Higgs Phenomenology in the Two Higgs Doublet Model of type II (Personal Notes)

Vol. III

Carlos A. Marín

Universidad San Francisco de Quito

July 2004

CONTENTS

Note: In these three volumenes, we present the detailed calculations of the results that appear in the thesis: "Higgs Phenomenology in the Two Higgs Doublet Model of type II".

Vol. I : Limits on the Two Higgs Doublet Model from meson decay, mixing and CP violation.

Vol. II: Mass constraints, production cross sections, and decay rates in the Two Higgs Doublet Model of type II.

Vol. III:Higgs production at a muon collider in the Two Higgs Doublet Model of type II.

Tree-level Higgs Potential

Tree-level Higgs Potential

Let's consider two Higgs Doublets:

$$H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} ; \quad H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \qquad (1)$$

and the potential : (of the neutral scalars)

$$\boxed{V^N = \frac{g^2+g'^2}{8}\left[|H_1^0|^2 - |H_2^0|^2 \right]^2 + (m_1^2 + |\mu|^2)|H_1^0|^2 + (m_2^2+|\mu|^2)|H_2^0|^2 - B\mu \, \mathrm{Re}\,(H_1^0 H_2^0) \qquad (2)}$$

$$H_i^0 = v_i + \phi_i$$

$$H_1^0 = V_1 + \phi_1 \; ; \; H_2^0 = V_2 + \phi_2$$

$$|H_1^0|^2 = (\phi_1 + V_1)(\phi_1 + V_1)^* = |\phi_1|^2 + \phi_1 V_1^* + V_1 \phi_1^* + |V_1|^2$$

$$|H_2^0|^2 = (V_2 + \phi_2)(V_2 + \phi_2)^* = |V_2|^2 + |\phi_2|^2 + V_2 \phi_2^* + \phi_2 V_2^*$$

$$\left[|H_1^0|^2 - |H_2^0|^2 \right]^2 = \left[|\phi_1|^2 + |V_1|^2 - |\phi_2|^2 - |V_2|^2 + \phi_1 V_1^* + V_1 \phi_1^* - V_2 \phi_2^* - \phi_2 V_2^* \right]^2$$

$$\phi_1 V_1^* + \phi_1^* V_1 = 2\,\mathrm{Re}\,(\phi_1 V_1^*)$$

$$\phi_2 V_2^* + \phi_2^* V_2 = 2\,\mathrm{Re}\,(\phi_2 V_2^*)$$

$$\Rightarrow \left[|H_1^0|^2 - |H_2^0|^2 \right]^2 = \left[|\phi_1|^2 + |V_1|^2 - |\phi_2|^2 - |V_2|^2 + \overbrace{2\,\mathrm{Re}\,(\phi_1 V_1^*) - 2\,\mathrm{Re}\,(\phi_2 V_2^*)}^{2\,\mathrm{Re}\,(\phi_1 V_1^* - \phi_2 V_2^*)} \right]^2$$

$$\approx 4\left[\mathrm{Re}\,(\phi_1 V_1^* - \phi_2 V_2^*) \right]^2 + 4\,\mathrm{Re}\,(\phi_1 V_1^* - \phi_2 V_2^*)\left[|\phi_1|^2 + |V_1|^2 - |\phi_2|^2 - |V_2|^2 \right]$$
$$+ 2\,(|\phi_1|^2 - |\phi_2|^2)(|V_1|^2 - |V_2|^2) + (|V_1|^2 - |V_2|^2)^2$$

$$\approx 4\left[\mathrm{Re}\,(\phi_1 V_1^* - \phi_2 V_2^*) \right]^2 + 2\,(|V_1|^2 - |V_2|^2)\left[|\phi_1|^2 - |\phi_2|^2 + 2\,\mathrm{Re}\,(\phi_1 V_1^* - \phi_2 V_2^*) \right] + \text{const}$$

$$|H_1^0|^2 = (V_1 + \phi_1)(V_1 + \phi_1)^* = |V_1|^2 + |\phi_1|^2 + 2\,\mathrm{Re}\,(\phi_1 V_1^*)$$

$$|H_2^0|^2 = (V_2 + \phi_2)(V_2 + \phi_2)^* = |V_2|^2 + |\phi_2|^2 + 2\,\mathrm{Re}\,(\phi_2 V_2^*)$$

$$H_1^0 H_2^0 = (V_1 + \phi_1)(V_2 + \phi_2) = (V_1 V_2 + V_1 \phi_2 + \phi_1 V_2 + \phi_1 \phi_2)$$

$$\Rightarrow \boxed{\begin{aligned} V^N = {} & \frac{g^2+g'^2}{4}\left[|V_1|^2 - |V_2|^2 \right]\left[|\phi_1|^2 - |\phi_2|^2 + 2\,\mathrm{Re}\,(\phi_1 V_1^* - \phi_2 V_2^*) \right] + \frac{g^2+g'^2}{2}\left[\mathrm{Re}\,(\phi_1 V_1^* - \phi_2 V_2^*) \right] \\ & + (m_1^2 + |\mu|^2)\left(2\,\mathrm{Re}\,(\phi_1 V_1^*) + |\phi_1|^2 \right) + (m_2^2 + |\mu|^2)\left(2\,\mathrm{Re}\,(\phi_2 V_2^*) + |\phi_2|^2 \right) \\ & - B\mu\,\mathrm{Re}\,(V_1 \phi_2 + V_2 \phi_1 + \phi_1 \phi_2) + \text{constant} \\ & \qquad \text{to second order in } \phi_i \end{aligned}}$$

(3)

For the v_i to be equilibrium values of the fields, the terms to first order in ϕ_i must vanish

$$\left\{\begin{array}{l} \frac{(g^2+g'^2)}{4}\,(|V_1|^2-|V_2|^2)\,V_1^* + (m_1^2+|\mu|^2)\,V_1^* - \frac{1}{2}B\mu V_2 = 0 \\ \frac{(g^2+g'^2)}{4}\,(-|V_1|^2+|V_2|^2)\,V_2^* + (m_2^2+|\mu|^2)\,V_2^* - \frac{1}{2}B\mu V_1 = 0 \end{array}\right. \qquad (4)$$

We may adjust the relative phases of the ϕ_i so that V_1 is real. Then V_2 is also real and :

$$\left\{\begin{array}{l} \frac{(g^2+g'^2)}{4}\,(V_1^2-V_2^2)\,V_1 + (m_1^2+|\mu|^2)V_1 - \frac{1}{2}B\mu V_2 = 0 \\ \frac{(g^2+g'^2)}{4}\,(V_2^2-V_1^2)\,V_2 + (m_2^2+|\mu|^2)V_2 - \frac{1}{2}B\mu V_1 = 0 \end{array}\right. \qquad (5)$$

The Lagrangian in the standard Model is :

$$\mathcal{L} = \left|\left(\partial_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu - \frac{ig'}{2}X_\mu\right)\overset{\nearrow H}{\phi}\right|^2 \qquad (6)$$

The relevant part of the Lagrangian is

$$\mathcal{L}' = \left|\left(-ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu - \frac{ig'}{2}X_\mu\right)\phi\right|^2$$

For two Higgs doublets

$$\mathcal{L}' = \frac{1}{4}\sum_{i=1}^{2}\left|\left(g\,\vec{\tau}\cdot\vec{W}_\mu + Y_i g' X_\mu\right)\overset{\nearrow H_i}{\phi_i}\right|^2$$

$$g\,\vec{\tau}\cdot\vec{W}_\mu + g' X_\mu Y_i = g\,(\tau_1 W_\mu^1 + \tau_2 W_\mu^2 + \tau_3 W_\mu^3) + g' X_\mu Y_i$$

$$= g\left[\begin{pmatrix}0&1\\1&0\end{pmatrix}W_\mu^1 + \begin{pmatrix}0&-i\\i&0\end{pmatrix}W_\mu^2 + \begin{pmatrix}1&0\\0&-1\end{pmatrix}W_\mu^3\right] + g' X_\mu \begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$= \begin{pmatrix} gW_\mu^3 + Y_i g' X_\mu & g(W_\mu^1 - iW_\mu^2) \\ (W_\mu^1 + iW_\mu^2)g & -gW_\mu^3 + g' X_\mu Y_i \end{pmatrix} \qquad (Y_i = Y_{iW})$$

but $W_\mu^{\pm} = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$

$$\Rightarrow\ g\,\vec{\tau}\cdot\vec{W}_\mu + g' X_\mu = \begin{pmatrix} gW_\mu^3 + g' X_\mu Y_i & \sqrt{2}\,W_\mu^+ g \\ \sqrt{2}\,g\,W_\mu^- & -gW_\mu^3 + g' X_\mu Y_i \end{pmatrix}$$

$H_2 \rightarrow Y_W = 1$

$H_1 \rightarrow Y_W = -1$

$$\mathcal{L}' = \frac{1}{4}\left|\begin{pmatrix} gW^3_\mu - g'X_\mu & \sqrt{2}W^+_\mu g \\ \sqrt{2}gW^-_\mu & -gW^3_\mu - g'X_\mu \end{pmatrix}\begin{pmatrix} v_1 \\ 0 \end{pmatrix}\right|^2 + \frac{1}{4}\left|\begin{pmatrix} gW^3_\mu + g'X_\mu & \sqrt{2}W^+_\mu g \\ \sqrt{2}gW^-_\mu & -gW^3_\mu + g'X_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}\right|$$

$$\mathcal{L}' = \frac{1}{4}\left|\begin{pmatrix} v_1(gW^3_\mu - g'X_\mu) \\ \sqrt{2}gv_1W^-_\mu \end{pmatrix}\right|^2 + \frac{1}{4}\left|\begin{pmatrix} \sqrt{2}gv_2W^+_\mu \\ v_2(-gW^3_\mu + g'X_\mu) \end{pmatrix}\right|^2$$

$$\mathcal{L}' = \frac{1}{4}\begin{pmatrix} v_1(gW^3_\mu - g'X_\mu) & \sqrt{2}gv_1W^+_\mu \end{pmatrix}\begin{pmatrix} v_1(gW^\mu_3 - g'X^\mu) \\ \sqrt{2}gv_1W^\mu_- \end{pmatrix}$$

$$+ \frac{1}{4}\begin{pmatrix} \sqrt{2}gv_2W^-_\mu & v_2(-gW^3_\mu + g'X_\mu) \end{pmatrix}\begin{pmatrix} \sqrt{2}gv_2W^\mu_+ \\ v_2(-gW^\mu_3 + g'X^\mu) \end{pmatrix}$$

$$\mathcal{L}' = \frac{1}{4}v_1^2(gW^3_\mu - g'X_\mu)(gW^\mu_3 - g'X^\mu) + \frac{1}{2}g^2v_1^2W^+_\mu W^\mu_- + \frac{1}{2}g^2v_2^2W^+_\mu W^\mu_-$$

$$+ \frac{1}{4}v_2^2(-gW^3_\mu + g'X_\mu)(-gW^\mu_3 + g'X^\mu)$$

$$\mathcal{L}' = \frac{1}{4}v_1^2(g^2W^3_\mu W^\mu_3 - 2gg'W^3_\mu X^\mu + g'^2X_\mu X^\mu)$$

$$+ \frac{1}{4}v_2^2(g^2W^3_\mu W^\mu_3 - 2gg'W^3_\mu X^\mu + g'^2X_\mu X^\mu) + \frac{1}{2}g^2(v_1^2+v_2^2)W^+_\mu W^\mu_-$$

$$\frac{1}{4}(v_1^2+v_2^2)(g^2+g'^2)Z_\mu Z^\mu + 0\,A_\mu A^\mu$$

$$\frac{1}{4}g^2(v_1^2+v_2^2)\left((W^1_\mu)^2 + (W^2_\mu)^2\right)$$

$$\Rightarrow \boxed{M_W^2 = \frac{1}{2}g^2(v_1^2+v_2^2)} = M_Z^2\cos^2\theta_W \qquad (7)$$

$$= \frac{1}{2}g^2v_1^2\sec^2\beta$$

$$\Rightarrow M_Z^2 = \frac{1}{2}(v_1^2+v_2^2)g^2\sec^2\theta_W = \frac{1}{2}(v_1^2+v_2^2)g^2\frac{1}{\cos^2\theta_W}$$

$$M_Z^2 = \frac{1}{2}(v_1^2+v_2^2)g^2(1+\tan^2\theta_W)$$

$$\boxed{M_Z^2 = \frac{1}{2}(v_1^2+v_2^2)(g^2+g'^2)} \qquad (8)$$

$$= \frac{1}{2}(g^2+g'^2)v_1^2\sec^2\beta$$

$$\tan\beta \equiv \frac{v_2}{v_1} \qquad (9)$$

$$m_A^2 \equiv m_1^2 + m_2^2 + 2|\mu|^2 \qquad (10)$$

multiplying (5) by V_2 and V_1 respectively :

$$\frac{(g^2+g'^2)}{4}(V_1^2-V_2^2)V_1V_2+(m_1^2+|\mu|^2)V_1V_2-\frac{1}{2}B\mu V_2^2=0$$

$$\frac{(g^2+g'^2)}{4}(V_2^2-V_1^2)V_2V_1+(m_2^2+|\mu|^2)V_2V_1-\frac{1}{2}B\mu V_1^2=0$$

Summing we get :

$$m_A^2V_1V_2-\frac{1}{2}B\mu(V_1^2+V_2^2)=0$$

$$\Rightarrow B\mu=\frac{2m_A^2V_1V_2}{V_1^2+V_2^2}=\frac{2m_A^2\left(\frac{V_2}{V_1}\right)}{1+\left(\frac{V_2}{V_1}\right)^2}$$

$$B\mu=\frac{2m_A^2\tan\beta}{\sec^2\beta}=2m_A^2\frac{\sin\beta}{\cancel{\cos\beta}}\cos^{\cancel{2}}\beta$$

$$\Rightarrow\boxed{B\mu=m_A^2\sin2\beta}\qquad(11)$$

The difference gives:

$$\frac{(g^2+g'^2)}{2}(V_1^2-V_2^2)V_1V_2+(m_1^2-m_2^2)V_1V_2-\frac{1}{2}B\mu(V_2^2-V_1^2)=0$$

$$V_1^2\frac{(g^2+g'^2)}{2}(1-\tan^2\beta)\tan\beta+(m_1^2-m_2^2)\tan\beta-\frac{1}{2}m_A^2\sin2\beta(\tan^2\beta-1)=0$$

$$\frac{(g^2+g'^2)}{2}V_1^2\frac{\cos2\beta}{\cos^2\beta}\tan\beta+(m_1^2-m_2^2)\tan\beta+\frac{1}{2}m_A^2\sin2\beta\frac{\cos2\beta}{\cos^2\beta}=0$$

$$M_Z^2=\frac{1}{2}(g^2+g'^2)\frac{V_1^2}{\cos^2\beta}$$

$$\Rightarrow M_Z^2\cos2\beta\tan\beta+\frac{1}{\cancel{2}}m_A^2\cancel{2}\frac{\sin\beta\cos\beta}{\cos^{\cancel{2}}\beta}\cos2\beta+(m_1^2-m_2^2)\tan\beta=0$$

$$M_Z^2\cos2\beta\cancel{\tan\beta}+m_A^2\cos2\beta\cancel{\tan\beta}+(m_1^2-m_2^2)\cancel{\tan\beta}=0$$

$$\Rightarrow\boxed{(m_1^2-m_2^2)=-(M_Z^2+m_A^2)\cos2\beta}\qquad(12)$$

∴

$$m_A^2=2|\mu|^2+m_1^2+m_2^2\qquad(10)$$

Adding (10) + (12)

$$\Rightarrow m_A^2-(M_Z^2+m_A^2)\cos2\beta=2|\mu|^2+2m_1^2$$

or $$\boxed{m_1^2+|\mu|^2=\frac{1}{2}m_A^2-\frac{1}{2}(M_Z^2+m_A^2)\cos2\beta}\qquad(13)$$

$-(12)+(10)$ give us :

$$2m_2^2+2|\mu|^2 = m_A^2 + (m_Z^2+m_A^2)\cos 2\beta$$

$$\Rightarrow \boxed{m_2^2+|\mu|^2 = \frac{1}{2}m_A^2+\frac{1}{2}(M_Z^2+m_A^2)\cos 2\beta} \qquad (14)$$

with linear terms concelling, the quadratic part of V^N is:

$$V^N_{quad} = \frac{(g^2+g'^2)}{4}\,[v_1^2-v_2^2]\,[|\phi_1|^2-|\phi_2|^2] + \frac{g^2+g'^2}{2}\,[\mathrm{Re}\,(\phi_1 v_1-\phi_2 v_2)]^2$$

$$+ (m_1^2+|\mu|^2)\,|\phi_1|^2 + (m_2^2+|\mu|^2)\,|\phi_2|^2 - B\mu\,\mathrm{Re}\,(\phi_1\phi_2) + \text{constant}$$

$$= \frac{1}{2}\cos 2\beta\, M_Z^2\,[|\phi_1|^2-|\phi_2|^2] + M_Z^2\,[\mathrm{Re}\,(\phi_1-\phi_2\tan\beta)]^2\cos^2\beta$$

$$+\left(\frac{1}{2}m_A^2-\frac{1}{2}(M_Z^2+m_A^2)\cos 2\beta\right)|\phi_1|^2 + \left(\frac{1}{2}m_A^2+\frac{1}{2}(M_Z^2+m_A^2)\cos 2\beta\right)|\phi_2|^2$$

$$- m_A^2\sin 2\beta\,\mathrm{Re}\,(\phi_1\phi_2) + \text{constant}$$

$$V^N_{quad.} = \frac{1}{2}M_Z^2\cos 2\beta\,[|\phi_1|^2-|\phi_2|^2] + M_Z^2\,[\mathrm{Re}\,(\phi_1\cos\beta-\phi_2\sin\beta)]^2$$

$$+\frac{1}{2}m_A^2\,(|\phi_1|^2+|\phi_2|^2) - \frac{1}{2}(M_Z^2+m_A^2)\cos 2\beta\,(|\phi_1|^2-|\phi_2|^2) - m_A^2\sin 2\beta\,\mathrm{Re}\,(\phi_1\phi_2) + \text{const}$$

(15)

$$\mathrm{Re}\,(\phi_1\phi_2) = \mathrm{Re}\,(\alpha+i\beta)(\gamma+i\delta) = \mathrm{Re}\,(\alpha\gamma+i\alpha\delta+i\beta\gamma-\beta\delta) = \alpha\gamma-\beta\delta$$

$$\mathrm{Re}\,(\phi_1^*\phi_2^*) = \alpha\gamma-\beta\delta = \mathrm{Re}\,\phi_1\,\mathrm{Re}\,\phi_2 - \mathrm{Im}\,\phi_1\,\mathrm{Im}\,\phi_2$$

$$\Rightarrow \mathrm{Re}\,(\phi_1\phi_2) = \mathrm{Re}\,(\phi_1^*\phi_2^*)\ ; \quad \mathrm{Re}\,\phi_i = \mathrm{Re}\,\phi_i^*$$

$$\Rightarrow \phi_i \rightarrow \phi_i^* \quad (CP)$$

$V^N \rightarrow V^N$ ∴ The potential is invariant under CP.

$$V^N_{quad} = \frac{1}{2}M_Z^2\cos 2\beta\,[(\mathrm{Re}\,\phi_1)^2+(\mathrm{Im}\,\phi_1)^2-(\mathrm{Re}\,\phi_2)^2-(\mathrm{Im}\,\phi_2)^2] + M_Z^2\,[\mathrm{Re}\,\phi_1\cos\beta-\mathrm{Re}\,\phi_2\sin\beta$$

$$+\frac{1}{2}m_A^2\,[(\mathrm{Re}\,\phi_1)^2+(\mathrm{Im}\,\phi_1)^2+(\mathrm{Re}\,\phi_2)^2+(\mathrm{Im}\,\phi_2)^2] - \frac{1}{2}(M_Z^2+m_A^2)\cos 2\beta\,[(\mathrm{Re}\,\phi_1)^2$$

$$+(\mathrm{Im}\,\phi_1)^2-(\mathrm{Re}\,\phi_2)^2-(\mathrm{Im}\,\phi_2)^2] - m_A^2\sin 2\beta\,\underset{\downarrow}{\mathrm{Re}\,(\phi_1\phi_2)} + \text{const} \qquad (15a)$$

$$(\mathrm{Re}\,\phi_1\,\mathrm{Re}\,\phi_2 - \mathrm{Im}\,\phi_1\,\mathrm{Im}\,\phi_2)$$

$\Rightarrow$ The mass-squared matrix of the imaginary parts of the ϕ_i is:

$$M^2_{im\phi} = \begin{pmatrix} \frac{1}{2} m_A^2 (1-\cos 2\beta) & \frac{1}{2} m_A^2 \sin 2\beta \\ \frac{1}{2} m_A^2 \sin 2\beta & \frac{1}{2} m_A^2 (1+\cos 2\beta) \end{pmatrix} \qquad (16)$$

$$\det M^2_{im\phi} = \frac{1}{4} m_A^4 \sin^2 2\beta - \frac{1}{4} m_A^2 \sin^2 2\beta = 0 \qquad (17)$$

$$\det (M^2_{im\phi} - \lambda I) = 0$$

$$\Rightarrow \begin{vmatrix} \frac{1}{2} m_A^2 (1-\cos 2\beta) - \lambda & \frac{1}{2} m_A^2 \sin 2\beta \\ \frac{1}{2} m_A^2 \sin 2\beta & \frac{1}{2} m_A^2 (1+\cos 2\beta) - \lambda \end{vmatrix} = 0$$

$$\frac{1}{4} m_A^4 \cancel{\sin^2 2\beta} - \lambda \frac{1}{2} m_A^2 (1-\cancel{\cos 2\beta}) - \lambda \frac{1}{2} m_A^2 (1+\cancel{\cos 2\beta}) + \lambda^2 - \frac{1}{4} m_A^2 \cancel{\sin^2 2\beta} = 0$$

$$\Rightarrow \quad -\lambda m_A^2 + \lambda^2 = 0$$

$$\therefore \begin{cases} \lambda_1 = 0 \rightarrow \text{Goldstone boson (zero mass neutral scalar)} \\ \lambda_2 = m_A^2 \rightarrow \text{mass of the neutral scalar } A^0. \end{cases}$$

(11) has a solution for $0 \leq \beta \leq \frac{\pi}{2}$

The mass squared matrix for the real scalars is:

$$M^2_{Re\phi} = \begin{pmatrix} \overset{\frac{1}{2} M_Z^2 (1+\cos 2\beta)}{\widehat{M_Z^2 \cos^2\beta}} + \frac{1}{2} m_A^2 (1-\cos 2\beta) & -\frac{1}{2} (m_A^2 + M_Z^2) \sin 2\beta \\ -\frac{1}{2} (m_A^2 + m_Z^2) \sin 2\beta & \overset{\frac{1}{2} M_Z^2 (1-\cos 2\beta)}{\widehat{M_Z^2 \sin^2\beta}} + \frac{1}{2} m_A^2 (1+\cos 2\beta) \end{pmatrix}$$

$$\left(\overset{\text{form}}{\approx} \begin{pmatrix} (Re\,\phi_1)^2 & \frac{1}{2} (Re\,\phi_1)(Re\,\phi_2) \\ \frac{1}{2} (Re\,\phi_1)(Re\,\phi_2) & (Re\,\phi_2)^2 \end{pmatrix} \right)$$

$$M_{Re\phi}^2 = \begin{pmatrix} \frac{1}{2} M_Z^2 (1+\cos 2\beta) + \frac{1}{2} m_A^2 (1-\cos 2\beta) & -\frac{1}{2} (m_A^2 + M_Z^2) \sin 2\beta \\ -\frac{1}{2} (m_A^2 + M_Z^2) \sin 2\beta & \frac{1}{2} M_Z^2 (1-\cos 2\beta) + \frac{1}{2} m_A^2 (1+\cos 2\beta) \end{pmatrix} \qquad (17)$$

$$\det (M_{Re\phi}^2 - \lambda I) = 0 \quad \Rightarrow$$

$$\begin{vmatrix} \frac{1}{2} M_Z^2 (1+\cos 2\beta) + \frac{1}{2} m_A^2 (1-\cos 2\beta) - \lambda & -\frac{1}{2} (m_A^2 + M_Z^2) \sin 2\beta \\ -\frac{1}{2} (m_A^2 + M_Z^2) \sin 2\beta & \frac{1}{2} M_Z^2 (1-\cos 2\beta) + \frac{1}{2} m_A^2 (1+\cos 2\beta) - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad \frac{1}{4} M_Z^4 \sin^2 2\beta + \frac{1}{4} m_A^2 M_Z^2 (1+\cos 2\beta)^2 - \frac{1}{2} M_Z^2 (1+\cos 2\beta)\lambda + \frac{1}{4} m_A^2 M_Z^2 (1-\cos 2\beta)^2$$

$$+ \frac{1}{4} m_A^4 \sin^2 2\beta - \frac{1}{2} m_A^2 (1-\cos 2\beta)\lambda - \frac{1}{2} M_Z^2 (1-\cos 2\beta)\lambda - \frac{1}{2} m_A^2 (1+\cos 2\beta)\lambda + \lambda^2$$

$$- \frac{1}{4} (m_A^2 + M_Z^2)^2 \sin^2 2\beta = 0$$

$$\frac{1}{4} M_Z^4 \cancel{\sin^2 2\beta} + \frac{1}{4} m_A^4 \cancel{\sin^2 2\beta} + \frac{1}{2} m_A^2 M_Z^2 (1+\cos^2 2\beta) - M_Z^2 \lambda - m_A^2 \lambda - \lambda^2 - \frac{1}{4} m_A^4 \cancel{\sin^2 2\beta}$$

$$- \frac{1}{4} M_Z^4 \cancel{\sin^2 2\beta} - \frac{1}{2} m_A^2 M_Z^2 \sin^2 2\beta = 0$$

$$\Rightarrow \quad m_A^2 M_Z^2 \cos^2 2\beta - M_Z^2 \lambda - m_A^2 \lambda + \lambda^2 = 0$$

$$\lambda^2 - \lambda (M_Z^2 + m_A^2) + m_A^2 M_Z^2 \cos^2 2\beta = 0$$

$$\lambda_{1,2} = \frac{(M_Z^2 + m_A^2) \pm \left[(M_Z^2 + m_A^2)^2 - 4 m_A^2 M_Z^2 \cos^2 2\beta \right]^{1/2}}{2}$$

$$\Rightarrow \quad \boxed{m_{H^0}^2 = \frac{1}{2} \left[M_Z^2 + m_A^2 + \left[(M_Z^2 + m_A^2)^2 - 4 m_A^2 M_Z^2 \cos^2 2\beta \right]^{1/2} \right]} \qquad (18)$$

$$m_{h^0}^2 = \frac{1}{2}\left[(M_Z^2+m_A^2) - \left[(M_Z^2+m_A^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta\right]^{1/2}\right] \qquad (19)$$

Let's consider the potential :

$$V = \frac{g^2}{2}\left|H_1^\dagger H_2\right|^2 + \frac{g^2+g'^2}{8}\left[(H_1^\dagger H_1 - H_2^\dagger H_2)\right]^2 + (m_1^2+|\mu|^2)(H_1^\dagger H_1) + (m_2^2+|\mu|^2)(H_2^\dagger H_2)$$
$$- B\mu\, \mathrm{Re}(H_1^T i\tau_2 H_2) \qquad (20)$$

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}; \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$$

$$H_1^\dagger H_2 = (H_1^{0\,*} \;\; H_1^{-\,*}) \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = H_1^{0\,*} H_2^+ + H_1^{-\,*} H_2^0$$

$$H_1^\dagger H_1 = (H_1^{0\,*} \;\; H_1^{-\,*}) \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = |H_1^0|^2 + |H_1^-|^2$$

$$H_2^\dagger H_2 = (H_2^{+\,*} \;\; H_2^{0\,*}) \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = |H_2^+|^2 + |H_2^0|^2$$

$$H_1^T i\tau_2 H_2 = (H_1^0 \;\; H_1^-)\, i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = (H_1^0 \;\; H_1^-) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$$

$$\ldots = (H_1^0 \;\; H_1^-) \begin{pmatrix} H_2^0 \\ -H_2^+ \end{pmatrix} = H_1^0 H_2^0 - H_1^- H_2^+$$

$$\Rightarrow V = \frac{g^2}{2}\left|H_1^{0\,*} H_2^+ + H_1^{-\,*} H_2^0\right|^2 + \frac{(g^2+g'^2)}{8}\left[|H_1^0|^2 - |H_2^0|^2 + |H_1^-|^2 - |H_2^+|^2\right]^2$$
$$+ (m_1^2+|\mu|^2)(|H_1^0|^2 + |H_1^-|^2) + (m_2^2+|\mu|^2)(|H_2^+|^2 + |H_2^0|^2) - B\mu\, \mathrm{Re}(H_1^0 H_2^0)$$
$$+ B\mu\, \mathrm{Re}(H_1^- H_2^+) \qquad (21)$$

From here we obtain (2) and the quadratic part of the charged scalar potential

$$V^c_{quad} = \frac{g^2}{2}\left|V_1 H_2^+ + (H_1^-)^* V_2\right|^2 + \frac{(g^2+g'^2)}{4}\left[|H_1^-|^2 - |H_2^+|^2\right](V_1^2 - V_2^2)$$
$$+ (m_1^2+|\mu|^2)|H_1^-|^2 + (m_2^2+|\mu|^2)|H_2^+|^2 + B\mu\, \mathrm{Re}(H_1^- H_2^+) \qquad (22)$$

with $H_1^{0\,*} = V_1$; $H_2^0 = V_2$

$$V^c_{quad} = \frac{g^2}{2}\left(v_1 H_2^+ + (H_1^-)^* v_2\right)\left(v_1 (H_2^+)^* + (H_1^-) v_2\right) + \frac{1}{4}(g^2+g'^2)\left[|H_1^-|^2 - |H_2^+|^2\right](v_1^2 - v_2^2)$$

$$+ (m_1^2 + |\mu|^2)|H_1^-|^2 + (m_2^2 + |\mu|^2)|H_2^+|^2 + B\mu \, \mathrm{Re}(H_1^- H_2^+)$$

$$V^c_{quad} = \frac{g^2}{2}\left(v_1^2 |H_2^+|^2 + v_1 v_2 H_2^+ H_1^- + v_2 v_1 (H_2^+ H_1^-)^* + v_2^2 |H_1^-|^2 \right) + \frac{1}{4}(g^2+g'^2)\left[|H_1^-|^2 - |H_2^+|^2\right]$$

$$(v_1^2 - v_2^2) + (m_1^2 + |\mu|^2)|H_1^-|^2 + (m_2^2 + |\mu|^2)|H_2^+|^2 + B\mu \, \mathrm{Re}(H_2^+ H_1^-)$$

$$= g^2 v_1 v_2 \, \mathrm{Re}(H_2^+ H_1^-) + |H_1^-|^2\left[\frac{g^2}{2} v_2^2 + \frac{1}{4}(g^2+g'^2)(v_1^2 - v_2^2)\right] + |H_2^+|^2\left[\frac{g^2}{2} v_1^2 - \right.$$

$$\left. - \frac{1}{4}(g^2+g'^2)(v_1^2 - v_2^2)\right] + (m_1^2 + |\mu|^2)|H_1^-|^2 + (m_2^2 + |\mu|^2)|H_2^+|^2 + B\mu \, \mathrm{Re}(H_2^+ H_1^-)$$

$$M_W^2 = \frac{1}{2} g^2 v_1^2 \sec^2\beta = \frac{1}{2}\frac{g^2 v_1^2}{\cos^2\beta} \;; \quad \frac{1}{2}(g^2+g'^2)(v_1^2 - v_2^2) = \frac{1}{2}(g^2+g'^2) v_1^2 \cos 2\beta \sec^2\beta$$

$$V^c_{quad} = g^2 v_1^2 \tan\beta \, \mathrm{Re}(H_2^+ H_1^-) + m_A^2 \sin 2\beta \, \mathrm{Re}(H_2^+ H_1^-) + |H_1^-|^2\left[\frac{g^2}{2} v_1^2 \cancel{\tan^2\beta} + \frac{1}{\cancel{2}} M_Z^2 \cos 2\beta\right]$$

$$+ |H_2^+|^2\left[\frac{g^2}{2} v_1^2 - \frac{1}{2} M_Z^2 \cos 2\beta\right] + \left(\frac{1}{2}\cancel{m_A^2} - \frac{1}{2}(M_Z^2 + m_A^2)\cos 2\beta\right)|H_1^-|^2$$

$$+ \left(\frac{1}{2} m_A^2 + \frac{1}{2}(M_Z^2 + m_A^2)\cos 2\beta\right)|H_2^+|^2$$

$$V^c_{quad} = \frac{2M_W^2}{\sec^2\beta}\tan\beta \, \mathrm{Re}(H_2^+ H_1^-) + m_A^2 \sin 2\beta \, \mathrm{Re}(H_2^+ H_1^-) + |H_1^-|^2\left[M_W^2 \cos^2\beta \tan^2\beta + \frac{1}{2} M_Z^2 \cancel{\cos 2\beta}\right.$$

$$\left. + \frac{1}{2} m_A^2 - \frac{1}{2}(\cancel{M_Z^2} + m_A^2)\cos 2\beta\right] + |H_2^+|^2\left[M_W^2 \cos^2\beta - \frac{1}{2} M_Z^2 \cancel{\cos 2\beta} + \frac{1}{2} m_A^2 + \frac{1}{2}(\cancel{M_Z^2} + m_A^2)\cos 2\right.$$

$$= (m_A^2 + M_W^2)\sin 2\beta \, \mathrm{Re}(H_2^+ H_1^-) + |H_1^-|^2\left[M_W^2 \left(\frac{1 - \cos 2\beta}{2}\right) + \frac{1}{2} m_A^2 (1 - \cos 2\beta)\right]$$

$$+ |H_2^+|^2\left[M_W^2 \left(\frac{\cos 2\beta + 1}{2}\right) + \frac{1}{2} m_A^2 (1 + \cos 2\beta)\right]$$

$$\boxed{V^c_{quad} = \frac{(M_W^2 + m_A^2)}{2}|H_1^-|^2(1 - \cos 2\beta) + \frac{(M_W^2 + m_A^2)}{2}|H_2^+|^2(1 + \cos 2\beta) + (M_W^2 + m_A^2)\sin 2\beta \, \mathrm{Re}(H_2^+ H_1 \qquad (23)}$$

The charged scalar mass matrix is then:

$$M_C^2 = \frac{1}{2}(M_W^2 + m_A^2)\begin{pmatrix} (1 - \cos 2\beta) & \sin 2\beta \\ \sin 2\beta & (1 + \cos 2\beta) \end{pmatrix} \qquad (24)$$

$$\left| M_c^2 - \lambda I \right| = 0 \quad \Rightarrow$$

$$\begin{vmatrix} \frac{1}{2}(M_W^2 + m_A^2)(1-\cos 2\beta) - \lambda & \sin 2\beta \, \frac{1}{2}(M_W^2 + m_A^2) \\ \sin 2\beta \, \frac{1}{2}(M_W^2+m_A^2) & \frac{1}{2}(M_W^2+m_A^2)(1+\cos 2\beta) - \lambda \end{vmatrix} = 0$$

$$\cancel{\frac{1}{4}(M_W^2+m_A^2)\sin^2 2\beta} - \lambda \frac{1}{2}(M_W^2+m_A^2)(1-\cancel{\cos 2\beta}) - \lambda \frac{1}{2}(M_W^2+m_A^2)(1+\cancel{\cos 2\beta}) + \lambda^2$$

$$\cancel{- \frac{1}{4}(M_W^2+m_A^2)\sin^2 2\beta} = 0$$

$$\Rightarrow \lambda^2 - \lambda(M_W^2+m_A^2) = 0$$

$$\lambda\left[\lambda - (M_W^2+m_A^2)\right] = 0$$

$\lambda_1 = 0 \rightarrow$ zero-mass charged scalar

$\lambda_2 = M_{H^\pm}^2 = M_W^2 + m_A^2$

$$\therefore \quad \boxed{M_{H^\pm}^2 = M_W^2 + m_A^2} \qquad (25)$$

$\lambda_1=0$, like the neutral Goldstone boson found earlier it is eliminated by the Higgs mechanism.

(25) implies:

$$\boxed{m_{A^0} < M_{H^\pm}} \qquad (26)$$

$$\boxed{M_{H^\pm} > M_W} \qquad (27)$$

(18) can also be written as:

$$\boxed{m_{H^0}^2 = \frac{1}{2}\left[M_Z^2 + m_A^2 + \left[(M_Z^2 - m_A^2)^2 + 4m_A^2 M_Z^2 \sin^2 2\beta\right]^{1/2}\right]} \qquad (28)$$

$0 \leq \beta \leq \pi/2$)

and (19) as:

$$\boxed{m_{h^0}^2 = \frac{1}{2}\left[M_Z^2 + m_A^2 - \left[(M_Z^2 - m_A^2)^2 + 4m_A^2 M_Z^2 \sin^2 2\beta\right]^{1/2}\right]} \qquad (29)$$

From (28) we see that

m_{H^0} is larger than the larger of M_Z and m_A

m_{h^0} is smaller than the smaller of M_Z and m_A

$$m_{H^0}^2 = \frac{1}{2}\left[M_Z^2 + m_A^2 + \left[(M_Z^2 - m_A^2)^2 + 4 m_A^2 M_Z^2 \sin^2 2\beta \right]^{1/2} \right]$$

$$0 \le \beta \le \pi/2$$

$$(M_Z^2 - m_A^2)^2 + 4 m_A^2 M_Z^2 \sin^2 2\beta \le (M_Z^2 - m_A^2)^2 + 4 m_A^2 M_Z^2$$

$$m_{H^0}^2 \le \frac{1}{2}\left[M_Z^2 + m_A^2 + \left[(M_Z^2 - m_A^2)^2 + 4 m_A^2 M_Z^2 \right]^{1/2} \right]$$

$$m_{H^0}^2 \le \frac{1}{2}\left[M_Z^2 + m_A^2 + (M_Z^2 + m_A^2) \right] = M_Z^2 + m_A^2$$

$$\boxed{m_{H^0}^2 \le M_Z^2 + m_A^2}$$

or $m_{H^0} \le (M_Z^2 + m_A^2)^{1/2}$.

$$(M_Z^2 - m_A^2)^2 + 4 m_A^2 M_Z^2 \sin^2 2\beta \geqslant (M_Z^2 - m_A^2)^2$$

$$\Rightarrow \quad m_{H^0}^2 \ge \frac{1}{2}\left[M_Z^2 + \cancel{m_A^2} + M_Z^2 - \cancel{m_A^2} \right] = M_Z^2$$

$$\Rightarrow \quad \boxed{m_{H^0} \ge M_Z}$$

$$m_{H^0}^2 \le M_Z^2 + M_{H^\pm}^2 - M_W^2 = M_{H^\pm}^2 + M_Z^2 - M_Z^2 \cos^2\theta_W = M_{H^\pm}^2 + M_Z^2 (1 - \cos^2\theta_W)$$

$$m_{H^0}^2 \le M_{H^\pm}^2 + M_Z^2 \sin^2\theta_W = M_{H^\pm}^2 + M_W^2 \tan^2\theta_W$$

but $M_W < M_{H^\pm}$

$\Rightarrow M_W^2 < M_{H^\pm}^2$

$\therefore M_W^2 \tan^2\theta_W < M_{H^\pm}^2 \tan^2\theta_W$

$$\Rightarrow \quad m_{H^0}^2 \le M_{H^\pm}^2 + M_{H^\pm}^2 \tan^2\theta_W = M_{H^\pm}^2 \sec^2\theta_W$$

$$\boxed{M_Z \le m_{H^0} \le \sec\theta_W M_{H^\pm}} = \frac{1}{(1 - \underset{\overset{\downarrow}{0.23113}}{\sin^2\theta_W})^{1/2}} M_{H^\pm} = 1.1404 M_{H^\pm}$$

$$m_{h^0}^2 = \frac{1}{2}(M_Z^2 + m_A^2) - \frac{1}{2}\left[M_Z^4 + m_A^4 - 2m_A^2 M_Z^2 + 4m_A^2 M_Z^2 \sin^2 2\beta\right]^{1/2}$$

$$= \frac{1}{2}(M_Z^2 + m_A^2) - \frac{1}{2}m_A^2\left[\frac{M_Z^4}{m_A^4} + 1 - \frac{2M_Z^2}{m_A^2} + \frac{4M_Z^2}{m_A^2}\sin^2 2\beta\right]^{1/2}$$

$$= \frac{1}{2}(M_Z^2 + m_A^2) - \frac{1}{2}m_A^2\left[1 + \left(\frac{M_Z^4}{m_A^4} - \frac{2M_Z^2}{m_A^2} + \frac{4M_Z^2}{m_A^2}\sin^2 2\beta\right)\right]^{1/2}$$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots$$

$$m_{h^0}^2 = \frac{1}{2}(M_Z^2 + m_A^2) - \frac{1}{2}m_A^2\left[1 + \frac{1}{2}\left(\frac{M_Z^4}{m_A^4} - 2\frac{M_Z^2}{m_A^2} + 4\frac{M_Z^2}{m_A^2}\sin^2 2\beta\right) - \frac{1}{8}\left(\frac{M_Z^4}{m_A^4}\right)(-2 + 4\sin^2 2\beta)^2 + \cdots\right]$$

$$= \cancel{\frac{1}{2}M_Z^2} + \cancel{\frac{1}{2}m_A^2} - \cancel{\frac{1}{2}m_A^2} - \frac{1}{4}m_A^2\cancel{\left(\frac{M_Z^4}{m_A^4}\right)} + \cancel{\frac{1}{2}M_Z^2} - M_Z^2\sin^2 2\beta + \frac{1}{16}m_A^2\left(\frac{M_Z^4}{m_A^4}\right)(\cancel{4} - 16\sin^2 2\beta$$
$$+ 16\sin^4 2\beta) + \cdots$$

$$= M_Z^2\cos^2 2\beta - m_A^2\left(\frac{M_Z^4}{m_A^4}\right)\sin^2 2\beta(1 - \sin^2 2\beta) + \cdots$$

$$= M_Z^2\cos^2 2\beta - m_A^2\left(\frac{M_Z^4}{m_A^4}\right)\sin^2 2\beta\cos^2 2\beta + \cdots$$

$$\boxed{m_{h^0}^2 = M_Z^2\cos^2 2\beta - \frac{1}{4}m_A^2\left(\frac{M_Z^4}{m_A^4}\right)\sin^2 4\beta + \cdots} \qquad (30)$$

$$\Rightarrow \boxed{m_{h^0} \le M_Z\,|\cos 2\beta|} \qquad (31)$$

$$\therefore \boxed{m_{h^0} \le M_Z} \qquad (31a)$$

If we take

$$H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}; \quad H_1 = \begin{pmatrix} H_{1\,0}^{*} \\ -H_1^- \end{pmatrix} \qquad ;$$

$$\mathrm{Re}\,(H_1^{0*} H_2^0) = \mathrm{Re}[(V_1^* + \phi_1^*)(V_2 + \phi_2)] = \mathrm{Re}\,[V_1^* V_2 + V_1^*\phi_2 + \phi_1^* V_2 + \phi_1^*\phi_2]$$

$$\boxed{V^N = \frac{g^2 + g'^2}{8}\left[|H_1^0|^2 - |H_2^0|^2\right]^2 + (m_1^2 + |\mu|^2)|H_1^0|^2 + (m_2^2 + |\mu|^2)|H_2^0|^2 - B\mu\,\mathrm{Re}\,(H_1^{0*}H_2^0)} \qquad (2a)$$

$$V^N = \frac{g^2 + g'^2}{4}\left[|V_1|^2 - |V_2|^2\right]\left[|\phi_1|^2 - |\phi_2|^2 + 2\,\mathrm{Re}\,(\phi_1^* V_1 - \phi_2 V_2^*)\right] + \frac{g^2 + g'^2}{2}\left[\mathrm{Re}\,(\phi_1 V_1^* - \phi_2 V_2^*)\right]$$
$$+ (m_1^2 + |\mu|^2)\left(2\,\mathrm{Re}\,(\phi_1^* V_1) + |\phi_1|^2\right) + (m_2^2 + |\mu|^2)\left(2\,\mathrm{Re}\,(\phi_2 V_2^*) + |\phi_2|^2\right)$$
$$- B\mu\,\mathrm{Re}\,(V_1^*\phi_2 + V_2\phi_1^* + \phi_1^*\phi_2) + \text{const.} \qquad (3a)$$

the terms to first order in ϕ_i or ϕ_i^* must vanish.

$$\begin{cases} \frac{(g^2+g'^2)}{4}\left(|v_1|^2-|v_2|^2\right)v_1 + (m_1^2+|\mu|^2)\,v_1 - \frac{B\mu}{2}v_2 = 0 \\ \frac{(g^2+g'^2)}{4}\left(|v_2|^2-|v_1|^2\right)v_2^* + (m_2^2+|\mu|^2)v_2^* - \frac{B\mu}{2}v_1^* = 0 \end{cases} \qquad (4a)$$

We may adjust the relative phases of the ϕ_i so that v_1 is real. Then v_2 is also real and we get again (5). We define $\tan\beta = \frac{v_2}{v_1}$.

The quadratic part of V^N is:

$$V^N_{quad} = \frac{g^2+g'^2}{4}(v_1^2-v_2^2)(|\phi_1|^2-|\phi_2|^2) + \frac{g^2+g'^2}{2}\left[\mathrm{Re}\,(\phi_1 v_1 - \phi_2 v_2)\right]^2 + (m_1^2+|\mu|^2)|\phi_1|^2$$
$$+ (m_2^2+|\mu|^2)|\phi_2|^2 - B\mu\,\mathrm{Re}\,(\phi_1^*\phi_2) + \text{const}.$$

$$= \frac{1}{2}M_Z^2\cos 2\beta\left[|\phi_1|^2-|\phi_2|^2\right] + M_Z^2\left[\mathrm{Re}\,(\phi_1\cos\beta - \phi_2\sin\beta)\right]^2 + \frac{1}{2}m_A^2\left(|\phi_1|^2+|\phi_2|^2\right)$$
$$-\frac{1}{2}(M_Z^2+m_A^2)\cos 2\beta\left(|\phi_1|^2-|\phi_2|^2\right) - m_A^2\sin 2\beta\,\mathrm{Re}\,(\phi_1^*\phi_2) + \text{const}.$$

$\phi_i \to \phi_i^*$ (CP)

$V^N \longrightarrow V^N$

because $\mathrm{Re}\,(\phi_1^*\phi_2) = \mathrm{Re}\,(\phi_1\phi_2^*)$

$$V^N_{quad} = \frac{1}{2}M_Z^2\cos 2\beta\left[(\mathrm{Re}\,\phi_1)^2+(\mathrm{Im}\,\phi_1)^2-(\mathrm{Re}\,\phi_2)^2-(\mathrm{Im}\,\phi_2)^2\right] + M_Z^2\left[\mathrm{Re}\,\phi_1\cos\beta - \mathrm{Re}\,\phi_2\sin\beta\right]^2$$
$$+\frac{1}{2}m_A^2\left[(\mathrm{Re}\,\phi_1)^2+(\mathrm{Im}\,\phi_1)^2+(\mathrm{Re}\,\phi_2)^2+(\mathrm{Im}\,\phi_2)^2\right] - \frac{1}{2}(M_Z^2+m_A^2)\cos 2\beta\left[(\mathrm{Re}\,\phi_1)^2+(\mathrm{Im}\,\phi_1)^2\right.$$
$$\left.-(\mathrm{Re}\,\phi_2)^2-(\mathrm{Im}\,\phi_2)^2\right] - m_A^2\sin 2\beta\,\mathrm{Re}\,(\phi_1^*\phi_2) + \text{const}.$$

$$\mathrm{Re}\,(\phi_1^*\phi_2) = \mathrm{Re}\,(\alpha+i\beta)^*(\gamma+i\delta) = \mathrm{Re}\,(\alpha-i\beta)(\gamma+i\delta)$$
$$= \alpha\gamma+\beta\delta$$

$$\mathrm{Re}\,(\phi_1^*\phi_2) = \mathrm{Re}\,(\phi_1)\,\mathrm{Re}\,(\phi_2) + \mathrm{Im}\,(\phi_1)\,\mathrm{Im}\,(\phi_2).$$

$$M_{img}^2 = \begin{pmatrix} \frac{1}{2} m_A^2 (1-\cos 2\beta) & -\frac{m_A^2}{2}\sin 2\beta \\ -\frac{m_A^2}{2}\sin 2\beta & \frac{1}{2} m_A^2 (1+\cos 2\beta) \end{pmatrix}$$

$$\det (M_{img}^2 - \lambda I) = 0 \Rightarrow$$

$$\begin{vmatrix} \frac{1}{2} m_A^2 (1-\cos 2\beta) - \lambda & -\frac{m_A^2}{2}\sin 2\beta \\ -\frac{m_A^2}{2}\sin 2\beta & \frac{1}{2} m_A^2 (1+\cos 2\beta) - \lambda \end{vmatrix} = 0$$

that give us

$$\lambda_1 = 0$$
$$\lambda_2 = m_A^2$$

For the real scalars the mass squared matrix is again (17). Then once we obtain (18) and (19).

↓ Let's consider the potential: ↙

$$\boxed{\begin{aligned} V = \frac{g^2}{2} |H_1^+ H_2|^2 + \frac{g^2+g'^2}{8} [(H_1^+ H_1 - H_2^+ H_2)]^2 + (m_1^2 + |\mu|^2)(H_1^+ H_1) + (m_2^2 + |\mu|^2)(H_2^+ H_2) \\ - B\mu \, \mathrm{Re} \, (H_1^T i \tau_2 H_2) \qquad (20) \end{aligned}}$$

↓ $H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$; $H_1 = \begin{pmatrix} H_1^{0*} \\ -H_1^- \end{pmatrix}$ ↙

(H_1^+ = H_1 dagger = Transpose conjugate of H_1)

$$H_1^+ H_2 = (H_1^0 \;\; -H_1^{-*}) \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = H_1^- H_2^+ - H_1^{-*} H_2^0$$

$$H_1^+ H_1 = (H_1^0 \;\; -H_1^{-*}) \begin{pmatrix} H_1^{0*} \\ -H_1^- \end{pmatrix} = |H_1^0|^2 + |H_1^-|^2$$

$$H_1^+ H_2 = (H_2^{+*} \;\; H_2^{0*}) \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = |H_2^+|^2 + |H_1^0|^2$$

$$H_1^T i \tau_2 H_2 = (H_1^{0*} \;\; -H_1^-) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = (H_1^{0*} \;\; -H_1^-) \begin{pmatrix} H_2^0 \\ -H_2^+ \end{pmatrix}$$

$$= H_1^{0*} H_2^0 + H_1^- H_2^+$$

$$\Rightarrow V = \frac{g^2}{2} | H_1^0 H_2^+ - H_1^{-*} H_2^0 |^2 + \frac{g^2+g'^2}{8} [|H_1^0|^2 - |H_2^0|^2 + |H_1^-|^2 - |H_2^+|^2]^2 + (m_1^2 + |\mu|^2)(|H_1^0|^2 + |H_1^-|^2)$$
$$+ (m_2^2 + |\mu|^2)(|H_2^0|^2 + |H_2^+|^2) - B\mu \, \mathrm{Re} \, (H_1^{0*} H_2^0 + H_1^- H_2^+) \qquad (21a)$$

From (21a) we obtain (20a) again.

$$V^c_{quad} = \frac{g^2}{2}|v_1 H_2^+ - v_2 H_1^{-*}|^2 + \frac{(g^2+g'^2)}{4}\left[|H_1^-|^2 - |H_2^+|^2\right](v_1^2 - v_2^2) + (m_1^2 + |\mu|^2)|H_1^-|^2$$

$$+ (m_2^2 + |\mu|^2)|H_2^+|^2 - B\mu\, Re\,(H_1^- H_2^+)$$

$$= \frac{g^2}{2}(v_1 H_2^+ - v_2 H_1^{-*})(v_1 H_2^{+*} - v_2 H_1^-) + \frac{(g^2+g'^2)}{4}\left[|H_1^-|^2 - |H_2^+|^2\right](v_1^2 - v_2^2) + (m_1^2 + |\mu|^2)|H_1^-|^2$$

$$+ (m_2^2 + |\mu|^2)|H_2^+|^2 - B\mu\, Re\,(H_1^- H_2^+)$$

$$= \frac{g^2}{2}\Big(v_1^2|H_2^+|^2 \overbrace{- v_1 v_2 \cancel{H_2^+} H_1^- - v_1 v_2 \cancel{H_2^{+*}} H_1^{-*}}^{-2v_1v_2 Re(H_2^+ H_1^-)} + v_2^2 |\cancel{H_1^-}|^2\Big) + \frac{(g^2+g'^2)}{4}\left[|\cancel{H_1^-}|^2 - |\cancel{H_2^+}|^2\right](v_1^2 - v_2^2)$$

$$+ (m_1^2 + |\mu|^2)|H_1^-|^2 + (m_2^2 + |\mu|^2)|H_2^+|^2 - B\mu\, Re\,(H_1^- H_2^+)$$

$$= -g^2 v_1 v_2\, Re\,(H_2^+ H_1^-) + |H_1^-|^2\left[\frac{g^2}{2}v_2^2 + \frac{(g^2+g'^2)}{4}(v_1^2 - v_2^2)\right] + |H_2^+|^2\left[\frac{g^2}{2}v_1^2 - \frac{(g^2+g'^2)}{4}(v_1^2 - v_2^2)\right]$$

$$+ (m_1^2 + |\mu|^2)|H_1^-|^2 + (m_2^2 + |\mu|^2)|H_2^+|^2 - B\mu\, Re\,(H_1^- H_2^+)$$

$$V^c_{quad} = -g^2 v_1^2 \tan\beta\, Re\,(H_2^+ \cancel{H_1^-}) - m_A^2 \sin\cancel{2}\beta\, Re\,(H_2^+ H_1^-) + |H_1^-|^2\Big[\overset{M_W^2\cos^2\beta}{\widetilde{\frac{g^2}{2}v_1^2}}\tan^2\beta + \frac{1}{2}M_Z^2\cos 2\beta\Big]$$

$$+ |H_2^+|^2\Big[\underset{}{\overset{M_W^2\cos^2\beta}{\frac{g^2}{2}v_1^2}} - \frac{1}{2}M_Z^2\cos 2\beta\Big] + \left(\frac{1}{2}m_A^2 - \frac{1}{2}(M_Z^2 + m_A^2)\cos 2\beta\right)|H_1^-|^2$$

$$+ \left(\frac{1}{2}m_A^2 + \frac{1}{2}(M_Z^2 + m_A^2)\cos 2\beta\right)|H_2^+|^2$$

$$= -(m_A^2 + M_W^2)\sin 2\beta\, Re\,(H_2^+ H_1^-) + |H_1^-|^2\Big[M_W^2\sin^2\beta + \frac{1}{2}\cancel{M_Z^2}\cos 2\beta + \frac{1}{2}m_A^2 - \frac{1}{2}(\cancel{M_Z^2} + m_A^2)\times$$

$$\cancel{\cos}2\beta\Big] + |H_2^+|^2\Big[M_W^2\cos^2\beta - \frac{1}{2}M_Z^2\cancel{\cos}2\beta + \frac{1}{2}m_A^2 + \frac{1}{2}(\cancel{M_Z^2} + m_A^2)\cos 2\beta\Big]$$

$$V^c_{quad} = -(m_A^2 + M_W^2)\sin 2\beta\, Re\,(H_2^+ H_1^-) + |H_1^-|^2\left[\frac{M_W^2 + m_A^2}{2}\right](1 - \cos 2\beta) + |H_2^+|^2\left[\frac{M_W^2 + m_A^2}{2}\right](1 + \cos 2\beta)$$

(23 a)

The charged scalar mass matrix is then:

$$M_c^2 = \frac{(m_A^2 + M_W^2)}{2}\begin{pmatrix} (1 - \cos 2\beta) & -\sin 2\beta \\ -\sin 2\beta & (1 + \cos 2\beta) \end{pmatrix} \qquad (24a)$$

$$|M_c^2 - \lambda I| = 0 \quad \Rightarrow$$

$$\begin{vmatrix} \frac{1}{2}(m_A^2+M_W^2)(1-\cos 2\beta)-\lambda & -\sin 2\beta\ \frac{1}{2}(m_A^2+M_W^2) \\ -\sin 2\beta\ \frac{1}{2}(m_A^2+M_W^2) & \frac{1}{2}(m_A^2+M_W^2)(1+\cos 2\beta)-\lambda \end{vmatrix} = 0$$

$$\cancel{\frac{1}{4}(m_A^2+M_W^2)^2\sin^2 2\beta} - \lambda\,\frac{1}{2}(m_A^2+M_W^2)(1-\cancel{\cos 2\beta}) - \lambda\,\frac{1}{2}(m_A^2+M_W^2)(1+\cancel{\cos 2\beta}) + \lambda^2$$

$$-\cancel{\frac{1}{4}(m_A^2+M_W^2)^2\sin^2 2\beta} = 0$$

$$-\lambda(m_A^2+M_W^2)+\lambda^2 = 0$$

$$\Rightarrow \lambda_1 = 0$$

$$\lambda_2 = m_A^2+M_W^2$$

$$\therefore \boxed{M_{H^\pm}^2 = M_W^2+m_A^2} \quad \text{again} \quad (25)$$

In conclusion:

$$\begin{cases} m_{A^0}^2 = M_{H^\pm}^2 - M_W^2 \\ m_{H^0}^2 = \frac{1}{2}\left\{ M_{H^\pm}^2 - M_W^2 + M_Z^2 + \left[(M_{H^\pm}^2 - M_W^2 + M_Z^2)^2 - 4M_Z^2(M_{H^\pm}^2 - M_W^2)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2 \right]^{1/2} \right\} \\ m_{h^0}^2 = \frac{1}{2}\left\{ M_{H^\pm}^2 - M_W^2 + M_Z^2 - \left[(M_{H^\pm}^2 - M_W^2 + M_Z^2)^2 - 4M_Z^2(M_{H^\pm}^2 - M_W^2)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2 \right]^{1/2} \right\} \end{cases}$$

This is because (32)

$$\sin 2\beta = \frac{\tan 2\beta}{(1+\tan^2 2\beta)^{1/2}} = \frac{2\tan\beta}{1+\tan^2\beta}$$

$$\cos 2\beta = \cos^2\beta - \sin^2\beta = \frac{1-\tan^2\beta}{1+\tan^2\beta}$$

Higgs bosons masses and radiative corrections

Taking into account radiative corrections we have:

$$m^2_{\substack{H^0\\h^0}} = \frac{1}{2}\left\{m_A^2 + m_Z^2 + \Delta_t + \Delta_b \pm \left[\left((m_A^2 - m_Z^2)\cos 2\beta + \Delta t - \Delta b\right)^2 + (m_A^2 + m_Z^2)^2 \sin^2 2\beta\right]^{1/}\right.$$

$$(*) \qquad (33)$$

where:

$$\Delta b = \frac{3\sqrt{2}\, m_b^4\, G_F}{2\pi^2 \cos^2\beta} \ln\left(\frac{M_{Sb}^2}{m_b^2}\right) = \frac{3\sqrt{2}\, m_b^4\, G_F\,(1+\tan^2\beta)}{2\pi^2} \ln\left(\frac{M_{Sb}^2}{m_b^2}\right) \qquad (34)$$

$$\Delta t = \frac{3\sqrt{2}\, m_t^4\, G_F}{2\pi^2 \sin^2\beta} \ln\left(\frac{M_{St}^2}{m_t^2}\right) = \frac{3\sqrt{2}\, m_t^4\, G_F\,(1+\tan^2\beta)}{2\pi^2 \tan^2\beta} \ln\left(\frac{M_{St}^2}{m_t^2}\right) \quad (\text{ref.}) \qquad (35)$$

M_{Sb} and M_{St} are the masses of the sbottom and stop (the scalar superpartners of the bottom and top quarks)

If $\Delta t = \Delta b = 0$ in $(*)$

we have:

$$m^2_{\substack{H^0\\h^0}} = \frac{1}{2}\left\{m_A^2 + m_Z^2 \pm \left[(m_A^2 - m_Z^2)^2 \cos^2 2\beta + (m_A^2 + m_Z^2)^2 \sin^2 2\beta\right]^{1/2}\right\}$$

$$= \frac{1}{2}\left\{m_A^2 + m_Z^2 \pm \left[(m_A^2 - m_Z^2)^2 \cos^2 2\beta + (m_A^2 + m_Z^2)^2 - (m_A^2 + m_Z^2)^2 \cos^2 2\beta\right]^{1/2}\right\}$$

$$= \frac{1}{2}\left\{m_A^2 + m_Z^2 \pm \left[(m_A^2 + m_Z^2)^2 - 4 m_A^2 m_Z^2 \cos^2 2\beta\right]^{1/2}\right\}$$

$m_A^2 = m_H^2 - M_W^2$ is practically not affected by radiative corrections.

$$\sin 2\beta = \frac{2\tan\beta}{1+\tan^2\beta} \quad ; \quad \cos 2\beta = \frac{1-\tan^2\beta}{1+\tan^2\beta} \qquad (36)$$

For $m_A \to \infty$, m_h reaches a finite upper bound:

$$\lim_{m_A\to\infty} m_{h^0}^2 = \frac{1}{2}\left\{m_A^2 + m_Z^2 + \Delta t + \Delta b - \left[(m_A^2 - m_Z^2)^2 \cos^2 2\beta + 2(\Delta t - \Delta b)(m_A^2 - m_Z^2)\cos 2\right.\right.$$

$$\left.\left. + (\Delta t - \Delta b)^2 + (m_A^2 + m_Z^2)^2 \sin^2 2\beta\right]^{1/2}\right\}$$

$$= \frac{1}{2}\left\{m_A^2 + m_Z^2 + \Delta t + \Delta b - \left[m_A^4 + m_Z^4 - 2 m_A^2 m_Z^2 \cos^2 2\beta + 2 m_A^2 m_Z^2 \sin^2 2\beta\right.\right.$$

$$\left.\left. + (\Delta t - \Delta b)^2 + 2(\Delta t - \Delta b)(m_A^2 - m_Z^2)\cos 2\beta\right]^{1/2}\right\}$$

$$= \frac{1}{2}\left\{m_A^2 + m_Z^2 + \Delta t + \Delta b - m_A^2\left[\left(1 + \cancelto{0}{\frac{m_Z^4}{m_A^4}} - \frac{2m_Z^2}{m_A^2}\cos^2 2\beta + \frac{2m_Z^2}{m_A^2}\sin^2 2\beta\right.\right.\right.$$

$$\left.\left.\left. + \cancelto{0}{\frac{(\Delta t - \Delta b)^2}{m_A^4}} + 2\frac{(\Delta t - \Delta b)\cos 2\beta}{m_A^2} - 2\cancelto{0}{\frac{(\Delta t - \Delta b)m_Z^2\cos 2\beta}{m_A^4}}\right]^{1/2}\right\}$$

$$\simeq \frac{1}{2}\left\{m_A^2 + m_Z^2 + \Delta t + \Delta b - m_A^2\left[1 - \frac{m_Z^2}{m_A^2}\cos^2 2\beta + \frac{m_Z^2}{m_A^2}\sin^2 2\beta + \frac{(\Delta t - \Delta b)\cos 2\beta}{m_A^2}\right]\right\}$$

$$= \frac{1}{2}\left\{m_Z^2 + \Delta t + \Delta b + m_Z^2(\cos^2 2\beta - \sin^2 2\beta) - (\Delta t - \Delta b)\cos 2\beta\right\}$$

$$= \frac{1}{2}\left\{2m_Z^2\cos^2 2\beta + \Delta t(1 - \cos 2\beta) + \Delta b(1 + \cos 2\beta)\right\}$$

$$= m_Z^2\cos^2 2\beta + \Delta t\sin^2\beta + \Delta b\cos^2\beta$$

$$\Rightarrow \boxed{m_h^2 \leq m_h^2(m_A \to \infty) = m_Z^2\cos^2 2\beta + \Delta t\sin^2\beta + \Delta b\cos^2\beta} \quad (**) \quad (37)$$

The effect is an increase in the mass of h^0

$m_{h^0} \leq m_{Z^0}$ is practically excluded by the bounds obtained by LEP and CDF

Taking $m_b = 4.3$ GeV, $m_t = 174.3$ GeV, $M_{st} \sim M_{sb} \sim 1$ TeV and $M_Z = 91.1876$ GeV
$G_F = 1.166391 \times 10^{-5}$ GeV^{-2}.
give us:

$$\Delta_b = \frac{3\sqrt{2} \times 4.3^4 \times 1.166391 \times 10^{-5}\,(1 + \tan^2\beta)}{2\pi^2}\ln\left(\frac{10^6}{4.3^2}\right)$$

$$\Delta_b = 9.34076 \times 10^{-3}\,(1 + \tan^2\beta) \qquad (38)$$

$$\Delta t = \frac{3\sqrt{2} \times 174.3^4 \times 1.166391 \times 10^{-5}\,(1 + \tan^2\beta)}{2\pi^2\tan^2\beta}\ln\left(\frac{10^6}{174.3^2}\right)$$

$$\Delta_t = \frac{8084.58\,(1 + \tan^2\beta)}{\tan^2\beta} \qquad (39)$$

in terms of $\tan\beta$ $(**)$ can be written as:

$$\boxed{m_h^2 \leq m_h^2(m_A \to \infty) = m_Z^2\left(\frac{1 - \tan^2\beta}{1 + \tan^2\beta}\right)^2 + \frac{\Delta t\,\tan^2\beta}{(1 + \tan^2\beta)} + \frac{\Delta b}{(1 + \tan^2\beta)}} \qquad (40)$$

$$\Rightarrow m_h^2 \le m_Z^2\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2 + 8084.58 + 9.34076\times10^{-3}$$

$$m_h^2 \le m_Z^2\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2 + 0.972268 m_Z^2$$

$$m_h \le m_Z\left[\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2 + 0.97227\right]^{1/2}$$

