f \mathrm{Tr}(\not{p}_1\not{p}_3\gamma^\mu\gamma^\nu\gamma^5) + m_f C_V^f C_A^f \mathrm{Tr}(\not{p}_1\not{p}_3\overset{0}{\cancel{\gamma^\mu}}\gamma^\nu\gamma^5\not{p}_3) \overset{\downarrow}{-} 2m_f C_V^f C_A^f\mathrm{Tr}(\not{p}_1\overset{0}{\cancel{\not{p}_3}}\gamma^5\gamma^\mu\not{p}_2\gamma^\nu)$$

$$+ C_A^f C_V^f \mathrm{Tr}(\not{p}_1\not{p}_3\gamma^5\gamma^\mu\not{p}_2\gamma^\nu\not{p}_3) + 2m_f (C_A^f)^2\mathrm{Tr}(\not{p}_1\not{p}_3\gamma^5\overset{0}{\cancel{\gamma^\mu}}\not{p}_2\gamma^\nu\gamma^5) - (C_A^f)^2\mathrm{Tr}(\not{p}_1\not{p}_3\gamma^5\gamma^\mu\not{p}_2\gamma^\nu\gamma^5\not{p}_3)$$

$$+2m_f^2 C_V^f C_A^f\mathrm{Tr}(\not{p}_1\not{p}_3\gamma^5\gamma^\mu\gamma^\nu) - m_f C_A^f C_V^f \mathrm{Tr}(\not{p}_1\not{p}_3\gamma^5\overset{0}{\cancel{\gamma^\mu}}\gamma^\nu\not{p}_3) - 2m_f^2 (C_A^f)^2\mathrm{Tr}(\not{p}_1\not{p}_3\gamma^5\gamma^\mu\gamma^\nu\gamma^5)$$

$$+ m_f (C_A^f)^2\mathrm{Tr}(\not{p}_1\not{p}_3\gamma^5\gamma^\mu\overset{0}{\cancel{\gamma^\nu}}\gamma^5\not{p}_3) \overset{\downarrow}{+} 4m_f^3 (C_V^f)^2\mathrm{Tr}(\overset{0}{\cancel{\gamma^\mu}}\not{p}_2\gamma^\nu) - 2m_f^2 (C_V^f)^2\mathrm{Tr}(\gamma^\mu\not{p}_2\gamma^\nu\not{p}_3)$$

$$-4m_f^3 C_V^f C_A^f\mathrm{Tr}(\gamma^\mu\not{p}_2\overset{0}{\cancel{\gamma^\nu}}\gamma^5) + 2m_f^2 C_V^f C_A^f\mathrm{Tr}(\gamma^\mu\not{p}_2\gamma^\nu\gamma^5\not{p}_3) - 4m_f^4 (C_V^f)^2\mathrm{Tr}(\gamma^\mu\gamma^\nu)$$

$$+2m_f^3 (C_V^f)^2\mathrm{Tr}(\gamma^\mu\overset{0}{\cancel{\gamma^\nu}}\not{p}_3) + 4m_f^4 C_V^f C_A^f \mathrm{Tr}(\overset{0}{\cancel{\gamma^\mu}}\gamma^\nu\gamma^5) - 2m_f^3 C_V^f C_A^f\mathrm{Tr}(\gamma^\mu\overset{0}{\cancel{\gamma^\nu}}\gamma^5\not{p}_3)$$

$$\downarrow + 4m_f^3 C_V^f C_A^f\mathrm{Tr}(\gamma^5\gamma^\mu\overset{0}{\cancel{\not{p}_2}}\gamma^\nu) - 2m_f^2 C_V^f C_A^f\mathrm{Tr}(\gamma^5\gamma^\mu\not{p}_2\gamma^\nu\not{p}_3) - 4m_f^3 (C_A^f)^2\mathrm{Tr}(\gamma^5\gamma^\mu\overset{0}{\cancel{\not{p}_2}}\gamma^\nu\gamma^5)$$

$$+2m_f^2 (C_A^f)^2\mathrm{Tr}(\gamma^5\gamma^\mu\not{p}_2\gamma^\nu\gamma^5\not{p}_3) - 4m_f^4 C_V^f C_A^f\mathrm{Tr}(\overset{0}{\cancel{\gamma^5}}\gamma^\mu\gamma^\nu) + 2m_f^3 C_V^f C_A^f\mathrm{Tr}(\gamma^5\overset{0}{\cancel{\gamma^\mu}}\gamma^\nu\not{p}_3)$$

$$+4m_f^4 (C_A^f)^2\mathrm{Tr}(\gamma^\mu\gamma^\nu) - 2m_f^3 (C_A^f)^2\mathrm{Tr}(\gamma^5\gamma^\mu\overset{0}{\cancel{\gamma^\nu}}\gamma^5\not{p}_3) \overset{\downarrow}{-} 2m_f^2 (C_V^f)^2\mathrm{Tr}(\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu)$$

$$+ m_f (C_V^f)^2\mathrm{Tr}(\not{p}_3\gamma^\mu\overset{0}{\cancel{\not{p}_2}}\gamma^\nu\not{p}_3) + 2m_f^2 C_V^f C_A^f\mathrm{Tr}(\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu\gamma^5) - m_f C_V^f C_A^f\mathrm{Tr}(\not{p}_3\gamma^\mu\overset{0}{\cancel{\not{p}_2}}\gamma^\nu\gamma^5\not{p}_3)$$

$$+2m_f^3 (C_V^f)^2\mathrm{Tr}(\overset{0}{\cancel{\not{p}_3}}\gamma^\mu\gamma^\nu) - m_f^2 (C_V^f)^2\mathrm{Tr}(\not{p}_3\gamma^\mu\gamma^\nu\not{p}_3) - 2m_f^3 C_V^f C_A^f\mathrm{Tr}(\overset{0}{\cancel{\not{p}_3}}\gamma^\mu\gamma^\nu\gamma^5)$$

$$+m_f^2 C_V^f C_A^f\mathrm{Tr}(\not{p}_3\gamma^\mu\overset{0}{\cancel{\gamma^\nu}}\gamma^5\not{p}_3) \overset{\downarrow}{-} 2m_f^2 C_V^f C_A^f\mathrm{Tr}(\not{p}_3\gamma^5\gamma^\mu\not{p}_2\gamma^\nu) + m_f C_V^f C_A^f\mathrm{Tr}(\not{p}_3\gamma^5\overset{0}{\cancel{\gamma^\mu}}\not{p}_2\gamma^\nu\not{p}_3)$$

$$+2m_f^2 (C_A^f)^2\mathrm{Tr}(\not{p}_3\gamma^5\gamma^\mu\not{p}_2\gamma^\nu\gamma^5) - m_f (C_A^f)^2\mathrm{Tr}(\not{p}_3\gamma^5\gamma^\mu\overset{0}{\cancel{\not{p}_2}}\gamma^\nu\gamma^5\not{p}_3) + 2m_f^3 C_V^f C_A^f\mathrm{Tr}(\overset{0}{\cancel{\not{p}_3}}\gamma^5\gamma^\mu\gamma^\nu)$$

$$- m_f^2 C_V^f C_A^f\mathrm{Tr}(\not{p}_3\overset{0}{\cancel{\gamma^5}}\gamma^\mu\gamma^\nu\not{p}_3) - 2m_f^3 (C_A^f)^2\mathrm{Tr}(\not{p}_3\gamma^5\overset{0}{\cancel{\gamma^\mu}}\gamma^\nu\gamma^5) + m_f^2 (C_A^f)^2\mathrm{Tr}(\not{p}_3\gamma^5\gamma^\mu\gamma^\nu\gamma^5\not{p}_3)$$

$$= 4m_f^2 (C_V^f)^2\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_2) - 4m_f^2 C_V^f C_A^f\mathrm{Tr}(\gamma^5\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu) + 2m_f^2 (C_V^f)^2\mathrm{Tr}((2p_1^\mu - \gamma^\mu\not{p}_1)\gamma^\nu\not{p}_3)$$

$$+2m_f^2 C_V^f \cancel{C_A^f}\mathrm{Tr}(\gamma^5\not{p}_1\gamma^\mu\gamma^\nu\not{p}_3) - 4m_f^2 C_V^f C_A^f\mathrm{Tr}(\gamma^5\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu) + 4m_f^2 (C_A^f)^2\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_2)$$

$$-2m_f^2 C_V^f C_A^f \cancel{\mathrm{Tr}}(\gamma^5\not{p}_1\gamma^\mu\gamma^\nu\not{p}_3) - 2m_f^2 (C_A^f)^2\mathrm{Tr}((2p_1^\mu-\gamma^\mu\not{p}_1)\gamma^\nu\not{p}_3) + (C_V^f)^2\cancel{\mathrm{Tr}}(\not{p}_3\not{p}_1\not{p}_3\gamma^\mu\cdot$$

$$\not{p}_2\gamma^\nu) - 2C_V^f C_A^f\mathrm{Tr}(\gamma^5\not{p}_3\not{p}_1\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu) + 2m_f^2 (C_V^f)^2\mathrm{Tr}(\overset{\downarrow}{\not{p}_1}(2p_3^\mu - \gamma^\mu\not{p}_3)\gamma^\nu)$$

$$-2m_f^2 C_V^f C_A^f\mathrm{Tr}(\gamma^5\not{p}_1\not{p}_3\gamma^\mu\gamma^\nu) + (C_A^f)^2\mathrm{Tr}(\not{p}_3\not{p}_1\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu) + 2m_f^2 C_V^f C_A^f\mathrm{Tr}(\gamma^5\not{p}_1\not{p}_3\gamma^\mu\gamma^\nu)$$

$$-2m_f^2 (C_A^f)^2\mathrm{Tr}(\not{p}_1(2p_3^\mu - \overset{\downarrow}{\gamma^\mu}\not{p}_3)\gamma^\nu) - 2m_f^2 (C_V^f)^2\mathrm{Tr}(\gamma^\mu\not{p}_2\gamma^\nu\not{p}_3) - 2m_f^2 C_V^f C_A^f\mathrm{Tr}(\gamma^5\gamma^\mu\not{p}_2\gamma^\nu\not{p}_3)$$

$$-16m_f^4 (C_V^f)^2\eta^{\mu\nu} - 4m_f^2 C_V^f C_A^f\mathrm{Tr}(\gamma^5\gamma^\mu\not{p}_2\gamma^\nu\not{p}_3) - 2m_f^2 (C_A^f)^2\mathrm{Tr}(\gamma^\mu\not{p}_2\gamma^\nu\not{p}_3)$$

$$+16m_f^4 (C_A^f)^2\eta^{\mu\nu} - 2m_f^2 (C_V^f)^2\mathrm{Tr}(\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu) - 4m_f^2 m_{ho}^2 (C_V^f)^2\eta^{\mu\nu} + 2m_f^2 C_V^f C_A^f\mathrm{Tr}(\gamma^5\not{p}_3$$

$$\gamma^\mu\not{p}_2\gamma^\nu) - 2m_f^2 (C_A^f)^2\mathrm{Tr}(\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu) + 4m_f^2 m_{ho}^2 (C_A^f)^2\eta^{\mu\nu}.$$

$$= 4m_f^2 \left[|c_V^+|^2 + |c_A^+|^2 \right] Tr(\gamma^\nu \not{p}_1 \gamma^\mu \not{p}_2) - 8m_f^2 c_V^+ c_A^+ Tr(\gamma^5 \not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$+ 2m_f^2 \left[|c_V^+|^2 - |c_A^+|^2 \right] Tr\left((2p_1^\mu - \gamma^\mu \not{p}_1) \gamma^\nu \not{p}_3 \right) - 4m_f^2 \left[|c_V^+|^2 + |c_A^+|^2 \right] Tr(\gamma^\mu \not{p}_2 \gamma^\nu \not{p}_3)$$

$$- 16 m_f^4 \left[|c_V^+|^2 - |c_A^+|^2 \right] n^{\mu\nu} - 4m_f^2 m_{h^0}^2 \left[|c_V^+|^2 - |c_A^+|^2 \right] n^{\mu\nu} + \left[|c_V^+|^2 + |c_A^+|^2 \right] .$$

$$. Tr\left(\not{p}_3 \; (-\not{p}_3 \not{p}_1 + 2(p_1 . p_3)) \gamma^\mu \not{p}_2 \gamma^\nu \right) + 2m_f^2 \left[|c_V^+|^2 - |c_A^+|^2 \right] Tr\left[\not{p}_1 (2p_3^\mu - \gamma^\mu \not{p}_3) \gamma^\nu \right]$$

$$- 2 c_V^+ c_A^+ Tr\left[\gamma^5 \not{p}_3 (-\not{p}_3 \not{p}_1 + 2(p_1 . p_3)) \gamma^\mu \not{p}_2 \gamma^\nu \right] .$$

$$= 4m_f^2 \left[|c_V^+|^2 + |c_A^+|^2 \right] Tr(\gamma^\nu \not{p}_1 \gamma^\mu \not{p}_2) - 8m_f^2 c_V^+ c_A^+ Tr(\gamma^5 \not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$+ 16 m_f^2 \left[|c_V^+|^2 - |c_A^+|^2 \right] p_1^\mu p_3^\nu - 2m_f^2 \left[|c_V^+|^2 - |c_A^+|^2 \right] Tr(\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_3)$$

$$- 4m_f^2 \left[|c_V^+|^2 + |c_A^+|^2 \right] Tr(\gamma^\mu \not{p}_2 \gamma^\nu \not{p}_3) - 16 m_f^4 \left[|c_V^+|^2 - |c_A^+|^2 \right] n^{\mu\nu}$$

$$- 4m_f^2 m_{h^0}^2 \left[|c_V^+|^2 - |c_A^+|^2 \right] n^{\mu\nu} - m_{h^0}^2 \left[|c_V^+|^2 + |c_A^+|^2 \right] Tr(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$+ 2(p_1 . p_3) \left[|c_V^+|^2 + |c_A^+|^2 \right] Tr(\not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu) + 16 m_f^2 \left[|c_V^+|^2 - |c_A^+|^2 \right] p_3^\mu p_1^\nu$$

$$- 2m_f^2 \left[|c_V^+|^2 - |c_A^+|^2 \right] Tr(\not{p}_1 \gamma^\mu \not{p}_3 \gamma^\nu) + 2 m_{h^0}^2 c_V^+ c_A^+ Tr(\gamma^5 \not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$- 4 (p_1 . p_3) c_V^+ c_A^+ Tr(\gamma^5 \not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$= (4m_f^2 - m_{h^0}^2) \left[|c_V^+|^2 + |c_A^+|^2 \right] Tr(\gamma^\nu \not{p}_1 \gamma^\mu \not{p}_2) + \left[2(p_1 . p_3) - 4m_f^2 \right] \left[|c_V^+|^2 + |c_A^+|^2 \right] .$$

$$. Tr(\gamma^\mu \not{p}_2 \gamma^\nu \not{p}_3) - 4m_f^2 (4m_f^2 + m_{h^0}^2) \left[|c_V^+|^2 - |c_A^+|^2 \right] n^{\mu\nu} + 2 c_V^+ c_A^+ (m_{h^0}^2 - 4m_f^2) .$$

$$. Tr(\gamma^5 \not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) + 16 m_f^2 \left[|c_V^+|^2 - |c_A^+|^2 \right] \left[p_1^\mu p_3^\nu + p_1^\nu p_3^\mu \right]$$

$$- 2m_f^2 \left[|c_V^+|^2 - |c_A^+|^2 \right] \left[Tr(\not{p}_1 \gamma^\mu \not{p}_3 \gamma^\nu) + Tr(\not{p}_3 \gamma^\mu \not{p}_1 \gamma^\nu) \right] - 4(p_1 . p_3) c_V^+ c_A^+ .$$

$$. Tr(\gamma^5 \not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu) .$$

$\Rightarrow Tr =$

$$4(4m_f^2 - m_{h^0}^2) \left[|c_V^+|^2 + |c_A^+|^2 \right] (p_1^\nu p_2^\mu + p_1^\mu p_2^\nu - (p_1 . p_2) n^{\mu\nu}) + 4 \left[2(p_1 . p_3) - 4m_f^2 \right] \left[|c_V^+|^2 + |c_A^+|^2 \right]$$

$$(p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - (p_2 . p_3) n^{\mu\nu}) - 4m_f^2 (4m_f^2 + m_{h^0}^2) \left[|c_V^+|^2 - |c_A^+|^2 \right] n^{\mu\nu} - 8i c_V^+ c_A^+ (m_{h^0}^2 - 4m_f^2)$$

$$\epsilon^{\alpha\mu\beta\nu} p_{1\alpha} p_{2\beta} + 16 m_f^2 \left[|c_V^+|^2 - |c_A^+|^2 \right] \left[p_1^\mu p_3^\nu + p_1^\nu p_3^\mu \right] - 16 m_f^2 \left[|c_V^+|^2 - |c_A^+|^2 \right] .$$

$$\left[p_1^\nu p_3^\mu + p_1^\mu p_3^\nu - (p_1 . p_3) n^{\mu\nu} \right] + 16i (p_1 . p_3) c_V^+ c_A^+ \epsilon^{\alpha\mu\beta\nu} p_{3\alpha} p_{2\beta} \qquad (50)$$

$$n_{\mu\nu}\,\varepsilon^{\alpha\mu\beta\nu} = 0$$

$$P_{4\mu}P_{4\nu}\,\varepsilon^{\alpha\mu\beta\nu} = 0$$

$$\Rightarrow |M_3|^2 = \frac{4g^4 m_f^2 C_{tf}^2}{16 M_W^2 \cos^2\theta_W}\left(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\Big\{(4m_f^2 - m_{h^0}^2)\,[\,|C_V^f|^2 + |C_A^f|^2\,]\,(P_1^\nu P_2^\mu + P_1^\mu P_2^\nu$$

$$-(P_1\cdot P_2)n^{\mu\nu}) + [\,2(P_1\cdot P_3) - 4m_f^2\,]\,[\,|C_V^f|^2 + |C_A^f|^2\,]\,(P_2^\mu P_3^\nu + P_2^\nu P_3^\mu - (P_2\cdot P_3)n^{\mu\nu})$$

$$- m_f^2(4m_f^2 + m_{h^0}^2)\,[\,|C_V^f|^2 - |C_A^f|^2\,]\,n^{\mu\nu} + 4m_f^2\,[\,|C_V^f|^2 - |C_A^f|^2\,]\,(P_1\cdot P_3)\,n^{\mu\nu}\Big\}\;. \quad (51)$$

$$|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2}{4 M_W^2\cos^2\theta_W}\Big\{-(4m_f^2 - m_{h^0}^2)\,[\,|C_V^f|^2 + |C_A^f|^2\,]\,(2(P_1\cdot P_2) - 4(P_1\cdot P_2)) + \frac{(4m_f^2 - m_{h^0}^2)}{M_Z^2}\cdot$$

$$[\,|C_V^f|^2 + |C_A^f|^2\,]\,[\,2(P_1\cdot P_4)(P_2\cdot P_4) - (P_1\cdot P_2)M_Z^2\,] - [\,2(P_1\cdot P_3) - 4m_f^2\,]\,[\,|C_V^f|^2 + |C_A^f|^2\,]\,[\,2(P_2\cdot P_3)$$

$$-4(P_2\cdot P_3)\,] + \frac{[\,2(P_1\cdot P_3) - 4m_f^2\,]}{M_Z^2}\,[\,|C_V^f|^2 + |C_A^f|^2\,]\,[\,2(P_2\cdot P_4)(P_3\cdot P_4) - M_Z^2(P_2\cdot P_3)\,]$$

$$+4m_f^2(4m_f^2 + m_{h^0}^2)\,[\,|C_V^f|^2 - |C_A^f|^2\,] - m_f^2(4m_f^2 + m_{h^0}^2)\,[\,|C_V^f|^2 - |C_A^f|^2\,]$$

$$-16m_f^2\,[\,|C_V^f|^2 - |C_A^f|^2\,]\,(P_1\cdot P_3) + 4m_f^2\,[\,|C_V^f|^2 - |C_A^f|^2\,]\,(P_1\cdot P_3)\Big\} \quad (52)$$

$$|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2}{4 M_W^2\cos^2\theta_W}\Big\{2(P_1\cdot P_2)(4m_f^2 - m_{h^0}^2)\,[\,|C_V^f|^2 + |C_A^f|^2\,] + \frac{(4m_f^2 - m_{h^0}^2)}{M_Z^2}\,[\,|C_V^f|^2 + |C_A^f|^2\,]\,\cdot$$

$$\cdot[\,2(P_1\cdot P_4)(P_2\cdot P_4) - (P_1\cdot P_2)M_Z^2\,] + 2(P_2\cdot P_3)\,[\,2(P_1\cdot P_3) - 4m_f^2\,]\,[\,|C_V^f|^2 + |C_A^f|^2\,] +$$

$$+\frac{[\,2(P_1\cdot P_3) - 4m_f^2\,]}{M_Z^2}\,[\,|C_V^f|^2 + |C_A^f|^2\,]\,[\,2(P_2\cdot P_4)(P_3\cdot P_4) - M_Z^2(P_2\cdot P_3)\,] + 3m_f^2(4m_f^2 + m_{h^0}^2)\,\cdot$$

$$\cdot[\,|C_V^f|^2 - |C_A^f|^2\,] - 12m_f^2\,[\,|C_V^f|^2 - |C_A^f|^2\,]\,(P_1\cdot P_3)\Big\} \quad (53)$$

$$|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2}{4M_W^2\cos^2\theta_W}\Big\{[\,|C_V^f|^2 + |C_A^f|^2\,]\Big\{2(P_1\cdot P_2)(4m_f^2 - m_{h^0}^2) + \frac{(4m_f^2 - m_{h^0}^2)}{M_Z^2}\,[\,2(P_1\cdot P_4)(P_2\cdot P_4)$$

$$-(P_1\cdot P_2)M_Z^2\,] + 2(P_2\cdot P_3)\,[\,2(P_1\cdot P_3) - 4m_f^2\,] + \frac{[\,2(P_1\cdot P_3) - 4m_f^2\,]}{M_Z^2}\,[\,\overset{\downarrow}{2}(P_2\cdot P_4)(P_3\cdot P_4) - M_Z^2(P_2\cdot P_3)\,]\Big\}$$

$$+3m_f^2\,[\,|C_V^f|^2 - |C_A^f|^2\,]\,\{4m_f^2 + m_{h^0}^2 - 4(P_1\cdot P_3)\}\Big\} \quad (54)$$

$$= \frac{g^4 m_f^2 C_{tf}^2}{4M_W^2\cos^2\theta_W}\Big\{[\,|C_V^f|^2 + |C_A^f|^2\,]\Big\{(s - 2m_f^2)(4m_f^2 - m_{h^0}^2) + \frac{(4m_f^2 - m_{h^0}^2)}{M_Z^2}\,[\,(m_f^2 + M_Z^2 - u)\frac{1}{2}$$

$$\cdot\,\overset{(m_f^2 + M_Z^2 - t)}{(s - m_{h^0}^2 - m_f^2 + u)} - \left(\frac{s}{2} - m_f^2\right)M_Z^2\,] + (m_f^2 + m_{h^0}^2 - u)\,[\,(m_f^2 + m_{h^0}^2 - t) - 4m_f^2\,]$$

$$+\frac{[\,(m_f^2 + m_{h^0}^2 - t) - 4m_f^2\,]}{M_Z^2}\,[\,(s - m_{h^0}^2 - m_f^2 + u)\frac{1}{2}(s - m_{h^0}^2 - M_Z^2) - \frac{M_Z^2}{2}(m_f^2 + m_{h^0}^2 - u)\,]\Big\}$$

$$+3m_f^2\,[\,|C_V^f|^2 - |C_A^f|^2\,]\,\{4\cancel{m_f^2} + \cancel{m_{h^0}^2} - 2(\cancel{m_f^2} + \cancel{m_{h^0}^2} - t)\}\Big\}$$

$$= \frac{g^4 m_f^2 C_{tf}^2}{4 M_W^2 \cos^2\theta_W} \Big\{ \big[|C_V^f|^2 + |C_A^f|^2 \big] \Big\{ (s-2m_f^2)(\cancel{4}m_f^2 - m_{h^0}^2) + \frac{(4m_f^2-m_{h^0}^2)}{2M_Z^2} \big[(m_f^2+M_Z^2)^2 -$$

$$-(u+t)(m_f^2+M_Z^2) + ut - (s-2\cancel{m_f^2}) M_Z^2 \big] + (m_f^2+m_{h^0}^2-u)(-3m_f^2+m_{h^0}^2-t)$$

$$+ \frac{(-3m_f^2+m_{h^0}^2-t)}{2M_Z^2} \big[\overset{(m_f^2+M_Z^2-t)}{(s-m_{h^0}^2-m_f^2+u)} (s-m_{h^0}^2) - sM_Z^2 \big] \Big\} + 3m_f^2 \big[|C_V^f|^2 - |C_A^f|^2 \big] \cdot$$

$$\big[2m_f^2 - m_{h^0}^2 + 2t \big] \Big\}$$

$$= \frac{g^4 m_f^2 C_{tf}^2}{4 M_W^2 \cos^2\theta_W} \Big\{ \big[|C_V^f|^2 + |C_A^f|^2 \big] \Big\{ \frac{1}{2}(s-2m_f^2)(4m_f^2-m_{h^0}^2) + \frac{(4m_f^2-m_{h^0}^2)}{2M_Z^2} \big[(m_f^2+M_Z^2)^2$$

$$-(u+t)(m_f^2+M_Z^2) + ut \big] + (m_f^2+m_{h^0}^2-u)(-3m_f^2+m_{h^0}^2-t) + \frac{(-3m_f^2+m_{h^0}^2-t)}{2M_Z^2} \cdot$$

$$\cdot \big[m_f^2 s - m_f^2 m_{h^0}^2 + \cancel{sM_Z^2} - m_{h^0}^2 M_Z^2 - st + t m_{h^0}^2 - \cancel{sM_Z^2} \big] \Big\} + 3m_f^2 \big[|C_V^f|^2 - |C_A^f|^2 \big] \cdot$$

$$\cdot \big[2m_f^2 - m_{h^0}^2 + 2t \big] \Big\}$$

$$s+t+u = 2m_f^2 + m_{h^0}^2 + M_Z^2$$

$$= \frac{g^4 m_f^2 C_{tf}^2}{4 M_W^2 \cos^2\theta_W} \Big\{ \big[|C_V^f|^2 + |C_A^f|^2 \big] \Big\{ \frac{1}{2}(s-2m_f^2)(4m_f^2-m_{h^0}^2) + \frac{(4m_f^2-m_{h^0}^2)}{2M_Z^2} \big[(m_f^2+M_Z^2) \cdot$$

$$(s-m_f^2-m_{h^0}^2) + ut \big] + (s+t-m_f^2-M_Z^2)(-3m_f^2+m_{h^0}^2-t) + \frac{(-3m_f^2+m_{h^0}^2-t)}{2M_Z^2} \cdot$$

$$\cdot \big[s m_f^2 - m_f^2 m_{h^0}^2 - m_{h^0}^2 M_Z^2 - st + t m_{h^0}^2 \big] \Big\} + 3m_f^2 \big[|C_V^f|^2 - |C_A^f|^2 \big] (2m_f^2 - m_{h^0}^2 + 2t$$

$$= \frac{g^4 m_f^2 C_{tf}^2}{8 M_W^4} \Big\{ \big[|C_V^f|^2 + |C_A^f|^2 \big] \Big\{ 4 s m_f^2 M_Z^2 - s m_{h^0}^2 M_Z^2 - 8 m_f^4 M_Z^2 + 2 m_f^2 m_{h^0}^2 M_Z^2$$

$$+ (4m_f^2 - m_{h^0}^2) \big[s m_f^2 - m_f^4 - m_f^2 m_{h^0}^2 + s M_Z^2 - m_f^2 M_Z^2 - m_{h^0}^2 M_Z^2 + ut \big]$$

$$+2 (-3 s m_f^2 + s m_{h^0}^2 - st - 3 t m_f^2 + t m_{h^0}^2 - t^2 + 3 m_f^4 - m_f^2 m_{h^0}^2 + t m_f^2 + 3 m_f^2 M_Z^2 - m_{h^0}^2 M_Z^2$$

$$+ t M_Z^2) M_Z^2 - 3 s m_f^4 + 3 m_f^4 m_{h^0}^2 + 3 m_f^2 m_{h^0}^2 M_Z^2 + 3 s t m_f^2 - 3 m_f^2 t m_{h^0}^2 + s m_f^2 m_{h^0}^2$$