If $\tan\beta = 10$

$m_h \le 1.39\, m_Z = 126.782$ GeV

If $\tan\beta = 30$

$m_h \le 1.4028\, m_Z = 127.917$ GeV

in the limit $\tan\beta \to \infty$

$m_h \le 1.40438\, m_Z = 128.062$ GeV

The upper bound on m_h is raised by radiative corrections from just below m_Z to 128.062 GeV for stop masses of order 1 TeV

Δt is much larger than Δb.

$$m^2_{\substack{H^0\\h^0}} \simeq \frac{1}{2}\Big\{m_A^2+m_Z^2+\Delta_t \pm \Big[(m_A^2-m_Z^2)^2\cos^2 2\beta + 2\Delta t(m_A^2-m_Z^2)\cos 2\beta + \Delta t^2$$
$$+ (m_A^2+m_Z^2)^2\sin^2 2\beta\Big]^{1/2}\Big\}$$

$$= \frac{1}{2}\Big\{m_A^2+m_Z^2+\Delta_t \pm \Big[(m_A^2+m_Z^2)^2 + \cos^2 2\beta\,(\cancel{m_A^4} - 2m_A^2m_Z^2 + \cancel{m_Z^4} - \cancel{m_A^4} - 2m_A^2m_Z^2 - \cancel{m_Z^4})$$
$$+ 2\Delta t(m_A^2-m_Z^2)\cos 2\beta + \Delta t^2\Big]^{1/2}\Big\}$$

$$m^2_{\substack{H^0\\h^0}} = \frac{1}{2}\Big\{m_A^2+m_Z^2+\Delta t \pm \Big[(m_A^2+m_Z^2)^2 - 4m_A^2m_Z^2\cos^2 2\beta + 2\Delta t(m_A^2-m_Z^2)\cos 2\beta$$
$$+ \Delta t^2\Big]^{1/2}\Big\}$$

$$= \frac{1}{2}\Big\{m_H^2-m_W^2+m_Z^2+\Delta t \pm \Big[(m_H^2-m_W^2+m_Z^2)^2 - 4(m_H^2-m_W^2)m_Z^2\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2$$
$$+ 2\Delta t(m_H^2-m_W^2-m_Z^2)\frac{(1-\tan^2\beta)}{(1+\tan^2\beta)} + \Delta t^2\Big]^{1/2}\Big\}$$

$$m^2_{\substack{H^0\\h^0}} = \frac{1}{2}m_H^2\Big\{1-\left(\frac{m_W^2}{m_H^2}\right)+\left(\frac{m_Z^2}{m_H^2}\right)+\left(\frac{\Delta t}{m_H^2}\right) \pm \Big[\left(1-\frac{m_W^2}{m_H^2}+\frac{m_Z^2}{m_H^2}\right)^2 - 4\left(1-\frac{m_W^2}{m_H^2}\right)\left(\frac{m_Z^2}{m_H^2}\right)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2$$
$$+ 2\left(\frac{\Delta t}{m_H^2}\right)\left(1-\frac{m_W^2}{m_H^2}-\frac{m_Z^2}{m_H^2}\right)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right) + \left(\frac{\Delta t}{m_H^2}\right)^2\Big]^{1/2}\Big\} \tag{41}$$

Defining

$$\boxed{g^{*}_{rc}\left(m_{H^\pm}^2, m_Z^2, m_W^2, \tan^2\beta, \Delta t\right) = \left[\left(1-\frac{m_W^2}{m_H^2}+\frac{m_Z^2}{m_H^2}\right)^2 - 4\left(1-\frac{m_W^2}{m_H^2}\right)\left(\frac{m_Z^2}{m_H^2}\right)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2 + 2\left(\frac{\Delta t}{m_H^2}\right)\left(1-\frac{m_W^2}{m_H^2}-\frac{m_Z^2}{m_H^2}\right)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right) + \left(\frac{\Delta t}{m_H^2}\right)^2\right]^{1/2}} \qquad (42)$$

$$\Rightarrow \boxed{m^2_{H^0,h^0} = \frac{1}{2} m_H^2 \left\{ 1-\left(\frac{m_W^2}{m_H^2}\right)+\left(\frac{m_Z^2}{m_H^2}\right)+\left(\frac{\Delta t}{m_H^2}\right) \pm g^{*}_{rc}\left(m_{H^\pm}^2, m_Z^2, m_W^2, \tan^2\beta, \Delta t\right)\right\}} \qquad (43)$$

$$\sin 2\alpha = -\frac{(m_A^2+m_Z^2)\sin 2\beta}{(m_{H^0}^2-m_{h^0}^2)} \qquad \left(-\frac{\pi}{2} \le \alpha \le 0\right) \qquad (44)$$

$$\cos 2\alpha = -\frac{\left(m_A^2-m_Z^2+\frac{\Delta t}{\cos 2\beta}\right)\cos 2\beta}{(m_{H^0}^2-m_{h^0}^2)} \qquad (45)$$

$$\sin 2\alpha = -\frac{2\tan\beta}{1+\tan^2\beta}\,\frac{(m_A^2+m_Z^2)}{(m_{H^0}^2-m_{h^0}^2)}$$

$$\cos 2\alpha = -\frac{\left[\cos 2\beta\,(m_A^2-m_Z^2)+\Delta t\right]}{(m_{H^0}^2-m_{h^0}^2)}$$

$$\cos 2\alpha = 1-2\sin^2\alpha$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\sin^2\alpha = \frac{1}{2}\left\{1+\frac{\left[\cos 2\beta\,(m_A^2-m_Z^2)+\Delta t\right]}{m_H^2\, g^{*}_{rc}}\right\}$$

$$\sin^2\alpha = \frac{1}{2}\left\{1+\frac{\left[\cos 2\beta\left(1-\frac{m_W^2}{m_H^2}-\frac{m_Z^2}{m_H^2}\right)+\frac{\Delta t}{m_H^2}\right]}{g^{*}_{rc}}\right\}$$

$$\boxed{\sin^2\alpha = \frac{1}{2}\left\{1+\frac{\left[\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)\left(1-\frac{m_W^2}{m_H^2}-\frac{m_Z^2}{m_H^2}\right)+\frac{\Delta t}{m_H^2}\right]}{g^{*}_{rc}}\right\}} \qquad (46)$$

$$\boxed{\cos^2\alpha = \frac{1}{2}\left\{1-\frac{\left[\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)\left(1-\frac{m_W^2}{m_H^2}-\frac{m_Z^2}{m_H^2}\right)+\frac{\Delta t}{m_H^2}\right]}{g^{*}_{rc}}\right\}} \qquad (47)$$

$$\boxed{\tan\alpha = - \frac{\left\{ 1 + \dfrac{\left[\left(\dfrac{1-\tan^2\beta}{1+\tan^2\beta} \right) \left(1 - \dfrac{M_W^2}{M_H^2} - \dfrac{M_Z^2}{M_H^2} \right) + \dfrac{\Delta t}{M_H^2} \right]}{g^{*}rc} \right\}^{1/2}}{\left\{ 1 - \dfrac{\left[\left(\dfrac{1-\tan^2\beta}{1+\tan^2\beta} \right) \left(1 - \dfrac{M_W^2}{M_H^2} - \dfrac{M_Z^2}{M_H^2} \right) + \dfrac{\Delta t}{M_H^2} \right]}{g^{*}rc} \right\}^{1/2}}} \tag{48}$$

$$\sin 2\alpha = - \frac{(m_H^2 - m_W^2 + m_Z^2)\sin 2\beta}{m_H^2 \, g^{*}_{rc}}$$

$$\boxed{\sin 2\alpha = \frac{-2\tan\beta}{1+\tan^2\beta} \; \frac{\left(1 - \dfrac{m_W^2}{m_H^2} + \dfrac{m_Z^2}{m_H^2} \right)}{g^{*}_{rc}}} \tag{49}$$

$$\boxed{\cos 2\alpha = - \frac{\left[\dfrac{(1-\tan^2\beta)}{(1+\tan^2\beta)} \left(1 - \dfrac{m_W^2}{m_H^2} - \dfrac{m_Z^2}{m_H^2} \right) + \left(\dfrac{\Delta t}{m_H^2} \right) \right]}{g^{*}_{rc}}} \tag{50}$$

$$\tan 2\alpha = \frac{\tan 2\beta \ (m_A^2 + m_Z^2)}{(m_A^2 - m_Z^2 + \epsilon / \cos 2\beta)} \ ; \qquad -\frac{\pi}{2} < \alpha \leq 0 \qquad (51)$$

$$\tan 2\alpha = a$$

$$\frac{2 \tan \alpha}{1 - \tan^2 \alpha} = a$$

$$2 \tan \alpha = a - a \tan^2 \alpha$$

$$a \tan^2 \alpha + 2 \tan \alpha - a = 0$$

$$\tan \alpha = \frac{-2 \pm [4 + 4a^2]^{1/2}}{2a}$$

$$\tan \alpha = \frac{-1 \pm [1 + a^2]^{1/2}}{a}$$

if $a < 0$: $$\tan \alpha = \frac{-1 + (1 + a^2)^{1/2}}{a} = -\frac{1}{a} - \left(1 + \frac{1}{a^2}\right)^{1/2} \qquad (\sqrt{a^2} = |a| = -a)$$

$$1 + a^{-2} = 1 + \frac{(m_A^2 - m_Z^2 + \epsilon / \cos 2\beta)^2}{\tan^2 2\beta \ (m_A^2 + m_Z^2)^2}$$

$$= \frac{\sin^2 2\beta \ (m_A^2 + m_Z^2)^2 + [(m_A^2 - m_Z^2) \cos 2\beta + \epsilon]^2}{\sin^2 2\beta \ (m_A^2 + m_Z^2)^2}$$

$$= [\sin^2 2\beta \ (m_A^2 + m_Z^2)^2 + \cos^2 2\beta (m_A^2 - m_Z^2)^2 + 2\epsilon (m_A^2 - m_Z^2) \cos 2\beta$$

$$+ \epsilon^2] \cdot \frac{1}{\sin^2 2\beta \ (m_A^2 + m_Z^2)^2}$$

$$= [(m_A^2 + m_Z^2)^2 + \cos^2 2\beta [\cancel{m_A^4} - 2 m_A^2 m_Z^2 + \cancel{m_Z^4} - \cancel{m_A^4} - 2 m_A^2 m_Z^2 - \cancel{m_Z^4}]$$

$$+ 2\epsilon (m_A^2 - m_Z^2) \cos 2\beta + \epsilon^2] \cdot \frac{1}{\sin^2 2\beta \ (m_A^2 + m_Z^2)^2}$$

$$= \frac{[(m_A^2 + m_Z^2)^2 - 4 m_A^2 m_Z^2 \cos^2 2\beta + 2\epsilon (m_A^2 - m_Z^2) \cos 2\beta + \epsilon^2]}{\sin^2 2\beta \ (m_A^2 + m_Z^2)^2}$$

$$\tan \alpha = -\frac{(m_A^2 - m_Z^2 + \epsilon / \cos 2\beta)}{\tan 2\beta \ (m_A^2 + m_Z^2)} - \frac{[(m_A^2 + m_Z^2)^2 - 4 m_A^2 m_Z^2 \cos^2 2\beta + 2\epsilon (m_A^2 - m_Z^2) \cos 2\beta + \epsilon^2}{(\sin^2 2\beta \ (m_A^2 + m_Z^2)^2 |^{1/2}}$$

$$\tan\alpha = -\left\{\left[(M_A^2 - M_Z^2)\cos 2\beta + \epsilon\right] + \left[(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta + 2\epsilon(M_A^2 - M_Z^2)\cos 2\right.\right.$$

$$\left.\left. + \epsilon^2\right]^{1/2}\right\} \cdot \frac{1}{\sin 2\beta\,(M_A^2 + M_Z^2)} \qquad (*)$$

$$\left\{\left[(M_A^2 - M_Z^2)\cos 2\beta + \epsilon\right] + \left[(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2\cos^2 2\beta + 2\epsilon(M_A^2 - M_Z^2)\cos 2\beta + \epsilon^2\right]^{1/2}\right.$$

$$\cdot \left\{-\left[(M_A^2 - M_Z^2)\cos 2\beta + \epsilon\right] + \left[\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\right]^{1/2}\right.$$

$$= -\left\{(M_A^2 - M_Z^2)^2\cos^2 2\beta + 2\epsilon\cancel{\cos 2\beta}(M_A^2 - M_Z^2) \cancel{+\epsilon^2} - (M_A^2 + M_Z^2)^2 + 4M_A^2 M_Z^2\cos^2 2\beta + \right.$$

$$\left. - 2\epsilon(M_A^2 \cancel{- M_Z^2})\cos 2\beta \cancel{-\epsilon^2}\right\}$$

$$= -\left\{\cos^2 2\beta\,(M_A^4 + M_Z^4 + 2M_A^2 M_Z^2) - (M_A^2 + M_Z^2)^2\right\}$$

$$= + \sin^2 2\beta\,(M_A^2 + M_Z^2)^2$$

$$\Rightarrow \tan\alpha = -\frac{\sin^2 2\beta\,(M_A^2 + M_Z^2)^2}{\sin 2\beta\,(M_A^2 + M_Z^2)} \times \frac{1}{\left\{-\left[(M_A^2 - M_Z^2)\cos 2\beta + \epsilon\right] + \left[\qquad\right]^{1/2}\right\}}$$

$$\tan\alpha = -\frac{\sin 2\beta\,(M_A^2 + M_Z^2)}{\left\{-\left[\qquad\right] + \left[\qquad\right]^{1/2}\right\}} \qquad (**)$$

$$\Rightarrow \tan^2\alpha = \frac{\left[\left[(M_A^2 - M_Z^2)\cos 2\beta + \epsilon\right] + \left[\qquad\right]^{1/2}\right]}{\left[-\left[(M_A^2 - M_Z^2)\cos 2\beta + \epsilon\right] + \left[\qquad\right]^{1/2}\right]} \qquad (* \cdot **)$$

$$\tan^2\alpha = \frac{\left[(M_H^2 - M_W^2 - M_Z^2)\cos 2\beta + \epsilon\right] + \left[\qquad\right]^{1/2}}{-\left[(M_H^2 - M_W^2 - M_t^2)\cos 2\beta + \epsilon\right] + \left[\qquad\right]^{1/2}}$$

$$\tan^2\alpha = \frac{\left[\left(1 - \frac{M_W^2}{M_H^2} - \frac{M_Z^2}{M_H^2}\right)\cos 2\beta + \frac{\epsilon}{M_H^2}\right] + g^*_{rc}}{-\left[\left(1 - \frac{M_W^2}{M_H^2} - \frac{M_Z^2}{M_H^2}\right)\cos 2\beta + \epsilon\right] + g^*_{rc}}$$

$$\boxed{\tan^2\alpha = \frac{\left\{1 + \frac{\left[\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)\left(1 - \frac{M_W^2}{M_H^2} - \frac{M_Z^2}{M_H^2}\right) + \frac{\epsilon}{M_H^2}\right]}{g^*_{rc}}\right\}}{\left\{1 - \frac{\left[\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)\left(1 - \frac{M_W^2}{M_H^2} - \frac{M_Z^2}{M_H^2}\right) + \frac{\epsilon}{M_H^2}\right]}{g^*_{rc}}\right\}}} \qquad \epsilon = At \qquad (52)$$

$$\Rightarrow \boxed{\tan^2\alpha = \frac{\left\{ 1 + \frac{\left[\left(\frac{1-\tan^2\beta}{1+\tan^2\beta} \right) \left(1 - \frac{m_W^2}{m_H^2} - \frac{m_Z^2}{m_H^2} \right) + \frac{\Delta t}{m_H^2} \right]}{g'' r c} \right\}}{\left\{ 1 - \frac{\left[\left(\frac{1-\tan^2\beta}{1+\tan^2\beta} \right) \left(1 - \frac{m_W^2}{m_H^2} - \frac{m_Z^2}{m_H^2} \right) + \frac{\Delta t}{m_H^2} \right]}{g^{\kappa} r c} \right\}}} \quad \text{if } a < 0 \quad (***)$$

α is the mixing angle between the two neutral scalar Higgs fields H^0, h^0. (see next page)

$$\tan^2\alpha = \frac{\left\{ 1 + \frac{\cos 2\beta \, (M_A^2 - M_Z^2)}{g} \right\}}{\left\{ 1 - \frac{\cos 2\beta \, (M_A^2 - M_Z^2)}{g} \right\}}$$

$(\tan\alpha < 0)$

ref : → Cern-Th/97-379 Michael Spira and Peter M. Zerwas (Dec. 1997

→ Weinberg

→ A. Krause, T. Plehn, M. Spira and P. M. Zerwas :

arXiv : hep-ph/9707430 V1 22 Jul 1997

(Answers in terms of $\sin^2\alpha$, $\cos^2\alpha$, $\sin 2\alpha$)

if $a > 0$:

$$\tan\alpha = \frac{-1 - (1+a^2)^{1/2}}{a} = -\frac{1}{a} - \left(1 + \frac{1}{a^2}\right)^{1/2}$$

we get the same result $(***)$.

$$\begin{cases} H^0 = \sqrt{2}\,[(\mathrm{Re}\,\phi_1)\cos\alpha + (\mathrm{Re}\,\phi_2)\sin\alpha] \\ h^0 = \sqrt{2}\,[-(\mathrm{Re}\,\phi_1)\sin\alpha + (\mathrm{Re}\,\phi_2)\cos\alpha] \end{cases} \qquad (53)$$

$$V^N_{\substack{quad\\ Re}} = [M_Z^2\cos^2\beta + \frac{1}{2}m_A^2(1-\cos 2\beta)](\mathrm{Re}\,\phi_1)^2 + [M_Z^2\sin^2\beta + \frac{1}{2}m_A^2(1+\cos 2\beta)](\mathrm{Re}$$

$$+(\mathrm{Re}\,\phi_1)(\mathrm{Re}\,\phi_2)[-\sin 2\beta\, M_Z^2 - m_A^2\sin^2\beta] \qquad (54)$$

$$\cos 2\beta = 2\cos^2\beta - 1 \quad ; \quad \cos 2\beta = 1 - 2\sin^2\beta$$

$$V^N_{\substack{quad\\ Re}} = [\frac{1}{2}M_Z^2(1+\cos 2\beta) + \frac{m_A^2}{2}(1-\cos 2\beta)](\mathrm{Re}\,\phi_1)^2 + [\frac{1}{2}M_Z^2(1-\cos 2\beta) + \frac{1}{2}m_A^2 \cdot$$

$$\cdot (1+\cos 2\beta)](\mathrm{Re}\,\phi_2)^2 - \sin 2\beta\,(m_A^2 + M_Z^2)(\mathrm{Re}\,\phi_1)(\mathrm{Re}\,\phi_2) \qquad (55)$$

$$\begin{pmatrix} \frac{H^0}{\sqrt{2}} \\ \frac{h^0}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \mathrm{Re}\,\phi_1 \\ \mathrm{Re}\,\phi_2 \end{pmatrix} \qquad (56)$$

$$\begin{pmatrix} \mathrm{Re}\,\phi_1 \\ \mathrm{Re}\,\phi_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \frac{H^0}{\sqrt{2}} \\ \frac{h^0}{\sqrt{2}} \end{pmatrix} \qquad (57)$$

$$\mathrm{Re}\,\phi_1 = \frac{H^0}{\sqrt{2}}\cos\alpha - \frac{h^0}{\sqrt{2}}\sin\alpha$$

$$\mathrm{Re}\,\phi_2 = \frac{H^0}{\sqrt{2}}\sin\alpha + \frac{h^0}{\sqrt{2}}\cos\alpha$$

$$(\mathrm{Re}\,\phi_1)^2 = \frac{(H^0)^2}{2}\cos^2\alpha + \frac{(h^0)^2}{2}\sin^2\alpha - \sin\alpha\cos\alpha\, H^0 h^0 \qquad (58)$$

$$(\mathrm{Re}\,\phi_2)^2 = \frac{(H^0)^2\sin^2\alpha}{2} + \frac{(h^0)^2}{2}\cos^2\alpha + \sin\alpha\cos\alpha\, H^0 h^0 \qquad (59)$$

$$(\mathrm{Re}\,\phi_1)(\mathrm{Re}\,\phi_2) = \frac{\sin\alpha\cos\alpha}{2}(H^0)^2 + \frac{\cos 2\alpha}{2}H^0 h^0 - \frac{\sin\alpha\cos\alpha}{2}(h^0)^2 \qquad (60)$$

$$V^{N}_{\substack{\text{quad}\\ \text{Re}}} = \frac{1}{2}\{ (M_Z^2+m_A^2)[(\text{Re}\,\phi_1)^2+(\text{Re}\,\phi_2)^2] + (M_Z^2-m_A^2)\cos2\beta\,[(\text{Re}\,\phi_1)^2-(\text{Re}\,\phi_2)^2]$$
$$-2\sin2\beta\,(m_A^2+M_Z^2)(\text{Re}\,\phi_1)(\text{Re}\,\phi_2)\} \qquad (61)$$

$$(\text{Re}\,\phi_1)^2+(\text{Re}\,\phi_2)^2 = \frac{1}{2}(H^0)^2+\frac{1}{2}(h^0)^2 \qquad (62)$$

$$(\text{Re}\,\phi_1)^2-(\text{Re}\,\phi_2)^2 = +\frac{1}{2}\cos2\alpha\,(H^0)^2 - \frac{(h^0)^2}{2}\cos2\alpha - \sin2\alpha\,H^0h^0 \qquad (63)$$

$$(\text{Re}\,\phi_1)(\text{Re}\,\phi_2) = \frac{1}{2}(\cos2\alpha)\,H^0h^0 + \frac{1}{4}(\sin2\alpha)\,(H^0)^2 - \frac{1}{4}\sin2\alpha\,(h^0)^2 \qquad (64)$$

the terms with $(H^0)^2$ are:

$$\left(\frac{1}{2}(M_Z^2+m_A^2)\frac{1}{2} + \frac{1}{2}(M_Z^2-m_A^2)\cos2\beta\,\frac{1}{2}\cos2\alpha - \sin2\beta\,(m_A^2+M_Z^2)\,\frac{1}{4}\sin2\alpha\right)(H^0)^2$$

$$= \frac{1}{4}\left[M_Z^2+m_A^2 + (M_Z^2-m_A^2)\cos2\beta\,\underline{\cos2\alpha - (m_A^2+M_Z^2)}\,\sin2\beta\,\sin2\alpha\right](H^0)^2 \qquad (65)$$

$$\sin2\alpha = \frac{2\mathcal{M}_{12}}{[(\mathcal{M}_{11}-\mathcal{M}_{22})^2+4\mathcal{M}_{12}^2]^{1/2}} \qquad (66)$$

$$\cos2\alpha = \frac{\mathcal{M}_{11}-\mathcal{M}_{22}}{[(\mathcal{M}_{11}-\mathcal{M}_{22})^2+4\mathcal{M}_{12}^2]^{1/2}} \qquad (67)$$

$$m^2_{H^0,h^0} = \frac{1}{2}\left[\mathcal{M}_{11}+\mathcal{M}_{22} \pm [(\mathcal{M}_{11}-\mathcal{M}_{22})^2+4\mathcal{M}_{12}^2]^{1/2}\right]. \qquad (68)$$

$$\mathcal{M}_{11} = \frac{1}{2}M_Z^2(1+\cos2\beta) + \frac{1}{2}m_A^2(1-\cos2\beta)\,; \quad \mathcal{M}_{22} = \frac{1}{2}M_Z^2(1-\cos2\beta) + \frac{1}{2}m_A^2(1+\cos2\beta)$$
$$\mathcal{M}_{12} = -\frac{1}{2}(m_A^2+M_Z^2)\sin2\beta \qquad (69)$$

$$\mathcal{M}_{11}+\mathcal{M}_{22} = M_Z^2+m_A^2 \qquad (70)$$

$$\mathcal{M}_{11} - \mathcal{M}_{22} = m_Z^2 \cos 2\beta - m_A^2 \cos 2\beta = (m_Z^2 - m_A^2)\cos 2\beta$$

$$\Rightarrow \sin 2\alpha = \frac{-(m_A^2 + m_Z^2)\sin 2\beta}{\left[(m_Z^2 - m_A^2)^2 \cos^2 2\beta + \cancel{4}\frac{1}{\cancel{4}}(m_A^2 + m_Z^2)^2 \sin^2 2\beta \right]^{1/2}}$$

$$\sin 2\alpha = \frac{-(m_A^2 + m_Z^2)\sin 2\beta}{\left[m_Z^4 - 2 m_Z^2 m_A^2 + m_A^4 - (m_Z^2 - m_A^2)^2 \sin^2 2\beta + (m_Z^2 + m_A^2)^2 \sin^2 2\beta \right]^{1/2}}$$

$$\boxed{\sin 2\alpha = \frac{-(m_A^2 + m_Z^2)\sin 2\beta}{\left[(m_A^2 - m_Z^2)^2 + 4 m_Z^2 m_A^2 \sin^2 2\beta \right]^{1/2}}} \qquad (71)$$

$$\boxed{\cos 2\alpha = \frac{-(m_A^2 - m_Z^2)\cos 2\beta}{\left[(m_A^2 - m_Z^2)^2 + 4 m_Z^2 m_A^2 \sin^2 2\beta \right]^{1/2}}} \qquad (72)$$

$$\boxed{m^2_{H^0, h^0} = \frac{1}{2}\left[m_A^2 + m_Z^2 \pm \left[(m_A^2 - m_Z^2)^2 + 4 m_Z^2 m_A^2 \sin^2 2\beta \right]^{1/2} \right]} \qquad (73)$$

From (65):

$$\Rightarrow \frac{1}{4}\left[m_Z^2 + m_A^2 + \frac{(m_Z^2 - m_A^2)^2 \cos^2 2\beta}{[\quad]^{1/2}} + \frac{(m_A^2 + m_Z^2)^2 \sin^2 2\beta}{[\quad]^{1/2}} \right] (H^0)^2$$

$$= \frac{1}{2}\,\frac{1}{2}\left[m_Z^2 + m_A^2 + \left[(m_Z^2 - m_A^2)^2 \cos^2 2\beta + (m_A^2 + m_Z^2)^2 \sin^2 2\beta \right]^{1/2} \right] (H^0)^2$$

$$= \frac{1}{2}\,\underbrace{\frac{1}{2}\left[m_A^2 + m_Z^2 + \left[(m_A^2 - m_Z^2)^2 + 4 m_A^2 m_Z^2 \sin^2 2\beta \right]^{1/2} \right]}_{(m_{H^0})^2} (H^0)^2 \qquad \underline{\underline{\text{OK}}} \qquad (74)$$

the part with $(h^0)^2$ is :

$$\frac{1}{2}(m_Z^2 + m_A^2)\frac{1}{2}(h^0)^2 + \frac{1}{2}(m_Z^2 - m_A^2)\cos 2\beta \, \frac{1}{2}\cos 2\alpha \, (h^0)^2 + \frac{1}{4}\sin 2\beta \, (m_A^2 + m_Z^2)\sin 2\alpha \, (h^0|$$

$$= \frac{1}{4}\left[m_A^2 + m_Z^2 - (m_Z^2 - m_A^2)\cos 2\beta \cos 2\alpha + (m_A^2 + m_Z^2)\sin 2\beta \sin 2\alpha \right] (h^0)^2$$

$$= \frac{1}{4}\Big[m_A^2 + M_Z^2 - \frac{(M_Z^2 - m_A^2)^2 \cos^2 2\beta}{[\quad]^{1/2}} - \frac{(m_A^2 + M_Z^2) \sin^2 2\beta}{[\quad]^{1/2}} \Big] (h^0)^2$$

$$\frac{1}{2}\ \underbrace{\frac{1}{2}\ [m_A^2 + M_Z^2 - [\quad]^{1/2}}_{(m_{h^0})^2}] (h^0)^2 \qquad \underline{\underline{\text{OK}}} \qquad (75)$$

$$\Rightarrow V^{N}_{\substack{\text{quad}\\ \text{Re}}} = \frac{1}{2} (m_{H^0})^2 (H^0)^2 + \frac{1}{2} (m_{h^0})^2 (h^0)^2 + \Big[\frac{1}{2} (M_Z^2 - m_A^2) \cos 2\beta \, (-1) \sin 2\alpha$$

$$- \sin 2\beta \, (m_A^2 + M_Z^2) \frac{1}{2} \cos 2\alpha \Big] H^0 h^0$$

$$= \frac{1}{2} (m_{H^0})^2 (H^0)^2 + \frac{1}{2} (m_{h^0})^2 (h^0)^2 + \Big[+ \frac{1}{2} \cancel{(M_Z^4 - m_A^4)} \cos 2\beta \sin 2\beta \frac{1}{[\ ]^{1/2}}$$

$$+ \frac{1}{2} \cancel{\frac{\sin 2\beta \cos 2\beta (m_A^4 - M_Z^4)}{[\quad]^{1/2}}} \Big] H^0 h^0$$

$$\Rightarrow \boxed{V^{\nu}_{\substack{\text{quad}\\ \text{Re}}} = \frac{1}{2} (m_{H^0})^2 (H^0)^2 + \frac{1}{2} (m_{h^0})^2 (h^0)^2} . \qquad (76)$$

$$M^2_{Re\phi} = \begin{pmatrix} \mu_{11} & \mu_{12} \\ \mu_{12} & \mu_{22} \end{pmatrix} \qquad (77)$$

$$\det\left(M^2_{Re\phi} - \lambda I\right) = 0 \qquad (78)$$

$$\begin{vmatrix} \mu_{11} - \lambda & \mu_{12} \\ \mu_{12} & \mu_{22} - \lambda \end{vmatrix} = 0$$

$$(\mu_{11} - \lambda)(\mu_{22} - \lambda) - \mu_{12}^2 = 0$$

$$\mu_{11}\mu_{22} - \lambda\mu_{11} - \lambda\mu_{22} + \lambda^2 - \mu_{12}^2 = 0$$

$$\lambda^2 - \lambda(\mu_{11} + \mu_{22}) + (\mu_{11}\mu_{22} - \mu_{12}^2) = 0$$

$$\lambda = \frac{(\mu_{11} + \mu_{22}) \pm \left[(\mu_{11} + \mu_{22})^2 - 4(\mu_{11}\mu_{22} - \mu_{12}^2)\right]^{1/2}}{2}$$

$$\lambda_{1,2} = \frac{1}{2}\left\{\mu_{11} + \mu_{22} \pm \left[(\mu_{11} - \mu_{22})^2 + 4\mu_{12}^2\right]^{1/2}\right\} \qquad (79)$$

If $\lambda = \lambda_1$

$$\begin{pmatrix} \frac{1}{2}\left(\mu_{11} - \mu_{22} - \left[(\mu_{11} - \mu_{22})^2 + 4\mu_{12}^2\right]^{1/2}\right) & \mu_{12} \\ \mu_{12} & -\frac{1}{2}\left(-\mu_{22} + \mu_{11} + \left[(\mu_{11} - \mu_{22})^2 + 4\mu_{12}^2\right]^{1/2}\right) \end{pmatrix}$$

$$\cdot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-\frac{1}{4}\;\frac{\left(\mu_{11}-\mu_{22}-[(\mu_{11}-\mu_{22})^2+4\mu_{12}^2]^{1/2}\right)\left(\mu_{11}-\mu_{22}+[(\mu_{11}-\mu_{22})^2+4\mu_{12}^2]^{1/2}\right)}{\mu_{12}}$$

$$=-\frac{1}{4}\left\{\frac{\cancel{(\mu_{11}-\mu_{22})^2}-\cancel{(\mu_{11}-\mu_{22})^2}-4\mu_{12}^2}{\mu_{12}}\right\}=\mu_{12}$$

$\Rightarrow$

$$\sim\begin{pmatrix}\frac{1}{2}[\mu_{11}-\mu_{22}-[\quad]^{1/2}] & \mu_{12}\\ \frac{1}{2}[\mu_{11}-\mu_{22}-[\quad]^{1/2}] & \mu_{12}\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$$

$$\Rightarrow\qquad a=\frac{-2b\mu_{12}}{[\mu_{11}-\mu_{22}-[\quad]^{1/2}]}$$

$$\therefore\qquad b\begin{pmatrix}\dfrac{-2\mu_{12}}{[\mu_{11}-\mu_{22}-[\quad]^{1/2}]}\\ 1\end{pmatrix}\times\frac{\sqrt{2}\,[\mu_{11}-\mu_{22}+[\quad]^{1/2}]^{1/2}}{[\mu_{11}-\mu_{22}+[\quad]^{1/2}]}\times\frac{\mu_{12}}{[\quad]^{1/4}}$$

$$\sim\begin{pmatrix}\dfrac{\sqrt{2}\,[\mu_{11}-\mu_{22}+[\quad]^{1/2}]^{1/2}}{2\,[\quad]^{1/4}}\\ \dfrac{\sqrt{2}\;\mu_{12}\;[-(\mu_{11}-\mu_{22})+[\quad]^{1/2}]^{1/2}}{[\quad]^{1/4}\,\underbrace{[\mu_{11}-\mu_{22}+[\quad]^{1/2}]^{1/2}\,[-(\mu_{11}-\mu_{22})+[\quad]^{1/2}]^{1/2}}_{(4\mu_{12}^2)^{1/2}=-2\mu_{12}}}\end{pmatrix}$$

$$=\begin{pmatrix}\sqrt{2}\,\dfrac{[\mu_{11}-\mu_{22}+[\quad]^{1/2}]^{1/2}}{2\,[\quad]^{1/4}}\\ -\sqrt{2}\,\dfrac{[-(\mu_{11}-\mu_{22})+[\quad]^{1/2}]^{1/2}\,\cancel{\mu_{12}}}{2\cancel{\mu_{12}}\,[\quad]^{1/4}}\end{pmatrix}$$

$$\vec{V}_1 = \begin{pmatrix} \frac{\sqrt{2}}{2} \left[\frac{\mu_{11} - \mu_{22} + [|\mu_{11} - \mu_{22}|^2 + 4\mu_{12}^2]^{1/2}}{[|\mu_{11} - \mu_{22}|^2 + 4\mu_{12}^2]^{1/4}} \right]^{1/2} \\ \\ -\frac{\sqrt{2}}{2} \left[\frac{[|\mu_{11} - \mu_{22}|^2 + 4\mu_{12}^2]^{1/2} - |\mu_{11} - \mu_{22}|}{[|\mu_{11} - \mu_{22}|^2 + 4\mu_{12}^2]^{1/4}} \right]^{1/2} \end{pmatrix} \qquad (80)$$

If $\lambda = \lambda_2$

$$\begin{pmatrix} \frac{1}{2}\left(\mu_{11} - \mu_{22} + [|\mu_{11} - \mu_{22}|^2 + 4\mu_{12}^2]^{1/2}\right) & \mu_{12} \\ \\ \mu_{12} & -\frac{1}{2}\left(-\mu_{22} + \mu_{11} - [(\mu_{11} - \mu_{22})^2 + 4\mu_{12}^2]^{1/2}\right) \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} \frac{1}{2}[\mu_{11} - \mu_{22} + [\quad]^{1/2}] & \mu_{12} \\ \\ \frac{1}{2}[\mu_{11} - \mu_{22} + [\quad]^{1/2}] & \mu_{12} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow a = \frac{-2b\mu_{12}}{[\mu_{11} - \mu_{22} + [\quad]^{1/2}]}$$

$$\therefore b \begin{pmatrix} \frac{-2\mu_{12}}{[\mu_{11} - \mu_{22} + [\quad]^{1/2}]} \\ \\ 1 \end{pmatrix} \times \frac{-[[\quad]^{1/2} - |\mu_{11} - \mu_{22})]^{1/2}\, \mu_{12}\sqrt{2}}{[\quad]^{1/4}\, [[\quad]^{1/2} - (\mu_{11} - \mu_{22})]}$$

$$\sim \begin{pmatrix} \frac{\sqrt{2}}{2}\left[\frac{[\quad]^{1/2}-(\mu_{11}-\mu_{22})}{[\quad]^{1/4}}\right]^{1/2} \\ \\ \frac{-\sqrt{2}\,\cancel{\mu_{12}}\,[[\quad]^{1/2}+(\mu_{11}-\mu_{22})]^{1/2}}{[\quad]^{1/4}\ \underset{\overset{\vee}{-2\cancel{\mu_{12}}}}{[4\mu_{12}^2]^{1/2}}} \end{pmatrix}$$

$$\vec{V}_2 = \begin{pmatrix} \frac{\sqrt{2}}{2}\left[\frac{[(\mu_{11}-\mu_{22})^2+4\mu_{12}^2]^{1/2}-(\mu_{11}-\mu_{22})}{[(\mu_{11}-\mu_{22})^2+4\mu_{12}^2]^{1/2}}\right]^{1/2} \\ \\ \frac{\sqrt{2}}{2}\left[\frac{[(\mu_{11}-\mu_{22})^2+4\mu_{12}^2]^{1/2}+(\mu_{11}-\mu_{22})}{[(\mu_{11}-\mu_{22})^2+4\mu_{12}^2]^{1/2}}\right]^{1/2} \end{pmatrix} \qquad (81)$$

Defining :

$$\boxed{\begin{aligned} \cos\alpha &= \frac{\sqrt{2}}{2}\left[\frac{\mu_{11}-\mu_{22}+[(\mu_{11}-\mu_{22})^2+4\mu_{12}^2]^{1/2}}{[(\mu_{11}-\mu_{22})^2+4\mu_{12}^2]^{1/4}}\right]^{1/2} \\ \sin\alpha &= -\frac{\sqrt{2}}{2}\left[\frac{[(\mu_{11}-\mu_{22})^2+4\mu_{12}^2]^{1/2}-(\mu_{11}-\mu_{22})}{[(\mu_{11}-\mu_{22})^2+4\mu_{12}^2]^{1/4}}\right]^{1/2} \end{aligned}} \qquad (82$$

(In fact: $\sin^2\alpha + \cos^2\alpha = 1$)

$$\Rightarrow \underset{\overset{\downarrow}{H^0}}{\vec{V}_1} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} ; \quad \underset{\overset{\downarrow}{h^0}}{\vec{V}_2} = \begin{pmatrix} -\sin\alpha \\ \cos\alpha \end{pmatrix} \qquad (83)$$

$\therefore$ we can write:

$$\begin{aligned} H^0 &= \sqrt{2}\,(\cos\alpha\,\mathrm{Re}\,\phi_1 + \sin\alpha\,\mathrm{Re}\,\phi_2) \\ h^0 &= \sqrt{2}\,(-\sin\alpha\,\mathrm{Re}\,\phi_1 + \cos\alpha\,\mathrm{Re}\,\phi_2) \end{aligned} \qquad (84)$$

$$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$

$$\cos 2\alpha = \frac{1}{\cancel{2}} \frac{\cancel{2}(\mathcal{M}_{11} - \mathcal{M}_{22})}{[(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2]^{1/2}}$$

$$\Rightarrow \boxed{\cos 2\alpha = \frac{\mathcal{M}_{11} - \mathcal{M}_{22}}{[(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2]^{1/2}}} \qquad \underline{\text{ok}}$$

$$\sin 2\alpha = 2 \sin\alpha \cos\alpha$$

$$\boxed{\sin 2\alpha = \frac{2\mathcal{M}_{12}}{[(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2]^{1/2}}}$$

$$\mathcal{M}_{11} - \mathcal{M}_{22} = (m_Z^2 - m_A^2) \cos 2\beta$$

$$\cos 2\alpha = \frac{(m_Z^2 - m_A^2)\cos 2\beta}{[(m_Z^2 - m_A^2)^2 \cos^2 2\beta + (m_Z^2 + m_A^2)^2 \sin^2 2\beta]^{1/2}}$$

$$\Rightarrow \boxed{\cos 2\alpha = \frac{-(m_A^2 - m_Z^2)\cos 2\beta}{[(m_A^2 - m_Z^2)^2 + 4 m_Z^2 m_A^2 \sin^2 2\beta]^{1/2}}}$$

$$\Rightarrow \boxed{\sin 2\alpha = \frac{-(m_A^2 + m_Z^2)\sin 2\beta}{[(m_A^2 - m_Z^2)^2 + 4 m_Z^2 m_A^2 \sin^2 2\beta]^{1/2}}}$$

$$\boxed{\cos\alpha = \frac{\sqrt{2}}{2} \frac{\left[(m_Z^2 - m_A^2)\cos 2\beta + [(m_A^2 - m_Z^2)^2 + 4 m_Z^2 m_A^2 \sin^2 2\beta]^{1/2}\right]^{1/2}}{[(m_A^2 - m_Z^2)^2 + 4 m_Z^2 m_A^2 \sin^2 2\beta]^{1/4}}} \qquad (85)$$

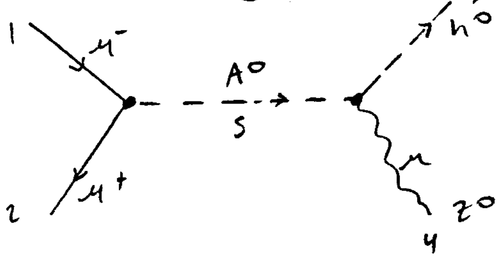
$$\boxed{\sin\alpha = -\frac{\sqrt{2}}{2} \frac{\left[[(m_A^2 - m_Z^2)^2 + 4 m_Z^2 m_A^2 \sin^2 2\beta]^{1/2} - (m_Z^2 - m_A^2)\cos 2\beta\right]^{1/2}}{[(m_A^2 - m_Z^2)^2 + 4 m_Z^2 m_A^2 \sin^2 2\beta]^{1/4}}} \qquad (86)$$

Production of h^0 , H^0

$\mu^-\mu^+ \to h^0 Z^0$

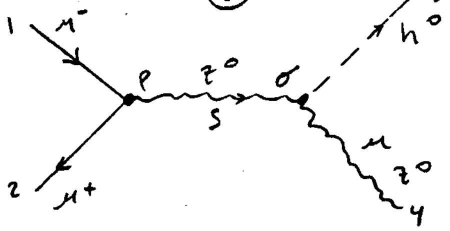
$\xrightarrow{\text{time}}$

①

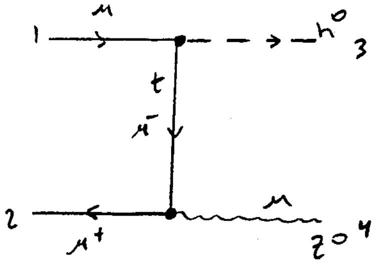


+

②

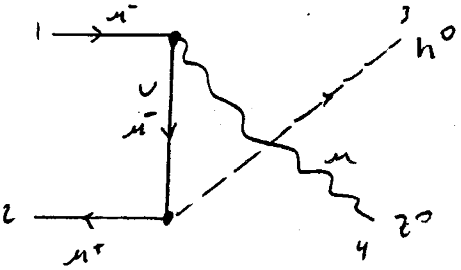


③

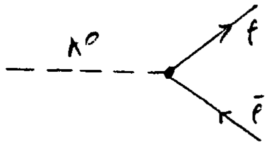


+

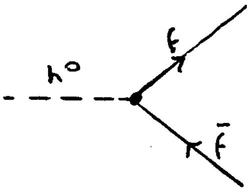
④



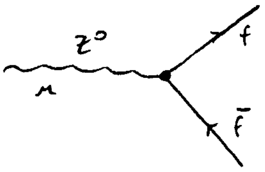
$f = e^-, \mu^-$



$-\frac{g m_f}{2M_W}\tan\beta\,\gamma^5$;



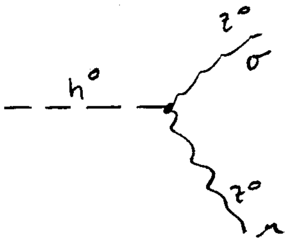
$\frac{i g m_f}{2M_W}\frac{\sin\alpha}{\cos\beta}$



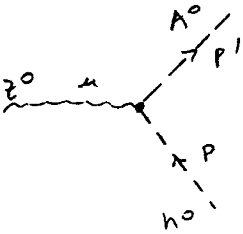
$-\frac{ig}{\cos\theta_W}\gamma^\mu\frac{1}{2}(c_V^f - c_A^f\gamma^5)$

$c_A^f = T_f^3$; $c_V^f = T_f^3 - 2\sin^2\theta_W Q_f$

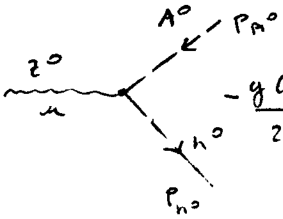
for $f = \mu^-, e^-$: $c_A^f = -\frac{1}{2}$; $c_V^f = -\frac{1}{2} + 2\sin^2\theta_W$



$\frac{i g M_Z}{\cos\theta_W}\sin(\beta-\alpha)\,\eta^{\sigma\mu}$



$\frac{g\cos(\beta-\alpha)}{2\cos\theta_W}(p+p')^\mu \Rightarrow$



$-\frac{g\cos(\beta-\alpha)}{2\cos\theta_W}(P_{h^0}+P_{A^0})^\mu$

$$(\not{p}_1 - m_\mu) U_1 = 0 \Rightarrow \not{p}_1 U_1 = m_\mu U_1$$

$$\bar{V}_2 (\not{p}_2 + m_\mu) = 0 \Rightarrow \bar{V}_2 \not{p}_2 = - m_\mu \bar{V}_2$$

$$(P_1 + P_2)_\rho \bar{V}_2 \gamma^\rho (C_V^\mu - C_A^\mu \gamma^5) U_1 = \bar{V}_2 \not{p}_1 (C_V^\mu - C_A^\mu \gamma^5) U_1 + \bar{V}_2 \not{p}_2 (C_V^\mu - C_A^\mu \gamma^5) U_1$$

$$= \bar{V}_2 (C_V^\mu + C_A^\mu \gamma^5) \not{p}_1 U_1 + \bar{V}_2 \not{p}_2 (C_V^\mu - C_A^\mu \gamma^5) U_1$$

$$= \bar{V}_2 (C_V^\mu + C_A^\mu \gamma^5) m_\mu U_1 - m_\mu \bar{V}_2 (C_V^\mu - C_A^\mu \gamma^5) U_1$$

$$= m_\mu \cancel{C_V^\mu \bar{V}_2 U_1} + m_\mu C_A^\mu \bar{V}_2 \gamma^5 U_1 - m_\mu \cancel{C_V^\mu \bar{V}_2 U_1} + m_\mu C_A^\mu \bar{V}_2 \gamma^5 U_1$$

$$= 2 m_\mu C_A^\mu \bar{V}_2 \gamma^5 U_1 \approx 0 \quad \text{(neglecting } m_\mu\text{)}$$

$$\rightarrow -i M_2 = \frac{-i g^2 M_Z \sin(\beta - \alpha)}{2 \cos^2 \theta_W} \cdot \frac{1}{(s - M_Z^2 + i M_Z \Gamma_Z)} \varepsilon^*_{4\mu} \bar{V}_2 \gamma^\mu (C_V^\mu - C_A^\mu \gamma^5) U_1 \quad (7)$$

$$-i M_3 = \bar{V}_2 \left(\frac{-ig}{\cos\theta_W} \gamma^\mu \frac{1}{2} (C_V^\mu - C_A^\mu \gamma^5) \right) \varepsilon^*_{4\mu} \, i \frac{((\not{p}_1 - \not{p}_3) + m_\mu)}{t - m_\mu^2} \left(\frac{i g m_\mu}{2 M_W} \frac{\sin\alpha}{\cos\beta} \right) U_1$$

$$\rightarrow -i M_3 = \frac{i g^2 m_\mu \sin\alpha}{4 M_W \cos\theta_W \cos\beta} \cdot \frac{1}{(t - m_\mu^2)} \cdot \varepsilon^*_{4\mu} \bar{V}_2 \gamma^\mu (C_V^\mu - C_A^\mu \gamma^5) ((\not{p}_1 - \not{p}_3) + m_\mu) U_1 \quad (8)$$

$$-i M_4 = \bar{V}_2 \left(\frac{i g m_\mu}{2 M_W} \frac{\sin\alpha}{\cos\beta} \right) i \frac{((\not{p}_1 - \not{p}_4) + m_\mu)}{(U - m_\mu^2)} \left(\frac{-ig}{\cos\theta_W} \gamma^\mu \frac{1}{2} (C_V^\mu - C_A^\mu \gamma^5) \right) U_1 \varepsilon^*_{4\mu}$$

$$\rightarrow -i M_4 = \frac{i g^2 m_\mu \sin\alpha}{4 M_W \cos\theta_W \cos\beta} \cdot \frac{1}{(U - m_\mu^2)} \cdot \varepsilon^*_{4\mu} \bar{V}_2 ((\not{p}_1 - \not{p}_4) + m_\mu) \gamma^\mu (C_V^\mu - C_A^\mu \gamma^5) U_1 \quad (9)$$

M_1, M_3, M_4 are proportional to m_μ. M_2 is proportional to M_Z. So M_2 is the only important term.

$$\therefore M \simeq M_2 \quad (10)$$

$$|M_1|^2 = \frac{g^4 m_\mu^2 |C_A|^2 \cos^2(\beta-\alpha)}{4 M_W^2 \cos^2\theta_W} \frac{s}{2} \left\{ -2s - 2m_{h^0}^2 + M_Z^2 + \frac{(s-m_{h^0}^2)^2}{M_Z^2} \right\}$$

$$|M_1|^2 = \frac{g^4 m_\mu^2 |C_A|^2 \cos^2(\beta-\alpha) s}{8 M_W^4} \left\{ -2s M_Z^2 - 2m_{h^0}^2 M_Z^2 + M_Z^4 + s^2 - 2s m_{h^0}^2 + m_{h^0}^4 \right\}$$

$$\boxed{|M_1|^2 = \frac{g^4 m_\mu^2 |C_A|^2 \cos^2(\beta-\alpha) s}{8 M_W^4} \lambda(s, M_Z^2, m_{h^0}^2)} \qquad (19) \qquad (\text{that is of order } m_\mu^2.)$$

The full expression for M_2 is:

$$M_2 = \frac{g^2 M_Z \sin(\beta-\alpha)}{2\cos^2\theta_W} \cdot \frac{1}{(s - M_Z^2 + i M_Z \Gamma_Z)} \varepsilon^*_{4\mu} \left\{ \bar{V}_2 \gamma^\mu (C_V^\mu - C_A^\mu \gamma^5) U_1 - 2\frac{(P_1+P_2)^\mu}{M_Z^2} m_\mu C_A^\mu \right.$$

$$\left. \bar{V}_2 \gamma^5 U_1 \right\} \qquad (20)$$

$$M_{2a} = \frac{g^2 M_Z \sin(\beta-\alpha)}{2\cos^2\theta_W} \cdot \frac{1}{(s - M_Z^2 + i M_Z \Gamma_Z)} \varepsilon^*_{4\mu} \bar{V}_2 \gamma^\mu (C_V^\mu - C_A^\mu \gamma^5) U_1$$

$$M_{2b} = -\frac{g^2 M_Z \sin(\beta-\alpha)}{2\cos^2\theta_W} \cdot \frac{1}{(s - M_Z^2 + i M_Z \Gamma_Z)} \varepsilon^*_{4\mu} \frac{2(P_1+P_2)^\mu}{M_Z^2} m_\mu C_A^\mu \bar{V}_2 \gamma^5 U_1$$

$$\text{defining}: \quad \frac{\sin(\beta-\alpha)}{(s - M_Z^2 + i M_Z \Gamma_Z)} = C_Z \qquad (21)$$

we have:

$$\left\{ \begin{array}{ll} M_{2a} = \dfrac{g^2 M_Z C_Z}{2\cos^2\theta_W} \varepsilon^*_{4\mu} \bar{V}_2 \gamma^\mu (C_V^\mu - C_A^\mu \gamma^5) U_1 & (22) \\ M_{2b} = -\dfrac{g^2 C_Z}{\cancel{2}\cos^2\theta_W} \varepsilon^*_{4\mu} \dfrac{\cancel{2}(P_1+P_2)^\mu}{M_Z} m_\mu C_A^\mu \bar{V}_2 \gamma^5 U_1 & (24) \end{array} \right. \Rightarrow M_2 = M_{2a} + M_{2b} \quad (23)$$

$$|M_2|^2 = (M_{2a} + M_{2b})^+ (M_{2a} + M_{2b}) = |M_{2a}|^2 + M_{2a}^+ M_{2b} + M_{2b}^+ M_{2a} + |M_{2b}|^2 \qquad (25)$$

$$M_{2a}^+ M_{2b} = -\frac{g^4 M_Z |C_Z|^2}{4\cos^4\theta_W} \frac{2(P_1+P_2)^\nu}{M_Z} m_\mu \left(\sum_\lambda \varepsilon_{4\mu} \varepsilon^*_{4\nu} \right) . C_A^\mu .$$

$$. \sum_s (\bar{V}_2 \gamma^\mu (C_V^\mu - C_A^\mu \gamma^5) U_1)^+ (\bar{V}_2 \gamma^5 U_1)$$

$$\sum_s (\;\;) = \sum_s (V_2^+ \gamma^0 \gamma^\mu (C_V^\mu - C_A^\mu \gamma^5) U_1)^+ (\bar{V}_2 \gamma^5 U_1)$$

$$= \sum_s (U_1^+ (C_V^\mu - C_A^\mu \gamma^5) \overset{\gamma^0\gamma^\mu}{\gamma^{\mu+}\gamma^0} V_2) (\bar{V}_2 \gamma^5 U_1) = \sum_s (\bar{U}_1 (C_V^\mu + C_A^\mu \gamma^5) \gamma^\mu V_2)$$

$$. (\bar{V}_2 \gamma^5 U_1)$$

In $M_2^+ M_4$ the relevant part is:

$$M_2^+ M_4 \propto \sum_s \bar{U}_1 (C_V^\mu + C_A^\mu \gamma^5) \gamma^\mu V_2 \, . \, \bar{V}_2 (\not{P}_1 - \not{P}_4) \gamma^\nu (C_V^\mu - C_A^\mu \gamma^5) U_1$$

$$= Tr [\not{P}_1 (C_V^\mu + C_A^\mu \gamma^5) \gamma^\mu \not{P}_2 (\not{P}_1 - \not{P}_4) \gamma^\nu (C_V^\mu - C_A^\mu \gamma^5)]$$

$$= Tr \Big[[C_V^\mu \not{P}_1 \gamma^\mu \not{P}_2 \not{P}_1 \gamma^\nu + \cancel{C_A^\mu \not{P}_1 \gamma^5 \gamma^\mu \not{P}_2 \not{P}_1 \gamma^\nu} - C_V^\mu \not{P}_1 \gamma^\mu \not{P}_2 \not{P}_4 \gamma^\nu$$
$$- C_A^\mu \not{P}_1 \gamma^5 \gamma^\mu \cancel{\not{P}_2 \not{P}_4 \gamma^\nu}] [C_V^\mu - C_A^\mu \gamma^5] \Big] \quad = 0$$

Trace (odd # of γ's) = 0.

$$\Rightarrow \quad \boxed{M_2^+ M_4 + M_4^+ M_2 = 0} \quad \text{to order } m_\mu. \qquad (31)$$

$$\therefore \quad \boxed{|M|^2 \approx |M_{2a}|^2} \text{ as we afirm in (10) to order } m_\mu.$$

($M_1^+ M_3$, $M_1^+ M_4$, $M_3^+ M_4$ are of order m_μ^2)

$$|M_{2a}|^2 = \frac{g^4 M_Z^2 |C_Z|^2}{4 \cos^4 \theta_W} \left(\sum_\lambda \varepsilon_{4\mu} \varepsilon^*_{4\nu} \right) \sum_s (\bar{V}_2 \gamma^\mu (C_V^\mu - C_A^\mu \gamma^5) U_1)^+ (\bar{V}_2 \gamma^\nu (C_V^\mu - C_A^\mu \gamma^5) U_1)$$

$$= \frac{g^4 M_Z^2 |C_Z|^2}{4 \cos^4 \theta_W} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_Z^2} \right) \sum_s (U_1^+ (C_V^\mu - C_A^\mu \gamma^5) \overset{\gamma^0 \gamma^\mu}{\overbrace{\gamma^{\mu +} \gamma^0}} V_2) (\bar{V}_2 \gamma^\nu (C_V^\mu - C_A^\mu \gamma^5) U_1$$

$$= \frac{g^4 M_Z^2 |C_Z|^2}{4 \cos^4 \theta_W} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_Z^2} \right) \sum_s (\bar{U}_1 (C_V^\mu + C_A^\mu \gamma^5) \gamma^\mu V_2) (\bar{V}_2 \gamma^\nu (C_V^\mu - C_A^\mu \gamma^5) U_1)$$

$$= \frac{g^4 M_Z^2 |C_Z|^2}{4 \cos^4 \theta_W} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_Z^2} \right) Tr \left[(\not{P}_1 + m_\mu) (C_V^\mu + C_A^\mu \gamma^5) \gamma^\mu (\not{P}_2 - m_\mu) \gamma^\nu (C_V^\mu - C_A^\mu \gamma^5) \right] \qquad (32)$$

neglecting m_μ in the trace

$$|M_{2a}|^2 = \frac{g^4 M_Z^2 |C_Z|^2}{4 \cos^4 \theta_W} \left(-\eta_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_Z^2} \right) Tr [\not{P}_1 (C_V^\mu + C_A^\mu \gamma^5) \gamma^\mu \not{P}_2 \gamma^\nu (C_V^\mu - C_A^\mu \gamma^5)] \qquad (33)$$

$$Tr [\not{P}_1 (C_V^\mu + C_A^\mu \gamma^5) \gamma^\mu \not{P}_2 \gamma^\nu (C_V^\mu - C_A^\mu \gamma^5)] = Tr \Big\{ [C_V^\mu (\not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu) + C_A^\mu (\not{P}_1 \gamma^5 \gamma^\mu \not{P}_2 \gamma^\nu)] \, . \, (C_V^\mu - C_A^\mu \gamma^5) \Big\}$$

$$= (C_V^\mu)^2 Tr (\not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu) - C_A^\mu C_V^\mu Tr (\gamma^5 \not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu) - C_A^\mu C_V^\mu Tr (\gamma^5 \not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu)$$
$$+ (C_A^\mu)^2 Tr (\not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu)$$

$$= [(C_A^\mu)^2 + (C_V^\mu)^2] Tr (\not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu) - 2 C_A^\mu C_V^\mu Tr (\gamma^5 \not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu)$$

$$= [(C_A^\mu)^2 + (C_V^\mu)^2] Tr (\gamma^\nu \not{P}_1 \gamma^\mu \not{P}_2) + 8 i C_A^\mu C_V^\mu \varepsilon^{\alpha\mu\beta\nu} P_{1\alpha} P_{2\beta}$$

replacing in (55) of $\mu^-\mu^+ \to H^\mp W^\pm$ $M_{H^\pm} \to m_{h^0}$; $M_W \to M_{Z^0}$ we have:

$$ut = m_{h^0}^2 M_Z^2 + \frac{1}{4}\lambda \sin^2\theta \qquad (40)$$

where $\lambda = \lambda(s, m_{h^0}^2, M_Z^2)$

$$\Rightarrow \quad |M|^2 = \frac{g^4 |C_Z|^2 [(|C_A^{\mu}|^2 + (|C_V^{\mu}|^2]}{2\cos^4\theta_W} \cdot [2sM_Z^2 + \frac{1}{4}\lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta] \qquad (41)$$

$$\boxed{\overline{|M|^2} = g^4 \frac{|C_Z|^2 [(|C_A^{\mu}|^2 + (|C_V^{\mu}|)^2]}{32\cos^4\theta_W} [8sM_Z^2 + \lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta]} \qquad (42)$$

$$\overline{|M|^2} = g^4 \frac{M_Z^4 |C_Z|^2 [(|C_A^{\mu}|^2 + (|C_V^{\mu}|^2]}{32 M_W^4} [8sM_Z^2 + \lambda(s, m_{h^0}^2, M_Z^2)\sin^2\theta] \qquad (43)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{64\pi^2 s}\frac{|\vec{P}_3|}{|\vec{P}_1|}\overline{|M|^2} \qquad (44)$$

$$|\vec{P}_1| = \frac{s^{1/2}}{2}; \qquad |\vec{P}_3| = \frac{\lambda^{1/2}(s, m_{h^0}^2, M_Z^2)}{2\sqrt{s}} \qquad (45)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{64\pi^2 s}\frac{\lambda^{1/2}(s, m_{h^0}^2, M_Z^2)}{s}\frac{g^4 M_Z^4 |C_Z|^2}{32 M_W^4}[(|C_A^{\mu}|^2 + (|C_V^{\mu}|^2][8sM_Z^2 + \lambda\sin^2\theta]$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \Rightarrow G_F^2 = \frac{g^4}{32 M_W^4}$$

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{64\pi^2 s^2} G_F^2 M_Z^4 |C_Z|^2 \lambda^{1/2} [(|C_A^{\mu}|^2 + (|C_V^{\mu}|)^2][8sM_Z^2 + \lambda\sin^2\theta]} \qquad (46)$$

$$\sim \frac{M_Z^4}{s}$$

$$d\Omega = 2\pi\, d\cos\theta$$

$$\int_{-1}^{1}\sin^2\theta\, d\cos\theta = \int_{-1}^{1}(1-\cos^2\theta)\,d\cos\theta = \int_{-1}^{1}(1-x^2)\,dx = \left(x - \frac{x^3}{3}\right)\Big|_{-1}^{1}$$

$$= \frac{2}{3}\times 2 = \frac{4}{3} \qquad (47)$$

$$\sigma(\mu^-\mu^+ \to h^0 Z^0) = \frac{1}{\underset{32\pi}{\cancel{64}\pi^2 s^2}}\cancel{2\pi}\, G_F^2 M_Z^4 |C_Z|^2 \lambda^{1/2} [(|C_A^{\mu}|^2 + (|C_V^{\mu}|^2][16sM_Z^2 + \frac{4}{3}\lambda]$$

$$= \frac{1}{24\pi s^2} G_F^2 M_Z^4 |C_Z|^2 \lambda^{1/2} [(|C_A^{\mu}|^2 + (|C_V^{\mu}|^2][12sM_Z^2 + \lambda]$$

$$\boxed{\tan\alpha = -\left\{\frac{1 + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{1-\frac{M_Z^2}{M_{H^\pm}^2}-\frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)}\right]}{1 - \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{1-\frac{M_Z^2}{M_{H^\pm}^2}-\frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)}\right]}\right\}^{1/2}} \qquad (51d)$$

$$\sigma(e^-e^+\to\mu^-\mu^+) = \frac{4}{3}\pi\frac{\alpha^2}{s} = \frac{4}{3}\frac{\pi}{s}\left(\frac{e^2}{4\pi}\right)^2$$

$$g = \frac{e}{\sin\theta_W} \Rightarrow e = g\sin\theta_W \Rightarrow e^4 = g^4\sin^4\theta_W$$

$$\therefore\ \sigma(e^-e^+\to\mu^-\mu^+) = \frac{1}{3s}\frac{1}{4\pi}g^4\sin^4\theta_W = \frac{1}{\underset{3}{\cancel{12}}\pi s}\overset{8}{\cancel{32}}M_W^4 G_F^2\sin^4\theta_W$$

$$\boxed{\sigma(e^-e^+\to\mu^-\mu^+) = \frac{8}{3\pi s}M_Z^4 G_F^2\sin^4\theta_W(1-\sin^2\theta_W)^2} \qquad (52)$$

$$\boxed{\sigma(e^-e^+\to h^0Z^0) \simeq \sigma(\mu^-\mu^+\to h^0Z^0)} \qquad (53)$$

$$\frac{\sigma(e^-e^+\to h^0Z^0)}{\sigma(e^-e^+\to\mu^-\mu^+)} = \frac{\frac{\cancel{G_F^2 M_Z^4}}{48\pi s^2}\frac{(\tan\beta-\tan\alpha)^2}{(1+\tan^2\alpha)(1+\tan^2\beta)}\frac{(1-4\sin^2\theta_W+8\sin^4\theta_W)}{[(s-M_Z^2)^2+M_Z^2\Gamma_Z^2]}[12sM_Z^2+\lambda]\lambda^{1/2}}{\frac{8}{3\pi s}\cancel{M_Z^4}\cancel{G_F^2}\sin^4\theta_W(1-\sin^2\theta_W)^2}$$

$$\boxed{\begin{aligned}\frac{\sigma(e^-e^+\to h^0Z^0)}{\sigma(e^-e^+\to\mu^-\mu^+)} &= \frac{1}{128s}\frac{(\tan\beta-\tan\alpha)^2}{(1+\tan^2\alpha)(1+\tan^2\beta)}\frac{(1-4\sin^2\theta_W+8\sin^4\theta_W)}{\sin^4\theta_W(1-\sin^2\theta_W)^2}\times\\ &\times\frac{[12sM_Z^2+\lambda(s,m_{h^0}^2,M_Z^2)]\lambda^{1/2}(s,m_{h^0}^2,M_Z^2)}{[(s-M_Z^2)^2+M_Z^2\Gamma_Z^2]}\end{aligned}} \qquad (54)$$

If $m_{h^0} = \sqrt{s} - M_Z$

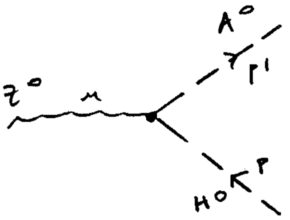
$$\begin{aligned}\lambda(s, m_{h^0}^2, M_Z^2) &= s^2 + m_{h^0}^4 + M_Z^4 - 2s\,m_{h^0}^2 - 2sM_Z^2 - 2m_{h^0}^2M_Z^2\\ &= s^2 + (\sqrt{s}-M_Z)^4 + M_Z^4 - 2s(\sqrt{s}-M_Z)^2 - 2sM_Z^2 - 2(\sqrt{s}-M_Z)^2M_Z^2\\ &= \cancel{s^2} + s^2 + \cancel{4sM_Z^2} + \cancel{M_Z^4} - 4\cancel{s^{3/2}}M_Z + \cancel{2sM_Z^2} - 4\cancel{s^{1/2}}M_Z^3 + \cancel{M_Z^4}\\ &\quad -\cancel{2s^2} + 4\cancel{s^{3/2}}M_Z - \cancel{2sM_Z^2} - \cancel{2sM_Z^2} - \cancel{2sM_Z^2} + \cancel{4s^{1/2}M_Z^3} - \cancel{2M_Z^4} = 0\end{aligned}$$

$$\sigma_{SM}(\mu^-\mu^+ \to \phi^0 Z^0) = \frac{G_F^2 M_Z^4}{48\pi s^2} \frac{(1-4\sin^2\theta_W + 8\sin^4\theta_W)}{[(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2]} \times$$

$$\times [12 s M_Z^2 + \lambda(s, m_{\phi^0}^2, M_Z^2)]\, \lambda^{1/2}(s, m_{\phi^0}^2, M_Z^2) \times$$

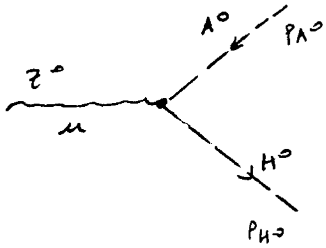
$$\times (3.893792914 \times 10^{11})\ \text{fb}.$$

$$\frac{(\tan\beta - \tan\alpha)^2}{(1+\tan^2\alpha)(1+\tan^2\beta)} = \sin^2(\beta-\alpha) \to 1$$

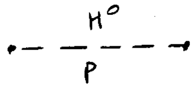


$$\frac{-g\sin(\beta-\alpha)}{2\cos\theta_W}(p+p')^\mu$$

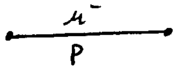
⇒



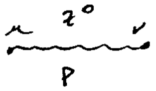
$$\frac{+g\sin(\beta-\alpha)}{2\cos\theta_W}(p_{H^0}+p_{A^0})^\mu$$



$$\frac{i}{p^2-M^2+iM\Gamma}$$



$$\frac{i}{\not p - m_\mu} = \frac{i(\not p + m_\mu)}{p^2-m_\mu^2}$$



$$: \quad -i\left(\eta_{\mu\nu}-\frac{p_\mu p_\nu}{M_Z^2}\right)\cdot\frac{1}{p^2-M_Z^2+iM_Z\Gamma_Z}$$

$$m^2_{H^0} = \frac{1}{2}\left\{m_{A^0}^2+M_Z^2+\left[(m_{A^0}^2+M_Z^2)^2-4M_Z^2m_{A^0}^2\cos^2 2\beta\right]^{1/2}\right\} \qquad (1)$$

$$\tan 2\alpha = \tan 2\beta\left(\frac{m_{A^0}^2+M_Z^2}{m_{A^0}^2-M_Z^2}\right) \qquad (2)$$

$$m_{A^0}^2 = M_{H^\pm}^2 - M_W^2 \qquad (3)$$

$$\boxed{\sigma(\mu^-\mu^+ \to H^0 Z^0) = \frac{1}{48\pi s^2}\,\frac{G_F^2 M_Z^4 \cos^2(\beta-\alpha)}{[(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2]}\,[1-4\sin^2\theta_W + 8\sin^4\theta_W] \times [12sM_Z^2 + \lambda(s, m_{H^0}^2, M_Z^2)]\,\lambda^{1/2}(s, m_{H^0}^2, M_Z^2)} \tag{13}$$

$$\cos(\beta-\alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha$$

$$\cos^2(\beta-\alpha) = (\cos\beta\cos\alpha + \sin\beta\sin\alpha)^2 = \cos^2\beta\cdot\cos^2\alpha\,(1+\tan\beta\tan\alpha)^2$$

$$= \frac{(1+\tan\beta\tan\alpha)^2}{(1+\tan^2\beta)(1+\tan^2\alpha)}$$

$$\boxed{\sigma(\mu^-\mu^+ \to H^0 Z^0) = \frac{G_F^2 M_Z^4}{48\pi s^2}\,\frac{(1+\tan\beta\tan\alpha)^2}{(1+\tan^2\beta)(1+\tan^2\alpha)}\,\frac{[1-4\sin^2\theta_W + 8\sin^4\theta_W]}{[(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2]} \times [12sM_Z^2 + \lambda(s, m_{H^0}^2, M_Z^2)]\,\lambda^{1/2}(s, m_{H^0}^2, M_Z^2) \times (3.893792914 \times 10^{11})\ fb} \tag{14}$$

$$\boxed{m_{H^0}^2 = \frac{1}{2}M_{H^\pm}^2\left\{1 - \frac{M_W^2}{M_{H^\pm}^2} + \frac{M_Z^2}{M_{H^\pm}^2} + g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)\right\}} \tag{14a}$$

$$\boxed{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta) = \left[\left(1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}\right)^2 - 4\left(\frac{M_Z^2}{M_{H^\pm}^2}\right)\left(1 - \frac{M_W^2}{M_{H^\pm}^2}\right)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2\right]^{1/2}} \tag{14b}$$

$$\boxed{\lambda(s, m_{H^0}^2, M_Z^2) = s^2 + m_{H^0}^4 + M_Z^4 - 2s\,m_{H^0}^2 - 2s\,M_Z^2 - 2m_{H^0}^2 M_Z^2} \tag{14c}$$

$$\boxed{\tan\alpha = \left\{\frac{1 + \dfrac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\dfrac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)}\right]}{1 - \dfrac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\dfrac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta}\right]}\right\}^{1/2}} \tag{14d}$$

Using:

$$\boxed{\sigma(e^-e^+ \to \mu^-\mu^+) = \frac{8}{3\pi s} M_Z^4 G_F^2 \sin^4\theta_W\,(1-\sin^2\theta_W)^2} \tag{15}$$

and because $\sigma(e^-e^+ \to H^0 Z^0) \approx \sigma(\mu^-\mu^+ \to H^0 Z^0)$ (16)

$$\frac{\sigma(e^-e^+ \to H^0Z^0)}{\sigma(e^-e^+ \to \mu^-\mu^+)} = \frac{\frac{\cancel{G_F^2}\,\cancel{M_Z^4}}{48\pi s^2}\,\frac{(1+\tan\beta\tan\alpha)^2}{(1+\tan^2\beta)(1+\tan^2\alpha)}\,\frac{[1-4\sin^2\theta_W+8\sin^4\theta_W]}{[(s-M_Z^2)^2+M_Z^2\Gamma_Z^2]}\,[12sM_Z^2+\lambda]\,\lambda^{1/2}}{\frac{8}{3\pi s}\,\cancel{M_Z^4}\,\cancel{G_F^2}\sin^4\theta_W(1-\sin^2\theta_W)^2}$$

$$\boxed{\begin{aligned}\frac{\sigma(e^-e^+ \to H^0Z^0)}{\sigma(e^-e^+ \to \mu^-\mu^+)} &= \frac{1}{128\,s}\,\frac{(1+\tan\beta\tan\alpha)^2}{(1+\tan^2\beta)(1+\tan^2\alpha)}\,\frac{(1-4\sin^2\theta_W+8\sin^4\theta_W)}{\sin^4\theta_W(1-\sin^2\theta_W)^2}\times\\ &\times\frac{[12sM_Z^2+\lambda(s,m_{H^0}^2,M_Z^2)]\,\lambda^{1/2}(s,m_{H^0}^2,M_Z^2)}{[(s-M_Z^2)^2+M_Z^2\Gamma_Z^2]}\end{aligned}} \qquad (17)$$

$$\left\{\begin{aligned}&\text{If}\quad m_{H^0}=\sqrt{s}-M_Z\,,\quad \lambda(s,m_{H^0}^2,M_Z^2)=0\\ &\sigma(\mu^-\mu^+\to H^0Z^0)=0\end{aligned}\right. \qquad (18) \qquad (\partial j \partial)$$

and $\frac{\sigma(e^-e^+ \to H^0Z^0)}{\sigma(e^-e^+ \to \mu^-\mu^+)} = 0$.

$(\tan\alpha = -1.4137934768)$

For $M_{H^\pm} = 120$ GeV, $\sqrt{s} = 500$ GeV, $\tan\beta = 30 \Rightarrow \frac{(1+\tan\beta\tan\alpha)^2}{(1+\tan^2\beta)(1+\tan^2\alpha)} = 0.634769749$

$$\sigma(\mu^-\mu^+\to H^0Z^0) = \frac{(1.166391\times10^{-5})^2\times(91.1876)^4}{48\pi\,(500)^4}\times\frac{0.634769749\times0.502848615}{5.841160477\times10^{10}}$$

$$\times\left[12\times(500)^2\times(91.1876)^2+5.399385943\times10^{10}\right]\times2.32365788\times10^5$$

$$\times\,(3.893792914\times10^{11})\text{ fb}$$

(without radiative corrections)

because:

$$m_{H^0}^2 = \frac{1}{2}(120)^2\left\{1-\left(\frac{80.423}{120}\right)^2+\left(\frac{91.1876}{120}\right)^2+0.079685444\right\}$$

$$m_{H^0} = 93.25982484$$

$$\lambda(s,m_{H^0}^2,M_Z^2) = (500)^4+(93.25982484)^4+(91.1876)^4-2(500)^2(93.25982484)^2-2(500)^2(91.1876)^2$$
$$-2(93.25982484)^2(91.1876)^2 = 5.399385943\times10^{10}$$

$$\lambda^{1/2}(s,m_{H^0}^2,M_Z^2) = 2.32365788\times10^5$$

$$\Rightarrow \sigma(\mu^-\mu^+\to H^0Z^0) = 9.980684664\times\cancel{10}^{-16}\times5.464549217\times10^{-12}\times[7.893939461\times10^{10}]\times$$
$$\times\,2.32365788\times\cancel{10}^{5}\times3.893792914\times\cancel{10}^{11}\text{ fb} = 38.95412952\text{ fb}.$$

with radiative corrections

$$\sigma(\mu^-\mu^+ \to H^0 Z^0) = 9.980684664 \times 10^{-16} \times \frac{0.990658217 \times 0.502848615}{5.841160477 \times 10^{10}} \times$$

$$\overbrace{\left[12 \times 500^2 \times (91.1876)^2 + 5.018711492 \times 10^{10} \right]}^{7.51326501 \times 10^{10}} \times 2.24024808 \times 10^5 \times 3.893792914 \times 10^{11} \text{ fb}$$

$$= 55.78532567 \text{ fb.}$$

because : $$\frac{(1 + \tan\beta \tan\alpha)^2}{(1+\tan^2\beta)(1+\tan^2\alpha)} = \frac{(1 - 30 \times 15.73107668)^2}{901 \times (1 + 15.73107668^2)} = 0.990658217$$

$$\lambda(s, m_{H^0}^2, M_Z^2) = 500^4 + 128.2291119^4 + 91.1876^4 - 2 \times 500^2 \times (128.2291119)^2$$

$$-2(500)^2 \times (91.1876)^2 - 2(128.2291119)^2 (91.1876)^2 = 5.018711492 \times 10^{10}$$

$$\lambda^{1/2} = 2.24024808 \times 10^5$$

$\dfrac{\sigma(e^-e^+ \to H^0 Z^0)}{\sigma(e^-e^+ \to \mu^-\mu^+)}$ with radiative corrections : ($\sqrt{s} = 500$ GeV; $\tan\beta = 30$; $M_{HS} = 120$ GeV)

$$\frac{\sigma(e^-e^+ \to H^0 Z^0)}{\sigma(e^-e^+ \to \mu^-\mu^+)} = \frac{1}{128(500)^2} \frac{(0.990658217) \times (0.502848615)}{0.031580461} \left[12 \times 500^2 \times (91.1876)^2 \right.$$

$$\left. + \frac{5.018711492 \times 10^{10} \,] \, 2.24024808 \times 10^5}{5.841160477 \times 10^{10}} \right.$$

$$= 4.929384109 \times 10^{-7} \times 7.51326501 \times 10^{10} \times 3.835279 \times 10^{-6}$$

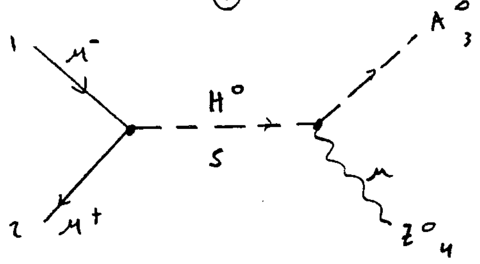
$$\frac{\sigma(e^-e^+ \to H^0 Z^0)}{\sigma(e^-e^+ \to \mu^-\mu^+)} = \underset{\sim\sim\sim}{0.1420425}$$

Production of A^0

$\mu^-\mu^+ \to A^0 Z^0$

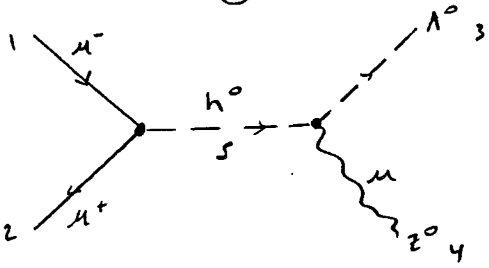
$\longrightarrow$ time

①

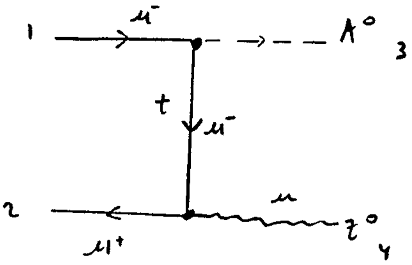


+

②

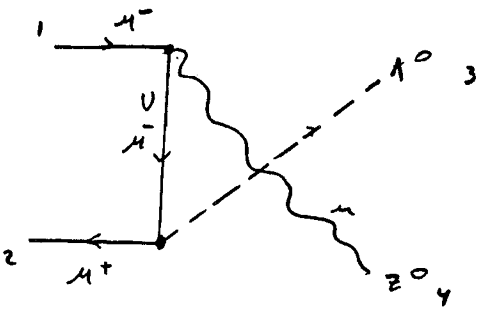


③

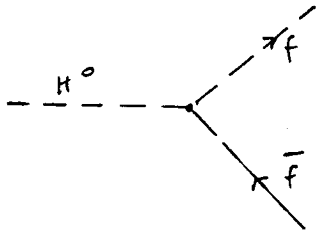


+

④

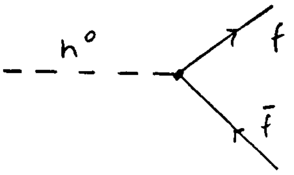


e^-, μ^-, τ^-

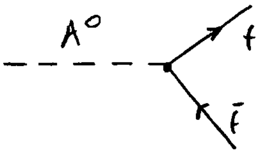


$$\frac{-ig m_f \cos\alpha}{2 M_W \cos\beta}$$

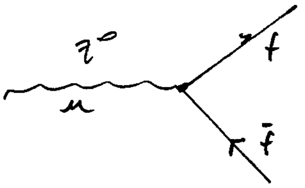
$f = e^-, \mu^-$



$$\frac{ig m_f}{2 M_W} \frac{\sin\alpha}{\cos\beta}$$



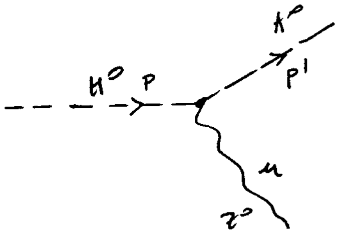
$$-\frac{g m_f}{2 M_W} \tan\beta\, \gamma^5$$



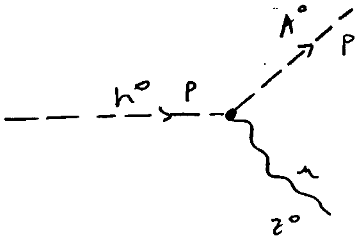
$$-\frac{ig}{\cos\theta_W}\gamma^\mu \frac{1}{2}(c_V^f - c_A^f\gamma^5)$$

$$c_A^f = T_f^3 \; ; \quad c_V^f = T_f^3 - 2\sin^2\theta_W Q_f$$

for $f = \mu^-$: $c_A^f = T_f^3 = -\frac{1}{2}$ and $c_V^f = -\frac{1}{2} + 2\sin^2\theta_W$



$$-\frac{g\sin(\beta-\alpha)}{2\cos\theta_W}(p+p')^\mu$$

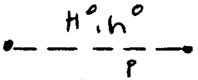


$$\frac{g\cos(\beta-\alpha)}{2\cos\theta_W}(p+p')^\mu$$

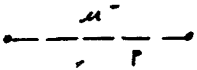
$$m^2_{\substack{H^0\\h^0}} = \frac{1}{2}\left\{m_{A^0}^2 + M_Z^2 \pm \left[(m_{A^0}^2 + M_Z^2)^2 - 4M_Z^2 m_{A^0}^2\cos^2 2\beta\right]^{1/2}\right\}$$

$$\tan 2\alpha = \tan 2\beta\left(\frac{m_{A^0}^2 + M_Z^2}{m_{A^0}^2 - M_Z^2}\right)$$

$$m_{A^0}^2 = M_{H^\pm}^2 - M_W^2$$



$$\frac{i}{p^2 - M^2 + iM\Gamma}$$



$$\frac{i}{\not{p} - m_\mu} = \frac{i(\not{p} + m_\mu)}{p^2 - m_\mu^2}$$

$$-i\mathcal{M} = -i(\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4) \qquad (1)$$

$$M_3 = \frac{-ig^2 m_\mu}{4 M_W \cos\theta_W} C_{\mu t}\ \bar{V}_2 \gamma^\mu (C_V^\mu - C_A^\mu \gamma^5)(\not{p}_1 - \not{p}_3 + m_\mu)\gamma^5 U_1\ \varepsilon^*_{4\mu} \qquad (13)$$

$$M_4 = \frac{-ig^2 m_\mu}{4 M_W \cos\theta_W} C_{\mu v}\ \bar{V}_2 \gamma^5 (\not{p}_1 - \not{p}_4 + m_\mu)\gamma^\mu (C_V^\mu - C_A^\mu \gamma^5) U_1\ \varepsilon^*_{4\mu} \qquad (14)$$

$$M = M^A + M_3 + M_4$$

$$\overline{|M|^2} = \frac{1}{4}(M^{A+} + M_3^+ + M_4^+)(M^A + M_3 + M_4)$$

$$\boxed{\overline{|M|^2} = \frac{1}{4}\left[\, |M^A|^2 + M^{A+}M_3 + M^{A+}M_4 + M_3^+ M^A + |M_3|^2 + M_3^+ M_4 + M_4^+ M^A + M_4^+ M_3 + |M_4|^2 \right]} \qquad (15)$$

$$|M^A|^2 = \frac{g^4 m_\mu^2}{16 M_W^2 \cos^2\theta_W} |C_H|^2 \sum_\lambda \varepsilon^\lambda_{4\mu}\, \varepsilon^{\lambda *}_{4\nu}\ (P_1+P_2+P_3)^\mu (P_1+P_2+P_3)^\nu \sum_s \bar{V}_2 U_1\ \bar{U}_1 V_2$$

$$|M^A|^2 = \frac{g^4 m_\mu^2}{16 M_W^2 \cos^2\theta_W} |C_H|^2 \left(-\eta_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_{Z^0}^2}\right)(P_1+P_2+P_3)^\mu (P_1+P_2+P_3)^\nu\ \mathrm{Tr}\left[(\not{p}_1 + m_\mu)(\not{p}_2 - m_\mu)\right]$$

$$|M^A|^2 = \frac{g^4 m_\mu^2}{16 M_W^2 \cos^2\theta_W} |C_H|^2 \left[-(P_1+P_2+P_3)^2 + \frac{((P_1+P_2+P_3)\cdot P_4)^2}{M_{Z^0}^2}\right]\left[4(P_1\cdot P_2) - 4m_\mu^2\right]$$

$$|M^A|^2 = \frac{g^4 m_\mu^2 |C_H|^2}{4 M_W^2 \cos^2\theta_W}\left[-(P_1+P_2+P_3)^2 + \frac{((P_1+P_2+P_3)\cdot P_4)^2}{M_Z^2}\right]\left((P_1\cdot P_2) - m_\mu^2\right) \qquad (16)$$

$$S = (P_1+P_2)^2 = 2m_\mu^2 + 2(P_1\cdot P_2)$$

$$\Rightarrow \boxed{(P_1\cdot P_2) = \frac{S}{2} - m_\mu^2 \approx \frac{S}{2}} \qquad (17)$$

$$(P_1+P_2+P_3)\cdot P_4 = (2P_3+P_4)\cdot P_4 = 2P_3\cdot P_4 + M_Z^2$$

$$S = (P_3+P_4)^2 = m_{A^0}^2 + M_Z^2 + 2(P_3\cdot P_4)$$

$$\boxed{2(P_3\cdot P_4) = S - m_{A^0}^2 - M_Z^2} \qquad (18)$$

$$\Rightarrow \boxed{(P_1+P_2+P_3)\cdot P_4 = S - m_{A^0}^2} \qquad (19)$$

$$(P_1+P_2+P_3)^2 = (2P_3+P_4)^2 = 4m_{A^0}^2 + M_Z^2 + 4(P_3\cdot P_4)$$

$$(P_1+P_2+P_3)^2 = 4\cancel{m}_{A^0}^2 + \cancel{M_Z^2} + 2S - 2\cancel{m}_{A^0}^2 - 2\cancel{M}_Z^2$$

$$\boxed{(P_1+P_2+P_3)^2 = 2M_{A^0}^2 - M_Z^2 + 2S} \qquad (20)$$

$$|M_A|^2 = \frac{g^4 m_\mu^2 |C_H|^2}{4M_W^2 \cos^2\theta_W}\left[-2M_{A^0}^2 + M_Z^2 - 2s + \frac{(s-m_{A^0}^2)^2}{M_Z^2}\right]\left[\frac{s}{2} - 2m_\mu^2\right]$$

$$= \frac{g^4 m_\mu^2 |C_H|^2}{4M_W^2 M_Z^2 \cos^2\theta_W}\left[-2M_{A^0}^2 M_Z^2 + M_Z^4 - 2sM_Z^2 + s^2 - 2sm_{A^0}^2 + m_{A^0}^4\right]\frac{(s-4m_\mu^2)}{2}$$

$$\boxed{|M_A|^2 = \frac{g^4 m_\mu^2 |C_H|^2}{8M_W^4}\lambda(s, M_Z^2, m_{A^0}^2)(s-4m_\mu^2) \approx \frac{g^4 m_\mu^2 |C_H|^2}{8M_W^4}\lambda(s, M_Z^2, m_{A^0}^2)\, s} \qquad (21)$$

$$|M_3|^2 = M_3^{+} M_3 = \frac{g^4 m_\mu^2}{16M_W^2\cos^2\theta_W} c_{\mu t}^2 \sum_\lambda \varepsilon_{4\mu}\varepsilon_{4\nu}^{*} \sum_s \left[\bar{V}_2\gamma^\mu(C_V^\mu - C_A^\mu\gamma^5)(\not{p}_1 - \not{p}_3 + m_\mu)\gamma^5 U_1\right]^{+}$$

$$\left[\bar{V}_2\gamma^\nu(C_V^\mu - C_A^\mu\gamma^5)(\not{p}_1 - \not{p}_3 + m_\mu)\gamma^5 U_1\right] \qquad (22)$$

$$\left[\bar{V}_2\gamma^\mu(C_V^\mu - C_A^\mu\gamma^5)(\not{p}_1 - \not{p}_3 + m_\mu)\gamma^5 U_1\right]^{+}$$

$$= \left[U_1^{+}\gamma^5(\gamma^{\alpha +}p_{1\alpha} - \gamma^{\alpha +}p_{3\alpha} + m_\mu)(C_V^\mu - C_A^\mu\gamma^5)\overset{\gamma^0\gamma^\mu}{\overbrace{\gamma^{\mu +}\gamma^0}} V_2\right]$$

$$= \left[U_1^{+}\gamma^5(\gamma^{\alpha +}p_{1\alpha} - \gamma^{\alpha +}p_{3\alpha} + m_\mu)\gamma^0(C_V^\mu + C_A^\mu\gamma^5)\gamma^\mu V_2\right]$$

$$= \left[U_1^{+}\gamma^5(\gamma^0\not{p}_1 - \gamma^0\not{p}_3 + m_\mu\gamma^0)(C_V^\mu + C_A^\mu\gamma^5)\gamma^\mu V_2\right]$$

$$= -\left[U_1^{+}\gamma^0\gamma^5(\not{p}_1 - \not{p}_3 + m_\mu)(C_V^\mu + C_A^\mu\gamma^5)\gamma^\mu V_2\right]$$

$$= -\bar{U}_1\gamma^5(\not{p}_1 - \not{p}_3 + m_\mu)(C_V^\mu + C_A^\mu\gamma^5)\gamma^\mu V_2 \qquad (23)$$

$$|M_3|^2 = \frac{-g^4 m_\mu^2}{16M_W^2\cos^2\theta_W} c_{\mu t}^2\left(-g_{\mu\nu} + \frac{p_{4\mu}p_{4\nu}}{M_Z^2}\right)\sum_s\left[\bar{U}_1\gamma^5(\not{p}_1 - \not{p}_3 + m_\mu)(C_V^\mu + C_A^\mu\gamma^5)\gamma^\mu V_2\right]$$

$$\cdot\left[\bar{V}_2\gamma^\nu(C_V^\mu - C_A^\mu\gamma^5)(\not{p}_1 - \not{p}_3 + m_\mu)\gamma^5 U_1\right] \qquad (24)$$

$$|M_3|^2 = \frac{-g^4 m_\mu^2}{16M_W^2\cos^2\theta_W} c_{\mu t}^2\left(-g_{\mu\nu} + \frac{p_{4\mu}p_{4\nu}}{M_Z^2}\right)\mathrm{Tr}\left[(\not{p}_1 + m_\mu)\gamma^5(\not{p}_1 - \not{p}_3 + m_\mu)(C_V^\mu + C_A^\mu\gamma^5)\gamma^\mu\right.$$

$$\left.(\not{p}_2 - m_\mu)\gamma^\nu(C_V^\mu - C_A^\mu\gamma^5)(\not{p}_1 - \not{p}_3 + m_\mu)\gamma^5\right] \qquad (25)$$

$$\mathrm{Tr}[\quad] = \mathrm{Tr}\left[(-\not{p}_1 + m_\mu)(\not{p}_1 - \not{p}_3 + m_\mu)(C_V^\mu + C_A^\mu\gamma^5)\gamma^\mu(\not{p}_2 - m_\mu)\gamma^\nu(C_V^\mu - C_A^\mu\gamma^5)\right.$$

$$\left.(\not{p}_1 - \not{p}_3 + m_\mu)\right]$$

$$= \mathrm{Tr}\left[(-\not{p}_1 + m_\mu)(C_V^\mu\not{p}_1 + C_A^\mu\not{p}_1\gamma^5 - C_V^\mu\not{p}_3 - C_A^\mu\not{p}_3\gamma^5 + m_\mu C_V^\mu + m_\mu C_A^\mu\gamma^5)\gamma^\mu\right.$$

$$\left.(\not{p}_2 - m_\mu)(C_V^\mu\gamma^\nu\not{p}_1 - C_V^\mu\gamma^\nu\not{p}_3 + m_\mu C_V^\mu\gamma^\nu - C_A^\mu\gamma^\nu\gamma^5\not{p}_1 + C_A^\mu\gamma^\nu\gamma^5\not{p}_3 - m_\mu C_A^\mu\gamma^\nu\gamma^5)\right]$$

$$= \mathrm{Tr}\Big[\big(\cancel{-C_V^{\mu} m_\mu^2} \cancel{- C_A^{\mu} m_\mu^2 \gamma^5} + C_V^{\mu} \not{p}_1 \not{p}_3 + C_A^{\mu} \not{p}_1 \not{p}_3 \gamma^5 \cancel{- m_\mu C_V^{\mu} \not{p}_1} \cancel{- m_\mu C_A^{\mu} \not{p}_1 \gamma^5}$$

$$\cancel{+ m_\mu C_V^{\mu} \not{p}_1} \cancel{+ m_\mu C_A^{\mu} \not{p}_1 \gamma^5} - m_\mu C_V^{\mu} \not{p}_3 - m_\mu C_A^{\mu} \not{p}_3 \gamma^5 \cancel{+ m_\mu^2 C_V^{\mu}} \cancel{+ m_\mu^2 C_A^{\mu} \gamma^5}\big)\gamma^\mu$$

$$\big(C_V^{\mu} \not{p}_2 \gamma^\nu \not{p}_1 - C_V^{\mu} \not{p}_2 \gamma^\nu \not{p}_3 + m_\mu C_V^{\mu} \not{p}_2 \gamma^\nu - C_A^{\mu} \not{p}_2 \gamma^\nu \gamma^5 \not{p}_1 + C_A^{\mu} \not{p}_2 \gamma^\nu \gamma^5 \not{p}_3$$

$$- m_\mu C_A^{\mu} \not{p}_2 \gamma^\nu \gamma^5 - m_\mu C_V^{\mu} \gamma^\nu \not{p}_1 + m_\mu C_V^{\mu} \gamma^\nu \not{p}_3 - m_\mu^2 C_V^{\mu} \gamma^\nu + m_\mu C_A^{\mu} \gamma^\nu \gamma^5 \not{p}_1$$

$$- m_\mu C_A^{\mu} \gamma^\nu \gamma^5 \not{p}_3 + m_\mu^2 C_A^{\mu} \gamma^\nu \gamma^5\big)\Big]$$

$$= \mathrm{Tr}\Big[\big(C_V^{\mu} \not{p}_1 \not{p}_3 + C_A^{\mu} \not{p}_1 \not{p}_3 \gamma^5 - m_\mu C_V^{\mu} \not{p}_3 - m_\mu \overset{\downarrow}{C_A^{\mu}} \not{p}_3 \gamma^5\big)\overset{\downarrow}{\gamma^\mu}\big(C_V^{\mu} \not{p}_2 \gamma^\nu \not{p}_1 - C_V^{\mu} \not{p}_2 \gamma^\nu \not{p}_3$$

$$+ m_\mu C_V^{\mu} \not{p}_2 \gamma^\nu - C_A^{\mu} \not{p}_2 \gamma^\nu \gamma^5 \not{p}_1 + C_A^{\mu} \not{p}_2 \gamma^\nu \gamma^5 \not{p}_3 - m_\mu C_A^{\mu} \not{p}_2 \gamma^\nu \overset{\swarrow}{\gamma^5} - m_\mu C_V^{\mu} \gamma^\nu \not{p}_1$$

$$+ m_\mu C_V^{\mu} \gamma^\nu \not{p}_3 - m_\mu^2 C_V^{\mu} \gamma^\nu + m_\mu C_A^{\mu} \gamma^\nu \gamma^5 \not{p}_1 - m_\mu C_A^{\mu} \gamma^\nu \gamma^5 \not{p}_3 + m_\mu^2 C_A^{\mu} \gamma^\nu \gamma^5\big)\Big]$$

$$= (C_V^{\mu})^2 m_\mu^2 \mathrm{Tr}(\not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu) - (C_V^{\mu})^2 \mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu \not{p}_3) + m_\mu (C_V^{\mu})^2 \mathrm{Tr}(\not{p}_1 \not{p}_3 \overset{0}{\cancel{\gamma^\mu \not{p}_2 \gamma^\nu}})$$

$$- C_V^{\mu} C_A^{\mu} m_\mu^2 \mathrm{Tr}(\not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5) + C_A^{\mu} C_V^{\mu} \mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5 \not{p}_3) - m_\mu C_V^{\mu} C_A^{\mu} \mathrm{Tr}(\overset{0}{\cancel{\not{p}_1 \not{p}_3 \gamma^\mu}}$$

$$\not{p}_2 \gamma^\nu \gamma^5) - m_\mu (C_V^{\mu})^2 \mathrm{Tr}(\not{p}_1 \not{p}_3 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu \not{p}_1) + m_\mu (C_V^{\mu})^2 \mathrm{Tr}(\not{p}_1 \not{p}_3 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu \not{p}_3)$$

$$- m_\mu^2 (C_V^{\mu})^2 \mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^\mu \gamma^\nu) + m_\mu^3 C_V^{\mu} C_A^{\mu} \mathrm{Tr}(\not{p}_3 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu \gamma^5) - m_\mu C_A^{\mu} C_V^{\mu} \mathrm{Tr}(\overset{0}{\cancel{\not{p}_1 \not{p}_3 \gamma^\mu}}$$

$$\gamma^\nu \gamma^5 \not{p}_3) + m_\mu^2 C_V^{\mu} C_A^{\mu} \mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^\mu \gamma^\nu \gamma^5) + C_A^{\mu} C_V^{\mu} \cancel{m_\mu^2 \mathrm{Tr}}(\not{p}_3 \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$- C_A^{\mu} C_V^{\mu} \mathrm{Tr}(\gamma^5 \not{p}_1 \not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu \not{p}_3) + m_\mu C_A^{\mu} C_V^{\mu} \mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^5 \overset{0}{\cancel{\gamma^\mu}} \not{p}_2 \gamma^\nu) - (C_A^{\mu})^2 \mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^5 \gamma^\mu$$

$$\not{p}_2 \gamma^\nu \gamma^5 \not{p}_1) + (C_A^{\mu})^2 \mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5 \not{p}_3) - m_\mu (C_A^{\mu})^2 \mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^5 \overset{0}{\cancel{\gamma^\mu}} \not{p}_2 \gamma^\nu \gamma^5)$$

$$- m_\mu^3 C_A^{\mu} C_V^{\mu} \mathrm{Tr}(\not{p}_3 \gamma^5 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu) + m_\mu C_A^{\mu} C_V^{\mu} \mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^5 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu \not{p}_3) - m_\mu^2 C_A^{\mu} C_V^{\mu} \mathrm{Tr}(\not{p}_1 \not{p}_3$$

$$\gamma^5 \gamma^\mu \gamma^\nu) + m_\mu^3 (C_A^{\mu})^2 \mathrm{Tr}(\not{p}_3 \gamma^5 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu \gamma^5) - m_\mu (C_A^{\mu})^2 \mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^5 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu \gamma^5 \not{p}_3)$$

$$+ m_\mu^2 (C_A^{\mu})^2 \mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^5 \gamma^\mu \gamma^\nu \gamma^5) - m_\mu (C_V^{\mu})^2 \mathrm{Tr}(\not{p}_3 \overset{0}{\cancel{\gamma^\mu \not{p}_2}} \gamma^\nu \not{p}_1) + m_\mu (C_V^{\mu})^2 m_{A^0}^2 \mathrm{Tr}(\overset{0}{\cancel{\gamma^\mu}} \not{p}_2 \gamma^\nu)$$

$$- m_\mu^2 (C_V^{\mu})^2 \mathrm{Tr}(\not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu) + m_\mu C_V^{\mu} C_A^{\mu} \mathrm{Tr}(\not{p}_3 \gamma^\mu \overset{0}{\cancel{\not{p}_2}} \gamma^\nu \gamma^5 \not{p}_1) - m_\mu C_V^{\mu} C_A^{\mu} m_{A^0}^2 \mathrm{Tr}(\gamma^\mu \overset{0}{\cancel{\not{p}_2}} \gamma^\nu \gamma^5)$$

$$+ m_\mu^2 C_V^{\mu} C_A^{\mu} \mathrm{Tr}(\not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5) + m_\mu^2 (C_V^{\mu})^2 \mathrm{Tr}(\not{p}_3 \gamma^\mu \gamma^\nu \not{p}_1) - m_\mu^2 (C_V^{\mu})^2 m_{A^0}^2 \mathrm{Tr}(\gamma^\mu \gamma^\nu)$$

$$+ m_\mu^3 (C_V^{\mu})^2 \mathrm{Tr}(\not{p}_3 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu) - m_\mu^2 C_V^{\mu} C_A^{\mu} \mathrm{Tr}(\not{p}_3 \gamma^\mu \gamma^\nu \gamma^5 \not{p}_1) + m_\mu^2 C_V^{\mu} C_A^{\mu} m_{A^0}^2 \mathrm{Tr}(\gamma^\mu \overset{0}{\cancel{\gamma^\nu}} \gamma^5)$$

$$- m_\mu^3 C_V^{\mu} C_A^{\mu} \mathrm{Tr}(\not{p}_3 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu \gamma^5) - m_\mu C_A^{\mu} C_V^{\mu} \mathrm{Tr}(\not{p}_3 \gamma^5 \overset{0}{\cancel{\gamma^\mu}} \not{p}_2 \gamma^\nu \not{p}_1) + m_\mu C_A^{\mu} C_V^{\mu} \mathrm{Tr}(\overset{0}{\cancel{\not{p}_3}} \gamma^5 \gamma^\mu$$

$$\not{p}_2 \gamma^\nu \not{p}_3) - m_\mu^2 C_A^{\mu} C_V^{\mu} \cancel{\mathrm{Tr}}(\not{p}_3 \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu) + m_\mu (C_A^{\mu})^2 \mathrm{Tr}(\not{p}_3 \gamma^5 \overset{0}{\cancel{\gamma^\mu}} \not{p}_2 \gamma^\nu \gamma^5 \not{p}_1)$$

$$- m_\mu (C_A^{\mu})^2 m_{A^0}^2 \mathrm{Tr}(\gamma^5 \gamma^\mu \overset{0}{\cancel{\not{p}_2}} \gamma^\nu \gamma^5) + m_\mu^2 (C_A^{\mu})^2 \mathrm{Tr}(\not{p}_3 \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5)$$

$$+ m_\mu^2 C_A^{\mu} C_V^{\mu} \mathrm{Tr}(\not{p}_3 \gamma^5 \gamma^\mu \gamma^\nu \not{p}_1) - m_\mu^2 C_A^{\mu} C_V^{\mu} m_{A^0}^2 \mathrm{Tr}(\gamma^5 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu) + m_\mu^3 C_A^{\mu} C_V^{\mu} \mathrm{Tr}(\not{p}_3 \overset{0}{\cancel{\gamma^5}} \gamma^\mu \gamma^\nu)$$

$$- m_\mu^2 (C_A^{\mu})^2 \mathrm{Tr}(\not{p}_3 \gamma^5 \gamma^\mu \gamma^\nu \gamma^5 \not{p}_1) + m_\mu^2 (C_A^{\mu})^2 m_{A^0}^2 \mathrm{Tr}(\gamma^5 \gamma^\mu \gamma^\nu \gamma^5) - m_\mu^3 (C_A^{\mu})^2 \mathrm{Tr}(\not{p}_3 \gamma^5 \overset{0}{\cancel{\gamma^\mu}} \gamma^\nu \gamma^5)$$

$$= (C_V^\mu)^2 m_\mu^2 \,\mathrm{Tr}(\not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu) - (C_V^\mu)^2 \,\mathrm{Tr}(\not{p}_3 \not{p}_1 \not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu) - C_V^\mu C_A^\mu \cancel{m_\mu} \,\mathrm{Tr}(\gamma^5 \not{p}_3 \gamma^\mu \quad (6$$

$$\not{p}_2 \gamma^\nu) + C_A^\mu C_V^\mu \,\mathrm{Tr}(\not{p}_3 \not{p}_1 \not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5) - m_\mu^2 (C_V^\mu)^2 \,\mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^\mu \gamma^\nu)$$

$$+ m_\mu^2 C_V^\mu C_A^\mu \,\cancel{\mathrm{Tr}(\gamma^5} \not{p}_1 \not{p}_3 \gamma^\mu \gamma^\nu) + C_A^\mu C_V^\mu \,\mathrm{Tr}(\not{p}_3 \not{p}_1 \not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5) - (C_A^\mu)^2 m_\mu^2$$

$$\mathrm{Tr}(\not{p}_3 \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5) + (C_A^\mu)^2 \,\mathrm{Tr}(\not{p}_3 \not{p}_1 \not{p}_3 \gamma^5 \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5) - m_\mu^2 C_A^\mu C_V^\mu \,\cancel{\mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^5 \gamma^\mu \gamma^\nu)}$$

$$+ m_\mu^2 (C_A^\mu)^2 \,\mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^\mu \gamma^\nu) - m_\mu^2 (C_V^\mu)^2 \,\mathrm{Tr}(\not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu) + m_\mu^2 C_A^\mu C_V^\mu \,\cancel{\mathrm{Tr}}(\gamma^5 \not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$+ m_\mu^2 (C_V^\mu)^2 \,\mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^\mu \gamma^\nu) - 4 m_\mu^2 (C_V^\mu)^2 m_{\Lambda^0}^2 \eta^{\mu\nu} - m_\mu^2 C_V^\mu C_A^\mu \,\cancel{\mathrm{Tr}}(\not{p}_1 \not{p}_3 \gamma^\mu \gamma^\nu \gamma^5)$$

$$- m_\mu^2 (C_A^\mu)^2 \,\mathrm{Tr}(\not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu) - m_\mu^2 C_A^\mu C_V^\mu \,\mathrm{Tr}(\cancel{\gamma^5} \not{p}_3 \gamma^\mu \gamma^\nu \not{p}_1) - m_\mu^2 (C_A^\mu)^2 \,\mathrm{Tr}(\not{p}_3 \gamma^\mu \gamma^\nu \not{p}_1)$$

$$+ 4 m_\mu^2 (C_A^\mu)^2 m_{\Lambda^0}^2 \eta^{\mu\nu} \qquad m_\mu^2 C_A^\mu C_V^\mu \,\mathrm{Tr}(\gamma^5 \not{p}_1 \not{p}_3 \gamma^\mu \gamma^\nu)$$

$$= (C_V^\mu)^2 m_\mu^2 \,\mathrm{Tr}(\cancel{\not{p}_3} \gamma^\mu \not{p}_2 \gamma^\nu) - (C_V^\mu)^2 \,\mathrm{Tr}(\not{p}_3 (-\not{p}_3 \not{p}_1 + 2 p_1 \cdot p_3) \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$+ 2 C_A^\mu C_V^\mu \,\mathrm{Tr}(\not{p}_3 (-\not{p}_3 \not{p}_1 + 2 p_1 \cdot p_3) \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5) - m_\mu^2 (C_V^\mu)^2 \,\mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^\mu \gamma^\nu)$$

$$+ (C_A^\mu)^2 m_\mu^2 \,\mathrm{Tr}(\cancel{\not{p}_3 \gamma^\mu} \not{p}_2 \gamma^\nu) - (C_A^\mu)^2 \,\mathrm{Tr}(\not{p}_3 (-\not{p}_3 \not{p}_1 + 2 p_1 \cdot p_3) \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$+ m_\mu^2 [(C_A^\mu)^2 + (C_V^\mu)^2] \,\mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^\mu \gamma^\nu) - m_\mu^2 (C_V^\mu)^2 \,\mathrm{Tr}(\cancel{\not{p}_3} \gamma^\mu \not{p}_2 \gamma^\nu) - 4 m_\mu^2 (C_V^\mu)^2 m_{\Lambda^0}^2 \eta^{\mu\nu}$$

$$- m_\mu^2 (C_A^\mu)^2 \,\mathrm{Tr}(\cancel{\not{p}_3 \gamma^\mu} \not{p}_2 \gamma^\nu) - m_\mu^2 (C_A^\mu)^2 \,\mathrm{Tr}(\not{p}_3 \gamma^\mu \gamma^\nu \not{p}_1) + 4 m_\mu^2 (C_A^\mu)^2 m_{\Lambda^0}^2 \eta^{\mu\nu}$$

$$= (C_V^\mu)^2 \cancel{m}_{\Lambda^0}^2 \,\mathrm{Tr}(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) - 2 (p_1 \cdot p_3) \cancel{(C_V^\mu)^2} \,\mathrm{Tr}(\not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$- 2 C_A^\mu C_V^\mu m_{\Lambda^0}^2 \,\mathrm{Tr}(\cancel{\gamma^5} \not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) + 4 (p_1 \cdot p_3) C_A^\mu C_V^\mu \,\mathrm{Tr}(\cancel{\not{p}_3} \gamma^\mu \not{p}_2 \gamma^\nu \gamma^5)$$

$$- m_\mu^2 \cancel{(C_V^\mu)^2} \,\mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^\mu \gamma^\nu) + (C_A^\mu)^2 m_{\Lambda^0}^2 \,\cancel{\mathrm{Tr}}(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) - 2 (p_1 \cdot p_3) \cancel{(C_A^\mu)^2} \,\mathrm{Tr}(\not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$+ m_\mu^2 [(C_A^\mu)^2 + (C_V^\mu)^2] \,\mathrm{Tr}(\not{p}_1 \not{p}_3 \gamma^\mu \gamma^\nu) - 4 m_\mu^2 (C_V^\mu)^2 \cancel{m}_{\Lambda^0}^2 \eta^{\mu\nu} - m_\mu^2 (C_A^\mu)^2 \,\mathrm{Tr}(\not{p}_3 \cancel{\gamma^\mu} \gamma^\nu p_1)$$

$$+ 4 m_\mu^2 (C_A^\mu)^2 m_{\Lambda^0}^2 \eta^{\mu\nu}$$

$$= m_{\Lambda^0}^2 [(C_A^\mu)^2 + (C_V^\mu)^2] \,\mathrm{Tr}(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) + m_\mu^2 [(C_A^\mu)^2 + \cancel{(C_V^\mu)^2}] \,\mathrm{Tr}(\gamma^\mu \gamma^\nu \not{p}_1 \not{p}_3)$$

$$- 2 (p_1 \cdot p_3) [(C_A^\mu)^2 + (C_V^\mu)^2] \,\mathrm{Tr}(\not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu) - m_\mu^2 [\cancel{(C_A^\mu)^2 + (C_V^\mu)^2}] \,\mathrm{Tr}(\gamma^\mu \gamma^\nu \not{p}_1 \not{p}_3)$$

$$- 2 C_A^\mu C_V^\mu m_{\Lambda^0}^2 \,\mathrm{Tr}(\gamma^5 \not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) + 4 (p_1 \cdot p_3) C_A^\mu C_V^\mu \,\mathrm{Tr}(\gamma^5 \not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu)$$

$$- 4 m_\mu^2 (C_V^\mu)^2 m_{\Lambda^0}^2 \eta^{\mu\nu} + 4 m_\mu^2 (C_A^\mu)^2 m_{\Lambda^0}^2 \eta^{\mu\nu}.$$

$$= [(C_A^\mu)^2 + (C_V^\mu)^2] [m_{\Lambda^0}^2 \,\mathrm{Tr}(\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu) - 2 (p_1 \cdot p_3) \,\mathrm{Tr}(\not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu)] - 2 C_A^\mu C_V^\mu m_{\Lambda^0}^2 \,\mathrm{Tr}(\gamma^5 \not{p}_1 \gamma^\mu$$

$$\not{p}_2 \gamma^\nu) + 4 (p_1 \cdot p_3) C_A^\mu C_V^\mu \,\mathrm{Tr}(\gamma^5 \not{p}_3 \gamma^\mu \not{p}_2 \gamma^\nu) - 4 m_\mu^2 (C_V^\mu)^2 m_{\Lambda^0}^2 \eta^{\mu\nu} + 4 m_\mu^2 (C_A^\mu)^2 m_{\Lambda^0}^2 \eta^{\mu\nu}$$

$$= [(C_A^\mu)^2 + (C_V^\mu)^2][-m_{A^0}^2 \mathrm{Tr}(\gamma^\mu \not{P}_2 \gamma^\nu \not{P}_1) - 2(P_1\cdot P_3)\mathrm{Tr}(\gamma^\mu \not{P}_2 \gamma^\nu \not{P}_3)]$$

$$-2C_A^\mu C_V^\mu m_{A^0}^2 \mathrm{Tr}(\gamma^5 \not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu) + 4(P_1\cdot P_3) C_A^\mu C_V^\mu \mathrm{Tr}(\gamma^5 \not{P}_3 \gamma^\mu \not{P}_2 \gamma^\nu)$$

$$-4m_\mu^2 (C_V^\mu)^2 m_{A^0}^2 \eta^{\mu\nu} + 4m_\mu^2 (C_A^\mu)^2 m_{A^0}^2 \eta^{\mu\nu}.$$

$$= ((C_A^\mu)^2 + (C_V^\mu)^2)[m_{A^0}^2 4(P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - \overset{\nearrow \frac{s}{2} - m_\mu^2}{(P_1\cdot P_2)}\eta^{\mu\nu}] - 2(P_1\cdot P_3)[(C_A^\mu)^2 + (C_V^\mu)^2]$$

$$4(P_2^\mu P_3^\nu + P_2^\nu P_3^\mu - (P_2\cdot P_3)\eta^{\mu\nu}) - 2C_A^\mu C_V^\mu m_{A^0}^2 \mathrm{Tr}(\gamma^5 \not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu)$$

$$+ 4(P_1\cdot P_3) C_A^\mu C_V^\mu \mathrm{Tr}(\gamma^5 \not{P}_3 \gamma^\mu \not{P}_2 \gamma^\nu) - 4m_\mu^2 (C_V^\mu)^2 m_{A^0}^2 \eta^{\mu\nu} + 4m_\mu^2 (C_A^\mu)^2 m_{A^0}^2 \eta^{\mu\nu}$$

$\Rightarrow$

$$\mathrm{Tr}[\ ] = 4m_{A^0}^2 (P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - \tfrac{s}{2}\eta^{\mu\nu})((C_A^\mu)^2 + (C_V^\mu)^2) + 8m_{A^0}^2 m_\mu^2 \eta^{\mu\nu}(C_A^\mu)^2$$

$$+ 4m_{A^0}^2 m_\mu^2 \cancel{\eta^{\mu\nu}}(C_V^\mu)^2 - 8(P_1\cdot P_3)[(C_A^\mu)^2 + (C_V^\mu)^2](P_2^\mu P_3^\nu + P_2^\nu P_3^\mu - (P_2\cdot P_3)\eta^{\mu\nu}]$$

$$-2C_A^\mu C_V^\mu m_{A^0}^2 \mathrm{Tr}(\gamma^5 \not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu) + 4(P_1\cdot P_3) C_A^\mu C_V^\mu \mathrm{Tr}(\gamma^5 \not{P}_3 \gamma^\mu \not{P}_2 \gamma^\nu) \cancel{-4m_{A^0}^2 m_\mu^2 (C_V^\mu)^2 \eta^{\mu\nu}} \quad (26)$$

Now

$$\eta_{\mu\nu}\mathrm{Tr}(\gamma^5 \not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu) = \eta_{\mu\nu}(-4i)\varepsilon^{\alpha\mu\beta\nu} P_{1\alpha} P_{2\beta} = 0$$

in fact

$$\eta_{\mu\nu}\varepsilon^{\alpha\mu\beta\nu} = \eta_{0\nu}\varepsilon^{\alpha 0\beta\nu} + \eta_{1\nu}\varepsilon^{\alpha 1\beta\nu} + \eta_{2\nu}\varepsilon^{\alpha 2\beta\nu} + \eta_{3\nu}\varepsilon^{\alpha 3\beta\nu}$$

$$= \eta_{00}\overset{0}{\cancel{\varepsilon^{\alpha 0\beta 0}}} + \eta_{11}\overset{0}{\cancel{\varepsilon^{\alpha 1\beta 1}}} + \eta_{22}\overset{0}{\cancel{\varepsilon^{\alpha 2\beta 2}}} + \eta_{33}\overset{0}{\cancel{\varepsilon^{\alpha 3\beta 3}}} = 0.$$

$$\eta_{\mu\nu}\mathrm{Tr}(\gamma^5 \not{P}_3 \gamma^\mu \not{P}_2 \gamma^\nu) = \eta_{\mu\nu}(-4i)\varepsilon^{\alpha\mu\beta\nu} P_{3\alpha} P_{2\beta} = 0.$$

$$P_{4\mu}P_{4\nu}\mathrm{Tr}(\gamma^5 \not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu) = (-4i)\varepsilon^{\alpha\mu\beta\nu} P_{1\alpha} P_{2\beta} P_{4\mu} P_{4\nu} = 0$$

$$P_{4\mu}P_{4\nu}\mathrm{Tr}(\gamma^5 \not{P}_3 \gamma^\mu \not{P}_2 \gamma^\nu) = (-4i)\varepsilon^{\alpha\mu\beta\nu} P_{3\alpha} P_{2\beta} P_{4\mu} P_{4\nu} = 0.$$

$$\Rightarrow |M_3|^2 = \frac{-g^4 m_\mu^2}{16M_W^2 \cos^2\theta_W} C_{\mu t}^2 \left(-\eta_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right) 4\{ m_{A^0}^2((C_A^\mu)^2 + (C_V^\mu)^2)(P_1^\mu P_2^\nu + P_1^\nu P_2^\mu$$

$$- \tfrac{s}{2}\eta^{\mu\nu}) + 2m_\mu^2 \eta^{\mu\nu}(C_A^\mu)^2 m_{A^0}^2 - 2(P_1\cdot P_3)((C_A^\mu)^2 + (C_V^\mu)^2)(P_2^\mu P_3^\nu + P_2^\nu P_3^\mu - (P_2\cdot P_3)\eta^{\mu\nu})\}$$

$$= \frac{-g^4 m_\mu^2 4}{16M_W^2 \cos^2\theta_W} C_{\mu t}^2 \Big\{ -m_{A^0}^2 [(C_A^\mu)^2 + (C_V^\mu)^2](2\overset{\nearrow s - 2m_\mu^2}{P_1\cdot P_2} - 2s) - 8m_\mu^2 m_{A^0}^2 (C_A^\mu)^2$$

$$+ 2(P_1\cdot P_3)[(C_A^\mu)^2 + (C_V^\mu)^2](2(P_2\cdot P_3) - 4(P_2\cdot P_3)) + \frac{m_{A^0}^2}{M_Z^2}((C_A^\mu)^2 + (C_V^\mu)^2)(2(P_1\cdot P_4)(P_2\cdot P_4)$$

$$-\frac{s}{2}M_Z^2\Big) + 2m_\mu^2 m_{A^0}^2 |C_A^{\mu}|^2 - \frac{2(P_1\cdot P_3)}{M_Z^2}\left[|C_A^{\mu}|^2 + |C_V^{\mu}|^2\right]\Big(2(P_2\cdot P_4)(P_3\cdot P_4) - (P_2\cdot P_3)M_Z^2\Big)\Big\}$$

$$= -\frac{g^4 m_\mu^2 C_{\mu t}^2}{4M_W^2\cos^2\theta_W}\Big\{ m_{A^0}^2 \cancel{s\,[}\,|C_A^{\mu}|^2 + |C_V^{\mu}|^2] + 2\cancel{m_\mu^2} m_{A^0}^2\left[|C_A^{\mu}|^2 + |C_V^{\mu}|^2\right]$$

$$-8\cancel{m_\mu^2} m_{A^0}^2 |C_A^{\mu}|^2 - 4(P_1\cdot P_3)(P_2\cdot P_3)\cancel{\left[|C_A^{\mu}|^2\right.} + |C_V^{\mu}|^2] + 2(P_1\cdot P_4)\cancel{(P_2\cdot P_4)\frac{m_{A^0}^2}{M_Z^2}}\left[|C_A^{\mu}|^2 +\right.$$

$$\left.+|C_V^{\mu}|^2\right] - \frac{s}{2}m_{A^0}^2\left[\cancel{|C_A^{\mu}|^2} + |C_V^{\mu}|^2\right] + 2\cancel{m_\mu^2 m_{A^0}^2}|C_A^{\mu}|^2 - 4\frac{\cancel{(P_1\cdot P_3)}(P_2\cdot P_4)(P_3\cdot P_4)}{\cancel{M_Z^2}}\cdot$$

$$\left[|C_A^{\mu}|^2 + |C_V^{\mu}|^2\right] + 2(P_1\cdot P_3)(P_2\cdot P_3)\left[|C_A^{\mu}|^2 + |C_V^{\mu}|^2\right]\Big\} \qquad (27)$$

neglecting the terms inside { } containing m_μ^2

$$\Rightarrow |M_3|^2 = -\frac{g^4 m_\mu^2 C_{\mu t}^2}{4M_W^2\cos^2\theta_W}\left[|C_A^{\mu}|^2 + |C_V^{\mu}|^2\right]\Big\{\frac{s}{2}m_{A^0}^2 - 2(P_1\cdot P_3)(P_2\cdot P_3) + 2(P_1\cdot P_4)(P_2\cdot P_4)\frac{m_{A^0}^2}{M_Z^2}$$

$$-4\frac{(P_1\cdot P_3)(P_2\cdot P_4)(P_3\cdot P_4)}{M_Z^2}\Big\} \qquad (28)$$

$$= -\frac{g^4 m_\mu^2 C_{\mu t}^2}{8M_W^4}\left[|C_A^{\mu}|^2 + |C_V^{\mu}|^2\right]\Big\{ s\,m_{A^0}^2 M_Z^2 - 4(P_1\cdot P_3)(P_2\cdot P_3)M_Z^2$$

$$+4(P_1\cdot P_4)(P_2\cdot P_4)m_{A^0}^2 - 8(P_1\cdot P_3)(P_2\cdot P_4)(P_3\cdot P_4)\Big\} \qquad (29)$$

$$t = (P_1 - P_3)^2 = m_\mu^2 + m_{A^0}^2 - 2(P_1\cdot P_3)$$

$$\Rightarrow \boxed{(P_1\cdot P_3) \approx \frac{m_{A^0}^2 - t}{2}} \qquad (30)$$

$$u = (P_1 - P_4)^2 = (P_3 - P_2)^2 = m_{A^0}^2 + m_\mu^2 - 2(P_2\cdot P_3)$$

$$\Rightarrow \boxed{(P_2\cdot P_3) \approx \frac{m_{A^0}^2 - u}{2}} \qquad (31)$$

$$u = m_\mu^2 + M_Z^2 - 2(P_1\cdot P_4)$$

$$\Rightarrow \boxed{(P_1\cdot P_4) \approx \frac{M_Z^2 - u}{2}} \qquad (32)$$

$$(P_2\cdot P_4) = P_2\cdot(P_1 + P_2 - P_3) = P_1\cdot P_2 + m_\mu^2 - P_2\cdot P_3 \approx \frac{s}{2} - \frac{(m_{A^0}^2 - u)}{2}$$

$$\boxed{(P_2\cdot P_4) = \frac{s - m_{A^0}^2 + u}{2}} \qquad (33)$$

$$(P_3\cdot P_4) = P_3\cdot(P_1 + P_2 - P_3) = (P_1\cdot P_3) + (P_2\cdot P_3) - m_{A^0}^2$$

$$(P_3\cdot P_4) = \frac{\cancel{m_{A^0}^2} - t}{\cancel{2}} + \frac{\cancel{m_{A^0}^2} - u}{\cancel{2}} - \cancel{m_{A^0}^2} = \frac{-u - t}{2}$$

$$\boxed{(P_3 . P_4) = \tfrac{1}{2}(-u - t)} \; (34) \; , \quad \boxed{s + t + u = m_{A^0}^2 + M_Z^2} \quad (35)$$

$$|M_3|^2 = -\frac{g^4 m_\mu^2 C_{\mu t}^2 \left[(\,(C_A^{\mu}|^2 + (\,(C_V^{\mu}|^2 \right]}{8 M_W^4} \Big\{ s\, m_{A^0}^2 M_Z^2 - (m_{A^0}^2 - t)(m_{A^0}^2 - u) M_Z^2$$

$$+ (M_Z^2 - u)(s + u - m_{A^0}^2)\, m_{A^0}^2 - (m_{A^0}^2 - t)(s + u - m_{A^0}^2)(-u - t) \Big\}$$

$$= -\frac{g^4 m_\mu^2 C_{\mu t}^2 \left[(\,(C_A^{\mu}|^2 + (\,(C_V^{\mu}|^2 \right]}{8 M_W^4} \Big\{ s\, m_{A^0}^2 M_Z^2 - m_{A^0}^4 M_Z^2 + m_{A^0}^2 M_Z^2 u + m_{A^0}^2 M_Z^2 t$$

$$- u t M_Z^2 + (M_Z^2 - u)(M_Z^2 - t)\, m_{A^0}^2 - (m_{A^0}^2 - t)(M_Z^2 - t)(s - m_{A^0}^2 - M_Z^2) \Big\}$$

$$= -\frac{g^4 m_\mu^2 C_{\mu t}^2 \left[(\,(C_A^{\mu}|^2 + (\,(C_V^{\mu}|^2 \right]}{8 M_W^4} \Big\{ s\, m_{A^0}^2 M_Z^2 - m_{A^0}^4 M_Z^2 + m_{A^0}^2 M_Z^2 u + m_{A^0}^2 M_Z^2 t$$

$$- u t M_Z^2 + M_Z^4 m_{A^0}^2 - M_Z^2 m_{A^0}^2 t - u M_Z^2 m_{A^0}^2 + u t\, m_{A^0}^2 - (m_{A^0}^2 M_Z^2 - m_{A^0}^2 t$$

$$- t M_Z^2 + t^2)(s - m_{A^0}^2 - M_Z^2) \Big\}$$

$$= -\frac{g^4 m_\mu^2 C_{\mu t}^2 \left[(\,(C_A^{\mu}|^2 + (\,(C_V^{\mu}|^2 \right]}{8 M_W^4} \Big\{ s\, m_{A^0}^2 M_Z^2 - m_{A^0}^4 M_Z^2 - u t M_Z^2 + M_Z^4 m_{A^0}^2 + u t\, m_{A^0}^2$$

$$- m_{A^0}^2 M_Z^2 s + m_{A^0}^4 M_Z^2 + m_{A^0}^2 M_Z^4 + m_{A^0}^2 s t - m_{A^0}^4 t - m_{A^0}^2 t M_Z^2 + s t M_Z^2$$

$$- t\, m_{A^0}^2 M_Z^2 - t M_Z^4 - t^2 s + t^2 m_{A^0}^2 + t^2 M_Z^2 \Big\}$$

$$s - m_{A^0}^2 - M_Z^2 = -u - t$$

$$= -\frac{g^4 m_\mu^2 C_{\mu t}^2 \left[(\,(C_A^{\mu}|^2 + (\,(C_V^{\mu}|^2 \right]}{8 M_W^4} \Big\{ - u t M_Z^2 + 2 M_Z^4 m_{A^0}^2 + u t\, m_{A^0}^2 + m_{A^0}^2 s t$$

$$+ M_Z^2 s t - t\, m_{A^0}^4 - t\, m_{A^0}^2 M_Z^2 - t\, m_{A^0}^2 M_Z^2 - t M_Z^4 - s t^2 + t^2 m_{A^0}^2 + t^2 M_Z^2 \Big\}$$

$$= -\frac{g^4 m_\mu^2 C_{\mu t}^2 \left[(\,(C_A^{\mu}|^2 + (\,(C_V^{\mu}|^2 \right]}{8 M_W^4} \Big\{ - u t M_Z^2 + 2 M_Z^4 m_{A^0}^2 + t\, m_{A^0}^2 (\cancel{m_{A^0}^2} + \cancel{M_Z^2} - \cancel{s} - \cancel{t})$$

$$+ m_{A^0}^2 s t + M_Z^2 s t - t\, m_{A^0}^4 - 2t\, m_{A^0}^2 M_Z^2 - t M_Z^4 - s t^2 + t^2 m_{A^0}^2 + t^2 M_Z^2 \Big\}$$

$$- t\, m_{A^0}^2 M_Z^2$$

$$= -\frac{g^4 m_\mu^2 C_{\mu t}^2 \left[(\,(C_A^{\mu}|^2 + (\,(C_V^{\mu}|^2 \right]}{8 M_W^4} \Big\{ - u t M_Z^2 + 2 M_Z^4 m_{A^0}^2 + M_Z^2 s t - t\, m_{A^0}^2 M_Z^2 - t M_Z^4$$

$$- s t^2 + t^2 M_Z^2 \Big\}$$

$$= -\frac{g^4 m_\mu^2 C_{\mu t}^2 [|C_A^\mu|^2 + |C_V^\mu|^2]}{8 M_W^4} \left\{ -ut M_Z^2 + 2 M_Z^4 m_{A^0}^2 - s t^2 + M_Z^2 t (s \underbrace{- m_{A^0}^2 - M_Z^2 + t}_{-u}) \right\}$$

$$= -\frac{g^4 m_\mu^2 C_{\mu t}^2 [|C_A^\mu|^2 + |C_V^\mu|^2]}{8 M_W^4} \left\{ -ut M_Z^2 + 2 M_Z^4 m_{A^0}^2 - s t^2 - ut M_Z^2 \right\}$$

$$\boxed{|M_3|^2 = \frac{g^4 m_\mu^2 C_{\mu t}^2 [|C_A^\mu|^2 + |C_V^\mu|^2]}{8 M_W^4} \left\{ +2ut M_Z^2 - 2 M_Z^4 m_{A^0}^2 + s t^2 \right\}} \qquad (36)$$