$$- m_f^2 m_{h^0}^4 - m_{h^0}^4 M_Z^2 - s t m_{h^0}^2 + t m_{h^0}^4 - s t m_f^2 + t m_f^2 m_{h^0}^2 + t m_{h^0}^2 M_Z^2 + s t^2 - t^2 m_{h^0}^2 \big]$$

$$+ 3 m_f^2 \big[|C_V^f|^2 - |C_A^f|^2 \big] (2m_f^2 - m_{h^0}^2 + 2t) \Big\}$$

$$= \frac{g^4 m_f^2 C_{tf}^2}{8M_W^4}\Big\{\left[|C_V^t|^2 + |C_A^t|^2\right]\Big\{\cancel{4sm_f^2M_Z^2} - \cancel{sm_{h^0}^2M_Z^2} - \cancel{8m_f^4M_Z^2} + \cancel{2m_f^2m_{h^0}^2M_Z^2}$$

$$+\cancel{4sm_f^4} - \cancel{4m_f^6} - \cancel{4m_f^4m_{h^0}^2} + \cancel{4m_f^2sM_Z^2} - \cancel{4m_f^4M_Z^2} - \cancel{4m_f^2m_{h^0}^2M_Z^2} + \cancel{4m_f^2ut}$$

$$-\cancel{sm_f^2m_{h^0}^2} + \cancel{m_f^4m_{h^0}^2} + \cancel{m_f^2m_{h^0}^4} - \cancel{sM_Z^2m_{h^0}^2} + \cancel{m_f^2m_{h^0}^2M_Z^2} + \cancel{m_{h^0}^4M_Z^2} - \cancel{m_{h^0}^2ut}$$

$$-\cancel{6sm_f^2M_Z^2} + \cancel{2sm_{h^0}^2M_Z^2} - \cancel{2stM_Z^2} - \cancel{6tm_f^2M_Z^2} + \cancel{2tm_{h^0}^2M_Z^2} - \cancel{2t^2M_Z^2} + \cancel{6m_f^4M_Z^2} - \cancel{2m_f^2m_{h^0}^2M_Z^2}$$

$$+\cancel{2tm_f^2M_Z^2} + \cancel{6m_f^2M_Z^4} - \cancel{2m_{h^0}^2M_Z^4} + \cancel{2tM_Z^4} - \cancel{3sm_f^4} + \cancel{3m_f^4m_{h^0}^2} + \cancel{3m_f^2m_{h^0}^2M_Z^2} + \cancel{3stm_f^2}$$

$$-\cancel{3m_f^2tm_{h^0}^2} + \cancel{sm_f^2m_{h^0}^2} - \cancel{m_f^2m_{h^0}^4} - \cancel{m_{h^0}^4M_Z^2} - \cancel{stm_{h^0}^2} + \cancel{tm_{h^0}^4} - \cancel{stm_f^2} + \cancel{tm_f^2m_{h^0}^2}$$

$$+\cancel{tm_{h^0}^2M_Z^2} + \cancel{st^2} - t^2m_{h^0}^2\Big\} + 3m_f^2\left[|C_V^t|^2 - |C_A^t|^2\right](2m_f^2 - m_{h^0}^2 + 2t)\Big\}$$

$$= \frac{g^4 m_f^2 C_{tf}^2}{8M_W^4}\Big\{\left[|C_V^t|^2 + |C_A^t|^2\right]\Big\{\cancel{2sm_f^2M_Z^2} - \cancel{6m_f^4M_Z^2} + \cancel{sm_f^4} - \cancel{4m_f^6} + \cancel{4m_f^2ut}$$

$$-\cancel{m_{h^0}^2ut} - \cancel{2stM_Z^2} - \cancel{4tm_f^2M_Z^2} + \cancel{3tm_{h^0}^2M_Z^2} - \cancel{2t^2M_Z^2} + \cancel{6m_f^2M_Z^4} - \cancel{2m_{h^0}^2M_Z^4} + \cancel{2tM_Z^4}$$

$$+\cancel{2stm_f^2} - \cancel{2tm_f^2m_{h^0}^2} - \cancel{stm_{h^0}^2} + \cancel{tm_{h^0}^4} + \cancel{st^2} - \cancel{t^2m_{h^0}^2}\Big\} + 3m_f^2\left[|C_V^t|^2 - |C_A^t|^2\right].$$

$$.(2m_f^2 - m_{h^0}^2 + 2t)\Big\}$$

$$-m_{h^0}^2ut = -m_{h^0}^2(2m_f^2 + m_{h^0}^2 + M_Z^2 - s - t)t = -\cancel{2m_f^2m_{h^0}^2t} - \cancel{m_{h^0}^4t} - \cancel{m_{h^0}^2M_Z^2t}$$

$$+\cancel{m_{h^0}^2st} + \cancel{m_{h^0}^2t^2}$$

$$4m_f^2ut = 4m_f^2t(2m_f^2 + m_{h^0}^2 + M_Z^2 - s - t) = \cancel{8m_f^4t} + \cancel{4m_f^2m_{h^0}^2t} + \cancel{4m_f^2M_Z^2t} - \cancel{4m_f^2st}$$

$$-\cancel{4m_f^2t^2}$$

$$\Rightarrow |M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2}{8M_W^4}\Big\{\left[|C_V^t|^2 + |C_A^t|^2\right]\Big\{\cancel{2sm_f^2M_Z^2} - \cancel{6m_f^4M_Z^2} + \cancel{sm_f^4} - \cancel{4m_f^6} + \cancel{8m_f^4t}$$

$$-\cancel{2m_f^2st} - \cancel{4m_f^2t^2} + \cancel{2tm_{h^0}^2M_Z^2} - \cancel{2stM_Z^2} - \cancel{2t^2M_Z^2} + \cancel{6m_f^2M_Z^4} - \cancel{2m_{h^0}^2M_Z^4} + \cancel{2tM_Z^4} + \cancel{st^2}\Big\}$$

$$+ 3m_f^2\left[|C_V^t|^2 - |C_A^t|^2\right]\cdot(2m_f^2 - m_{h^0}^2 + 2t)\Big\} \qquad (55)$$

neglecting terms with m_f^2, m_f^4, m_f^6 inside $\{\ \}$ we have:

$$\therefore \quad |M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2}{8M_W^4}\left[|C_V^t|^2 + |C_A^t|^2\right]\Big\{\cancel{2tm_{h^0}^2M_Z^2} - \cancel{2stM_Z^2} - \cancel{2t^2M_Z^2} - 2m_{h^0}^2M_Z^4$$

$$+\cancel{2tM_Z^4} + st^2\Big\}$$

$$-2stM_Z^2 + 2tm_{h^0}^2M_Z^2 - 2t^2M_Z^2 + 2tM_Z^4 = 2tM_Z^2(\overbrace{m_{h^0}^2 - t + M_Z^2 - s}^{\simeq +u}) = +2utM_Z^2$$

$$\Rightarrow |M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2}{8M_W^4}\left[|C_V^t|^2 + |C_A^t|^2\right]\left[2utM_Z^2 - 2m_{h^0}^2M_Z^4 + st^2\right]$$

but $ut - m_{h^0}^2 M_Z^2 = \frac{1}{4}\lambda \sin^2\theta$

$$\Rightarrow \quad |M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2}{8 M_W^4}\left[|C_V^t|^2 + |C_A^t|^2\right]\left[\frac{1}{2}\lambda \sin^2\theta\, M_Z^2 + s t^2\right]$$

$$\boxed{|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2}{8 M_W^4}\left[|C_V^t|^2 + |C_A^t|^2\right]\left[\frac{1}{2}\lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta\, M_Z^2 + s t^2\right]}$$

or : $$\boxed{|M_3|^2 = \frac{g^4 m_f^2 C_{tf}^2\, s}{8 M_W^4}\left[|C_V^t|^2 + |C_A^t|^2\right]\left[t^2 + \frac{\lambda(s, m_{h^0}^2, M_Z^2)}{2s}\sin^2\theta\, M_Z^2\right]} \quad (56)$$

$$|M_4|^2 = \frac{g^4 m_f{}^2 C_{V_f}^2}{16 M_W{}^2 \cos^2\theta_W} \Big(\sum_\lambda \varepsilon_{4\mu} \varepsilon^*_{4\nu}\Big) \sum_s \Big(\bar{V}_2 (2m_f + \not{p}_3)\gamma^\mu (C_V^f - C_A^f\gamma^5) U_1\Big)^+ \Big(\bar{V}_2 (2m_f + \not{p}_3)\gamma^\nu \cdot$$

$$\cdot (C_V^f - C_A^f\gamma^5) U_1\Big) \qquad (57)$$

$$\Big[\underset{\substack{\downarrow \\ V_2^+\gamma^0}}{\bar{V}_2} (2m_f + \not{p}_3)\gamma^\mu (C_V^f - C_A^f\gamma^5) U_1\Big]^+ = U_1^+ (C_V^f - C_A^f\gamma^5)\gamma^{\mu+}(2m_f + \gamma^{\alpha+} p_{3\alpha})\gamma^0 V_2$$

$$= U_1^+ (C_V^f - C_A^f\gamma^5) \underset{\substack{\checkmark \\ \gamma^0\gamma^\mu}}{\gamma^{\mu+}\gamma^0} (2m_f + \not{p}_3) V_2$$

$$= \bar{U}_1 (C_V^f + C_A^f\gamma^5)\gamma^\mu (2m_f + \not{p}_3) V_2 \qquad (58)$$

$$\Rightarrow |M_4|^2 = \frac{g^4 m_f{}^2 C_{V_f}^2}{16 M_W{}^2 \cos^2\theta_W} \Big(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z{}^2}\Big) \sum_s \Big(\bar{U}_1 (C_V^f + C_A^f\gamma^5)\gamma^\mu (2m_f + \not{p}_3) V_2\Big)\Big(\bar{V}_2 (2m_f + \not{p}_3)$$

$$\gamma^\nu (C_V^f - C_A^f\gamma^5) U_1\Big) \qquad (59)$$

$$|M_4|^2 = \frac{g^4 m_f{}^2 C_{V_f}^2}{16 M_W{}^2 \cos^2\theta_W} \Big(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z{}^2}\Big) \mathrm{Tr}\Big\{(\not{p}_1 + m_f)(C_V^f + C_A^f\gamma^5)\gamma^\mu (2m_f + \not{p}_3)(\not{p}_2 - m_f)(2m_f + \not{p}_3$$

$$\gamma^\nu (C_V^f - C_A^f\gamma^5)\Big\} \qquad (60)$$

neglecting m_f in the trace :

$$|M_4|^2 = \frac{g^4 m_f{}^2 C_{V_f}^2}{16 M_W{}^2 \cos^2\theta_W} \Big(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z{}^2}\Big) \mathrm{Tr}\Big\{\not{p}_1 (C_V^f + C_A^f\gamma^5)\gamma^\mu \not{p}_3 \not{p}_2 \not{p}_3 \gamma^\nu (C_V^f - C_A^f\gamma^5)\Big\} \quad (61$$

$$\mathrm{Tr}\{\;\} = (C_V^f)^2 \mathrm{Tr}[\not{p}_1\gamma^\mu\not{p}_3\not{p}_2\not{p}_3\gamma^\nu] - C_V^f C_A^f \mathrm{Tr}[\not{p}_1\gamma^\mu\not{p}_3\not{p}_2\not{p}_3\gamma^\nu\gamma^5]$$

$$+ C_A^f C_V^f \mathrm{Tr}[\not{p}_1\gamma^5\gamma^\mu\not{p}_3\not{p}_2\not{p}_3\gamma^\nu] - (C_A^f)^2 \mathrm{Tr}[\not{p}_1\gamma^5\gamma^\mu\not{p}_3\not{p}_2\not{p}_3\gamma^\nu\gamma^5]$$

$$= (C_V^f)^2 \mathrm{Tr}[\not{p}_1\gamma^\mu\not{p}_3(-\not{p}_3\not{p}_2 + 2(P_2\cdot P_3))\gamma^\nu] - C_V^f C_A^f \mathrm{Tr}[\gamma^5\not{p}_1\gamma^\mu\not{p}_3(-\not{p}_3\not{p}_2 + 2(P_2\cdot P_3))$$

$$\gamma^\nu] - C_V^f C_A^f \mathrm{Tr}[\gamma^5\not{p}_1\gamma^\mu\not{p}_3(-\not{p}_3\not{p}_2 + (2P_2\cdot P_3))\gamma^\nu] + (C_A^f)^2 \mathrm{Tr}[\not{p}_1\gamma^\mu\not{p}_3 \cdot$$

$$(-\not{p}_3\not{p}_2 + 2(P_2\cdot P_3))\gamma^\nu]$$

$$= -mho^2 (C_V^f)^2 \mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu) + 2(P_2\cdot P_3)(C_V^f)^2 \mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_3\gamma^\nu)$$

$$+ mho^2 C_V^f C_A^f \mathrm{Tr}(\gamma^5\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu) - 2(P_2\cdot P_3) C_V^f C_A^f \mathrm{Tr}(\gamma^5\not{p}_1\gamma^\mu\not{p}_3\gamma^\nu)$$

$$+ mho^2 C_V^f C_A^f \mathrm{Tr}(\gamma^5\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu) - 2(P_2\cdot P_3) C_V^f C_A^f \mathrm{Tr}(\gamma^5\not{p}_1\gamma^\mu\not{p}_3\gamma^\nu)$$

$$- mho^2 (C_A^f)^2 \mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu) + 2(P_2\cdot P_3)(C_A^f)^2 \mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_3\gamma^\nu)$$

$$= -m_{h^0}^2\left[(C_V^f)^2+(C_A^f)^2\right]\mathrm{Tr}\left(\gamma^\nu \not{p}_1\gamma^\mu\not{p}_2\right) + 2(P_2\cdot P_3)\left[(C_V^f)^2+(C_A^f)^2\right]\mathrm{Tr}\left(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_3\right)$$

$$+ 2m_{h^0}^2 C_V^f C_A^f\,\mathrm{Tr}\left(\gamma^5\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu\right) - 4(P_2\cdot P_3)C_V^f C_A^f\,\mathrm{Tr}\left(\gamma^5\not{p}_1\gamma^\mu\not{p}_3\gamma^\nu\right)$$

$$\Rightarrow \mathrm{Tr} = 4\left[(C_V^f)^2+(C_A^f)^2\right]\Big\{-m_{h^0}^2\left[P_1^\nu P_2^\mu + P_1^\mu P_2^\nu - (P_1\cdot P_2)\eta^{\mu\nu}\right] + 2(P_2\cdot P_3)\left[P_1^\nu P_3^\mu + P_1^\mu P_3^\nu\right.$$

$$\left.- (P_1\cdot P_3)\eta^{\mu\nu}\right]\Big\} - 8i\, m_{h^0}^2 C_V^f C_A^f\,\epsilon^{\alpha\mu\beta\nu}P_{1\alpha}P_{2\beta} + 16i(P_2\cdot P_3)C_V^f C_A^f\,\epsilon^{\alpha\mu\beta\nu}P_{1\alpha}P_{3\beta}$$

(62)

$$\eta_{\mu\nu}\,\epsilon^{\alpha\mu\beta\nu} = 0$$

$$P_{4\mu}P_{4\nu}\,\epsilon^{\alpha\mu\beta\nu} = 0$$

$$\Rightarrow |M_4|^2 = \frac{g^4 m_f^2 C_{Vf}^2}{16 M_W^2\cos^2\theta_W}\left(-\eta_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)4\left[(C_V^f)^2+(C_A^f)^2\right]\Big\{-m_{h^0}^2\left[P_1^\nu P_2^\mu + P_1^\mu P_2^\nu - (P_1\cdot P_2)\eta^{\mu\nu}\right]$$

$$+ 2(P_2\cdot P_3)\left[P_1^\nu P_3^\mu + P_1^\mu P_3^\nu - (P_1\cdot P_3)\eta^{\mu\nu}\right]\Big\} \qquad (63)$$

$$|M_4|^2 = \frac{g^4 m_f^2 C_{Vf}^2}{4 M_W^2\cos^2\theta_W}\left[(C_V^f)^2+(C_A^f)^2\right]\Big\{+m_{h^0}^2\left[2(P_1\cdot P_2) - 4(P_1\cdot P_2)\right] - \frac{m_{h^0}^2}{M_Z^2}\left[2(P_1\cdot P_4)(P_2\cdot P_4) - (P_1\cdot P_2)M_Z^2\right]$$

$$- 2(P_2\cdot P_3)\left[2(P_1\cdot P_3) - 4(P_1\cdot P_3)\right] + \frac{2(P_2\cdot P_3)}{M_Z^2}\left[2(P_1\cdot P_4)(P_3\cdot P_4) - (P_1\cdot P_3)M_Z^2\right]\Big\} \qquad (64)$$

$$|M_4|^2 = \frac{g^4 m_f^2 C_{Vf}^2}{4 M_W^2\cos^2\theta_W}\left[(C_V^f)^2+(C_A^f)^2\right]\Big\{-2m_{h^0}^2(P_1\cdot P_2) - \frac{m_{h^0}^2}{M_Z^2}\left[2(P_1\cdot P_4)(P_2\cdot P_4) - (P_1\cdot P_2)M_Z^2\right]$$

$$+ 4(P_1\cdot P_3)(P_2\cdot P_3) + \frac{2(P_2\cdot P_3)}{M_Z^2}\left[2(P_1\cdot P_4)(P_3\cdot P_4) - (P_1\cdot P_3)M_Z^2\right]\Big\}$$

$$|M_4|^2 = \frac{g^4 m_f^2 C_{Vf}^2}{4 M_W^2\cos^2\theta_W}\left[(C_V^f)^2+(C_A^f)^2\right]\Big\{-s\,\cancel{m}_{h^0}^2 - \frac{m_{h^0}^2}{M_Z^2}\left[\frac{(M_Z^2-u)}{2}(\overbrace{s-m_{h^0}^2+u}^{(M_Z^2-t)}) - \frac{s}{\cancel{2}}M_Z^2\right]$$

$$+ (m_{h^0}^2-t)(m_{h^0}^2-u) + \frac{(m_{h^0}^2-u)}{M_Z^2}\left[(M_Z^2-u)\frac{(\overbrace{s-m_{h^0}^2}^{-u-t}-M_Z^2)}{2} - \frac{(m_{h^0}^2-t)}{2}M_Z^2\right]\Big\}$$

$$s+t+u \approx m_{h^0}^2 + M_Z^2$$

$$|M_4|^2 = \frac{g^4 m_f^2 C_{Vf}^2}{4 M_W^2\cos^2\theta_W}\left[(C_V^f)^2+(C_A^f)^2\right]\Big\{-\frac{s}{2}m_{h^0}^2 - \frac{m_{h^0}^2}{2M_Z^2}(M_Z^2-u)(M_Z^2-t)$$

$$+ (m_{h^0}^2-t)(m_{h^0}^2-u) + \frac{(m_{h^0}^2-u)}{2M_Z^2}\left[-uM_Z^2 \cancel{-t}M_Z^2 + u^2 + ut - m_{h^0}^2M_Z^2 + t\cancel{M_Z^2}\right]\Big\}$$

$$|M_4|^2 = \frac{g^4 m_f^2 C_{Vf}^2}{4 M_W^2\cos^2\theta_W}\left[(C_V^f)^2+(C_A^f)^2\right]\Big\{-\frac{s}{2}m_{h^0}^2 - \frac{m_{h^0}^2}{2M_Z^2}(M_Z^4 - tM_Z^2 - uM_Z^2 + ut)$$

$$+ m_{h^0}^4 - u\,m_{h^0}^2 - t\,m_{h^0}^2 + ut + \frac{(m_{h^0}^2-u)}{2M_Z^2}\left[u(m_{h^0}^2-s) - m_{h^0}^2M_Z^2\right]\Big\}$$

$$|M_4|^2 = \frac{g^4 m_f^2 C_{Vf}^2}{4 M_W^2\cos^2\theta_W}\,\frac{\left[(C_V^f)^2+(C_A^f)^2\right]}{2M_Z^2}\Big\{-s\,\cancel{m_{h^0}^2}M_Z^2 - \cancel{m_{h^0}^2}M_Z^4 + t\,\cancel{m_{h^0}^2}M_Z^2 + \cancel{u}\,m_{h^0}^2M_Z^2$$

$$- \cancel{m_{h^0}^2}ut + 2m_{h^0}^4M_Z^2 - 2u\,m_{h^0}^2M_Z^2 - 2t\,\cancel{m_{h^0}^2}M_Z^2 + 2\cancel{u}tM_Z^2 + u\,m_{h^0}^4 - us\,\cancel{m_{h^0}^2} - m_{h^0}^4M_Z^2$$

$$-u^2 m_{h^0}^2 + \cancel{u^2 s} + u\, m_{h^0}^2 M_Z^2 \Big\}$$

$$|M_4|^2 = \frac{g^4 m_f^2 C_{uf}^2}{4 M_W^2 \cos^2\theta_W} \left[\frac{|C_V^f|^2 + |C_A^f|^2}{2 M_Z^2}\right] \Big\{ -s\, m_{h^0}^2 M_Z^2 - m_{h^0}^2 M_Z^4 - m_{h^0}^2 M_Z^2 (m_{h^0}^2 + M_Z^2 - s)$$

$$\cancel{- m_{h^0}^2 ut} + 2 m_{h^0}^4 M_Z^2 + 2ut M_Z^2 + u\, m_{h^0}^2 \underbrace{\cancel{(m_{h^0}^2 - s - u + M_Z^2)}}_{}^{\;t} - m_{h^0}^4 M_Z^2 + u^2 s \Big\}$$

$$|M_4|^2 = \frac{g^4 m_f^2 C_{uf}^2}{8 M_W^4} \left[|C_V^f|^2 + |C_A^f|^2 \right] \Big\{ -\cancel{s\, m_{h^0}^2 M_Z^2} - m_{h^0}^2 M_Z^4 - \cancel{m_{h^0}^4 M_Z^2} - m_{h^0}^2 M_Z^4 + \cancel{s\, m_{h^0}^2 M_Z^2}$$

$$+ \cancel{2 m_{h^0}^4 M_Z^2} + 2ut M_Z^2 - \cancel{m_{h^0}^4 M_Z^2} + u^2 s \Big\}$$

$$|M_4|^2 = \frac{g^4 m_f^2 C_{uf}^2}{8 M_W^4} \left[|C_V^f|^2 + |C_A^f|^2 \right] \left[-2 m_{h^0}^2 M_Z^4 + 2ut M_Z^2 + u^2 s \right] \qquad (65)$$

$$ut = m_{h^0}^2 M_Z^2 + \frac{1}{4} \lambda \sin^2\theta$$

$$|M_4|^2 = \frac{g^4 m_f^2 C_{uf}^2}{8 M_W^4} \left[|C_V^f|^2 + |C_A^f|^2 \right] \left[2 M_Z^2 (ut - m_{h^0}^2 M_Z^2) + u^2 s \right]$$

$$|M_4|^2 = \frac{g^4 m_f^2 C_{uf}^2}{8 M_W^4} \left[|C_V^f|^2 + |C_A^f|^2 \right] \left[\frac{\lambda M_Z^2 \sin^2\theta}{2} + u^2 s \right]$$

$$\boxed{|M_4|^2 = \frac{s g^4 m_f^2 C_{uf}^2}{8 M_W^4} \left[|C_V^f|^2 + |C_A^f|^2 \right] \left[u^2 + \frac{\lambda(s, m_{h^0}^2, M_Z^2) \sin^2\theta\, M_Z^2}{2s} \right]} \qquad (66)$$

$$M_{2a} M_{12b}^* = + \frac{g^4 C_Z m_f}{2 \cos^4\theta_W} \left(\sum_\lambda \varepsilon_{4\mu}^* \varepsilon_{4\nu} \right) \left\{ \frac{A_f C_A^{1*}}{4} (P_1 + P_2 + P_3)^\nu + C_Z^* C_A^f (P_1 + P_2)^\nu \right\}$$

$$\sum_s \overline{V}_2 \gamma^\mu (C_V^f - C_A^f \gamma^5) U_1 \,.\, \overline{U}_1 \gamma^5 V_2 \qquad (67)$$

$$M_{2a} M_{12b}^* = \frac{g^4 C_Z m_f}{2 \cos^4\theta_W} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_Z^2} \right) \left\{ \frac{A_f C_A^{1*}}{4} (P_1 + P_2 + P_3)^\nu + C_Z^* C_A^f (P_1 + P_2)^\nu \right\}$$

$$T_r \left[(\not{P}_1 + m_f) \gamma^5 (\not{P}_2 - m_f) \gamma^\mu (C_V^f - C_A^f \gamma^5) \right] \qquad (68)$$

$$T_r = T_r \left[(\overset{\text{ok}}{\not{P}_1 \gamma^5 \not{P}_2} - m_f \overset{\text{ok}}{\not{P}_1 \gamma^5} + m_f \overset{\text{ok}}{\gamma^5 \not{P}_2} - m_f^2 \gamma^5)(C_V^f \gamma^\mu - C_A^f \gamma^\mu \gamma^5) \right]$$

$$= C_V^f \overset{0}{\cancel{T_r (\not{P}_1 \gamma^5 \not{P}_2 \gamma^\mu)}} - C_A^f \overset{0}{\cancel{T_r (\not{P}_1 \gamma^5 \not{P}_2 \gamma^\mu \gamma^5)}} - m_f C_V^f \overset{0}{\cancel{T_r (\not{P}_1 \gamma^5 \gamma^\mu)}} + m_f C_A^f T_r (\not{P}_1 \gamma^5 \gamma^\mu \gamma^5)$$

$$+ m_f C_V^f \overset{0}{\cancel{T_r (\gamma^5 \not{P}_2 \gamma^\mu)}} - m_f C_A^f T_r (\gamma^5 \not{P}_2 \gamma^\mu \gamma^5) - m_f^2 C_V^f \overset{0}{\cancel{T_r (\gamma^5 \gamma^\mu)}} + m_f^2 C_A^f \overset{0}{\cancel{T_r (\gamma^5 \gamma^\mu \gamma^5)}}$$

$$\Rightarrow$$

$$T_r[\;] = -m_f C_A^f T_r (\not{P}_1 \gamma^\mu) - m_f C_A^f T_r (\not{P}_2 \gamma^\mu) = -4 m_f C_A^f P_1^\mu - 4 m_f C_A^f P_2^\mu = -4 m_f C_A^f (P_1 + P_2)^\mu \qquad (69)$$

because $T_r (\gamma^5 \gamma^\mu \gamma^\nu) = 0$; Trace (odd # of γ's) = 0

$\Rightarrow$

$$M_{2a} M_{12b}^{*} = -\frac{g^4 C_Z m_f^2}{2\cos^4\theta_W}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\left\{\frac{A_f C_A^{\prime *}}{4}(P_1+P_2+P_3)^\nu + C_Z^{*} C_A^{f} (P_1+P_2)^\nu\right\} 4 C_A^{f} (P_1+P_2)^\mu \qquad (70)$$

$$= \frac{-g^4 C_Z m_f^2}{2\cos^4\theta_W}\left\{-\frac{A_f C_A^{\prime *}}{4}(P_1+P_2+P_3)_\mu - C_Z^{*} C_A^{f}(P_1+P_2)_\mu + \frac{A_f C_A^{\prime *}}{4M_Z^2} P_{4\mu}\left[(P_1+P_2+P_3)\cdot P_4\right]\right.$$

$$\left. + \frac{C_Z^{*} C_A^{f} P_{4\mu}}{M_Z^2}\left[(P_1+P_2)\cdot P_4\right]\right\} 4 C_A^{f} (P_1+P_2)^\mu \qquad (71)$$

$$= -\frac{2g^4 C_Z m_f^2 C_A^{f}}{\cos^4\theta_W}\left\{-\frac{A_f C_A^{\prime *}}{4}\left[(P_1+\cancel{P_2}+P_3)\cdot(P_1+P_2)\right] - C_Z^{*} C_A^{f}(P_1+P_2)^2\right.$$

$$\left. + \frac{A_f C_A^{\prime *}}{4M_Z^2}\left[(P_1+P_2)\cdot P_4\right]\left[(\cancel{P_1}+P_2+P_3)\cdot P_4\right] + \frac{C_Z^{*} C_A^{f}}{M_Z^2}\left[(P_1+P_2)\cdot P_4\right]^2\right\} \qquad (72)$$

$$= \frac{-2g^4 C_Z m_f^2 C_A^{f}}{\cos^4\theta_W}\left\{\frac{A_f C_A^{\prime *}}{4M_Z^2}\left[\frac{(s-m_{h^0}^2+M_Z^2)}{2}(s-m_{h^0}^2) - M_Z^2\left(s+\frac{(s+m_{h^0}^2-M_Z^2)}{2}\right)\right]\right.$$

$$\left. + \frac{C_Z^{*} C_A^{f}}{M_Z^2}\left[\frac{1}{4}(s-m_{h^0}^2+M_Z^2)^2 - M_Z^2 s\right]\right\}$$

$$= -\frac{2g^4 C_Z m_f^2 C_A^{f}}{\cos^4\theta_W}\left\{\frac{A_f C_A^{\prime *}}{8M_Z^2}\left[\cancel{s^2} - \cancel{s m_{h^0}^2} - \cancel{s m_{h^0}^2} + \cancel{m_{h^0}^4} + \cancel{s M_Z^2} - \cancel{M_Z^2 m_{h^0}^2} - 3\cancel{s M_Z^2}\right.\right.$$

$$\left.\left. - M_Z^2 \cancel{m_{h^0}^2} + \cancel{M_Z^4}\right] + \frac{C_Z^{*} C_A^{f}}{4M_Z^2}\left[\cancel{s^2} + \cancel{m_{h^0}^4} + \cancel{M_Z^4} - \cancel{2s m_{h^0}^2} + \cancel{2s M_Z^2} - \cancel{2m_{h^0}^2 M_Z^2} - \cancel{4s M_Z^2}\right]\right\}$$

$$= -\frac{g^4 C_Z m_f^2 C_A^{f} M_Z^2}{2M_W^4}\left\{\frac{1}{2}A_f C_A^{\prime *}\left[s^2 - 2s m_{h^0}^2 + m_{h^0}^4 - 2s M_Z^2 - 2M_Z^2 m_{h^0}^2 + M_Z^4\right]\right.$$

$$\left. + C_Z^{*} C_A^{f}\left[s^2 + m_{h^0}^4 + M_Z^4 - 2s m_{h^0}^2 - 2s M_Z^2 - 2m_{h^0}^2 M_Z^2\right]\right\}$$

$$= -\frac{g^4 C_Z m_f^2 C_A^{f} M_Z^2}{2M_W^4}\left\{\frac{1}{2}A_f C_A^{\prime *}\lambda(s, m_{h^0}^2, M_Z^2) + C_Z^{*} C_A^{f}\lambda(s, m_{h^0}^2, M_Z^2)\right\}$$

$$\boxed{M_{2a} M_{12b}^{*} = -\frac{g^4 C_Z m_f^2 M_Z^2 C_A^{f}}{4M_W^4}\lambda(s, m_{h^0}^2, M_Z^2)\left[A_f C_A^{\prime *} + 2C_Z^{*} C_A^{f}\right]} \qquad (73)$$

$$\boxed{M_{12b} M_{2a}^{*} = \frac{-g^4 C_Z^{*} m_f^2 M_Z^2 C_A^{f}}{4M_W^4}\lambda(s, m_{h^0}^2, M_Z^2)\left[A_f C_A^{\prime} + 2C_Z C_A^{f}\right]} \qquad (74)$$

$$\Rightarrow M_{2a} M_{12b}^{*} + M_{12b} M_{2a}^{*} = -\frac{g^4 m_f^2 M_Z^2 C_A^{f}}{2M_W^4}\lambda(s, m_{h^0}^2, M_Z^2)\,\mathrm{Re}\left[C_Z\left[A_f C_A^{\prime *} + 2C_Z^{*} C_A^{f}\right]\right]$$

$$\boxed{M_{2a} M_{12b}^* + M_{12b} M_{2a}^* = -\frac{g^4 m_f^2 M_Z^2 C_A^f}{2 M_W^4} \lambda(s, m_{no}^2, M_Z^2) \left\{ A_f \, \mathrm{Re}(C_Z C_A'^*) + 2|C_Z|^2 C_A^f \right\}} \quad (75)$$

$$M_3 M_{12b}^* = -\frac{g^4 m_f^2 C_{tf}}{4 M_W^2 \cos^2\theta_W} \left(\sum_\lambda \varepsilon_{4\mu}^* \varepsilon_{4\nu} \right) \left\{ \frac{A_f C_A'^*}{4} (P_1 + P_2 + P_3)^\nu + C_Z^* C_A^f (P_1 + P_2)^\nu \right\} .$$

$$\cdot \sum_s \bar{V}_2 \gamma^\mu (C_V^f - C_A^f \gamma^5)(2m_f - \not{P}_3) U_1 . \bar{U}_1 \gamma^5 V_2 \qquad (76)$$

$$M_3 M_{12b}^* = \frac{-g^4 m_f^2 C_{tf}}{4 M_W^2 \cos^2\theta_W} \left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_Z^2} \right) \left\{ \frac{A_f C_A'^*}{4} (P_1 + P_2 + P_3)^\nu + C_Z^* C_A^f (P_1 + P_2)^\nu \right\} .$$

$$\mathrm{Tr} \left\{ (\not{P}_2 - m_f) \gamma^\mu (C_V^f - C_A^f \gamma^5)(2m_f - \not{P}_3)(\not{P}_1 + m_f) \gamma^5 \right\} \qquad (77)$$

Neglecting m_f in the Trace :

$$\mathrm{Tr}\{ \} = -\mathrm{Tr} \left\{ \not{P}_2 \gamma^\mu (C_V^f - C_A^f \gamma^5) \not{P}_3 \not{P}_1 \gamma^5 \right\}$$

$$= -C_V^f \, \mathrm{Tr}(\not{P}_2 \gamma^\mu \not{P}_3 \not{P}_1 \gamma^5) + C_A^f \, \mathrm{Tr}(\not{P}_2 \gamma^\mu \gamma^5 \not{P}_3 \not{P}_1 \gamma^5)$$

$$\mathrm{Tr}\{ \} = -C_V^f \, \mathrm{Tr}(\not{P}_2 \gamma^\mu \not{P}_3 \not{P}_1 \gamma^5) + C_A^f \, \mathrm{Tr}(\not{P}_2 \gamma^\mu \not{P}_3 \not{P}_1) \qquad (78)$$

$$M_3 M_{12b}^* = \frac{-g^4 m_f^2 C_{tf}}{4 M_W^2 \cos^2\theta_W} \left\{ -\frac{A_f C_A'^*}{4} (P_1 + P_2 + P_3)_\mu - C_Z^* C_A^f (P_1 + P_2)_\mu + \frac{A_f C_A'^*}{4 M_Z^2} P_{4\mu} [(P_1 + P_2 + P_3) . P_4 \right.$$

$$\left. + \frac{C_Z^* C_A^f}{M_Z^2} P_{4\mu} [(P_1 + P_2) . P_4] \right\} \left[-C_V^f \, \mathrm{Tr}(\gamma^5 \not{P}_2 \gamma^\mu \not{P}_3 \not{P}_1) + C_A^f \, \mathrm{Tr}(\not{P}_2 \gamma^\mu \not{P}_3 \not{P}_1) \right] \quad (79)$$

$$(P_1 + P_2 + P_3)_\mu \, \mathrm{Tr}(\gamma^5 \not{P}_2 \gamma^\mu \not{P}_3 \not{P}_1) = \mathrm{Tr}[\gamma^5 \not{P}_2 (\not{P}_1 + \not{P}_2 + \not{P}_3) \not{P}_3 \not{P}_1]$$

$$= \mathrm{Tr}(\gamma^5 \not{P}_2 \not{P}_1 \not{P}_3 \not{P}_1) + \mathrm{Tr}(\gamma^5 \not{P}_2 \not{P}_2 \not{P}_3 \not{P}_1) + \mathrm{Tr}(\gamma^5 \not{P}_2 \not{P}_3 \not{P}_3 \not{P}_1)$$

$$= \overset{0}{\cancel{-4i \, \varepsilon^{\mu\nu\rho\sigma} P_{2\mu} P_{1\nu} P_{3\rho} P_{1\sigma}}} + m_f^2 \, \overset{0}{\cancel{\mathrm{Tr}(\gamma^5 \not{P}_3 \not{P}_1)}} + m_{no}^2 \, \overset{0}{\cancel{\mathrm{Tr}(\gamma^5 \not{P}_2 \not{P}_1)}} = 0$$

$$\Rightarrow \boxed{(P_1 + P_2 + P_3)_\mu \, \mathrm{Tr}(\gamma^5 \not{P}_2 \gamma^\mu \not{P}_3 \not{P}_1) = 0} \qquad (80)$$

$$(P_1 + P_2)_\mu \, \mathrm{Tr}(\gamma^5 \not{P}_2 \gamma^\mu \not{P}_3 \not{P}_1) = \overset{0}{\cancel{\mathrm{Tr}(\gamma^5 \not{P}_2 \not{P}_1 \not{P}_3 \not{P}_1)}} + \overset{0}{\cancel{\mathrm{Tr}(\gamma^5 \not{P}_2 \not{P}_2 \not{P}_3 \not{P}_1)}} = 0 .$$

$$\Rightarrow \boxed{(P_1 + P_2)_\mu \, \mathrm{Tr}(\gamma^5 \not{P}_2 \gamma^\mu \not{P}_3 \not{P}_1) = 0} \qquad (81)$$

$$\boxed{P_{4\mu} \, \mathrm{Tr}(\gamma^5 \not{P}_2 \gamma^\mu \not{P}_3 \not{P}_1) = (P_1 + P_2 - P_3)_\mu \, \mathrm{Tr}(\gamma^5 \not{P}_2 \gamma^\mu \not{P}_3 \not{P}_1) = 0} \qquad (82)$$

$$M_3 M_{12b}^{*} = \frac{-g^4 m_f^2 C_{tf} C_A^f}{4 M_W^2 \cos^2\theta_W} \left\{ -\frac{A_f C_A'^{*}}{4} (p_1+p_2+p_3)_\mu - C_Z^{*} C_A^f (p_1+p_2)_\mu + \frac{A_f C_A'^{*}}{4 M_Z^2} p_{4\mu} \cdot \right.$$

$$\left. [(p_1+p_2+p_3)\cdot p_4] + \frac{C_Z^{*} C_A^f}{M_Z^2} p_{4\mu} [(p_1+p_2)\cdot p_4] \right\} Tr(\not{p}_2 \gamma^\mu \not{p}_3 \not{p}_1) \qquad (83)$$

$$Tr(\not{p}_2 \gamma^\mu \not{p}_3 \not{p}_1) = Tr(\gamma^\nu \gamma^\mu \gamma^\alpha \gamma^\beta) p_{2\nu} p_{3\alpha} p_{1\beta}$$

$$= 4[n^{\nu\mu} n^{\alpha\beta} - n^{\nu\alpha} n^{\mu\beta} + n^{\nu\beta} n^{\mu\alpha}] p_{2\nu} p_{3\alpha} p_{1\beta}$$

$$= 4[p_2^\mu (p_1\cdot p_3) - p_1^\mu (p_2\cdot p_3) + p_3^\mu (p_1\cdot p_2)] \qquad (84)$$

$$M_3 M_{12b}^{*} = \frac{-g^4 m_f^2 C_{tf} C_A^f}{M_W^2 \cos^2\theta_W} \left\{ -\frac{A_f C_A'^{*}}{4} [(p_1\cdot p_3)((p_1+p_2+p_3)\cdot p_2) - (p_2\cdot p_3)((p_1+p_2+p_3)\cdot p_1) \right.$$

$$+ (p_1\cdot p_2)((p_1+p_2+p_3)\cdot p_3)] - C_Z^{*} C_A^f [(p_1\cdot p_3)((p_1+p_2)\cdot p_2) - (p_2\cdot p_3)((p_1+p_2)\cdot p_1) + (p_1\cdot p_2)\cdot$$

$$((p_1+p_2)\cdot p_3)] + \frac{A_f C_A'^{*}}{4 M_Z^2} [(p_1+p_2+p_3)\cdot p_4] [(p_1\cdot p_3)(p_2\cdot p_4) - (p_2\cdot p_3)(p_1\cdot p_4) + (p_1\cdot p_2)(p_3\cdot p_4)]$$

$$\left. + \frac{C_Z^{*} C_A^f}{M_Z^2} [(p_1+p_2)\cdot p_4] [(p_1\cdot p_3)(p_2\cdot p_4) - (p_2\cdot p_3)(p_1\cdot p_4) + (p_1\cdot p_2)(p_3\cdot p_4)] \right\} \qquad (85)$$

$$= \frac{-g^4 m_f^2 C_{tf} C_A^f}{M_W^2 \cos^2\theta_W} \left\{ -\frac{A_f C_A'^{*}}{4} [(p_1\cdot p_3)((p_1+p_3)\cdot p_2) - (p_2\cdot p_3)((p_2+p_3)\cdot p_1) \right.$$

$$+ (p_1\cdot p_2)((p_1+p_2+p_3)\cdot p_3)] - C_Z^{*} C_A^f [(p_1\cdot p_3)(p_1\cdot p_2) - \cancel{(p_2\cdot p_3)}(p_1\cdot p_2) + (p_1\cdot p_2)(p_1\cdot p_3)$$

$$+ (p_1\cdot p_2)\cancel{(p_2\cdot p_3)}] + \frac{[(p_1\cdot p_3)(p_2\cdot p_4) - (p_2\cdot p_3)(p_1\cdot p_4) + (p_1\cdot p_2)(p_3\cdot p_4)]}{M_Z^2} \left[\frac{A_f C_A'^{*}}{4} ((p_1+p_2+p_3)\cdot p_4) \right.$$

$$\left.\left. + C_Z^{*} C_A^f [(p_1+p_2)\cdot p_4] \right] \right\}$$

$$= -\frac{g^4 m_f^2 C_{tf} C_A^f}{M_W^2 \cos^2\theta_W} \left\{ -\frac{A_f C_A'^{*}}{4} [2(p_1\cdot p_2)(p_1\cdot p_3) + m_{h^0}^2 (p_1\cdot p_2)] - 2 C_Z^{*} C_A^f (p_1\cdot p_2)(p_1\cdot p_3) \right.$$

$$+ \frac{[(p_1\cdot p_3)(p_2\cdot p_4) - (p_2\cdot p_3)(p_1\cdot p_4) + (p_1\cdot p_2)(p_3\cdot p_4)]}{M_Z^2} \left[\frac{A_f C_A'^{*}}{4} ((p_1+p_2+p_3)\cdot p_4) \right.$$

$$\left.\left. + C_Z^{*} C_A^f [(p_1+p_2)\cdot p_4] \right] \right\} \qquad (86)$$

$$= \frac{-g^4 m_f^2 C_{tf} C_A^f}{M_W^2 \cos^2\theta_W} \left\{ -\frac{A_f C_A'^*}{4} \frac{s}{2} \left[2m_{h^0}^2 - t\right] - C_Z^* C_A^f s \frac{1}{2} (m_{h^0}^2 - t) + \right.$$

$$+ \frac{1}{M_Z^2} \left[\frac{(m_{h^0}^2 - t)}{2} \frac{(u + s - m_{h^0}^2)}{2} - \frac{(m_{h^0}^2 - u)}{2} \frac{(M_Z^2 - u)}{2} + \frac{s}{2} \frac{(s - m_{h^0}^2 - M_Z^2)}{2} \right] \left[\frac{A_f C_A'^*}{4} (s - m_{h^0}^2) \right.$$

$$\left. \left. + C_Z^* C_A^f (s - m_{h^0}^2 + M_Z^2) \frac{1}{2} \right] \right\} \qquad (87)$$

$$s + t + u \approx m_{h^0}^2 + M_Z^2$$

$$= \frac{-g^4 m_f^2 C_{tf} C_A^f}{M_W^2 \cos^2\theta_W} \left\{ -\frac{A_f C_A'^*}{8} s (2m_{h^0}^2 - t) - \frac{C_Z^* C_A^f s}{2} (m_{h^0}^2 - t) + \frac{1}{4M_Z} \left[(s + u - M_Z^2)(s + u - m_{h^0}^2) \right. \right.$$

$$\left. \left. - (m_{h^0}^2 - u)(M_Z^2 - u) + s^2 - s m_{h^0}^2 - s M_Z^2 \right] \left[\frac{A_f C_A'^*}{4} (s - m_{h^0}^2) + \frac{1}{2} C_Z^* C_A^f (s - m_{h^0}^2 + M_Z^2) \right] \right\}$$

$$= \frac{-g^4 m_f^2 C_{tf} C_A^f}{M_W^2 \cos^2\theta_W} \left\{ -\frac{A_f C_A'^*}{8} s (2m_{h^0}^2 - t) - \frac{C_Z^* C_A^f s}{2} (m_{h^0}^2 - t) + \frac{1}{4} \left[\cancel{s^2} + 2\cancel{u}s + \cancel{u^2} - \cancel{m_{h^0}^2 s} \right. \right.$$

$$- \cancel{m_{h^0}^2 u} - \cancel{M_Z^2 s} - \cancel{M_Z^2 u} + \cancel{M_Z^2 m_{h^0}^2} - \cancel{m_{h^0}^2 M_Z^2} + \cancel{u m_{h^0}^2} + \cancel{u M_Z^2} - \cancel{u^2} + \cancel{s^2} - \cancel{s m_{h^0}^2} - \left. \cancel{s M_Z^2} \right] \cdot \frac{1}{M_Z}$$

$$\left. \cdot \left[\frac{A_f C_A'^*}{4} (s - m_{h^0}^2) + \frac{1}{2} C_Z^* C_A^f (s - m_{h^0}^2 + M_Z^2) \right] \right\}$$

$$= -\frac{g^4 m_f^2 C_{tf} C_A^f}{M_W^2 \cos^2\theta_W} \left\{ -\frac{A_f C_A'^*}{8} s (2m_{h^0}^2 - t) - \frac{C_Z^* C_A^f s}{2} (m_{h^0}^2 - t) + \frac{1}{4} \left[\overbrace{2s^2 + 2us - 2s m_{h^0}^2}^{2s(\overbrace{s + u - m_{h^0}^2 - M_Z^2}^{-2st})} \right. \right.$$

$$\left. \left. - 2s M_Z^2 \right] \frac{1}{M_Z^2} \left[\frac{A_f C_A'^*}{4} (s - m_{h^0}^2) + \frac{1}{2} C_Z^* C_A^f (s - m_{h^0}^2 + M_Z^2) \right] \right\} \qquad (88)$$

$$= \frac{-g^4 m_f^2 C_{tf} C_A^f}{M_W^2 \cos^2\theta_W} \left\{ -\frac{A_f C_A'^*}{8} s (2m_{h^0}^2 - t) - \frac{C_Z^* C_A^f s}{2} (m_{h^0}^2 - t) - \frac{st}{2M_Z^2} \left[\frac{A_f C_A'^*}{4} (s - m_{h^0}^2) \right. \right.$$

$$\left. \left. + \frac{1}{2} C_Z^* C_A^f (s - m_{h^0}^2 + M_Z^2) \right] \right\}$$

$$= -\frac{g^4 m_f^2 C_{tf} C_A^f}{M_W^2 \cos^2\theta_W} \left\{ -\frac{A_f C_A'^*}{8} s \left[2m_{h^0}^2 - t + \frac{(st - t m_{h^0}^2)}{M_Z^2} \right] - \frac{C_Z^* C_A^f}{2} s \left[m_{h^0}^2 - t + \right. \right.$$

$$\left. \left. + \frac{t}{2M_Z^2} (s - m_{h^0}^2 + M_Z^2) \right] \right\}$$

$$= \frac{g^4 m_f^2 C_{tf} C_A^f s}{M_W^2 \cos^2\theta_W} \left\{ \frac{A_f C_A'^*}{8} \left[\frac{2m_{h^0}^2 M_Z^2 - ut - t^2}{M_Z^2} \right] + \frac{C_Z^* C_A^f}{2} \left[\frac{\overbrace{2m_{h^0}^2 M_Z^2 - t M_Z^2 + st - t m_{h^0}^2}^{2m_{h^0}^2 M_Z^2 - ut - t^2}}{2M_Z^2} \right] \right.$$

$$= \frac{g^4 m_f^2 C_{tf} C_A^f s}{8 M_W^4} (2m_{h^0}^2 M_Z^2 - ut - t^2) \left[A_f C_A'^* + 2 C_Z^* C_A^f \right]$$

$$\boxed{M_3 M_{12b}^* = \frac{g^4 m_f^2 C_{tf} C_A^f s}{8 M_W^4} \left(m_{h^0}^2 M_Z^2 - \frac{1}{4} \lambda \sin^2\theta - t^2 \right) \left[A_f C_A'^* + 2 C_Z^* C_A^f \right]} \quad (89)$$

$$\boxed{M_{12b} M_3^* = \frac{g^4 m_f^2 C_{tf} C_A^f s}{8 M_W^4} \left(m_{h^0}^2 M_Z^2 - \frac{1}{4} \lambda \sin^2\theta - t^2 \right) \left[A_f C_A' + 2 C_Z C_A^f \right]} \quad (90)$$

$$\boxed{\begin{aligned} M_3 M_{12b}^* + M_{12b} M_3^* = {} & \frac{g^4 m_f^2 C_{tf} C_A^f s}{4 M_W^4} \left(m_{h^0}^2 M_Z^2 - \frac{1}{4} \lambda (s, m_{h^0}^2, M_Z^2) \sin^2\theta - t^2 \right) \cdot \\ & \cdot \mathrm{Re} \left[A_f C_A' + 2 C_Z C_A^f \right] \end{aligned}} \quad (91) \quad \checkmark$$

$$M_4 M_{12b}^* = \frac{-g^4 m_f^2 C_{Vf}}{4 M_W^2 \cos^2\theta_W} \left(\sum_\lambda \varepsilon_{4\mu}^* \varepsilon_{4\nu} \right) \sum_s \left(\bar{V}_2 (2 m_f + \not{p}_3) \gamma^\mu (C_V^f - C_A^f \gamma^5) U_1 \bar{U}_1 \gamma^5 V_2 \right) .$$

$$\cdot \left\{ \frac{A_f C_A'^*}{4} (P_1 + P_2 + P_3)^\nu + C_Z^* C_A^f (P_1 + P_2)^\nu \right\} \quad (92)$$

$$M_4 M_{12b}^* = \frac{-g^4 m_f^2 C_{Vf}}{4 M_W^2 \cos^2\theta_W} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_Z^2} \right) \left\{ \frac{A_f C_A'^*}{4} (P_1 + P_2 + P_3)^\nu + C_Z^* C_A^f (P_1 + P_2)^\nu \right\} .$$

$$\cdot \mathrm{Tr} \left[(\not{P}_1 + m_f) \gamma^5 (\not{P}_2 - m_f)(2 m_f + \not{P}_3) \gamma^\mu (C_V^f - C_A^f \gamma^5) \right] \quad (93)$$

neglecting m_f inside the trace :

$$\mathrm{Tr}[\quad] = \mathrm{Tr} \left[\not{P}_1 \gamma^5 \not{P}_2 \not{P}_3 \gamma^\mu (C_V^f - C_A^f \gamma^5) \right]$$

$$= \mathrm{Tr} (\not{P}_1 \gamma^5 \not{P}_2 \not{P}_3 \gamma^\mu) C_V^f - C_A^f \mathrm{Tr} (\not{P}_1 \gamma^5 \not{P}_2 \not{P}_3 \gamma^\mu \gamma^5)$$

$$= - C_V^f \mathrm{Tr} (\gamma^5 \not{P}_1 \not{P}_2 \not{P}_3 \gamma^\mu) + C_A^f \mathrm{Tr} (\not{P}_1 \not{P}_2 \not{P}_3 \gamma^\mu) \quad (94)$$

$$M_4 M_{12b}^* = \frac{-g^4 m_f^2 C_{Vf}}{4 M_W^2 \cos^2\theta_W} \left\{ -\frac{A_f C_A'^*}{4} (P_1 + P_2 + P_3)_\mu - C_Z^* C_A^f (P_1 + P_2)_\mu + \frac{A_f C_A'^*}{4} \frac{P_{4\mu}}{M_Z^2} \left[(P_1 + P_2 + P_3) . P_4 \right] \right.$$

$$\left. + \frac{C_Z^* C_A^f}{M_Z^2} P_{4\mu} \left[(P_1 + P_2) . P_4 \right] \right\} \left[- C_V^f \mathrm{Tr} (\gamma^5 \not{P}_1 \not{P}_2 \not{P}_3 \gamma^\mu) + C_A^f \mathrm{Tr} (\not{P}_1 \not{P}_2 \not{P}_3 \gamma^\mu) \right] \quad (95)$$

$$(P_1 + P_2 + P_3)_\mu \mathrm{Tr} (\gamma^5 \not{P}_1 \not{P}_2 \not{P}_3 \gamma^\mu) = \mathrm{Tr} (\gamma^5 \overset{0}{\cancel{\not{P}_1 \not{P}_2}} \not{P}_3 \not{P}_1) + \mathrm{Tr} (\gamma^5 \overset{0}{\cancel{\not{P}_1 \not{P}_2}} \not{P}_3 \not{P}_2)$$

$$+ \mathrm{Tr} (\gamma^5 \overset{0}{\cancel{\not{P}_1 \not{P}_2}} \not{P}_3 \not{P}_3) = 0$$

$$(p_1+p_2)_\mu \mathrm{Tr}[\gamma^5 \not{p}_1 \not{p}_2 \not{p}_3 \gamma^\mu] = \mathrm{Tr}(\gamma^5 \not{p}_1 \cancelto{0}{\not{p}_2} \not{p}_3 \not{p}_1) + \mathrm{Tr}(\gamma^5 \not{p}_1 \cancelto{0}{\not{p}_2} \not{p}_3 \not{p}_2) = 0.$$

$$p_{4\mu} \mathrm{Tr}(\gamma^5 \not{p}_1 \not{p}_2 \not{p}_3 \gamma^\mu) = (p_1+p_2-p_3)_\mu \mathrm{Tr}(\gamma^5 \not{p}_1 \not{p}_2 \not{p}_3 \gamma^\mu) = \mathrm{Tr}(\gamma^5 \cancelto{0}{\not{p}_1} \not{p}_2 \not{p}_3 \not{p}_1) + \mathrm{Tr}(\gamma^5 \cancelto{0}{\not{p}_1} \not{p}_2 \not{p}_3 \not{p}_2)$$

$$- \mathrm{Tr}(\gamma^5 \not{p}_1 \cancelto{0}{\not{p}_2} \not{p}_3 \not{p}_3) = 0.$$