From $\mu^- \mu^+ \to H^\mp W^\pm$ we can obtain

ut replacing $M_H^2 \to m_{A^0}$; $M_W \to M_Z$

$$\Rightarrow \quad ut = m_{A^0}^2 M_Z^2 + \frac{1}{4} \lambda(s, m_{A^0}^2, M_Z^2) \sin^2\theta \qquad (37)$$

$$2ut M_Z^2 = 2 m_{A^0}^2 M_Z^4 + \frac{M_Z^2}{2} \lambda(s, m_{A^0}^2, M_Z^2) \sin^2\theta$$

$$\Rightarrow \quad |M_3|^2 = \frac{g^4 m_\mu^2 C_{\mu t}^2 [|C_A^\mu|^2 + |C_V^\mu|^2]}{8 M_W^4} \left\{ \frac{M_Z^2}{2} \lambda(s, m_{A^0}^2, M_Z^2) \sin^2\theta + s t^2 \right\}$$

$$\boxed{|M_3|^2 = \frac{g^4 m_\mu^2 s\, C_{\mu t}^2 [|C_A^\mu|^2 + |C_V^\mu|^2]}{8 M_W^4} \left\{ t^2 + \frac{M_Z^2}{2s} \lambda(s, M_Z^2, m_{A^0}^2) \sin^2\theta \right\}} \qquad (38)$$

$$|M_4|^2 = \frac{g^4 m_\mu^2 C_{\mu\nu}^2}{16 M_W^2 \cos^2\theta_W} \sum_\lambda \varepsilon_{4\mu} \varepsilon_{4\nu}^* \sum_s [\bar{V}_2 \gamma^5 (\not{p}_1 - \not{p}_4 + m_\mu) \gamma^\mu (C_V^\mu - C_A^\mu \gamma^5) U_1]^\dagger \cdot$$
$$\cdot [\bar{V}_2 \gamma^5 (\not{p}_1 - \not{p}_4 + m_\mu) \gamma^\nu (C_V^\mu - C_A^\mu \gamma^5) U_1] \qquad (39)$$

$$[\bar{V}_2 \gamma^5 (\not{p}_1 - \not{p}_4 + m_\mu) \gamma^\mu (C_V^\mu - C_A^\mu \gamma^5) U_1]^\dagger = [U_1^\dagger (C_V^\mu - C_A^\mu \gamma^5) \gamma^{\mu\dagger} (\gamma^{\alpha\dagger} p_{1\alpha} - \gamma^{\alpha\dagger} p_{4\alpha} + m_\mu)$$
$$\gamma^5 \gamma^0 V_2] = -U_1^\dagger (C_V^\mu - C_A^\mu \gamma^5) \gamma^{\mu\dagger} (\underset{\gamma^0\gamma^\alpha}{\underset{\vee}{\gamma^{\alpha\dagger}\gamma^0}} p_{1\alpha} - \underset{\gamma^0\gamma^\alpha}{\underset{\vee}{\gamma^{\alpha\dagger}\gamma^0}} p_{4\alpha} + m_\mu) \gamma^5 V_2$$

$$= -U_1^\dagger (C_V^\mu - C_A^\mu \gamma^5) \gamma^0 \gamma^\mu (\not{p}_1 - \not{p}_4 + m_\mu) \gamma^5 V_2$$
$$= -\bar{U}_1 (C_V^\mu + C_A^\mu \gamma^5) \gamma^\mu (\not{p}_1 - \not{p}_4 + m_\mu) \gamma^5 V_2 \qquad (40)$$

$$|M_4|^2 = -\frac{g^4 m_\mu^2 C_{\mu\nu}^2}{16 M_W^2 \cos^2\theta_W} \left(-g_{\mu\nu} + \frac{p_{4\mu} p_{4\nu}}{M_Z^2} \right) \sum_s [\bar{U}_1 (C_V^\mu + C_A^\mu \gamma^5) \gamma^\mu (\not{p}_1 - \not{p}_4 + m_\mu) \gamma^5 V_2] \cdot$$
$$\cdot [\bar{V}_2 \gamma^5 (\not{p}_1 - \not{p}_4 + m_\mu) \gamma^\nu (C_V^\mu - C_A^\mu \gamma^5) U_1] \qquad (41)$$

$$|M_4|^2 = -\frac{g^4 m_\mu^2 C_{\mu\nu}^2}{16 M_W^2 \cos^2\theta_W}\left(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\mathrm{Tr}\Big[(\not{p}_1+m_\mu)(C_V^\mu + C_A^\mu\gamma^5)\gamma^\mu(\not{p}_1-\not{p}_4+m_\mu)\gamma^5 .$$

$$(\not{p}_2-m_\mu)\gamma^5(\not{p}_1-\not{p}_4+m_\mu)\gamma^\nu(C_V^\mu - C_A^\mu\gamma^5)\Big] \qquad (42)$$

$$|M_4|^2 = +\frac{g^4 m_\mu^2 C_{\mu\nu}^2}{16 M_W^2 \cos^2\theta_W}\left(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\mathrm{Tr}\Big[(\not{p}_1+m_\mu)(C_V^\mu + C_A^\mu\gamma^5)\gamma^\mu(\not{p}_1-\not{p}_4+m_\mu)(\not{p}_2+m_\mu)$$

$$(\not{p}_1-\not{p}_4+m_\mu)\gamma^\nu(C_V^\mu - C_A^\mu\gamma^5)\Big] \qquad (43)$$

$$|M_4|^2 = \frac{g^4 m_\mu^2 C_{\mu\nu}^2}{16 M_W^2 \cos^2\theta_W}\left(-n_{\mu\nu} + \frac{P_{4\nu}P_{4\nu}}{M_Z^2}\right)\mathrm{Tr}\Big[\gamma^\nu(C_V^\mu - C_A^\mu\gamma^5)(\not{p}_1+m_\mu)(C_V^\mu + C_A^\mu\gamma^5)\gamma^\mu$$

$$(\not{p}_1-\not{p}_4+m_\mu)(\not{p}_2+m_\mu)(\not{p}_1-\not{p}_4+m_\mu)\Big] \qquad (44)$$

$$|M_4|^2 = \frac{g^4 m_\mu^2 C_{\mu\nu}^2}{16 M_W^2 \cos^2\theta_W}\left(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\mathrm{Tr}\Big[\gamma^\nu(C_V^\mu - C_A^\mu\gamma^5)(C_V^\mu\not{p}_1 + C_A^\mu\not{p}_1\gamma^5 + m_\mu C_V^\mu$$

$$+ m_\mu C_A^\mu\gamma^5)\gamma^\mu(\not{p}_1-\not{p}_4+m_\mu)(\not{p}_2+m_\mu)(\not{p}_1-\not{p}_4+m_\mu)\Big] \qquad (45)$$

$$= \frac{g^4 m_\mu^2 C_{\mu\nu}^2}{16 M_W^2 \cos^2\theta_W}\left(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\mathrm{Tr}\Big[\gamma^\nu\big((C_V^\mu|^2\not{p}_1 + C_V^\mu C_A^\mu\not{p}_1\gamma^5 + m_\mu(C_V^\mu|^2$$

$$+ m_\mu C_V^\mu C_A^\mu\gamma^5 - C_A^\mu C_V^\mu\gamma^5\not{p}_1 - (C_A^\mu|^2\gamma^5\not{p}_1\gamma^5 - m_\mu C_V^\mu C_A^\mu\gamma^5 - m_\mu(C_A^\mu|^2\big)\gamma^\mu$$

$$(\not{p}_1-\not{p}_4+m_\mu)\,(\not{p}_2\not{p}_1 - \not{p}_2\not{p}_4 + m_\mu\not{p}_2 + m_\mu\not{p}_1 - m_\mu\not{p}_4 + m_\mu^2)\Big]$$

$$= \frac{g^4 m_\mu^2 C_{\mu\nu}^2}{16 M_W^2 \cos^2\theta_W}\left(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\mathrm{Tr}\Big[\gamma^\nu\big((C_V^\mu|^2\not{p}_1 + 2C_V^\mu C_A^\mu\not{p}_1\gamma^5 + m_\mu(C_V^\mu|^2 + \cancel{m_\mu C_V^\mu C_A^\mu\gamma^5}$$

$$+ (C_A^\mu|^2\not{p}_1 - \cancel{m_\mu C_V^\mu C_A^\mu\gamma^5} - m_\mu(C_A^\mu|^2\big)\gamma^\mu\big(\not{p}_1\not{p}_2\not{p}_1 - \not{p}_1\not{p}_2\not{p}_4 + m_\mu\not{p}_1\not{p}_2 + m_\mu^3$$

$$- m_\mu\not{p}_1\not{p}_4 + m_\mu^2\not{p}_1 - \not{p}_4\not{p}_2\not{p}_1 + \not{p}_4\not{p}_2\not{p}_4 - m_\mu\not{p}_4\not{p}_2 - m_\mu\not{p}_4\not{p}_1 + m_\mu M_Z^2 - m_\mu^2\not{p}_4$$

$$+ m_\mu\not{p}_2\not{p}_1 - m_\mu\not{p}_2\not{p}_4 + m_\mu^2\not{p}_2 + m_\mu^2\not{p}_1 - m_\mu^2\not{p}_4 + m_\mu^3\big)\Big]$$

$$= \frac{g^4 m_\mu^2 C_{\mu\nu}^2}{16 M_W^2 \cos^2\theta_W}\left(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\mathrm{Tr}\Big[\big((C_A^\mu)^2 + (C_V^\mu|^2\big)\gamma^\nu\not{p}_1\gamma^\mu + 2C_V^\mu C_A^\mu\gamma^\nu\not{p}_1\gamma^5\gamma^\mu$$

$$+ m_\mu\big((C_V^\mu|^2 - (C_A^\mu|^2\big)\gamma^\nu\gamma^\mu\Big)\Big(\not{p}_1\not{p}_2\not{p}_1 - \not{p}_1\not{p}_2\not{p}_4 + 2m_\mu(p_1\cdot p_2) + 2m_\mu^3 - 2m_\mu(p_1\cdot p_4)$$

$$+ 2m_\mu^2\not{p}_1 - \not{p}_4\not{p}_2\not{p}_1 + \not{p}_4\not{p}_2\not{p}_4 - 2m_\mu(p_2\cdot p_4) + m_\mu M_Z^2 - 2m_\mu^2\not{p}_4 + m_\mu^2\not{p}_2\Big)\Big] \qquad (46)$$

$$\mathrm{Tr}[\quad] = \big((C_A^\mu|^2 + (C_V^\mu|^2\big)\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_1\not{p}_2\not{p}_1) - \big((C_A^\mu)^2 + (C_V^\mu|^2\big)\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_1\not{p}_2\not{p}_4)$$

$$+ \big(2m_\mu(p_1\cdot p_2) + 2m_\mu^3 - 2m_\mu(p_1\cdot p_4) - 2m_\mu(p_2\cdot p_4) + m_\mu M_Z^2\big)\,\mathrm{Tr}\big(\overset{0}{\cancel{\gamma^\nu\not{p}_1\gamma^\mu}}\big)\,\big((C_A^\mu|^2 + (C_V^\mu|^2\big)$$

$$+ 2m_\mu^2\big((C_A^\mu|^2 + (C_V^\mu|^2\big)\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_1) - \big((C_A^\mu|^2 + (C_V^\mu|^2\big)\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_4\not{p}_2\not{p}_1)$$

$$+ \big((C_A^\mu|^2 + (C_V^\mu|^2\big)\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_4\not{p}_2\not{p}_4) - 2m_\mu^2\big((C_A^\mu|^2 + (C_V^\mu|^2\big)\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_4)$$

$$+ m_\mu^2\big((C_A^\mu|^2 + (C_V^\mu|^2\big)\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_2) + 2C_V^\mu C_A^\mu\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^5\gamma^\mu\not{p}_1\not{p}_2\not{p}_1)$$

$$-2C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^\nu \not{p}_1\gamma^5\gamma^\mu\not{p}_1\not{p}_2\not{p}_4) + 2C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,(2m_\mu(P_1\cdot P_2) + 2m_\mu^3 - 2m_\mu(P_1\cdot P_4) - 2m_\mu(P_2\cdot P_4)$$

$$+m_\mu M_Z^2)\,\mathrm{Tr}(\gamma^\nu \overset{0}{\cancel{\not{p}_1\gamma^5\gamma^\mu}}) + 4m_\mu^2 C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^5\gamma^\mu\not{p}_1) - 2C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^5\gamma^\mu\not{p}_4\not{p}_2\not{p}_1)$$

$$+2C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^5\gamma^\mu\not{p}_4\not{p}_2\not{p}_4) - 4m_\mu^2 C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^5\gamma^\mu\not{p}_4) + 2m_\mu^2 C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^5\gamma^\mu$$

$$\not{p}_2) + m_\mu\left(|C_{\vec V}^{\mu}|^2 - |C_{\vec A}^{\mu}|^2\right)\mathrm{Tr}\left(\gamma^\nu\gamma^\mu(\not{p}_1\not{p}_2\not{p}_1 - \not{p}_1\not{p}_2\not{p}_4 + 2m_\mu^2\overset{0}{\not{p}_1} - \not{p}_4\not{p}_2\not{p}_1 + \not{p}_4\not{p}_2\not{p}_4\right.$$

$$\left.-2m_\mu^2\not{p}_4 + m_\mu^2\not{p}_2)\right) + 4m_\mu\left(|C_{\vec V}^{\mu}|^2 - |C_{\vec A}^{\mu}|^2\right)(2m_\mu(P_1\cdot P_2) + 2m_\mu^3 - 2m_\mu(P_1\cdot P_4)$$

$$-2m_\mu(P_2\cdot P_4) + m_\mu M_Z^2)\,\eta^{\mu\nu}$$

$$\mathrm{Tr}[\;\;] = \left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_1(-\not{p}_1\not{p}_2 + 2(P_1\cdot P_2))) - \left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\not{p}_1$$

$$(2P_1^\mu - \not{p}_1\gamma^\mu)\not{p}_2\not{p}_4) + 2m_\mu^2\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)4(2P_1^\mu P_1^\nu - m_\mu^2\eta^{\mu\nu}) - \left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\cdot$$

$$\mathrm{Tr}(\not{p}_1(2P_1^\nu - \not{p}_1\gamma^\nu)\gamma^\mu\not{p}_4\not{p}_2) + \left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_4(-\not{p}_4\not{p}_2 + 2P_2\cdot P_4)))$$

$$-2m_\mu^2\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_4) + m_\mu^2\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_2)$$

$$+2C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^5\gamma^\mu\not{p}_1(-\not{p}_1\not{p}_2 + 2P_1\cdot P_2)) - 2C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_1(2P_1^\mu - \not{p}_1\gamma^\mu)\not{p}_2\not{p}_4))$$

$$+4m_\mu^2 C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\overset{0}{\cancel{\gamma^5}}\gamma^\nu\not{p}_1\gamma^\mu\not{p}_1) + 2C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\not{p}_1(2P_1^\nu - \not{p}_1\gamma^\nu)\gamma^\mu\not{p}_4\not{p}_2)$$

$$+2C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_1\gamma^\mu\not{p}_4(-\not{p}_4\not{p}_2 + 2P_2\cdot P_4)) - 4m_\mu^2 C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_1\gamma^\mu\not{p}_4)$$

$$+2m_\mu^2 C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_1\gamma^\mu\not{p}_2) + 4m_\mu\left(|C_{\vec V}^{\mu}|^2 - |C_{\vec A}^{\mu}|^2\right)(2m_\mu(P_1\cdot P_2) + 2m_\mu^3 - 2m_\mu(P_1\cdot P_4)$$

$$-2m_\mu(P_2\cdot P_4) + m_\mu M_Z^2)\,\eta^{\mu\nu}$$

$$= -\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)m_\mu^2\,\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_2) + 2(P_1\cdot P_2)\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_1)$$

$$-2P_1^\mu\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\not{p}_1\not{p}_2\not{p}_4) + m_\mu^2\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\gamma^\mu\not{p}_2\not{p}_4)$$

$$+16m_\mu^2\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)P_1^\mu P_1^\nu - 8m_\mu^4\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\eta^{\mu\nu} - 2\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)P_1^\nu\,\mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_4\not{p}_2)$$

$$+m_\mu^2\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\gamma^\mu\not{p}_4\not{p}_2) - M_Z^2\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_2)$$

$$+2(P_2\cdot P_4)\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_4) - 2m_\mu^2\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_4)$$

$$+m_\mu^2\left(|C_{\vec A}^{\mu}|^2 + |C_{\vec V}^{\mu}|^2\right)\mathrm{Tr}(\gamma^\nu\not{p}_1\gamma^\mu\not{p}_2) - 2m_\mu^2 C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_1\gamma^\mu\not{p}_2) + 4(P_1\cdot P_2)C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\overset{0}{\cancel{\gamma^5}}\gamma^\nu\not{p}_1$$

$$\gamma^\mu\not{p}_1) - 4P_1^\mu C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_1\not{p}_2\not{p}_4) + 2m_\mu^2 C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\gamma^\nu\gamma^\mu\not{p}_2\not{p}_4)$$

$$+4P_1^\nu C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\underset{}{\overset{= \mathrm{Tr}(\gamma^5\gamma^\mu\not{p}_1\not{p}_2\not{p}_4)}{\mathrm{Tr}(\gamma^5\not{p}_1\gamma^\mu\not{p}_4\not{p}_2)}} - 2m_\mu^2 C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\gamma^\nu\gamma^\mu\overset{-\not{p}_2\not{p}_4 + 2(P_2\cdot P_4)}{\not{p}_4\not{p}_2}) - 2M_Z^2 C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\gamma^\nu$$

$$\not{p}_1\gamma^\mu\not{p}_2) + 4(P_2\cdot P_4)C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_1\gamma^\mu\not{p}_4) - 4m_\mu^2 C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_1\gamma^\mu\not{p}_4)$$

$$+2m_\mu^2 C_{\vec V}^{\mu}C_{\vec A}^{\mu}\,\mathrm{Tr}(\gamma^5\gamma^\nu\not{p}_1\gamma^\mu\not{p}_2) + 4m_\mu\left(|C_{\vec V}^{\mu}|^2 - |C_{\vec A}^{\mu}|^2\right)(2m_\mu(P_1\cdot P_2) + 2m_\mu^3 - 2m_\mu(P_1\cdot P_4)$$

$$-2m_\mu(P_2\cdot P_4) + m_\mu M_Z^2)\,\eta^{\mu\nu}.$$

$$\mathrm{Tr}(\gamma^5 \not{P}_1 \gamma^\mu \not{P}_4 \not{P}_2) = \mathrm{Tr}(\gamma^5 \not{P}_1 \gamma^\mu (-\not{P}_2 \not{P}_4 + 2(P_2\cdot P_4)))$$

$$= -\mathrm{Tr}(\gamma^5 \not{P}_1 \gamma^\mu \not{P}_2 \not{P}_4) + 2(P_2\cdot P_4)\,\overset{0}{\cancel{\mathrm{Tr}(\gamma^5 \not{P}_1 \gamma^\mu)}}$$

$$= -\mathrm{Tr}(\gamma^5 (2P_1^\mu - \gamma^\mu \not{P}_1) \not{P}_2 \not{P}_4) = -2P_1^\mu\, \overset{0}{\cancel{\mathrm{Tr}(\gamma^5 \not{P}_2 \not{P}_4)}}$$

$$+ \mathrm{Tr}(\gamma^5 \gamma^\mu \not{P}_1 \not{P}_2 \not{P}_4)$$

$$\Rightarrow \mathrm{Tr}[\quad] = 8(P_1\cdot P_2)\,(|C_A^\mu|^2 + |C_V^\mu|^2)\,(2P_1^\mu P_1^\nu - m_\mu^2 n^{\mu\nu}) - 2P_1^\mu\,(|C_A^\mu|^2 + |C_V^\mu|^2) \times \mathrm{Tr}(\gamma^\nu \not{P}_1 \not{P}_2 \not{P}_4) + 2m_\mu^2\,(|C_A^\mu|^2 + |C_V^\mu|^2)\,(P_2\cdot P_4)\,\mathrm{Tr}(\gamma^\nu\gamma^\mu) + 16 m_\mu^2 (|C_A^\mu|^2 + |C_V^\mu|^2)\,P_1^\mu P_1^\nu$$
$$-8m_\mu^4\,(|C_A^\mu|^2 + |C_V^\mu|^2)\,n^{\mu\nu} - 2\,(|C_A^\mu|^2 + |C_V^\mu|^2)\,P_1^\nu\,\mathrm{Tr}(\not{P}_1 \gamma^\mu \not{P}_4 \not{P}_2) + 2(P_2\cdot P_4)\,(|C_A^\mu|^2 + |C_V^\mu|^2)\,\mathrm{Tr}(\gamma^\nu \not{P}_1 \gamma^\mu \not{P}_4)$$
$$-M_Z^2\,(|C_A^\mu|^2 + |C_V^\mu|^2)\,\mathrm{Tr}(\gamma^\nu \not{P}_1 \gamma^\mu \not{P}_2) - 4P_1^\mu C_V^\mu C_A^\mu\,\mathrm{Tr}(\gamma^5\gamma^\nu \not{P}_1 \not{P}_2 \not{P}_4) + 4P_1^\nu C_V^\mu C_A^\mu\,\mathrm{Tr}(\gamma^5\gamma^\mu \not{P}_1 \not{P}_2 \not{P}_4)$$
$$-2m_\mu^2\,(|C_A^\mu|^2 + |C_V^\mu|^2)\,\mathrm{Tr}(\gamma^\nu \not{P}_1 \gamma^\mu \not{P}_4) + 4m_\mu^2 C_V^\mu C_A^\mu\,\mathrm{Tr}(\gamma^5\gamma^\nu\gamma^\mu \not{P}_2 \not{P}_4) - 2M_Z^2 C_V^\mu C_A^\mu\,\mathrm{Tr}(\gamma^5\gamma^\nu \not{P}_1 \gamma^\mu \not{P}_2)$$
$$+4(P_2\cdot P_4)\,C_V^\mu C_A^\mu\,\mathrm{Tr}(\gamma^5\gamma^\nu \not{P}_1 \gamma^\mu \not{P}_4) - 4m_\mu^2 C_V^\mu C_A^\mu\,\mathrm{Tr}(\gamma^5\gamma^\nu \not{P}_1 \gamma^\mu \not{P}_4) + 4m_\mu\,(|C_V^\mu|^2 - |C_A^\mu|^2) \times$$
$$\times (2m_\mu (P_1\cdot P_2) + 2m_\mu^3 - 2m_\mu (P_1\cdot P_4) - 2m_\mu (P_2\cdot P_4) + m_\mu M_Z^2)\,n^{\mu\nu}. \qquad (47)$$

$$\left(-P_1^\mu\,\mathrm{Tr}(\gamma^5\gamma^\nu \not{P}_1 \not{P}_2 \not{P}_4) + P_1^\nu\,\mathrm{Tr}(\gamma^5\gamma^\mu \not{P}_1 \not{P}_2 \not{P}_4)\right)\left(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)$$

$$= \left(4i P_1^\mu \varepsilon^{\nu\alpha\beta\sigma} P_{1\alpha} P_{2\beta} P_{4\sigma} - 4i P_1^\nu \varepsilon^{\mu\alpha\beta\sigma} P_{1\alpha} P_{2\beta} P_{4\sigma}\right)\left(-n_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)$$

$$= -4i\, P_{1\alpha} P_{2\beta} P_{4\sigma}\,(P_1^\mu \varepsilon^{\nu\alpha\beta\sigma} - P_1^\nu \varepsilon^{\mu\alpha\beta\sigma})\,n_{\mu\nu} + \frac{4i}{M_Z^2} P_{1\alpha} P_{2\beta} P_{4\sigma}\,(P_1^\mu \varepsilon^{\nu\alpha\beta\sigma} - P_1^\nu \varepsilon^{\mu\alpha\beta\sigma})\,P_{4\mu} P_{4\nu}$$

$$(P_1^\mu \varepsilon^{\nu\alpha\beta\sigma} - P_1^\nu \varepsilon^{\mu\alpha\beta\sigma})\,n_{\mu\nu} = (P_1^0 \varepsilon^{\nu\alpha\beta\sigma} - P_1^\nu \varepsilon^{0\alpha\beta\sigma})\,n_{0\nu}$$
$$+ (P_1^1 \varepsilon^{\nu\alpha\beta\sigma} - P_1^\nu \varepsilon^{1\alpha\beta\sigma})\,n_{1\nu} + (P_1^2 \varepsilon^{\nu\alpha\beta\sigma} - P_1^\nu \varepsilon^{2\alpha\beta\sigma})\,n_{2\nu}$$
$$+ (P_1^3 \varepsilon^{\nu\alpha\beta\sigma} - P_1^\nu \varepsilon^{3\alpha\beta\sigma})\,n_{3\nu} = (P_1^0 \cancel{\varepsilon^{0\alpha\beta\sigma}} - P_1^0 \cancel{\varepsilon^{0\alpha\beta\sigma}}).1$$
$$+ (P_1^1 \cancel{\varepsilon^{1\alpha\beta\sigma}} - P_1^1 \cancel{\varepsilon^{1\alpha\beta\sigma}})(-1) + (P_1^2 \cancel{\varepsilon^{2\alpha\beta\sigma}} - P_1^2 \cancel{\varepsilon^{2\alpha\beta\sigma}})(-1)$$
$$+ (P_1^3 \cancel{\varepsilon^{3\alpha\beta\sigma}} - P_1^3 \cancel{\varepsilon^{3\alpha\beta\sigma}})(-1) = 0$$

$$\therefore \boxed{(P_1^\mu \varepsilon^{\nu\alpha\beta\sigma} - P_1^\nu \varepsilon^{\mu\alpha\beta\sigma})\,n_{\mu\nu} = 0} \qquad (48)$$

$$(P_1^{\mu}\epsilon^{\nu\alpha\rho\sigma}-P_1^{\nu}\epsilon^{\mu\alpha\rho\sigma})P_{4\mu}P_{4\nu}$$

$$=(P_1\cdot P_4)\,\epsilon^{\nu\alpha\rho\sigma}P_{4\nu}-(P_1\cdot P_4)\,\epsilon^{\mu\alpha\rho\sigma}P_{4\mu}$$

$$=(P_1\cdot P_4)\,\epsilon^{\nu\alpha\rho\sigma}P_{4\nu}-(P_1\cdot P_4)\,\epsilon^{\nu\alpha\rho\sigma}P_{4\nu}=0$$

$$\therefore\quad \boxed{(P_1^{\mu}\epsilon^{\nu\alpha\rho\sigma}-P_1^{\nu}\epsilon^{\mu\alpha\rho\sigma})P_{4\mu}P_{4\nu}=0}\qquad(49)$$

$$\Rightarrow\quad \boxed{\left(-P_1^{\mu}Tr(\gamma^5\gamma^{\nu}\not{p}_1\not{p}_2\not{p}_4)+P_1^{\nu}Tr(\gamma^5\gamma^{\mu}\not{p}_1\not{p}_2\not{p}_4)\right)\left(-\eta_{\mu\nu}+\frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)=0}\qquad(50)$$

also:

$$Tr(\gamma^5\gamma^{\nu}\gamma^{\mu}\not{p}_2\not{p}_4)\left(-\eta_{\mu\nu}+\frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)=-4i\,\epsilon^{\nu\mu\alpha\beta}P_{2\alpha}P_{4\beta}\left(-\eta_{\mu\nu}+\frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)=0.\qquad(51)$$

$$Tr(\gamma^5\gamma^{\nu}\not{p}_1\gamma^{\mu}\not{p}_2)\left(-\eta_{\mu\nu}+\frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)=0\qquad(52)$$

$$Tr(\gamma^5\gamma^{\nu}\not{p}_1\gamma^{\mu}\not{p}_4)\left(-\eta_{\mu\nu}+\frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)=0\qquad(53)$$

$$|M_4|^2=\frac{g^4m_\mu^2C_{\mu V}^2}{16M_W^2\cos^2\theta_W}\left(-\eta_{\mu\nu}+\frac{P_{4\mu}P_{4\nu}}{M_Z^2}\right)\cdot\Big\{\left[(C_A^{\mu})^2+(C_V^{\mu})^2\right]\Big[16(P_1\cdot P_2)P_1^{\mu}P_1^{\nu}-8(P_1\cdot P_2)m_\mu^2$$

$$\eta^{\mu\nu}-2P_1^{\mu}Tr(\gamma^{\nu}\not{p}_1\not{p}_2\not{p}_4)+8m_\mu^2(P_2\cdot P_4)\eta^{\mu\nu}+16m_\mu^2P_1^{\mu}P_1^{\nu}-8m_\mu^4\eta^{\mu\nu}-2P_1^{\nu}Tr(\not{p}_1\gamma^{\mu}\not{p}_4\not{p}_2)$$

$$-M_Z^2Tr(\gamma^{\nu}\not{p}_1\gamma^{\mu}\not{p}_2)+2(P_2\cdot P_4)Tr(\gamma^{\nu}\not{p}_1\gamma^{\mu}\not{p}_4)-2m_\mu^2Tr(\gamma^{\nu}\not{p}_1\gamma^{\mu}\not{p}_4)\Big]+4m_\mu\left[(C_V^{\mu})^2-(C_A^{\mu})^2\right]$$

$$\left(2m_\mu(P_1\cdot P_2)+2m_\mu^3-2m_\mu(P_1\cdot P_4)-2m_\mu(P_2\cdot P_4)+m_\mu M_Z^2\right)\eta^{\mu\nu}\Big\}$$

$$=\frac{g^4m_\mu^2C_{\mu V}^2}{16M_W^2\cos^2\theta_W}\Big\{\left[(C_A^{\mu})^2+(C_V^{\mu})^2\right]\Big[-16m_\mu^2\cancel{(P_1\cdot P_2)}+32\cancel{(P_1\cdot P_2)}\overset{8m_\mu^2(P_2\cdot P_4)}{m_\mu^2}+2m_\mu^2Tr(\not{p}_2\not{p}_4)$$

$$-32m_\mu^2\cancel{(P_2\cdot P_4)}-16\cancel{m_\mu^4}+32\cancel{m_\mu^4}+2\overset{8m_\mu^2(P_2\cdot P_4)}{m_\mu^2}\cancel{Tr(\not{p}_4\not{p}_2)}+M_Z^2\eta_{\mu\nu}4\left[\cancel{P_1^{\mu}P_2^{\nu}}+\cancel{P_1^{\nu}P_2^{\mu}}-\right.$$

$$\left.\cancel{(P_1\cdot P_2)\eta^{\mu\nu}}\right]-8(P_2\cdot P_4)\left[\cancel{P_1^{\nu}P_4^{\mu}}+\cancel{P_1^{\mu}P_4^{\nu}}-\cancel{(P_1\cdot P_4)\eta^{\mu\nu}}\right]\eta_{\mu\nu}+8m_\mu^2\left[\cancel{P_1^{\nu}P_4^{\mu}}+\cancel{P_1^{\mu}P_4^{\nu}}-\cancel{(P_1\cdot P_4)\eta^{\mu\nu}}\right]\eta_{\mu\nu}\Big]$$

$$-16m_\mu\left[(C_V^{\mu})^2-(C_A^{\mu})^2\right]\left[2m_\mu(P_1\cdot P_2)+2m_\mu^3-2m_\mu(P_1\cdot P_4)-2m_\mu(P_2\cdot P_4)+m_\mu M_Z^2\right]$$

$$+\frac{\left[(C_A^{\mu})^2+(C_V^{\mu})^2\right]}{M_Z^2}\Big\{16(P_1\cdot P_2)(P_1\cdot P_4)^2-8m_\mu^2(P_1\cdot P_2)M_Z^2-8(P_1\cdot P_4)M_Z^2(P_1\cdot P_2)$$

$$+8m_\mu^2(P_2\cdot P_4)M_Z^2+16m_\mu^2(P_1\cdot P_4)^2-8m_\mu^4M_Z^2-8(P_1\cdot P_4)M_Z^2(P_1\cdot P_2)-4M_Z^2(P_1^{\mu}P_2^{\nu}+P_1^{\nu}P_2^{\mu}$$

$$-(P_1\cdot P_2)\eta^{\mu\nu})P_{4\mu}P_{4\nu}+8M_Z^2(P_2\cdot P_4)(P_1\cdot P_4)-8m_\mu^2M_Z^2(P_1\cdot P_4)\Big\}+4m_\mu\left[(C_V^{\mu})^2-(C_A^{\mu})^2\right]\cdot\Big\{2m_\mu(P_1\cdot P_2)$$

$$+2m_\mu^3-2m_\mu(P_1\cdot P_4)-2m_\mu(P_2\cdot P_4)+m_\mu M_Z^2\Big\}\Big\}$$

$$= \frac{g^4 m_\mu^2 C_{\mu\nu}^2}{16 M_W^2 \cos^2\theta_W} \Big\{ \left[|C_A^\mu|^2 + |C_V^\mu|^2 \right] \Big\{ 16 m_\mu^2 \cancel{(p_1\cdot p_2)} - 16 m_\mu^2 \cancel{(p_2\cdot p_4)} + 16 \cancel{m_\mu^4}$$

$$- 8 \cancel{(p_1\cdot p_2)} M_Z^2 + 16 \cancel{(p_1\cdot p_4)}(p_2\cdot p_4) - 16 m_\mu^2 \cancel{(p_1\cdot p_4)} \Big\} - 16 m_\mu \left[\cancel{|C_V^\mu|^2 - |C_A^\mu|^2} \right] \Big\{$$

$$2 m_\mu \cancel{(p_1\cdot p_2)} + \cancel{2 m_\mu^3} - \cancel{2 m_\mu (p_1\cdot p_4)} - 2 m_\mu \cancel{(p_2\cdot p_4)} \cancel{+ m_\mu M_Z^2} \Big\} + \frac{\left[|C_A^\mu|^2 + |C_V^\mu|^2 \right]}{M_Z^2} \Big\{$$

$$16 (p_1\cdot p_2) \cancel{(p_1\cdot p_4)^2} - 8 m_\mu^2 \xcancel{(p_1\cdot p_2)} M_Z^2 - 8 (p_1\cdot p_4) \xcancel{M_Z^2} (p_1\cdot p_2) + 8 m_\mu^2 \cancel{(p_2\cdot p_4)} M_Z^2 + 16 m_\mu^2 \cancel{(p_1\cdot p_4)^2}$$

$$- 8 \cancel{m_\mu^4} M_Z^2 - 8 M_Z^2 \cancel{(p_1\cdot p_2)} (p_1\cdot p_4) - 8 M_Z^2 \cancel{(p_1\cdot p_4)} (p_2\cdot p_4) + 4 M_Z^4 \cancel{(p_1\cdot p_2)} + 8 M_Z^2 \cancel{(p_2\cdot p_4)} (p_1\cdot p_4)$$

$$- 8 m_\mu^2 M_Z^2 \cancel{(p_1\cdot p_4)} \Big\} + 4 \cancel{m_\mu} \left[|C_V^\mu|^2 - |C_A^\mu|^2 \right] \Big\{ \cancel{2 m_\mu} (p_1\cdot p_2) + \cancel{2 m_\mu^3} - 2 m_\mu \cancel{(p_1\cdot p_4)}$$

$$- 2 m_\mu \cancel{(p_2\cdot p_4)} + m_\mu \cancel{M_Z^2} \Big\} \Big\} \qquad (54)$$

neglecting the terms containing m_μ inside $\{\}$ we have:

$$|M_4|^2 = \frac{g^4 m_\mu^2 C_{\mu\nu}^2}{16 M_W^2 \cos^2\theta_W} \left[|C_A^\mu|^2 + |C_V^\mu|^2 \right] \Big\{ -8 \cancel{(p_1\cdot p_2)} M_Z^2 + 16 \cancel{(p_1\cdot p_4)} (p_2\cdot p_4) + \frac{16 \cancel{(p_1\cdot p_2)} (p_1\cdot p_4)^2}{M_Z^2}$$

$$- 8 \cancel{(p_1\cdot p_4)} (p_1\cdot p_2) - 8 \cancel{(p_1\cdot p_2)} (p_1\cdot p_4) - 8 \cancel{(p_1\cdot p_4)} \cancel{(p_2\cdot p_4)} + 4 M_Z^2 \cancel{(p_1\cdot p_2)} + 8 \cancel{(p_2\cdot p_4)} (p_1\cdot p_4) \Big\}$$

$$= \frac{g^4 m_\mu^2 C_{\mu\nu}^2}{16 M_W^2 \cos^2\theta_W} \left[|C_A^\mu|^2 + |C_V^\mu|^2 \right] \Big\{ 16 (p_1\cdot p_4)(p_2\cdot p_4) - 16 (p_1\cdot p_2)(p_1\cdot p_4) - 4 M_Z^2 (p_1\cdot p_2)$$

$$+ 16 \frac{(p_1\cdot p_2)(p_1\cdot p_4)^2}{M_Z^2} \Big\}$$

$$= \frac{\cancel{4} g^4 m_\mu^2 C_{\mu\nu}^2}{\underset{4}{\cancel{16}} M_W^4} \left[|C_A^\mu|^2 + |C_V^\mu|^2 \right] \Big\{ M_Z^2 (M_Z^2 - u)(s + u - m_{A^0}^2) - M_Z^2 s (M_Z^2 - u) - M_Z^2 \frac{s}{2} M_Z^2$$

$$+ \cancel{4} \frac{s}{2} \frac{(M_Z^2 - u)^2}{\cancel{4}} \Big\}$$

$$= \frac{g^4 m_\mu^2 C_{\mu\nu}^2}{8 M_W^4} \left[|C_A^\mu|^2 + |C_V^\mu|^2 \right] \Big\{ 2 M_Z^2 (M_Z^2 - u)(s + u - m_{A^0}^2) - 2 s (M_Z^4 - u M_Z^2) - s M_Z^4$$

$$+ s (M_Z^2 - u)^2 \Big\}$$

$$= \frac{g^4 m_\mu^2 C_{\mu\nu}^2}{8 M_W^4} \left[|C_A^\mu|^2 + |C_V^\mu|^2 \right] \Big\{ 2 (M_Z^4 - M_Z^2 u)(M_Z^2 - t) - 2 s M_Z^4 + 2 \cancel{u s} M_Z^2 - \cancel{s M_Z^4}$$

$$+ \cancel{s M_Z^4} - 2 \cancel{u s} M_Z^2 + s u^2 \Big\}$$

$$= \frac{g^4 m_\mu^2 C_{\mu\nu}^2}{8 M_W^4} \left[|C_A^\mu|^2 + |C_V^\mu|^2 \right] \Big\{ 2 M_Z^6 - 2 M_Z^4 t - 2 u M_Z^4 + 2 \cancel{M_Z^4} u t - 2 s M_Z^4$$

$$+ s \cancel{u}^2 \Big\}$$

71

$$J = (p_1+p_2+p_3)^\mu \left(-n_{\mu\nu} + \frac{p_{4\mu}p_{4\nu}}{m_Z^2}\right) \mathrm{Tr}\,[\ \ ] = \left(-(p_1+p_2+p_3)_\nu + \frac{((p_1+p_2+p_3)\cdot p_4)\,p_{4\nu}}{m_Z^2}\right) \times$$

$$\times\, \mathrm{Tr}\,[\ \ ]$$

$$J = \left(-(p_1+p_2+p_3)_\nu + \frac{((p_1+p_2+p_3)\cdot p_4)\,p_{4\nu}}{m_Z^2}\right) \times \left[4i\,C_V^\mu\,\varepsilon^{\alpha\rho\nu\sigma} p_{1\alpha} p_{2\rho} p_{3\sigma} - C_A^\mu \mathrm{Tr}(\not p_1 \not p_2 \gamma^\nu \not p\right.$$

$$(p_1+p_2+p_3)_\nu\,\varepsilon^{\alpha\rho\nu\sigma} p_{1\alpha} p_{2\rho} p_{3\sigma}$$

$$= p_{1\nu} p_{1\alpha} \overset{0}{\cancel{p_{2\rho} p_{3\sigma}}}\,\varepsilon^{\alpha\rho\nu\sigma} + p_{2\nu} p_{2\rho} \overset{0}{\cancel{p_{1\alpha} p_{3\sigma}\,\varepsilon^{\alpha\rho\nu\sigma}}} + p_{1\alpha} p_{2\rho} \overset{0}{\cancel{p_{3\nu} p_{3\sigma}}}\,\varepsilon^{\alpha\rho\nu\sigma} = 0$$

$$p_{4\nu}\,\varepsilon^{\alpha\rho\nu\sigma} p_{1\alpha} p_{2\rho} p_{3\sigma} = \overset{0}{\cancel{(p_1+p_2-p_3)_\nu\,\varepsilon^{\alpha\rho\nu\sigma}}}\, p_{1\alpha} p_{2\rho} p_{3\sigma} = 0$$

$$\Rightarrow J = C_A^\mu\,\mathrm{Tr}(\not p_1 \not p_2 (\not p_1 + \not p_2 + \not p_3) \not p_3) - \frac{C_A^\mu}{m_Z^2}((p_1+p_2+p_3)\cdot p_4)\,\mathrm{Tr}(\not p_1 \not p_2 \not p_4 \not p_3)$$

$$J = C_A^\mu\,\mathrm{Tr}(\not p_1 \not p_2 \not p_1 \not p_3) + C_A^\mu\,\mathrm{Tr}(\not p_1 \not p_2 \not p_2 \not p_3) + C_A^\mu\,\mathrm{Tr}(\not p_1 \not p_2 \not p_3 \not p_3)$$
$$- \frac{C_A^\mu}{m_Z^2}((p_1+p_2+p_3)\cdot p_4)\,\mathrm{Tr}(\not p_1 \not p_2 \not p_4 \not p_3)$$

$$J = C_A^\mu\,\mathrm{Tr}(\not p_1 (-\not p_1 \not p_2 + 2(p_1\cdot p_2))\not p_3) + \overset{\sim 0}{\cancel{m_\mu^2}}\, C_A^\mu\,\mathrm{Tr}(\not p_1 \not p_3) + m_{A^0}^2\, C_A^\mu\, 4(p_1\cdot p_2)$$
$$- \frac{4C_A^\mu}{m_Z^2}((p_1+p_2+p_3)\cdot p_4)\,[(p_1\cdot p_2)(p_3\cdot p_4) - (p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4)]$$

$$J = -\overset{\sim 0}{\cancel{m_\mu^2}}\, C_A^\mu\,\mathrm{Tr}(\not p_2 \not p_3) + 8(p_1\cdot p_2)\, C_A^\mu\,(p_1\cdot p_3) + 4C_A^\mu m_{A^0}^2 (p_1\cdot p_2)$$
$$- \frac{4C_A^\mu}{m_Z^2}((p_1+p_2+p_3)\cdot p_4)\,[(p_1\cdot p_2)(p_3\cdot p_4) - (p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4)]$$

$$J = 2s\,C_A^\mu (m_{A^0}^2 - t) + 2s\,C_A^\mu m_{A^0}^2 - \frac{C_A^\mu}{m_Z^2}(s - m_{A^0}^2)\,[\,s\,(s - m_{A^0}^2 - m_Z^2) - (m_Z^2 - u)\times$$

$$\times (m_{A^0}^2 - u) + (m_{A^0}^2 - t)(s - m_{A^0}^2 + u)]$$

$$J = C_A^\mu \left\{ 4s\,m_{A^0}^2 - 2st - \frac{(s - m_{A^0}^2)}{m_Z^2}\,[\,s^2 - \cancel{s\,m_{A^0}^2} - s\,m_Z^2 - m_Z^2 m_{A^0}^2 + m_Z^2 u + u\,m_{A^0}^2\right.$$
$$\left. - u^2 + \cancel{s\,m_{A^0}^2} - m_{A^0}^4 + u\,m_{A^0}^2 - st + t\,m_{A^0}^2 - ut\,] \right\}$$

$$s + t + u = m_{A^0}^2 + m_Z^2$$

$$M_A^{+} M_4 = \frac{i g^2 m_\mu}{4 M_W \cos\theta_W} C_H^{*} (p_1+p_2+p_3)^{\mu} \left(\sum_{\lambda} \varepsilon_{4\mu}\, \varepsilon^{*}_{4\nu}\right) \left(\frac{-i g^2 m_\mu}{4 M_W \cos\theta_W}\right) C_{\mu\nu} \sum_{s} \bar{U}_1 V_2 \bar{V}_2 \gamma^5$$

$$(\not{p}_1 - \not{p}_4 + m_\mu)\gamma^{\nu} (C_V^{\mu} - C_A^{\mu}\gamma^5) U_1 \qquad (65)$$

$$M_A^{+} M_4 = \frac{g^4 m_\mu^2}{16 M_W^2 \cos^2\theta_W} C_H^{*} C_{\mu\nu} (p_1+p_2+p_3)^{\mu} \left(-n_{\mu\nu} + \frac{p_{4\mu} p_{4\nu}}{M_Z^2}\right) Tr\left[(\not{p}_2 - m_\mu)\gamma^5 (\not{p}_1 - \not{p}_4 + m_\mu)\right.$$

$$\left.\gamma^{\nu}(C_V^{\mu} - C_A^{\mu}\gamma^5)(\not{p}_1 + m_\mu)\right] \qquad (66)$$

neglecting m_μ inside the trace :

$$M_A^{+} M_4 = \frac{g^4 m_\mu^2}{16 M_W^2 \cos^2\theta_W} C_H^{*} C_{\mu\nu} (p_1+p_2+p_3)^{\mu} \left(-n_{\mu\nu} + \frac{p_{4\mu} p_{4\nu}}{M_Z^2}\right) Tr\left[\not{p}_1 \not{p}_2 \gamma^5 (\not{p}_1 - \not{p}_4)\gamma^{\nu}(C_V^{\mu} - C_A^{\mu}\gamma^5)\right] \qquad (67)$$

$$Tr[\quad] = Tr\left[(\not{p}_1 \not{p}_2 \gamma^5 \not{p}_1 \gamma^{\nu} - \not{p}_1 \not{p}_2 \gamma^5 \not{p}_4 \gamma^{\nu})(C_V^{\mu} - C_A^{\mu}\gamma^5)\right]$$

$$= Tr(\not{p}_1 \not{p}_2 \overset{0}{\cancel{\gamma^5 \not{p}_1}} \gamma^{\nu}) C_V^{\mu} - C_A^{\mu} Tr[\not{p}_1 \not{p}_2 \gamma^5 \not{p}_1 \gamma^{\nu}\gamma^5] - C_V^{\mu} Tr[\not{p}_1 \not{p}_2 \gamma^5 \not{p}_4 \gamma^{\nu}]$$

$$+ C_A^{\mu} Tr[\not{p}_1 \not{p}_2 \gamma^5 \not{p}_4 \gamma^{\nu}\gamma^5]$$

$$= -C_A^{\mu} Tr[\not{p}_1 \not{p}_2 \not{p}_1 \gamma^{\nu}] - C_V^{\mu} Tr[\gamma^5 \not{p}_1 \not{p}_2 \not{p}_4 \gamma^{\nu}] + C_A^{\mu} Tr[\not{p}_1 \not{p}_2 \not{p}_4 \gamma^{\nu}]$$

$$= -C_A^{\mu} Tr[\not{p}_1 (-\not{p}_1 \not{p}_2 + 2(p_1 \cdot p_2))\gamma^{\nu}] + 4i C_V^{\mu} \varepsilon^{\alpha\beta\gamma\nu} p_{1\alpha} p_{2\beta} p_{4\gamma}$$

$$+ C_A^{\mu} p_{1\alpha} p_{2\beta} p_{4\gamma}\, Tr(\gamma^{\alpha}\gamma^{\beta}\gamma^{\gamma}\gamma^{\nu})$$

$$= \overset{\to 0}{\cancel{m_\mu^2}} C_A^{\mu} Tr(\not{p}_2 \gamma^{\nu}) - 2(p_1 \cdot p_2) C_A^{\mu} Tr(\not{p}_1 \gamma^{\nu}) + 4i C_V^{\mu} \varepsilon^{\alpha\beta\gamma\nu} p_{1\alpha} p_{2\beta} p_{4\gamma}$$

$$+ 4 C_A^{\mu} p_{1\alpha} p_{2\beta} p_{4\gamma} [n^{\alpha\beta} n^{\gamma\nu} - n^{\alpha\gamma} n^{\beta\nu} + n^{\alpha\nu} n^{\beta\gamma}]$$

$$= -8(p_1 \cdot p_2) C_A^{\mu} p_1^{\nu} + 4i C_V^{\mu} \varepsilon^{\alpha\beta\gamma\nu} p_{1\alpha} p_{2\beta} p_{4\gamma}$$

$$+ 4 C_A^{\mu} [(p_1 \cdot p_2) p_4^{\nu} - (p_1 \cdot p_4) p_2^{\nu} + (p_2 \cdot p_4) p_1^{\nu}] \qquad (68)$$

$$K = (p_1 + p_2 + p_3)^{\mu} \left(-n_{\mu\nu} + \frac{p_{4\mu} p_{4\nu}}{M_Z^2}\right) Tr[\quad] \qquad (69)$$

$$K = \left(-(p_1+p_2+p_3)_{\nu} + \frac{((p_1+p_2+p_3)\cdot p_4) p_{4\nu}}{M_Z^2}\right)\left[-8(p_1 \cdot p_2) C_A^{\mu} p_1^{\nu} + 4i C_V^{\mu} \varepsilon^{\alpha\beta\gamma\nu} p_{1\alpha} p_{2\beta} p_{4\gamma}\right.$$

$$\left. + 4 C_A^{\mu} [(p_1 \cdot p_2) p_4^{\nu} - (p_1 \cdot p_4) p_2^{\nu} + (p_2 \cdot p_4) p_1^{\nu}]\right] \qquad (70)$$

$$(p_1 + p_2 + \underset{0}{p_3})_{\nu}\, \varepsilon^{\alpha\beta\gamma\nu} p_{1\alpha} p_{2\beta} p_{4\gamma} = (2p_1 + 2p_2 - p_4)_{\nu}\, \varepsilon^{\alpha\beta\gamma\nu} p_{1\alpha} p_{2\beta} p_{4\gamma}$$

$$= 2\varepsilon^{\alpha\beta\gamma\nu} \overset{0}{\cancel{p_{1\nu} p_{1\alpha}}} p_{2\beta} p_{4\gamma} + 2\varepsilon^{\alpha\beta\gamma\nu} \overset{0}{\cancel{p_{1\alpha} p_{2\nu}}} p_{2\beta} p_{4\gamma} - \varepsilon^{\alpha\beta\gamma\nu} \overset{0}{\cancel{p_{1\alpha} p_{2\beta}}} p_{4\nu} p_{4\gamma}$$

$$-s^2 - st + sM_Z^2 - us = s(-s-t-u+M_Z^2) = -s\,m_{A^0}^2$$

because : $s+t+u = m_{A^0}^2 + M_Z^2$

$$K = C_A^{*}\Big[4s\cancel{m}_{A^0}^2 \overset{\downarrow}{-s\,m_{A^0}^2} \overset{\downarrow}{+M_Z^2 m_{A^0}^2} - u\cancel{M}_Z^2 \overset{\downarrow}{-2u\cancel{m}_{A^0}^2} - \cancel{u}s + \cancel{u}^2 + \cancel{u}t \overset{\downarrow}{+m_{A^0}^4} \overset{\downarrow}{-m_{A^0}^2 t}$$

$$+\frac{2us^2}{M_Z^2} - \frac{2us\,m_{A^0}^2}{M_Z^2}\Big]$$

$$-s\cancel{m}_{A^0}^2 + M_Z^2\cancel{m}_{A^0}^2 - u\cancel{m}_{A^0}^2 + \cancel{m}_{A^0}^4 - m_{\cancel{A^0}}^2 t$$

$$= m_{A^0}^2(-s-u-t \overset{0}{\cancel{+M_Z^2}} + m_{A^0}^2) = 0$$

$$\Rightarrow K = C_A^{*}\left[4s\,m_{A^0}^2 - uM_Z^2 - us + u^2 + ut - u\,m_{A^0}^2 + \frac{2us^2}{M_Z^2} - \frac{2us\,m_{A^0}^2}{M_Z^2}\right]$$

$$K = \frac{C_A^{*}}{M_Z^2}\left[4s\cancel{m}_{A^0}^2 M_Z^2 \overset{\downarrow}{-uM_Z^4} - u\cancel{s}M_Z^2 \overset{\downarrow}{+u^2M_Z^2} \overset{\downarrow}{+ut\,M_Z^2} \overset{\downarrow}{-u\,m_{A^0}^2 M_Z^2} + 2\cancel{u}s^2 - 2us\,m_{A^0}^2\right]$$

$$-u\cancel{M}_Z^4 + u^2\cancel{M}_Z^2 + ut\cancel{M}_Z^2 - u\,m_{\cancel{A^0}}^2 M_Z^2 = uM_Z^2(\underbrace{-M_Z^2 - m_{A^0}^2 + u}_{-s} + t) = -usM_Z^2$$

$$K = \frac{C_A^{*}}{M_Z^2}\left[4s\,m_{A^0}^2 M_Z^2 - 2usM_Z^2 + 2us^2 - 2us\,m_{A^0}^2\right]$$

$$K = \frac{2s\,C_A^{*}}{M_Z^2}\left[2m_{A^0}^2 M_Z^2 \overset{\downarrow}{-uM_Z^2} \overset{\downarrow}{+us} \overset{\downarrow}{-u\,m_{A^0}^2}\right]$$

$$-uM_Z^2 + us - u\,m_{A^0}^2 = u(s - m_{A^0}^2 - M_Z^2) = u(-t-u) = -ut - u^2$$

$$\Rightarrow K = \frac{2s\,C_A^{*}}{M_Z^2}\left[2m_{A^0}^2 M_Z^2 - ut - u^2\right] \qquad (73)$$

$$\therefore\ M_A^{+} M_4 = \frac{g^4 m_u^2}{\cancel{16}M_W^2\cos^2\theta_W}\, C_H^{*}\, C_{au}\, \frac{\cancel{2}s\,C_A^{*}}{M_Z^2}\left[2m_{A^0}^2 M_Z^2 - ut - u^2\right]$$

$$\boxed{M_A^{+} M_4 = \frac{g^4 m_u^2 C_H^{*} C_{au}\, s\, C_A^{*}}{8M_W^4}\left[2m_{A^0}^2 M_Z^2 - ut - u^2\right]} \qquad (74)$$

or
$$\boxed{M_A^{+} M_4 = \frac{g^4 m_u^2 C_H^{*} C_{au}\, s\, C_A^{*}}{8M_W^4}\left[m_{A^0}^2 M_Z^2 - \frac{1}{4}\lambda(s, M_Z^2, m_{A^0}^2)\sin^2\theta - u^2\right]} \qquad (75)$$

$$\boxed{M_4^{+} M_A = (M_A^{+} M_4)^{+} = \frac{g^4 m_u^2 C_H C_{au}\, s\, C_A^{*}}{8M_W^4}\left[m_{A^0}^2 M_Z^2 - \frac{1}{4}\lambda(s, M_Z^2, m_{A^0}^2)\sin^2\theta - u^2\right]} \qquad (76)$$

$$\Rightarrow \boxed{M_A^{+} M_4 + M_4^{+} M_A = \frac{2\,\mathrm{Re}(C_H)\, s\, g^4 m_u^2 C_{au} C_A^{*}}{8M_W^4}\left[m_{A^0}^2 M_Z^2 - \frac{1}{4}\lambda(s, M_Z^2, m_{A^0}^2)\sin^2\theta - u^2\right]} \qquad (77)$$

$$K' = \left(-\eta_{\mu\nu} + \frac{p_{4\mu}p_{4\nu}}{M_Z^2}\right) \mathrm{Tr}(\;) \qquad (83)$$

$$= \left[(C_V^{\mu})^2 - (C_A^{\mu})^2 \right] \left(-\eta_{\mu\nu} + \frac{p_{4\mu}p_{4\nu}}{M_Z^2}\right) \left[m_{A^0}^2 \mathrm{Tr}(\gamma^\nu \not{p}_1 \gamma^\mu \not{p}_2) - 2(p_2\cdot p_3)\mathrm{Tr}(\gamma^\nu \not{p}_1 \gamma^\mu \not{p}_3)\right.$$

$$\left. + 2p_3^\nu \mathrm{Tr}(\not{p}_1 \gamma^\mu \not{p}_3 \not{p}_2) \right]$$

$$= [\quad] \left\{ - m_{A^0}^2 \mathrm{Tr}(\gamma^\mu \not{p}_1 \gamma_\mu \not{p}_2) + 2(p_2\cdot p_3)\mathrm{Tr}(\gamma^\mu \not{p}_1 \gamma_\mu \not{p}_3) - 2\mathrm{Tr}(\not{p}_1 \not{p}_3 \not{p}_3 \not{p}_2) \right.$$

$$\left. + \frac{1}{M_Z^2}\left[m_{A^0}^2 \mathrm{Tr}(\not{p}_4 \not{p}_1 \not{p}_4 \not{p}_2) - 2(p_2\cdot p_3)\mathrm{Tr}(\not{p}_4 \not{p}_1 \not{p}_4 \not{p}_3) + 2(p_3\cdot p_4)\mathrm{Tr}(\not{p}_1 \not{p}_4 \not{p}_3 \not{p}_2)\right]\right\}$$

$$= [\quad] \left\{ 8\cancel{m_{A^0}^2}(p_1\cdot p_2) - 16(p_2\cdot p_3)(p_1\cdot p_3) - 8\cancel{m_{A^0}^2}(p_1\cdot p_2) \right.$$

$$+ \frac{1}{M_Z^2}\left[m_{A^0}^2 \mathrm{Tr}(\not{p}_4(-\not{p}_4\not{p}_1 + 2(p_1\cdot p_4))\not{p}_2) - 2(p_2\cdot p_3)\mathrm{Tr}(\not{p}_4(-\not{p}_4\not{p}_1 + 2p_1\cdot p_4)\not{p}_3) \right.$$

$$\left.\left. + 2(p_3\cdot p_4)\mathrm{Tr}(\not{p}_1 \not{p}_4 \not{p}_3 \not{p}_2) \right]\right\}$$

$$= [\quad] \left\{ -16\cancel{(p_2\cdot p_3)}(p_1\cdot p_3) + \frac{1}{M_Z^2}\left[-4M_Z^2(p_1\cdot \cancel{p_2})m_{A^0}^2 + 8(p_1\cdot p_4)(p_2\cdot p_4)m_{A^0}^2 + 8(p_2\cdot p_3)\cancel{M_Z^2}(p_1\cdot p_3) \right.\right.$$

$$\left.\left. - 16(p_2\cdot p_3)(p_1\cdot p_4)(p_3\cdot p_4) + 8(p_3\cdot p_4)\left[(p_1\cdot p_4)(p_3\cdot p_2) - (p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_2)(p_3\cdot p_4)\right]\right]\right\}$$

$$= [\quad] \left\{ -8(p_2\cdot p_3)(p_1\cdot p_3) - 4m_{A^0}^2(p_1\cdot p_2) + \frac{1}{M_Z^2}\left[8m_{A^0}^2(p_1\cdot p_4)(p_2\cdot p_4) \right.\right.$$

$$\left.\left. - 8(p_3\cdot p_4)(p_1\cdot p_4)(p_2\cdot p_3) - 8(p_3\cdot p_4)(p_1\cdot p_3)(p_2\cdot p_4) + 8(p_3\cdot p_4)(p_1\cdot p_2)(p_3\cdot p_4)\right]\right\}$$

$$= [\quad] \left\{ -2(m_{A^0}^2 - u)(m_{A^0}^2 - t) - 2s\,m_{A^0}^2 + \frac{1}{M_Z^2}\left[2m_{A^0}^2(M_Z^2 - u)(\overbrace{s+u-m_{A^0}^2}^{M_Z^2-t}) \right.\right.$$

$$\left.\left. + (u+t)\left[(M_Z^2-u)(m_{A^0}^2-u) + (m_{A^0}^2-t)(\overbrace{s+u-m_{A^0}^2}^{M_Z^2-t}) + s(u+t)\right]\right]\right\}$$

$$= [\quad] \left\{ -2m_{A^0}^4 + 2\cancel{m_{A^0}^2}t + 2\cancel{m_{A^0}^2}u - 2ut - 2s\,m_{A^0}^2 + 2m_{A^0}^2M_Z^2 - 2\cancel{m_{A^0}^2}t - 2\cancel{u}m_{A^0}^2 \right.$$

$$+ \frac{1}{M_Z^2}\left[2m_{A^0}^2ut + (u+t)\left[m_{A^0}^2\cancel{M_Z^2} - u\cancel{M_Z^2} - u\,m_{A^0}^2 + u^2 + m_{A^0}^2\cancel{M_Z^2} - m_{A^0}^2t \right.\right.$$

$$\left.\left.\left. - t\cancel{M_Z^2} + t^2 + us + st\right]\right]\right\}$$

$$= [\quad] \left\{ -2\cancel{m_{A^0}^4} - 2\cancel{u}t - 2s\cancel{m_{A^0}^2} + 2\cancel{m_{A^0}^2}M_Z^2 + \cancel{m_{A^0}^2}u + \cancel{m_{A^0}^2}t - \cancel{u^2} - \cancel{u}t + u\cancel{m_{A^0}^2} + \cancel{t}m_{A^0}^2 \right.$$

$$- \cancel{u}t - \cancel{t^2} + \frac{1}{M_Z^2}\left[2\cancel{m_{A^0}^2}ut - \overset{\downarrow}{u^2m_{A^0}^2} - ut\cancel{m_{A^0}^2} + \overset{\downarrow}{u^3} + \overset{\downarrow}{u^2t} - \cancel{m_{A^0}^2}ut - \cancel{m_{A^0}^2}t^2 + u\cancel{t^2} \right.$$

$$\left.\left. + \cancel{t^3} + \overset{\downarrow}{u^2s} + u\cancel{st} + u\cancel{st} + s\cancel{t^2}\right]\right\}$$

$$-u^2m_{A^0}^2 + u^3 + u^2t + u^2s = u^2(s+t+u-m_{A^0}^2) = u^2M_Z^2$$

$$\Rightarrow \overline{|M|^2} = \frac{1}{4}\left\{ \frac{g^4 m_\mu^2 |C_H|^2}{8M_W^4} \lambda s + \frac{g^4 m_\mu^2 s\, C_{\mu t}^2 [(C_A^{\mu})^2 + (C_V^{\mu})^2]}{8M_W^4} \left[t^2 + \frac{M_Z^2}{2s}\lambda \sin^2\theta\right] \right.$$

$$+ \frac{g^4 m_\mu^2 s\, C_{\mu u}^2}{8M_W^4}[(C_A^{\mu})^2 + (C_V^{\mu})^2]\left[u^2 + \frac{M_Z^2}{2s}\lambda\sin^2\theta\right] + \frac{2g^4 m_\mu^2 s}{8M_W^4} C_A^{\mu} \mathrm{Re}(C_H) C_{\mu t}[$$

$$m_{A^0}^2 M_Z^2 - \frac{1}{4}\lambda\sin^2\theta - t^2] + \frac{2\,\mathrm{Re}(C_H)\, s\, g^4 m_\mu^2 C_{\mu u} C_A^{\mu}}{8M_W^4}\left[m_{A^0}^2 M_Z^2 - \frac{1}{4}\lambda\sin^2\theta - u^2\right]$$

$$\left. + \frac{2g^4 m_\mu^2 s\, C_{\mu u} C_{\mu t}}{8M_W^4}[(C_V^{\mu})^2 - (C_A^{\mu})^2]\left[m_{A^0}^2 M_Z^2 - \frac{1}{4}\lambda\sin^2\theta + \frac{M_Z^2}{2s}\lambda\sin^2\theta\right]\right\}$$

$$\overline{|M|^2} = \frac{g^4 m_\mu^2 s}{32 M_W^4}\left\{ |C_H|^2\lambda + [(C_A^{\mu})^2 + (C_V^{\mu})^2]\left[C_{\mu t}^2\left[t^2 + \frac{M_Z^2}{2s}\lambda\sin^2\theta\right]\right.\right.$$

$$\left. + C_{\mu u}^2\left[u^2 + \frac{M_Z^2}{2s}\lambda\sin^2\theta\right]\right] + 2C_A^{\mu} C_{\mu t}\mathrm{Re}(C_H)\left[m_{A^0}^2 M_Z^2 - \frac{1}{4}\lambda\sin^2\theta - t^2\right]$$

$$+ 2C_A^{\mu} C_{\mu u}\mathrm{Re}(C_H)\left[m_{A^0}^2 M_Z^2 - \frac{1}{4}\lambda\sin^2\theta - u^2\right] + 2C_{\mu u}C_{\mu t}[(C_V^{\mu})^2 - (C_A^{\mu})^2]\,\cdot$$

$$\left.\cdot\left[m_{A^0}^2 M_Z^2 - \frac{1}{4}\lambda\sin^2\theta + \frac{M_Z^2}{2s}\lambda\sin^2\theta\right]\right\}$$

where $\lambda = \lambda(s, M_Z^2, m_{A^0}^2)$ (88)

$|\vec{P}_1| = \frac{s^{1/2}}{2}$; $|\vec{P}_3| = \frac{\lambda^{1/2}(s, M_Z^2, m_{A^0}^2)}{2\sqrt{s}}$ (see $\mu^-\mu^+ \to H^\pm W^\mp$ (86), (87))