$\Rightarrow$

$$M_4 M_{12b}^* = \frac{-g^4 m_f^2 C_{vf} C_A^f}{\cancel{4} M_W^2 \cos^2\theta_W} \{ -\frac{A_f C_A^{\prime *}}{4}(p_1+p_2+p_3)_\mu - C_Z^* C_A^f (p_1+p_2)_\mu + \frac{A_f C_A^{\prime *}}{4}\frac{p_{4\mu}}{M_Z^2}[(p_1+p_2+p_3)\cdot p_4]$$

$$+ \frac{C_Z^* C_A^f}{M_Z^2} p_{4\mu}[(p_1+p_2)\cdot p_4]\} \cancel{4}[\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha}] p_{1\nu} p_{2\alpha} p_{3\beta} \qquad (96)$$

$$M_4 M_{12b}^* = \frac{-g^4 m_f^2 C_{vf} C_A^f}{M_W^2 \cos^2\theta_W} \{ -\frac{A_f C_A^{\prime *}}{4}(p_1+p_2+p_3)_\mu - C_Z^* C_A^f (p_1+p_2)_\mu + \frac{A_f C_A^{\prime *}}{4}\frac{p_{4\mu}}{M_Z^2}[(p_1+p_2+p_3)\cdot p_4]$$

$$+ \frac{C_Z^* C_A^f}{M_Z^2} p_{4\mu}[(p_1+p_2)\cdot p_4]\} [p_1^\mu (p_2\cdot p_3) - p_2^\mu (p_1\cdot p_3) + p_3^\mu (p_1\cdot p_2)] \qquad (97)$$

$$M_4 M_{12b}^* = \frac{-g^4 m_f^2 C_{vf} C_A^f}{M_W^2 \cos^2\theta_W} \{ -\frac{A_f C_A^{\prime *}}{4}[(p_1\cdot(p_2+p_3))(p_2\cdot p_3) - (p_2\cdot(p_1+\cancel{p_3}))(p_1\cdot p_3) + ((p_1+p_2+p_3)\cdot p_3)(p_1\cdot p_2)] -$$

$$- C_Z^* C_A^f [(p_1\cdot p_2)(p_2\cdot p_3) - (p_1\cdot p_2)(p_1\cdot p_3) \cancel{+} ((p_1+p_2)\cdot p_3)(p_1\cdot p_2)] + \frac{A_f C_A^{\prime *}}{4M_Z^2}[(p_1+p_2+p_3)\cdot p_4][(p_1\cdot p_4)(p_2\cdot p_3)$$

$$- (p_2\cdot p_4)(p_1\cdot p_3) + (p_3\cdot p_4)(p_1\cdot p_2)] + \frac{C_Z^* C_A^f}{M_Z^2}[(p_1+p_2)\cdot p_4][(p_1\cdot p_4)(p_2\cdot p_3) - (p_2\cdot p_4)(p_1\cdot p_3) + (p_3\cdot p_4)(p_1\cdot p_2)]\}$$

$$= \frac{-g^4 m_f^2 C_{vf} C_A^f}{M_W^2 \cos^2\theta_W} \{ -\frac{A_f C_A^{\prime *}}{4}(p_1\cdot p_2)[2(p_2\cdot p_3) + m_{h^0}^2] - 2 C_Z^* C_A^f (p_1\cdot p_2)(p_2\cdot p_3) +$$

$$+ \frac{A_f C_A^{*}}{4M_Z^2}[(p_1+p_2+p_3)\cdot p_4][(p_1\cdot p_4)(p_2\cdot p_3) - (p_2\cdot p_4)(p_1\cdot p_3) + (p_3\cdot p_4)(p_1\cdot p_2)] + \frac{C_Z^* C_A^f}{M_Z^2}[(p_1+p_2)\cdot p_4] \cdot$$

$$\cdot [(p_1\cdot p_4)(p_2\cdot p_3) - (p_2\cdot p_4)(p_1\cdot p_3) + (p_3\cdot p_4)(p_1\cdot p_2)]\}$$

$$= \frac{-g^4 m_f^2 C_{vf} C_A^f}{M_W^2 \cos^2\theta_W} \{ -\frac{A_f C_A^{\prime *} s}{8}[2m_{h^0}^2 - u] - \frac{C_Z^* C_A^f}{2} s (m_{h^0}^2 - u) + [\frac{(M_Z^2-u)}{2}\frac{(m_{h^0}^2-u)}{2}$$

$$- \frac{(s-m_{h^0}^2+u)}{2}\frac{(m_{h^0}^2-t)}{2} + \frac{(s-m_{h^0}^2-M_Z^2)}{2}\frac{s}{2}][\frac{A_f C_A^{\prime *}}{4M_Z^2}(s-m_{h^0}^2) + \frac{C_Z^* C_A^f}{M_Z^2}\frac{(s-m_{h^0}^2+M_Z^2)}{2}]\}$$

$$= \frac{-g^4 m_f^2 C_{vf} C_A^f}{M_W^2 \cos^2\theta_W} \{ -\frac{A_f C_A^{\prime *}}{8} s (2m_{h^0}^2 - u) - \frac{C_Z^* C_A^f}{2} s (m_{h^0}^2 - u) + \frac{1}{4}[(s+t-m_{h^0}^2)(m_{h^0}^2-u)$$

$$- (s-m_{h^0}^2+u)(m_{h^0}^2-t) + s^2 - s m_{h^0}^2 - s M_Z^2][\frac{A_f C_A^{\prime *}}{4M_Z^2}(s-m_{h^0}^2) + \frac{C_Z^* C_A^f}{2M_Z^2}(s-m_{h^0}^2+M_Z^2)]\}$$

$$= -\frac{g^4 m_f^2 C_{vf} C_A^f}{M_W^2 \cos^2\theta_W}\Big\{ -\frac{A_f C_A'^* s}{8}(2m_{h^0}^2-u) - \frac{C_Z^* C_A^f}{2} s(m_{h^0}^2-u) + \frac{1}{4}\big[\cancel{s m_{h^0}^2} - \cancel{us} + \cancel{t m_{h^0}^2} - \cancel{u}$$

$$- \cancel{m_{h^0}^4} + \cancel{u m_{h^0}^2} - \cancel{s m_{h^0}^2} + \cancel{st} + \cancel{m_{h^0}^4} - \cancel{m_{h^0}^2 t} - \cancel{u m_{h^0}^2} + \cancel{ut} + \cancel{s^2} - \cancel{s m_{h^0}^2} - \cancel{s M_Z^2}\big]\cdot\Big[\frac{A_f C_A'^*}{4M_Z^2}\cdot$$

$$\cdot(s-m_{h^0}^2) + \frac{C_Z^* C_A^f}{2M_Z^2}(s-m_{h^0}^2+M_Z^2)\Big]\Big\}$$

$$-us + s(\overbrace{s+t-m_{h^0}^2-M_Z^2}^{-us})$$

$$= -\frac{g^4 m_f^2 C_{vf} C_A^f}{M_W^2 \cos^2\theta_W}\Big\{ -\frac{A_f C_A'^* s}{8}(2m_{h^0}^2-u) - \frac{C_Z^* C_A^f}{2} s(m_{h^0}^2-u) + \frac{1}{4}(\overbrace{-us + st + s^2 - s m_{h^0}^2 - s M_Z}$$

$$\Big[\frac{A_f C_A'^*}{4M_Z^2}(s-m_{h^0}^2) + \frac{C_Z^* C_A^f}{2M_Z^2}(s-m_{h^0}^2+M_Z^2)\Big]\Big\}$$

$$= \frac{-g^4 m_f^2 C_{vf} C_A^f}{M_W^2 \cos^2\theta_W}\Big\{ -\frac{A_f C_A'^* s}{8}(2\cancel{m_{h^0}^2}-u) - \cancel{\frac{C_Z^* C_A^f s}{2}(m_{h^0}^2-u)} - \frac{us}{2}\Big[\frac{A_f C_A'^*}{\cancel{4M_Z^2}}(s-m_{h^0}^2) +$$

$$+ \frac{C_Z^* C_A^f}{2M_Z^2}(s-m_{h^0}^2+M_Z^2)\Big]\Big\}$$

$$= -\frac{g^4 m_f^2 C_{vf} C_A^f}{8 M_W^4}\Big\{ A_f C_A'^*\big[-us(s-m_{h^0}^2) - sM_Z^2(2m_{h^0}^2-u)\big] - C_Z^* C_A^f\big[4sM_Z^2(m_{h^0}^2-u)$$

$$+ 2us(s-m_{h^0}^2+M_Z^2)\big]\Big\}$$

$$= -\frac{g^4 m_f^2 C_{vf} C_A^f s}{8M_W^4}\Big\{-A_f C_A'^*(us - u m_{h^0}^2 + 2m_{h^0}^2 M_Z^2 - uM_Z^2) - 2C_Z^* C_A^f(2m_{h^0}^2 M_Z^2 - 2uM_Z^2$$

$$+ us - u m_{h^0}^2 + uM_Z^2)\Big\}$$

$$= -\frac{g^4 m_f^2 C_{vf} C_A^f s}{8M_W^4}\Big\{-A_f C_A'^*(2m_{h^0}^2 M_Z^2 + u(-u-t)) - 2C_Z^* C_A^f(2m_{h^0}^2 M_Z^2 - 2\cancel{M_Z^2}u$$

$$+ u(\cancel{M_Z^2}-u-t) + u\cancel{M_Z^2})\Big\}$$

$$= \frac{-g^4 m_f^2 C_{vf} C_A^f s}{8M_W^4}\Big\{-A_f C_A'^*(2m_{h^0}^2 M_Z^2 - u^2 - ut) - 2C_Z^* C_A^f(2m_{h^0}^2 M_Z^2 - u^2 - ut)\Big\}$$

$$\Rightarrow M_4 M_{12b}^* = \frac{g^4 m_f^2 C_{vf} C_A^f s}{8M_W^4}(2m_{h^0}^2 M_Z^2 - u^2 - ut)(A_f C_A'^* + 2C_Z^* C_A^f) \quad (98)$$

$$\boxed{M_4 M_{12b}^* = \frac{g^4 m_f^2 C_{vf} C_A^f s}{8M_W^4}(m_{h^0}^2 M_Z^2 - \frac{1}{4}\lambda \sin^2\theta - u^2)(A_f C_A'^* + 2C_Z^* C_A^f)} \quad (99)$$

$$\boxed{M_{12b} M_4^* = \frac{g^4 m_f^2 C_{vf} C_A^f s}{8M_W^4}(m_{h^0}^2 M_Z^2 - \frac{1}{4}\lambda \sin^2\theta - u^2)(A_f C_A' + 2C_Z C_A^f)} \quad (100)$$

$$\therefore \boxed{M_4 M_{12b}^* + M_{12b} M_4^* = \frac{g^4 m_f^2 C_{vf} C_A^f s}{4M_W^4}(m_{h^0}^2 M_Z^2 - \frac{1}{4}\lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta - u^2)\cdot \operatorname{Re}(A_f C_A' + 2C_Z C_A^f)} \quad (101)$$

$$M_3 M_{2a}^{*} = -\frac{g^4 m_f M_Z C_{tf} C_Z^{*}}{8 M_W \cos^3\theta_W}\left(\sum_\lambda \varepsilon_{4\mu}^{*}\varepsilon_{4\nu}\right)\sum_s \bar{V}_2 \gamma^\mu (C_V^f - C_A^f\gamma^5)(2m_f - \not{p}_3) U_1 . \bar{U}_1 (C_V^f + C_A^f\gamma^5)$$

$$\gamma^\nu V_2 \qquad (102)$$

$$M_3 M_{2a}^{*} = -\frac{g^4 m_f M_Z C_{tf} C_Z^{*}}{8 M_W \cos^3\theta_W}\left(-\eta_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\mathrm{Tr}\left[(\not{p}_1 + m_f)(C_V^f + C_A^f\gamma^5)\gamma^\nu(\overset{\downarrow}{\not{p}_2} - m_f)\gamma^\mu\right.$$

$$\left.(C_V^f - C_A^f\gamma^5)(2m_f - \not{p}_3)\right] \qquad (103)$$

$$\mathrm{Tr}[\;] = \mathrm{Tr}\left[(C_V^f\not{p}_1\gamma^\nu + C_A^f\not{p}_1\gamma^5\gamma^\nu + m_f C_V^f\gamma^\nu + m_f C_A^f\gamma^5\gamma^\nu)(C_V^f\not{p}_2\gamma^\mu - C_A^f\not{p}_2\gamma^\mu\gamma^5\right.$$

$$\left. - m_f C_V^f\gamma^\mu + m_f C_A^f\gamma^\mu\gamma^5)(2m_f - \not{p}_3)\right] \qquad (104)$$

$$= \mathrm{Tr}\left[(C_V^f\overset{\downarrow}{\not{p}_1}\gamma^\nu + C_A^f\overset{\downarrow}{\not{p}_1}\gamma^5\gamma^\nu + m_f \overset{\downarrow}{C_V^f}\gamma^\nu + m_f \overset{\downarrow}{C_A^f}\gamma^5\gamma^\nu)(2m_f C_V^f\not{p}_2\gamma^\mu - 2m_f C_A^f\not{p}_2\gamma^\mu\gamma^5\right.$$

$$\left. - 2m_f^2 C_V^f\gamma^\mu + 2m_f^2 C_A^f\gamma^\mu\gamma^5 - C_V^f\not{p}_2\gamma^\mu\not{p}_3 + C_A^f\not{p}_2\gamma^\mu\gamma^5\not{p}_3 + m_f C_V^f\gamma^\mu\not{p}_3 - m_f C_A^f\gamma^\mu\gamma^5\not{p}_3)\right]$$

$$= 2m_f (C_V^f)^2\,\mathrm{Tr}(\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu) - 2m_f C_V^f C_A^f\,\mathrm{Tr}(\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu\gamma^5) - 2m_f^2 (C_V^f)^2\,\overset{0}{\cancel{\mathrm{Tr}(\not{p}_1\gamma^\nu\gamma^\mu)}}$$

$$+ 2m_f^2 C_V^f C_A^f\,\overset{0}{\cancel{\mathrm{Tr}(\not{p}_1\gamma^\nu\gamma^\mu\gamma^5)}} - (C_V^f)^2\,\overset{0}{\cancel{\mathrm{Tr}(\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu\not{p}_3)}} + C_V^f C_A^f\,\overset{0}{\cancel{\mathrm{Tr}(\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu\gamma^5\not{p}_3)}}$$

$$+ m_f (C_V^f)^2\,\mathrm{Tr}(\not{p}_1\gamma^\nu\gamma^\mu\not{p}_3) - m_f C_V^f C_A^f\,\mathrm{Tr}(\not{p}_1\gamma^\nu\gamma^\mu\gamma^5\not{p}_3) + 2m_f C_V^f C_A^f\,\mathrm{Tr}(\not{p}_1\gamma^5\gamma^\nu\not{p}_2\gamma^\mu)$$

$$- 2m_f (C_A^f)^2\,\mathrm{Tr}(\not{p}_1\gamma^5\gamma^\nu\not{p}_2\gamma^\mu\gamma^5) - 2m_f^2 C_V^f C_A^f\,\overset{0}{\cancel{\mathrm{Tr}(\not{p}_1\gamma^5\gamma^\nu\gamma^\mu)}} + 2m_f^2 (C_A^f)^2\,\overset{0}{\cancel{\mathrm{Tr}(\not{p}_1\gamma^5\gamma^\nu\gamma^\mu\gamma^5)}}$$

$$- C_V^f C_A^f\,\overset{0}{\cancel{\mathrm{Tr}(\not{p}_1\gamma^5\gamma^\nu\not{p}_2\gamma^\mu\not{p}_3)}} + (C_A^f)^2\,\overset{0}{\cancel{\mathrm{Tr}(\not{p}_1\gamma^5\gamma^\nu\not{p}_2\gamma^\mu\gamma^5\not{p}_3)}} + m_f C_V^f C_A^f\,\mathrm{Tr}(\not{p}_1\gamma^5\gamma^\nu\gamma^\mu\not{p}_3)$$

$$- m_f (C_A^f)^2\,\mathrm{Tr}(\not{p}_1\gamma^5\gamma^\nu\gamma^\mu\gamma^5\not{p}_3) + 2m_f^2 (C_V^f)^2\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^\nu\not{p}_2\gamma^\mu)}} - 2m_f^2 C_V^f C_A^f\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^\nu\not{p}_2\gamma^\mu\gamma^5)}}$$

$$- 2m_f^3 (C_V^f)^2\,\mathrm{Tr}(\gamma^\nu\gamma^\mu) + 2m_f^3 C_V^f C_A^f\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^5\gamma^\nu\gamma^\mu)}} - m_f (C_V^f)^2\,\mathrm{Tr}(\gamma^\nu\not{p}_2\gamma^\mu\not{p}_3)$$

$$+ m_f C_V^f C_A^f\,\mathrm{Tr}(\gamma^\nu\not{p}_2\gamma^\mu\gamma^5\not{p}_3) + m_f^2 (C_V^f)^2\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^\nu\gamma^\mu\not{p}_3)}} - m_f^2 C_V^f C_A^f\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^\nu\gamma^\mu\gamma^5\not{p}_3)}}$$

$$+ 2m_f^2 C_V^f C_A^f\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_2\gamma^\mu)}} - 2m_f^2 (C_A^f)^2\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_2\gamma^\mu\gamma^5)}} - 2m_f^3 C_V^f C_A^f\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^5\gamma^\nu\gamma^\mu)}}$$

$$+ 2m_f^3 (C_A^f)^2\,\mathrm{Tr}(\gamma^5\gamma^\nu\gamma^\mu\gamma^5) - m_f C_V^f C_A^f\,\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_2\gamma^\mu\not{p}_3) + m_f (C_A^f)^2\,\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_2\gamma^\mu\gamma^5\not{p}_3)$$

$$+ m_f^2 C_V^f C_A^f\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^5\gamma^\nu\gamma^\mu\not{p}_3)}} - m_f^2 (C_A^f)^2\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^5\gamma^\nu\gamma^\mu\gamma^5\not{p}_3)}}$$

$$= 2m_f\left[(C_V^f)^2 + (C_A^f)^2\right]\mathrm{Tr}(\gamma^\mu\not{p}_1\gamma^\nu\not{p}_2) - 4m_f C_V^f C_A^f\,\mathrm{Tr}(\gamma^5\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu)$$

$$+ m_f\left[(C_V^f)^2 - (C_A^f)^2\right]\mathrm{Tr}(\not{p}_1\gamma^\nu\gamma^\mu\not{p}_3) - 2m_f^3\left[(C_V^f)^2 - (C_A^f)^2\right]\mathrm{Tr}(\gamma^\nu\gamma^\mu)$$

$$- m_f\left[(C_V^f)^2 + (C_A^f)^2\right]\mathrm{Tr}(\gamma^\nu\not{p}_2\gamma^\mu\not{p}_3) - 2m_f C_V^f C_A^f\,\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_2\gamma^\mu\not{p}_3)$$

$$\mathrm{Tr}(\not{p}_1\gamma^\nu\gamma^\mu\not{p}_3) = \mathrm{Tr}\left((-\gamma^\nu\not{p}_1 + 2P_1^\nu)\gamma^\mu\not{p}_3\right) = -\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_3) + 2P_1^\nu\,\overset{8P_1^\nu P_3^\mu}{\mathrm{Tr}(\gamma^\mu\not{p}_3)}$$

$\Rightarrow$

$$Tr[\ ] = 8m_f\,[|c_V^f|^2 + |c_A^f|^2]\,(P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - (P_1\cdot P_2)\eta^{\mu\nu}) + 16 i m_f\, c_V^f c_A^f\, \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} P_{2\beta}$$

$$-4m_f\,[|c_V^f|^2 - |c_A^f|^2]\,(P_1^\nu P_3^\mu + P_1^\mu P_3^\nu - (P_1\cdot P_3)\eta^{\mu\nu}) + 8m_f\,[|c_V^f|^2 - |c_A^f|^2]\,P_1^\nu P_3^\mu$$

$$-8m_f^3\,[|c_V^f|^2 - |c_A^f|^2]\,\eta^{\mu\nu} - 4m_f\,[|c_V^f|^2 + |c_A^f|^2]\,(P_2^\nu P_3^\mu + P_2^\mu P_3^\nu - (P_2\cdot P_3)\eta^{\mu\nu})$$

$$+ 8 i m_f\, c_V^f c_A^f\, \varepsilon^{\nu\alpha\mu\beta} P_{2\alpha} P_{3\beta}\,. \qquad (105)$$

$$\eta_{\mu\nu}\varepsilon^{\alpha\nu\beta\mu} = 0$$

$$P_{4\mu} P_{4\nu}\varepsilon^{\alpha\nu\beta\mu} = 0$$

$$\Rightarrow M_3 M_{2a}^* = -\frac{g^4 m_f M_Z C_{tf} C_Z^*}{8 M_W \cos^3\theta_W}\Big\{ 16 m_f\,[|c_V^f|^2 + |c_A^f|^2]\,(P_1\cdot P_2) - 16 m_f\,[|c_V^f|^2 - |c_A^f|^2]\,(P_1\cdot P_3)$$

$$+ 32 m_f^3\,[|c_V^f|^2 - |c_A^f|^2] - 8m_f\,[|c_V^f|^2 + |c_A^f|^2]\,(P_2\cdot P_3) + \frac{1}{M_Z^2}\Big\{ 8m_f\,[|c_V^f|^2 + |c_A^f|^2]\,(2(P_1\cdot P_4)$$

$$(P_2\cdot P_4) - M_Z^2 (P_1\cdot P_2)) - 4m_f\,[|c_V^f|^2 - |c_A^f|^2]\,(2(P_1\cdot P_4)(P_3\cdot P_4) - M_Z^2(P_1\cdot P_3)) + 8m_f\,[|c_V^f|^2 - |c_A^f|^2]\,(P_1\cdot P_4)\times$$

$$(P_3\cdot P_4) - 8m_f^3 M_Z^2\,[|c_V^f|^2 - |c_A^f|^2] - 4m_f\,[|c_V^f|^2 + |c_A^f|^2]\,(2(P_2\cdot P_4)(P_3\cdot P_4) - M_Z^2(P_2\cdot P_3))\Big\}\Big\} \qquad (106)$$

$$= \frac{-g^4 m_f M_Z C_{tf} C_Z^*}{8 M_W \cos^3\theta_W}\Big\{ 16 m_f\,[|c_V^f|^2 + |c_A^f|^2]\,(P_1\cdot P_2) - 16 m_f\,[|c_V^f|^2 - |c_A^f|^2]\,(P_1\cdot P_3)$$

$$+ 32 m_f^3\,[|c_V^f|^2 - |c_A^f|^2] - 8m_f\,[|c_V^f|^2 + |c_A^f|^2]\,(P_2\cdot P_3) + \frac{1}{M_Z^2}\Big\{ 8m_f\,[|c_V^f|^2 + |c_A^f|^2]\times$$

$$(2(P_1\cdot P_4)(P_2\cdot P_4) - M_Z^2(P_1\cdot P_2)) + 4m_f M_Z^2\,[|c_V^f|^2 - |c_A^f|^2]\,(P_1\cdot P_3) - 8m_f^3 M_Z^2\,[|c_V^f|^2 - |c_A^f|^2]$$

$$-4m_f\,[|c_V^f|^2 + |c_A^f|^2]\,(2(P_2\cdot P_4)(P_3\cdot P_4) - M_Z^2(P_2\cdot P_3))\Big\}\Big\}$$

$$= \frac{-g^4 m_f^2 M_Z C_{tf} C_Z^*}{8 M_W \cos^3\theta_W}\,4\Big\{ 2\,[|c_V^f|^2 + |c_A^f|^2]\,(P_1\cdot P_2) - 3\,[|c_V^f|^2 - |c_A^f|^2]\,(P_1\cdot P_3)$$

$$+ 6 m_f^2\,[|c_V^f|^2 - |c_A^f|^2] - [|c_V^f|^2 + |c_A^f|^2]\,(P_2\cdot P_3) + \frac{[|c_V^f|^2 + |c_A^f|^2]}{M_Z^2}\,[4(P_1\cdot P_4)(P_2\cdot P_4)$$

$$-2(P_2\cdot P_4)(P_3\cdot P_4)]\Big\} \qquad (107)$$

neglecting terms with m_f^2 inside $\{\ \}$

$$M_3 M_{2a}^* = -\frac{g^4 m_f^2 M_Z C_{tf} C_Z^*}{2 M_W \cos^3\theta_W}\Big\{ [|c_V^f|^2 + |c_A^f|^2]\,s - 3\,[|c_V^f|^2 - |c_A^f|^2]\,\frac{(m_{h^0}^2 - t)}{2}$$

$$- [|c_V^f|^2 + |c_A^f|^2]\,\frac{(m_{h^0}^2 - u)}{2} + \frac{[|c_V^f|^2 + |c_A^f|^2]}{M_Z^2}\Big[(M_Z^2 - u)(s - m_{h^0}^2 + u) - (s - m_{h^0}^2 + u)\times$$

$$\frac{(s - m_{h^0}^2 - M_Z^2)}{2}\Big]\Big\} \qquad (108)$$

$$= \frac{-g^4 m_f^2 M_Z C_{tf} C_Z^*}{2M_W\cos^3\theta_W}\Big\{ [|C_V^f|^2 + |C_A^f|^2]\Big\{ s - \frac{(m_{h^0}^2-u)}{2} + \frac{1}{M_Z^2}(s-m_{h^0}^2+u)(M_Z^2-u)$$

$$- \frac{(s-m_{h^0}^2+u)}{2M_Z^2}(s-m_{h^0}^2-M_Z^2)\Big\} - 3[|C_V^f|^2 - |C_A^f|^2]\frac{(m_{h^0}^2-t)}{2}\Big\}$$

$$= \frac{-g^4 m_f^2 M_Z C_{tf} C_Z^*}{2M_W\cos^3\theta_W}\Big\{ [|C_V^f|^2 + |C_A^f|^2]\Big\{ s - \frac{(m_{h^0}^2-u)}{2} + \frac{1}{M_Z^2}(M_Z^2-t)(M_Z^2-u)$$

$$+ \frac{(M_Z^2-t)(u+t)}{2M_Z^2}\Big\} - 3[|C_V^f|^2 - |C_A^f|^2]\frac{(m_{h^0}^2-t)}{2}\Big\}$$

$$= -\frac{g^4 m_f^2 M_Z C_{tf} C_Z^*}{4M_W^3\cos\theta_W}\Big\{[|C_V^f|^2 + |C_A^f|^2]\Big\{2s\cancel{M_Z}^2 - m_{h^0}^2\cancel{M_Z}^2 + u\cancel{M_Z}^2 + 2\cancel{M_Z}^4 - 2u\cancel{M_Z}^2 - 2t\cancel{M_Z}^2$$

$$+2\cancel{u}t + \xcancel{uM_Z^2} + t\cancel{M_Z}^2 - \cancel{u}t - \cancel{t}^2\Big\} - 3[|C_V^f|^2 - |C_A^f|^2]M_Z^2(m_{h^0}^2-t)\Big\}$$

$$= -\frac{g^4 m_f^2 M_Z C_{tf} C_Z^*}{4M_W^3\cos\theta_W}\Big\{[|C_V^f|^2 + |C_A^f|^2]\Big\{2s\cancel{M_Z}^2 - m_{h^0}^2\cancel{M_Z}^2 + 2\cancel{M_Z}^4 - t\cancel{M_Z}^2 + u\cancel{t} - t^2\Big\}$$

$$- 3[|C_V^f|^2 - |C_A^f|^2]M_Z^2(m_{h^0}^2-t)\Big\} \qquad (109)$$

$$ut - m_{h^0}^2 M_Z^2 = \frac{1}{4}\lambda\sin^2\theta$$

$$\mathcal{M}_3\mathcal{M}_{2a}^* - \frac{g^4 m_f^2 M_Z^2 C_{tf} C_Z^*}{4M_W^4}\Big\{[|C_V^f|^2 + |C_A^f|^2]\Big[2sM_Z^2 + \frac{1}{4}\lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta + 2M_Z^4 - tM_Z^2$$

$$- t^2\Big] - 3[|C_V^f|^2 - |C_A^f|^2]M_Z^2(m_{h^0}^2-t)\Big\} \qquad (110)$$

$$\mathcal{M}_{2a}\mathcal{M}_3^* = -\frac{g^4 m_f^2 M_Z^2 C_{tf} C_Z}{4M_W^4}\Big\{[|C_V^f|^2 + |C_A^f|^2]\Big[2sM_Z^2 + \frac{1}{4}\lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta + 2M_Z^4 - tM_Z^2$$

$$- t^2\Big] - 3[|C_V^f|^2 - |C_A^f|^2]M_Z^2(m_{h^0}^2-t)\Big\} \qquad (111)$$

$$\boxed{\begin{aligned}\mathcal{M}_3\mathcal{M}_{2a}^* + \mathcal{M}_{2a}\mathcal{M}_3^* = -\frac{g^4 m_f^2 M_Z^2 C_{tf}\,\mathrm{Re}(C_Z)}{2M_W^4}\Big\{[|C_V^f|^2 + |C_A^f|^2]\Big[2sM_Z^2 + \frac{1}{4}\lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta \\ + 2M_Z^4 - tM_Z^2 - t^2\Big] - 3[|C_V^f|^2 - |C_A^f|^2]M_Z^2(m_{h^0}^2-t)\Big\} \qquad (112)\end{aligned}}$$

$$M_4 M_{2a}^* = \frac{-g^4 m_f M_Z C_{uf} C_Z^*}{8 M_W \cos^3\theta_W}\left(\sum_\lambda \varepsilon_{4\mu}^* \varepsilon_{4\nu}\right)\sum_s \bar{V}_2 (2m_f + \not{p}_3)\gamma^\mu (C_V^f - C_A^f\gamma^5)U_1 \cdot \bar{U}_1 (C_V^f + C_A^f\gamma^5)\gamma^\nu V_2$$