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{64\pi^2 s}\frac{|\vec{P}_3|}{|\vec{P}_1|}\overline{|M|^2} \qquad (89)$$

$$\boxed{\begin{aligned}\left.\frac{d\sigma}{d\Omega}\right|_{CM} = {} & \frac{1}{64\pi^2 s}\frac{\lambda^{1/2} g^4 m_\mu^2}{32 M_W^4}\left\{ |C_H|^2\lambda + [(C_A^{\mu})^2 + (C_V^{\mu})^2]\left[C_{\mu t}^2\left[t^2 + \frac{M_Z^2}{2s}\lambda\sin^2\theta\right]\right.\right.\\ & \left. + C_{\mu u}^2\left[u^2 + \frac{M_Z^2}{2s}\lambda\sin^2\theta\right]\right] + 2C_A^{\mu} C_{\mu t}\mathrm{Re}(C_H)\left[m_{A^0}^2 M_Z^2 - \frac{1}{4}\lambda\sin^2\theta - t^2\right]\\ & + 2C_A^{\mu} C_{\mu u}\mathrm{Re}(C_H)\left[m_{A^0}^2 M_Z^2 - \frac{1}{4}\lambda\sin^2\theta - u^2\right] + 2C_{\mu u}C_{\mu t}[(C_V^{\mu})^2 - (C_A^{\mu})^2]\,\cdot\\ & \left.\cdot\left[m_{A^0}^2 M_Z^2 - \frac{1}{4}\lambda\sin^2\theta + \frac{M_Z^2}{2s}\lambda\sin^2\theta\right]\right\} \qquad (90)\end{aligned}}$$

$$d\Omega = 2\pi \sin\theta\, d\theta. = -2\pi\, d\cos\theta$$

$$\Omega = 2\pi\int_{-1}^{1} d\cos\theta$$

$$\boxed{\int_{-1}^{1} t \, d\cos\theta = (m_{A^0}^2 + M_Z^2 - s)} \qquad (102)$$

$$\boxed{\int_{-1}^{1} \frac{d\cos\theta}{u} = \frac{2}{\lambda^{1/2}} \ln \left| \frac{m_{A^0}^2 + M_Z^2 - s + \lambda^{1/2}}{m_{A^0}^2 + M_Z^2 - s - \lambda^{1/2}} \right|} \qquad (103)$$

$$\boxed{\int_{-1}^{1} \frac{\sin^2\theta \, d\cos\theta}{u} = \frac{4}{\lambda} \left[(m_{A^0}^2 + M_Z^2 - s) - \frac{2 m_{A^0}^2 M_Z^2}{\lambda^{1/2}} \ln \left| \frac{m_{A^0}^2 + M_Z^2 - s + \lambda^{1/2}}{m_{A^0}^2 + M_Z^2 - s - \lambda^{1/2}} \right| \right]} \qquad (104)$$

$$\boxed{\int_{-1}^{1} u \, d\cos\theta = (m_{A^0}^2 + M_Z^2 - s)} \qquad (105)$$

$$\int_{-1}^{1} \frac{d\cos\theta}{ut} = \int_{-1}^{1} \frac{d\cos\theta}{(a - b\cos\theta)(a + b\cos\theta)} = \int_{-1}^{1} \frac{dx}{(a-bx)(a+bx)}$$

$$\frac{1}{(a-bx)(a+bx)} = \frac{A}{a-bx} + \frac{B}{a+bx} = \frac{Aa + \cancel{Abx} + Ba - B\cancel{b}x}{(a-bx)(a+bx)}$$

$$= \frac{b(A-B)x + a(A+B)}{(a-bx)(a+bx)}$$

$$\Rightarrow \quad b(A-B) = 0 \quad \therefore \quad A = B$$

$$\text{and} \quad (A+B)a = 1 \quad \Rightarrow \quad 2Aa = 1 \quad \Rightarrow \quad A = \frac{1}{2a} = B$$

$$\int_{-1}^{1} \frac{dx}{(a-bx)(a+bx)} = \int_{-1}^{1} \frac{1}{2a(a-bx)} dx + \int_{-1}^{1} \frac{1}{2a(a+bx)} dx$$

$$= \frac{1}{2a} \left[-\frac{\ln|a-bx|}{b} \Big|_{-1}^{1} + \frac{\ln|a+bx|}{b} \Big|_{-1}^{1} \right]$$

$$= \frac{1}{2a} \left[-\frac{\ln|a-b|}{b} + \frac{\ln|a+b|}{b} + \frac{\ln|a+b|}{b} - \frac{\ln|a-b|}{b} \right]$$

$$\int_{-1}^{1} \frac{dx}{(a-bx)(a+bx)} = \frac{1}{ab} \ln \left| \frac{a+b}{a-b} \right|$$

$$\boxed{\int_{-1}^{1} \frac{d\cos\theta}{ut} = \frac{4}{\lambda^{1/2} (m_{A^0}^2 + M_Z^2 - s)} \ln \left| \frac{m_{A^0}^2 + M_Z^2 - s + \lambda^{1/2}}{m_{A^0}^2 + M_Z^2 - s - \lambda^{1/2}} \right|} \qquad (106)$$

$$\sigma_{CH} = \frac{\cancel{2\pi}}{\underset{32\pi}{\cancel{64\pi^2}}\, s} \; \frac{\lambda^{1/2} g^4 m_\mu^2}{32 M_W^4} \Big\{ 2|C_H|^2 \lambda + [|C_A^\mu|^2 + |C_V^\mu|^2] \tan^2\beta \Big[4 + \frac{M_Z^2 \cancel{\lambda}}{\underset{1}{\cancel{2s}}} \cdot \frac{\overset{4}{\cancel{8}}}{\cancel{\lambda}} \cdot$$

$$\cdot \Big(-2 + \cancel{\frac{(m_{A^0}^2 + M_Z^2 - s)}{\lambda^{1/2}}} \cdot f \Big) + \frac{M_Z^2}{\underset{1}{\cancel{2s}}} \cancel{\lambda} \cdot \frac{\overset{4}{\cancel{8}}}{\cancel{\lambda}} \Big(-2 + \cancel{\frac{(m_{A^0}^2 + M_Z^2 - s)}{\lambda^{1/2}}} \cdot f \Big) \Big] + 2 C_A^\mu \tan\beta \, \mathrm{Re}(C_H)$$

$$\cdot \Big[m_{A^0}^2 M_Z^2 \frac{2}{\lambda^{1/2}} \cdot f - \frac{1}{\cancel{4}} \lambda \Big(\frac{4}{\cancel{\lambda}}\Big) \Big((m_{A^0}^2 + M_Z^2 - s) - \frac{2 m_{A^0}^2 M_Z^2}{\lambda^{1/2}} \cdot f \Big) - (m_{A^0}^2 + M_Z^2 - s) \Big]$$

$$+ 2 C_A^\mu \tan\beta \, \mathrm{Re}(C_H) \Big[m_{A^0}^2 M_Z^2 \frac{2}{\lambda^{1/2}} \cdot f - \frac{1}{\cancel{4}} \cancel{\lambda} \Big(\frac{4}{\cancel{\lambda}}\Big) \cdot \Big((m_{A^0}^2 + M_Z^2 - s) - \frac{2 m_{A^0}^2 M_Z^2}{\lambda^{1/2}} \cdot f \Big) -$$

$$- (m_{A^0}^2 + M_Z^2 - s) \Big] + 2 \tan^2\beta \, [|C_V^\mu|^2 - |C_A^\mu|^2] \Big[m_{A^0}^2 M_Z^2 \frac{4}{\lambda^{1/2} (m_{A^0}^2 + M_Z^2 - s)} \cdot f - \frac{1}{\cancel{4}} \cancel{\lambda} \cdot \Big(\frac{8}{\cancel{\lambda}} \Big)$$

$$\cdot \Big(1 - \frac{2 m_{A^0}^2 M_Z^2}{\lambda^{1/2} (m_{A^0}^2 + M_Z^2 - s)} \cdot f \Big) + \frac{M_Z^2}{2s} \cancel{\lambda} \cdot \Big(\frac{8}{\cancel{\lambda}} \Big) \Big(1 - \frac{2 m_{A^0}^2 M_Z^2}{\lambda^{1/2} (m_{A^0}^2 + M_Z^2 - s)} \cdot f \Big) \Big] \Big\} \qquad (109)$$

The terms containing $[|C_V^\mu|^2 - |C_A^\mu|^2]$ is:

$$\frac{4 m_{A^0}^2 M_Z^2}{\lambda^{1/2} (m_{A^0}^2 + M_Z^2 - s)} \cdot f - 2 + \frac{4 m_{A^0}^2 \cancel{M_Z^2}}{\lambda^{1/2} \cancel{(m_{A^0}^2 + M_Z^2 - s)}} \cdot f + \frac{4 M_Z^2}{s} - \frac{8 M_Z^2}{s} \frac{m_{A^0}^2 \cancel{M_Z^2}}{\lambda^{1/2} \cancel{(m_{A^0}^2 + M_Z^2 - s)}} \cdot f$$

$$= \frac{8 m_{A^0}^2 M_Z^2}{\lambda^{1/2} (m_{A^0}^2 + M_Z^2 - s)} \cdot f - \frac{8 M_Z^2}{s} \frac{m_{A^0}^2 M_Z^2}{\lambda^{1/2} (m_{A^0}^2 + M_Z^2 - s)} \cdot f - 2 + 4 \frac{M_Z^2}{s}$$

$$= \frac{8 m_{A^0}^2 M_Z^2}{\lambda^{1/2} (m_{A^0}^2 + M_Z^2 - s)} \cdot f \Big(1 - \frac{M_Z^2}{s} \Big) - 2 \Big(1 - \frac{2 M_Z^2}{s} \Big)$$

$$\sigma_{CH} = \frac{\lambda^{1/2} g^4 m_\mu^2}{32^2 \pi s M_W^4} \Big\{ 2 |C_H|^2 \lambda + 4 [|C_A^\mu|^2 + |C_V^\mu|^2] \tan^2\beta \Big[1 + \frac{2 M_Z^2}{s} \cdot \Big(-2 +$$

$$+ \frac{(m_{A^0}^2 + M_Z^2 - s)}{\lambda^{1/2}} f \Big) \Big] + 2 C_A^\mu \tan\beta \, \mathrm{Re}(C_H) \Big[\frac{4 m_{A^0}^2 M_Z^2}{\lambda^{1/2}} f - 2 (m_{A^0}^2 + M_Z^2 - s) \Big]$$

$$+ 2 C_A^\mu \tan\beta \, \mathrm{Re}(C_H) \Big[\frac{4 m_{A^0}^2 M_Z^2}{\lambda^{1/2}} \cdot f - 2 (m_{A^0}^2 + M_Z^2 - s) \Big] + 2 \tan^2\beta \, [|C_V^\mu|^2 - |C_A^\mu|^2] \cdot$$

$$\cdot \Big[\frac{8 m_{A^0}^2 M_Z^2}{\lambda^{1/2} (m_{A^0}^2 + M_Z^2 - s)} \cdot f \Big(1 - \frac{M_Z^2}{s} \Big) - 2 \Big(1 - 2 \frac{M_Z^2}{s} \Big) \Big] \Big\}$$

but $G_F^2 = \dfrac{g^4}{32 M_W^4}$

$\Rightarrow$

$$|C_A^\mu|^2 + |C_V^\mu|^2 = \frac{1}{4} + \left(-\frac{1}{2} + 2\sin^2\theta_W\right)^2$$

$$= \frac{1}{4} + \frac{1}{4} - 2\sin^2\theta_W + 4\sin^4\theta_W$$

$$= \frac{1}{2} - 2\sin^2\theta_W + 4\sin^4\theta_W$$

$$\boxed{|C_A^\mu|^2 + |C_V^\mu|^2 = \frac{1}{2}\left(1 - 4\sin^2\theta_W + 8\sin^4\theta_W\right)} \qquad (111)$$

$$|C_A^\mu|^2 - |C_V^\mu|^2 = \frac{1}{4} - \left(-\frac{1}{2} + 2\sin^2\theta_W\right)^2$$

$$= \frac{1}{4} - \left(\frac{1}{4} - 2\sin^2\theta_W + 4\sin^4\theta_W\right)$$

$$= 2\sin^2\theta_W - 4\sin^4\theta_W = 2\sin^2\theta_W\left(1 - 2\sin^2\theta_W\right)$$

$$\boxed{|C_A^\mu|^2 - |C_V^\mu|^2 = 2\sin^2\theta_W\left(1 - 2\sin^2\theta_W\right)} \qquad (112)$$

$$\sigma(\mu^-\mu^+ \to A^0 Z^0)_{C_H} = \frac{G_F^2 m_\mu^2}{16\pi s^2}\Big\{\lambda^{1/2}(s, m_{A^0}^2, M_Z^2)\Big[s|C_H|^2\lambda(s, m_{A^0}^2, M_Z^2)$$

$$+ 4\tan^2\beta\,\sin^2\theta_W(1 - 2\sin^2\theta_W)(s - 2M_Z^2) + 2\tan\beta\, s\,\mathrm{Re}(C_H)(m_{A^0}^2 + M_Z^2 - s)$$

$$+ (1 - 4\sin^2\theta_W + 8\sin^4\theta_W)\tan^2\beta\,(s - 4M_Z^2)\Big] + 4M_Z^2\tan\beta\, f(s, m_{A^0}^2, M_Z^2)\Big[-s\,\mathrm{Re}(C_H)\,m_{A^0}^2$$

$$+ \frac{1}{2}\tan\beta\,(1 - 4\sin^2\theta_W + 8\sin^4\theta_W)(m_{A^0}^2 + M_Z^2 - s) - \frac{4\sin^2\theta_W(1 - 2\sin^2\theta_W)\tan\beta\, m_{A^0}^2(s - M_Z^2)}{(m_{A^0}^2 + M_Z^2 - s)}\Big]\Big\} \qquad (113)$$

---

if $m_{A^0} = \sqrt{s} - M_Z$

$$\lambda(s, m_{A^0}^2, M_Z^2) = s^2 + m_{A^0}^4 + M_Z^4 - 2s\,m_{A^0}^2 - 2sM_Z^2 - 2m_{A^0}^2 M_Z^2$$

$$= s^2 + (\sqrt{s} - M_Z)^4 + M_Z^4 - 2s(\sqrt{s} - M_Z)^2 - 2sM_Z^2 - 2M_Z^2(\sqrt{s} - M_Z)^2$$

$$= s^2 + (s - 2\sqrt{s}M_Z + M_Z^2)^2 + M_Z^4 - 2s(s - 2\sqrt{s}M_Z + M_Z^2) - 2sM_Z^2$$

$$-2M_Z^2(s - 2\sqrt{s}M_Z + M_Z^2)$$

$$= \cancel{s^2} + \cancel{s^2} + \cancel{4sM_Z^2} + \cancel{M_Z^4} - \cancel{4s\sqrt{s}M_Z} + \cancel{2sM_Z^2} - \cancel{4\sqrt{s}M_Z^3} + \cancel{M_Z^4} - \cancel{2s^2}$$

$$+ \cancel{4s\sqrt{s}M_Z} - \cancel{2sM_Z^2} - \cancel{2sM_Z^2} - \cancel{2sM_Z^2} + \cancel{4\sqrt{s}M_Z^3} - \cancel{2M_Z^4}$$

$$\lambda(s, m_{A^0}^2, M_Z^2) = 0$$

$$\sigma(\mu^-\mu^+ \to A^0 Z^0)\Big|_{CM} = (3.893792914 \times 10^{11})\,\text{fb}\ \frac{G_F^2\, m_\mu^2\, m_{A^0}^4}{16\pi s^2} \left\{ \lambda^{*\,1/2}\left(1, \frac{s}{m_{A^0}^2}, \frac{M_Z^2}{m_{A^0}^2}\right) \cdot \right.$$

$$\cdot \left[\left(\frac{s}{m_{A^0}^2}\right) (C_{HA})^2\, \lambda^*\left(1, \frac{s}{m_{A^0}^2}, \frac{M_Z^2}{m_{A^0}^2}\right) + 4\tan^2\beta \sin^2\theta_W (1 - 2\sin^2\theta_W)\left(\frac{s}{m_{A^0}^2} - \frac{2M_Z^2}{m_{A^0}^2}\right)\right.$$

$$\left. + 2\tan\beta \left(\frac{s}{m_{A^0}^2}\right) C_{HA}\left(1 + \frac{M_Z^2}{m_{A^0}^2} - \frac{s}{m_{A^0}^2}\right) + \left(1 - 4\sin^2\theta_W + 8\sin^4\theta_W\right)\tan^2\beta\left(\frac{s}{m_{A^0}^2} - \frac{4M_Z^2}{m_{A^0}^2}\right)\right]$$

$$+ 4\left(\frac{M_Z^2}{m_{A^0}^2}\right)\tan\beta\,(f)\left[-\left(\frac{s}{m_{A^0}^2}\right) C_{HA} + \frac{1}{2}\tan\beta\,(1 - 4\sin^2\theta_W + 8\sin^4\theta_W)\left(1 + \frac{M_Z^2}{m_{A^0}^2} - \frac{s}{m_{A^0}^2}\right)\right.$$

$$\left.\left. - \frac{4\sin^2\theta_W (1 - 2\sin^2\theta_W)\tan\beta \left(\frac{s}{m_{A^0}^2} - \frac{M_Z^2}{m_{A^0}^2}\right)}{\left(1 + \frac{M_Z^2}{m_{A^0}^2} - \frac{s}{m_{A^0}^2}\right)} \right]\right\} \qquad (120)$$

where λ^* is defined in (116), (f) is given in (117), C_{HA} is given in (119)

$$\boxed{\sin^2\alpha = \frac{1}{2}\left\{1 + \frac{(1 - \tan^2\beta)}{(1 + \tan^2\beta)}\left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)}\right]\right\}} \qquad (121)$$

$$\boxed{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta) = \left[\left(1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}\right)^2 - 4\left(\frac{M_Z^2}{M_{H^\pm}^2}\right)\left(1 - \frac{M_W^2}{M_{H^\pm}^2}\right)\left(\frac{1 - \tan^2\beta}{1 + \tan^2\beta}\right)^2\right]^{1/2}} \qquad (122)$$

$$\boxed{\sin 2\alpha = -\left(\frac{2\tan\beta}{1 + \tan^2\beta}\right)\left[\frac{1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta)}\right]} \qquad (123)$$

$$\boxed{M_{H^\pm}^2 = m_{A^0}^2 + M_W^2} \qquad (124)$$

$$\boxed{\frac{s - m_{h^0}^2}{M_{H^\pm}^2} = \frac{s}{M_{H^\pm}^2} - \frac{1}{2}\left\{1 - \frac{M_W^2}{M_{H^\pm}^2} + \frac{M_Z^2}{M_{H^\pm}^2} - g^*\right\}} \qquad (125)$$

$$\boxed{\frac{s - m_{H^0}^2}{M_{H^\pm}^2} = \frac{s}{M_{H^\pm}^2} - \frac{1}{2}\left\{1 - \frac{M_W^2}{M_{H^\pm}^2} + \frac{M_Z^2}{M_{H^\pm}^2} + g^*\right\}} \qquad (126)$$

$$(127) \qquad \boxed{\frac{s - m_{h^0}^2}{m_{A^0}^2} = \frac{M_{H^\pm}^2}{m_{A^0}^2}\left(\frac{s - m_{h^0}^2}{M_{H^\pm}^2}\right) = \frac{M_{H^\pm}^2}{m_{A^0}^2}\left\{\frac{s}{M_{H^\pm}^2} - \frac{1}{2}\left\{1 - \frac{M_W^2}{M_{H^\pm}^2} + \frac{M_Z^2}{M_{H^\pm}^2} - g^*\right\}\right\}}$$

$$(128) \qquad \boxed{\frac{s - m_{H^0}^2}{m_{A^0}^2} = \frac{M_{H^\pm}^2}{m_{A^0}^2}\left(\frac{s - m_{H^0}^2}{M_{H^\pm}^2}\right) = \frac{M_{H^\pm}^2}{m_{A^0}^2}\left\{\frac{s}{M_{H^\pm}^2} - \frac{1}{2}\left\{1 - \frac{M_W^2}{M_{H^\pm}^2} + \frac{M_Z^2}{M_{H^\pm}^2} + g^*\right\}\right\}}$$

$$\frac{s-m_{H^0}^2}{m_{A^0}^2} = \left(\frac{120}{89.06256829}\right)^2 \left\{\left(\frac{500}{120}\right)^2 - \frac{1}{2}\left\{1-\left(\frac{80.423}{120}\right)^2 + \left(\frac{91.1876}{120}\right)^2 + 0.079685444\right\}\right\}$$

$$\Rightarrow \frac{s-m_{H^0}^2}{m_{A^0}^2} = 30.42086656$$

$$C_{HA} = \left(\frac{\frac{1}{2}(-0.942902278) + 30(0.666534611)}{30.56552743}\right) - \left(\frac{\frac{1}{2}(-0.942902278) - 30(0.333465389)}{30.42086656}\right)$$

$$\Rightarrow C_{HA} = 0.638778023 + 0.344349586 = 0.983127609$$

$$\Rightarrow 2\sin^2\theta_W(1-2\sin^2\theta_W) = 2(0.23113)(1-2\times 0.23113) = 0.248575692$$

$$\Rightarrow \frac{1}{2}(1-4\sin^2\theta_W + 8\sin^4\theta_W) = \frac{1}{2}(1-4\times 0.23113 + 8\times(0.23113)^2) = 0.251424307$$

$$\Rightarrow \left(\frac{s-2M_Z^2}{m_{A^0}^2}\right) = \left(\frac{500}{89.06256829}\right)^2 - 2\left(\frac{91.1876}{89.06256829}\right)^2 = 29.42076309$$

$$\Rightarrow \left(1+\frac{M_Z^2}{m_{A^0}^2} - \frac{s}{m_{A^0}^2}\right) = \left(1+\left(\frac{91.1876}{89.06256829}\right)^2 - \left(\frac{500}{89.06256829}\right)^2\right) = -29.46905236$$

$$\Rightarrow \left(\frac{s-4M_Z^2}{m_{A^0}^2}\right) = \left(\frac{500}{89.06256829}\right)^2 - 4\left(\frac{91.1876}{89.06256829}\right)^2 = 27.32418454$$

$$\Rightarrow \left(\frac{s-M_Z^2}{m_{A^0}^2}\right) = \left(\frac{500}{89.06256829}\right)^2 - \left(\frac{91.1876}{89.06256829}\right)^2 = 30.46905236 \; ; \; \frac{s}{m_{A^0}^2} = 31.5173416$$

$$\Rightarrow \frac{G_F^2 m_\mu^2 m_{A^0}^4}{16\pi s^2} = \frac{(1.166391\times 10^{-5})^2(0.105658357)^2(89.06256829)^4}{16\pi(500)^4} = 3.041774481\times 10^{-17}$$

$$\Rightarrow \sigma(\mu^-\mu^+ \to A^0Z^0) = (3.893792914\times 10^{11})\times 3.041774481\times 10^{-17}\left\{29.39782118\,\cdot\right.$$

$$\cdot\left[\left(\frac{500}{89.06256829}\right)^2(0.983127609)^2(864.2318899) + \overset{\downarrow}{2}\times 900\times 0.248575692\times 29.42076309\right.$$

$$\left.+ 2\times 30\times\left(\frac{500}{89.06256829}\right)^2\times 0.983127609\,(-29.46905236) + \overset{\downarrow}{2}\times 0.251424307\times 900\times 27.32418454\right]$$

$$+ 4\left(\frac{91.1876}{89.06256829}\right)^2\times 30\times(-6.717103149)\left[-\left(\frac{500}{89.06256829}\right)^2\times 0.983127609 + \right.$$

$$\left.\left.+ 30\times \overset{\downarrow}{0.251424307}\times(-29.46905236) - \frac{\overset{\downarrow}{2}\times 0.248575692\times 30\times 30.46905236}{(-29.46905236)}\right]\right\}$$

(91)

$$C_{HA} = \frac{\left(\frac{1}{2}(-0.126625186) + 30\times(0.995975317)\right)}{30.52162344} - \frac{\left(\frac{1}{2}(-0.126625186) - 30\times(4.024683\times10^{-3}\right.}{29.44442021}$$

$$C_{HA} = 0.976879456 + 6.25086456 7\times10^{-3}$$

$$\rightarrow \quad C_{HA} = 0.98313032$$

$$\rightarrow 2\sin^2\theta_W(1-2\sin^2\theta_W) = 0.248575692$$

$$\rightarrow \frac{1}{2}(1-4\sin^2\theta_W + 8\sin^4\theta_W) = 0.251424307$$

$$\rightarrow \left|\frac{s-2m_Z^2}{m_{A^0}^2}\right| = 29.42076309$$

$$\rightarrow \left(1+\frac{m_Z^2}{m_{A^0}^2} - \frac{s}{m_{A^0}^2}\right) = -29.46905236$$

$$\rightarrow \left(\frac{s-4m_Z^2}{m_{A^0}^2}\right) = 27.32418454$$

$$\rightarrow \left(\frac{s-m_Z^2}{m_{A^0}^2}\right) = 30.46905236$$

$$\rightarrow \frac{s}{m_{A^0}^2} = 31.51734163$$

$$\rightarrow \frac{G_F^2 m_\mu^2 m_{A^0}^4}{16\pi s^2} = 3.041774481\times10^{-17}$$

$$\Rightarrow \sigma(\mu^-\mu^+\rightarrow A^0Z^0) = (3.893792914\times10^{11})\{6\times(3.041774481\times10^{-17})\left\{29.39782118\,\cdot\right.$$

$$\left[(31.51734163)\times(0.98313032)^2\times(864.2318899) + 2\times900\times0.248575692\times(29.42076309)\right.$$

$$\left. + 60(31.51734163)(0.98313032)(-29.46905236) + 2\times(0.251424307)900(27.32418454)\right]$$

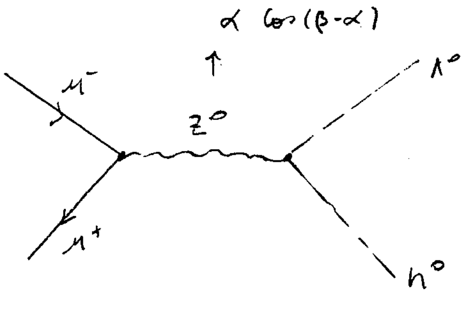
$$+4\left(\frac{91.1876}{89.06256829}\right)^2\times30\times(-6.717103149)\left[-31.51734163\times0.98313032 + 30\times0.251424307\times\right.$$

$$\left.\left.\times(-29.46905236) - \frac{2\times(0.248575692)\times30\times(30.46905236)}{(-29.46905236)}\right]\right\}$$

$\mu^-\mu^+ \to A^0 h^0$:

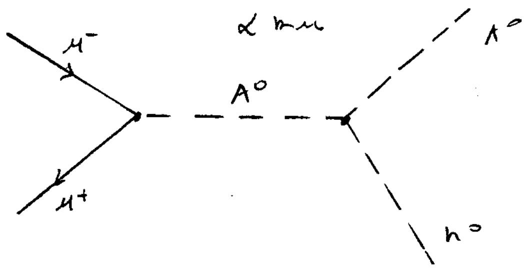
→ time

①

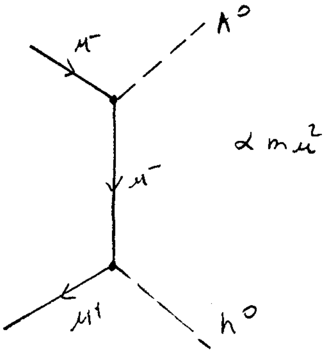


+

②

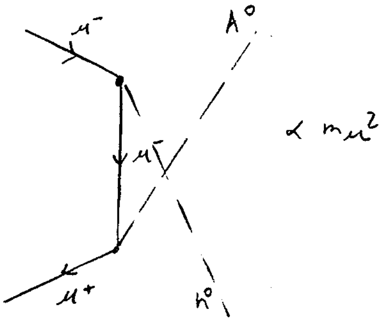


③



+

④



only the first diagram is important.

$$\sigma(\mu^-\mu^+ \to A^0 h^0) \propto \cos^2(\beta-\alpha) = \frac{(1+\tan\beta\tan\alpha)^2}{(1+\tan^2\beta)(1+\tan^2\alpha)}$$

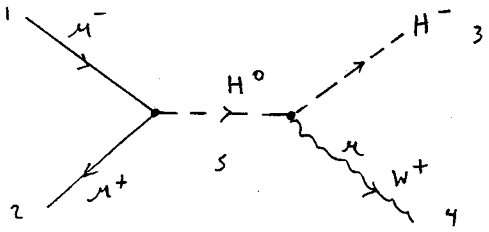
σ decreases as the mass of the A⁰ increases.

Production of $H^{\pm}$

$\mu^-\mu^+ \to H^\mp W^\pm$

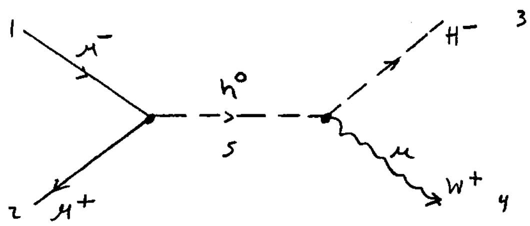
$\longrightarrow$ time

①



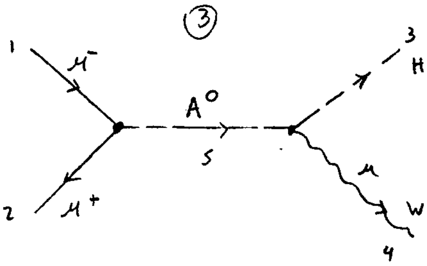
\+

②



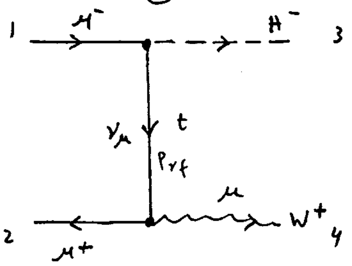
\+

③



\+

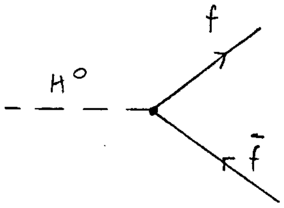
④



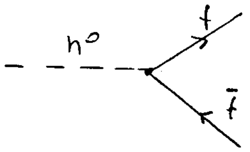
$$m^2_{\substack{H^0\\h^0}} = \frac{1}{2}\left\{m_{A^0}^2 + M_Z^2 \pm \left[(m_{A^0}^2 + M_Z^2)^2 - 4M_Z^2 m_{A^0}^2 \cos^2 2\beta\right]^{1/2}\right\} \; ; \quad \tan 2\alpha = \tan 2\beta \left(\frac{m_{A^0}^2 + M_Z^2}{m_{A^0}^2 - M_Z^2}\right)$$

and $m_{A^0}^2 = M_{H^\pm}^2 - M_W^2$

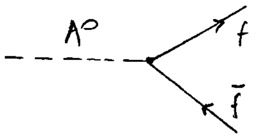
$f = e^-, \mu^-, \tau^-$



$$-\frac{i g m_f \cos\alpha}{2 M_W \cos\beta}$$



$$\frac{i g m_f \sin\alpha}{2 M_W \cos\beta}$$



$$-\frac{g m_f \tan\beta}{2 M_W}\gamma^5$$

$$-iM = -i(M_1 + M_2 + M_3 + M_4)$$

$$\overline{|M|^2} = \frac{1}{4} M^+ M = \frac{1}{4}\left(M_1^+ + M_2^+ + M_3^+ + M_4^+\right)(M_1 + M_2 + M_3 + M_4)$$

$$= \frac{1}{4}\Big(|M_1|^2 + M_1^+ M_2 + M_1^+ M_3 + M_1^+ M_4 + M_2^+ M_1 + |M_2|^2 + M_2^+ M_3 + M_2^+ M_4$$

$$+ M_3^+ M_1 + M_3^+ M_2 + |M_3|^2 + M_3^+ M_4 + M_4^+ M_1 + M_4^+ M_2 + M_4^+ M_3 + |M_4|^2\Big)$$

$$-iM_1 = i\,\varepsilon^*_{\mu 4}\, \frac{g}{2} \sin(\alpha-\beta)(P_3 + P_{H^0})^\mu \frac{i}{P^2_{H^0} - m^2_{H^0} + i m_{H^0}\Gamma_{H^0}}\; \bar{V}_2 \left(\frac{-ig\, m_f \cos\alpha}{2M_W \cos\beta}\right) U_1 \qquad (1)$$

$$-iM_2 = i\,\varepsilon^*_{\mu 4}\, \frac{g}{2} \cos(\alpha-\beta)\,(P_3 + P_{h^0})^\mu \frac{i}{P^2_{h^0} - m^2_{h^0} + i m_{h^0}\Gamma_{h^0}}\; \bar{V}_2 \left(\frac{ig\, m_f \sin\alpha}{2M_W \cos\beta}\right) U_1 \qquad (2)$$

$$-iM_3 = \varepsilon^*_{\mu 4}\, \frac{g}{2} (P_3 + P_{A^0})^\mu \frac{i}{P^2_{A^0} - m^2_{A^0} + i m_{A^0}\Gamma_{A^0}}\; \bar{V}_2 \left(-\frac{g\, m_f \tan\beta}{2M_W}\gamma^5\right) U_1 \qquad (3)$$

$$-iM_4 = \varepsilon^*_{\mu 4}\bar{V}_2\left(-\frac{ig}{\sqrt{2}}\gamma^\mu \frac{1}{2}(1-\gamma^5)\right) \frac{i\,\not{P}_{\nu_f}}{P^2_{\nu_f}}\; \frac{ig\; m_f \tan\beta}{2\sqrt{2}M_W}(1+\gamma^5)\, U_1 \qquad (4)$$

$$M_1 = -\frac{g^2 m_f \cos\alpha\, \sin(\alpha-\beta)}{4M_W \cos\beta}\; \frac{1}{P^2_{H^0} - m^2_{H^0} + i m_{H^0}\Gamma_{H^0}}\; (P_3 + P_{H^0})^\mu (\bar{V}_2 U_1)\, \varepsilon^*_{\mu 4}$$

$$M_2 = \frac{g^2 m_f \sin\alpha\, \cos(\alpha-\beta)}{4M_W \cos\beta}\; \frac{1}{P^2_{h^0} - m^2_{h^0} + i m_{h^0}\Gamma_{h^0}}\; (P_3 + P_{h^0})^\mu (\bar{V}_2 U_1)\, \varepsilon^*_{\mu 4}$$

$$M_3 = \frac{g^2 m_f \tan\beta}{4M_W}\; \frac{1}{P^2_{A^0} - m^2_{A^0} + i m_{A^0}\Gamma_{A^0}}\; (P_3 + P_{A^0})^\mu\; (\bar{V}_2 \gamma^5 U_1)\, \varepsilon^*_{\mu 4}$$

$$M_4 = \frac{-g^2 m_f \tan\beta}{8M_W\, \underset{\substack{\downarrow \\ t}}{P^2_{\nu_f}}}\; \bar{V}_2 \gamma^\mu (1-\gamma^5)\, \not{P}_{\nu_f}\, (1+\gamma^5) U_1\;\; \varepsilon^*_{\mu 4}$$

$$\Rightarrow \boxed{M_3 = \frac{g^2 m_f}{4M_W} c_A (P_1+P_2+P_3)^\mu (\bar{V}_2 \gamma^5 U_1) \varepsilon^*_{\mu 4}} \qquad (15)$$

$$\boxed{M_4 = \frac{-g^2 m_f}{8M_W} c_V \bar{V}_2 \gamma^\mu (1-\gamma^5) \not{P}_{\gamma f} (1+\gamma^5) U_1 \varepsilon^*_{\mu 4}} \qquad (16)$$

$$|M^H|^2 = M^{H+} M^H = \sum_{s,\lambda} \frac{g^4 m_f^2}{16 M_W^2} |C_H|^2 (P_1+P_2+P_3)^\nu \varepsilon_{\nu 4} (\bar{U}_1 V_2) (P_1+P_2+P_3)^\mu (\bar{V}_2 U_1) \varepsilon^*_{\mu 4} \qquad (17)$$

$$|M^H|^2 = \frac{g^4 m_f^2}{16 M_W^2} |C_H|^2 \left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) (P_1+P_2+P_3)^\mu (P_1+P_2+P_3)^\nu \, \mathrm{Tr}\left[(\not{P}_2 - m_f)(\not{P}_1 + m_f)\right]$$

$$|M^H|^2 = \frac{g^4 m_f^2}{16 M_W^2} |C_H|^2 \left(-(P_1+P_2+P_3)^2 + \frac{1}{M_W^2}\left((P_1+P_2+P_3).P_4\right)^2\right) \left[4(P_1.P_2) - 4m_f^2\right]$$

$$\boxed{|M^H|^2 = \frac{g^4 m_f^2 |C_H|^2}{4 M_W^2} \left(-(P_1+P_2+P_3)^2 + \frac{1}{M_W^2}\left((P_1+P_2+P_3).P_4\right)^2\right) \left[P_1.P_2 - m_f^2\right]} \qquad (18)$$

$$s = (P_1+P_2)^2 = 2m_f^2 + 2P_1.P_2$$

$$\Rightarrow \boxed{P_1.P_2 = \frac{s - 2m_f^2}{2} = \frac{s}{2} - m_f^2} \approx \frac{s}{2} \qquad (19)$$

$$(P_1+P_2+P_3).P_4 = (P_3+P_4+P_3).P_4 = 2(P_3.P_4) + M_W^2$$

$$s = (P_3+P_4)^2 = M_{H^\pm}^2 + M_W^2 + 2(P_3.P_4)$$

$$\Rightarrow \boxed{(P_3.P_4) = \frac{s - M_{H^\pm}^2 - M_W^2}{2}} \qquad (20)$$

$$\Rightarrow (P_1+P_2+P_3).P_4 = s - M_{H^\pm}^2 - \cancel{M_W^2} + \cancel{M_W^2} = s - M_{H^\pm}^2$$

$$\boxed{(P_1+P_2+P_3).P_4 = (s - M_{H^\pm}^2)} \qquad (21)$$

$$(P_1+P_2+P_3)^2 = (2P_3+P_4)^2 = 4M_{H^\pm}^2 + M_W^2 + 4P_3.P_4 = 4\cancel{M_{H^\pm}^2} + \cancel{M_W^2} + 2s - 2\cancel{M_{H^\pm}^2} - 2\cancel{M_W^2}$$

$$\boxed{(P_1+P_2+P_3)^2 = 2M_{H^\pm}^2 + 2s - M_W^2} \qquad (22)$$

$$|M^H|^2 = \frac{g^4 m_f^2 |C_H|^2}{4 M_W^2} \left(-2M_{H^\pm}^2 - 2s + M_W^2 + \frac{1}{M_W^2}(s - M_{H^\pm}^2)^2\right)\left(\frac{s}{2} - 2m_f^2\right)$$

$$= \frac{g^4 m_f^2 |C_H|^2}{8 M_W^4} \left(-2M_{H^\pm}^2 M_W^2 - 2s M_W^2 + M_W^4 + s^2 - 2s M_{H^\pm}^2 + M_{H^\pm}^4\right)(s - 4m_f^2)$$

$$\boxed{|M^H|^2 = \frac{g^4 m_f^2 |C_H|^2}{8 M_W^4} \lambda(s, M_W^2, M_{H^\pm}^2)(s - 4m_f^2)} \approx \frac{g^4 m_f^2 |C_H|^2}{8 M_W^4} \lambda(s, M_W^2, M_{H^\pm}^2)\, s \qquad (23)$$

$$|M_3|^2 = \sum_{\lambda, s} - \frac{g^4 m_f^2}{16 M_W^2} |C_A|^2 (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu 4} (\bar{V_2} \gamma^5 U_1) (P_1 + P_2 + P_3)^\nu \varepsilon_{\nu 4} (\bar{U_1} \gamma^5 V_2) \qquad (24)$$

$$|M_3|^2 = - \frac{g^4 m_f^2}{16 M_W^2} |C_A|^2 (P_1 + P_2 + P_3)^\mu (P_1 + P_2 + P_3)^\nu \left(- g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) \sum_s [\bar{V_2} \gamma^5 U_1 . \bar{U_1} \gamma^5 V_2]$$

$$|M_3|^2 = - \frac{g^4 m_f^2}{16 M_W^2} |C_A|^2 \left[- (P_1 + P_2 + P_3)^2 + \frac{1}{M_W^2} ((P_1 + P_2 + P_3) . P_4)^2 \right] Tr[(\not{P}_1 + m_f) \gamma^5 (\not{P}_2 - m_f) \gamma^5]$$

$$Tr[(\not{P}_1 + m_f) \gamma^5 (\not{P}_2 - m_f) \gamma^5] = Tr[(\not{P}_1 \gamma^5 + m_f \gamma^5)(\not{P}_2 \gamma^5 - m_f \gamma^5)]$$

$$= Tr[\not{P}_1 \gamma^5 \not{P}_2 \gamma^5 - m_f \not{P}_1 - m_f \not{P}_2 - m_f^2] = Tr[- \not{P}_1 \not{P}_2 - m_f^2]$$

$$= - 4 (P_1 . P_2) - 4 m_f^2$$

$$|M_3|^2 = \frac{g^4 m_f^2 |C_A|^2}{4 M_W^2} \left[-2 M_{H\pm}^2 - 2S + M_W^2 + \frac{1}{M_W^2} (S - M_{H\pm}^2)^2 \right] (\overset{\frac{S}{2} - m_f^2}{\widehat{P_1 . P_2}} + m_f^2)$$

$$|M_3|^2 = \frac{S g^4 m_f^2 |C_A|^2}{8 M_W^4} [-2 M_{H\pm}^2 M_W^2 - 2S M_W^2 + M_W^4 + S^2 - 2S M_{H\pm}^2 + M_{H\pm}^4]$$

$$\boxed{|M_3|^2 = \frac{g^4 m_f^2 |C_A|^2 S}{8 M_W^4} \lambda (S, M_W^2, M_{H\pm}^2)} \qquad (25)$$

$$|M_4|^2 = \frac{g^4 m_f^2}{64 M_W^2} |C_\nu|^2 \sum_\lambda \varepsilon^*_{\mu 4} \varepsilon_{\nu 4} \sum_s [\bar{V_2} \gamma^\mu (1-\gamma^5) \gamma^\alpha P_{\nu_f \alpha} (1+\gamma^5) U_1][U_1^+ (1+\gamma^5) \gamma^{\beta +} P_{\nu_f \beta} (1-\gamma^5) \overset{\gamma^0 \gamma^\nu}{\overbrace{\gamma^{\nu +}}} \gamma^0 V_2] \qquad (26)$$

$$|M_4|^2 = \frac{g^4 m_f^2}{64 M_W^2} |C_\nu|^2 \sum_\lambda \varepsilon^*_{\mu 4} \varepsilon_{\nu 4} \sum_s [\bar{V_2} \gamma^\mu (1-\gamma^5) \gamma^\alpha P_{\nu_f \alpha} (1+\gamma^5) U_1] [U_1^+ (1+\gamma^5) \overset{\gamma^0 \gamma^\beta}{\overbrace{\gamma^{\beta +}}} \gamma^0 P_{\nu_f \beta} (1+\gamma^5) \gamma^\nu V_2]$$

$$|M_4|^2 = \frac{g^4 m_f^2}{64 M_W^2} |C_\nu|^2 \sum_\lambda \varepsilon^*_{\mu 4} \varepsilon_{\nu 4} \sum_s [\bar{V_2} \gamma^\mu (1-\gamma^5) \gamma^\alpha P_{\nu_f \alpha} (1+\gamma^5) U_1] [\bar{U_1} (1-\gamma^5) \gamma^\beta P_{\nu_f \beta} (1+\gamma^5) \gamma^\nu V_2]$$

$$|M_4|^2 = 4 \frac{g^4 m_f^2}{64 M_W^2} |C_\nu|^2 \left(- g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) \sum_s [\bar{V_2} \gamma^\mu (1-\gamma^5) \not{P}_{\nu_f} U_1] [\bar{U_1} (1-\gamma^5) \not{P}_{\nu_f} \gamma^\nu V_2]$$

$$|M_4|^2 = \frac{g^4 m_f^2}{16 M_W^2} |C_\nu|^2 \left(- g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) Tr[(\not{P}_1 + m_f) (1-\gamma^5) \not{P}_{\nu_f} \gamma^\nu (\not{P}_2 - m_f) \gamma^\mu (1-\gamma^5) \not{P}_{\nu_f}]$$

$$Tr[(\not{P}_1 + m_f) (1-\gamma^5) \not{P}_{\nu_f} \gamma^\nu (\not{P}_2 - m_f) \gamma^\mu (1-\gamma^5) \not{P}_{\nu_f}]$$

$$= Tr[(1-\gamma^5) \not{P}_{\nu_f} (\not{P}_1 + m_f)(1-\gamma^5) \not{P}_{\nu_f} \gamma^\nu (\not{P}_2 - m_f) \gamma^\mu]$$

$$= Tr[2 (1-\gamma^5) \not{P}_{\nu_f} \not{P}_1 \not{P}_{\nu_f} \gamma^\nu (\not{P}_2 - m_f) \gamma^\mu + \overset{0}{\cancel{(1-\gamma^5)(1+\gamma^5)}} m_f \not{P}_{\nu_f} \not{P}_{\nu_f} \gamma^\nu (\not{P}_2 - m_f) \gamma^\mu]$$

$$= 2\,\mathrm{Tr}\left[\,(1-\gamma^5)\,\not{p}_{\nu f}\,\underbrace{\not{p}_1\,\not{p}_{\nu f}}_{\checkmark \atop -\not{p}_{\nu f}\not{p}_1 + 2(P_1 . P_{\nu f})}\,\gamma^\nu\,(\not{p}_2 - m_f)\,\gamma^\mu\,\right]$$

$$= 2\,\mathrm{Tr}\left[\,(1-\gamma^5)\,\not{p}_{\nu f}\,\left(-\not{p}_{\nu f}\not{p}_1 + 2\,(P_1 . P_{\nu f})\right)\gamma^\nu\,(\not{p}_2 - m_f)\,\gamma^\mu\right]$$

$$= 2\,\mathrm{Tr}\left[\,-(1-\gamma^5)\,P_{\nu f}^2\,\not{p}_1\,\gamma^\nu\,(\not{p}_2 - m_f)\gamma^\mu + 2\,(P_1 . P_{\nu f})\,(1-\gamma^5)\,\not{p}_{\nu f}\,\gamma^\nu(\not{p}_2 - m_f)\gamma^\mu\right]$$

$$= 2\,\mathrm{Tr}\left[\,-P_{\nu f}^2\,(\not{p}_1\gamma^\nu - \gamma^5\not{p}_1\gamma^\nu)(\not{p}_2\gamma^\mu - m_f\gamma^\mu) + 2\,(P_1 . P_{\nu f})\,(\not{p}_{\nu f}\gamma^\nu - \gamma^5\not{p}_{\nu f}\gamma^\nu)(\not{p}_2\gamma^\mu - m_f\gamma^\mu)\right]$$

$$= 2\,\mathrm{Tr}\left[\,-P_{\nu f}^2\,(\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu - m_f\,\cancel{\not{p}_1\gamma^\nu\gamma^\mu} - \gamma^5\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu + m_f\,\gamma^5\,\cancel{\not{p}_1\gamma^\nu\gamma^\mu})\right.$$
$$\left. + 2\,(P_1 . P_{\nu f})\left(\not{p}_{\nu f}\gamma^\nu\not{p}_2\gamma^\mu - m_f\,\cancel{\not{p}_{\nu f}\gamma^\nu\gamma^\mu} - \gamma^5\not{p}_{\nu f}\gamma^\nu\not{p}_2\gamma^\mu + m_f\,\gamma^5\,\cancel{\not{p}_{\nu f}\gamma^\nu\gamma^\mu}\right)\right]$$

$$= -2P_{\nu f}^2\,\mathrm{Tr}\,(\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu) + 2P_{\nu f}^2\,\mathrm{Tr}\,(\gamma^5\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu) + 4\,(P_1 . P_{\nu f})\,\mathrm{Tr}\,(\not{p}_{\nu f}\gamma^\nu\not{p}_2\gamma^\mu)$$
$$- 4(P_1 . P_{\nu f})\,\mathrm{Tr}\,(\gamma^5\not{p}_{\nu f}\gamma^\nu\not{p}_2\gamma^\mu)$$

$$= -2P_{\nu f}^2\;4\,(P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - (P_1 . P_2)\,g^{\mu\nu}) + 2\,P_{1\alpha}P_{2\beta}\,(-4i\,\varepsilon^{\alpha\nu\beta\mu})P_{\nu f}^2 + 4\,(P_1 . P_{\nu f})\,4\,(P_2^\mu P_{\nu f}^\nu + P_2^\nu P_{\nu f}^\mu$$
$$-(P_2 . P_{\nu f})g^{\mu\nu}) \quad - 4\,(P_1 . P_{\nu f})\,(-4i\,\varepsilon^{\alpha\nu\beta\mu})\,P_{\nu f\,\alpha}P_{2\beta}$$

$\Rightarrow$

$$\mathrm{Tr}[\,] = -8P_{\nu f}^2\,(P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - (P_1 . P_2)\,g^{\mu\nu}) - 8i\,P_{1\alpha}P_{2\beta}\,\varepsilon^{\alpha\nu\beta\mu}P_{\nu f}^2 + 16\,(P_1 . P_{\nu f})\,(P_2^\mu P_{\nu f}^\nu + P_2^\nu P_{\nu f}^\mu - (P_2 . P_{\nu f})\,g^{\mu\nu})$$
$$+ 16i\,(P_1 . P_{\nu f})\,\varepsilon^{\alpha\nu\beta\mu}\,P_{\nu f\,\alpha}P_{2\beta} \qquad (27)$$

$(g_{\mu\nu}\,g^{\mu\nu} = 4)$

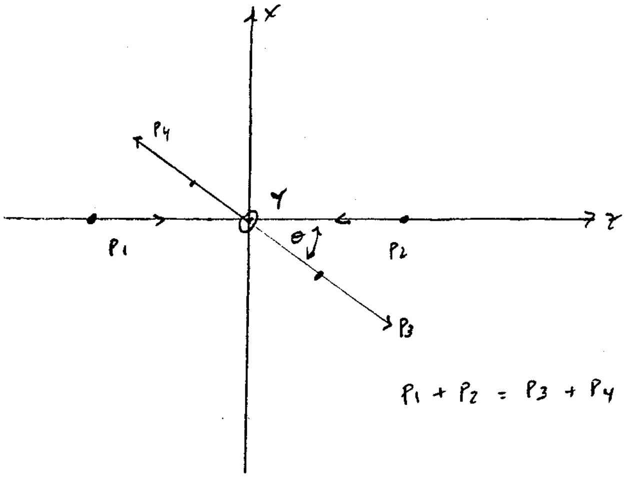
$$|M_4|^2 = \frac{g^4 m_f^2}{16\,M_W^2}\,|C_\nu|^2 \cdot 8\left\{\,P_{\nu f}^2\,\left(2\,(P_1 . P_2) - 4\,(P_1 . P_2)\right) - 2(P_1 . P_{\nu f})\,\left(2\,(P_2 . P_{\nu f}) - 4\,(P_2 . P_{\nu f})\right)\right.$$
$$- \frac{1}{M_W^2}\,P_{\nu f}^2\,\left(2(P_1 . P_4)\,(P_2 . P_4) - M_W^2\,(P_1 . P_2)\right) - \frac{i}{M_W^2}P_{\nu f}^2\,P_{1\alpha}P_{2\beta}\,P_{4\mu}P_{4\nu}\,\varepsilon^{\alpha\nu\beta\mu} + \frac{2(P_1 . P_{\nu f})}{M_W^2}\left(2(P_2 . P_4)\,(P_{\nu f} . P_4)\right.$$
$$\left.\left. - M_W^2\,(P_2 . P_{\nu f})\right) + 2i\,\frac{(P_1 . P_{\nu f})\;P_{\nu f\,\alpha}P_{2\beta}}{M_W^2}\;P_{4\mu}P_{4\nu}\,\varepsilon^{\alpha\nu\beta\mu}\right\} \qquad (28)$$

but

$$P_{4\mu}P_{4\nu}\,\varepsilon^{\alpha\nu\beta\mu} = 0 \qquad (29)$$

$$\Rightarrow |M_4|^2 = \frac{g^4 m_f^2\,|C_\nu|^2}{2\,M_W^2}\left\{-2t\,\cancel{(P_1 . P_2)} + 4\,\cancel{(P_1 . P_{\nu f})}(P_2 . P_{\nu f}) - \frac{2t}{M_W^2}\,\cancel{(P_1 . P_4)}(P_2 . P_4) + t\,\cancel{(P_1 . P_2)}\right.$$
$$\left. + 4\,\frac{(P_1 . P_{\nu f})}{M_W^2}\,\cancel{(P_2 . P_4)}\,(P_{\nu f} . P_4) - 2\,\cancel{(P_1 . P_{\nu f})}(P_2 . P_{\nu f})\right\}$$

$$|M_4|^2 = \frac{g^4 m_f^2\,|C_\nu|^2}{2\,M_W^2}\left\{-t\,(P_1 . P_2) + 2(P_1 . P_{\nu f})\,(P_2 . P_{\nu f}) - \frac{2t}{M_W^2}\,(P_1 . P_4)(P_2 . P_4) + 4\,\frac{(P_1 . P_{\nu f})}{M_W^2}\,(P_2 . P_4)\,(P_{\nu f} . P_4)\right\}$$
$$(30)$$



$P_1 + P_2 = P_3 + P_4$

$P_{\nu f} = P_1 - P_3$

$\Rightarrow P_3 = P_1 - P_{\nu f} \quad \Rightarrow P_3^2 = (P_1 - P_{\nu f})^2$

$M_{H^\pm}^2 = m_f^2 - 2 P_1 . P_{\nu f} + t$

$\Rightarrow \boxed{P_1 . P_{\nu f} = \frac{m_f^2 + t - M_{H^\pm}^2}{2}} \approx \frac{t - M_{H^\pm}^2}{2}$ (31)

$P_2 + P_{\nu f} = P_4 \quad \Rightarrow (P_2 + P_{\nu f})^2 = P_4^2$

$m_f^2 + t + 2 P_2 . P_{\nu f} = M_W^2$

$\Rightarrow \boxed{P_2 . P_{\nu f} = \frac{M_W^2 - m_f^2 - t}{2}} \approx \frac{M_W^2 - t}{2}$ (32)

$s + t + u = 2 m_f^2 + M_{H^\pm}^2 + M_W^2 \approx M_{H^\pm}^2 + M_W^2$

$P_2 = P_4 - P_{\nu f} \quad \Rightarrow P_2^2 = (P_4 - P_{\nu f})^2$

$m_f^2 = M_W^2 + t - 2 P_4 . P_{\nu f}$

$\Rightarrow \boxed{P_4 . P_{\nu f} = \frac{M_W^2 - m_f^2 + t}{2}} \approx \frac{M_W^2 + t}{2}$ (33)

$u = (P_1 - P_4)^2 = m_f^2 + M_W^2 - 2 P_1 . P_4$

$\Rightarrow P_1 . P_4 = \frac{m_f^2 + M_W^2 - u}{2} = \frac{\cancel{m_f^2} + \cancel{M_W^2} - (2\cancel{m_f^2} + M_{H^\pm}^2 + \cancel{M_W^2} - s - t)}{2}$

$\Rightarrow \boxed{P_1 . P_4 = \frac{- m_f^2 - M_{H^\pm}^2 + s + t}{2}} \approx \frac{s + t - M_{H^\pm}^2}{2}$ (34)

$P_2 . P_4 = (P_3 + P_4 - P_1) . P_4 = P_3 . P_4 + M_W^2 - P_1 . P_4$

$P_2 . P_4 = \frac{\cancel{s} - \cancel{M_{H^\pm}^2} - M_W^2}{2} + M_W^2 + \frac{m_f^2 + \cancel{M_{H^\pm}^2} - \cancel{s} - t}{2}$

$\boxed{P_2 . P_4 = \frac{- \cancel{M_W^2} + 2 \cancel{M_W^2} + m_f^2 - t}{2} = \frac{M_W^2 + m_f^2 - t}{2}} \approx \frac{M_W^2 - t}{2} \approx P_2 . P_{\nu f}$ (35)

$$|M_4|^2 = \frac{g^4 m_f^2 |C_\nu|^2}{2M_W^2} \left\{ -t\,(P_1 \cdot P_2) + 2(P_2 \cdot P_4)\left[(P_1 \cdot P_{\nu f}) - \frac{t}{M_W^2}(P_1 \cdot P_4) + \frac{2}{M_W^2}(P_1 \cdot P_{\nu f})(P_4 \cdot P_{\nu f}) \right] \right\}$$

$$|M_4|^2 = \frac{g^4 m_f^2 |C_\nu|^2}{2M_W^2} \left\{ -t\,\frac{s}{2} + \cancel{2}\frac{(M_W^2 - t)}{\cancel{2}}\left[\frac{(t - M_{H^\pm}^2)}{2} - \frac{t}{M_W^2}\frac{(s + t - M_{H^\pm}^2)}{2} + \frac{\cancel{2}}{M_W^2}\frac{(t - M_{H^\pm}^2)}{\cancel{2}}\frac{(t + M_W^2)}{2} \right] \right\}$$

$$|M_4|^2 = \frac{g^4 m_f^2 |C_\nu|^2}{4M_W^2} \left\{ -st + \frac{(M_W^2 - t)}{M_W^2}\left[t\cancel{M_W^2} - M_{H^\pm}^2\cancel{M_W^2} - st - \cancel{t^2} + t\cancel{M_{H^\pm}^2} + \cancel{t^2} + t\cancel{M_W^2} - t\cancel{M_{H^\pm}^2} - M_{H^\pm}^2\cancel{M_W^2} \right] \right\}$$

$$|M_4|^2 = \frac{g^4 m_f^2 |C_\nu|^2}{4M_W^2} \left\{ -st + \frac{(M_W^2 - t)}{M_W^2}\left[2t M_W^2 - 2M_{H^\pm}^2 M_W^2 - st \right] \right\}$$

$$|M_4|^2 = \frac{g^4 m_f^2 |C_\nu|^2}{4M_W^4} \left\{ -st\cancel{M_W^2} + 2t\cancel{M_W^4} - 2M_{\cancel{W}}^4 M_{H^\pm}^2 - s\cancel{t}M_W^2 - 2t^{\cancel{2}} M_W^2 + 2t \cancel{M_{H^\pm}^2} M_W^2 + st^2 \right\}$$

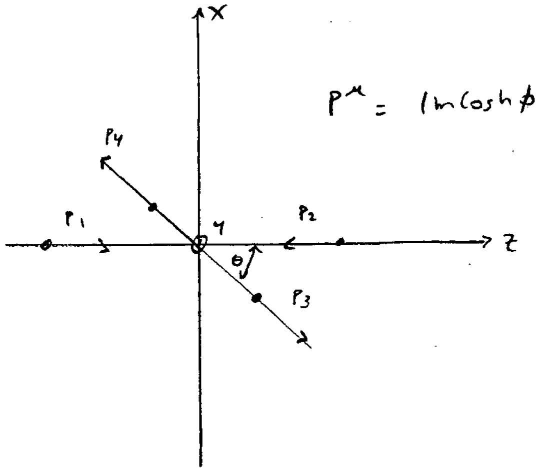
$$|M_4|^2 = \frac{g^4 m_f^2 |C_\nu|^2}{4M_W^4} \left\{ -2st M_W^2 + 2t M_W^4 - 2M_W^4 M_{H^\pm}^2 - 2t^2 M_W^2 + 2t M_{H^\pm}^2 M_W^2 + st^2 \right\}$$

but $s = M_W^2 + M_{H^\pm}^2 - t - u$

$$\Rightarrow -2st M_W^2 = -2t M_W^2 (M_W^2 + M_{H^\pm}^2 - t - u)$$

$$|M_4|^2 = \frac{g^4 m_f^2 |C_\nu|^2}{4M_W^4} \left\{ -\xcancel{2t} M_W^4 - 2t\xcancel{M_W^2} M_{H^\pm}^2 + \xcancel{2t^2} M_W^2 + 2M_W^2 ut + \xcancel{2t} M_W^4 - 2M_{\xcancel{W}}^4 M_{H^\pm}^2 - 2t^2 \xcancel{M_W^2} + 2t \xcancel{M_{H^\pm}^2} M_W^2 + s\xcancel{t}^2 \right\}$$

$$|M_4|^2 = \frac{g^4 m_f^2 |C_\nu|^2}{4M_W^4} \left\{ 2M_W^2 ut - 2M_W^4 M_{H^\pm}^2 + st^2 \right\} \qquad (36)$$



$$P^\mu = (m\cosh\phi, -m\sin\theta\,\sinh\phi, 0, m\cos\theta\,\sinh\phi)$$

$$P_1 = (m_f\cosh\phi_1, 0, 0, m_f\sinh\phi_1) \qquad (37)$$

$$P_2 = (m_f\cosh\phi_2, 0, 0, -m_f\sinh\phi_2) \qquad (38)$$

$$P_3 = (M_{H^\pm}\cosh\phi_3, -M_{H^\pm}\sin\theta\,\sinh\phi_3, 0, M_{H^\pm}\cos\theta\,\sinh\phi_3) \qquad (39)$$

$$P_4 = (M_W \cosh\phi_4,\ M_W \sin\theta \sinh\phi_4,\ 0,\ -M_W \cos\theta \sinh\phi_4) \qquad (40)$$

$$P_1 + P_2 = P_3 + P_4$$

$$\Rightarrow \quad \boxed{M_{H^\pm} \sinh\phi_3 = M_W \sinh\phi_4} \qquad (41)$$

$$\Rightarrow \quad \boxed{\sinh\phi_1 = \sinh\phi_2} \quad \Rightarrow \phi_1 = \phi_2 \qquad (42)$$

$$S = (P_1 + P_2)^2 = m_f^2 (\cosh\phi_1 + \cosh\phi_2)^2$$

$$S = 4 m_f^2 \cosh^2\phi_1$$

$$\boxed{\cosh\phi_1 = \frac{\sqrt{S}}{2m_f}} \quad ; \quad \cosh^2\phi_1 - \sinh^2\phi_1 = 1 \qquad (43)$$

$$\boxed{\sinh\phi_1 = \left(\frac{S}{4m_f^2} - 1\right)^{1/2} = \frac{(S-4m_f^2)^{1/2}}{2m_f}} \qquad (44)$$

$$t = (P_1 - P_3)^2 = (m_f \cosh\phi_1 - M_{H^\pm}\cosh\phi_3,\ M_{H^\pm}\sin\theta \sinh\phi_3,\ 0,\ m_f \sinh\phi_1 - M_{H^\pm}\cos\theta \sinh\phi_3)^2$$

$$t = m_f^2 \cancel{\cosh^2}\phi_1 - 2m_f M_{H^\pm} \cosh\phi_1 \cosh\phi_3 + M_{H^\pm}^2 \cosh^2\phi_3 - M_{H^\pm}^2 \sin^2\theta \sinh^2\phi_3 - m_f^2 \cancel{\sinh^2}\phi_1$$
$$+ 2 m_f M_{H^\pm} \cos\theta \sinh\phi_1 \sinh\phi_3 - M_{H^\pm}^2 \cos^2\theta \sinh^2\phi_3 \quad \swarrow\!\!\nearrow - M_{H^\pm}^2 \sinh^2\phi_3$$

$$t = m_f^2 + M_{H^\pm}^2 + 2 m_f M_{H^\pm} (\cos\theta \sinh\phi_1 \sinh\phi_3 - \cosh\phi_1 \cosh\phi_3)$$

$$t = m_f^2 + M_{H^\pm}^2 + M_{H^\pm} \cos\theta \sinh\phi_3 (S - 4m_f^2)^{1/2} - M_{H^\pm} \cosh\phi_3 \sqrt{S} \qquad (45)$$

on the other hand

$$S = (P_3 + P_4)^2 = (M_{H^\pm} \cosh\phi_3 + M_W \cosh\phi_4)^2 \qquad (46)$$

$$M_{H^\pm} \sinh\phi_3 = M_W \sinh\phi_4 \Rightarrow M_{H^\pm}^2 \sinh^2\phi_3 = M_W^2 \sinh^2\phi_4$$

$$M_{H^\pm}^2 (\cosh^2\phi_3 - 1) = M_W^2 (\cosh^2\phi_4 - 1)$$

$$\cosh\phi_4 = \left(\frac{M_{H^\pm}^2}{M_W^2} (\cosh^2\phi_3 - 1) + 1\right)^{1/2} \qquad (47)$$

$$S = M_{H^\pm}^2 \cosh^2\phi_3 + 2 M_{H^\pm} M_W \cosh\phi_3 \cosh\phi_4 + M_W^2 \cosh^2\phi_4$$

$$S = M_{H^\pm}^2 \cosh^2\phi_3 + 2 M_{H^\pm} M_W \cosh\phi_3 \left(\frac{M_{H^\pm}^2}{M_W^2}(\cosh^2\phi_3 - 1) + 1\right)^{1/2} + M_W^2 \left[\frac{M_{H^\pm}^2}{M_W^2}(\cosh^2\phi_3 - 1) + 1\right]$$

$$S = 2M_{H^\pm}^2 \cosh^2\phi_3 - M_{H^\pm}^2 + M_W^2 + 2 M_{H^\pm} M_W \cosh\phi_3 \left(\frac{M_{H^\pm}^2}{M_W^2}(\cosh^2\phi_3 - 1) + 1\right)^{1/2}$$

$$(S - 2M_{H^\pm}^2 \cosh^2\phi_3 + M_{H^\pm}^2 - M_W^2)^2 = 4 M_{H^\pm}^2 M_W^2 \cosh^2\phi_3 \left(\frac{M_{H^\pm}^2}{M_W^2} \cosh^2\phi_3 - \frac{M_{H^\pm}^2}{M_W^2} + 1\right)$$

$$s^2 + 4 M_{H^\pm}^4 \cancel{\cosh^4\phi_3} + M_{H^\pm}^4 + M_W^4 - 4 s M_{H^\pm}^2 \cosh^2\phi_3 + 2 s M_{H^\pm}^2 - 2 s M_W^2 - 4 M_{H^\pm}^4 \cancel{\cosh^2\phi_3}$$

$$+ 4 M_{H^\pm}^2 \cancel{M_W^2} \cosh^2\phi_3 - 2 M_{H^\pm}^2 M_W^2 = 4 M_{H^\pm}^4 \cancel{\cosh^4\phi_3} - 4 M_{H^\pm}^4 \cancel{\cosh^2\phi_3} + 4 M_{H^\pm}^2 \cancel{M_W^2} \cosh^2\phi_3$$

$$\cosh^2\phi_3 = \frac{s^2 + M_{H^\pm}^4 + M_W^4 + 2 s M_{H^\pm}^2 - 2 s M_W^2 - 2 M_{H^\pm}^2 M_W^2}{4 s M_{H^\pm}^2}$$

$$\boxed{\cosh^2\phi_3 = \frac{(s + M_{H^\pm}^2 - M_W^2)^2}{4 s M_{H^\pm}^2}} \Rightarrow \cosh\phi_3 = \frac{s + M_{H^\pm}^2 - M_W^2}{2\sqrt{s}\, M_{H^\pm}} \qquad (48)$$

$$\sinh\phi_3 = \left(\frac{(s + M_{H^\pm}^2 - M_W^2)^2}{4 s M_{H^\pm}^2} - 1 \right)^{1/2}$$

$$\boxed{\sinh\phi_3 = \frac{\lambda^{1/2}(s, M_{H^\pm}^2, M_W^2)}{2\sqrt{s}\, M_{H^\pm}}} \qquad \lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$$