(113)

$$= -\frac{g^4 m_f M_Z C_{uf} C_Z^*}{8 M_W \cos^3\theta_W}\left(-\eta_{\mu\nu} + \frac{p_{4\mu}p_{4\nu}}{M_Z^2}\right) \mathrm{Tr}\Big[(\not{p}_2 - m_f)(2m_f + \not{p}_3)\gamma^\mu (C_V^f - C_A^f\gamma^5)(\not{p}_1 + m_f)$$

$$(C_V^f + C_A^f\gamma^5)\gamma^\nu \Big] \qquad (114)$$

$$= \frac{-g^4 m_f M_Z C_{uf} C_Z^*}{8 M_W \cos^3\theta_W}\left(-\eta_{\mu\nu} + \frac{p_{4\mu}p_{4\nu}}{M_Z^2}\right) \mathrm{Tr}\Big[(2m_f \not{p}_2\gamma^\mu + \not{p}_2\not{p}_3\gamma^\mu - 2m_f^2\gamma^\mu - m_f\not{p}_3\gamma^\mu)$$

$$(C_V^f - C_A^f\gamma^5)(C_V^f\not{p}_1\gamma^\nu + C_A^f\not{p}_1\gamma^5\gamma^\nu + m_f C_V^f\gamma^\nu + m_f C_A^f\gamma^5\gamma^\nu)\Big] \qquad (115)$$

$$= -\frac{g^4 m_f M_Z C_{uf} C_Z^*}{8 M_W \cos^3\theta_W}\left(-\eta_{\mu\nu} + \frac{p_{4\mu}p_{4\nu}}{M_Z^2}\right) \mathrm{Tr}\Big[(2m_f \overset{\downarrow}{C_V^f}\not{p}_2\gamma^\mu + \overset{\downarrow}{C_V^f}\not{p}_2\not{p}_3\gamma^\mu - 2m_f^2 \overset{\downarrow}{C_V^f}\gamma^\mu - m_f \overset{\downarrow}{C_V^f}\not{p}_3\gamma^\mu$$

$$\overset{\downarrow}{-2m_f} C_A^f\not{p}_2\gamma^\mu\gamma^5 \overset{\downarrow}{-} C_A^f\not{p}_2\not{p}_3\gamma^\mu\gamma^5 \overset{\downarrow}{+} 2m_f^2 C_A^f\gamma^\mu\gamma^5 + m_f \overset{\downarrow}{C_A^f}\not{p}_3\gamma^\mu\gamma^5)(C_V^f\not{p}_1\gamma^\nu + C_A^f\not{p}_1\gamma^5\gamma^\nu$$

$$+ m_f C_V^f\gamma^\nu + m_f C_A^f\gamma^5\gamma^\nu)\Big] \qquad (116)$$

$$\mathrm{Tr}[\;] = 2m_f (C_V^f)^2 \mathrm{Tr}(\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu) + 2m_f C_V^f C_A^f \mathrm{Tr}(\not{p}_2\gamma^\mu\not{p}_1\gamma^5\gamma^\nu) + 2m_f^2 (C_V^f)^2 \mathrm{Tr}(\not{p}_2\overset{0}{\cancel{\gamma^\mu}}\gamma^\nu)$$

$$+ 2m_f^2 C_V^f C_A^f \mathrm{Tr}(\not{p}_2\overset{0}{\cancel{\gamma^\mu}}\gamma^5\gamma^\nu) + (C_V^f)^2 \mathrm{Tr}(\not{p}_2\not{p}_3\overset{0}{\cancel{\gamma^\mu}}\not{p}_1\gamma^\nu) + C_V^f C_A^f \mathrm{Tr}(\not{p}_2\not{p}_3\overset{0}{\cancel{\gamma^\mu}}\not{p}_1\gamma^5\gamma^\nu)$$

$$+ m_f (C_V^f)^2 \cancel{\mathrm{Tr}(\not{p}_2\not{p}_3\gamma^\mu\gamma^\nu)} + m_f C_V^f C_A^f \mathrm{Tr}(\cancel{\not{p}_2\not{p}_3\gamma^\mu\gamma^5\gamma^\nu}) - 2m_f^2 (C_V^f)^2 \mathrm{Tr}(\gamma^\mu\overset{0}{\cancel{\not{p}_1}}\gamma^\nu)$$

$$-2m_f^2 C_V^f C_A^f \mathrm{Tr}(\gamma^\mu\overset{0}{\cancel{\not{p}_1}}\gamma^5\gamma^\nu) - 2m_f^3 \cancel{(C_V^f)^2 \mathrm{Tr}(\gamma^\mu\gamma^\nu)} - 2m_f^3 C_V^f C_A^f \mathrm{Tr}(\gamma^\mu\overset{0}{\cancel{\gamma^5}}\gamma^\nu)$$

$$- m_f (C_V^f)^2 \mathrm{Tr}(\not{p}_3\gamma^\mu\not{p}_1\gamma^\nu) - m_f C_V^f C_A^f \mathrm{Tr}(\not{p}_3\gamma^\mu\not{p}_1\gamma^5\gamma^\nu) - m_f^2 (C_V^f)^2 \mathrm{Tr}(\not{p}_3\overset{0}{\cancel{\gamma^\mu}}\gamma^\nu) - m_f^2 C_V^f C_A^f \cdot$$

$$\cdot \mathrm{Tr}(\not{p}_3\overset{0}{\cancel{\gamma^\mu}}\gamma^5\gamma^\nu) - 2m_f C_V^f C_A^f \mathrm{Tr}(\not{p}_2\gamma^\mu\gamma^5\not{p}_1\gamma^\nu) - 2m_f (C_A^f)^2 \mathrm{Tr}(\not{p}_2\gamma^\mu\gamma^5\not{p}_1\gamma^5\gamma^\nu)$$

$$-2m_f^2 C_V^f C_A^f \mathrm{Tr}(\not{p}_2\overset{0}{\cancel{\gamma^\mu}}\gamma^5\gamma^\nu) - 2m_f^2 (C_A^f)^2 \mathrm{Tr}(\not{p}_2\gamma^\mu\overset{0}{\cancel{\gamma^5}}\gamma^5\gamma^\nu) - C_V^f C_A^f \mathrm{Tr}(\not{p}_2\not{p}_3\overset{0}{\cancel{\gamma^\mu}}\gamma^5\not{p}_1\gamma^\nu)$$

$$-(C_A^f)^2 \mathrm{Tr}(\not{p}_2\not{p}_3\gamma^\mu\overset{0}{\cancel{\gamma^5}}\not{p}_1\gamma^5\gamma^\nu) - m_f C_V^f C_A^f \mathrm{Tr}(\cancel{\not{p}_2}\cancel{\not{p}_3}\gamma^\mu\gamma^5\gamma^\nu) - m_f (C_A^f)^2 \mathrm{Tr}(\cancel{\not{p}_2\not{p}_3\gamma^\mu}\gamma^5\gamma^5\gamma^\nu)$$

$$+ 2m_f^2 C_V^f C_A^f \mathrm{Tr}(\gamma^\mu\gamma^5\overset{0}{\cancel{\not{p}_1}}\gamma^\nu) + 2m_f^2 (C_A^f)^2 \mathrm{Tr}(\gamma^\mu\gamma^5\overset{0}{\cancel{\not{p}_1}}\gamma^5\gamma^\nu) + 2m_f^3 C_V^f C_A^f \mathrm{Tr}(\gamma^\mu\overset{0}{\cancel{\gamma^5}}\gamma^\nu)$$

$$+ 2m_f^3 \cancel{(C_A^f)^2} \mathrm{Tr}(\gamma^\mu\gamma^\nu) + m_f C_V^f C_A^f \mathrm{Tr}(\not{p}_3\gamma^\mu\gamma^5\not{p}_1\gamma^\nu) + m_f (C_A^f)^2 \mathrm{Tr}(\not{p}_3\gamma^\mu\gamma^5\not{p}_1\gamma^5\gamma^\nu)$$

$$+ m_f^2 C_V^f C_A^f \mathrm{Tr}(\not{p}_3\overset{0}{\cancel{\gamma^\mu}}\gamma^5\gamma^\nu) + m_f^2 (C_A^f)^2 \mathrm{Tr}(\not{p}_3\gamma^\mu\overset{0}{\cancel{\gamma^5}}\gamma^5\gamma^\nu)$$

$$= 2m_f[(C_V^f)^2 + (C_A^f)^2]\,\mathrm{Tr}(\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu) - 4m_f C_V^f C_A^f \mathrm{Tr}(\gamma^5\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu) + m_f[(C_V^f)^2 - (C_A^f)^2] \times$$

$$\times \mathrm{Tr}(\not{p}_2\not{p}_3\gamma^\mu\gamma^\nu) - 2m_f^3[(C_V^f)^2 - (C_A^f)^2]\,\mathrm{Tr}(\gamma^\mu\gamma^\nu) - m_f[(C_V^f)^2 + (C_A^f)^2]\,\mathrm{Tr}(\not{p}_3\gamma^\mu\not{p}_1\gamma^\nu)$$

$$+ 2m_f C_V^f C_A^f \mathrm{Tr}(\gamma^5\not{p}_3\gamma^\mu\not{p}_1\gamma^\nu)$$

$\Rightarrow$

$$Tr = 8m_f\left[|C_V^t|^2 + |C_A^t|^2\right]\left[P_2^\nu P_1^\mu + P_2^\mu P_1^\nu - (P_1\cdot P_2)\eta^{\mu\nu}\right] + 16 i m_f C_V^t \cancel{C_A^t \varepsilon^{\alpha\mu\beta\nu} P_{2\alpha} P_{1\beta}}$$

$$+ m_f\left[|C_V^t|^2 - |C_A^t|^2\right] Tr\left(\gamma^\nu \not{P}_2\left(-\gamma^\mu \not{P}_3 + 2P_3^\mu\right)\right) - 8m_f^3\left[|C_V^t|^2 - |C_A^t|^2\right]\eta^{\mu\nu}$$

$$-4m_f\left[|C_V^t|^2 + |C_A^t|^2\right]\left(P_3^\nu P_1^\mu + P_3^\mu P_1^\nu - (P_1\cdot P_3)\eta^{\mu\nu}\right) - 8 i m_f C_V^t \cancel{C_A^t \varepsilon^{\alpha\mu\beta\nu} P_{3\alpha} P_{1\beta}} \quad (117)$$

$$\varepsilon^{\alpha\mu\beta\nu}\eta_{\mu\nu} = 0 \quad \text{and} \quad \varepsilon^{\alpha\mu\beta\nu} P_{4\mu} P_{4\nu} = 0$$

$\Rightarrow$

$$M_4 M_{2u}^* = -\frac{g^4 m_f M_Z C_{uf} C_Z^*}{8M_W \cos^3\theta_W}\left(-\eta_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\Big[8m_f\left[|C_V^t|^2 + |C_A^t|^2\right]\left(P_2^\nu P_1^\mu + P_2^\mu P_1^\nu - (P_1\cdot P_2)\eta^{\mu\nu}\right)$$

$$-4m_f\left[|C_V^t|^2 - |C_A^t|^2\right]\left(P_2^\nu P_3^\mu + P_2^\mu P_3^\nu - (P_2\cdot P_3)\eta^{\mu\nu}\right) + 8m_f\left[|C_V^t|^2 - |C_A^t|^2\right] P_2^\nu P_3^\mu$$

$$-8m_f^3\left[|C_V^t|^2 - |C_A^t|^2\right]\eta^{\mu\nu} - 4m_f\left[|C_V^t|^2 + |C_A^t|^2\right]\left[P_3^\nu P_1^\mu + P_3^\mu P_1^\nu - (P_1\cdot P_3)\eta^{\mu\nu}\right]\Big] \quad (118)$$

$$M_4 M_{2u}^* = -\frac{g^4 m_f M_Z C_{uf} C_Z^*}{8M_W \cos^3\theta_W}\Big\{16m_f\left[|C_V^t|^2 + |C_A^t|^2\right](P_1\cdot P_2) - 16m_f\left[|C_V^t|^2 - |C_A^t|^2\right](P_2\cdot P_3)$$

$$+32m_f^3\left[|C_V^t|^2 - |C_A^t|^2\right] - 8m_f\left[|C_V^t|^2 + |C_A^t|^2\right](P_1\cdot P_3) + \frac{8m_f}{M_Z^2}\left[|C_V^t|^2 + |C_A^t|^2\right]\left[2(P_1\cdot P_4)(P_2\cdot P_4)\right.$$

$$\left. -(P_1\cdot P_2)M_Z^2\right] - \frac{4m_f}{M_Z^2}\left[|C_V^t|^2 - |C_A^t|^2\right]\left[2(P_2\cdot P_4)(P_3\cdot P_4) - (P_2\cdot P_3)M_Z^2\right] + \frac{8m_f}{M_Z^2}\left[|C_V^t|^2 - |C_A^t|^2\right]\times$$

$$\times (P_2\cdot P_4)(P_3\cdot P_4) - 8m_f^3\left[|C_V^t|^2 - |C_A^t|^2\right] - \frac{4m_f}{M_Z^2}\left[|C_V^t|^2 + |C_A^t|^2\right]\left[2(P_3\cdot P_4)(P_1\cdot P_4) - (P_1\cdot P_3)M_Z^2\right]\Big\}$$

$$M_4 M_{2u}^* = -\frac{g^4 m_f M_Z C_{uf} C_Z^*}{8M_W \cos^3\theta_W}\Big\{8m_f\left[|C_V^t|^2 + |C_A^t|^2\right](P_1\cdot P_2) - 12m_f\left[|C_V^t|^2 - |C_A^t|^2\right](P_2\cdot P_3)$$

$$+24m_f^3\left[|C_V^t|^2 - |C_A^t|^2\right] - 4m_f\left[|C_V^t|^2 + |C_A^t|^2\right](P_1\cdot P_3) + \frac{16m_f}{M_Z^2}\left[|C_V^t|^2 + |C_A^t|^2\right](P_1\cdot P_4)(P_2\cdot P_4)$$

$$-\frac{8m_f}{M_Z^2}\left[|C_V^t|^2 + |C_A^t|^2\right](P_1\cdot P_4)(P_3\cdot P_4)\Big\}$$

$$= -\frac{g^4 m_f^2 M_Z C_{uf} C_{Z4}^*}{8M_W \cos^3\theta_W}\Big\{\left[|C_V^t|^2 + |C_A^t|^2\right]\Big[2(P_1\cdot P_2) - (P_1\cdot P_3) + \frac{4}{M_Z^2}(P_1\cdot P_4)(P_2\cdot P_4)$$

$$-\frac{2}{M_Z^2}(P_1\cdot P_4)(P_3\cdot P_4)\Big] + 3\left[|C_V^t|^2 - |C_A^t|^2\right]\left[2m_f^2 - (P_2\cdot P_3)\right]\Big\} \quad (119)$$

$$= -\frac{g^4 m_f^2 M_Z C_{uf} C_Z^*}{2M_W \cos^3\theta_W}\Big\{\left[|C_V^t|^2 + |C_A^t|^2\right]\Big[s - \frac{(m_{h^0}^2 - t)}{2} + \frac{1}{M_Z^2}(M_Z^2 - u)\big(\overset{(M_Z^2 - t)}{\overbrace{s + u - m_{h^0}^2}}\big)$$

$$-\frac{1}{2M_Z^2}(M_Z^2 - u)\big(\overset{(-u-t)}{\overbrace{s - m_{h^0}^2}} - M_Z^2\big)\Big] - \frac{3}{2}\left[|C_V^t|^2 - |C_A^t|^2\right](m_{h^0}^2 - u)\Big\} \quad (120)$$

(we have neglected terms with m_f^2 inside { }

$$= -\frac{g^4 m_f^2 M_Z^2 C_{uf} C_Z^*}{4 M_W^4}\Big\{[|C_V^f|^2+|C_A^f|^2][2s\cancel{M_Z^2} - m_{h^0}^2\cancel{M_Z^2} + t\cancel{M_Z^2} + 2\cancel{M_Z^4} - \cancel{2M_Z^2}t - 2\cancel{M_Z^2}u + 2\cancel{u}$$

$$+ M_Z^2\cancel{u} + M_Z^2\cancel{t} - \cancel{u}^2 - \cancel{u}t] - 3[|C_V^f|^2 - |C_A^f|^2]M_Z^2(m_{h^0}^2 - u)\Big\}$$

$$= -\frac{g^4 m_f^2 M_Z^2 C_{uf} C_Z^*}{4 M_W^4}\Big\{[|C_V^f|^2+|C_A^f|^2][2\cancel{s}M_Z^2 - m_{h^0}^2\cancel{M_Z^2} + 2\cancel{M_Z^4} + \cancel{u}t - u\cancel{M_Z^2} - \cancel{u}^2]$$

$$-3[|C_V^f|^2 - |C_A^f|^2]M_Z^2(m_{h^0}^2 - u)\Big\}$$

$$\boxed{M_4 M_{1a}^* = \frac{-g^4 m_f^2 M_Z^2 C_{uf} C_Z^*}{4M_W^4}\Big\{[|C_V^f|^2+|C_A^f|^2][2sM_Z^2 + \frac{1}{4}\lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta + 2M_Z^4 - uM_Z^2 - u^2] -3[|C_V^f|^2 - |C_A^f|^2]M_Z^2(m_{h^0}^2 - u)\Big\}} \qquad (121)$$

$$\boxed{M_{1a} M_4^* = -\frac{g^4 m_f^2 M_Z^2 C_{uf} C_Z}{4M_W^4}\Big\{[|C_V^f|^2+|C_A^f|^2][2sM_Z^2 + \frac{1}{4}\lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta + 2M_Z^4 - uM_Z^2 - u^2] -3[|C_V^f|^2 - |C_A^f|^2]M_Z^2(m_{h^0}^2 - u)\Big\}} \qquad (122)$$

$$\boxed{\Rightarrow M_4 M_{1a}^* + M_{1a} M_4^* = -\frac{g^4 m_f^2 M_Z^2 C_{uf}\,\mathrm{Re}(C_Z)}{2M_W^4}\Big\{[|C_V^f|^2+|C_A^f|^2][2sM_Z^2 + \frac{1}{4}\lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta + 2M_Z^4 - uM_Z^2 - u^2] -3[|C_V^f|^2 - |C_A^f|^2]M_Z^2(m_{h^0}^2 - u)\Big\}} \qquad (123)$$

$$M_3 M_4^* = \frac{g^4 m_f^2 C_{tf} C_{uf}}{16 M_W^2 \cos^2\theta_W}\Big(\sum_\lambda \varepsilon_{4\mu}^* \varepsilon_{4\nu}\Big)\sum_s [\bar{V}_2 \gamma^\mu (C_V^f - C_A^f\gamma^5)(2m_f - \not{p}_3)U_1]\cdot[\bar{V}_2(2m_f + \not{p}_3)\gamma^\nu(C_V^f - C_A^f\gamma^5)U \qquad (124)$$

$$[\bar{V}_2(2m_f + \not{p}_3)\gamma^\nu(C_V^f - C_A^f\gamma^5)U_1]^+ = U_1^+(C_V^f - C_A^f\gamma^5)\gamma^{\nu+}(2m_f + \overbrace{\gamma^{\alpha+}p_{3\alpha}}^{\gamma^0\gamma^\alpha})\gamma^0 V_2$$

$$= U_1^+(C_V^f - C_A^f\gamma^5)\gamma^{\nu+}(2m_f\gamma^0 + \gamma^0\not{p}_3)V_2$$

$$= U_1^+(C_V^f - C_A^f\gamma^5)\gamma^0\gamma^\nu(2m_f + \not{p}_3)V_2 = \bar{U}_1(C_V^f + C_A^f\gamma^5)\gamma^\nu(2m_f + \not{p}_3)V_2$$

$$\Rightarrow \boxed{[\bar{V}_2(2m_f + \not{p}_3)\gamma^\nu(C_V^f - C_A^f\gamma^5)U_1]^+ = \bar{U}_1(C_V^f + C_A^f\gamma^5)\gamma^\nu(2m_f + \not{p}_3)V_2} \qquad (125)$$

$$M_3 M_4^* = \frac{g^4 m_f^2 C_{tf} C_{uf}}{16 M_W^2 \cos^2\theta_W}\Big(-\eta_{\mu\nu} + \frac{p_{4\mu}p_{4\nu}}{M_Z^2}\Big)\sum_s [\bar{V}_2\gamma^\mu(C_V^f - C_A^f\gamma^5)(2m_f - \not{p}_3)U_1]\cdot[\bar{U}_1(C_V^f + C_A^f\gamma^5)\gamma^\nu(2m_f + \not{p}_3)V_2] \qquad (126)$$

$$= \frac{g^4 m_f^2 C_{tf} C_{uf}}{16 M_W^2 \cos^2\theta_W}\Big(-\eta_{\mu\nu} + \frac{p_{4\mu}p_{4\nu}}{M_Z^2}\Big)\mathrm{Tr}\Big[(\not{p}_2 - m_f)\gamma^\mu(C_V^f - C_A^f\gamma^5)(2m_f - \not{p}_3)(\not{p}_1 + m_f)\cdot(C_V^f + C_A^f\gamma^5)\gamma^\nu(2m_f + \not{p}_3)\Big] \qquad (127)$$

neglecting m_t in the trace

$$\mathrm{Tr}[\ ] = -\mathrm{Tr}[\not{p}_2 \gamma^\mu (C_V^t - C_A^t\gamma^5)\not{p}_3 \not{p}_1 (C_V^t + C_A^t\gamma^5)\gamma^\nu \not{p}_3]$$

$$= -(C_V^t|^2 \mathrm{Tr}[\not{p}_2\gamma^\mu \not{p}_3 \not{p}_1 \gamma^\nu \not{p}_3] - C_V^t C_A^t \mathrm{Tr}(\cancel{\not{p}_2\gamma^\mu} \not{p}_3\not{p}_1\gamma^5\gamma^\nu\not{p}_3)$$

$$+ C_A^t C_V^t \mathrm{Tr}[\not{p}_2 \cancel{\gamma^\mu\gamma^5} \not{p}_3 \not{p}_1 \gamma^\nu \not{p}_3] + (C_A^t|^2 \mathrm{Tr}[\not{p}_2\gamma^\mu\gamma^5 \not{p}_3 \not{p}_1 \gamma^5 \gamma^\nu \not{p}_3]$$

$$\mathrm{Tr}[\ ] = [(C_A^t|^2 - (C_V^t|^2]\mathrm{Tr}[\not{p}_2\gamma^\mu\not{p}_3\not{p}_1\gamma^\nu\not{p}_3]$$

$$= -[(C_V^t|^2 - (C_A^t|^2]\mathrm{Tr}[\not{p}_2\gamma^\mu\not{p}_3\not{p}_1(-\not{p}_3\gamma^\nu + 2P_3^\nu)]$$

$$= -[(C_V^t|^2 - (C_A^t|^2]\{-\mathrm{Tr}[\not{p}_2\gamma^\mu\not{p}_3\not{p}_1\not{p}_3\gamma^\nu] + 2P_3^\nu \mathrm{Tr}[\not{p}_2\gamma^\mu\not{p}_3\not{p}_1]\}$$

$$= -[(C_V^t|^2 - (C_A^t|^2]\{-\mathrm{Tr}[\not{p}_2\gamma^\mu\not{p}_3(-\not{p}_3\not{p}_1 + 2(P_1.P_3))\gamma^\nu] + 2P_3^\nu \mathrm{Tr}[\not{p}_2\gamma^\mu\not{p}_3\not{p}_1]$$

$$= -[(C_V^t|^2 - (C_A^t|^2]\{m_{no}^2\mathrm{Tr}(\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu) - 2(P_1.P_3)\mathrm{Tr}(\not{p}_2\gamma^\mu\not{p}_3\gamma^\nu) + 2P_3^\nu\mathrm{Tr}(\not{p}_2\gamma^\mu\not{p}_3\not{p}_1)\}$$

$$= -[(C_V^t|^2 - (C_A^t|^2]\{m_{no}^2\mathrm{Tr}(\gamma^\nu\not{p}_2\gamma^\mu\not{p}_1) - 2(P_1.P_3)\mathrm{Tr}(\gamma^\nu\not{p}_2\gamma^\mu\not{p}_3) + 2P_3^\nu P_{1\alpha}\mathrm{Tr}(\not{p}_2\gamma^\mu\not{p}_3\gamma^\alpha)\}$$

$$= -[(C_V^t|^2 - (C_A^t|^2]\{m_{no}^2\mathrm{Tr}(\gamma^\nu\not{p}_2\gamma^\mu\not{p}_1) - 2(P_1.P_3)\mathrm{Tr}(\gamma^\nu\not{p}_2\gamma^\mu\not{p}_3) + 2P_3^\nu P_{1\alpha}\mathrm{Tr}(\gamma^\alpha\not{p}_2\gamma^\mu\not{p}_3)\}$$

$$\mathrm{Tr}[\,] = -4[(C_V^t|^2 - (C_A^t|^2]\{m_{no}^2[P_2^\nu P_1^\mu + P_2^\mu P_1^\nu - (P_1.P_2)\eta^{\mu\nu}] - 2(P_1.P_3)[P_2^\nu P_3^\mu + P_2^\mu P_3^\nu$$

$$- (P_2.P_3)\eta^{\mu\nu}] + 2P_3^\nu P_{1\alpha}[P_2^\alpha P_3^\mu + P_2^\mu P_3^\alpha - (P_2.P_3)\eta^{\mu\alpha}] \qquad (128)$$

$$\Rightarrow M_3 M_4^* = \frac{-4g^4 m_t^2 C_{tf} C_{uf}}{16 M_W^2 \cos^2\theta_W}[(C_V^t|^2 - (C_A^t|^2]\left(-\eta_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\{m_{no}^2[P_2^\nu P_1^\mu + P_2^\mu P_1^\nu - (P_1.P_2)\eta^{\mu\nu}]$$

$$-2(P_1.P_3)[P_2^\nu P_3^\mu + P_2^\mu P_3^\nu - (P_2.P_3)\eta^{\mu\nu}] + 2P_3^\nu P_{1\alpha}[P_2^\alpha P_3^\mu + P_2^\mu P_3^\alpha - (P_2.P_3)\eta^{\mu\alpha}]\} \qquad (129)$$

$$= -\frac{4g^4 m_t^2 C_{tf} C_{uf}}{16 M_W^2 \cos^2\theta_W}[(C_V^t|^2 - (C_A^t|^2]\{2(P_1.\cancel{P_2)}m_{no}^2 - 4(\cancel{P_1}.P_3)(P_2.P_3) - 2(\cancel{P_1}.P_2)m_{no}^2$$

$$+\frac{m_{no}^2}{M_Z^2}[2(P_1.\cancel{P_4)}(P_2.P_4) - M_Z^2(\cancel{P_1}.P_2)] - \frac{2(P_1.P_3)}{M_Z^2}[2(P_2.P_4)(P_3.P_4) - \cancel{M_Z^2}(P_2.P_3)]$$

$$+\frac{2}{M_Z^2}(P_3.P_4)[(P_1.\cancel{P_2})(P_3.P_4) + (P_1.P_3)(P_2.P_4) - (P_2.P_3)(P_1.P_4)]\} \qquad (130)$$

$$= -\frac{4g^4 m_t^2 C_{tf} C_{uf}[(C_V^t|^2 - (C_A^t|^2]}{16 M_W^2 \cos^2\theta_W}\{-2(P_1.P_3)(P_2.P_3) - m_{no}^2(P_1.P_2) + \frac{2m_{no}^2}{M_Z^2}(P_1.P_4)(P_2.P_4)$$

$$+\frac{2}{M_Z^2}(P_3.P_4)[(P_1.P_2)(P_3.P_4) - (P_1.P_3)(P_2.P_4) - (P_2.P_3)(P_1.P_4)]\} \qquad (131)$$

$$= \frac{-g^4 m_t^2 C_{tf} C_{uf}[(C_V^t|^2 - (C_A^t|^2]}{4M_W^2\cos^2\theta_W}\left\{-\frac{(m_{no}^2 - t)(m_{no}^2 - u)}{2} - \frac{s}{2}m_{no}^2 + \frac{\cancel{2}m_{no}^2}{M_Z^2}\frac{(M_Z^2 - u)}{\cancel{2}}\frac{(s+u-m_{no}^2)}{2}\right.$$

$$\left.+\frac{1}{M_Z^2}(s - m_{no}^2 - M_Z^2)\left[\frac{s}{2}\frac{(s - m_{no}^2 - M_Z^2)}{2} - \frac{(m_{no}^2 - t)}{2}\frac{(s+u-m_{no}^2)}{2} - \frac{(m_{no}^2 - u)}{2}\frac{(M_Z^2 - u)}{2}\right]\right\}$$

$$= \frac{-g^4 m_f^2 C_{tf} C_{uf} \left[|C_V^f|^2 - |C_A^f|^2 \right]}{4 M_W^2 \cos^2\theta_W} \Big\{ -\frac{(m_{h^0}^2 - t)(m_{h^0}^2 - u)}{2} - \frac{s}{2} m_{h^0}^2 + \frac{m_{h^0}^2}{2M_Z^2} (M_Z^2 - u)(M_Z^2 - t)$$

$$+ \frac{1}{4M_Z^2} (s - m_{h^0}^2 - M_Z^2) \left[s(-u-t) - (m_{h^0}^2 - t)(M_Z^2 - t) - (m_{h^0}^2 - u)(M_Z^2 - u) \right] \Big\}$$

$$s(-u-t) - (m_{h^0}^2 - t)(M_Z^2 - t) - (m_{h^0}^2 - u)(M_Z^2 - u) = -us - st - m_{h^0}^2 M_Z^2 + t m_{h^0}^2 + t M_Z^2$$

$$- t^2 - m_{h^0}^2 M_Z^2 + u m_{h^0}^2 + u M_Z^2 - u^2 = u (\overbrace{m_{h^0}^2 + M_Z^2 - u - s}^{t}) + t (\overbrace{m_{h^0}^2 + M_Z^2 - s}^{u} - t)$$

$$- 2 m_{h^0}^2 M_Z^2 = 2ut - 2 m_{h^0}^2 M_Z^2 \qquad (132)$$

$$\Rightarrow M_3 M_4^* = - \frac{g^4 m_f^2 C_{tf} C_{uf} \left[|C_V^f|^2 - |C_A^f|^2 \right]}{4 M_W^2 \cos^2\theta_W} \Big\{ -\frac{1}{2} (m_{h^0}^2 - t)(m_{h^0}^2 - u) - \frac{s}{2} m_{h^0}^2$$

$$+ \frac{m_{h^0}^2}{2M_Z^2} (M_Z^2 - u)(M_Z^2 - t) + \frac{1}{2M_Z^2} (-u-t) \left[ut - m_{h^0}^2 M_Z^2 \right] \Big\} \qquad (133)$$

$$= \frac{-g^4 m_f^2 C_{tf} C_{uf} \left[|C_V^f|^2 - |C_A^f|^2 \right]}{4 M_W^2 \cos^2\theta_W} \Big\{ -\frac{1}{2} (m_{h^0}^2 - t)(m_{h^0}^2 - u) - \frac{s}{2} m_{h^0}^2$$

$$+ \frac{1}{2M_Z^2} \Big[m_{h^0}^2 M_Z^4 - \cancel{M_Z^2 t m_{h^0}^2} - \cancel{M_Z^2 u m_{h^0}^2} + ut m_{h^0}^2 - u^2 t + \cancel{u m_{h^0}^2 M_Z^2} - ut^2$$

$$+ \cancel{t m_{h^0}^2 M_Z^2} \Big] \Big\}$$

$$= \frac{-g^4 m_f^2 C_{tf} C_{uf} \left[|C_V^f|^2 - |C_A^f|^2 \right]}{4 M_W^2 \cos^2\theta_W} \Big\{ -\frac{1}{2} (m_{h^0}^2 - t)(m_{h^0}^2 - u) - \frac{s}{2} m_{h^0}^2$$

$$+ \frac{1}{2M_Z^2} \Big[m_{h^0}^2 M_Z^4 + \underbrace{ut m_{h^0}^2 - u^2 t - ut^2}_{ut(\underbrace{m_{h^0}^2 - u - t}_{s - M_Z^2})} \Big]$$

$$= \frac{-g^4 m_f^2 C_{tf} C_{uf} \left[|C_V^f|^2 - |C_A^f|^2 \right]}{4 M_W^2 \cos^2\theta_W} \Big\{ -\frac{1}{2} (m_{h^0}^2 - t)(m_{h^0}^2 - u) - \frac{s}{2} m_{h^0}^2$$

$$+ \frac{1}{2M_Z^2} \left[m_{h^0}^2 M_Z^4 + ut (s - M_Z^2) \right] \Big\}$$

$$= - \frac{g^4 m_f^2 C_{tf} C_{uf} \left[|C_V^f|^2 - |C_A^f|^2 \right]}{8 M_W^4} \Big\{ - M_Z^2 (\overbrace{m_{h^0}^4 - u m_{h^0}^2 - t m_{h^0}^2}^{\overset{m_{h^0}^2 (s - M_Z^2)}{m_{h^0}^2 (m_{h^0}^2 - u - t)}} + ut) - s m_{h^0}^2 M_Z^2$$

$$+ m_{h^0}^2 M_Z^4 + ust - ut M_Z^2 \Big\}$$

$$= \frac{-g^4 m_f^2 C_{tf} C_{uf} \left[(|C_V^t|^2 - (|C_A^t|)^2 \right]}{8 M_W^4} \Big\{ -s \cancel{m}_{h^0}^2 M_Z^2 + m_{h^0}^{\cancel{2}} M_Z^4 - ut \cancel{M_Z}^2 - s\, m_{h^0}^{\cancel{2}} M_Z^2 + m_{h^0}^{\cancel{2}} M_Z^4$$

$$+ \cancel{u}st - ut \cancel{M_Z}^2 \Big\}$$

$$= \frac{-g^4 m_f^2 C_{tf} C_{uf} \left[(|C_V^t|^2 - (|C_A^t|^2 \right]}{8 M_W^4} \Big\{ -2s\, m_{h^0}^2 M_Z^2 + 2 m_{h^0}^{\cancel{2}} M_Z^4 - 2ut \cancel{M_Z}^2 + ust \Big\} \qquad (134$$

$$= \frac{-g^4 m_f^2 C_{tf} C_{uf} \left[(|C_V^t|^2 - (|C_A^t|^2 \right]}{8 M_W^4} \Big\{ 2M_Z^2 \left(-\frac{1}{4} \lambda \sin^2\theta \right) - 2s\, m_{h^0}^2 M_Z^2 + s\, (m_{h^0}^2 M_Z^2 +$$

$$+ \frac{1}{4} \lambda \sin^2\theta) \Big\}$$

$$= \frac{-g^4 m_f^2 C_{tf} C_{uf} \left[(|C_V^t|^2 - (|C_A^t|^2 \right]}{8 M_W^4} \Big\{ -\frac{1}{2} M_Z^2 \lambda \sin^2\theta - s\, m_{h^0}^2 M_Z^2 + \frac{1}{4} \lambda s \sin^2\theta \Big\}$$

$$M_3 M_4^* = \frac{s g^4 m_f^2 C_{tf} C_{uf} \left[(|C_V^t|^2 - (|C_A^t|)^2 \right]}{8 M_W^4} \Big\{ m_{h^0}^2 M_Z^2 - \frac{1}{4} \lambda (s, m_{h^0}^2, M_Z^2) \sin^2\theta + \frac{M_Z^2}{2s} \lambda (s, m_{h^0}^2, M_Z^2) \sin^2\theta \Big\}$$

(135)

$$M_4 M_3^* = \frac{s g^4 m_f^2 C_{tf} C_{uf} \left[(|C_V^t|^2 - (|C_A^t|^2 \right]}{8 M_W^4} \Big\{ \nearrow \Big\} \qquad (136)$$

$\swarrow$

$\Rightarrow$

$$M_3 M_4^* + M_4 M_3^* = \frac{s g^4 m_f^2 C_{tf} C_{uf} \left[(|C_V^t|^2 - (|C_A^t|^2 \right]}{4 M_W^4} \Big\{ m_{h^0}^2 M_Z^2 - \frac{1}{4} \lambda (s, m_{h^0}^2, M_Z^2) \sin^2\theta + \frac{M_Z^2}{2s} \lambda (s, m_{h^0}^2, M_Z^2) \sin^2$$

(137)

$$\overline{|M|^2} = \frac{1}{4} \Bigg\{ \frac{\hat{s} g^4 m_f^2}{2 M_W^4} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \Big[\frac{A_f^2}{4} |C_A'|^2 + A_f C_A^t \, \mathrm{Re}(C_A'^* C_Z) + |C_Z|^2 |C_A^t|^2 \Big]$$

$$+ \frac{\hat{s} g^4 |C_Z|^2}{8 \cos^4\theta_W} (|C_V^t|^2 + |C_A^t|^2) \Big[8 M_Z^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{\hat{s}} \sin^2\theta \Big]$$

$$+ \frac{\hat{s} g^4 m_f^2 C_{tf}^2}{8 M_W^4} (|C_V^t|^2 + |C_A^t|^2) \Big[\hat{t}^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{2\hat{s}} M_Z^2 \sin^2\theta \Big]$$

$$+ \frac{\hat{s} g^4 m_f^2 C_{uf}^2}{8 M_W^4} (|C_V^t|^2 + |C_A^t|^2) \Big[\hat{u}^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{2\hat{s}} M_Z^2 \sin^2\theta \Big]$$

$$- \frac{g^4 m_f^2 M_Z^2 C_A^t}{2 M_W^4} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \Big[A_f \, \mathrm{Re}(C_Z C_A'^*) + 2 |C_Z|^2 C_A^t \Big]$$

$$+ \frac{g^4 m_f^2 C_{tf} C_A^t \hat{s}}{4 M_W^4} \Big[m_{h^0}^2 M_Z^2 - \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta - \hat{t}^2 \Big] . \mathrm{Re}\Big[A_f C_A' + 2 C_Z C_A^t \Big]$$

$$+ \frac{g^4 m_f^2 C_{uf} C_A^t \hat{s}}{4 M_W^4} \Big[m_{h^0}^2 M_Z^2 - \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta - \hat{u}^2 \Big] . \mathrm{Re}\Big[A_f C_A' + 2 C_Z C_A^t \Big]$$

$$- \frac{g^4 m_f^2 M_Z^2 C_{tf} \, \mathrm{Re}(C_Z)}{2 M_W^4} \Big\{ \Big[|C_V^t|^2 + |C_A^t|^2 \Big] \Big[2\hat{s} M_Z^2 + \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta$$

$$+ 2 M_Z^4 - \hat{t} M_Z^2 - \hat{t}^2 \Big] - 3 \Big[|C_V^t|^2 - |C_A^t|^2 \Big] M_Z^2 (m_{h^0}^2 - \hat{t}) \Big\}$$

$$- \frac{g^4 m_f^2 M_Z^2 C_{uf} \, \mathrm{Re}(C_Z)}{2 M_W^4} \Big\{ \Big[|C_V^t|^2 + |C_A^t|^2 \Big] \Big[2\hat{s} M_Z^2 + \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta$$

$$+ 2 M_Z^4 - \hat{u} M_Z^2 - \hat{u}^2 \Big] - 3 \Big[|C_V^t|^2 - |C_A^t|^2 \Big] M_Z^2 (m_{h^0}^2 - \hat{u}) \Big\}$$

$$+ \frac{\hat{s} g^4 m_f^2 C_{tf} C_{uf} \Big[|C_V^t|^2 - |C_A^t|^2 \Big]}{4 M_W^4} \Big\{ m_{h^0}^2 M_Z^2 - \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta + \frac{M_Z^2}{2\hat{s}} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2$$

(138)

$$\overline{|M|^2} = \hat{s}\, G_F^2\, m_t^2 \Big\{ 4\lambda(\hat{s}, m_{h^0}^2, M_Z^2) \Big[\frac{A_t^2}{4} \cancel{|C_A'|^2} + A_t \cancel{C_A^t \mathrm{Re}}\, (C_A'^{*} C_Z) + |C_Z|^2 \cancel{|C_A^t|^2} \Big]$$

$$+ \frac{M_Z^4}{m_t^2} |C_Z|^2 \left[|C_V^t|^2 + |C_A^t|^2 \right] \left[8 M_Z^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{\hat{s}} \sin^2\theta \right]$$

$$+ C_{tf}^2 \left[|C_V^t|^2 + |C_A^t|^2 \right] \left[\hat{t}^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{2\hat{s}} M_Z^2 \sin^2\theta \right]$$

$$+ C_{uf}^2 \left[|C_V^t|^2 + |C_A^t|^2 \right] \left[\hat{u}^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{2\hat{s}} M_Z^2 \sin^2\theta \right]$$

$$- 4 \frac{M_Z^2}{\hat{s}} C_A^t \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \left[A_t \cancel{\mathrm{Re}} \cancel{(} C_Z C_A'^{*}) + 2 |C_Z|^2 \cancel{C_A^t} \right]$$

$$+ 2 C_{tf} C_A^t \left[m_{h^0}^2 M_Z^2 - \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta - \hat{t}^2 \right] \mathrm{Re} \left[\cancel{A_t} C_A' + 2 \cancel{C_Z} C_A^t \right]$$

$$+ 2 C_{uf} C_A^t \left[m_{h^0}^2 M_Z^2 - \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta - \hat{u}^2 \right] \mathrm{Re} \left[\cancel{A_t} \cancel{C_A'} + 2 \cancel{C_Z} C_A^t \right]$$

$$- 4 M_Z^2 C_{tf} \mathrm{Re}(C_Z) \Big\{ \left[|C_V^t|^2 + |C_A^t|^2 \right] \Big[2 M_Z^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{4\hat{s}} \sin^2\theta + \frac{(2M_Z^4 - \hat{t} M_Z^2 - \hat{t}^2)}{\hat{s}} \Big]$$

$$- 3 \left[|C_V^t|^2 \cancel{-|C_A^t|^2} \right] \frac{M_Z^2}{\hat{s}} (m_{h^0}^2 - \hat{t}) \Big\} - 4 M_Z^2 C_{uf} \mathrm{Re}(C_Z) \Big\{ \left[|C_V^t|^2 \cancel{+|C_A^t|^2} \right] \Big[2 M_Z^2 +$$

$$+ \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{4\hat{s}} \sin^2\theta + \frac{(2M_Z^4 - \hat{u} M_Z^2 - \hat{u}^2)}{\hat{s}} \Big] - 3 \left[|C_V^t|^2 - \cancel{|C_A^t|^2} \right] \frac{M_Z^2}{\hat{s}} (m_{h^0}^2 - \hat{u}) \Big\}$$

$$+ 2 C_{tf} C_{uf} \left[|C_V^t|^2 - |C_A^t|^2 \right] \Big[m_{h^0}^2 M_Z^2 \cancel{- \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta} + \frac{M_Z^2}{2\hat{s}} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta \Big] \Big\} \qquad (139)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{64\pi^2 s} \frac{P_f}{P_i} \overline{|M|^2} \qquad (140)$$

$$\hat{t} = \frac{2m_t^2 + m_{h^0}^2 + M_Z^2 - \hat{s}}{2} + \frac{1}{2} \cos\theta\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_Z^2) \frac{(\hat{s} - 4m_t^2)^{1/2}}{\sqrt{\hat{s}}} \qquad (141)$$

$$\hat{u} = \frac{2m_t^2 + m_{h^0}^2 + M_Z^2 - \hat{s}}{2} - \frac{1}{2} \cos\theta\, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_Z^2) \frac{(\hat{s} - 4m_t^2)^{1/2}}{\sqrt{\hat{s}}} \qquad (142)$$

$$d\hat{t} = \frac{1}{2} \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_Z^2)\, d\cos\theta\, \frac{(\hat{s} - 4m_t^2)^{1/2}}{\sqrt{\hat{s}}} \qquad (143)$$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{2\pi\, d\cos\theta} = \frac{d\sigma}{d\hat{t}} \frac{d\hat{t}}{d\cos\theta} \frac{1}{2\pi} = \frac{1}{2\pi} \frac{1}{2} \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_Z^2) \frac{d\sigma}{d\hat{t}} \frac{(\hat{s} - 4m_t^2)^{1/2}}{\sqrt{\hat{s}}}$$

$$P_i = |\vec{P}_1| = |\vec{P}_2| = m_t \sinh\phi_1 = \frac{1}{2} (\hat{s} - 4m_t^2)^{1/2} \qquad (144)$$

$$P_f = |\vec{P}_3| = |\vec{P}_4| = \frac{\lambda^{1/2}(\hat{s}, m_{h^0}^2, M_Z^2)}{2\sqrt{\hat{s}}} \qquad (145)$$

$$\Rightarrow \frac{d\sigma}{d\hat{t}} = \frac{\pi}{P_i P_f} \left(\frac{d\sigma}{d\Omega} \right) = \frac{\pi}{P_i \cancel{P_f}} \frac{1}{64\pi^2 \hat{s}} \frac{\cancel{P_f}}{P_i} \overline{|M|^2} = \frac{1}{64\pi \hat{s} P_i^2} \overline{|M|^2}$$

$$\boxed{\frac{d\sigma}{d\hat{t}} = \frac{1}{64\pi \hat{s}\, p_i^2} \overline{|M|^2} \;\Big|\; = \frac{1}{16\pi \hat{s}^2} \overline{|M|^2}} \tag{146}$$

$$p_i^2 \approx \frac{\hat{s}}{4} \tag{147}$$

neglecting m_t^2 in (141), (142) we have:

$$\hat{t} = \frac{m_{h^0}^2 + M_Z^2 - \hat{s}}{2} + \frac{1}{2} \cos\theta \, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_Z^2) \tag{148}$$

$$\hat{u} = \frac{m_{h^0}^2 + M_Z^2 - \hat{s}}{2} - \frac{1}{2} \cos\theta \, \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_Z^2) \tag{149}$$

from here

$$\hat{u}\,\hat{t} = m_{h^0}^2 M_Z^2 + \frac{1}{4} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta \tag{150}$$

$$C_Z = \frac{\sin(\beta-\alpha)}{(\hat{s} - M_Z^2 + i M_Z \Gamma_Z)} = \frac{\sin(\beta-\alpha)\,(\hat{s} - M_Z^2 - i M_Z \Gamma_Z)}{(\hat{s} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

$$\boxed{\mathrm{Re}(C_Z) = \frac{(\hat{s} - M_Z^2)\,\sin(\beta-\alpha)}{(\hat{s} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}} \tag{151}$$

$$\boxed{|C_Z|^2 = \frac{\sin^2(\beta-\alpha)}{(\hat{s} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}} \tag{152}$$

$$\boxed{C_{uf} \approx \frac{C_f}{\hat{u}}} \tag{153}$$

$$\boxed{C_{tf} \approx \frac{C_f}{\hat{t}}} \tag{154}$$

$$C_A' = \frac{\cos(\beta-\alpha)}{(\hat{s} - m_{A^0}^2 + i m_{A^0} \Gamma_{A^0})}$$

neglecting Γ_{A^0}

$$\boxed{C_A' \approx \frac{\cos(\beta-\alpha)}{\hat{s} - m_{A^0}^2}} \tag{155}$$

$$
\begin{aligned}
\overline{|M|^2} = {} & \frac{\hat{s}\, G_F^2 m_f^2}{C_f'} \Bigg\{ \left[|C_V^f|^2 + |C_A^f|^2 \right] \Bigg\{ \frac{M_Z^4}{m_f^2} |C_Z|^2 \left[8M_Z^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{\hat{s}} \sin^2\theta \right] \\
& + \frac{C_f^2}{\hat{t}^2} \left[\hat{t}^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{2\hat{s}} M_Z^2 \sin^2\theta \right] + \frac{C_f^2}{\hat{u}^2} \left[\hat{u}^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{2\hat{s}} M_Z^2 \sin^2\theta \right] \\
& - 4M_Z^2 \frac{C_f}{\hat{u}} \mathrm{Re}(C_Z) \left[2M_Z^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{4\hat{s}} \sin^2\theta + \frac{(2M_Z^4 - \hat{u} M_Z^2 - \hat{u}^2)}{\hat{s}} \right] \\
& - 4M_Z^2 \frac{C_f}{\hat{t}} \mathrm{Re}(C_Z) \left[2M_Z^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{4\hat{s}} \sin^2\theta + \frac{(2M_Z^4 - \hat{t} M_Z^2 - \hat{t}^2)}{\hat{s}} \right] \Bigg\} \\
& + \left[|C_V^f|^2 - |C_A^f|^2 \right] \Bigg\{ \frac{2C_f^2}{\hat{u}\,\hat{t}} \left[m_{h^0}^2 M_Z^2 - \frac{1}{4}\lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta + \frac{M_Z^2}{2\hat{s}} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta \right] \\
& + 12 M_Z^4 \frac{C_f}{\hat{t}} \mathrm{Re}(C_Z) \frac{(m_{h^0}^2 - \hat{t})}{\hat{s}} + 12 M_Z^4 \frac{C_f}{\hat{u}} \mathrm{Re}(C_Z) \frac{(m_{h^0}^2 - \hat{u})}{\hat{s}} \Bigg\} + 4|C_A^f|^2 \Bigg\{ \lambda(\hat{s}, m_{h^0}^2, M_Z^2) |C_Z|^2 \\
& - 2\frac{M_Z^2}{\hat{s}} |C_Z|^2 \lambda(\hat{s}, m_{h^0}^2, M_Z^2) + \frac{C_f}{\hat{t}} \mathrm{Re}(C_Z) \left[m_{h^0}^2 M_Z^2 - \frac{1}{4}\lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta - \hat{t}^2 \right] \\
& + \frac{C_f}{\hat{u}} \mathrm{Re}(C_Z) \left[m_{h^0}^2 M_Z^2 - \frac{1}{4}\lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta - \hat{u}^2 \right] \Bigg\} + 2C_A^f \Bigg\{ 2\lambda(\hat{s}, m_{h^0}^2, M_Z^2) A_f C_A' \\
& \mathrm{Re}(C_Z) - 2\frac{M_Z^2}{\hat{s}} \lambda(\hat{s}, m_{h^0}^2, M_Z^2) A_f C_A' \mathrm{Re}(C_Z) + \frac{C_f}{\hat{t}} A_f C_A' \left[m_{h^0}^2 M_Z^2 - \frac{1}{4}\lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta \right. \\
& \left. - \hat{t}^2 \right] + \frac{C_f}{\hat{u}} A_f C_A' \left[m_{h^0}^2 M_Z^2 - \frac{1}{4}\lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta - \hat{u}^2 \right] \Bigg\} + A_f^2 (C_A')^2 \lambda(\hat{s}, m_{h^0}^2, M_Z^2) \Bigg\}
\end{aligned}
\tag{156}
$$

for $f = e^-, \mu^-, d, s, u$:

$$
\boxed{\overline{|M|^2} = \frac{\hat{s}\, G_F^2 M_Z^4 |C_Z|^2}{C_f'} \left[|C_V^f|^2 : |C_A^f|^2 \right] \left[8M_Z^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{\hat{s}} \sin^2\theta \right]} \tag{157}
$$

$$
\Rightarrow \boxed{\frac{d\sigma}{d\hat{t}} = \frac{1}{16\pi \hat{s}} \frac{G_F^2 M_Z^4 |C_Z|^2}{C_f'} \left[(C_V^f)^2 + (C_A^f)^2 \right] \left[8M_Z^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{\hat{s}} \sin^2\theta \right]} \tag{158}
$$

where $|C_Z|^2 = \dfrac{\sin^2(\beta - \alpha)}{(\hat{s} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$

$\Gamma_Z = 2.4952 \pm 0.0023$ GeV ; $\sin^2\theta_W = 0.23113$; $M_Z = 91.1876$ GeV

$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$; $G_F = 1.166391 \times 10^{-5}$ GeV^{-2}

$C_A^f = T_f^3$; $C_V^f = T_f^3 - 2\sin^2\theta_W Q_f$

$C_f' = 3$ for quarks and 1 for leptons.

	T_f^3	Q_f	c_A^f	c_V^f
e^- μ^- τ^-	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}+2\sin^2\theta_W$
u c t	$\frac{1}{2}$	$+\frac{2}{3}$	$+\frac{1}{2}$	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$
d s b	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$

For any f: $(f\bar{f} \to h^0 Z^0)$ (including τ^-, c, b)

$$\frac{d\sigma}{d\hat{t}} = \frac{1}{16\pi\hat{s}}\,\frac{G_F^2 m_f^2}{C_f'}\Bigg\{\left[(C_V^f)^2+(C_A^f)^2\right]\Bigg\{\frac{M_Z^4}{m_f^2}|C(z)|^2\left[8M_Z^2+\frac{\lambda(\hat{s},m_{h^0}^2,M_Z^2)}{\hat{s}}\sin^2\theta\right]+$$

$$+2C_f^2+\frac{C_f^2}{2\hat{s}}\lambda(\hat{s},m_{h^0}^2,M_Z^2)M_Z^2\sin^2\theta\left(\frac{1}{\hat{t}^2}+\frac{1}{\hat{u}^2}\right)-4M_Z^2C_f\,\mathrm{Re}\,C(z)\Big[2M_Z^2+$$

$$+\frac{\lambda(\hat{s},m_{h^0}^2,M_Z^2)}{4\hat{s}}\sin^2\theta\Big]\left(\frac{1}{\hat{u}}+\frac{1}{\hat{t}}\right)-4M_Z^2C_f\,\mathrm{Re}\,C(z)\Big[\frac{(2M_Z^4-\hat{u}M_Z^2-\hat{u}^2)}{\hat{u}\hat{s}}$$

$$+\frac{(2M_Z^4-\hat{t}M_Z^2-\hat{t}^2)}{\hat{t}\hat{s}}\Big]\Bigg\}+\left[(C_V^f)^2-(C_A^f)^2\right]\Bigg\{\frac{2C_f^2}{\hat{u}\hat{t}}\Big[m_{h^0}^2M_Z^2-\frac{1}{4}\lambda(\hat{s},m_{h^0}^2,M_Z^2)\sin^2\theta$$

$$+\frac{M_Z^2}{2\hat{s}}\lambda(\hat{s},m_{h^0}^2,M_Z^2)\sin^2\theta\Big]+12M_Z^4C_f\,\mathrm{Re}\,C(z)\left[\frac{(m_{h^0}^2-\hat{t})}{\hat{t}\hat{s}}+\frac{(m_{h^0}^2-\hat{u})}{\hat{u}\hat{s}}\right]\Bigg\}+$$

$$+4(C_A^f)^2\Bigg\{\lambda(\hat{s},m_{h^0}^2,M_Z^2)|C(z)|^2\left[1-\frac{2M_Z^2}{\hat{s}}\right]+C_f\,\mathrm{Re}\,C(z)\left(\frac{1}{\hat{t}}+\frac{1}{\hat{u}}\right)\cdot\Big[m_{h^0}^2M_Z^2-$$

$$-\frac{1}{4}\lambda(\hat{s},m_{h^0}^2,M_Z^2)\sin^2\theta\Big]-C_f\,\mathrm{Re}\,C(z)(\hat{u}+\hat{t})\Bigg\}+2C_A^f\Bigg\{2\lambda(\hat{s},m_{h^0}^2,M_Z^2)A_fC_A'\,\mathrm{Re}\,C(z$$

$$\times\left(1-\frac{M_Z^2}{\hat{s}}\right)+C_fA_fC_A'\Big[m_{h^0}^2M_Z^2-\frac{1}{4}\lambda(\hat{s},m_{h^0}^2,M_Z^2)\sin^2\theta\Big]\left(\frac{1}{\hat{t}}+\frac{1}{\hat{u}}\right)-C_fA_fC_A'(\hat{u}+\hat{t})$$

$$+A_f^2(C_A')^2\lambda(\hat{s},m_{h^0}^2,M_Z^2)\Bigg\} \qquad (159)$$

Where $\mathrm{Re}\,C(z) = \dfrac{(\hat{s}-M_Z^2)\sin(\beta-\alpha)}{(\hat{s}-M_Z^2)^2+M_Z^2\Gamma_Z^2}$

$$|C(z)|^2 = \frac{\sin^2(\beta-\alpha)}{(\hat{s}-M_Z^2)^2+M_Z^2\Gamma_Z^2}$$

$$C_A' = \frac{\cos(\beta-\alpha)}{\hat{s}-m_{A^0}^2}\ ;\quad m_{A^0}^2 = M_{H^\pm}^2 - M_W^2$$

$$A_f = \begin{cases}\cot\beta & f = u, c\\ \tan\beta & f = d, s, b, e^-, \mu^-, \tau^-\end{cases}$$

$$C_f = \begin{cases}\dfrac{\sin\alpha}{\cos\beta} & f = d, s, b\ e^-, \mu^-, \tau^-\\[2mm] -\dfrac{\cos\alpha}{\sin\beta} & f = u, c\end{cases}$$

$$\lambda(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc$$

$$\rightarrow \quad \hat{t} = \frac{m_{h^0}^2 + M_Z^2 - \hat{s}}{2} + \frac{1}{2} \cos\theta \, \lambda^{1/2} (\hat{s}, m_{h^0}^2, M_Z^2) \, .$$

$$\rightarrow \quad \hat{U} = \frac{m_{h^0}^2 + M_Z^2 - \hat{s}}{2} - \frac{1}{2} \cos\theta \, \lambda^{1/2} (\hat{s}, m_{h^0}^2, M_Z^2)$$

$$\rightarrow \quad \hat{U} \hat{t} = m_{h^0}^2 M_Z^2 + \frac{1}{4} \lambda (\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta$$

$$\rightarrow m_{h^0}^2 = \frac{1}{2} M_{H^\pm}^2 \left\{ 1 - \frac{M_W^2}{M_{H^\pm}^2} + \frac{M_Z^2}{M_{H^\pm}^2} - g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta) \right\}$$

$$\rightarrow \quad g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta) = \left[\left(1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2} \right)^2 - 4 \left(\frac{M_Z^2}{M_{H^\pm}^2} \right) \left(1 - \frac{M_W^2}{M_{H^\pm}^2} \right) \left(\frac{1 - \tan^2\beta}{1 + \tan^2\beta} \right)^2 \right]^{1/2}$$