(49)

$$\cosh\phi_4 = \left(\frac{\cancel{M_{H^\pm}^2}}{M_W^2} \left(\frac{\lambda(s, M_{H^\pm}^2, M_W^2)}{4 s \cancel{M_{H^\pm}^2}} \right) + 1 \right)^{1/2}$$

$$\boxed{\cosh\phi_4 = \left(\frac{\lambda(s, M_{H^\pm}^2, M_W^2)}{4 s M_W^2} + 1 \right)^{1/2}} \qquad (50)$$

$$t = m_f^2 + M_{H^\pm}^2 - \cancel{M_{H^\pm}} \frac{\sqrt{s}\,(s + M_{H^\pm}^2 - M_W^2)}{2\sqrt{s}\, M_{H^\pm}} + \cancel{M_{H^\pm}} \frac{\cos\theta\, \lambda^{1/2}(s, M_{H^\pm}^2, M_W^2)}{2\sqrt{s}\, \cancel{M_{H^\pm}}} (s - 4m_f^2)^{1/2}$$

$$\boxed{t = \frac{2m_f^2 + M_{H^\pm}^2 + M_W^2 - s}{2} + \frac{1}{2} \cos\theta\, \lambda^{1/2}(s, M_{H^\pm}^2, M_W^2) \frac{(s - 4m_f^2)^{1/2}}{\sqrt{s}}} \qquad (51)$$

$$s + t + U = 2m_f^2 + M_{H^\pm}^2 + M_W^2$$

$$U = 2\cancel{m_f}^2 + M_{H^\pm}^{\cancel{2}} + \cancel{M}_W^2 - s - \frac{(2\cancel{m_f}^2 + M_{W^\pm}^{\cancel{2}} + M_W^2 - s)}{2} - \frac{1}{2}\cos\theta\, \lambda^{1/2} \frac{(s - 4m_f^2)^{1/2}}{\sqrt{s}}$$

$$\boxed{U = \frac{2m_f^2 + M_{H^\pm}^2 + M_W^2 - s}{2} - \frac{1}{2} \cos\theta\, \lambda^{1/2}(s, M_{H^\pm}^2, M_W^2) \frac{(s - 4m_f^2)^{1/2}}{\sqrt{s}}} \qquad (52)$$

if $m_f \approx 0$

$$t = \frac{M_{H^\pm}^2 + M_W^2 - s}{2} + \frac{1}{2}\cos\theta\,\lambda^{1/2}(s, M_{H^\pm}^2, M_W^2) \qquad (53)$$

$$u = \frac{M_{H^\pm}^2 + M_W^2 - s}{2} - \frac{1}{2}\cos\theta\,\lambda^{1/2}(s, M_{H^\pm}^2, M_W^2) \qquad (54)$$

$$ut = \frac{1}{4}(M_{H^\pm}^2 + M_W^2 - s)^2 - \frac{1}{4}\lambda(s, M_{H^\pm}^2, M_W^2)\cos^2\theta$$

$$ut = \frac{1}{4}(\cancel{M_{H^\pm}^4} + \cancel{M_W^4} + \cancel{s^2} + 2M_{H^\pm}^2 M_W^2 - \cancel{2sM_{H^\pm}^2} - \cancel{2sM_W^2}) - \frac{1}{4}(\cancel{s^2} + \cancel{M_{H^\pm}^4} + \cancel{M_W^4} - \cancel{2sM_{H^\pm}^2} - \cancel{2sM_W^2}$$
$$- 2\cancel{M_{H^\pm}^2 M_W^2}) + \frac{1}{4}\lambda\sin^2\theta$$

$$\boxed{ut = M_{H^\pm}^2 M_W^2 + \frac{1}{4}\lambda\sin^2\theta} \qquad \text{(introducing in } |M_4|^2) \qquad (55)$$

$$\Rightarrow |M_4|^2 = \frac{g^4 m_f^2 |C_V|^2}{4M_W^4}\left\{2\cancel{M_W^4} M_{H^\pm}^2 + \frac{1}{2}\lambda\sin^2\theta M_W^2 - 2\cancel{M_W^4 M_{H^\pm}^2} + st^2\right\}$$

$$\boxed{|M_4|^2 = \frac{s g^4 m_f^2 |C_V|^{2}}{4M_W^4}\left\{\frac{\lambda(s, M_{H^\pm}^2, M_W^2)\sin^2\theta\, M_W^2}{2s} + t^2\right\}} \qquad (56)$$

$$\overset{\nearrow \text{real } \#}{}$$

$$M_H^+ M_3 = \sum_{s,\lambda}\left(\frac{g^2 m_f}{4M_W}\right)^2 C_H^*(p_1+p_2+p_3)^\mu \varepsilon_{\mu 4}(\bar{U}_1 V_2) .\ C_A (p_1+p_2+p_3)^\nu \varepsilon^*_{\nu 4}(\bar{V}_2 \gamma^5 U_1) \qquad (57)$$

$$M_H^+ M_3 = \left(\frac{g^2 m_f}{4M_W}\right)^2 C_H^* C_A (p_1+p_2+p_3)^\mu (p_1+p_2+p_3)^\nu \left(-g_{\mu\nu} + \frac{p_{4\mu} p_{4\nu}}{M_W^2}\right)\mathrm{Tr}\left[(\not{p}_2 - m_f)\gamma^5(\not{p}_1 + m_f)\right]$$

$$\mathrm{Tr}\left[(\not{p}_2 - m_f)\gamma^5(\not{p}_1 + m_f)\right] = \mathrm{Tr}\left[(\not{p}_2\gamma^5 - m_f\gamma^5)(\not{p}_1 + m_f)\right]$$

$$= \mathrm{Tr}\left[\not{p}_2\gamma^5\not{p}_1 + m_f\not{p}_2\gamma^5 - m_f\gamma^5\not{p}_1 - m_f^2\gamma^5\right]$$

$$= 0 \qquad (\text{because :· } \mathrm{Tr}(\gamma^5\not{a}\not{b}) = 0,\ \mathrm{Tr}(\text{odd \# of } \gamma\text{'s} = 0,\ \mathrm{Tr}(\gamma^5) = 0) \qquad (58)$$

$$\Rightarrow \boxed{M_H^+ M_3 = 0} \quad \text{and} \quad \boxed{M_3^+ M_H = 0} \qquad (59)$$

$$M_H{}^+ M_4 = \sum_{s,\lambda} - \left|\frac{g^2 m_f}{4M_W}\right|^2 \frac{C_H{}^*}{2} (P_1+P_2+P_3)^\mu \, \varepsilon_{\mu 4} \, (\bar{U}_1 V_2) \, C_\nu \, (\bar{V}_2 \gamma^\nu (1-\gamma^5) \not{P}_{\nu f} (1+\gamma^5) U_1) \, \varepsilon^*_{\nu 4} \qquad (60)$$

$$M_H{}^+ M_4 = -\left(\frac{g^2 m_f}{4M_W}\right)^2 \frac{C_H{}^* C_\nu}{2} (P_1+P_2+P_3)^\mu \left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W{}^2}\right) \sum_s (\bar{U}_1 \; V_2)(\bar{V}_2 \gamma^\nu (1-\gamma^5) \not{P}_{\nu f} (1+\gamma^5) U_1)$$

$$M_H{}^+ M_4 = -\left(\frac{g^2 m_f}{4M_W}\right)^2 \frac{C_H{}^* C_\nu}{2} (P_1+P_2+P_3)^\mu \left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W{}^2}\right) Tr\left[(\not{P}_2 - m_f)\gamma^\nu (1-\gamma^5) \not{P}_{\nu f} (1+\gamma^5)(\not{P}_1 + m_f)\right] \qquad (61)$$

$$Tr\left[(\not{P}_2 - m_f)\gamma^\nu (1-\gamma^5) \not{P}_{\nu f} (1+\gamma^5)(\not{P}_1 + m_f)\right] \qquad \left((1-\gamma^5)^2 = 2(1-\gamma^5)\right)$$

$$= 2\,Tr\left[(\not{P}_2 - m_f)\gamma^\nu (1-\gamma^5) \not{P}_{\nu f} (\not{P}_1 + m_f)\right]$$

$$= 2\,Tr\left[(\not{P}_2 - m_f)(\gamma^\nu - \gamma^\nu\gamma^5)(\not{P}_{\nu f}\not{P}_1 + \not{P}_{\nu f} m_f)\right]$$

$$= 2\,Tr\left[(\not{P}_2\gamma^\nu - \not{P}_2\gamma^\nu\gamma^5 - m_f\gamma^\nu + m_f\gamma^\nu\gamma^5)(\not{P}_{\nu f}\not{P}_1 + \not{P}_{\nu f} m_f)\right]$$

$$= 2\,Tr\left[\not{P}_2\gamma^\nu\not{P}_{\nu f}\not{P}_1 + m_f \not{P}_2 \cancel{\gamma^\nu} \not{P}_{\nu f} - \not{P}_2 \gamma^\nu \gamma^5 \not{P}_{\nu f} \not{P}_1 - \not{P}_2 \gamma^\nu \cancel{\gamma^5} \not{P}_{\nu f} m_f - m_f \cancel{\gamma^\nu} \not{P}_{\nu f} \not{P}_1\right.$$
$$\left. - m_f^2 \gamma^\nu \not{P}_{\nu f} + m_f \cancel{\gamma^\nu} \gamma^5 \not{P}_{\nu f} \not{P}_1 + m_f{}^2 \gamma^\nu \cancel{\gamma^5} \not{P}_{\nu f}\right]$$

$$= 2\,Tr\left[\not{P}_2\gamma^\nu\not{P}_{\nu f}\not{P}_1 - \not{P}_2\gamma^\nu\gamma^5\not{P}_{\nu f}\not{P}_1 - m_f{}^2\gamma^\nu\not{P}_{\nu f}\right]$$

$$(Tr(\gamma^5 \not{a} \not{b}) = 0)$$

$$= 2\,Tr\left[\gamma^\alpha P_{1\alpha} \not{P}_2 \gamma^\nu \not{P}_{\nu f}\right] - 2\,Tr\left[\gamma^5 \not{P}_2 \gamma^\nu \not{P}_{\nu f} \not{P}_1\right] - 2m_f{}^2\,Tr\left[\gamma^\nu\gamma^\alpha P_{\nu f\alpha}\right]$$

$$= 2P_{1\alpha} Tr\left[\gamma^\alpha \not{P}_2 \gamma^\nu \not{P}_{\nu f}\right] - 2\,Tr\left[\gamma^5\gamma^\alpha\gamma^\nu\gamma^\rho\gamma^\sigma\right] P_{2\alpha} P_{\nu f\rho} P_{1\sigma} - 2m_f{}^2 P_{\nu f\alpha}\, Tr\left[\gamma^\nu\gamma^\alpha\right]$$

$$= 2P_{1\alpha}\, 4\left[P_2{}^\alpha P_{\nu f}^\nu + P_2^\nu P_{\nu f}^\alpha - (P_2 \cdot P_{\nu f}) g^{\alpha\nu}\right] - 2(-4i\,\varepsilon^{\alpha\nu\rho\sigma}) P_{2\alpha} P_{\nu f\rho} P_{1\sigma} - 2m_f{}^2 P_{\nu f\alpha}\, 4 g^{\nu\alpha}$$

$$= 8P_{1\alpha}\left[P_2{}^\alpha P_{\nu f}^\nu + P_2^\nu P_{\nu f}^\alpha - (P_2 \cdot P_{\nu f}) g^{\alpha\nu}\right] + 8i\,\varepsilon^{\alpha\nu\rho\sigma} P_{2\alpha}(P_{1\rho} - P_{3\rho}) P_{1\sigma} - 8m_f{}^2 P_{\nu f}^\nu$$

but $\varepsilon^{\alpha\nu\rho\sigma} P_{1\rho} P_{1\sigma} = 0$

$$Tr[\,] = 8\left[(P_1 \cdot P_2) P_{\nu f}^\nu + P_2^\nu (P_1 \cdot P_{\nu f}) - (P_2 \cdot P_{\nu f}) P_1{}^\nu\right] - 8i\,\varepsilon^{\alpha\nu\rho\sigma} P_{2\alpha} P_{3\rho} P_{1\sigma} - 8m_f{}^2 P_{\nu f}^\nu \qquad (62)$$

$\Rightarrow$

$$M_H^+ M_4 = -\left(\frac{g^2 m_f}{4M_W}\right)^2 \frac{C_H{}^* C_\nu}{2} 8\left(-(P_1+P_2+P_3)_\nu + \frac{((P_1+P_2+P_3)\cdot P_4) P_{4\nu}}{M_W{}^2}\right)\left[(P_1 \cdot P_2) P_{\nu f}^\nu + (P_1 \cdot P_{\nu f}) P_2{}^\nu\right.$$
$$\left. - (P_2 \cdot P_{\nu f}) P_1{}^\nu - i\,\varepsilon^{\alpha\nu\rho\sigma} P_{2\alpha} P_{3\rho} P_{1\sigma} - m_f{}^2 P_{\nu f}^\nu\right] \qquad (63)$$

but $$(P_1+P_2+P_3)_\nu\, \varepsilon^{\alpha\nu\rho\sigma} P_{2\alpha} P_{3\rho} P_{1\sigma} = \varepsilon^{\alpha\nu\rho\sigma} \cancelto{0}{P_{1\nu} P_{2\alpha}} P_{3\rho} P_{1\sigma}$$

$$+ \varepsilon^{\alpha\nu\rho\sigma} P_{2\nu} \cancelto{0}{P_{2\alpha}} P_{3\rho} P_{1\sigma} + \varepsilon^{\alpha\nu\rho\sigma} P_{3\nu} \cancelto{0}{P_{2\alpha} P_{3\rho}} P_{1\sigma} = 0 \qquad (64)$$

and $$\varepsilon^{\alpha\nu\rho\sigma} P_{2\alpha} P_{3\rho} P_{1\sigma} P_{4\nu} = \varepsilon^{\alpha\nu\rho\sigma} P_{2\alpha} P_{3\rho} P_{1\sigma} (P_{1\nu} + P_{2\nu} - P_{3\nu})$$

$$= \varepsilon^{\alpha\nu\rho\sigma} P_{2\alpha} \cancelto{0}{P_{3\rho}} P_{1\sigma} P_{1\nu} + \varepsilon^{\alpha\nu\rho\sigma} P_{2\alpha} \cancelto{0}{P_{3\rho}} P_{1\sigma} P_{2\nu} - \cancelto{0}{\varepsilon^{\alpha\nu\rho\sigma}} P_{2\alpha} P_{3\rho} P_{1\sigma} P_{3\nu} \qquad (65)$$

$$\Rightarrow M_H^{+} M_4 = -\frac{g^4 m_f^2}{4 M_W^2} C_H^{*} C_\nu \Big[-(P_1\cdot P_2)\big((P_1+P_2+P_3)\cdot P_{\nu f}\big) - (P_1\cdot P_{\nu f})\big((P_1+P_2+P_3)\cdot P_2\big)$$

$$+ (P_2\cdot P_{\nu f})\big((P_1+P_2+P_3)\cdot P_1\big) + m_f^2\big((P_1+P_2+P_3)\cdot P_{\nu f}\big) + \frac{1}{M_W^2}(P_1\cdot P_2)\big((P_1+P_2+P_3)\cdot P_4\big)(P_4\cdot P_{\nu f})$$

$$+ \frac{1}{M_W^2}(P_1\cdot P_{\nu f})(P_2\cdot P_4)\big((P_1+P_2+P_3)\cdot P_4\big) - \frac{1}{M_W^2}(P_2\cdot P_{\nu f})(P_1\cdot P_4)\big((P_1+P_2+P_3)\cdot P_4\big) - \frac{m_f^2}{M_W^2}(P_4\cdot P_{\nu f})\big((P_1+P_2+P_3)\cdot P_4\big)\Big] \qquad (66)$$

$$(P_1+P_2+P_3)\cdot P_{\nu f} = (P_1+P_2+P_3)\cdot(P_1-P_3) = \cancel{m_f}^2 - (\cancel{P_1}\cdot P_3) + (P_1\cdot P_2) - (P_2\cdot P_3) + (\cancel{P_1}\cdot P_3) - \cancel{M_{H^\pm}^2}$$

$$(P_1+P_2+P_3)\cdot P_{\nu f} = \cancel{m_f}^2 - M_{H^\pm}^2 + \frac{s}{2} - \cancel{m_f}^2 - (P_2\cdot P_3) = -M_{H^\pm}^2 + \frac{s}{2} - (P_2\cdot P_3)$$

$$U = |P_1 - P_4|^2 = (P_3 - P_2)^2 = m_f^2 + M_{H^\pm}^2 - 2(P_2\cdot P_3)$$

$$(P_2\cdot P_3) = \frac{m_f^2 + M_{H^\pm}^2 - U}{2} = \frac{\cancel{m_f^2} + \cancel{M_{H^\pm}^2} - (2\cancel{m_f}^2 + \cancel{M_{H^\pm}^2} + M_W^2 - s - t)}{2}$$

$$\boxed{(P_2\cdot P_3) = \frac{-m_f^2 - M_W^2 + s + t}{2}} \qquad (67)$$

$$\Rightarrow (P_1+P_2+P_3)\cdot P_{\nu f} = -M_{H^\pm}^2 + \frac{\cancel{s}}{\cancel{2}} + \frac{m_f^2 + M_W^2 - \cancel{s} - t}{\cancel{2}}$$

$$\boxed{(P_1+P_2+P_3)\cdot P_{\nu f} = \frac{-2M_{H^\pm}^2 + m_f^2 + M_W^2 - t}{2}} \boxed{\approx \frac{M_W^2 - t - 2M_{H^\pm}^2}{2}} \qquad (68)$$

$$(P_1+P_2+P_3)\cdot P_2 = \frac{s}{2} - \cancel{m_f}^2 + \cancel{m_f}^2 + \frac{(-m_f^2 - M_W^2 + s + t)}{2}$$

$$\boxed{(P_1+P_2+P_3)\cdot P_2 = \frac{2s - m_f^2 - M_W^2 + t}{2} \approx \frac{2s - M_W^2 + t}{2}} \qquad (69)$$

$$(P_1+P_2+P_3)\cdot P_1 = \cancel{m_f}^2 + \frac{s}{2} - \cancel{m_f}^2 + (P_1\cdot P_3)$$

$$t = (P_1 - P_3)^2 = m_f^2 + M_{H^\pm}^2 - 2(P_1\cdot P_3)$$

$$\boxed{(P_1\cdot P_3) = \frac{m_f^2 + M_{H^\pm}^2 - t}{2}} \approx \frac{M_{H^\pm}^2 - t}{2} \qquad (70)$$

$$\Rightarrow (P_1+P_2+P_3)\cdot P_1 = \frac{s}{2} + \frac{m_f^2 + M_{H^\pm}^2 - t}{2}$$

$$\boxed{(P_1+P_2+P_3)\cdot P_1 = \frac{m_f^2 + M_{H^\pm}^2 + s - t}{2} \approx \frac{M_{H^\pm}^2 + s - t}{2}} \qquad (71)$$

$$M_H^+ M_4 = \frac{-g^4 m_f^2 C_H^* C_V}{4M_W^2}\Big[-\frac{s}{2}\Big(\frac{M_W^2 - t - 2M_{H^\pm}^2}{2}\Big) - \frac{(t - M_{H^\pm}^2)}{2}\frac{(2s - M_W^2 + t)}{2} + \frac{(M_W^2 - t)}{2}\frac{(M_{H^\pm}^2 + s - t)}{2}$$

$$+ \frac{1}{M_W^2}\frac{s}{2}\frac{(M_W^2 + t)}{2}(s - M_{H^\pm}^2) + \frac{1}{M_W^2}\frac{(t - M_{H^\pm}^2)}{2}\frac{(M_W^2 - t)}{2}(s - M_{H^\pm}^2) - \frac{1}{M_W^2}\frac{(M_W^2 - t)}{2}\frac{(s + t - M_{H^\pm}^2)}{2}(s - M_{H^\pm}^2)\Big] \quad (72)$$

$$M_H^+ M_4 = \frac{-g^4 m_f^2 C_H^* C_V}{4 \times 4M_W^2}\Big[\cancel{-sM_W^2} + \cancel{st} + \cancel{2sM_{H^\pm}^2} - \cancel{2st} + \cancel{M_W^2 t} - \cancel{t^2} + \cancel{2sM_{H^\pm}^2} - \cancel{M_W^2 M_{H^\pm}^2} + \cancel{tM_{H^\pm}^2} + \cancel{M_W^2 M_{H^\pm}^2}$$

$$+ \cancel{sM_W^2} - \cancel{tM_W^2} - \cancel{tM_{H^\pm}^2} - \cancel{st} + \cancel{t^2} + \frac{(s - M_{H^\pm}^2)}{M_W^2}\big[\cancel{sM_W^2} + st + \cancel{tM_W^2} - \cancel{t^2}$$

$$- \cancel{M_W^2 M_{H^\pm}^2} + \cancel{tM_{H^\pm}^2} - \cancel{sM_W^2} - \cancel{tM_W^2} + \cancel{M_W^2 M_{H^\pm}^2} + \cancel{st} + \cancel{t^2} - \cancel{tM_{H^\pm}^2}\big]\Big]$$

$$M_H^+ M_4 = \frac{-g^4 m_f^2 C_H^* C_V}{16 M_W^2}\Big[-2st + 4sM_{H^\pm}^2 + \frac{(s - M_{H^\pm}^2)}{M_W^2}(2st)\Big]$$

$$M_H^+ M_4 = \frac{-g^4 m_f^2 C_H^* C_V}{8M_W^2}\Big[-st + 2sM_{H^\pm}^2 + \frac{s^2 t}{M_W^2} - \frac{stM_{H^\pm}^2}{M_W^2}\Big]$$

$$= \frac{-g^4 m_f^2 C_H^* C_V}{8M_W^4}\Big[-stM_W^2 + 2sM_W^2 M_{H^\pm}^2 + s^2 t - stM_{H^\pm}^2\Big]$$

$$= -\frac{g^4 s m_f^2 C_H^* C_V}{8M_W^4}\Big[-t(\cancel{s} + t + u) + 2M_W^2 M_{H^\pm}^2 + \cancel{st}\Big]$$

$$\boxed{M_H^+ M_4 = -\frac{g^4 s m_f^2 C_H^* C_V}{8M_W^4}\Big[-t^2 - ut + 2M_W^2 M_{H^\pm}^2\Big]} \quad (73)$$

$$= \frac{-g^4 s m_f^2 C_H^* C_V}{8M_W^4}\Big[-\cancel{t}^2 - M_{H^\pm}^2 M_W^2 - \frac{1}{4}\lambda\,\cancel{s}\mathrm{in}^2\theta + \cancel{2M_W^2} M_{H^\pm}^2\Big]$$

$$\boxed{M_H^+ M_4 = -\frac{g^4 s m_f^2 C_H^* C_V}{8M_W^4}\Big[-t^2 - \frac{1}{4}\lambda \sin^2\theta + M_W^2 M_{H^\pm}^2\Big]} \quad (74)$$

$$\boxed{(M_H^+ M_4)^+ = M_4^+ M_H = -\frac{g^4 s m_f^2 C_H C_V}{8M_W^4}\Big[-t^2 - \frac{1}{4}\lambda \sin^2\theta + M_W^2 M_{H^\pm}^2\Big]} \quad (75)$$

$$\boxed{M_H^+ M_4 + M_4^+ M_H = -\frac{g^4 s m_f^2 C_V}{8M_W^4} \cdot 2\,\mathrm{Re}(C_H)\Big[-t^2 - \frac{1}{4}\lambda(s, M_{H^\pm}^2, M_W^2)\sin^2\theta + M_W^2 M_{H^\pm}^2\Big]} \quad (76)$$

$$M_3^+ M_4 = +\frac{1}{2}\left(\frac{g^2 m_f}{4M_W}\right)^2 \sum_{\lambda, s} C_A^* C_\nu (P_1+P_2+P_3)^\mu \varepsilon_{\mu 4} (\bar{U}_1 \gamma^5 V_2)(\bar{V}_2 \gamma^\nu (1-\gamma^5) \not{P}_{\nu f} (1+\gamma^5) U_1) \varepsilon_{\nu 4}^* \quad (77)$$

$$= \frac{1}{2}\left(\frac{g^2 m_f}{4M_W}\right)^2 C_A^* C_\nu (P_1+P_2+P_3)^\mu \left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right) Tr\left[(\not{P}_2 - m_f)\gamma^\nu(1-\gamma^5)\not{P}_{\nu f}(1+\gamma^5)(\not{P}_1+m_f)\gamma^5\right] \quad (78)$$

$$Tr\left[(\not{P}_2 - m_f)\gamma^\nu(1-\gamma^5)\not{P}_{\nu f}(1+\gamma^5)(\not{P}_1+m_f)\gamma^5\right] = Tr\left[(\not{P}_2 - m_f)\gamma^\nu(1-\gamma^5)\not{P}_{\nu f}(1+\gamma^5)\gamma^5(-\not{P}_1+m_f)\right]$$

$$= -Tr\left[(\not{P}_2 - m_f)\gamma^\nu(1-\gamma^5)\not{P}_{\nu f}(1+\gamma^5)(\not{P}_1 - m_f)\right]$$

$$= -2Tr\left[(\not{P}_2 - m_f)\gamma^\nu(1-\gamma^5)\not{P}_{\nu f}(\not{P}_1 - m_f)\right]$$

$$= -2\,Tr\left[(\not{P}_2 - m_f)(\gamma^\nu - \gamma^\nu\gamma^5)(\not{P}_{\nu f}\not{P}_1 - m_f\not{P}_{\nu f})\right]$$

$$= -2Tr\left[(\not{P}_2\gamma^\nu - \not{P}_2\gamma^\nu\gamma^5 - m_f\gamma^\nu + m_f\gamma^\nu\gamma^5)(\not{P}_{\nu f}\not{P}_1 - m_f\not{P}_{\nu f})\right]$$

$$= -2Tr\left[\not{P}_2\gamma^\nu\not{P}_{\nu f}\not{P}_1 - \cancel{m_f\not{P}_2\gamma^\nu\not{P}_{\nu f}} - \not{P}_2\gamma^\nu\gamma^5\not{P}_{\nu f}\not{P}_1 + \cancel{m_f\not{P}_2\gamma^\nu\gamma^5\not{P}_{\nu f}}\right.$$

$$\left. - \cancel{m_f\gamma^\nu\not{P}_{\nu f}\not{P}_1} + m_f^2\gamma^\nu\not{P}_{\nu f} + \cancel{m_f\gamma^\nu\gamma^5\not{P}_{\nu f}\not{P}_1} - \cancel{m_f^2\gamma^\nu\gamma^5\not{P}_{\nu f}}\right]$$

Tr (odd # of γ's = 0)

$$= -2\,Tr\left[\not{P}_2\gamma^\nu\not{P}_{\nu f}\not{P}_1 - \not{P}_2\gamma^\nu\gamma^5\not{P}_{\nu f}\not{P}_1 + m_f^2\gamma^\nu\not{P}_{\nu f}\right]$$

$$= -2\,Tr\left[\gamma^\alpha\not{P}_2\gamma^\nu\not{P}_{\nu f}\right]P_{1\alpha} + 2\,Tr\left[\gamma^5\gamma^\alpha\gamma^\nu\gamma^\rho\gamma^\sigma\right]P_{2\alpha}P_{\nu f\rho}P_{1\sigma}$$

$$- 2m_f^2\,Tr\left[\gamma^\nu\gamma^\alpha\right]P_{\nu f\alpha}$$

$$= -8P_{1\alpha}\left[P_2^\alpha P_{\nu f}^\nu + P_2^\nu P_{\nu f}^\alpha - (P_2\cdot P_{\nu f})g^{\alpha\nu}\right] + 2(-4i\,\varepsilon^{\alpha\nu\rho\sigma})P_{2\alpha}P_{\nu f\rho}P_{1\sigma}$$

$$-2m_f^2\,4\,g^{\nu\alpha}P_{\nu f\alpha}$$

$$Tr[\ ] = -8\left[(P_1\cdot P_2)P_{\nu f}^\nu + (P_1\cdot P_{\nu f})P_2^\nu - (P_2\cdot P_{\nu f})P_1^\nu + i\,\varepsilon^{\alpha\nu\rho\sigma}P_{2\alpha}P_{\nu f\rho}P_{1\sigma} + m_f^2 P_{\nu f}^\nu\right] \quad (79)$$

$$\Rightarrow M_3^+ M_4 = -\frac{1}{2}\cdot 8\left(\frac{g^2 m_f}{4M_W}\right)^2 C_A^* C_\nu\left(-(P_1+P_2+P_3)_\nu + \frac{((P_1+P_2+P_3)\cdot P_4)P_{4\nu}}{M_W^2}\right)\left[(P_1\cdot P_2)P_{\nu f}^\nu + (P_1\cdot P_{\nu f})P_2^\nu\right.$$

$$\left.-(P_2\cdot P_{\nu f})P_1^\nu + i\,\varepsilon^{\alpha\nu\rho\sigma}P_{2\alpha}P_{\nu f\rho}P_{1\sigma} + m_f^2 P_{\nu f}^\nu\right] \quad (80)$$

but $(P_1+P_2+P_3)_\nu\, \varepsilon^{\alpha\nu\rho\sigma} P_{2\alpha} \underset{\substack{\downarrow \\ (P_1-P_3)_\rho}}{P_{\nu f\rho}} P_{1\sigma} = 0$

and $\underset{\substack{\downarrow \\ (P_1+P_2-P_3)_\nu}}{P_{4\nu}}\, \varepsilon^{\alpha\nu\rho\sigma} P_{2\alpha} \underset{\substack{\downarrow \\ (P_1-P_3)_\rho}}{P_{\nu f\rho}} P_{1\sigma} = 0$

$$M_3^+ M_4 = -4\left(\frac{g^2 m_f}{4M_W}\right)^2 C_A^* C_V \Big[-(P_1\cdot P_2)\big((P_1+P_2+P_3)\cdot P_{\nu f}\big) - (P_1\cdot P_{\nu f})\big((P_1+P_2+P_3)\cdot P_2\big) + (P_2\cdot P_{\nu f})\big((P_1+P_2+P_3)\cdot P_1$$

$$- m_f^2 (P_1+P_2+P_3)\cdot P_{\nu f} + \frac{1}{M_W^2}\big((P_1+P_2+P_3)\cdot P_4\big)(P_1\cdot P_2)(P_4\cdot P_{\nu f}) + \frac{1}{M_W^2}\big((P_1+P_2+P_3)\cdot P_4\big)(P_1\cdot P_{\nu f})(P_2\cdot P_4)$$

$$-\frac{1}{M_W^2}\big((P_1+P_2+P_3)\cdot P_4\big)(P_2\cdot P_{\nu f})(P_1\cdot P_4) + \frac{m_f^2}{M_W^2}\big((P_1+P_2+P_3)\cdot P_4\big)(P_4\cdot P_{\nu f})\Big] \quad (81)$$

neglecting terms with m_f^2

$$M_3^+ M_4 = -\frac{g^4 m_f^2 s}{8M_W^4} C_A^* C_V \left[-t^2 - ut + 2M_W^2 M_{H^\pm}^2\right]$$

$$\boxed{M_3^+ M_4 = -\frac{g^4 m_f^2 s}{8M_W^4} C_A^* C_V \left[-t^2 - \frac{1}{4}\lambda\sin^2\theta + M_W^2 M_{H^\pm}^2\right]} \quad (82)$$

$$\boxed{(M_3^+ M_4)^+ = M_4^+ M_3 = -\frac{g^4 m_f^2 s\, C_A C_V}{8M_W^4}\left[-t^2 - \frac{1}{4}\lambda(s, M_{H^\pm}^2, M_W^2)\sin^2\theta + M_V^2 M_{H^\pm}^2\right]} \quad (83)$$

$$\Rightarrow \boxed{M_3^+ M_4 + M_4^+ M_3 = -\frac{g^4 s\, m_f^2}{8M_W^4} C_V . 2\,\mathrm{Re}(C_A)\left[-t^2 - \frac{1}{4}\lambda(s, M_{H^\pm}^2, M_W^2)\sin^2\theta + M_W^2 M_{H^\pm}^2\right]} \quad (84)$$

$$\therefore \overline{|M|^2} = \frac{g^4 m_f^2 s}{32M_W^4}\Big\{ |C_H|^2\lambda(s, M_{H^\pm}^2, M_W^2) + |C_A|^2\lambda(s, M_{H^\pm}^2, M_W^2) + 2C_V^2\left[\frac{\lambda(s, M_{H^\pm}^2, M_W^2)\sin^2\theta M_W^2}{2s} + t^2\right.$$

$$-2C_V\,\mathrm{Re}(C_A + C_H)\left[-t^2 - \frac{1}{4}\lambda(s, M_{H^\pm}^2, M_W^2)\sin^2\theta + M_W^2 M_{H^\pm}^2\right]\Big\}$$

$$\boxed{\begin{aligned}\overline{|M|^2} = \frac{g^4 m_f^2 s}{32M_W^4}\Big\{ &\left[|C_H|^2 + |C_A|^2\right]\lambda(s, M_{H^\pm}^2, M_W^2) + 2C_V^2\left[\frac{\lambda(s, M_{H^\pm}^2, M_W^2)\sin^2\theta M_W^2}{2s} + t^2\right] \\ &-2C_V\,\mathrm{Re}(C_A + C_H)\left[-t^2 - \frac{1}{4}\lambda(s, M_{H^\pm}^2, M_W^2)\sin^2\theta + M_W^2 M_{H^\pm}^2\right]\Big\}\end{aligned}}$$

(85)

(37) and (44) $\Rightarrow$

$$\boxed{|\vec{P}_1| = m_f \sinh\phi_1 = \frac{(s - 4m_f^2)^{1/2}}{2} \approx \frac{s^{1/2}}{2}} \quad (86)$$

(39) and (49)

$$\boxed{|\vec{P}_3| = M_{H^\pm}\sinh\phi_3 = \frac{\lambda^{1/2}(s, M_{H^\pm}^2, M_W^2)}{2\sqrt{s}}} \quad (87)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{64\pi^2 s}\frac{P_f}{P_i}\overline{|M|^2} = \frac{1}{64\pi^2 s}\frac{|\vec{P}_3|}{|\vec{P}_1|}\overline{|M|^2} \quad (88)$$

$$\Rightarrow \quad I = -\frac{2}{b^2} + \int_{-1}^{1} \frac{\frac{2a}{b^2}}{(a+bx)}\,dx + \int_{-1}^{1} \frac{\left(1-\frac{a^2}{b^2}\right)}{(a+bx)^2}\,dx$$

$$I = -\frac{2}{b^2} + \frac{2a}{b^2}\,\frac{1}{b}\,\ln|a+bx|\Big|_{-1}^{1} - \frac{1}{b}\cdot\left(\frac{1}{a+bx}\right)\Big|_{-1}^{1}\cdot\left(1-\frac{a^2}{b^2}\right)$$

$$I = -\frac{2}{b^2} + \frac{2a}{b^3}\ln\left|\frac{a+b}{a-b}\right| - \frac{1}{b^3}(b^2-a^2)\left[\frac{1}{a+b} - \frac{1}{a-b}\right]$$

$$I = -\frac{2}{b^2} + \frac{2a}{b^3}\ln\left|\frac{a+b}{a-b}\right| + \frac{(\cancel{a-b})(\cancel{b+a})}{b^3}\left(\frac{-2b}{(\cancel{a-b})(\cancel{a+b})}\right)$$

$$I = -\frac{4}{b^2} + \frac{2a}{b^3}\ln\left|\frac{a+b}{a-b}\right|$$

$$\Rightarrow \quad \boxed{\int_{-1}^{1}\frac{\sin^2\theta\, d\cos\theta}{(a+b\cos\theta)^2} = \frac{2}{b^2}\left[-2 + \frac{a}{b}\ln\left|\frac{a+b}{a-b}\right|\right]} \qquad (91)$$

$$\Rightarrow \quad \int_{-1}^{1}\frac{\sin^2\theta\, d\cos\theta}{t^2} = \;?$$

$$a = \frac{M_H^2 + M_W^2 - s}{2} \;; \qquad b = \frac{1}{2}\lambda^{1/2}$$

$$\therefore \quad \boxed{\int_{-1}^{1}\frac{\sin^2\theta\, d\cos\theta}{t^2} = \frac{8}{\lambda}\left[-2 + \frac{(M_H^2+M_W^2-s)}{\lambda^{1/2}}\ln\left|\frac{M_H^2+M_W^2-s+\lambda^{1/2}}{M_H^2+M_W^2-s-\lambda^{1/2}}\right|\right]} \qquad (92)$$

$$I^* = \int_{-1}^{1}\frac{\sin^2\theta\, d\cos\theta}{(a+b\cos\theta)} = \int_{-1}^{1}\frac{(1-\cos^2\theta)\, d\cos\theta}{(a+b\cos\theta)} = \int_{-1}^{1}\frac{(1-x^2)\,dx}{(a+bx)}$$

$$\begin{array}{lll|l}
-\cancel{x^2} & + & 1 & bx + a \\ \cline{4-4}
+\cancel{x^2} & + & x\frac{a}{b} & -\frac{x}{b} + \frac{a}{b^2} \\ \cline{1-3}
 & & \cancel{x}\frac{a}{b} + 1 & \\
 & & -\cancel{x}\frac{a}{b} - \frac{a^2}{b^2} & \\ \cline{3-3}
 & & 1 - a^2/b^2 &
\end{array}$$

$$\Rightarrow \quad \frac{(1-x^2)}{(a+bx)} = -\frac{x}{b} + \frac{a}{b^2} + \frac{(b^2-a^2)}{b^2(a+bx)}$$

Defining: $$\boxed{\ln\left|\frac{M_{H^\pm}^2+M_W^2-s+\lambda^{1/2}}{M_{H^\pm}^2+M_W^2-s-\lambda^{1/2}}\right| \equiv f\,(s, M_{H^\pm}^2, M_W^2)} \qquad (99)$$

$\Rightarrow$

$$\sigma = \frac{1}{64\pi^2 s}\,\frac{\lambda^{1/2} g^4 m_f^2}{32 M_W^4}\left\{ 4\pi\lambda\left[|C_H|^2+|C_A|^2\right] + \frac{\tan^2\beta\,\cancel{\lambda}\,M_W^2}{s}\,\frac{8}{\cancel{\lambda}}\left[-2+\frac{(M_{H^\pm}^2+M_W^2-s)}{\lambda^{1/2}}\,f\right]2\pi \right.$$

$$+\,2\tan^2\beta\cdot 4\pi + 2\tan\beta\,\mathrm{Re}\,(C_A+C_H)\,(M_{H^\pm}^2+M_W^2-s)2\pi + \frac{1}{2}\tan\beta\,\cancel{\lambda}\,\mathrm{Re}\,(C_A+C_H)\cdot\frac{4}{\cancel{\lambda}}\cdot 2\pi\,\cdot$$

$$\left.\cdot\left[(M_{H^\pm}^2+M_W^2-s) - \frac{2M_W^2 M_{H^\pm}^2}{\lambda^{1/2}}\,f\right] - 4\pi\tan\beta\,\mathrm{Re}\,(C_A+C_H)\,M_W^2 M_{H^\pm}^2\,\frac{2}{\lambda^{1/2}}\,f\right\}$$

$$\sigma = \frac{1}{64\pi^2 s}\,\frac{\lambda^{1/2} g^4 m_f^2}{32 M_W^4}\left\{ 4\pi\lambda\left[|C_H|^2+|C_A|^2\right] - \frac{16}{s}\tan^2\beta\,M_W^2\,2\pi + \frac{8\tan^2\beta\,M_W^2}{s\lambda^{1/2}}(M_{H^\pm}^2+M_W^2-s)\,f\,2\pi \right.$$

$$+\,8\pi\tan^2\beta + 4\pi\tan\beta\,\mathrm{Re}\,(C_A+C_H)\,(M_{H^\pm}^2+M_W^2-s) + 2\tan\beta\,\mathrm{Re}\,(C_A+C_H)\,(M_W^2+M_{H^\pm}^2-s)\,2\pi$$

$$\left. -\,4\,\frac{\tan\beta\,\mathrm{Re}\,(C_A+C_H)\,M_W^2 M_{H^\pm}^2}{\lambda^{1/2}}\,f\cdot 2\pi - 8\pi\,\frac{\tan\beta\,\mathrm{Re}\,(C_A+C_H)\,M_W^2 M_{H^\pm}^2\,f}{\lambda^{1/2}}\right\}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \quad\Rightarrow\quad G_F^2 = \frac{g^4}{32 M_W^4}$$

$$\sigma = \frac{m_f^2 G_F^2}{16\pi s}\left\{ \lambda^{3/2}\left[|C_H|^2+|C_A|^2\right] - \frac{8}{s}\tan^2\beta\,M_W^2\,\lambda^{1/2} + \frac{4\tan^2\beta\,M_W^2(M_{H^\pm}^2+M_W^2-s)\,f}{s}\right.$$

$$+\,2\tan^2\beta\,\lambda^{1/2} + \tan\beta\,\mathrm{Re}\,(C_A+C_H)\,(M_{H^\pm}^2+M_W^2-s)\,\lambda^{1/2} + \tan\beta\,\mathrm{Re}\,(C_A+C_H)\,(M_W^2+M_{H^\pm}^2-s)\,\lambda^{1/2}$$

$$\left. -\,4\tan\beta\,\mathrm{Re}\,(C_A+C_H)\,M_W^2 M_{H^\pm}^2\,f\right\}$$

$$\sigma = \frac{G_F^2 m_f^2}{16\pi s^2}\left\{ s\lambda^{3/2}\left[|C_H|^2+|C_A|^2\right] + 2\tan^2\beta\,\lambda^{1/2}(s-4M_W^2) + 2\tan\beta\,\mathrm{Re}\,(C_A+C_H)\cdot s\,\cdot\right.$$

$$(M_{H^\pm}^2+M_W^2-s)\,\lambda^{1/2} + 4M_W^2\tan\beta\,f\,(s, M_{H^\pm}^2, M_W^2)\left[\tan\beta\,(M_{H^\pm}^2+M_W^2-s)\right.$$

$$\left. -\,\mathrm{Re}\,(C_A+C_H)\,s\,M_{H^\pm}^2\right]$$

$$\Rightarrow \quad \sin 2\alpha = -\left(\frac{2\tan\beta}{1+\tan^2\beta}\right)\left(\frac{m_{A^0}^2 + M_Z^2}{m_{H^0}^2 - m_{h^0}^2}\right)$$

$$\sin 2\alpha = -\left(\frac{2\tan\beta}{1+\tan^2\beta}\right)\left(\frac{M_{H^\pm}^2 + M_Z^2 - M_W^2}{\left[(m_{A^0}^2 + M_Z^2)^2 - 4 m_{A^0}^2 M_Z^2 \cos^2 2\beta\right]^{1/2}}\right)$$

$$\boxed{\sin 2\alpha = -\left(\frac{2\tan\beta}{1+\tan^2\beta}\right)\left[\frac{M_{H^\pm}^2 + M_Z^2 - M_W^2}{\left((M_{H^\pm}^2 + M_Z^2 - M_W^2)^2 - 4(M_{H^\pm}^2 - M_W^2)M_Z^2\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2\right)^{1/2}}\right]} \tag{106}$$

$$\boxed{\cos 2\alpha = -\frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{M_{H^\pm}^2 - M_W^2 - M_Z^2}{\left((M_{H^\pm}^2 + M_Z^2 - M_W^2)^2 - 4(M_{H^\pm}^2 - M_W^2)M_Z^2\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2\right)^{1/2}}\right]} \tag{107}$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\boxed{\sin^2\alpha = \frac{1}{2}\left\{1 + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{M_{H^\pm}^2 - M_W^2 - M_Z^2}{\left((M_{H^\pm}^2 + M_Z^2 - M_W^2)^2 - 4(M_{H^\pm}^2 - M_W^2)M_Z^2\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2\right)^{1/2}}\right]\right\}} \tag{108}$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\boxed{\cos^2\alpha = \frac{1}{2}\left\{1 - \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{M_{H^\pm}^2 - M_W^2 - M_Z^2}{\left((M_{H^\pm}^2 + M_Z^2 - M_W^2)^2 - 4(M_{H^\pm}^2 - M_W^2)M_Z^2\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2\right)^{1/2}}\right]\right\}} \tag{109}$$

$$m_{h^0}^2 = \frac{1}{2}\left\{ M_{H^\pm}^2 - M_W^2 + M_Z^2 - g\left(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta\right) \right\}$$

$$\boxed{\sin 2\alpha = -\left(\frac{2\tan\beta}{1+\tan^2\beta}\right)\left[\frac{M_{H^\pm}^2 + M_Z^2 - M_W^2}{g\left(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta\right)}\right]}$$

$$\boxed{\sin^2\alpha = \frac{1}{2}\left\{ 1 + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{M_{H^\pm}^2 - M_W^2 - M_Z^2}{g\left(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta\right)}\right]\right\}}$$

$$\boxed{\cos^2\alpha = \frac{1}{2}\left\{ 1 - \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{M_{H^\pm}^2 - M_W^2 - M_Z^2}{g\left(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta\right)}\right]\right\}}$$

$1\,fb = 10^{-15}$ barn

$G_F = 1.166391 \times 10^{-5}$ GeV^{-2}

$M_W = 80.419$ GeV/c^2

$M_Z = 91.1882$ GeV/c^2

$m_{\mu^-} = .105658357$ GeV/c^2

$m_{e^-} = 0.510998902$ MeV/c^2

$\sqrt{s} = 500$ GeV

$\pi = 3.141592654$

$20 \leq \tan\beta \leq 50$

$120 \leq M_{H^\pm} \leq 420$

$$g\left(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta\right) = M_{H^\pm}^2\left[\left(1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}\right)^2 - 4\frac{M_Z^2}{M_{H^\pm}^2}\left(1 - \frac{M_W^2}{M_{H^\pm}^2}\right)\cdot\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2\right]^{1/2} = M_{H^\pm}^2\, g^*$$

$$\Rightarrow \boxed{\sin^2\alpha = \frac{1}{2}\left\{ 1 + \frac{(1-\tan^2\beta)}{(1+\tan^2\beta)}\left[\frac{1 - \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*\left(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta\right)}\right]\right\}} \qquad (114)$$

with $g^*\left(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta\right) = \left[\left(1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}\right)^2 - 4\left(\frac{M_Z^2}{M_{H^\pm}^2}\right)\left(1 - \frac{M_W^2}{M_{H^\pm}^2}\right)\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)^2\right]^{1/2}$ (11

$$\boxed{\sin 2\alpha = -\left(\frac{2\tan\beta}{1+\tan^2\beta}\right)\left[\frac{1 + \frac{M_Z^2}{M_{H^\pm}^2} - \frac{M_W^2}{M_{H^\pm}^2}}{g^*\left(M_{H^\pm}^2, M_Z^2, M_W^2, \tan^2\beta\right)}\right]} \qquad (116)$$

$$\boxed{\begin{aligned}\sigma(\mu^-\mu^+ \to H^\mp W^\pm) = {} & (3.893792914 \times 10^{11})\ \frac{G_F^2\, m_f^2\, M_{H^\pm}^4}{8\pi s^2} \Bigg\{ \left(\frac{s}{M_{H^\pm}^2}\right) \lambda^{*\,3/2} \left[|C_H^{v*}|^2 + |C_A^{v*}|^2\right] \\ & + 2\tan\beta\, \lambda^{*\,1/2} \left[\tan\beta \left(\frac{s}{M_{H^\pm}^2} - \frac{4M_W^2}{M_{H^\pm}^2} \right) + \left(\frac{s}{M_{H^\pm}^2}\right) \overset{\downarrow \text{not necessary}}{\mathrm{Re}}\, (C_A^{v*} + C_H^{v*}) . \left(1 + \frac{M_W^2}{M_{H^\pm}^2} - \frac{s}{M_{H^\pm}^2}\right) \right] \\ & + 4\left(\frac{M_W^2}{M_{H^\pm}^2}\right) \tan\beta \cdot f \cdot \left[\tan\beta \left(1 + \frac{M_W^2}{M_{H^\pm}^2} - \frac{s}{M_{H^\pm}^2}\right) - \overset{\downarrow \text{no}}{\mathrm{Re}}\, (C_A^{v*} + C_H^{v*}) \frac{s}{M_{H^\pm}^2} \right] \Bigg\}\end{aligned}} \qquad (121)$$

$$\boxed{\frac{s - m_{h^0}^2}{M_{H^\pm}^2} = \frac{s}{M_{H^\pm}^2} - \frac{1}{2}\left\{1 - \frac{M_W^2}{M_{H^\pm}^2} + \frac{M_Z^2}{M_{H^\pm}^2} - g^*\right\}} \qquad (122)$$

$$\boxed{\frac{s - m_{H^0}^2}{M_{H^\pm}^2} = \frac{s}{M_{H^\pm}^2} - \frac{1}{2}\left\{1 - \frac{M_W^2}{M_{H^\pm}^2} + \frac{M_Z^2}{M_{H^\pm}^2} + g^*\right\}} \qquad (123)$$

$$\boxed{\frac{s - m_{A^0}^2}{M_{H^\pm}^2} = \frac{s}{M_{H^\pm}^2} - \left(1 - \frac{M_W^2}{M_{H^\pm}^2}\right)} \qquad (124)$$

If $M_{H^\pm} = \sqrt{s} - M_W$

$$\lambda(s, M_{H^\pm}^2, M_W^2) = s^2 + M_{H^\pm}^4 + M_W^4 - 2sM_{H^\pm}^2 - 2sM_W^2 - 2M_W^2 M_{H^\pm}^2$$

$$M_{H^\pm}^2 = s - 2\sqrt{s}M_W + M_W^2$$

$$M_{H^\pm}^4 = s^2 + 4\cancel{s}M_W^2 + M_W^4 - 4s^{3/2}M_W + 2\cancel{s}M_W^2 - 4\sqrt{s}M_W^3$$

$$M_{H^\pm}^4 = s^2 + 6sM_W^2 + M_W^4 - 4s^{3/2}M_W - 4\sqrt{s}M_W^3$$

$$\Rightarrow \lambda = \cancel{s^2} + \cancel{s^2} + \cancel{6sM_W^2} + \cancel{M_W^4} \cancel{-4s^{3/2}M_W} - 4\sqrt{s}\cancel{M_W^3} + \cancel{M_W^4} - \cancel{2s^2} \cancel{+4s^{3/2}M_W} - 2\cancel{s}M_W^2$$
$$- 2\cancel{s}M_W^2 - \cancel{2M_W^2 s} + \cancel{4M_W^3\sqrt{s}} - 2\cancel{M_W^4}$$

$\therefore$ $\boxed{\lambda = 0 \text{ and } \lambda^* = 0 \text{ if } M_{H^\pm} = \sqrt{s} - M_W}$ (125)

Then $\boxed{f(s, M_{H^\pm}^2, M_W^2) = 0}$ (see 113)

So $\boxed{\sigma(\mu^-\mu^+ \to H^\mp W^\pm) = 0 \text{ if } M_{H^\pm} = \sqrt{s} - M_W}$ (126)

$\approx 419.581\ \mathrm{GeV}/c^2$

$\mu^-\mu^+ \rightarrow H^\mp W^\pm$ with rad. corrections

$$\lambda^{1/2}\left(1, \frac{s}{m_{H^\pm}^2}, \frac{m_W^2}{m_{H^\pm}^2}\right) = 15.85539846$$

$$f(s, m_{H^\pm}^2, m_W^2) = \ln\left|\frac{-15.91195424 + 15.85539846}{-15.91195424 - 15.85539846}\right| = -6.330966989$$

$g^*_{rc} = 0.593369997$ ((chequear $m_{W^\pm} = 80.423$))

$\sin 2\alpha = -0.126625186$

$\sin^2\alpha = 0.995975317$

$m_{H^0} = 128.2291119$

$m_{h^0} = 88.87168943$

$$\frac{s - m_{h^0}^2}{m_{H^\pm}^2} = 16.81262658 \quad ; \quad \frac{s}{m_{H^\pm}^2} = 17.36111111$$

$$\frac{s - m_{H^0}^2}{m_{H^\pm}^2} = 16.21925659$$

$$\frac{s - m_{A^0}^2}{m_{H^\pm}^2} = \frac{500^2 - (120^2 - 80.423^2)}{120^2} = 16.81026798$$

$$C_H^{xx} = \left\{ 1.77342587 + 0.011347812 \right\} = 1.784773683$$

$$C_A^{xx} = 1.784623543$$

$$\Rightarrow \sigma(\mu^-\mu^+ \rightarrow H^\mp W^\pm) = (3.893792914 \times 10^{11})(2.004938858 \times 10^{-16}) \{ 440827.5895$$

$$+ 951.3239076 [466.9345089 - 986.043148] - 341.2316781 . [-477.3586272$$

$$- 61.96870184] \}$$

$$= 10.22870205$$

124

$$\int_{-1}^{1} \frac{d\cos\theta}{(a-b\cos\theta)} = -\frac{1}{b}\ln\left|\frac{a-b}{a+b}\right| = \frac{1}{b}\ln\left|\frac{a+b}{a-b}\right| \qquad (133)$$

$$\boxed{\int_{-1}^{1} \frac{d\cos\theta}{U} = \frac{2}{\lambda^{1/2}}\ln\left|\frac{M_{H^\pm}^2 + M_W^2 - s + \lambda^{1/2}}{M_{H^\pm}^2 + M_W^2 - s - \lambda^{1/2}}\right|} \qquad (134)$$

$$\int_{-1}^{1} (a - b\cos\theta)\, d\cos\theta = 2a \qquad (135)$$

$$\boxed{\int_{-1}^{1} U\, d\cos\theta = (M_{H^\pm}^2 + M_W^2 - s)} \qquad (136)$$

$$\boxed{\int_{-1}^{1} d\cos\theta = 2} \qquad (137)$$

$$\sigma(\mu^-\mu^+ \to H^+W^-) = \frac{1\,\overset{1}{\cancel{2\pi}}}{\underset{32}{\cancel{64}}\,\cancel{\pi^2}s}\,\frac{\lambda^{1/2} g^4 m_f^2}{32 M_W^4}\Big\{2[|C_H|^2 + |C_A|^2]\lambda + \frac{\cancel{\lambda}\tan^2\beta\, M_W^2}{s}\frac{8}{\cancel{\lambda}}\Big[-2 + \frac{(M_{H^\pm}^2 + M_W^2 - s)}{\lambda^{1/2}} f\Big]$$

$$+ 4\tan^2\beta + 2\tan\beta\, \mathrm{Re}\,(C_A + C_H)(M_{H^\pm}^2 + M_W^2 - s) + \frac{\cancel{\lambda}}{\cancel{2}}\tan\beta\, \mathrm{Re}\,(C_A + C_H)\frac{\overset{2}{\cancel{4}}}{\cancel{\lambda}}\Big[M_{H^\pm}^2 + M_W^2 - s - \frac{2M_W^2 M_{H^\pm}^2}{\lambda^{1/2}} f\Big]$$

$$- 2\tan\beta\, \mathrm{Re}\,(C_A + C_H) M_W^2 M_{H^\pm}^2 \frac{2}{\lambda^{1/2}} f\Big\}$$

$$= \frac{1\,\cancel{2}}{\underset{16}{\cancel{32}}\pi s}\,\frac{g^4 m_f^2}{32 M_W^4}\Big\{\lambda^{3/2}[|C_H|^2 + |C_A|^2] + \frac{4\lambda^{1/2}\tan^2\beta\, M_W^2}{s}\Big[-2 + \frac{(M_{H^\pm}^2 + M_W^2 - s)}{\lambda^{1/2}} f\Big]$$

$$+ 2\tan^2\beta\,\lambda^{1/2} + \tan\beta\, \mathrm{Re}\,(C_A + C_H)(M_{H^\pm}^2 + M_W^2 - s)\lambda^{1/2} + \lambda^{1/2}\tan\beta\, \mathrm{Re}\,(C_A + C_H)\Big[M_{H^\pm}^2 + M_W^2 - s - \frac{2M_W^2 M_{H^\pm}^2}{\lambda^{1/2}} f\Big]$$

$$- 2\tan\beta\, \mathrm{Re}\,(C_A + C_H) M_W^2 M_{H^\pm}^2 f\Big\}$$

$$= \frac{G_F^2 m_f^2}{16\pi s}\Big\{\lambda^{3/2}[|C_H|^2 + |C_A|^2] - \frac{8\lambda^{1/2}\tan^2\beta\, M_W^2}{s} + \frac{4\tan^2\beta\, M_W^2}{s}(M_{H^\pm}^2 + M_W^2 - s) f$$

$$+ 2\tan^2\beta\,\lambda^{1/2} + \tan\beta\, \mathrm{Re}\,(C_A + \cancel{C_H})(M_{H^\pm}^2 + M_W^2 - s)\lambda^{1/2} + \tan\beta\, \mathrm{Re}\,(C_A + C_H)(\cancel{M_{H^\pm}^2 +} M_W^2 - s)\lambda^{1/2}$$

$$- 2\tan\beta\, \mathrm{Re}\,(C_A + C_H)\cancel{M_W^2 M_{H^\pm}^2} f - 2\tan\beta\, \cancel{\mathrm{Re}}\,(C_A + C_H) M_W^2 M_{H^\pm}^2 f\Big\}$$

$$\sigma(\mu^-\mu^+ \to H^+W^-) = \frac{G_F^2 m_f^2}{16\pi s^2}\Big\{\lambda^{3/2} s\,[|C_H|^2 + |C_A|^2] - 8\lambda^{1/2}\tan^2\beta\, M_W^2 + 4\tan^2\beta\, M_W^2(M_{H^\pm}^2 + M_W^2 - s) f$$

$$+ 2s\tan^2\beta\,\lambda^{1/2} + 2s\tan\beta\, \mathrm{Re}\,(C_A + C_H)(M_{H^\pm}^2 + M_W^2 - s)\lambda^{1/2}$$

$$- 4s\tan\beta\, \mathrm{Re}\,(C_A + C_H) M_W^2 M_{H^\pm}^2 f\Big\} \qquad (138)$$

that is identical to eq. (100)

$$\Rightarrow \quad \boxed{\sigma(\mu^-\mu^+ \to H^-W^+) = \sigma(\mu^-\mu^+ \to H^+W^-)} \qquad (139)$$

and then $$\boxed{\sigma(\mu^-\mu^+ \to H^\mp W^\pm) = 2\,\sigma(\mu^-\mu^+ \to H^-W^+)} \qquad (140)$$

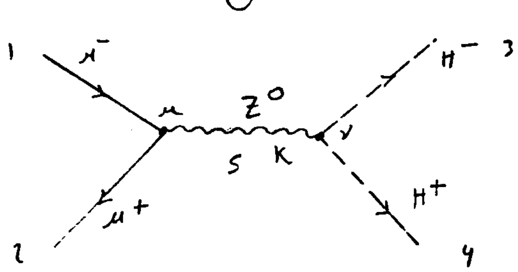
Production of charged Higgs boson pairs

$\mu^+\mu^- \to H^+H^-$ (Model II)

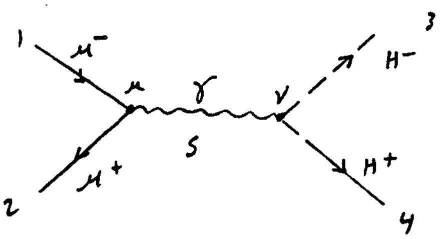
or $\mu^-\mu^+ \to H^-H^+ \leftarrow$

$\longrightarrow$ time

① ②

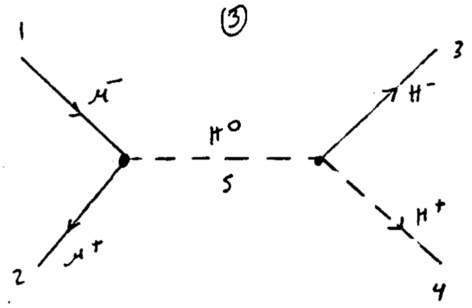


+

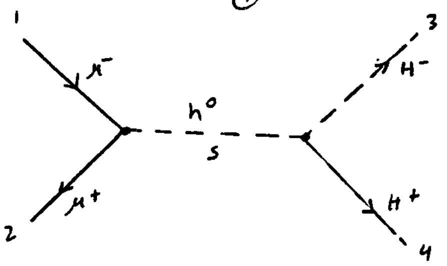


③ ④

+

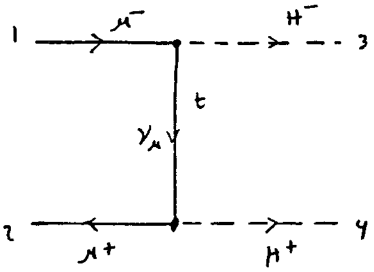


+

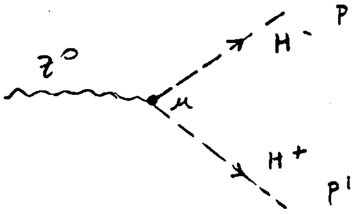


⑤

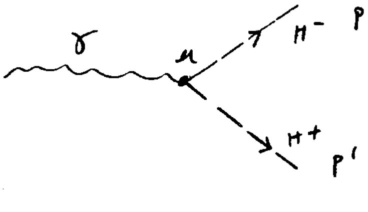
+



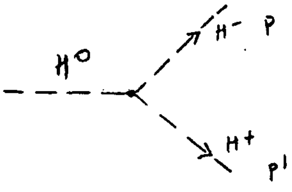
$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu$$



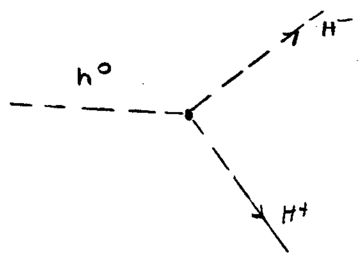
$$-\frac{ig\cos(2\theta_W)}{2\cos\theta_W}\,(p'-p)^\mu$$



$$-ie\,(p'-p)^\mu$$

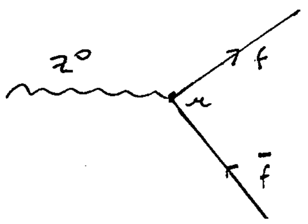


$$-ig\left[M_W\cos(\beta-\alpha) - \frac{M_Z}{2\cos\theta_W}\cos 2\beta\,\cos(\alpha+\beta)\right]$$



$$-ig\left[M_W \sin(\beta-\alpha) + \frac{M_Z}{2\cos\theta_W}\cos 2\beta \,\sin(\alpha+\beta)\right]$$

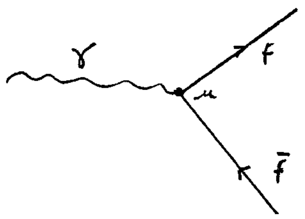
$f = e^-, \mu^-, \tau^-$



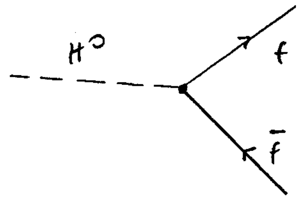
$$-\frac{ig}{\cos\theta_W}\gamma^\mu \frac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)$$

$$c_A^f = T_f^3 = -\frac{1}{2}$$

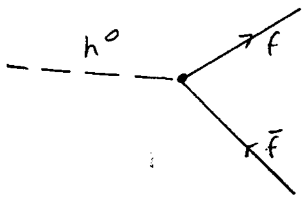
$$c_V^f = T_f^3 - 2\sin^2\theta_W Q_f = -\frac{1}{2} + 2\sin^2\theta_W$$



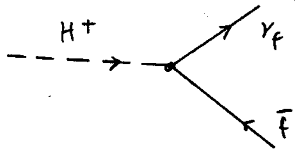
$$ie\gamma^\mu$$



$$-\frac{ig\, m_f \cos\alpha}{2 M_W \cos\beta}$$

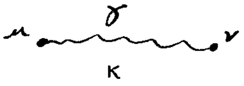


$$\frac{ig\, m_f \sin\alpha}{2 M_W \cos\beta}$$



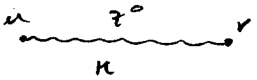
$$\frac{ig}{2\sqrt{2}\, M_W}\, m_f \tan\beta\,(1+\gamma^5)$$

$$\left(\frac{-i\left(g_{\mu\nu}+\frac{(\xi-1)k_\mu k_\nu}{k^2}\right)}{k^2+i\epsilon}\right)$$

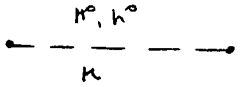


$$\frac{-ig_{\mu\nu}}{k^2+i\epsilon}$$ Feynman gauge

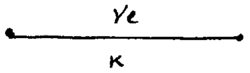
$$\left(\frac{-i\left(g_{\mu\nu}+\frac{(\xi-1)k_\mu k_\nu}{k^2-\xi M^2}\right)}{k^2-M^2+iM\Gamma}\right)$$



$$\frac{-ig_{\mu\nu}}{k^2-M_{Z^0}^2+iM_Z\Gamma_Z}$$ Feynman 't Hooft gauge



$$\frac{i}{k^2-M^2+iM\Gamma}$$



$$\frac{i}{\not k+i\epsilon}=\frac{i\not k}{k^2+i\epsilon}$$

$$-iM_1 = -\frac{ig\cos(2\theta_W)(P_4-P_3)^\nu}{2\cos\theta_W}\left[\frac{-i\left(g_{\mu\nu}+\frac{(\xi-1)k_\mu k_\nu}{k^2-\xi M_Z^2}\right)}{k^2-M_Z^2+iM_Z\Gamma_Z}\right]\bar V_2\left(\frac{-ig}{\cos\theta_W}\gamma^\mu\frac{1}{2}\cdot\right.$$

$$\left.\cdot\left(C_V^f-C_A^f\gamma^5\right)\right)U_1 \qquad (1)$$

$$\bar V_2\gamma^\mu \overset{k_\mu}{\widehat{(P_1+P_2)}}_\mu U_1 = \bar V_2\gamma^\mu P_{2\mu}U_1+\bar V_2\gamma^\mu P_{1\mu}U_1 = 0 \quad \text{because :}$$

but $\not P_1 U_1\approx 0 \Rightarrow \gamma^\mu U_1 P_{1\mu}=0$ (m_f is small)