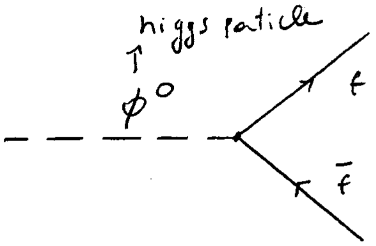
$$\rightarrow \quad \lambda (\hat{s}, m_{h^0}^2, M_Z^2) = \hat{s}^2 + m_{h^0}^4 + M_Z^4 - 2\hat{s}\, m_{h^0}^2 - 2\hat{s}\, M_Z^2 - 2 m_{h^0}^2 M_Z^2 .$$

$$\rightarrow \quad \cos(\beta - \alpha) = \frac{(1 + \tan\beta \tan\alpha)}{(1 + \tan^2\beta)^{1/2} (1 + \tan^2\alpha)^{1/2}}$$

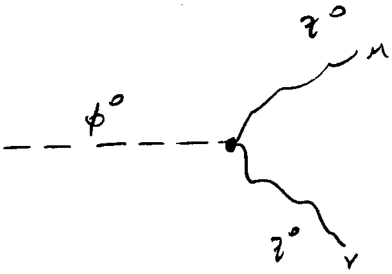
$$\rightarrow \quad \sin(\beta - \alpha) = \frac{(\tan\beta - \tan\alpha)}{(1 + \tan^2\beta)^{1/2} (1 + \tan^2\alpha)^{1/2}}$$

$$\rightarrow \quad \tan\alpha = - \left\{ \frac{1 + \frac{(1 - \tan^2\beta)}{(1 + \tan^2\beta)} \left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)} \right]}{1 - \frac{(1 - \tan^2\beta)}{(1 + \tan^2\beta)} \left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)} \right]} \right\}^{1/2}$$

In the standard Model : $f\bar{f} \to \phi^0 Z^0$

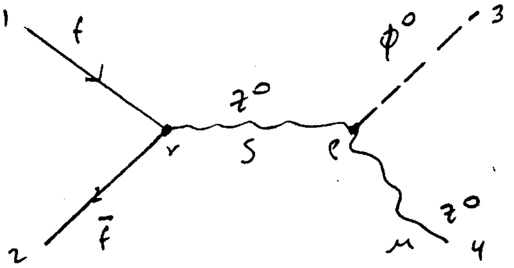


$$-\frac{igm_f}{2M_W}$$

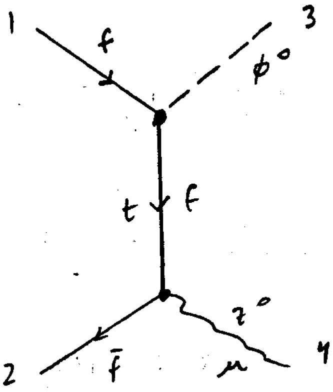


$$\frac{igM_Z}{\cos\theta_W}\,\eta^{\mu\nu}$$

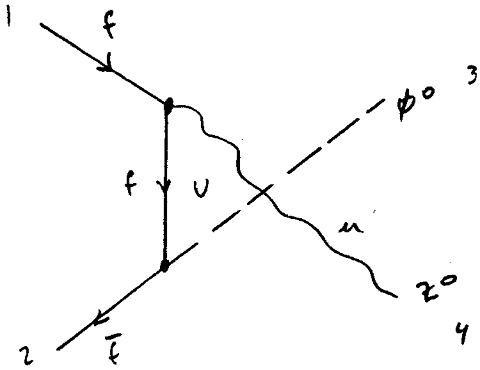
$f\bar{f} \to \phi^0 Z^0$:



+



+



$\frac{d\sigma}{d\hat{t}}$ in the standard model is obtained from $\frac{d\sigma}{d\hat{t}}$ in the two Higgs doublet model replacing: $A_f \to 0$; $\sin(\beta-\alpha) \to 1$; $C_f \to -1$; $m_{h^0} \to m_{\phi^0}$

$\therefore$ for $f = e^-, \mu^-, d, s, u$:

$$\boxed{\left.\frac{d\sigma}{d\hat{t}}\right|_{SM} = \frac{1}{16\pi\hat{s}}\,\frac{G_F^2 M_Z^4}{C_f}\,\frac{1}{(\hat{s}-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\left[(C_V^f)^2 + (C_A^f)^2\right]\left[8M_Z^2 + \frac{\lambda(\hat{s}, \overset{m_{\phi^0}^2}{\overset{\uparrow}{m_{h^0}^2}}, M_Z^2)}{\hat{s}}\sin^2\theta\right.}$$

(160)

for any f (including τ^-, b, c

$$\left.\frac{d\theta}{d\hat{t}}\right|_{SM} = \frac{1}{16\pi\hat{s}}\,\frac{G_F^2 m_f^2}{C_f}\Bigg\{\left[(C_V^f)^2 + (C_A^f)^2\right]\Bigg\{\frac{M_Z^4}{m_f^2}\cdot\frac{1}{(\hat{s}-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\left[8M_Z^2 + \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{\hat{s}}\sin^2\theta\right.$$

$$+ 2 + \frac{1}{2\hat{s}}\lambda(\hat{s}, m_{h^0}^2, M_Z^2)\,M_Z^2\sin^2\theta\left(\frac{1}{\hat{t}^2} + \frac{1}{\hat{u}^2}\right) + \frac{4M_Z^2(\hat{s}-M_Z^2)}{(\hat{s}-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\Big[2M_Z^2 +$$

$$+ \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{4\hat{s}}\sin^2\theta\Big]\left(\frac{1}{\hat{u}} + \frac{1}{\hat{t}}\right) + \frac{4M_Z^2(\hat{s}-M_Z^2)}{(\hat{s}-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\Bigg[\frac{(2M_Z^4 - \hat{u}M_Z^2 - \hat{u}^2)}{\hat{u}\hat{s}}$$

$$+ \frac{(2M_Z^4 - \hat{t}M_Z^2 - \hat{t}^2)}{\hat{t}\hat{s}}\Bigg]\Bigg\} + \left[(C_V^f)^2 - (C_A^f)^2\right]\Bigg\{\frac{2}{\hat{u}\hat{t}}\Big[m_{h^0}^2 M_Z^2 - \frac{1}{4}\lambda(\hat{s}, m_{h^0}^2, M_Z^2)\sin^2\theta\,+$$

$$+ \frac{M_Z^2}{2\hat{s}}\lambda(\hat{s}, m_{h^0}^2, M_Z^2)\sin^2\theta\Big] - \frac{12M_Z^4(\hat{s}-M_Z^2)}{(\hat{s}-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\left[\frac{(m_{h^0}^2 - \hat{t})}{\hat{t}\hat{s}} + \frac{(m_{h^0}^2 - \hat{u})}{\hat{u}\hat{s}}\right]\Bigg\} +$$

$$+ 4(C_A^f)^2\Bigg\{\frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}{(\hat{s}-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\left(1 - \frac{2M_Z^2}{\hat{s}}\right) - \frac{(\hat{s}-M_Z^2)}{(\hat{s}-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}}\right)\cdot\Big[m_{h^0}^2 M_Z^2 -$$

$$- \frac{1}{4}\lambda(\hat{s}, m_{h^0}^2, M_Z^2)\sin^2\theta\Big] + \frac{(\hat{s}-M_Z^2)}{(\hat{s}-M_Z^2)^2 + M_Z^2\Gamma_Z^2}(\hat{u} + \hat{t})\Bigg\}\Bigg\} \qquad (161)$$

$p\bar{p} \to h^0 Z^0 X$:

$$\boxed{\frac{d^2\sigma}{dY\, dP_T^2}(p\bar{p} \to h^0 Z^0 X) = \sum_{(q\,\mathrm{or}\,g)} \int_{x_{a\,min}}^{1} dx_a\, F_q(x_a, M_a^2)\, F_q(x_b, M_b^2)\, \frac{x_b \hat{s}}{m_{h^0}^2 - \hat{u}} \cdot \frac{d\sigma}{d\hat{t}}\left(\begin{matrix} q\bar{q} \\ gg \end{matrix} \to h^0 Z^0\right) \qquad (162)}$$

where
$$x_{a\,min} = \frac{\sqrt{s}\, m_T\, e^{Y} + m_{h^0}^2 - M_Z^2}{s - \sqrt{s}\, m_T\, e^{-Y}} \qquad (163)$$

$$m_T = (M_Z^2 + P_T^2)^{1/2} \qquad (164)$$

$$x_b = \frac{x_a \sqrt{s}\, m_T\, e^{-Y} + m_{h^0}^2 - M_Z^2}{x_a s - \sqrt{s}\, m_T\, e^{Y}} \qquad (165)$$

$$\hat{s} = x_a x_b s \qquad (166)$$

$$P_T^2 = \frac{\lambda(\hat{s}, m_{h^0}^2, M_Z^2) \sin^2\theta}{4\hat{s}} \qquad (167)$$

$$\hat{u}\hat{t} = m_{h^0}^2 M_Z^2 + \hat{s} P_T^2 \qquad (168)$$

Y is the rapidity of Z^0 and P_T is the transverse momentum of Z^0.

(see $\frac{d^2\sigma}{dY\, dP_T^2}(AB \to H^{\pm} W^{\mp} X)$)

$F_q(x, M^2)$ is the parton (or gluon) density function. M^2 is the factorization scale

$M^2 = \hat{s}$ (invariant mass of the pair $h^0 Z^0$).

$$\cos\theta = \left(1 - \frac{4\hat{s} P_T^2}{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}\right)^{1/2} \qquad (169)$$

$$\Rightarrow \quad \hat{t} = \frac{m_{h^0}^2 + M_Z^2 - \hat{s}}{2} + \frac{1}{2}\left(1 - \frac{4\hat{s} P_T^2}{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}\right)^{1/2} \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_Z^2) \qquad (170)$$

$$\hat{u} = \frac{m_{h^0}^2 + M_Z^2 - \hat{s}}{2} - \frac{1}{2}\left(1 - \frac{4\hat{s} P_T^2}{\lambda(\hat{s}, m_{h^0}^2, M_Z^2)}\right)^{1/2} \lambda^{1/2}(\hat{s}, m_{h^0}^2, M_Z^2) \qquad (171)$$

Running coupling constants and Grand Unification

For the $SU(3) \times SU(2) \times U(1)$ couplings with n_F fermion generations:

$$\beta_3 = \mu \frac{d}{d\mu} g_s(\mu) = \frac{g_s^3(\mu)}{4\pi^2}\left(-\frac{11}{4} + \frac{n_F}{3}\right) \qquad (1)$$

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$$\beta_2 = \mu \frac{d}{d\mu} g(\mu) = \frac{g^3(\mu)}{4\pi^2}\left(-\frac{11}{6} + \frac{n_F}{3}\right) \qquad (2)$$

$$\beta_1 = \mu \frac{d}{d\mu} g'(\mu) = \frac{g'^3(\mu)}{4\pi^2}\left(\frac{5}{9} n_F\right) \qquad (3)$$

$$\left(\beta_i(g_i)\right)_{\text{scalar}} = \frac{g_i^3 C_{2i}}{48\pi^2} \qquad (4) \qquad \left(\mathrm{Tr}(t_A t_B) = g_i^2 C_{2i} \delta_{AB}\right)$$

For $SU(2)$ the constant $C_{2i} = \frac{n_s}{2}$ (5)

For $U(1)$ " " " $= \frac{n_s}{2}$ (6)

$$\Rightarrow \beta_2(g)_{\text{scalar}} = \frac{g^3 n_s}{96\pi^2} \qquad (7)$$

n_s: # of Higgs doublets.

$$\Rightarrow \beta_1(g')_{\text{scala}} = \frac{g'^3 n_s}{96\pi^2} \qquad (8)$$

Including n_s doublets we have:

$$\Rightarrow \beta_3 = \mu \frac{d}{d\mu} g_s(\mu) = \frac{g_s^3(\mu)}{4\pi^2}\left(-\frac{11}{4} + \frac{n_F}{3}\right) \qquad (9)$$

$$\beta_2 = \mu \frac{d}{d\mu} g(\mu) = \frac{g^3(\mu)}{4\pi^2}\left(-\frac{11}{6} + \frac{n_F}{3} + \frac{n_s}{24}\right) \qquad (10)$$

$$\beta_1 = \mu \frac{d}{d\mu} g'(\mu) = \frac{g'^3(\mu)}{4\pi^2}\left(\frac{5}{9} n_F + \frac{n_s}{24}\right) \qquad (11)$$

$$\int_{g_s(\mu)}^{g_s(M_X)} \frac{dg_s}{g_s^3} = \int_{\mu}^{M_X} \frac{d\mu}{\mu} \cdot \frac{1}{4\pi^2}\left(-\frac{11}{4} + \frac{n_F}{3}\right)$$

$$-\frac{1}{2 g_s^2}\Bigg|_{g_s(\mu)}^{g_s(M_X)} = \ln\left(\frac{M_X}{\mu}\right) \frac{1}{4\pi^2}\left(-\frac{11}{4} + \frac{n_F}{3}\right)$$

$$\boxed{\frac{1}{g_s^2(\mu)} = \frac{1}{g_s^2(M_X)} + \frac{1}{8\pi^2}\left(-11 + \frac{4n_F}{3}\right)\ln\left(\frac{M_X}{\mu}\right)} \qquad (12)$$

$$\int_{g(\mu)}^{g(M_X)} \frac{dg}{g^3} = \int_{\mu}^{M_X} \frac{d\mu}{\mu}\,\frac{1}{4\pi^2}\left(-\frac{11}{6}+\frac{n_F}{3}+\frac{n_S}{24}\right)$$

$$-\frac{1}{2g^2}\Bigg|_{g(\mu)}^{g(M_X)} = \ln\left(\frac{M_X}{\mu}\right)\frac{1}{4\pi^2}\left(-\frac{11}{6}+\frac{n_F}{3}+\frac{n_S}{24}\right)$$

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(M_X)} + \ln\left(\frac{M_X}{\mu}\right)\frac{1}{2\pi^2}\left(-\frac{11.2}{2.6}+\frac{4n_F}{12}+\frac{n_S}{6.4}\right)$$

$$\boxed{\frac{1}{g^2(\mu)} = \frac{1}{g^2(M_X)} + \frac{1}{8\pi^2}\ln\left(\frac{M_X}{\mu}\right)\left[-\frac{22}{3}+\frac{4}{3}n_F+\frac{n_S}{6}\right]} \qquad (13)$$

$$\int_{g'(\mu)}^{g'(M_X)} \frac{dg'}{g'^3} = \int_{\mu}^{M_X} \frac{d\mu}{\mu}\,\frac{1}{4\pi^2}\left(\frac{5}{9}n_F+\frac{n_S}{24}\right)$$

$$-\frac{1}{2g'^2}\Bigg|_{g'(\mu)}^{g'(M_X)} = \ln\left(\frac{M_X}{\mu}\right)\frac{1}{4\pi^2}\left(\frac{5}{9}n_F+\frac{n_S}{24}\right)$$

$$\frac{1}{g'^2(\mu)} = \frac{1}{g'^2(M_X)} + \ln\left(\frac{M_X}{\mu}\right)\frac{1}{2\pi^2}\left(\frac{5.4}{4.9}n_F+\frac{n_S}{6.4}\right)$$

$$\boxed{\frac{1}{g'^2(\mu)} = \frac{1}{g'^2(M_X)} + \frac{1}{8\pi^2}\ln\left(\frac{M_X}{\mu}\right)\left(\frac{20}{9}n_F+\frac{n_S}{6}\right)} \qquad (14)$$

It is convenient to take $\mu = M_Z$

Using $g_s^2 = g^2 = \frac{5}{3}g'^2$ at energies $\geq M_X$. (15)

M_X is the scale at which G is spontaneously broken. $(G \supset SU(3)\times SU(2) \times U(1))$

(12) - (13) give us:

$$\frac{1}{g_s^2(\mu)} - \frac{1}{g^2(\mu)} = \frac{1}{8\pi^2}\left(-11+\cancel{\frac{4}{3}n_F}\right)\ln\left(\frac{M_X}{\mu}\right) - \frac{1}{8\pi^2}\left(-\frac{22}{3}+\cancel{\frac{4}{3}n_F}+\frac{n_S}{6}\right)\ln\left(\frac{M_X}{\mu}\right)$$

$$\frac{1}{g_s^2(\mu)} - \frac{1}{g^2(\mu)} = \frac{1}{8\pi^2}\left(-\frac{11}{3}-\frac{n_S}{6}\right)\ln\left(\frac{M_X}{\mu}\right)$$

$$\frac{1}{g_s^2(\mu)} - \frac{1}{g^2(\mu)} = -\frac{1}{24\pi^2}\left(11 + \frac{n_s}{2}\right)\ln\left(\frac{M_X}{\mu}\right)$$

$$\boxed{\frac{1}{g_s^2(M_Z)} - \frac{1}{g^2(M_Z)} = -\frac{1}{24\pi^2}\left(11 + \frac{n_s}{2}\right)\ln\left(\frac{M_X}{M_Z}\right)} \qquad (16)$$

$$(13) - \frac{3}{5}\cdot(14)$$

$$\frac{1}{g^2(\mu)} - \frac{3}{5}\cdot\frac{1}{g'^2(\mu)} = \frac{1}{8\pi^2}\ln\left(\frac{M_X}{\mu}\right)\left[-\frac{22}{3} \cancel{+\frac{4}{3}n_F} + \frac{n_s}{6} - \frac{\cancel{3}}{\cancel{5}}\,\frac{\overset{4}{\cancel{20}}}{\underset{3}{\cancel{9}}}n_F - \frac{3}{5}\frac{n_s}{6}\right]$$

$$\frac{1}{g^2(\mu)} - \frac{3}{5}\,\frac{1}{g'^2(\mu)} = \frac{1}{8\pi^2}\ln\left(\frac{M_X}{\mu}\right)\left(-\frac{22}{3} + \frac{n_s}{15}\right)$$

$$\boxed{\frac{1}{g^2(M_Z)} - \frac{3}{5g'^2(M_Z)} = \frac{1}{24\pi^2}\left(-22 + \frac{n_s}{5}\right)\ln\left(\frac{M_X}{M_Z}\right)} \qquad (17)$$

$$\frac{(17)}{(16)} \Rightarrow$$

$$\frac{\dfrac{1}{g^2(M_Z)} - \dfrac{3}{5g'^2(M_Z)}}{\dfrac{1}{g_s^2(M_Z)} - \dfrac{1}{g^2(M_Z)}} = \frac{-22 + \dfrac{n_s}{5}}{-11 - \dfrac{n_s}{2}}$$

$$\frac{1}{g^2(M_Z)} - \frac{3}{5g'^2(M_Z)} = \left(\frac{1}{g_s^2(M_Z)} - \frac{1}{g^2(M_Z)}\right)\left(\frac{-22 + \frac{n_s}{5}}{-11 - \frac{n_s}{2}}\right)$$

$$\frac{3}{5g'^2(M_Z)} = \frac{1}{g^2(M_Z)} - \left(\frac{1}{g_s^2(M_Z)} - \frac{1}{g^2(M_Z)}\right)\left(\frac{22 - \frac{n_s}{5}}{11 + \frac{n_s}{2}}\right)$$

$$\frac{\cancel{3}}{5g'^2(M_Z)} = \frac{1}{g^2(M_Z)}\left[\frac{\cancel{33} + \cancel{\frac{3}{10}}n_s}{11 + \frac{n_s}{2}}\right] - \frac{1\,\cancel{3}}{3\,g_s^2(M_Z)}\left(\frac{22 - \frac{n_s}{5}}{11 + \frac{n_s}{2}}\right)$$

$$\frac{1}{g'^2(M_Z)} = \frac{5}{g^2(M_Z)}\left(\frac{11 + \frac{n_s}{10}}{11 + \frac{n_s}{2}}\right) - \frac{5}{3g_s^2(M_Z)}\left(\frac{22 - \frac{n_s}{5}}{11 + \frac{n_s}{2}}\right)$$

$$\frac{1 - \sin^2\theta_W}{e^2} = \frac{5\sin^2\theta_W}{e^2}\left(\frac{11 + \frac{n_s}{10}}{11 + \frac{n_s}{2}}\right) - \frac{5}{3g_s^2(M_Z)}\left(\frac{22 - \frac{n_s}{5}}{11 + \frac{n_s}{2}}\right)$$

$$1-\sin^2\theta_W = 5\sin^2\theta_W\left(\frac{11+\frac{n_s}{10}}{11+\frac{n_s}{2}}\right) - \frac{5e^2}{3g_s^2(M_Z)}\left(\frac{22-\frac{n_s}{5}}{11+\frac{n_s}{2}}\right)$$

$$\boxed{\sin^2\theta_W = \frac{1+\frac{5e^2(M_Z)}{3g_s^2(M_Z)}\left(\frac{22-\frac{n_s}{5}}{11+\frac{n_s}{2}}\right)}{1+5\left(\frac{11+\frac{n_s}{10}}{11+\frac{n_s}{2}}\right)}} = \frac{11+\frac{n_s}{2}+\frac{5}{3}\frac{e^2}{g_s^2}\left(22-\frac{n_s}{5}\right)}{66+n_s}$$

because $g=\frac{e}{\sin\theta_W}$; $g'=\frac{e}{\cos\theta_W}$; $e=e(M_Z)$

Using (17)

$$\frac{\sin^2\theta_W}{e^2} - \frac{3(1-\sin^2\theta_W)}{5e^2} = \frac{1}{24\pi^2}\left(-22+\frac{n_s}{5}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$\frac{8}{5}\sin^2\theta_W - \frac{3}{5} = \frac{e^2}{24\pi^2}\left(-22+\frac{n_s}{5}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$8\left\{\frac{1+\frac{5e^2}{3g_s^2(M_Z)}\left(\frac{22-\frac{n_s}{5}}{11+\frac{n_s}{2}}\right)}{\underbrace{1+5\left(\frac{11+\frac{n_s}{10}}{11+\frac{n_s}{2}}\right)}_{\gamma}}\right\} - 3 = \frac{5e^2}{24\pi^2}\left(-22+\frac{n_s}{5}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$\frac{1}{\gamma}\left\{\cancel{8}+\frac{40}{3}\frac{e^2}{g_s^2(M_Z)}\left(\frac{22-\frac{n_s}{5}}{11+\frac{n_s}{2}}\right) - \cancel{3} - 15\left(\cancel{\frac{11+\frac{n_s}{10}}{11+\frac{n_s}{2}}}\right)\right\} = \frac{5e^2}{24\pi^2}\left(-22+\frac{n_s}{5}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$\frac{1}{\gamma}\left\{\frac{-\cancel{(-n_s+110)}}{\left(11+\frac{n_s}{2}\right)} + \frac{40e^2}{15g_s^2(M_Z)}\frac{\cancel{(110-n_s)}}{\left(11+\frac{n_s}{2}\right)}\right\} = -\frac{\cancel{5}e^2}{24\pi^2}\left(\frac{\cancel{110-n_s}}{\cancel{5}}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$\frac{1}{\gamma}\left\{-\frac{1}{\left(11+\frac{n_s}{2}\right)} + \frac{\overset{8}{\cancel{40}}e^2}{\underset{3}{\cancel{15}}g_s^2(M_Z)\left(11+\frac{n_s}{2}\right)}\right\} = -\frac{e^2}{24\pi^2}\ln\left(\frac{M_X}{M_Z}\right)$$

$$\Rightarrow \ln\left(\frac{M_X}{M_Z}\right) = \frac{24\pi^2}{e^2(M_Z)}\cdot\frac{1}{\left(11+\frac{n_s}{2}\right)}\left[1-\frac{8e^2(M_Z)}{3g_s^2(M_Z)}\right]\frac{1}{\left(1+5\left(\frac{11+\frac{n_s}{10}}{11+\frac{n_s}{2}}\right)\right)}$$

$$= \frac{24\pi^2}{e^2}\frac{\left(1-\frac{8}{3}\frac{e^2}{g_s^2}\right)}{(66+n_s)} \qquad (19)$$

If $n_s = 0$

$$\boxed{\sin^2\theta_W = \frac{1}{6} + \frac{5\, e^2(M_Z)}{9 g_s^2(M_Z)}} \qquad (20)$$

and

$$\boxed{\ln\left(\frac{M_X}{M_Z}\right) = \frac{4\pi^2}{11\, e^2(M_Z)} \left(1 - \frac{8\, e^2(M_Z)}{3 g_s^2(M_Z)}\right)} \qquad (21)$$

If $n_s \neq 0$

$$\boxed{M_X = M_Z \exp\left\{ \frac{24\pi^2}{e^2(M_Z)} \cdot \frac{1}{\left(11 + \frac{n_s}{2}\right)} \left[1 - \frac{8 e^2(M_Z)}{3 g_s^2(M_Z)}\right] \frac{1}{\left[1 + 5\left(\frac{11 + \frac{n_s}{10}}{11 + \frac{n_s}{2}}\right)\right]} \right\}} \qquad (22)$$

$$\alpha(M_Z) = \frac{e^2(M_Z)}{4\pi} = \frac{1}{127.922} \qquad (23)$$

$$\alpha_s(M_Z) = \frac{g_s^2(M_Z)}{4\pi} = 0.1172 \qquad (24)$$

(Particle Physics Booklet 2002)

$$\frac{\alpha(M_Z)}{\alpha_s(M_Z)} = \frac{e^2(M_Z)}{g_s^2(M_Z)} = 0.06670 \qquad (25)$$

$M_Z = 91.1876$ GeV

$\Rightarrow$ For $n_s = 2$

$$\sin^2\theta_W = \frac{1 + \frac{5}{3}\, 0.06670 \left(\frac{22 - \frac{2}{5}}{11 + 1}\right)}{1 + 5\left(\frac{11 + \frac{1}{5}}{11 + 1}\right)} = \frac{1 + \frac{5}{3} \times 0.06670 \times 1.8}{1 + 5 \times 0.93333\ldots}$$

$\sin^2\theta_W = 0.211$ $\qquad \left(\sin^2\theta_W^{\text{experiment}} = 0.23113\right)$

$$M_X = 91.1876 \exp\left\{ \frac{6\pi \times 127.922}{12} \left[1 - \frac{8}{3}\, 0.06670\right] \frac{1}{\left[1 + 5\left(\frac{11 + \frac{1}{3}}{12}\right)\right]} \right\}$$

$M_X = 4.1766 \times 10^{14}$ GeV

For $n_S = 0$

$$\sin^2\theta_W = \frac{1}{6} + \frac{5}{9} \times 0.06670 = 0.2037$$

$$M_X = M_Z \exp\left\{ \frac{4\pi^2}{11\, e^2(M_Z)} \left(1 - \frac{8}{3} \frac{\alpha(M_Z)}{\alpha_S(M_Z)} \right) \right\}$$

$$M_X = 91.1876 \exp\left\{ \frac{\pi}{11} \times 127.922 \left(1 - \frac{8}{3} \times 0.06670 \right) \right\}$$

$$M_X = 10^{15}\ \text{GeV}$$

For $n_S = 4$

$$\sin^2\theta_W = \frac{1 + \frac{5}{3}\, 0.06670 \left(\frac{22 - \frac{4}{5}}{13} \right)}{1 + 5 \left(\frac{11 + \frac{2}{5}}{13} \right)} = 0.22$$

$$M_X = 91.1876 \exp\left\{ \frac{6\pi \times 127.922}{13} \left(1 - \frac{8}{3}\, 0.06670 \right) \cdot \frac{1}{\left[1 + 5 \cdot \left(\frac{11 + \frac{2}{5}}{13} \right)\right]} \right\}$$

$$M_X = 1.816 \times 10^{14}\ \text{GeV}$$

n_S	$\sin^2\theta_W$	M_X (GeV)
0	0.2037	10^{15}
2	0.211	4.176×10^{14}
4	0.22	1.816×10^{14}
6	0.2265	8.27×10^{13}

In the Minimal Supersymmetric Standard Model:

$$\boxed{\sin^2\theta = \frac{18 + 3n_S + \frac{e^2(M_Z)}{g_S^2(M_Z)} (60 - 2n_S)}{108 + 6n_S}} \qquad (26)$$

$$\boxed{M_X = M_Z \exp\left\{ \frac{8\pi^2}{e^2(M_Z)} \left[\frac{1 - \frac{8}{3} \frac{e^2(M_Z)}{g_S^2(M_Z)}}{18 + n_S} \right] \right\}} \qquad (27)$$

The Quantum Theory of Fields Volume III Supersymmetry, Steven Weinberg (Cambridge Univ. Press)

For $n_s = 0$

$$\boxed{\sin^2\theta = \frac{1}{6} + \frac{5}{9}\frac{e^2(M_Z)}{g_s^2(M_Z)}} \qquad (28)$$

$$\boxed{\ln\left(\frac{M_X}{M_Z}\right) = \frac{4\pi^2}{9e^2(M_Z)}\left(1 - \frac{8}{3}\frac{e^2(M_Z)}{g_s^2(M_Z)}\right)} \qquad (29)$$

$\sin^2\theta = 0.2037$

$M_X = 8 \times 10^{17}$ GeV

For $n_s = 2$

$\sin^2\theta = 0.23113$

$M_X = 2.037 \times 10^{16}$ GeV

For $n_s = 4$

$\sin^2\theta = 0.2535$

$M_X = 1.01 \times 10^{15}$ GeV

n_s	$\sin^2\theta_W$	M_X (GeV)
0	0.2037	8×10^{17} GeV
→ 2	0.2311	2.037×10^{16} GeV
4	0.2535	1.01×10^{15} GeV

In the two Higgs doublet model

For $n_s = 6$

$\sin^2\theta_W = 0.2265$

$M_X = 8.27 \times 10^{13}$ GeV

For $n_s = 7$

$\sin^2\theta_W = 0.23$

$M_X = 5.67 \times 10^{13}$ GeV

In the MSSM: (weinberg)

$$\frac{1}{g_s^2(\mu)} = \frac{1}{g_s^2(M_X)} + \frac{1}{2\pi^2}\left(-\frac{9}{4} + \frac{n_f}{2}\right)\ln\left(\frac{M_X}{\mu}\right) \qquad (30)$$

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(M_X)} + \frac{1}{2\pi^2}\left(-\frac{3}{2} + \frac{n_f}{2} + \frac{n_s}{8}\right)\ln\left(\frac{M_X}{\mu}\right) \qquad (31)$$

$$\frac{1}{g'^2(\mu)} = \frac{1}{g'^2(M_X)} + \frac{1}{2\pi^2}\left(\frac{5n_f}{6} + \frac{n_s}{8}\right)\ln\left(\frac{M_X}{\mu}\right) \qquad (32)$$

$\mu = M_Z$

$n_f =$ # of generations of quarks and leptons

$g_s^2 = g^2 = \frac{5}{3}g'^2$ at energies $\gtrsim M_X$.