$\bar V_2\not P_2\approx 0 \Rightarrow \bar V_2\gamma^\mu P_{2\mu}=0$

$$\bar V_2\gamma^\mu k_\mu\gamma^5U_1 = \bar V_2\gamma^\mu(P_{1\mu}+P_{2\mu})\gamma^5U_1 = \bar V_2\gamma^\mu P_{2\mu}\gamma^5U_1-\bar V_2\gamma^5\gamma^\mu P_{1\mu}U_1$$
$$=0$$

$$\Rightarrow\quad M_1 = -\frac{g^2\cos(2\theta_W)}{4\cos^2\theta_W}(P_4-P_3)^\nu\left[\frac{g_{\mu\nu}}{k^2-M_Z^2+iM_Z\Gamma_Z}\right]\bar V_2\gamma^\mu\left(C_V^f-C_A^f\gamma^5\right)U_1$$

$$\boxed{M_1 = -\frac{g^2\cos(2\theta_W)(P_4-P_3)_\mu}{4\cos^2\theta_W}\frac{1}{(k^2-M_Z^2+iM_Z\Gamma_Z)}\bar V_2\gamma^\mu\left(C_V^f-C_A^f\gamma^5\right)U_1} \qquad (2)$$

$$-iM_2 = -ie\,(P_4-P_3)^\nu \left[\frac{-i\left(g_{\mu\nu} + (\xi-1)\frac{K_\mu K_\nu}{K^2}\right)}{K^2} \right] \bar{V}_2\; ie\gamma^\mu U_1 \qquad (3)$$

again $\bar{V}_2 \gamma^\mu K_\mu U_1 = 0$ (4)

$$\Rightarrow \quad M_2 = \frac{e^2\,(P_4-P_3)_\mu}{K^2}\, \bar{V}_2 \gamma^\mu U_1$$

$$g = \frac{e}{\sin\theta_W} \quad \Rightarrow \quad e^2 = g^2 \sin^2\theta_W$$

$$\boxed{M_2 = \frac{g^2 \sin^2\theta_W}{K^2}\,(P_4-P_3)_\mu\, \bar{V}_2 \gamma^\mu U_1} \qquad (5)$$

$$-iM_3 = -ig\left[M_W \cos(\beta-\alpha) - \frac{M_Z}{2\cos\theta_W}\cos 2\beta \cos(\alpha+\beta)\right] \frac{i}{K^2 - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}}\, \bar{V}_2 \left(\frac{-ig\, m_f \cos\alpha}{2M_W \cos\beta}\right) U_1$$

$$\boxed{M_3 = \frac{g^2 m_f \cos\alpha}{2M_W\cos\beta}\left[M_W \cos(\beta-\alpha) - \frac{M_Z}{2\cos\theta_W}\cos 2\beta \cos(\alpha+\beta)\right] \cdot \frac{1}{K^2 - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} \cdot \bar{V}_2 U_1} \qquad (6)$$

$$-iM_4 = -ig\left[M_W \sin(\beta-\alpha) + \frac{M_Z}{2\cos\theta_W}\cos 2\beta \sin(\alpha+\beta)\right] \frac{i}{K^2 - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}}\, \bar{V}_2 \left(\frac{ig\, m_f \sin\alpha}{2M_W \cos\beta}\right) U_1$$

$$\boxed{M_4 = -\frac{g^2 m_f \sin\alpha}{2M_W\cos\beta}\left[M_W \sin(\beta-\alpha) + \frac{M_Z}{2\cos\theta_W}\cos 2\beta \sin(\alpha+\beta)\right] \cdot \frac{1}{K^2 - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}} \cdot \bar{V}_2 U_1} \qquad (7)$$

$$-iM_5 = \bar{V}_2 \left(\frac{ig}{2\sqrt{2}M_W}\right) m_f \tan\beta\,(1-\gamma^5)\; \frac{i\not{K}}{K^2} \left(\frac{ig}{2\sqrt{2}M_W}\right) m_f \tan\beta\,(1+\gamma^5)\, U_1$$

$$M_5 = \frac{g^2 m_f^2 \tan^2\beta}{8M_W^2 K^2}\, \bar{V}_2 (1-\gamma^5)\not{K}(1+\gamma^5) U_1 \simeq 0$$

$$(1-\gamma^5)\gamma^\mu(1+\gamma^5) = \gamma^\mu(1+\gamma^5)(1+\gamma^5) = 2(1-\gamma^5)\gamma^\mu$$

$$\Rightarrow \boxed{M_5 \approx 0} \qquad (8)$$

$$M_1 = \frac{-g^2 \cos(2\theta_W)}{4\cos^2\theta_W} (P_4 - P_3)_\mu \frac{1}{s - M_Z^2 + i M_Z \Gamma_Z} \bar{V}_2 \gamma^\mu (C_V^f - C_A^f \gamma^5) U_1$$

$$\boxed{C_1 \equiv \frac{\cos(2\theta_W)}{\cos^2\theta_W} \cdot \frac{1}{s - M_Z^2 + i M_Z \Gamma_Z}} \qquad (9)$$

$$\Rightarrow \boxed{M_1 = -\frac{g^2}{4} C_1 (P_4 - P_3)_\mu \bar{V}_2 \gamma^\mu (C_V^f - C_A^f \gamma^5) U_1} \qquad (10)$$

$$M_2 = \frac{g^2 \sin^2\theta_W}{s} (P_4 - P_3)_\mu \bar{V}_2 \gamma^\mu U_1$$

$$\boxed{C_2 \equiv \frac{\sin^2\theta_W}{s}} \qquad (11)$$

$$\Rightarrow \boxed{M_2 = g^2 C_2 (P_4 - P_3)_\mu \bar{V}_2 \gamma^\mu U_1} \qquad (12)$$

$$\boxed{C_{H^0} \equiv \frac{\cos\alpha}{\cos\beta M_W} \left[M_W \cos(\beta - \alpha) - \frac{M_Z}{2\cos\theta_W} \cos 2\beta \cos(\alpha + \beta) \right] \cdot \frac{1}{s - m_{H^0}^2 + i m_{H^0} \Gamma_{H^0}}} \qquad (13)$$

$$\boxed{C_{h^0} \equiv \frac{\sin\alpha}{\cos\beta M_W} \left[M_W \sin(\beta - \alpha) + \frac{M_Z}{2\cos\theta_W} \cos 2\beta \sin(\alpha + \beta) \right] \cdot \frac{1}{s - m_{h^0}^2 + i m_{h^0} \Gamma_{h^0}}} \qquad (14)$$

$$\Rightarrow M_3 = \frac{g^2 m_f}{2} \bar{V}_2 U_1 C_{H^0} \qquad (15)$$

$$M_4 = -\frac{g^2}{2} m_f \bar{V}_2 U_1 C_{h^0} \qquad (16)$$

$$M_3 + M_4 = \frac{g^2}{2} m_f \bar{V}_2 U_1 (C_{H^0} - C_{h^0})$$

$$\boxed{C^{Hh} \equiv C_{H^0} - C_{h^0}} \; (17) \qquad \Rightarrow \boxed{M_3 + M_4 = \frac{g^2}{2} m_f C^{Hh} \bar{V}_2 U_1} \equiv M_{34} \qquad (18)$$

$$\Rightarrow \overline{|M|^2} = \frac{1}{4}\,|M_1 + M_2 + M_{34}|^2 = \frac{1}{4}\,(M_1^+ + M_2^+ + M_{34}^+)(M_1 + M_2 + M_{34})$$

$$\boxed{\overline{|M|^2} = \frac{1}{4}\left(|M_1|^2 + M_1^+ M_2 + M_1^+ M_{34} + M_2^+ M_1 + |M_2|^2 + M_2^+ M_{34} + M_{34}^+ M_1 + M_{34}^+ M_2 + |M_{34}|^2 \right)} \quad (19)$$

$$|M_1|^2 = M_1^+ M_1 = \frac{g^4}{16}\,|C_1|^2\,(P_4-P_3)_\mu \sum_s \left(\bar{V}_2 \gamma^\mu (C_V^f - C_A^f\gamma^5) U_1\right)(P_4-P_3)_\nu \left(\bar{V}_2 \gamma^\nu (C_V^f - C_A^f \gamma^5) U_1\right)^+$$

$$(\underbrace{\bar{V}_2}_{V_2^+\gamma^0} \gamma^\nu (C_V^f - C_A^f\gamma^5) U_1)^+ = (U_1^+ (C_V^f - C_A^f\gamma^5)\gamma^{\nu +}\gamma^0 V_2)$$

$$= (U_1^+ (C_V^f - C_A^f\gamma^5)\gamma^0\gamma^\nu V_2) = (\bar{U}_1 (C_V^f + C_A^f\gamma^5)\gamma^\nu V_2)$$

$$= (\bar{U}_1 \gamma^\nu (C_V^f - C_A^f\gamma^5) V_2)$$

$$|M_1|^2 = \frac{g^4}{16}\,|C_1|^2\,(P_4-P_3)_\mu (P_4-P_3)_\nu \sum_s (\bar{V}_2\gamma^\mu (C_V^f - C_A^f\gamma^5) U_1)(\bar{U}_1 \gamma^\nu (C_V^f - C_A^f\gamma^5) V_2).$$

$$= \frac{g^4}{16}\,|C_1|^2\,(P_4-P_3)_\mu (P_4-P_3)_\nu \,\mathrm{Tr}\left[(\not{P}_1 + m_f)\gamma^\nu (C_V^f - C_A^f\gamma^5)\,(\not{P}_2 - m_f)\gamma^\mu (C_V^f - C_A^f\gamma^5)\right] \quad (20)$$

$$\mathrm{Tr}[\ \ ] = \mathrm{Tr}\left[\gamma^\mu (C_V^f - C_A^f\gamma^5)(\not{P}_1 + m_f)\gamma^\nu (C_V^f - C_A^f\gamma^5)(\not{P}_2 - m_f)\right]$$

$$= \mathrm{Tr}\left[\gamma^\mu (C_V^f - C_A^f\gamma^5)\,(\not{P}_1 + m_f)\,(C_V^f + C_A^f\gamma^5)\gamma^\nu (\not{P}_2 - m_f)\right]$$

$$(C_V^f - C_A^f\gamma^5)\not{P}_1 (C_V^f + C_A^f\gamma^5) + m_f (C_V^f - C_A^f\gamma^5)(C_V^f + C_A^f\gamma^5)$$

$$= (C_V^f - C_A^f\gamma^5)^2 \not{P}_1 + m_f((C_V^f)^2 - (C_A^f)^2)$$

$$= [(C_V^f)^2 - 2C_V^f C_A^f\gamma^5 + (C_A^f)^2]\not{P}_1 + m_f[(C_V^f)^2 - (C_A^f)^2]$$

$$\Rightarrow \mathrm{Tr}[\ \ ] = \mathrm{Tr}\left\{\gamma^\mu \left\{[(C_V^f)^2 - 2C_V^f C_A^f\gamma^5 + (C_A^f)^2]\not{P}_1 + m_f[(C_V^f)^2 - (C_A^f)^2]\right\} (\gamma^\nu \not{P}_2 - m_f\gamma^\nu)\right\}$$

$$= \mathrm{Tr}\Big\{\gamma^\mu [(C_V^f)^2 + (C_A^f)^2]\not{P}_1 \gamma^\nu \not{P}_2 - m_f \gamma^\mu [(C_V^f)^2 + (C_A^f)^2]\not{P}_1\gamma^\nu$$
$$- 2C_V^f C_A^f \gamma^\mu\gamma^5\not{P}_1\gamma^\nu\not{P}_2 + 2m_f C_V^f C_A^f \gamma^\mu\gamma^5\not{P}_1\gamma^\nu + m_f[(C_V^f)^2 - (C_A^f)^2].$$

$$\boxed{P_1\cdot(P_4-P_3) = \frac{1}{2}\cos\theta\,\lambda^{1/2}(s, M_{H^\pm}^2, M_{H^\pm}^2)\,\frac{(s-4m_f^2)^{1/2}}{\sqrt{s}}} \approx \frac{1}{2}\cos\theta\,\lambda^{1/2} \qquad (28)$$

$$(P_4-P_3)^2 = 2M_{H^\pm}^2 - 2P_3\cdot P_4$$

$$s = (P_3+P_4)^2 = 2M_{H^\pm}^2 + 2P_3\cdot P_4$$

$$\Rightarrow \quad 2P_3\cdot P_4 = s - 2M_{H^\pm}^2$$

$$\therefore \quad (P_4-P_3)^2 = 2M_{H^\pm}^2 - s + 2M_{H^\pm}^2$$

$$\boxed{(P_4-P_3)^2 = 4M_{H^\pm}^2 - s} \qquad (29)$$

$$P_2\cdot(P_4-P_3) = P_2\cdot(P_1+P_2-2P_3) = \frac{s-2m_f^2}{2} + m_f^2 - 2P_2\cdot P_3 = \frac{s}{2} - 2P_2\cdot P_3$$

$$U = (P_1-P_4)^2 = (P_3-P_2)^2$$

$$U = M_{H^\pm}^2 + m_f^2 - 2P_2\cdot P_3$$

$$-2P_2\cdot P_3 = U - M_{H^\pm}^2 - m_f^2$$

$$\Rightarrow \boxed{P_2\cdot(P_4-P_3) = \frac{s}{2} + U - M_{H^\pm}^2 - m_f^2 \approx \frac{s}{2} + U - M_{H^\pm}^2} \qquad (30)$$

but $$U = \frac{2m_f^2 + 2M_{H^\pm}^2 - s}{2} - \frac{1}{2}\cos\theta\,\lambda^{1/2}(s, M_{H^\pm}^2, M_{H^\pm}^2)\,\frac{(s-4m_f^2)^{1/2}}{\sqrt{s}} \qquad (31)$$

(see (52))

$$U \approx M_{H^\pm}^2 - \frac{s}{2} - \frac{1}{2}\cos\theta\,\lambda^{1/2}(s, M_{H^\pm}^2, M_{H^\pm}^2)$$

$$\therefore \boxed{P_2\cdot(P_4-P_3) = -\frac{1}{2}\cos\theta\,\lambda^{1/2}(s, M_{H^\pm}^2, M_{H^\pm}^2)\,\frac{(s-4m_f^2)^{1/2}}{\sqrt{s}} \approx -\frac{1}{2}\cos\theta\,\lambda^{1/2}} \qquad (32)$$

$$\boxed{\begin{aligned}|M_1|^2 = \frac{g^4}{4}|C_1|^2 \Big\{ &\left[(C_V^f)^2 + (C_A^f)^2\right]\left[-\frac{1}{2}\cos^2\theta\,\frac{\lambda(s, M_{H^\pm}^2, M_{H^\pm}^2)}{s}(s-4m_f^2) - \frac{(s-2m_f^2)}{2}\cdot\right.\\ &\left.\cdot(4M_{H^\pm}^2 - s)\right] - m_f^2\left[(C_V^f)^2 - (C_A^f)^2\right](4M_{H^\pm}^2 - s)\Big\}\end{aligned}} \qquad (33)$$

$$\lambda(s, M_{H^\pm}^2, M_{H^\pm}^2) = s^2 + \cancel{2M_{H^\pm}^4} - 4sM_{H^\pm}^2 - \cancel{2M_{H^\pm}^4} = s(s-4M_{H^\pm}^2) \quad (\text{ok}) \qquad (34)$$

$$|M_1|^2 \approx \frac{g^4}{4}|C_1|^2\left\{\left[(C_V^f)^2 + (C_A^f)^2\right]\left[+\frac{1}{2}\cos^2\theta\, s\,(4M_{H^\pm}^2 - s) - \frac{1}{2}s\,(4M_{H^\pm}^2 - s)\right]\right\}$$

$$|M_1|^2 \approx \frac{g^4}{4}|C_1|^2\left\{-\left[(C_V^f)^2 + (C_A^f)^2\right]\frac{s}{2}(4M_{H^\pm}^2 - s)\sin^2\theta\right\}$$

$$M_2^\dagger M_1 = -\frac{g^4}{4} C_1 C_2 \sum_s (P_4 - P_3)_\mu \left(\bar{U}_1 \gamma^\mu V_2\right) (P_4 - P_3)_\nu \left(\bar{V}_2 \gamma^\nu (C_V^f - C_A^f \gamma^5) U_1\right)$$

$$M_2^\dagger M_1 = -\frac{g^4}{4} C_1 C_2 (P_4 - P_3)_\mu (P_4 - P_3)_\nu \sum_s (\bar{U}_1 \gamma^\mu V_2)(\bar{V}_2 \gamma^\nu (C_V^f - C_A^f \gamma^5) U_1)$$

$$= -\frac{g^4}{4} C_1 C_2 (P_4 - P_3)_\mu (P_4 - P_3)_\nu \, Tr\left[(\not{P}_2 - m_f) \gamma^\nu (C_V^f - C_A^f \gamma^5)(\not{P}_1 + m_f) \gamma^\mu \right]$$

$$= -\frac{g^4}{4} C_1 C_2 (P_4 - P_3)_\mu (P_4 - P_3)_\nu \, Tr\left[\gamma^\mu (\not{P}_2 - m_f) \gamma^\nu (C_V^f - C_A^f \gamma^5)(\not{P}_1 + m_f) \right] \quad (39)$$

$$Tr[\;] = Tr\left[\gamma^\mu (\not{P}_2 - m_f)(C_V^f + C_A^f \gamma^5) \gamma^\nu (\not{P}_1 + m_f) \right]$$

$$= Tr\left[(\gamma^\mu \not{P}_2 - m_f \gamma^\mu)(C_V^f + C_A^f \gamma^5)(\gamma^\nu \not{P}_1 + m_f \gamma^\nu) \right]$$

$$= Tr\left[(\gamma^\mu \not{P}_2 C_V^f + C_A^f \gamma^\mu \not{P}_2 \gamma^5 - m_f C_V^f \gamma^\mu - m_f C_A^f \gamma^\mu \gamma^5)(\gamma^\nu \not{P}_1 + m_f \gamma^\nu) \right]$$

$$= \cancel{Tr}\Big[C_V^f \gamma^\mu \not{P}_2 \gamma^\nu \not{P}_1 + m_f C_V^f \gamma^\mu \cancel{\not{P}_2} \gamma^\nu + C_A^f \gamma^\mu \not{P}_2 \gamma^5 \gamma^\nu \not{P}_1 + m_f C_A^f \gamma^\mu \cancel{\not{P}_2} \gamma^5 \gamma^\nu$$
$$- m_f C_V^f \gamma^\mu \cancel{\gamma^\nu} \not{P}_1 - m_f^2 C_V^f \gamma^\mu \gamma^\nu - m_f C_A^f \gamma^\mu \cancel{\gamma^5} \gamma^\nu \not{P}_1 + m_f^2 C_A^f \gamma^5 \gamma^\mu \gamma^\nu \Big]$$

$$= C_V^f \, Tr(\gamma^\mu \not{P}_2 \gamma^\nu \not{P}_1) + C_A^f \, Tr(\gamma^\mu \not{P}_2 \gamma^5 \gamma^\nu \not{P}_1) - m_f^2 C_V^f \, Tr(\gamma^\mu \gamma^\nu)$$
$$+ m_f^2 C_A^f \, Tr\,\cancelto{0}{(\gamma^5 \gamma^\mu \gamma^\nu)}$$

$$= C_V^f \, Tr(\gamma^\mu \not{P}_2 \gamma^\nu \not{P}_1) + C_A^f \, Tr(\gamma^5 \gamma^\mu \not{P}_2 \gamma^\nu \not{P}_1) - m_f^2 C_V^f \, Tr(\gamma^\mu \gamma^\nu)$$

$$Tr[\;] = 4 C_V^f \left[P_2^\mu P_1^\nu + P_2^\nu P_1^\mu - (P_1 \cdot P_2) g^{\mu\nu} \right] - 4i C_A^f \epsilon^{\mu\alpha\nu\beta} P_{2\alpha} P_{1\beta} - 4 m_f^2 C_V^f g^{\mu\nu} \quad (40)$$

but $(P_4 - P_3)_\mu (P_4 - P_3)_\nu \, \epsilon^{\mu\alpha\nu\beta} = 0$

$$\Rightarrow M_2^\dagger M_1 = -\frac{g^4}{\cancel{4}} C_1 C_2 \cancel{4} \left\{ C_V^f \left[2(P_1 \cdot (P_4 - P_3))(P_2 \cdot (P_4 - P_3)) - (P_1 \cdot P_2)(P_4 - P_3)^2 \right] - m_f^2 C_V^f (P_4 - P_3)^2 \right\}$$

$$M_2^\dagger M_1 = -g^4 C_1 C_2 \left\{ C_V^f \left[-\frac{1}{2} \cos^2\theta \, \lambda \frac{(s - 4m_f^2)}{s} - \frac{(s - 2m_f^2)}{2} (4 M_{H^\pm}^2 - s) \right] \right.$$
$$\left. - m_f^2 C_V^f (4 M_{H^\pm}^2 - s) \right\} \quad (41)$$

$$M_2^\dagger M_1 \approx -g^4 C_1 C_2 \left\{ C_V^f \left[-\frac{1}{2} \cos^2\theta \, s(s - 4 M_{H^\pm}^2) + \frac{s}{2}(s - 4 M_{H^\pm}^2) \right] \right\} \quad (42)$$

because $\mathrm{Tr}\,(\text{odd \# of } \gamma\text{'s}) = 0$

$$\mathrm{Tr}\,(\gamma^5 \not{a}\, \not{b}) = 0$$

$$M_{34}^{+} M_1 = -\frac{g^4}{8} m_f C^{Hh^*} C_1 (P_4 - P_3)_\mu \left\{ 4 m_f C_V^f P_{2\alpha} g^{\mu\alpha} - 4 m_f C_V^f P_{1\alpha} g^{\mu\alpha} \right\}$$

$$= -\frac{g^4}{8} 4 m_f^2 C_V^f C^{Hh^*} C_1 [(P_4 - P_3)_\mu (P_2 - P_1)^\mu]$$

$$= -\frac{g^4}{2} m_f^2 C_V^f C^{Hh^*} C_1 [(P_1 + P_2 - 2P_3) \cdot (P_2 - P_1)]$$

$$= -\frac{g^4}{2} m_f^2 C_V^f C^{Hh^*} C_1 [\cancel{(P_1 \cdot P_2)} - \cancel{m_f^2} + \cancel{m_f^2} - \cancel{(P_1 \cdot P_2)} - 2(P_2 \cdot P_3) + 2(P_1 \cdot P_3)]$$

$$= -g^4 m_f^2 C_V^f C^{Hh^*} C_1 [(P_1 \cdot P_3) - (P_2 \cdot P_3)] \qquad (48)$$

$$t = (P_1 - P_3)^2 = m_f^2 + M_{H^\pm}^2 - 2(P_1 \cdot P_3)$$

$$v = (P_1 - P_4)^2 = (P_3 - P_2)^2 = M_{H^\pm}^2 + m_f^2 - 2(P_2 \cdot P_3)$$

$$\Rightarrow \boxed{(P_1 \cdot P_3) = \frac{m_f^2 + M_{H^\pm}^2 - t}{2}} \qquad (49)$$

$$\Rightarrow \boxed{(P_2 \cdot P_3) = \frac{m_f^2 + M_{H^\pm}^2 - v}{2}} \qquad (50)$$

$$\therefore M_{34}^{+} M_1 = -g^4 m_f^2 C_V^f C^{Hh^*} C_1 \left[\frac{v - t}{2}\right]$$

$$= \frac{g^4}{2} m_f^2 C_V^f C^{Hh^*} C_1 \cos\theta\, \lambda^{1/2} \frac{(s - 4m_f^2)^{1/2}}{\sqrt{s}} \qquad (51)$$

$$\boxed{M_{34}^{+} M_1 \approx \frac{g^4}{2} m_f^2 C_V^f C^{Hh^*} C_1 \cos\theta \left[s(s - 4M_{H^\pm}^2)\right]^{1/2}} \qquad (52)$$

$$\boxed{(M_{34}^{+} M_1)^{+} = M_1^{+} M_{34} = \frac{g^4}{2} m_f^2 C_V^f C^{Hh} C_1^{*} \cos\theta \left[s(s - 4M_{H^\pm}^2)\right]^{1/2}} \qquad (53)$$

$$\boxed{M_{34}^{+} M_1 + M_1^{+} M_{34} = \frac{g^4}{2} m_f^2 C_V^f \cos\theta \left[s(s - 4M_{H^\pm}^2)\right]^{1/2} \left[C^{Hh^*} C_1 + C^{Hh} C_1^{*}\right]} \quad (5$$

If we don't neglect m_f^2 in $|M_1|^2$, we have:

$$|M_1|^2 = \frac{g^4}{4} |C_1|^2 \left\{ \left[|C_V^f|^2 + |C_A^f|^2 \right] \left[-\frac{1}{2} \cos^2\theta \, (S-4m_f^2)(S-4M_{H^\pm}^2) + \frac{(S-2m_f^2)}{2} (S-4M_{H^\pm}^2) \right] \right.$$

$$\left. + m_f^2 \left[|C_V^f|^2 - |C_A^f|^2 \right] (S-4M_{H^\pm}^2) \right\}$$

$$= \frac{g^4}{4} |C_1|^2 (S-4M_{H^\pm}^2) \left\{ \left[|C_V^f|^2 + |C_A^f|^2 \right] \left[-\frac{1}{2} \cos^2\theta \, (S-4m_f^2) + \frac{(S-2m_f^2)}{2} \right] \right.$$

$$\left. + m_f^2 \left[|C_V^f|^2 - |C_A^f|^2 \right] \right\}$$

$$= \frac{g^4}{4} |C_1|^2 (S-4M_{H^\pm}^2) \left\{ \frac{S}{2} \sin^2\theta \left[|C_V^f|^2 + |C_A^f|^2 \right] + |C_V^f|^2 \left[2m_f^2 \cos^2\theta - \cancel{m_f^2} \right.\right.$$

$$\left.\left. + \cancel{m_f^2} \right] + |C_A^f|^2 \left[2m_f^2 \cos^2\theta - m_f^2 - m_f^2 \right] \right.$$

$$\boxed{|M_1|^2 = \frac{g^4}{4} |C_1|^2 (S-4M_{H^\pm}^2) \left\{ \frac{S}{2} \sin^2\theta \left[|C_V^f|^2 + |C_A^f|^2 \right] + 2m_f^2 \cos^2\theta \, |C_V^f|^2 - 2m_f^2 \sin^2\theta \, |C_A^f|^2 \right\}} \quad (59)$$

$$|M_2|^2 = 4g^4 C_2^2 \left\{ -\frac{1}{2} \cos^2\theta \, (S-4M_{H^\pm}^2)(S-4m_f^2) + \frac{(S-2m_f^2)}{2} (S-4M_{H^\pm}^2) + m_f^2 (S-4M_{H^\pm}^2) \right\}$$

$$|M_2|^2 = 4g^4 C_2^2 (S-4M_{H^\pm}^2) \left\{ -\frac{1}{2} \cos^2\theta \, (S-4m_f^2) + \frac{(S-2\cancel{m_f^2})}{\cancel{2}} + \cancel{m_f^2} \right\}$$

$$\boxed{|M_2|^2 = 4g^4 C_2^2 (S-4M_{H^\pm}^2) \left\{ \frac{S}{2} \sin^2\theta + 2 \cos^2\theta \, m_f^2 \right\}} \quad (60)$$

$$M_2^+ M_1 = -g^4 C_1 C_2 \left\{ C_V^f \left[-\frac{1}{2} \cos^2\theta \, (S-4M_{H^\pm}^2)(S-4m_f^2) + \frac{(S-2m_f^2)}{2} (S-4M_{H^\pm}^2) \right] \right.$$

$$\left. + m_f^2 C_V^f (S-4M_{H^\pm}^2) \right\}$$

$$M_2^+ M_1 = -g^4 C_1 C_2 (S-4M_{H^\pm}^2) C_V^f \left[-\frac{1}{2} \cos^2\theta \, (S-4m_f^2) + \frac{1}{2} (S-2\cancel{m_f^2}) + \cancel{m_f^2} \right]$$

$$\boxed{M_2^+ M_1 = -g^4 C_1 C_2 (S-4M_{H^\pm}^2) C_V^f \left\{ \frac{S}{2} \sin^2\theta + 2m_f^2 \cos^2\theta \right\}} \quad (61)$$

$$\boxed{M_1^+ M_2 = -g^4 C_1^* C_2 (S-4M_{H^\pm}^2) C_V^f \left\{ \frac{S}{2} \sin^2\theta + 2m_f^2 \cos^2\theta \right\}} \quad (62)$$

$$\Rightarrow \boxed{M_2^+ M_1 + M_1^+ M_2 = -2g^4 C_2 \, \mathrm{Re}(C_1) (S-4M_{H^\pm}^2) C_V^f \left\{ \frac{S}{2} \sin^2\theta + 2m_f^2 \cos^2\theta \right\}} \quad (63)$$

$$\sigma(\mu^+\mu^- \to H^+H^-) = 2 \cdot \frac{1}{3}\, \frac{M_W^4 G_F^2}{\pi s} \left(s - 4M_{H^\pm}^2\right)^{3/2} \frac{s^{1/2}}{s^2} \left\{ \frac{\cos^2 2\theta_W}{64 \cos^4\theta_W} \cdot \frac{1}{\left[\left(1-\frac{M_Z^2}{s}\right)^2 + \frac{M_Z^2}{s^2}\Gamma_Z^2\right]} \right.$$

$$\left. \cdot \left[\left(4\sin^2\theta_W - 1\right)^2 + 1\right] + \sin^4\theta_W - \frac{\sin^2\theta_W \left(1-\frac{M_Z^2}{s}\right)}{4\left[\left(1-\frac{M_Z^2}{s}\right)^2 + \frac{M_Z^2\Gamma_Z^2}{s^2}\right]} \left(4\sin^2\theta_W - 1\right) \frac{\cos 2\theta_W}{\cos^2\theta_W} \right\}$$

$$= \frac{2}{3}\, \frac{M_W^4 G_F^2}{\pi s} \left(1 - \frac{4M_{H^\pm}^2}{s}\right)^{3/2} \sin^4\theta_W \left\{ 1 + \frac{\cos^2 2\theta_W}{64 \sin^4\theta_W \cos^4\theta_W} \cdot \right.$$

$$\cdot \frac{1}{\left[\left(1-\frac{M_Z^2}{s}\right)^2 + \frac{M_Z^2\Gamma_Z^2}{s^2}\right]} \cdot \left[\left(4\sin^2\theta_W - 1\right)^2 + 1\right] - \frac{\left(1-\frac{M_Z^2}{s}\right)\left(4\sin^2\theta_W - 1\right)\left(1 - 2\sin^2\theta_W\right)}{4\sin^2\theta_W \left[\left(1-\frac{M_Z^2}{s}\right)^2 + \frac{M_Z^2\Gamma_Z^2}{s^2}\right] \cos^2\theta_W}$$

$$\sigma(\mu^+\mu^- \to H^+H^-) = \frac{2}{3}\, \frac{M_W^4 G_F^2}{\pi s} \left(1 - \frac{4M_{H^\pm}^2}{s}\right)^{3/2} \sin^4\theta_W \left\{ 1 + \frac{\left(1 - 2\sin^2\theta_W\right)^2}{64 \sin^4\theta_W \cos^4\theta_W} \cdot \right.$$

$$\left. \cdot \frac{1}{\left[\left(1-\frac{M_Z^2}{s}\right)^2 + \frac{M_Z^2\Gamma_Z^2}{s^2}\right]} \cdot \left[\left(4\sin^2\theta_W - 1\right)^2 + 1\right] - \frac{\left(1-\frac{M_Z^2}{s}\right)\left(4\sin^2\theta_W - 1\right)\left(1 - 2\sin^2\theta_W\right)}{4\sin^2\theta_W \cos^2\theta_W \left[\left(1-\frac{M_Z^2}{s}\right)^2 + \frac{M_Z^2\Gamma_Z^2}{s^2}\right]} \right\}$$

---

(67)

$$\sigma(\mu^+\mu^- \to H^+H^-) = \left(3.893792914 \times 10^{11}\right) fb \times \frac{2}{3}\, \frac{G_F^2 M_W^4}{\pi s} \sin^4\theta_W \left(1 - \frac{4M_{H^\pm}^2}{s}\right)^{3/2} \cdot$$

$$\cdot \left\{ 1 + \frac{\left(1 - 2\sin^2\theta_W\right)^2}{64 \sin^4\theta_W \cos^4\theta_W} \cdot \left[\left(4\sin^2\theta_W - 1\right)^2 + 1\right] \cdot \frac{1}{\left[\left(1-\frac{M_Z^2}{s}\right)^2 + \left(\frac{M_Z\Gamma_Z}{s}\right)^2\right]} \right. -$$

$$\left. - \frac{\left(4\sin^2\theta_W - 1\right)\left(1 - 2\sin^2\theta_W\right)\left(1-\frac{M_Z^2}{s}\right)}{4\sin^2\theta_W \cos^2\theta_W \left[\left(1-\frac{M_Z^2}{s}\right)^2 + \left(\frac{M_Z\Gamma_Z}{s}\right)^2\right]} \right\}$$

$$\frac{{}_F \;\; M_W}{8\pi^2 S}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{64\pi^2 S} \frac{(S-4M_{H^\pm}^2)^{1/2}}{S^{1/2}} \frac{g^4}{4} \Big\{ S(S-4M_{H^\pm}^2)\sin^2\theta \Big[\frac{1}{8}|C_1|^2 \left((C_V^f)^2+(C_A^f)^2\right) + 2C_2^2$$

$$- C_2 \operatorname{Re}(C_1) C_V^f \Big] + 2m_f^2 \Big[(S-4M_{H^\pm}^2) \Big[\frac{|C_1|^2}{4} \left(\cos^2\theta \, (C_V^f)^2 - \sin^2\theta \, (C_A^f)^2\right)$$

$$+ 4C_2^2\cos^2\theta - 2C_2 \operatorname{Re}(C_1) C_V^f \cos^2\theta \Big] + \frac{1}{4} (C^{Hh})^2 S + [S(S-4M_{H^\pm}^2)]^{1/2} \cos\theta \, C^{Hh} \cdot$$

$$\cdot \Big[\frac{1}{2} C_V^f \operatorname{Re}(C_1) - 2C_2 \Big]\Big]\Big\} \qquad (71)$$

$$\int_{-1}^{1} \sin^2\theta \, d\cos\theta = \int_{-1}^{1} d\cos\theta - \int_{-1}^{1} \cos^2\theta \, d\cos\theta : 2 - \left.\frac{x^3}{3}\right|_{-1}^{1} = 2 - \frac{2}{3} \cdot \frac{4}{3}$$

$$\int_{-1}^{1} \cos^2\theta \, d\cos\theta = \left.\frac{x^3}{3}\right|_{-1}^{1} = \frac{2}{3}$$

$$\int_{-1}^{1} d\cos\theta = 2$$

$$\int_{-1}^{1} \cos\theta \, d\cos\theta = \left.\frac{x^2}{2}\right|_{-1}^{1} = 0$$

$$\Rightarrow \sigma(\mu^+\mu^- \to H^+H^-) = \frac{1}{4} \frac{(S-4M_{H^\pm}^2)^{1/2}}{S^{1/2}} \frac{M_W^4 G_F^2}{\pi S} \Big\{ \frac{4}{3} S(S-4M_{H^\pm}^2) \Big[\frac{1}{8}|C_1|^2 \left((C_V^f)^2+(C_A^f)^2\right)$$

$$+ 2C_2^2 - C_2 \operatorname{Re}(C_1) C_V^f \Big] + 2m_f^2 \Big[(S-4M_{H^\pm}^2) \Big[\frac{|C_1|^2}{4} \left(\frac{2}{3}(C_V^f)^2 - \frac{4}{3}(C_A^f)^2\right)$$

$$+ 4C_2^2 \frac{2}{3} - 2C_2 \operatorname{Re}(C_1) C_V^f \frac{2}{3} \Big] + \frac{1}{4} \cdot 2 \, (C^{Hh})^2 S \Big]\Big\}$$

$$\sigma(\mu^+\mu^- \to H^+H^-) = \frac{1}{4\pi S} \frac{(S-4M_{H^\pm}^2)^{1/2}}{S^{1/2}} M_W^4 G_F^2 \Big\{ \frac{4}{3} S(S-4M_{H^\pm}^2) \Big[\frac{1}{8}|C_1|^2 \left((C_V^f)^2+(C_A^f)^2\right)$$

$$+ 2C_2^2 - C_2 \operatorname{Re}(C_1) C_V^f \Big] + 2m_f^2 \Big[\frac{2}{3}(S-4M_{H^\pm}^2) \Big[\frac{|C_1|^2}{4} \left((C_V^f)^2 - 2(C_A^f)^2\right)$$

$$+ 4C_2^2 - 2C_2 \operatorname{Re}(C_1) C_V^f \Big] + \frac{1}{2} S (C^{Hh})^2 \Big]\Big\} \qquad (72)$$

$$= \frac{1}{4\pi S} \frac{(S-4M_{H^\pm}^2)^{3/2}}{S^{3/2}} M_W^4 G_F^2 \Big\{ \frac{4}{3} S^2 \Big[\frac{1}{8}|C_1|^2 \left((C_V^f)^2+(C_A^f)^2\right)$$

$$+ 2C_2^2 - C_2 \operatorname{Re}(C_1) C_V^f \Big] + \frac{4}{3} S m_f^2 \Big[\frac{|C_1|^2}{4} \left((C_V^f)^2 - 2(C_A^f)^2\right) + 4C_2^2 - 2C_2 \operatorname{Re}(C_1) C_V^f \cdot$$

$$+ m_f^2 \frac{S^2}{(S-4M_{H^\pm}^2)} (C^{Hh})^2 \Big\}$$

$$= \frac{2}{3\pi s}\Big(1-4\frac{M_{H^\pm}^2}{s}\Big)^{3/2} M_W^4 G_F^2 \sin^4\theta_W \Big\{\Big[1+\frac{\cos^2(2\theta_W)}{16\sin^4\theta_W\cos^4\theta_W}\cdot\frac{((c_V^f)^2+(c_A^f)^2)}{\Big[\Big(1-\frac{M_Z^2}{s}\Big)^2+\frac{M_Z^2\Gamma_Z^2}{s^2}\Big]}$$

$$-\frac{\cos(2\theta_W)}{2\sin^2\theta_W\cos^2\theta_W}\,\frac{\Big(1-\frac{M_Z^2}{s}\Big)c_V^f}{\Big[\Big(1-\frac{M_Z^2}{s}\Big)^2+\frac{M_Z^2\Gamma_Z^2}{s^2}\Big]}\Big]+\frac{m_f^2}{s}\Big[\Big((c_V^f)^2-2(c_A^f)^2\Big)\cdot\frac{\cos^2(2\theta_W)}{8\sin^4\theta_W\cos^4\theta_W}\cdot$$

$$\cdot\frac{1}{\Big[\Big(1-\frac{M_Z^2}{s}\Big)^2+\frac{M_Z^2\Gamma_Z^2}{s^2}\Big]}+2-\frac{\cos(2\theta_W)}{\sin^2\theta_W\cos^2\theta_W}\,\frac{\Big(1-\frac{M_Z^2}{s}\Big)c_V^f}{\Big[\Big(1-\frac{M_Z^2}{s}\Big)^2+\frac{M_Z^2\Gamma_Z^2}{s^2}\Big]}+$$

$$+\frac{3}{4}\frac{(c_x^{Hh})^2}{\Big(1-4\frac{M_{H^\pm}^2}{s}\Big)}\Big]\Big\}$$

$$(c_A^f)^2=\frac{1}{4}$$

$$(c_V^f)=\frac{4\sin^2\theta_W-1}{2}$$

$$(c_V^f)^2=\frac{(4\sin^2\theta_W-1)^2}{4}\quad\Rightarrow\quad(c_A^f)^2+(c_V^f)^2=\frac{1+(4\sin^2\theta_W-1)^2}{4}$$

$$\Rightarrow\quad(c_V^f)^2-2(c_A^f)^2=\frac{(4\sin^2\theta_W-1)^2-2}{4}$$

$$\sigma(\mu^+\mu^-\longrightarrow H^+H^-)=\frac{2}{3}\,\frac{M_W^4G_F^2}{\pi s}\Big(1-4\frac{M_{H^\pm}^2}{s}\Big)^{3/2}\sin^4\theta_W\Big\{\Big[1+\frac{(1-2\sin^2\theta_W)^2}{64\sin^4\theta_W\cos^4\theta_W}\cdot\frac{[1+(4\sin^2\theta_W-1)^2]}{\Big[\Big(1-\frac{M_Z^2}{s}\Big)^2+\Big(\frac{M_Z\Gamma_Z}{s}\Big)^2\Big]}$$

$$-\frac{(1-2\sin^2\theta_W)(4\sin^2\theta_W-1)}{4\sin^2\theta_W\cos^2\theta_W}\,\frac{\Big(1-\frac{M_Z^2}{s}\Big)}{\Big[\Big(1-\frac{M_Z^2}{s}\Big)^2+\Big(\frac{M_Z\Gamma_Z}{s}\Big)^2\Big]}\Big]+\frac{m_f^2}{s}\Big[\frac{[(4\sin^2\theta_W-1)^2-2](1-2\sin^2\theta_W)^2}{32\sin^4\theta_W\cos^4\theta_W}\cdot$$

$$\cdot\frac{1}{\Big[\Big(1-\frac{M_Z^2}{s}\Big)^2+\Big(\frac{M_Z\Gamma_Z}{s}\Big)^2\Big]}+2-\frac{(1-2\sin^2\theta_W)(4\sin^2\theta_W-1)}{2\sin^2\theta_W\cos^2\theta_W}\,\frac{\Big(1-\frac{M_Z^2}{s}\Big)}{\Big[\Big(1-\frac{M_Z^2}{s}\Big)^2+\Big(\frac{M_Z\Gamma_Z}{s}\Big)^2\Big]}+$$

$$+\frac{3}{4}\frac{(c_*^{Hh})^2}{\Big(1-4\frac{M_{H^\pm}^2}{s}\Big)}\Big]\Big\}\qquad(73)$$

---

where $$C_*^{Hh}=\frac{a_1}{1-\frac{m_{H^0}^2}{s}}-\frac{a_2}{1-\frac{m_{h^0}^2}{s}}\qquad(74)$$

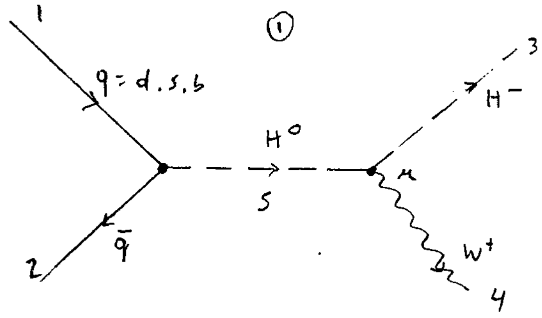
$H^{\mp} W^{\pm}$ production at a Hadron Collider

$q\bar{q} \rightarrow H^{\pm}W^{\mp}$

a) $q\bar{q} \rightarrow H^- W^+$

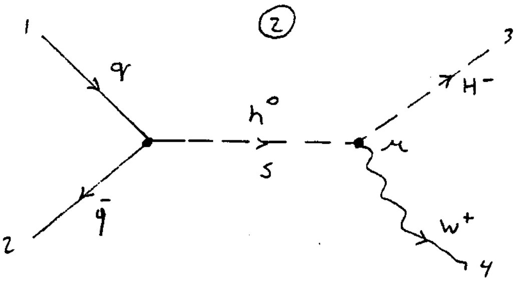
time $\longrightarrow$

①



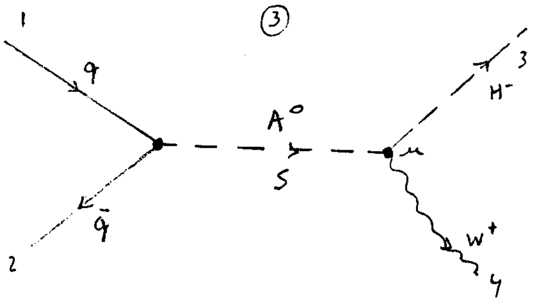
\+

②



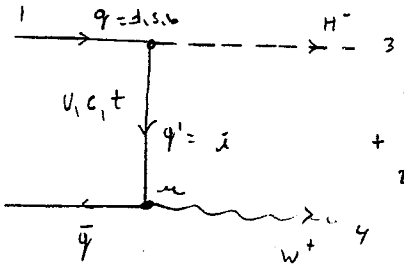
\+

③



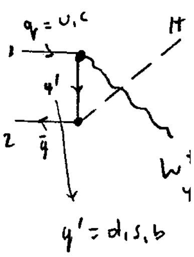
\+

④



\+

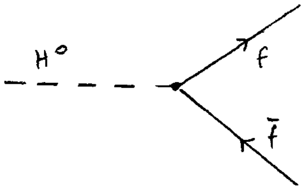
⑤ *



$q = u, d, c, s, b$

$q'_i = u, d, c, s, b, t$

$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$

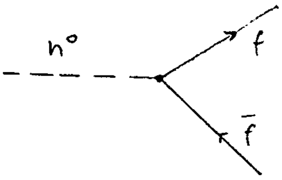


$f = d, s, b$

$$-\frac{igm_f \cos\alpha}{2M_W \cos\beta}$$

$f = u, c, t$

$$-\frac{igm_f \sin\alpha}{2M_W \sin\beta}$$

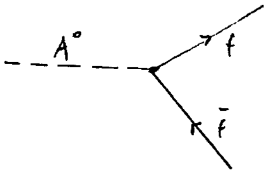


$f = d, s, b$

$$\frac{igm_f \sin\alpha}{2M_W \cos\beta}$$

$f = u, c, t$

$$-\frac{igm_f \cos\alpha}{2M_W \sin\beta}$$

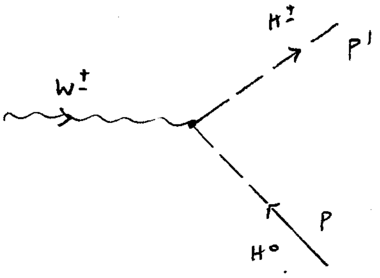


$f = d, s, b$

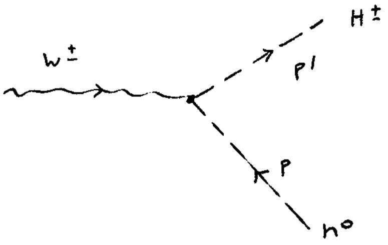
$$-\frac{gm_f \tan\beta}{2M_W}\gamma^5$$

$f = u, c, t$

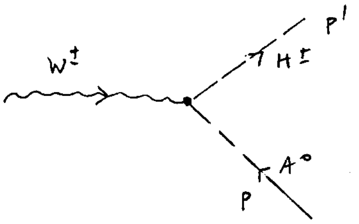
$$-\frac{gm_f \cot\beta}{2M_W}\gamma^5$$



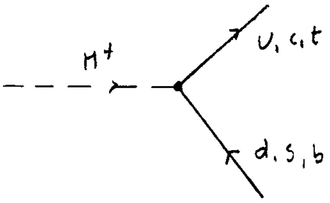
$$\pm \frac{ig}{2} \sin(\beta-\alpha)(P+P')^\mu$$



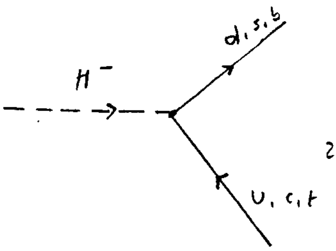
$$\mp \frac{ig}{2} \cos(\beta-\alpha)(P+P')^\mu$$



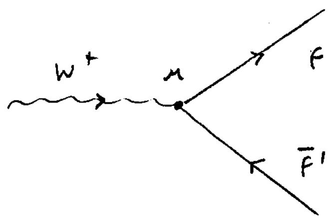
$$\frac{g}{2}(P+P')^\mu$$



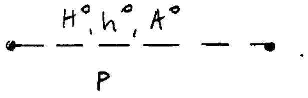
$$\frac{ig}{2\sqrt{2}M_W} V_{ud} \left\{ m_d \tan\beta(1+\gamma^5) + m_u \cot\beta(1-\gamma^5) \right\}$$



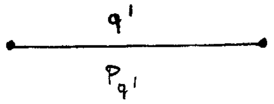
$$\frac{ig}{2\sqrt{2}M_W} V^*_{ud} \left\{ m_d \tan\beta(1-\gamma^5) + m_u \cot\beta(1+\gamma^5) \right\}$$



$$-\frac{ig}{\sqrt{2}}\gamma^\mu \frac{1}{2}(1-\gamma^5)$$



$$\frac{i}{P^2 - M^2 + iM\Gamma}$$



$$\frac{i}{\not{P}_{q'} - m_{q'} + i\epsilon} = \frac{i(\not{P}_{q'} + m_{q'})}{P_{q'}^2 - m_{q'}^2 + i\epsilon}$$

For $q = d, s, b$; $q' = u, c, t$

$$-iM_1 = \epsilon^*_{\mu 4} \frac{ig}{2} \sin(\alpha - \beta)(P_3 + P_{H^0})^\mu \frac{i}{s - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} \bar{V}_2 \left(\frac{-ig\, m_q \cos\alpha}{2M_W \cos\beta}\right) U_1 \qquad (1)$$

$$-iM_2 = \epsilon^*_{\mu 4} \frac{ig}{2} \cos(\beta - \alpha)(P_3 + P_{h^0})^\mu \frac{i}{s - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}} \bar{V}_2 \left(\frac{ig\, m_q \sin\alpha}{2M_W \cos\beta}\right) U_1 \qquad (2)$$

$$-iM_3 = \epsilon^*_{\mu 4} \frac{g}{2} (P_3 + P_{A^0})^\mu \frac{i}{s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0}} \bar{V}_2 \left(\frac{-g\, m_q \tan\beta\, \gamma^5}{2M_W}\right) U_1 \qquad (3)$$

$$-iM_4 = \bar{V}_2 \epsilon^*_{\mu 4} \left(-\frac{ig}{\sqrt{2}}\gamma^\mu \frac{1}{2}(1-\gamma^5)\right) \sum_{q'} \frac{i(\not{P}_{q'} + m_{q'})}{t - m_{q'}^2 + i\epsilon} \frac{ig}{2\sqrt{2}M_W} V_{q'q} \left\{ m_q \tan\beta (1+\gamma^5) + m_{q'} \cot\beta (1-\gamma^5) \right\} U_1$$

$(q\bar{q} \to H^- W^+)$ (4)

For $q = u, c, t$; $q' = d, s, b$ we have to change $\frac{\cos\alpha}{\cos\beta} \to \frac{\sin\alpha}{\sin\beta}$ in (1);

$\frac{\sin\alpha}{\cos\beta} \to -\frac{\cos\alpha}{\sin\beta}$ in (2) ; $\tan\beta \to \cot\beta$ in (3)

equation (4) reads :

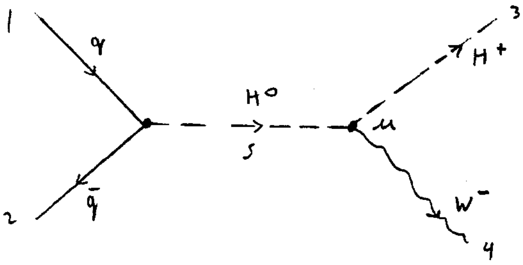
$$-iM_4 = \sum_{q'} \bar{V}_2 \frac{ig}{2\sqrt{2}M_W} V_{qq'} \left\{ m_{q'} \tan\beta (1+\gamma^5) + m_q \cot\beta (1-\gamma^5) \right\} \frac{i(\not{P}_{q'} + m_{q'})}{U - m_{q'}^2 + i\epsilon} \cdot \qquad (5)$$

$$\cdot \left(-\frac{ig}{\sqrt{2}}\gamma^\mu \frac{1}{2}(1-\gamma^5)\right) U_1 \epsilon^*_{\mu 4}$$

b) $q\bar{q} \to H^+W^-$

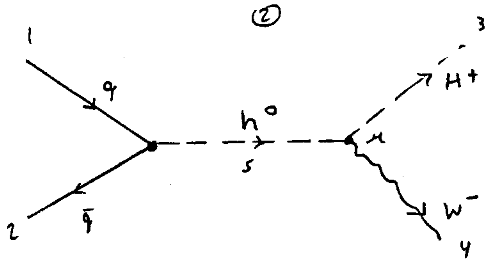
time →

①



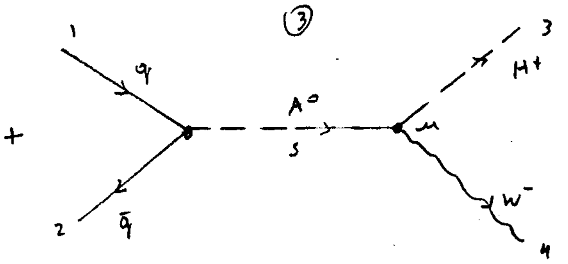
+

②



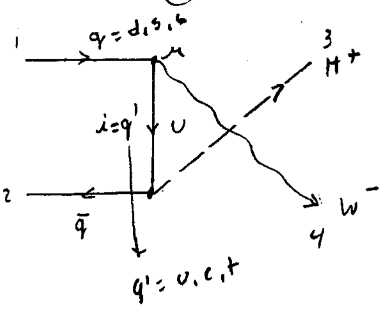
+

③



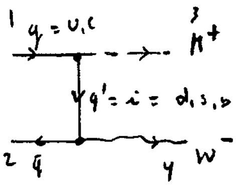
+

④



+

⑤ *



if $q = d,s,b$; $q' = u,c,t$

$$-iM'_1 = -\varepsilon^{*}_{\mu 4}\left(\frac{ig}{2}\sin(\alpha-\beta)(P_3+P_{H^0})^{\mu}\right)\frac{i}{s-m_{H^0}^2+im_{H^0}\Gamma_{H^0}}\,\bar{V}_2\left(\frac{-igm_q\cos\alpha}{2M_W\cos\beta}\right)U_1 \qquad (6)$$

$$-iM'_2 = \varepsilon^{*}_{\mu 4}\left(-\frac{ig}{2}\cos(\alpha-\beta)(P_3+P_{h^0})^{\mu}\right)\frac{i}{s-m_{h^0}^2+im_{h^0}\Gamma_{h^0}}\,\bar{V}_2\left(\frac{igm_q\sin\alpha}{2M_W\cos\beta}\right)U_1 \qquad (7)$$

$$-iM'_3 = \varepsilon^{*}_{\mu 4}\left(\frac{g}{2}(P_3+P_{A^0})^{\mu}\right)\frac{i}{s-m_{A^0}^2+im_{A^0}\Gamma_{A^0}}\,\bar{V}_2\left(\frac{-gm_q\tan\beta\,\gamma^5}{2M_W}\right)U_1 \qquad (8)$$

$$-iM'_4 = \bar{V}_2\,\frac{ig}{2\sqrt{2}M_W}\sum_{q'}V^{*}_{q'q}\left\{m_q\tan\beta(1-\gamma^5)+m_{q'}\cot\beta(1+\gamma^5)\right\}\frac{i(\not{P}_{q'}+m_{q'})}{U-m_{q'}^2+i\varepsilon}\left(-\frac{ig}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right)U_1\,\varepsilon^{*}_{\mu 4} \qquad (9)$$

For $q = u,c$; $q' = d,s,b$ we have to replace : $\frac{\cos\alpha}{\cos\beta} \to \frac{\sin\alpha}{\sin\beta}$ in (6); $\frac{\sin\alpha}{\cos\beta} \to -\frac{\cos\alpha}{\sin\beta}$ in (7) ; $\tan\beta \to \cot\beta$ in (8) . Equation (9) reads:

$$-iM'_4 = \sum_{q'}\varepsilon^{*}_{\mu 4}\,\bar{V}_2\left(-\frac{ig}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right)\frac{i(\not{P}_{q'}+m_{q'})}{t-m_{q'}^2+i\varepsilon}\,\frac{ig}{2\sqrt{2}M_W}V^{*}_{qq'}\left\{m_{q'}\tan\beta(1-\gamma^5)+ m_q\cot\beta(1+\gamma^5)\right\}U_1 \qquad (10)$$

Returning to a) For $q\bar{q} \to H^- W^+$ with $q = d, s, b$; $q' = u, c, t$

$$M_1 = -\frac{g^2 m_q \cos\alpha \sin(\alpha-\beta)}{4 M_W \cos\beta} \frac{1}{s - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} (\bar{V}_2 U_1)(P_1+P_2+P_3)^\mu \varepsilon^*_{\mu 4} \qquad (11)$$

$$M_2 = \frac{g^2 m_q \sin\alpha \cos(\alpha-\beta)}{4 M_W \cos\beta} \frac{1}{s - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}} (\bar{V}_2 U_1)(P_1+P_2+P_3)^\mu \varepsilon^*_{\mu 4} \qquad (12)$$

$$M_3 = \frac{g^2 m_q \tan\beta}{4 M_W} \frac{1}{s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0}} (\bar{V}_2 \gamma^5 U_1)(P_1+P_2+P_3)^\mu \varepsilon^*_{\mu 4} \qquad (13)$$

$$M_4 = -\sum_{q'} \frac{g^2 V_{q'q}}{8 M_W} \cdot \frac{1}{t - m_{q'}^2} \cdot \varepsilon^*_{\mu 4} \bar{V}_2 \gamma^\mu (1-\gamma^5)(\not{p}_{q'} + m_{q'})\{m_q \tan\beta (1+\gamma^5) + m_{q'}\cot\beta (1-\gamma^5)\} U_1$$

if $q = u, c$; $q' = d, s, b$ $\frac{\cos\alpha}{\cos\beta} \to \frac{\sin\alpha}{\sin\beta}$ in (11) ; $\frac{\sin\alpha}{\cos\beta} \to -\frac{\cos\alpha}{\sin\beta}$ in (12) (14)

$\tan\beta \to \cot\beta$ in (13)

$$M_4 = -\sum_{q'} \frac{g^2 V_{qq'}}{8 M_W} \cdot \frac{1}{u - m_{q'}^2 + i\varepsilon} \bar{V}_2 \{m_{q'} \tan\beta (1+\gamma^5) + m_q \cot\beta (1-\gamma^5)\} (\not{p}_{q'} + m_{q'}) \gamma^\mu (1-\gamma^5) U_1 \varepsilon^*_{\mu 4}$$

(15)

Returning to b) For $q\bar{q} \to H^+ W^-$ with $q = d, s, b$; $q' = u, c, t$

$$M_1' = \frac{g^2 m_q \cos\alpha \sin(\alpha-\beta)}{4 M_W \cos\beta} \cdot \frac{1}{s - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} \cdot (\bar{V}_2 U_1)(P_1+P_2+P_3)^\mu \varepsilon^*_{\mu 4} \qquad (16)$$

$$M_2' = -\frac{g^2 m_q \sin\alpha \cos(\alpha-\beta)}{4 M_W \cos\beta} \cdot \frac{1}{s - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}} \cdot (\bar{V}_2 U_1)(P_1+P_2+P_3)^\mu \varepsilon^*_{\mu 4} \qquad (17)$$

$$M_3' = \frac{g^2 m_q \tan\beta}{4 M_W} \cdot \frac{1}{s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0}} \cdot (\bar{V}_2 \gamma^5 U_1)(P_1+P_2+P_3)^\mu \varepsilon^*_{\mu 4} \qquad (18)$$

$$M_4' = \sum_{q'} -\frac{g^2 V^*_{q'q}}{8 M_W} \cdot \frac{1}{u - m_{q'}^2} \bar{V}_2 \{m_q \tan\beta (1-\gamma^5) + m_{q'} \cot\beta (1+\gamma^5)\} (\not{p}_{q'} + m_{q'}) \gamma^\mu (1-\gamma^5) U_1 \varepsilon^*_{\mu 4} \qquad (19)$$

if $q = u, c$; $q' = d, s, b$ replace $\frac{\cos\alpha}{\cos\beta} \to \frac{\sin\alpha}{\sin\beta}$ in (16); $\frac{\sin\alpha}{\cos\beta} \to -\frac{\cos\alpha}{\sin\beta}$ in (17)

$\tan\beta \to \cot\beta$ in (18). (19) reads:

$$M_4' = -\sum_{q'} \frac{g^2 V^*_{qq'}}{8 M_W} \cdot \frac{1}{t - m_{q'}^2} \varepsilon^*_{\mu 4} \bar{V}_2 \gamma^\mu (1-\gamma^5)(\not{p}_{q'} + m_{q'})\{m_{q'} \tan\beta (1-\gamma^5) + m_q \cot\beta (1+\gamma^5)\} U_1 \qquad (20)$$

Returning to a) For $q\bar{q} \to H^- W^+$ with $q = d, s, b$; $q' = u, c, t$:

$$M_1 + M_2 = \frac{g^2 m_q}{4 M_W} \cdot \frac{1}{\cos\beta} \left[\frac{\sin\alpha \cos(\alpha-\beta)}{s - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}} - \frac{\cos\alpha \sin(\alpha-\beta)}{s - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} \right] (\bar{V}_2 U_1)\, (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu 4}$$

$$\boxed{C_{H_b} \equiv \frac{1}{\cos\beta} \left[\frac{\sin\alpha \cos(\alpha-\beta)}{s - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}} - \frac{\cos\alpha \sin(\alpha-\beta)}{s - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} \right]} \qquad (21)$$

$$\Rightarrow \boxed{M_b^H \equiv M_1 + M_2 = \frac{g^2 m_q}{4 M_W} C_{H_b} (\bar{V}_2 U_1)\, (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu 4}} \qquad (22)$$

$$\boxed{C_{A_b} \equiv \frac{\tan\beta}{s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0}}} \; ; \; \boxed{C_{At} \equiv \frac{\cot\beta}{s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0}}} \qquad (23)$$

$$\boxed{C_{t_1} \equiv \frac{\tan\beta}{t - m_{q'}^2} \; ; \quad C_{t_2} \equiv \frac{\cot\beta}{t - m_{q'}^2}} \qquad (24)$$

$$\boxed{C_{u_1} \equiv \frac{\tan\beta}{u - m_{q'}^2} \; ; \quad C_{u_2} \equiv \frac{\cot\beta}{u - m_{q'}^2}} \qquad (25)$$

$$\Rightarrow \boxed{M_{3_b} = \frac{g^2 m_q}{4 M_W} C_{A_b} (\bar{V}_2 \gamma^5 U_1)\, (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu 4}} \qquad (26)$$

$$\Rightarrow \boxed{\begin{array}{c} M_4 = -\sum_{q'} \frac{g^2 V_{q'q}}{8 M_W} \varepsilon^*_{\mu 4} \, \bar{V}_2 \gamma^\mu (1-\gamma^5)(\not{P}_{q'} + m_{q'}) \{ m_q C_{t_1} (1+\gamma^5) + m_{q'} C_{t_2} (1-\gamma^5) \} U_1 \\ \text{for } q\bar{q} \to H^- W^+ \text{ with } q = d, s, b \; ; \; q' = u, c, t \end{array}} \qquad (27)$$

if $q = u, c$; $q' = d, s, b$:

$$\boxed{C_{Ht} \equiv -\frac{1}{\sin\beta} \left[\frac{\sin\alpha \sin(\alpha-\beta)}{s - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} + \frac{\cos\alpha \cos(\alpha-\beta)}{s - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}} \right]} \qquad (21b)$$

$$\Rightarrow \boxed{M_t^H \equiv \frac{g^2 m_q}{4 M_W} C_{Ht} (\bar{V}_2 U_1)\, (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu 4}} = M_1 + M_2 \qquad (29)$$

$$\boxed{M_4 = -\sum_{q'} \frac{g^2 V_{qq'}}{8 M_W} \bar{V}_2 \{ m_{q'} C_{u_1} (1+\gamma^5) + m_q C_{u_2} (1-\gamma^5) \} (\not{P}_{q'} + m_{q'}) \gamma^\mu (1-\gamma^5) U_1 \varepsilon^*_{\mu 4}} \qquad (28)$$

$$\boxed{M_{3_t} = \frac{g^2 m_q}{4 M_W} C_{A_t} (\bar{V}_2 \gamma^5 U_1)\, (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu 4}} \qquad (30)$$

For $q\bar{q} \to H^+ W^-$ with $q = d, s, b$; $q' = u, c, t$:

$$\boxed{M_b^{H'} = M_1' + M_2' = -\frac{g^2 m_q}{4 M_W} C_{Hb} (\bar{V}_2 U_1) (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu_4} = - M_b^H} \quad (31a)$$

$$\boxed{M_{3_b}' = M_{3b}} \quad (31b)$$

$$\boxed{M_4' = -\sum_{q'} \frac{g^2 V^*_{q'q}}{8 M_W} \bar{V}_2 \{ m_q C_{U1} (1-\gamma^5) + m_{q'} C_{U2} (1+\gamma^5) \} (\not{P}_{q'} + m_{q'}) \gamma^\mu (1-\gamma^5) U_1 \varepsilon^*_{\mu_4}} \quad (31c)$$