(31) - (30) give us:

$$\boxed{\frac{1}{g^2(\mu)} - \frac{1}{g_s^2(\mu)} = \frac{1}{2\pi^2}\ln\left(\frac{M_X}{\mu}\right)\left[\frac{3}{4} + \frac{n_s}{8}\right]} \qquad (33)$$

(31) $-\frac{3}{5}$ (32) $\Rightarrow$

$$\frac{1}{g^2(\mu)} - \frac{3}{5g'^2(\mu)} = \frac{1}{2\pi^2}\left[-\frac{3}{2} + \frac{\cancel{n_f}}{2} + \frac{n_s}{8} - \frac{\cancel{3}}{\cancel{5}}\frac{\cancel{5}\cancel{n_f}}{\underset{2}{\cancel{6}}} - \frac{3}{5}\frac{n_s}{8}\right]\ln\left(\frac{M_X}{\mu}\right)$$

$$\boxed{\frac{1}{g^2(\mu)} - \frac{3}{5g'^2(\mu)} = \frac{1}{2\pi^2}\left[-\frac{3}{2} + \frac{n_s}{20}\right]\ln\left(\frac{M_X}{\mu}\right)} \qquad (34)$$

$\frac{(34)}{(33)} \Rightarrow$

$$\frac{\frac{1}{g^2(\mu)} - \frac{3}{5g'^2(\mu)}}{\frac{1}{g^2(\mu)} - \frac{1}{g_s^2(\mu)}} = \frac{-\frac{3}{2} + \frac{n_s}{20}}{\frac{3}{4} + \frac{n_s}{8}}$$

$$\frac{1}{g^2(\mu)} - \frac{3}{5g'^2(\mu)} = \left(\frac{1}{g^2(\mu)} - \frac{1}{g_s^2(\mu)}\right)\frac{\left(\frac{2n_s - 60}{40}\right)}{\left(\frac{24 + 4n_s}{32}\right)}$$

$$\frac{1}{g^2(\mu)} - \frac{3}{5g'^2(\mu)} = \left(\frac{1}{g^2(\mu)} - \frac{1}{g_s^2(\mu)}\right)\frac{2}{5}\frac{(n_s - 30)}{(6 + n_s)}$$

$$\frac{3}{5g'^2(\mu)} = \frac{1}{g^2(\mu)}\left(1 - \frac{2}{5}\,\frac{(n_s-30)}{(6+n_s)}\right) + \frac{2}{5}\,\frac{(n_s-30)}{(6+n_s)}\,\frac{1}{g_s^2(\mu)}$$

$$\frac{1}{g'^2(M_Z)} = \frac{5}{3}\,\frac{1}{g^2(M_Z)}\left(1 - \frac{2}{5}\,\frac{(n_s-30)}{(6+n_s)}\right) + \frac{2}{3}\,\frac{1}{g_s^2(M_Z)}\,\frac{(n_s-30)}{(6+n_s)}$$

$$\frac{1-\sin^2\theta_W}{e^2(M_Z)} = \frac{\cancel{8}}{3}\,\frac{\sin^2\theta_W}{e^2(M_Z)}\left(\frac{90+3n_s}{\cancel{5}(6+n_s)}\right) + \frac{2}{3}\,\frac{1}{g_s^2(M_Z)}\,\frac{(n_s-30)}{(6+n_s)}$$

$$1-\sin^2\theta_W = \frac{\sin^2\theta_W\,(30+n_s)}{(6+n_s)} + \frac{2}{3}\,\frac{e^2(M_Z)}{g_s^2(M_Z)}\,\frac{(n_s-30)}{(6+n_s)}$$

$$\sin^2\theta_W\left(1 + \frac{30+n_s}{6+n_s}\right) = 1 - \frac{2}{3}\,\frac{e^2(M_Z)}{g_s^2(M_Z)}\,\frac{(n_s-30)}{(n_s+6)}$$

$$\sin^2\theta_W\left(\frac{36+2n_s}{6+n_s}\right) = 1 - \frac{2}{3}\,\frac{e^2(M_Z)}{g_s^2(M_Z)}\,\frac{(n_s-30)}{(n_s+6)}$$

$$\sin^2\theta_W = \frac{3n_s + 18 - 2(n_s-30)\,e^2/g_s^2}{3(36+2n_s)}$$

$$\boxed{\sin^2\theta_W = \frac{18 + 3n_s + \left(e^2(M_Z)/g_s^2(M_Z)\right)(60-2n_s)}{108+6n_s}}$$

that is: (26)

From (34)

$$\frac{\sin^2\theta_W}{e^2} - \frac{3}{5}\,\frac{(1-\sin^2\theta_W)}{e^2} = \frac{1}{2\pi^2}\left(-\frac{3}{2} + \frac{n_s}{20}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$\frac{8}{5}\left[\frac{18+3n_s + e^2/g_s^2\,(60-2n_s)}{108+6n_s}\right] - \frac{3}{5} = \frac{e^2}{2\pi^2}\left(\frac{-60+2n_s}{40}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$\frac{1}{5}\left(\frac{1\cancel{4}4 + 24n_s - 3\cancel{2}4 - 18n_s}{108+6n_s}\right) + \frac{8}{5}\left(\frac{e^2}{g_s^2}\right)\frac{(30-n_s)}{(54+3n_s)} = \frac{e^2}{2\pi^2}\left(\frac{-30+n_s}{10}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$\frac{1}{5}\left(\frac{-180+6n_s}{108+6n_s}\right) + \frac{8}{5}\left(\frac{e^2}{g_s^2}\right)\frac{(30-n_s)}{(54+3n_s)} = \frac{e^2}{2\pi^2}\left(\frac{-30+n_s}{20}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$-\frac{6}{\cancel{6}}\left(\frac{\cancel{30-n_s}}{18+n_s}\right)\frac{1}{\cancel{5}} + \frac{8}{\cancel{5}}\left(\frac{e^2}{g_s^2}\right)\frac{\cancel{(30-n_s)}}{\underset{\downarrow}{(54+3n_s)}} = -\frac{e^2}{\underset{8}{\cancel{40}}\pi^2}\,\cancel{(30-n_s)}\,\ln\left(\frac{M_X}{M_Z}\right)$$

$$-\frac{1}{(18+n_s)} + \frac{8}{3}\left(\frac{e^2}{g_s^2}\right)\frac{1}{(18+n_s)} = -\frac{e^2}{8\pi^2}\cdot\ln\left(\frac{M_X}{M_Z}\right)$$

$$\Rightarrow \boxed{\ln\left(\frac{M_X}{M_Z}\right) = \frac{8\pi^2}{e^2(M_Z)}\left[\frac{1-\frac{8}{3}\left(\frac{e^2(M_Z)}{g_s^2(M_Z)}\right)}{18+n_s}\right]}$$

again
(27)

<u>In the Two Higgs Doublet Model:</u>

$$\alpha_s^{-1}(\mu) = \alpha_s^{-1}(M_X) + \frac{1}{2\pi}\left(-11 + \frac{4n_F}{3}\right)\ln\left(\frac{M_X}{\mu}\right) \qquad (a)$$

$$\alpha_s^{-1}(M_Z) = \alpha_s^{-1}(M_X) + \frac{1}{2\pi}\left(-11 + \frac{4n_F}{3}\right)\ln\left(\frac{M_X}{M_Z}\right) \qquad (b)$$

$$\alpha_s = g_s^2/4\pi$$

(a)-(b) :

$$\Rightarrow \alpha_s^{-1}(\mu) - \alpha_s^{-1}(M_Z) = \frac{1}{2\pi}\left(-11 + \frac{4n_F}{3}\right)\ln\left(\frac{M_Z}{\mu}\right)$$

$$\boxed{\alpha_s^{-1}(\mu) = \alpha_s^{-1}(M_Z) + \frac{1}{2\pi}\left(11 - \frac{4}{3}n_F\right)\ln\left(\frac{\mu}{M_Z}\right)} \qquad (35)$$

$\alpha_s(M_Z) = 0.1172$

$M_Z = 91.1876\ \text{GeV}$

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(M_X)} + \frac{1}{8\pi^2}\ln\left(\frac{M_X}{\mu}\right)\left[-\frac{22}{3} + \frac{4}{3}n_F + \frac{n_S}{6}\right]$$

$$\frac{1}{g^2(M_Z)} = \frac{1}{g^2(M_X)} + \frac{1}{8\pi^2}\ln\left(\frac{M_X}{M_Z}\right)\left[-\frac{22}{3} + \frac{4}{3}n_F + \frac{n_S}{6}\right]$$

$$\Rightarrow \frac{1}{g^2(\mu)} - \frac{1}{g^2(M_Z)} = \frac{1}{8\pi^2}\ln\left(\frac{M_Z}{\mu}\right)\left[-\frac{22}{3} + \frac{4}{3}n_F + \frac{n_S}{6}\right]$$

$$\alpha_2 \equiv \frac{g^2}{4\pi}$$

$$\boxed{\alpha_2^{-1}(\mu) = \alpha_2^{-1}(M_Z) + \frac{1}{2\pi}\left[\frac{22}{3} - \frac{4}{3}n_F - \frac{n_S}{6}\right]\ln\left(\frac{\mu}{M_Z}\right)} \qquad (36)$$

<u>where</u> $\alpha_2^{-1}(M_Z) = \dfrac{4\pi}{g^2(M_Z)} = \dfrac{4\pi\sin^2\theta_W}{e^2(M_Z)} = \dfrac{\sin^2\theta_W}{\alpha(M_Z)}$; $\alpha(M_Z) = \dfrac{1}{127.922}$

where $\sin^2\theta_W$ is given in (18)

$$\frac{1}{g'^2(\mu)} = \frac{1}{g'^2(M_X)} + \frac{1}{8\pi^2}\ln\left(\frac{M_X}{\mu}\right)\left(\frac{20}{9}n_F + \frac{n_S}{6}\right)$$

$$\frac{1}{g'^2(M_Z)} = \frac{1}{g'^2(M_X)} + \frac{1}{8\pi^2}\ln\left(\frac{M_X}{M_Z}\right)\left(\frac{20}{9}n_F + \frac{n_S}{6}\right)$$

$$\Rightarrow \frac{1}{g'^2(\mu)} - \frac{1}{g'^2(M_Z)} = \frac{1}{8\pi^2}\ln\left(\frac{M_Z}{\mu}\right)\left(\frac{20}{9}n_F + \frac{n_S}{6}\right)$$

$$\alpha_1 \equiv \frac{g'^2}{4\pi}\cdot\frac{5}{3}$$

$$\boxed{\alpha_1^{-1}(\mu) = \alpha_1^{-1}(M_Z) - \frac{1}{2\pi}\left(\frac{20}{9}n_F + \frac{n_S}{6}\right)\ln\left(\frac{\mu}{M_Z}\right)\cdot\frac{3}{5}} \qquad (37)$$

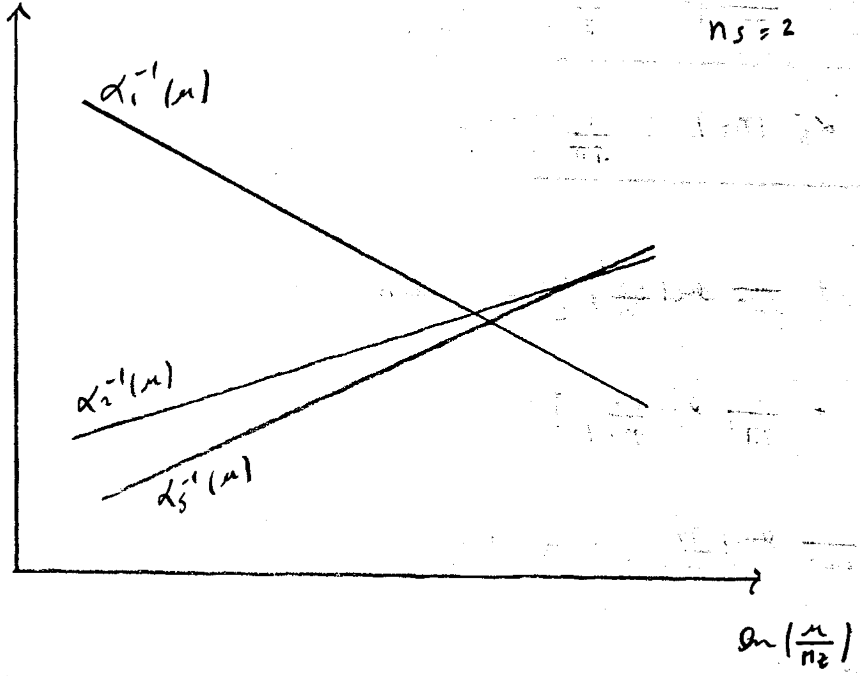
where: $\alpha_1^{-1}(M_Z) = \frac{4\pi}{g_1^2(M_Z)}\frac{3}{5} = \frac{4\pi}{e^2(M_Z)}(1-\sin^2\theta_W)\cdot\frac{3}{5}$

$$\boxed{\alpha_1^{-1}(M_Z) = \alpha^{-1}(M_Z)\,(1-\sin^2\theta_W)\,\frac{3}{5}}$$

$\sin^2\theta_W = 0.23113$

$n_f = 3$

$n_s = 2$



In the MSSM:

$$\frac{1}{g_s^2(\mu)} = \frac{1}{g_s^2(M_X)} + \frac{1}{2\pi^2}\left(-\frac{9}{4}+\frac{n_f}{2}\right)\ln\left(\frac{M_X}{\mu}\right)$$

$$\frac{1}{g_s^2(M_Z)} = \frac{1}{g_s^2(M_X)} + \frac{1}{2\pi^2}\left(-\frac{9}{4}+\frac{n_f}{2}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$\alpha_s^{-1}(\mu) = \alpha_s^{-1}(M_Z) + \frac{2}{\pi}\left(-\frac{9}{4}+\frac{n_f}{2}\right)\ln\left(\frac{M_Z}{\mu}\right)$$

$$\boxed{\alpha_s^{-1}(\mu) = \alpha_s^{-1}(M_Z) + \frac{1}{\pi}\left(\frac{9}{2}-n_f\right)\ln\left(\frac{\mu}{M_Z}\right)} \qquad (38)$$

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(M_X)} + \frac{1}{2\pi^2}\left(-\frac{3}{2}+\frac{n_f}{2}+\frac{n_s}{8}\right)\ln\left(\frac{M_X}{\mu}\right)$$

$$\frac{1}{g^2(M_Z)} = \frac{1}{g^2(M_X)} + \frac{1}{2\pi^2}\left(-\frac{3}{2}+\frac{n_f}{2}+\frac{n_s}{8}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$\Rightarrow \quad \alpha_2^{-1}(\mu) = \alpha_2^{-1}(M_Z) + \frac{2}{\pi}\left(-\frac{3}{2}+\frac{n_f}{2}+\frac{n_s}{8}\right)\ln\left(\frac{M_Z}{\mu}\right)$$

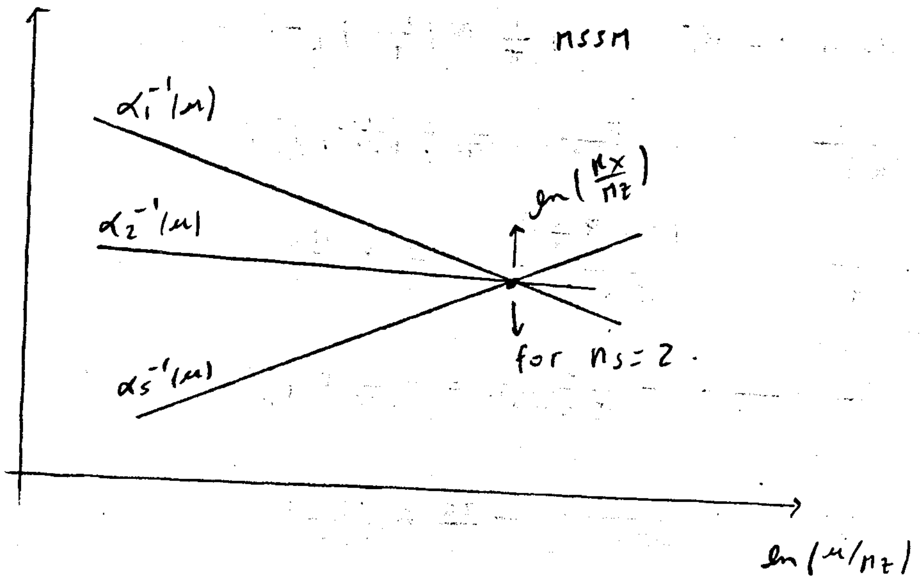
$$\boxed{\alpha_2^{-1}(\mu) = \alpha_2^{-1}(M_Z) + \frac{1}{\pi}\left(3-n_f-\frac{n_s}{4}\right)\ln\left(\frac{\mu}{M_Z}\right)} \qquad (39)$$

$$\frac{}{g'^2(\mu)} = \frac{}{g'^2(M_X)} + \frac{}{2\pi^2}\left(\frac{}{6} + \frac{}{8}\right)\ln\left(\frac{}{\mu}\right)$$

$$\frac{1}{g'^2(M_Z)} = \frac{1}{g'^2(M_X)} + \frac{1}{2\pi^2}\left(\frac{5n_f}{6} + \frac{n_s}{8}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$\alpha_1^{-1}(\mu) = \alpha_1^{-1}(M_Z) + \frac{2}{\pi}\left(\frac{5n_f}{6} + \frac{n_s}{8}\right)\ln\left(\frac{M_Z}{\mu}\right)\cdot\frac{3}{5}$$

$$\boxed{\alpha_1^{-1}(\mu) = \alpha_1^{-1}(M_Z) - \frac{1}{\pi}\left(\frac{5n_f}{3} + \frac{n_s}{4}\right)\ln\left(\frac{\mu}{M_Z}\right)\frac{3}{5}} \quad (40)$$



$$\alpha_1^{-1}(M_X) = \alpha_2^{-1}(M_X) = \alpha_2^{-1}(M_Z) + \frac{1}{\pi}\left(3 - n_f - \frac{n_s}{4}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$= \alpha_1^{-1}(M_Z) - \frac{1}{\pi}\left(n_f + \frac{3n_s}{20}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$\Rightarrow \quad \alpha_2^{-1}(M_Z) - \alpha_1^{-1}(M_Z) = \frac{1}{\pi}\left(-3 + \frac{n_s}{10}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

$$\frac{\sin^2\theta_W}{\alpha(M_Z)} - \frac{3}{5}\,\frac{(1-\sin^2\theta_W)}{\alpha(M_Z)} = \frac{1}{\pi}\left(-3 + \frac{n_s}{10}\right)\ln\left(\frac{M_X}{M_Z}\right)$$

for $n_s = 2$ $\quad \sin^2\theta_W = \dfrac{3 + 7\frac{\alpha}{\alpha_s}}{15}$

$$\Rightarrow \boxed{\ln\left(\frac{M_X}{M_Z}\right) = \frac{\pi}{10\alpha}\left(1 - \frac{8}{3}\frac{\alpha}{\alpha_s}\right)} \quad (41)$$

From (27) we see that this is the correct answer.

From (38), (39)

$$\alpha_s^{-1}(M_X) = \alpha_s^{-1}(M_Z) + \frac{1}{\pi}\left(\frac{9}{2} - n_f\right) \ln\left(\frac{M_X}{M_Z}\right)$$

$$\alpha_2^{-1}(M_X) = \alpha_2^{-1}(M_Z) + \frac{1}{\pi}\left(3 - n_f - \frac{n_s}{4}\right) \ln\left(\frac{M_X}{M_Z}\right)$$

$$\alpha_s^{-1}(M_X) = \alpha_2^{-1}(M_X) \Rightarrow$$

$$\alpha_s^{-1}(M_Z) + \frac{1}{\pi}\left(\frac{9}{2} - n_f\right) \ln\left(\frac{M_X}{M_Z}\right) = \alpha_2^{-1}(M_Z) + \frac{1}{\pi}\left(3 - n_f - \frac{n_s}{4}\right) \ln\left(\frac{M_X}{M_Z}\right)$$

$$\alpha_s^{-1}(M_Z) - \alpha_2^{-1}(M_Z) = \frac{1}{\pi} \ln\left(\frac{M_X}{M_Z}\right)\left[-\frac{3}{2} - \frac{n_s}{4}\right]$$

$$\frac{1}{\alpha_s(M_Z)} - \frac{\sin^2\theta_W}{\alpha(M_Z)} = \frac{1}{\pi} \ln\left(\frac{M_X}{M_Z}\right)\left(-\frac{3}{2} - \frac{n_s}{4}\right)$$

$$\frac{1}{\alpha_s(M_Z)} - \frac{\left(3 + 7\frac{\alpha}{\alpha_s}\right)}{15\,\alpha(M_Z)} = -\frac{1}{\pi} \ln\left(\frac{M_X}{M_Z}\right)\left(\frac{3}{2} + \frac{n_s}{4}\right)$$

If $n_s = 2$

$$\frac{8}{15} \cdot \frac{1}{\alpha_s(M_Z)} - \frac{3}{15\,\alpha(M_Z)} = -\frac{1}{\pi} \ln\left(\frac{M_X}{M_Z}\right) \cdot 2$$

$$\frac{8}{\alpha_s(M_Z)} - \frac{3}{\alpha(M_Z)} = -\frac{30}{\pi} \ln\left(\frac{M_X}{M_Z}\right)$$

$$\frac{1}{\alpha(M_Z)} - \frac{8}{3}\frac{1}{\alpha_s(M_Z)} = \frac{10}{\pi} \ln\left(\frac{M_X}{M_Z}\right)$$

$$\ln\left(\frac{M_X}{M_Z}\right) = \frac{\pi}{10}\left(\frac{1}{\alpha} - \frac{8}{3}\frac{1}{\alpha_s}\right)$$

$$\Rightarrow \boxed{\ln\left(\frac{M_X}{M_Z}\right) = \frac{\pi}{10\,\alpha(M_Z)}\left(1 - \frac{8}{3}\frac{\alpha(M_Z)}{\alpha_s(M_Z)}\right)}$$

(41) again.

1 doublet	4 - 3 = 1	higgs
2 doublets	4·2 - 3 = 5	"
3 doublets	4·3 - 3 = 9	"
⋮		
7 "	4·7 - 3 = 25	"

Proton decay:

$$\mu^- \to \nu_\mu e^- \bar{\nu}_e$$

$$\Gamma_\mu = \frac{G^2 m_\mu^5}{192\pi^3}$$

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2} \Rightarrow G^2 = \frac{g^4}{32M_W^4}$$

$$\Gamma_\mu = \frac{g^4 m_\mu^5}{192\pi^3 . 32 M_W^4} = \frac{e^4 m_\mu^5}{192\pi^3\, 32\, M_W^4 \sin^4\theta_W}$$

$$\alpha = \frac{e^2}{4\pi} \Rightarrow \alpha^2 = \frac{e^4}{16\pi^2}$$

$$\Gamma_\mu = \frac{\alpha^2}{384\pi} \frac{m_\mu^5}{M_W^4 \sin^4\theta_W} \propto \frac{m_\mu^5}{M_W^4}$$

$$p \to \pi^0 e^+$$

$$\frac{G_G}{\sqrt{2}} = \frac{g_G^2}{8M_X^2} \Rightarrow G_G^2 = \frac{g_G^4}{32M_X^4}$$

$$\Rightarrow \Gamma_p \propto \frac{m_p^5}{M_X^4} \qquad \Rightarrow M_X = \left(\frac{m_p^5}{\Gamma_p}\right)^{1/4}$$

$$\Rightarrow \boxed{\tau_p \approx \frac{M_X^4}{m_p^5}}$$

In the MSSM with $n_s = 2$ $M_X = 2.037 \times 10^{16}$ GeV

and $$\tau_p \approx \frac{(2.037 \times 10^{16})^4}{(.9383)^5} \times 6.58212 \times 10^{-25} = 1.56 \times 10^{41} \text{ sec}$$

$$\tau_p \approx 4.94 \times 10^{33} \text{ years} \qquad \underline{\text{OK}}$$

$$\tau_p > 10^{31} \text{ to } 10^{33} \text{ y.} \qquad \text{(mode dependent)}$$

In the SM with 7 doublets $M_X = 5.67 \times 10^{13}$ GeV

and

$$\tau_p \approx \frac{(5.67 \times 10^{13})^4}{(.9383)^5} \times 6.58212 \times 10^{-25} = 9.35 \times 10^{30} \text{ sec}$$

$$= 2.97 \times 10^{23} \text{ y.} \qquad \underline{\text{Conflict}}$$

In the two higgs doublet model

$M_X = 4.176 \times 10^{14}$ GeV

$\tau_p \approx 8.73 \times 10^{26}$ years Conflict

$$M_X = (\tau_p \, m_p^5)^{1/4}$$

$$M_X = \left(10^{31} \times 365 \times 86400 \times (6.58212 \times 10^{-25})^{-1} \times (0.9383)^5\right)^{1/4} = 4.32 \times 10^{15} \text{ GeV}$$

$$M_X = \left(10^{33} \times 365 \times 86400 \times (6.58212 \times 10^{-25})^{-1} \times (0.9338)^5\right)^{1/4} = 1.366 \times 10^{16} \text{ GeV}$$

Model dependent :

$$M_X > 4.3 \times 10^{15} — 1.36 \times 10^{16} \text{ GeV}$$