If $q = u, c$; $q' = d, s, b$:

$$\boxed{M_t^{H'} = M_1' + M_2' = -\frac{g^2 m_q}{4 M_W} C_{Ht} (\bar{V}_2 U_1) (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu_4} = - M_t^H} \quad (32a)$$

$$\boxed{M_{3_t}' = M_{3t}} \quad (32b)$$

$$\boxed{M_4' = -\sum_{q'} \frac{g^2 V^*_{qq'}}{8 M_W} \varepsilon^*_{\mu_4} \bar{V}_2 \gamma^\mu (1-\gamma^5) (\not{P}_{q'} + m_{q'}) \{ m_{q'} C_{t1} (1-\gamma^5) + m_q C_{t2} (1+\gamma^5) \} U_1} \quad (32c)$$

On the other hand:

$$(1-\gamma^5)(\not{P}_{q'} + m_{q'}) \{ m_q C_{t1} (1+\gamma^5) + m_{q'} C_{t2} (1-\gamma^5) \} = m_q C_{t1} (1-\gamma^5) \not{P}_{q'} (1+\gamma^5)$$

$$+ m_q m_{q'} C_{t1} \overset{0}{\cancel{(1-\gamma^5)(1+\gamma^5)}} + m_{q'} C_{t2} (1-\gamma^5) \not{P}_{q'} (1-\gamma^5) + m_{q'}^2 C_{t2} . 2 (1-\gamma^5)$$

$$= 2 m_q C_{t1} (1-\gamma^5) \not{P}_{q'} + m_{q'} C_{t2} \overset{0}{\cancel{(1-\gamma^5)(1+\gamma^5)}} \not{P}_{q'} + 2 m_{q'}^2 C_{t2} (1-\gamma^5)$$

$$= 2 (1-\gamma^5) [m_q C_{t1} \not{P}_{q'} + m_{q'}^2 C_{t2}]$$

$$\Rightarrow \boxed{M_4 = -\sum_{q'} \frac{g^2 V_{q'q}}{4 M_W} \varepsilon^*_{\mu_4} \bar{V}_2 \gamma^\mu (1-\gamma^5) [m_q C_{t1} \not{P}_{q'} + m_{q'}^2 C_{t2}] U_1} \quad (27b)$$

$q\bar{q} \to H^- W^+$ $q = d, s, b$
$q' = u, c, t$

also:

$$\{ m_{q'} C_{U1} (1+\gamma^5) + m_q C_{U2} (1-\gamma^5) \} (\not{P}_{q'} + m_{q'}) \gamma^\mu (1-\gamma^5)$$

$$= \{ m_{q'} C_{U1} (1+\gamma^5) + m_q C_{U2} (1-\gamma^5) \} (\not{P}_{q'} + m_{q'}) (1+\gamma^5) \gamma^\mu$$

$$= \{ m_{q'} C_{U1} (1+\gamma^5) \not{P}_{q'} (1+\gamma^5) + m_{q'}^2 C_{U1} 2 (1+\gamma^5) + m_q C_{U2} (1-\gamma^5) \not{P}_{q'} (1+\gamma^5)$$

$$+ m_q m_{q'} C_{U2} \underset{0}{\cancel{(1-\gamma^5)(1+\gamma^5)}} \} \gamma^\mu$$

$$= \{ m_{q'} C_{U1} \overset{-}{\cancel{(1+\gamma^5)}} (1-\gamma^5) \not{P}_{q'} + 2 m_{q'}^2 C_{U1} (1+\gamma^5) + 2 m_q C_{U2} (1-\gamma^5) \not{P}_{q'} \} \gamma^\mu$$

$$= 2 \{ m_{q'}^2 C_{U1} + m_q C_{U2} \not{P}_{q'} \} \gamma^\mu (1-\gamma^5)$$

$\Rightarrow$ For $q\bar{q} \rightarrow H^- W^+$; $q = u, c$; $q' = d, s, b$.

$$\boxed{M_4 = -\sum_{q'} \frac{g^2 V_{qq'}}{4M_W} \bar{v}_2 \left\{ m_{q'}^2 C_{v_1} + m_q C_{v_2} \not{p}_{q'} \right\} \gamma^\mu (1-\gamma^5) u_1 \varepsilon^*_{\mu 4} =} M_{4_{III}} \quad (28b)$$

and

$$\left\{ m_q C_{v_1} (1-\gamma^5) + m_{q'} C_{v_2} (1+\gamma^5) \right\} (\not{p}_{q'} + m_{q'}) \gamma^\mu (1-\gamma^5)$$

$$= \left\{ m_q C_{v_1} (1-\gamma^5) + m_{q'} C_{v_2} (1+\gamma^5) \right\} (\not{p}_{q'} + m_{q'}) (1+\gamma^5) \gamma^\mu$$

$$= \left\{ m_q C_{v_1} (1-\gamma^5) \not{p}_{q'} (1+\gamma^5) + m_q m_{q'} C_{v_1} \overset{0}{\cancel{(1-\gamma^5)(1+\gamma^5)}} + m_{q'} C_{v_2} (1+\gamma^5) \not{p}_{q'} (1+\gamma^5) \right.$$

$$\left. + m_{q'}^2 C_{v_2} \, 2 (1+\gamma^5) \right\} \gamma^\mu$$

$$= \left\{ 2 m_q C_{v_1} (1-\gamma^5) \not{p}_{q'} + m_{q'} C_{v_2} \overset{0}{\cancel{(1+\gamma^5)(1-\gamma^5)}} \not{p}_{q'} + 2 m_{q'}^2 C_{v_2} (1+\gamma^5) \right\} \gamma^\mu$$

$$= 2 \left\{ m_q C_{v_1} \not{p}_{q'} + m_{q'}^2 C_{v_2} \right\} \gamma^\mu (1-\gamma^5)$$

$$\left(\gamma^5 \gamma^\mu + \gamma^\mu \gamma^5 = 0 \; ; \quad (1 \mp \gamma^5)^2 = 2 (1 \mp \gamma^5) \right)$$

$\Rightarrow$ For $q\bar{q} \rightarrow H^+ W^-$; $q = d, s, b$; $q' = u, c, t$

$$\boxed{M'_4 = -\sum_{q'} \frac{g^2 V^*_{q'q}}{4M_W} \bar{v}_2 \left\{ m_q C_{v_1} \not{p}_{q'} + m_{q'}^2 C_{v_2} \right\} \gamma^\mu (1-\gamma^5) u_1 \varepsilon^*_{\mu 4} =} M_{4_{II}} \quad (31\,d)$$

additionally :

$$(1-\gamma^5)(\not{p}_{q'} + m_{q'}) \left\{ m_{q'} C_{t_1} (1-\gamma^5) + m_q C_{t_2} (1+\gamma^5) \right\}$$

$$= m_{q'} C_{t_1} (1-\gamma^5) \not{p}_{q'} (1-\gamma^5) + m_q C_{t_2} (1-\gamma^5) \not{p}_{q'} (1+\gamma^5) + m_{q'}^2 C_{t_1} (1-\gamma^5)^2$$

$$+ m_q m_{q'} C_{t_2} \overset{0}{\cancel{(1-\gamma^5)(1+\gamma^5)}}$$

$$= m_{q'} C_{t_1} \overset{0}{\cancel{(1-\gamma^5)(1+\gamma^5)}} \not{p}_{q'} + 2 m_q C_{t_2} (1-\gamma^5) \not{p}_{q'} + 2 m_{q'}^2 C_{t_1} (1-\gamma^5)$$

$$= 2 (1-\gamma^5) \left\{ m_q C_{t_2} \not{p}_{q'} + m_{q'}^2 C_{t_1} \right\}$$

$\Rightarrow$ For $q\bar{q} \rightarrow H^+ W^-$; $q = u, c$; $q' = d, s, b$

$$\boxed{M'_4 = -\sum_{q'} \frac{g^2 V^*_{qq'}}{4M_W} \varepsilon^*_{\mu 4} \bar{v}_2 \gamma^\mu (1-\gamma^5) \left\{ m_q C_{t_2} \not{p}_{q'} + m_{q'}^2 C_{t_1} \right\} u_1} \quad (32\,d)$$

$$= M_{4_{IV}}$$

I)

For $q\bar{q} \rightarrow H^- W^+$ with $q = d, s, b$; $q' = u, c, t$

$$M_{\mathrm{I}} = M_1 + M_2 + M_3 + M_4 = M_b^H + M_3 + M_4 \tag{33}$$

$$M_b^H = M_1 + M_2 = \frac{g^2 m_q}{4 M_W} C_{Hb} (\bar{V}_2 U_1) (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu_4} \tag{34}$$

$$M_{3_b} = \frac{g^2 m_q}{4 M_W} C_{A_b} (\bar{V}_2 \gamma^5 U_1) (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu_4} \tag{35}$$

$$M_4 = -\sum_{q'} \frac{g^2 V_{q'q}}{4 M_W} \varepsilon^*_{\mu_4} \bar{V}_2 \gamma^\mu (1-\gamma^5) [m_q C_{t1} \not{p}_{q'} + m_{q'}^2 C_{t2}] U_1 \tag{36}$$

ok

II)

For $q\bar{q} \rightarrow H^+ W^-$ with $q = d, s, b$; $q' = u, c, t$

$$M_{\mathrm{II}} = M_1' + M_2' + M_{3b}' + M_4' = -M_b^H + M_{3_b} + M_4' \tag{37}$$

$$M_1' + M_2' = -M_b^H = -\frac{g^2 m_q}{4 M_W} C_{Hb} (\bar{V}_2 U_1) (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu_4} \tag{38}$$

$$M_{3b}' = M_{3_b} = \frac{g^2 m_q}{4 M_W} C_{Ab} (\bar{V}_2 \gamma^5 U_1) (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu_4} \tag{39}$$

$$M_{4_{\mathrm{II}}} = -\sum_{q'} \frac{g^2 V^*_{q'q}}{4 M_W} \bar{V}_2 [m_q C_{U1} \not{p}_{q'} + m_{q'}^2 C_{U2}] \gamma^\mu (1-\gamma^5) U_1 \varepsilon^*_{\mu_4} \tag{40}$$

ok

III)

For $q\bar{q} \rightarrow H^- W^+$ with $q = u, c$; $q' = d, s, b$.

$$M_{\mathrm{III}} = M_1 + M_2 + M_3 + M_4 = M_t^H + M_{3_t} + M_{4\,\mathrm{III}} \tag{41}$$

$$M_1 + M_2 = M_t^H = \frac{g^2 m_q C_{Ht}}{4 M_W} (\bar{V}_2 U_1) (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu_4} \tag{42}$$

$$M_{3_t} = \frac{g^2 m_q}{4 M_W} C_{A_t} (\bar{V}_2 \gamma^5 U_1) (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu_4} \tag{43}$$

$$M_{4_{\mathrm{III}}} = -\sum_{q'} \frac{g^2 V_{qq'}}{4 M_W} \bar{V}_2 [m_q C_{U2} \not{p}_{q'} + m_{q'}^2 C_{U1}] \gamma^\mu (1-\gamma^5) U_1 \varepsilon^*_{\mu_4} \tag{44}$$

ok

IV)

For $q\bar{q} \rightarrow H^+ W^-$ with $q = u, c$; $q' = d, s, b$

$$M_{\mathrm{IV}} = M_1' + M_2' + M_{3_t}' + M_{4_{\mathrm{IV}}} = -M_t^H + M_{3_t} + M_{4_{\mathrm{IV}}} \tag{45}$$

$$M_t^{H'} = M_1' + M_2' = -M_t^H = \frac{-g^2 m_q C_{Ht}}{4 M_W} (\bar{V}_2 U_1) (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu_4} \tag{46}$$

$$M_{3_t}' = M_{3_t} = \frac{g^2 m_q}{4 M_W} C_{A_t} (\bar{V}_2 \gamma^5 U_1) (P_1 + P_2 + P_3)^\mu \varepsilon^*_{\mu_4} \tag{47}$$

$$M_{4_{\mathrm{IV}}} = -\sum_{q'} \frac{g^2 V^*_{qq'}}{4 M_W} \varepsilon^*_{\mu_4} \bar{V}_2 \gamma^\mu (1-\gamma^5) [m_q C_{t2} \not{p}_{q'} + m_{q'}^2 C_{t1}] U_1 \tag{48}$$

ok

I)

$$M_I = M_b^H + M_3 + M_4$$

$$|M_I|^2 = M_I^+ M_I = (M_b^H + M_3 + M_4)^+ (M_b^H + M_3 + M_4)$$

$$|M_I|^2 = M_b^{H+} M_b^H + M_b^{H+} M_3 + M_b^{H+} M_4 + M_3^+ M_b^H + M_3^+ M_3 + M_3^+ M_4 + M_4^+ M_b^H + M_4^+ M_3 + |M_4|^2$$

$$|M_I|^2 = |M_b^H|^2 + M_b^{H+} M_3 + M_3^+ M_b^H + M_b^{H+} M_4 + M_4^+ M_b^H + |M_3|^2 + M_3^+ M_4 + M_4^+ M_3 + |M_4|^2$$

$$\boxed{|M_I|^2 = |M_b^H|^2 + |M_3|^2 + |M_4|^2 + (M_b^{H+} M_3 + M_3^+ M_b^H) + (M_b^{H+} M_4 + M_4^+ M_b^H) + (M_3^+ M_4 + M_4^+ M_3)} \tag{49}$$

II)

$$M_{II} = - M_b^H + M_{3_b} + M_4' \qquad (M_4' = M_{4\,II})$$

$$|M_{II}|^2 = (-M_b^H + M_{3_b} + M_4')^+ (-M_b^H + M_{3_b} + M_4')$$

$$\boxed{|M_{II}|^2 = |M_b^H|^2 + |M_{3_b}|^2 + |M_4'|^2 - (M_b^{H+} M_{3_b} + M_{3_b}^+ M_b^H) - (M_b^{H+} M_4' + M_4'^+ M_b^H) + (M_4'^+ M_{3_b} + M_{3_b}^+ M_4')} \tag{50}$$

III)

$$M_{III} = M_t^H + M_{3_t} + M_{4\,III}$$

$$|M_{III}|^2 = (M_t^H + M_{3_t} + M_{4\,III})^+ (M_t^H + M_{3_t} + M_{4\,III})$$

$$\boxed{\begin{aligned}|M_{III}|^2 = {} & |M_t^H|^2 + |M_{3_t}|^2 + |M_{4\,III}|^2 + (M_t^{H+} M_{3_t} + M_{3_t}^+ M_t^H) + (M_t^{H+} M_{4\,III} + M_{4\,III}^+ M_t^H) \\ & + (M_{3_t}^+ M_{4\,III} + M_{4\,III}^+ M_{3_t})\end{aligned}} \tag{51}$$

IV)

$$M_{IV} = - M_t^H + M_{3_t} + M_{4\,IV}$$

$$|M_{IV}|^2 = (-M_t^H + M_{3_t} + M_{4\,IV})^+ (-M_t^H + M_{3_t} + M_{4\,IV})$$

$$\boxed{\begin{aligned}|M_{IV}|^2 = {} & |M_t^H|^2 + |M_{3_t}|^2 + |M_{4\,IV}|^2 - (M_t^{H+} M_{3_t} + M_{3_t}^+ M_t^H) - (M_t^{H+} M_{4\,IV} + M_{4\,IV}^+ M_t^H) \\ & + (M_{4\,IV}^+ M_{3_t} + M_{3_t}^+ M_{4\,IV})\end{aligned}} \tag{52}$$

$$\boxed{|M^H|^2 = \frac{g^4 m_q^2 |C_H|^2}{8 M_W^4} \lambda(s, M_W^2, M_{H^\pm}^2)(s - 4m_q^2)} \qquad (53)$$

with

$C_H = C_{Hb}$ for $\overbrace{q = d, s, b \, ; \, q' = u, c, t}^{(I, II)}$

$C_H = C_{Ht}$ for $\underbrace{q = u, c \, ; \, q' = d, s, b}_{(III, IV)}$

see (23) $\mu^-\mu^+ \to H^\mp W^\pm$

$$\boxed{|M_3|^2 = \frac{g^4 m_q^2 |C_A|^2 s}{8 M_W^4} \lambda(s, M_W^2, M_{H^\pm}^2)} \qquad (54)$$

with $C_A = C_{Ab}$ for I, II

$C_A = C_{At}$ for III, IV

see (25)

$$\boxed{M^{H^+} M_3 = M_3^+ M^H = 0} \qquad (55)$$

for I, II, III, IV

(see 59)

$$M_b^{A+} M_4 = -\sum_{q'} \frac{g^4 m_q C_{H_b}^* V_{q'q}}{16 M_W^2} \sum_\lambda \varepsilon_{\mu 4}^* \varepsilon_{\nu 4} (P_1+P_2+P_3)^\mu \sum_s (\overline{U}_1 V_2) \overline{V}_2 \gamma^\nu (1-\gamma^5) [m_q C_{t_1} \not{P}_{q'} + m_{q'}^2 C_{t_2}] U_1$$

$$= -\sum_{q'} \frac{g^4 m_q C_{H_b}^* V_{q'q}}{16 M_W^2} \left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) (P_1+P_2+P_3)^\mu \, Tr\left[(\not{P}_2 - m_q)\gamma^\nu (1-\gamma^5)(m_q C_{t_1} \not{P}_{q'} + m_{q'}^2 C_{t_2})(\not{P}_1 + m_q)\right] \quad (76)$$

$$Tr\left[(\not{P}_2 - m_q) \overbrace{\gamma^\nu (1-\gamma^5)}^{(\gamma^\nu - \gamma^\nu\gamma^5)} (m_q C_{t_1} \not{P}_{q'} + m_{q'}^2 C_{t_2})(\not{P}_1 + m_q)\right]$$

$$= Tr\left[(\not{P}_2 \gamma^\nu - \not{P}_2 \gamma^\nu\gamma^5 - m_q \gamma^\nu + m_q \gamma^\nu\gamma^5)(m_q C_{t_1} \not{P}_{q'} \not{P}_1 + m_q^2 C_{t_1} \not{P}_{q'} + m_{q'}^2 C_{t_2} \not{P}_1 + m_q m_{q'}^2 C_{t_2})\right]$$

$$= m_q C_{t_1} Tr(\not{P}_2 \gamma^\nu \not{P}_{q'} \not{P}_1) + m_q^2 C_{t_1} \cancelto{0}{Tr(\not{P}_2 \gamma^\nu \not{P}_{q'})} + m_{q'}^2 C_{t_2} \cancelto{0}{Tr(\not{P}_2 \gamma^\nu \not{P}_1)} + m_q m_{q'}^2 C_{t_2} Tr(\not{P}_2 \gamma^\nu)$$
$$- m_q C_{t_1} Tr(\not{P}_2 \gamma^\nu \gamma^5 \not{P}_{q'} \not{P}_1) - m_q^2 C_{t_1} \cancelto{0}{Tr(\not{P}_2 \gamma^\nu \gamma^5 \not{P}_{q'})} - m_{q'}^2 C_{t_2} \cancelto{0}{Tr(\not{P}_2 \gamma^\nu \gamma^5 \not{P}_1)}$$
$$- m_q m_{q'}^2 C_{t_2} \cancelto{0}{Tr(\not{P}_2 \gamma^\nu \gamma^5)} - m_q^2 C_{t_1} \cancelto{0}{Tr(\gamma^\nu \not{P}_{q'} \not{P}_1)} - m_q^3 C_{t_1} Tr(\gamma^\nu \not{P}_{q'}) - m_q m_{q'}^2 C_{t_2} Tr(\gamma^\nu \not{P}_1)$$
$$- m_q^2 m_{q'}^2 C_{t_2} \cancelto{0}{Tr(\gamma^\nu)} + m_q^2 C_{t_1} \cancelto{0}{Tr(\gamma^\nu\gamma^5 \not{P}_{q'} \not{P}_1)} + m_q^3 C_{t_1} \cancelto{0}{Tr(\gamma^\nu\gamma^5 \not{P}_{q'})} + m_q m_{q'}^2 C_{t_2} \cancelto{0}{Tr(\gamma^\nu\gamma^5 \not{P}_1)}$$
$$+ m_q^2 m_{q'}^2 C_{t_2} \cancelto{0}{Tr(\gamma^\nu\gamma^5)}$$

because $Tr(\gamma^5\gamma^\mu\gamma^\nu) = 0$; Tr (odd # of γ's) $= 0$; $\gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0$

$$= m_q C_{t_1} Tr(\not{P}_2 \gamma^\nu \not{P}_{q'} \not{P}_1) + m_q m_{q'}^2 C_{t_2} Tr(\not{P}_2 \gamma^\nu) - m_q C_{t_1} Tr(\gamma^5 \not{P}_2 \gamma^\nu \not{P}_{q'} \not{P}_1)$$
$$- m_q^3 C_{t_1} Tr(\gamma^\nu \not{P}_{q'}) - m_q m_{q'}^2 C_{t_2} Tr(\gamma^\nu \not{P}_1)$$

$$= m_q C_{t_1} Tr(\not{P}_2 \gamma^\nu \not{P}_{q'} \gamma^\alpha) P_{1\alpha} + m_q m_{q'}^2 C_{t_2} Tr(\gamma^\alpha\gamma^\nu) P_{2\alpha} - m_q C_{t_1} Tr(\gamma^5\gamma^\alpha\gamma^\nu\gamma^\rho\gamma^\beta) \cdot$$
$$P_{2\alpha} P_{q'\rho} P_{1\beta} - m_q^3 C_{t_1} Tr(\gamma^\nu\gamma^\alpha) P_{q'\alpha} - m_q m_{q'}^2 C_{t_2} Tr(\gamma^\nu\gamma^\alpha) P_{1\alpha}$$

$$Tr() = 4 m_q C_{t_1} [P_2^\alpha P_{q'}^\nu + P_2^\nu P_{q'}^\alpha - (P_2 \cdot P_{q'}) g^{\alpha\nu}] P_{1\alpha} + 4 m_q m_{q'}^2 C_{t_2} \overbrace{g^{\alpha\nu} P_{2\alpha}}^{P_2^\nu} - 4 i m_q C_{t_1} \varepsilon^{\alpha\nu\rho\beta} \cdot$$
$$\cdot P_{2\alpha} P_{q'\rho} P_{1\beta} - 4 m_q^3 C_{t_1} \overbrace{g^{\nu\alpha} P_{q'\alpha}}^{P_{q'}^\nu} - 4 m_q m_{q'}^2 C_{t_2} \overbrace{g^{\nu\alpha} P_{1\alpha}}^{P_1^\nu} \quad (77)$$

$$g_{\mu\nu} (P_1+P_2+P_3)^\mu \varepsilon^{\alpha\nu\rho\beta} P_{2\alpha} P_{q'\rho} P_{1\beta} = (P_1+P_2+P_3)_\nu \varepsilon^{\alpha\nu\rho\beta} P_{2\alpha} P_{q'\rho} P_{1\beta}$$
$$= (P_1+P_2+P_3)_\nu \varepsilon^{\alpha\nu\rho\beta} P_{2\alpha} (P_{1\rho} - P_{3\rho}) P_{1\beta}$$
$$= P_{3\nu} \varepsilon^{\alpha\nu\rho\beta} P_{2\alpha} (P_{1\rho} - P_{3\rho}) P_{1\beta} = 0$$

because $P_\alpha P_\rho \varepsilon^{\alpha\nu\rho\beta} = 0$

$$P_{4\nu}\,\varepsilon^{\alpha\nu\rho\beta}\,P_{2\alpha}P_{q'\rho}P_{1\beta} = (P_1+P_2-P_3)_\nu\,P_{2\alpha}P_{q'\rho}P_{1\beta}\,\varepsilon^{\alpha\nu\rho\beta} = -P_{3\nu}P_{2\alpha}(P_{1\rho}-P_{3\rho})P_{1\beta}\,\varepsilon^{\alpha\nu\rho\beta} = 0$$

$$\Rightarrow M_b^{H^+}M_4 = -\sum_{q'}\frac{g^4 m_q C^*_{H_b}V_{q'q}}{16M_W^2}\,4\left(-(P_1+P_2+P_3)_\nu + \frac{((P_1+P_2+P_3).P_4)}{M_W^2}P_{4\nu}\right)\Big\{m_q C_{t_1}\left[(P_1.P_2)P_{q'}^\nu\right.$$
$$\left.+(P_1.P_{q'})P_2^\nu - (P_2.P_{q'})P_1^\nu\right] + m_q m_{q'}^2 C_{t_2}P_2^\nu - m_q^3 C_{t_1}P_{q'}^\nu - m_q m_{q'}^2 C_{t_2}P_1^\nu\Big\} \quad (78)$$

$$= -\sum_{q'}\frac{g^4 m_q C^*_{H_b}V_{q'q}}{4M_W^2}\Big\{-m_q C_{t_1}(P_1.P_2)(P_{q'}.(P_1+P_2+P_3)) - m_q C_{t_1}(P_1.P_{q'})(P_2.(P_1+P_2+P_3))$$
$$+ m_q C_{t_1}(P_2.P_{q'})(P_1.(P_1+P_2+P_3)) - m_q m_{q'}^2 C_{t_2}(P_2.(P_1+P_2+P_3))$$
$$+ m_q^3 C_{t_1}(P_{q'}.(P_1+P_2+P_3)) + m_q m_{q'}^2 C_{t_2}(P_1.(P_1+P_2+P_3))$$
$$+\frac{m_q C_{t_1}}{M_W^2}(P_1.P_2)(P_4.P_{q'})(P_4.(P_1+P_2+P_3)) + \frac{m_q C_{t_1}}{M_W^2}(P_1.P_{q'})(P_2.P_4)(P_4.(P_1+P_2+P_3))$$
$$-\frac{m_q C_{t_1}}{M_W^2}(P_2.P_{q'})(P_1.P_4)(P_4.(P_1+P_2+P_3)) + \frac{m_q m_{q'}^2}{M_W^2}C_{t_2}(P_2.P_4)(P_4.(P_1+P_2+P_3))$$
$$-\frac{m_q^3 C_{t_1}}{M_W^2}(P_4.P_{q'})(P_4.(P_1+P_2+P_3)) - \frac{m_q m_{q'}^2}{M_W^2}C_{t_2}(P_1.P_4)(P_4.(P_1+P_2+P_3))\Big\} \quad (79)$$

$$P_{q'}.(P_1+P_2+P_3) = (P_1-P_3).(P_1+P_2+P_3) = m_q^2 + (P_1.P_2) + (\cancel{P_1}.P_3) - (\cancel{P_1}.P_3) - (P_2.P_3) - M_{H^\pm}^2$$

$$P_{q'}.(P_1+P_2+P_3) = \cancel{m_q^2} + \frac{s}{2} - \cancel{m_q^2} - (P_2.P_3) - M_{H^\pm}^2$$

$$U = (P_1-P_4)^2 = (P_3-P_2)^2 = M_{H^\pm}^2 + m_q^2 - 2(P_2.P_3)$$

$$U = 2m_q^2 + M_{H^\pm}^2 + M_W^2 - s - t$$

$$\Rightarrow 2m_q^2 + \cancel{M_{H^\pm}^2} + M_W^2 - s - t = \cancel{M_{H^\pm}^2} + \cancel{m_q^2} - 2(P_2.P_3)$$

$$\boxed{(P_2.P_3) = \frac{s+t-M_W^2-m_q^2}{2}} \quad (80)$$

$$\Rightarrow P_{q'}.(P_1+P_2+P_3) = \frac{\cancel{s}}{\cancel{2}} - \frac{(\cancel{s}+t-M_W^2-m_q^2)}{\cancel{2}} - M_{H^\pm}^2$$

$$\boxed{P_{q'}.(P_1+P_2+P_3) = \frac{M_W^2+m_q^2-2M_{H^\pm}^2-t}{2}} \quad (81)$$

$$P_2 . (P_1 + P_2 + P_3) = (P_1 . P_2) + m_q^2 + (P_2 . P_3)$$

$$= \frac{s}{2} - \cancel{m_q^2} + \cancel{m_q^2} + \frac{s}{2} + \frac{(t - M_W^2 - m_q^2)}{2}$$

$$\boxed{P_2 . (P_1 + P_2 + P_3) = \frac{2s + t - M_W^2 - m_q^2}{2}} \qquad (82)$$

$$P_1 . (P_1 + P_2 + P_3) = m_q^2 + (P_1 . P_2) + (P_1 . P_3)$$

$$(P_1 . P_3) = ?$$

$$t = (P_1 - P_3)^2$$

$$t = m_q^2 + M_{H^\pm}^2 - 2(P_1 . P_3)$$

$$\boxed{(P_1 . P_3) = \frac{m_q^2 + M_{H^\pm}^2 - t}{2}} \qquad (83)$$

$$P_1 . (P_1 + P_2 + P_3) = \cancel{m_q^2} + \frac{s}{2} - \cancel{m_q^2} + \frac{m_q^2 + M_{H^\pm}^2 - t}{2}$$

$$\boxed{P_1 . (P_1 + P_2 + P_3) = \frac{s + m_q^2 + M_{H^\pm}^2 - t}{2}} \qquad (84)$$

$$P_4 . (P_1 + P_2 + P_3) = (P_1 + P_2 - P_3) . (P_1 + P_2 + P_3) = \cancel{m_q^2} + \cancel{(P_1 . P_2)} + \cancel{(P_1 . P_3)} + \cancel{(P_1 . P_2)} + \cancel{m_q^2} + \cancel{(P_2 . P_3)}$$
$$- \cancel{(P_1 . P_3)} - \cancel{(P_2 . P_3)} - M_{H^\pm}^2$$

$$P_4 . (P_1 + P_2 + P_3) = 2\cancel{m_q^2} + 2\left(\frac{s}{2} - \cancel{m_q^2}\right) - M_{H^\pm}^2$$

$$\boxed{P_4 . (P_1 + P_2 + P_3) = s - M_{H^\pm}^2} \qquad (85)$$

$$M_b^{H^+} M_4 = -\sum_{q'} \frac{g^4 m_q C^{*}_{H_b} V_{q'q}}{4 M_W^2} \Big\{ -m_q C_{t1} \overset{\frac{1}{2}(s - 2m_q^2)}{\left(\frac{s}{2} - m_q^2\right)} \frac{1}{2}\left(M_W^2 + m_q^2 - 2M_{H^\pm}^2 - t\right) - m_q C_{t1} \frac{1}{2}\left(m_q^2 + t - M_{H^\pm}^2\right) \frac{1}{2} .$$

$$. \frac{(2s + t - M_W^2 - m_q^2)}{2} + m_q C_{t1} \frac{1}{2} (M_W^2 - m_q^2 - t) \frac{(s + m_q^2 + M_{H^\pm}^2 - t)}{2}$$

$$- m_q m_{q'}^2 C_{t2} \frac{1}{2} (2s + t - M_W^2 - m_q^2) + m_q^3 C_{t1} \frac{1}{2} (M_W^2 + m_q^2 - 2M_{H^\pm}^2 - t)$$

$$+ m_q m_{q'}^2 C_{t2} \frac{1}{2} (s + m_q^2 + M_{H^\pm}^2 - t) + \frac{m_q C_{t1}}{M_W^2} (s - M_{H^\pm}^2) \Big[\overset{\frac{1}{2}(s - 2m_q^2)}{\left(\frac{s}{2} - m_q^2\right)} \frac{(M_W^2 - m_q^2 + t)}{2} + \frac{(m_q^2 + t - M_{H^\pm}^2)}{2} ,$$

$$. \frac{1}{2} (M_W^2 + m_q^2 - t) - \frac{(M_W^2 - m_q^2 - t)}{2} \frac{(s + t - m_q^2 - M_{H^\pm}^2)}{2} \Big]$$

$$+\frac{m_q\, m_{q'}^2}{M_W^2} C_{t2}\,(s - M_{H^\pm}^2)\left[\frac{(M_W^2 + \cancel{m_q^2} - \cancel{t})}{2} - \frac{(s + \cancel{t} - \cancel{m_q^2} - M_{H^\pm}^2)}{2}\right] - \frac{m_q^3 C_{t1}}{M_W^2}(s - M_{H^\pm}^2)\frac{1}{2}(M_W^2 - m_q^2 + t)\Big\}$$

$$(86)$$

$$\begin{aligned}
M_b^{H^+} M_4 = -\sum_{q'} \frac{g^4 m_q C_{H_b}^{*} V_{q'q}}{4M_W^2} \Big\{ & -\frac{m_q C_{t1}}{4}\Big[\cancel{sM_W^2} + \cancel{s m_q^2} - \cancel{2sM_{H^\pm}^2} - \cancel{st} - \cancel{2m_q^2 M_W^2} - \cancel{2m_q^4} + \cancel{4m_q^2 M_{H^\pm}^2} \\
& + \cancel{2m_q^2 t} + \cancel{2s m_q^2} + \cancel{t m_q^2} - \cancel{M_W^2 m_q^2} - \cancel{m_q^4} + \cancel{2st} + \cancel{t^2} - \cancel{tM_W^2} - \cancel{t m_q^2} - \cancel{2sM_{H^\pm}^2} - \cancel{tM_{H^\pm}^2} \\
& + \cancel{M_W^2 M_{H^\pm}^2} + \cancel{m_q^2 M_{H^\pm}^2} - \cancel{M_W^2 s} - \cancel{M_W^2 m_q^2} - \cancel{M_W^2 M_{H^\pm}^2} + \cancel{tM_W^2} + \cancel{s m_q^2} + \cancel{m_q^4} + \cancel{m_q^2 M_{H^\pm}^2} - \cancel{t m_q^2} \\
& -+\cancel{st} + \cancel{t m_q^2} + \cancel{tM_{H^\pm}^2} - \cancel{t^2} - \cancel{2m_q^2 M_W^2} - \cancel{2m_q^4} + \cancel{4m_q^2 M_{H^\pm}^2} + \cancel{2m_q^2 t}\Big] \\
& + m_q m_{q'}^2 \frac{C_{t2}}{2}\left(-s + M_{H^\pm}^2 + M_W^2 + 2m_q^2 - 2t\right) + \frac{m_q C_{t1}}{4M_W^2}(s - M_{H^\pm}^2)\Big[\cancel{sM_W^2} - \cancel{s m_q^2} + \cancel{st} \\
& - \cancel{2m_q^2 M_W^2} + \cancel{2m_q^4} - \cancel{2m_q^2 t} + \cancel{m_q^2 M_W^2} + \cancel{m_q^4} - \cancel{m_q^2 t} + \cancel{tM_W^2} + \cancel{t m_q^2} - \cancel{t^2} - \cancel{M_W^2 M_{H^\pm}^2} - \cancel{m_q^2 M_{H^\pm}^2} \\
& + \cancel{tM_{H^\pm}^2} - \cancel{M_W^2 s} - \cancel{M_W^2 t} + \cancel{M_W^2 m_q^2} + \cancel{M_W^2 M_{H^\pm}^2} + \cancel{s m_q^2} + \cancel{t m_q^2} - \cancel{m_q^4} - \cancel{m_q^2 M_{H^\pm}^2} + \cancel{st} + \cancel{t^2} - \cancel{t m_q^2} \\
& - \cancel{tM_{H^\pm}^2}\Big] + \frac{m_q m_{q'}^2}{2M_W^2} C_{t2}(s - M_{H^\pm}^2)\left[M_W^2 + 2m_q^2 - 2t - s + M_{H^\pm}^2\right] - \frac{m_q^3 C_{t1}}{2M_W^2}(s - M_{H^\pm}^2)(M_W^2 - m_q^2 + t)\Big\}
\end{aligned}$$

$$\begin{aligned}
M_b^{H^+} M_4 = -\sum_{q'} \frac{g^4 m_q C_{H_b}^{*} V_{q'q}}{4M_W^2} \Big\{ & -\frac{m_q C_{t1}}{4}\Big[4s m_q^2 - 4sM_{H^\pm}^2 + 2st - 6m_q^2 M_W^2 - 4m_q^4 + 10 m_q^2 M_{H^\pm}^2 \\
& + 4m_q^2 t\Big] + m_q m_{q'}^2 \frac{C_{t2}}{2}\left(-s + M_{H^\pm}^2 + M_W^2 + 2m_q^2 - 2t\right) + \frac{m_q C_{t1}}{4M_W^2}(s - M_{H^\pm}^2)\Big[2st \\
& + 2m_q^4 - 2m_q^2 t - 2m_q^2 M_{H^\pm}^2\Big] + \frac{m_q m_{q'}^2}{2M_W^2} C_{t2}(s - M_{H^\pm}^2)\left[-s + M_{H^\pm}^2 + M_W^2 + 2m_q^2 - 2t\right] \\
& - \frac{m_q^3 C_{t1}}{2M_W^2}(s - M_{H^\pm}^2)(M_W^2 - m_q^2 + t)\Big\}
\end{aligned} \qquad (87)$$

The terms containing C_{t1} are:

$$\begin{aligned}
\frac{m_q C_{t1}}{4M_W^2}\Big\{ & -\cancel{4s m_q^2 M_W^2} + \cancel{4sM_{H^\pm}^2 M_W^2} - \cancel{2st M_W^2} + \cancel{6m_q^2 M_W^4} + \cancel{4m_q^4 M_W^2} - \cancel{10 m_q^2 M_{H^\pm}^2 M_W^2} \\
& - \cancel{4m_q^2 t M_W^2} + \cancel{2s^2 t} + \cancel{2s m_q^4} - \cancel{2st m_q^2} - \cancel{2s m_q^2 M_{H^\pm}^2} - \cancel{2st M_{H^\pm}^2} - \cancel{2m_q^4 M_{H^\pm}^2} \\
& + \cancel{2m_q^2 t M_{H^\pm}^2} + 2m_q^2 \cancel{M_{H^\pm}^4} - \cancel{2m_q^2 s M_W^2} + \cancel{2s m_q^4} - \cancel{2m_q^2 st} + \cancel{2m_q^2 M_W^2 M_{H^\pm}^2} \\
& - \cancel{2m_q^4 M_{H^\pm}^2} + \cancel{2m_q^2 t M_{H^\pm}^2}\Big\}
\end{aligned}$$

$$\frac{m_q C_{t1}}{4 M_W^2}\Big\{ -6s m_q^2 M_W^2 + 4s M_{H^\pm}^2 M_W^2 \cancel{- 2st M_W^2} + 6 m_q^2 M_W^4 + 4 m_q^4 M_W^2 - 8 M_{H^\pm}^2 M_W^2 m_q^2$$
$$-4 m_q^2 t M_W^2 \cancel{+ 2s^2 t} + 4s m_q^4 \cancel{- 4st m_q^2} - 2s m_q^2 M_{H^\pm}^2 \cancel{- 2st M_{H^\pm}^2} - 4 m_q^4 M_{H^\pm}^2 + 4 m_q^2 t M_{H^\pm}^2$$
$$+ 2 m_q^2 M_{H^\pm}^4 \Big\} \qquad (88)$$

that for m_q small is:

$$\approx \frac{m_q C_{t1}}{4 M_W^2}\Big\{ 4s M_{H^\pm}^2 M_W^2 - 2st M_W^2 + 2s^2 t - 2st M_{H^\pm}^2 \Big\} \qquad (89)$$

The terms containing c_{t2} are:

$$\frac{m_q m_{q1}^2 C_{t2}}{2 M_W^2}\Big(\cancel{-s M_W^2} + \cancel{M_{H^\pm}^2 M_W^2} + \cancel{M_W^4} + \cancel{2 m_q^2 M_W^2} \cancel{-2t M_W^2} \cancel{- s^2} + \cancel{s M_{H^\pm}^2} + \cancel{s M_W^2} + \cancel{2 m_q^2 s}$$
$$\cancel{-2ts} + \cancel{s M_{H^\pm}^2} \cancel{- M_{H^\pm}^4} \cancel{- M_{H^\pm}^2 M_W^2} \cancel{- 2 m_q^2 M_{H^\pm}^2} + \cancel{2t M_{H^\pm}^2} \Big)$$

$$= \frac{m_q m_{q1}^2 C_{t2}}{2 M_W^2}\Big(M_W^4 + 2 m_q^2 M_W^2 - 2t M_W^2 - s^2 + 2s M_{H^\pm}^2 + 2 m_q^2 s - 2st - M_{H^\pm}^4 - 2 m_q^2 M_{H^\pm}^2$$
$$+ 2t M_{H^\pm}^2 \Big) \qquad (90)$$

for m_q small is:

$$\approx \frac{m_q m_{q1}^2 C_{t2}}{2 M_W^2}\Big(M_W^4 - 2t M_W^2 - s^2 + 2s M_{H^\pm}^2 - 2st - M_{H^\pm}^4 + 2t M_{H^\pm}^2 \Big) \qquad (91)$$

$$s + t + u = 2 m_q^2 + M_W^2 + M_{H^\pm}^2$$

$$\Rightarrow 2s^2 t + 2st^2 + 2sut = 4st\, m_q^2 + 2st M_W^2 + 2st M_{H^\pm}^2 \qquad (92)$$

$\therefore$ (88) is:

$$\frac{m_q C_{t1}}{4 M_W^2}\Big\{ -2st^2 - 2sut - 6s m_q^2 M_W^2 + 4s M_{H^\pm}^2 M_W^2 + 6 m_q^2 M_W^4 + 4 m_q^4 M_W^2 - 8 m_q^2 M_{H^\pm}^2 M_W^2$$
$$-4 m_q^2 t M_W^2 + 4s m_q^4 - 2s m_q^2 M_{H^\pm}^2 - 4 m_q^4 M_{H^\pm}^2 + 4 m_q^2 t M_{H^\pm}^2 + 2 m_q^2 M_{H^\pm}^4 \Big\} \qquad (93)$$

for m_q small (93) is:

$$\frac{m_q C_{t1}}{4 M_W^2}\Big\{ -2st^2 - 2sut + 4s M_{H^\pm}^2 M_W^2 \Big\} \qquad (94)$$

$$2sut = 2s m_q^4 - 2s m_q^2 M_{H^\pm}^2 - 2s m_q^2 M_W^2 + 2s M_{H^\pm}^2 M_W^2 + 2 m_q^2 M_{H^\pm}^4 + 2 m_q^2 M_W^4 - 4 m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{2} \sin^2\theta\, \lambda\, (s - 4m_q^2) \qquad (95)$$

$\Rightarrow$ (93) is:

$$\frac{m_q C_{t1}}{4 M_W^2} \Big\{ \cancel{-2st^2} \cancel{- 2s m_q^4} \cancel{+ 2s m_q^2 M_{H^\pm}^2} \cancel{+ 2s m_q^2 M_W^2} \cancel{- 2s M_{H^\pm}^2 M_W^2} \cancel{- 2 m_q^2 M_{H^\pm}^4} \cancel{- 2 m_q^2 M_W^4}$$
$$\cancel{+ 4 m_q^2 M_{H^\pm}^2 M_W^2} \cancel{- \frac{1}{2} \sin^2\theta\, \lambda\, (s - 4m_q^2)} \cancel{- 6s m_q^2 M_W^2} \cancel{+ 4s M_{H^\pm}^2 M_W^2} \cancel{+ 6 m_q^2 M_W^4}$$
$$\cancel{+ 4 m_q^4 M_W^2} \cancel{- 8 m_q^2 M_{H^\pm}^2 M_W^2} \cancel{- 4 m_q^2 t M_W^2} \cancel{+ 4s m_q^4} \cancel{- 2s m_q^2 M_{H^\pm}^2} \cancel{- 4 m_q^4 M_{H^\pm}^2}$$
$$\cancel{+ 4 m_q^2 t M_{H^\pm}^2} \cancel{+ 2 m_q^2 M_{H^\pm}^4} \Big\}$$

$$\boxed{\begin{aligned} = \frac{m_q C_{t1}}{4 M_W^2} \Big\{ & -2st^2 + 2s m_q^4 - 4s m_q^2 M_W^2 + 2s M_{H^\pm}^2 M_W^2 + 4 m_q^2 M_W^4 - 4 m_q^2 M_{H^\pm}^2 M_W^2 \\ & + 4 m_q^4 M_W^2 - 4 m_q^2 t M_W^2 - 4 m_q^4 M_{H^\pm}^2 + 4 m_q^2 t M_{H^\pm}^2 - \frac{1}{2} \sin^2\theta\, \lambda\, (s - 4m_q^2) \Big\} \end{aligned}} \qquad (96)$$

for m_q small

$$\boxed{\simeq \frac{m_q C_{t1} s}{4 M_W^2} \Big\{ -2t^2 + 2 M_{H^\pm}^2 M_W^2 - \frac{1}{2} \sin^2\theta\, \lambda \Big\}} \qquad (97)$$

(90) can be written as:

$$= \frac{m_q m_{q'}^2 c_{t2}}{2 M_W^2} \Big\{ \cancel{M_W^4} + \cancel{2 m_q^2 M_W^2} - 2t \cancel{M_W^2} - \cancel{s^2} + \cancel{2s M_{H^\pm}^2} + \cancel{2s m_q^2} - 2t \big(2\cancel{m_q^2} + \cancel{M_W^2} + \cancel{M_{H^\pm}^2} - \cancel{t} - u \big)$$
$$- \cancel{M_{H^\pm}^4} - \cancel{2 m_q^2 M_{H^\pm}^2} + 2t \cancel{M_{H^\pm}^2} \Big\}$$

$$= \frac{m_q m_{q'}^2 C_{t2}}{2 M_W^2} \Big\{ M_W^4 + 2 m_q^2 M_W^2 - 4t M_W^2 - s^2 + 2s M_{H^\pm}^2 + 2s m_q^2 - 4t m_q^2 + 2t^2 + 2ut$$
$$- M_{H^\pm}^4 - 2 m_q^2 M_{H^\pm}^2 \Big\}$$

$$= \frac{m_q m_{q'}^2 C_{t2}}{2 M_W^2} \Big\{ \cancel{M_W^4} + \cancel{2 m_q^2 M_W^2} - 4t \cancel{M_W^2} - \cancel{s^2} + \cancel{2s M_{H^\pm}^2} + \cancel{2s m_q^2} - \cancel{4t m_q^2} + \cancel{2t^2} - \cancel{M_{H^\pm}^4} - \cancel{2 m_q^2 M_{H^\pm}^2}$$
$$+ \cancel{2 m_q^4} - \cancel{2 m_q^2 M_{H^\pm}^2} - \cancel{2 m_q^2 M_W^2} + \cancel{2 M_{H^\pm}^2 M_W^2} + \frac{2}{s} \Big[\cancel{m_q^2 M_{H^\pm}^4} + \cancel{m_q^2 M_W^4} - 2 m_q^2 M_{H^\pm}^2 M_W^2$$
$$+ \frac{1}{4} \sin^2\theta\, \lambda\, (s - 4m_q^2) \Big] \Big\}$$

$$\boxed{\begin{aligned} = \frac{m_q m_{q'}^2 C_{t2}}{2 M_W^2} \Big\{ & M_W^4 - 4t M_W^2 - s^2 + 2s M_{H^\pm}^2 + 2s m_q^2 - 4t m_q^2 + 2t^2 - M_{H^\pm}^4 - 4 m_q^2 M_{H^\pm}^2 + 2 m_q^4 \\ & + 2 M_{H^\pm}^2 M_W^2 + \frac{2}{s} \Big[m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2 m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4} \sin^2\theta\, \lambda\, (s - 4m_q^2) \Big] \Big\} \end{aligned}} \qquad (98)$$

For m_q small (98) is:

$$\boxed{\frac{m_q m_{q'}^2 C_{t2}}{2M_W^2}\left\{M_W^4 - 4tM_W^2 - s^2 + 2sM_{H^\pm}^2 + 2t^2 - M_{H^\pm}^4 + 2M_{H^\pm}^2 M_W^2 + \frac{1}{2}\sin^2\theta\,\lambda\right\}} \qquad (99)$$

then

$$M_b^{H+} M_q = -\sum_{q'} \frac{g^4 m_q^2 C_{H_b}^* V_{q'q}}{16 M_W^4} \Big\{ C_{t1}\big\{ -2st^2 + 2sm_q^4 - 4sm_q^2 M_W^2 + 2sM_{H^\pm}^2 M_W^2 + 4m_q^2 M_W^4$$
$$- 4m_q^2 M_{H^\pm}^2 M_W^2 + 4m_q^4 M_W^2 - 4m_q^2 t M_W^2 - 4m_q^4 M_{H^\pm}^2 + 4m_q^2 t M_{H^\pm}^2 - \frac{1}{2}\sin^2\theta\,\lambda\,(s - 4m_q^2)\big\}$$
$$+ 2m_{q'}^2 C_{t2}\big\{ M_W^4 - 4tM_W^2 - s^2 + 2sM_{H^\pm}^2 + 2sm_q^2 - 4tm_q^2 + 2t^2 - M_{H^\pm}^4 - 4m_q^2 M_{H^\pm}^2 + 2m_q^4$$
$$+ 2M_{H^\pm}^2 M_W^2 + \frac{2}{s}\big[m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4}\sin^2\theta\,\lambda\,(s - 4m_q^2)\big]\big\}\Big\} \qquad (100)$$

for m_q small:

$$\boxed{\begin{aligned} M_b^{H+} M_q = -\sum_{q'} \frac{g^4 m_q^2 C_{H_b}^* V_{q'q}}{16 M_W^4} \Big\{ & sC_{t1}\big[-2t^2 + 2M_{H^\pm}^2 M_W^2 - \tfrac{1}{2}\sin^2\theta\,\lambda\big] + 2m_{q'}^2 C_{t2}\big[M_W^4 - \\ & - 4tM_W^2 - s^2 + 2sM_{H^\pm}^2 + 2t^2 - M_{H^\pm}^4 + 2M_{H^\pm}^2 M_W^2 + \tfrac{1}{2}\sin^2\theta\,\lambda\big]\Big\}\end{aligned}} \qquad (101)$$

$$M_q^+ M_b^H = (M_b^{H+} M_q)^+ = -\sum_{q'} \frac{g^4 m_q^2 C_{H_b} V_{q'q}^*}{16 M_W^4} \Big\{ C_{t1}\big\{ -2st^2 + 2sm_q^4 - 4sm_q^2 \overset{\text{the same}}{M_W^2} + 2sM_{H^\pm}^2 M_W^2 + 4m_q^2 M_W^4$$
$$- 4m_q^2 M_{H^\pm}^2 M_W^2 + 4m_q^4 M_W^2 - 4m_q^2 t M_W^2 + \cdots\big\} + 2m_{q'}^2 C_{t2}\big\{ M_W^4 - 4t\overset{\text{the same}}{M_W^2} + \cdots\big\}\Big\} \qquad (102)$$

$$M_q^+ M_b^H + M_b^{H+} M_q = -\sum_{q'} \frac{g^4 m_q^2}{16 M_W^4}\big[C_{H_b}^* V_{q'q} + C_{H_b} V_{q'q}^*\big] \Big\{ C_{t1}\big\{ -2st^2 + 2sm_q^4 - 4sm_q^2 M_W^2 + 2sM_{H^\pm}^2 M_W^2$$
$$+ 4m_q^2 M_W^4 - 4m_q^2 M_{H^\pm}^2 M_W^2 + 4m_q^4 M_W^2 - 4m_q^2 t M_W^2 - 4m_q^4 M_{H^\pm}^2 + 4m_q^2 t M_{H^\pm}^2 - \frac{1}{2}\sin^2\theta\,\lambda\,(s - 4m_q^2)\big\}$$
$$+ 2m_{q'}^2 C_{t2}\big\{ M_W^4 - 4tM_W^2 - s^2 + 2sM_{H^\pm}^2 + 2sm_q^2 - 4tm_q^2 + 2t^2 - M_{H^\pm}^4 - 4m_q^2 M_{H^\pm}^2 + 2m_q^4$$
$$+ 2M_{H^\pm}^2 M_W^2 + \frac{2}{s}\big[m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4}\sin^2\theta\,\lambda\,(s - 4m_q^2)\big]\big\}\Big\}$$

(103)

for m_q small :

$$\boxed{\begin{aligned} M_b^{H\,+} M_4 + M_4^{+} M_b^{H} = -\sum_{q'} \frac{g^4 m_q^2}{16 M_W^4} \left[C_{H_b}^{*} V_{q'q} + C_{H_b} V_{q'q}^{*} \right] \Big\{ s\, C_{t_1} \left[-2t^2 + 2 M_{H^\pm}^2 M_W^2 - \tfrac{1}{2} \sin^2\theta\, \lambda \right] \\ + 2 m_{q'}^2 C_{t_2} \left[M_W^4 - 4t M_W^2 - s^2 + 2s M_{H^\pm}^2 + 2t^2 - M_{H^\pm}^4 + 2 M_{H^\pm}^2 M_W^2 + \tfrac{1}{2} \sin^2\theta\, \lambda \right] \Big\} \end{aligned}}$$

(104)

$$M_{3_b}^{+} M_4 = -\sum_{q'} \frac{g^4 m_q V_{q'q}}{16 M_W^2} C_{A_b}^{*} \sum_{\lambda} \varepsilon_{\nu 4}^{*} \varepsilon_{\mu 4} (P_1 + P_2 + P_3)^{\mu} \sum_{S} \left[V_2^{+} \gamma^0 \gamma^5 U_1 \right]^{+} \bar{V}_2 \gamma^{\nu} (1-\gamma^5) \cdot$$

$$\cdot \left[m_q C_{t_1} \not{p}_{q'} + m_{q'}^2 C_{t_2} \right] U_1 \qquad (105)$$

$$= \sum_{q'} \frac{+ g^4 m_q V_{q'q}}{16 M_W^2} C_{A_b}^{*} \left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2} \right) (P_1 + P_2 + P_3)^{\mu} \sum_{S} \left[\bar{U}_1 \gamma^5 V_2 \right] \bar{V}_2 \gamma^{\nu} (1-\gamma^5) \cdot$$

$$\cdot \left[m_q C_{t_1} \not{p}_{q'} + m_{q'}^2 C_{t_2} \right] U_1$$

$$= \sum_{q'} \frac{g^4 m_q V_{q'q}}{16 M_W^2} C_{A_b}^{*} \left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2} \right) (P_1 + P_2 + P_3)^{\mu} \, \mathrm{Tr} \Big[(\not{P}_2 - m_q) \gamma^{\nu} (1-\gamma^5) \cdot$$

$$\cdot \left[m_q C_{t_1} \not{p}_{q'} + m_{q'}^2 C_{t_2} \right] (\not{P}_1 + m_q) \gamma^5 \Big] \qquad (106)$$

$$\mathrm{Tr} \left[(\not{P}_2 - m_q) \gamma^{\nu} (1-\gamma^5) \left[m_q C_{t_1} \not{p}_{q'} + m_{q'}^2 C_{t_2} \right] (\not{P}_1 + m_q) \gamma^5 \right] =$$

$$\mathrm{Tr} \left[\gamma^5 (\not{P}_2 - m_q) \gamma^{\nu} (1-\gamma^5) \left[m_q C_{t_1} \not{p}_{q'} + m_{q'}^2 C_{t_2} \right] (\not{P}_1 + m_q) \right]$$

$$= \mathrm{Tr} \left[(-\not{P}_2 - m_q) \gamma^5 \gamma^{\nu} (1-\gamma^5) \left[m_q C_{t_1} \not{p}_{q'} + m_{q'}^2 C_{t_2} \right] (\not{P}_1 + m_q) \right]$$

$$= -\mathrm{Tr} \left[(\not{P}_2 + m_q) \gamma^{\nu} (1-\gamma^5) \left[m_q C_{t_1} \not{p}_{q'} + m_{q'}^2 C_{t_2} \right] (\not{P}_1 + m_q) \right] \qquad (107)$$

$$\Rightarrow M_{3_b}^{+} M_4 = \sum_{q'} \frac{g^4 m_q V_{q'q}}{16 M_W^2} C_{A_b}^{*} \left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2} \right) (P_1 + P_2 + P_3)^{\mu} \, \mathrm{Tr} \Big\{ (-\not{P}_2 - m_q) \gamma^{\nu} (1-\gamma^5) \Big[m_q C_{t_1} \not{p}_{q'}$$

$$+ m_{q'}^2 C_{t_2} \Big] (\not{P}_1 + m_q) \Big] \Big\} \qquad (108)$$

The last trace is the same that appears in (76) if we change $-P_2$ by P_2

So we have:

$$M_3^{\dagger} M_4 = \sum_{q'} \frac{g^4 m_q V_{q'q}}{16 M_W^2} C^*_{A_b} 4 \Big\{ -(p_1+p_2+p_3)_\nu + \frac{1}{M_W^2}\left[p_4 . (p_1+p_2+p_3)\right] p_{4\nu}\Big) \Big\{ m_q C_{t_1} \big[-(p_1 . p_2) p_{q'}^{\nu}$$

$$- (p_1 . p_{q'}) p_2^{\nu} + (p_2 . p_{q'}) p_1^{\nu} \big] - m_q m_{q'}^2 C_{t_2} p_2^{\nu} - m_q^3 C_{t_1} p_{q'}^{\nu} - m_q m_{q'}^2 C_{t_2} p_1^{\nu} \Big\} \qquad (109)$$

$$= \sum_{q'} 4 \frac{g^4 m_q V_{q'q}}{16 M_W^2} C^*_{A_b} \Big\{ m_q C_{t_1} (p_1 . p_2)(p_{q'} . (p_1+p_2+p_3)) + m_q C_{t_1} (p_1 . p_{q'})(p_2 . (p_1+p_2+p_3))$$

$$- m_q C_{t_1} (p_2 . p_{q'})(p_1 . (p_1+p_2+p_3)) + m_q m_{q'}^2 C_{t_2} (p_2 . (p_1+p_2+p_3)) + m_q^3 C_{t_1} (p_{q'} . (p_1+p_2+p_3))$$

$$+ m_q m_{q'}^2 C_{t_2} (p_1 . (p_1+p_2+p_3)) - \frac{m_q C_{t_1}}{M_W^2} (p_1 . p_2)(p_{q'} . p_4)(p_4 . (p_1+p_2+p_3))$$

$$- \frac{m_q C_{t_1}}{M_W^2} (p_1 . p_{q'})(p_2 . p_4)(p_4 . (p_1+p_2+p_3)) + \frac{m_q C_{t_1}}{M_W^2} (p_2 . p_{q'})(p_1 . p_4)(p_4 . (p_1+p_2+p_3))$$

$$- \frac{m_q m_{q'}^2 C_{t_2}}{M_W^2} (p_2 . p_4)(p_4 . (p_1+p_2+p_3)) - \frac{m_q^3 C_{t_1}}{M_W^2} (p_4 . p_{q'})(p_4 . (p_1+p_2+p_3)) - \frac{m_q m_{q'}^2 C_{t_2}}{M_W^2} (p_1 . p_4)(p_4 . (p_1+p_2+p_3)) \Big\}$$

$$(110)$$

$$= \sum_{q'} \frac{g^4 m_q V_{q'q}}{4 M_W^2} C^*_{A_b} \Big\{ m_q C_{t_1} \overset{\frac{1}{2}(s-2m_q^2)}{\left(\frac{s}{2} - m_q^2\right)} \frac{1}{2}(M_W^2 + m_q^2 - 2M_{H^\pm}^2 - t) + m_q C_{t_1} \frac{1}{2}(m_q^2 + t - M_{H^\pm}^2) .$$

$$. (2s + t - M_W^2 - m_q^2)\frac{1}{2} - m_q C_{t_1} \frac{1}{2}(M_W^2 - m_q^2 - t)\frac{1}{2}(s + m_q^2 + M_{H^\pm}^2 - t) + m_q m_{q'}^2 C_{t_2} \frac{1}{2}(2s + t - M_W^2 - m_q^2)$$

$$+ m_q^3 C_{t_1} \frac{1}{2}(M_W^2 + m_q^2 - 2M_{H^\pm}^2 - t) + m_q m_{q'}^2 C_{t_2} \frac{1}{2}(s + m_q^2 + M_{H^\pm}^2 - t) - \frac{m_q C_{t_1}}{M_W^2} \overset{\frac{1}{2}(s-2m_q^2)}{\left(\frac{s}{2} - m_q^2\right)} \frac{1}{2}(M_W^2 - m_q^2 + t)$$

$$(s - M_{H^\pm}^2) - \frac{m_q C_{t_1}}{M_W^2} \frac{1}{2}(m_q^2 + t - M_{H^\pm}^2) \frac{1}{2}(M_W^2 + m_q^2 - t)(s - M_{H^\pm}^2) + \frac{m_q C_{t_1}}{M_W^2} \frac{1}{2}(M_W^2 - m_q^2 - t)\frac{1}{2}(s + t - m_q^2 - M_{H^\pm}^2)$$

$$\swarrow$$

$$(s - M_{H^\pm}^2) - \frac{m_q m_{q'}^2 C_{t_2}}{M_W^2}\frac{1}{2}(M_W^2 + m_q^2 - t)(s - M_{H^\pm}^2) - \frac{m_q^3 C_{t_1}}{M_W^2}\frac{1}{2}(M_W^2 - m_q^2 + t)(s - M_{H^\pm}^2)$$

$$\swarrow$$

$$- \frac{m_q m_{q'}^2 C_{t_2}}{M_W^2}\frac{1}{2}(s + t - m_q^2 - M_{H^\pm}^2)(s - M_{H^\pm}^2) \Big\}$$

$$= \sum_{q'} \frac{g^4 m_q V_{q'q}}{4 M_W^2} C^*_{A_b} \Big\{ \frac{m_q C_{t_1}}{4} \big[\cancel{s M_W^2} + \cancel{s m_q^2} - \cancel{2s M_{H^\pm}^2} - \cancel{st} - \cancel{2m_q^2 M_W^2} - \cancel{2m_q^4} + \cancel{4m_q^2 M_{H^\pm}^2} + \cancel{2m_q^2 t} + \cancel{2s m_q^2}$$

$$- \cancel{t m_q^2} - \cancel{m_q^2 M_W^2} - \cancel{m_q^4} + \cancel{2st} + \cancel{t^2} - \cancel{M_W^2 t} - \cancel{m_q^2 t} - \cancel{2s M_{H^\pm}^2} - \cancel{t M_{H^\pm}^2} + \cancel{M_W^2 M_{H^\pm}^2} + \cancel{m_q^2 M_{H^\pm}^2} - \cancel{s M_W^2} - \cancel{M_W^2 m_q^2}$$

$$- \cancel{M_W^2 M_{H^\pm}^2} + \cancel{t M_W^2} + \cancel{s m_q^2} + \cancel{m_q^4} + \cancel{m_q^2 M_{H^\pm}^2} - \cancel{m_q^2 t} + \cancel{st} + \cancel{t m_q^2} + \cancel{t M_{H^\pm}^2} - \cancel{t^2} + \cancel{2m_q^2 M_W^2} + \cancel{2m_q^4} - \cancel{4 m_q^2 M_{H^\pm}^2}$$

$$- \cancel{2m_q^2 t} \big] + \frac{m_q m_{q'}^2 C_{t_2}}{2} \big[\cancel{2s} + \cancel{t} - \cancel{M_W^2} - \cancel{m_q^2} + \cancel{s} + \cancel{m_q^2} + \cancel{M_{H^\pm}^2} - \cancel{t} \big] + \frac{m_q C_{t_1}}{4 M_W^2}(s - M_{H^\pm}^2)\big[-\cancel{s M_W^2} + \cancel{s m_q^2} - \cancel{st}$$

$$+ \cancel{2m_q^2 M_W^2} - \cancel{2m_q^4} + \cancel{2m_q^2 t} - \cancel{m_q^2 M_W^2} - \cancel{m_q^4} + \cancel{m_q^2 t} - \cancel{t M_W^2} - \cancel{t m_q^2} + \cancel{t^2} + \cancel{M_W^2 M_{H^\pm}^2} + \cancel{m_q^2 M_{H^\pm}^2} - \cancel{t M_{H^\pm}^2} + \cancel{s M_W^2} + \cancel{t M_W^2}$$

$$- \cancel{m_q^2 M_W^2} - \cancel{M_W^2 M_{H^\pm}^2} - \cancel{s m_q^2} - \cancel{t m_q^2} + \cancel{m_q^4} + \cancel{m_q^2 M_{H^\pm}^2} - \cancel{st} - \cancel{t^2} + \cancel{t m_q^2} + \cancel{t M_{H^\pm}^2} - \cancel{2 m_q^2 M_W^2} + \cancel{2m_q^4} - \cancel{2m_q^2 t} \big]$$

$$- \frac{m_q m_{q'}^2 C_{t_2}}{2 M_W^2}(s - M_{H^\pm}^2)\big[\cancel{M_W^2} + \cancel{m_q^2} - \cancel{t} + \cancel{s} + \cancel{t} - \cancel{m_q^2} - \cancel{M_{H^\pm}^2} \big] \Big\}$$

$$= \sum_{q'} \frac{g^4 m_q V_{q'q}}{4M_W^2} C^*_{A_b} \Big\{ \frac{m_q C_{t1}}{4} \left[4s m_q^2 - 4s M_{H^\pm}^2 + 2st - 2m_q^2 M_W^2 + 2m_q^2 M_{H^\pm}^2 \right] + \frac{m_q m_{q'}^2 C_{t2}}{2} \left[3s + M_{H^\pm}^2 - M_W^2 \right]$$

$$+ \frac{m_q C_{t1}}{4M_W^2} (s - M_{H^\pm}^2) \left[-2st - 2m_q^2 M_W^2 + 2m_q^2 M_{H^\pm}^2 \right] - \frac{m_q m_{q'}^2 C_{t2}}{2M_W^2} (s - M_{H^\pm}^2) \left[M_W^2 + s - M_{H^\pm}^2 \right]$$

$$= \sum_{q'} \frac{g^4 m_q V_{q'q}}{4M_W^2} C^*_{A_b} \Big\{ \frac{m_q C_{t1}}{4M_W^2} \Big[4s\cancel{m_q^2} M_W^2 - 4s \cancel{M_{H^\pm}^2} M_W^2 + 2st \cancel{M_W^2} - 2\cancel{m_q^2} M_W^4 + 2m_q^2 \cancel{M_{H^\pm}^2} M_W^2 - 2\cancel{s^2}t - 2s\cancel{m_q^2} M_W^2$$

$$+ 2s\cancel{m_q^2} M_{H^\pm}^2 + 2st\cancel{M_{H^\pm}^2} + 2m_q^2 \cancel{M_W^2} M_{H^\pm}^2 - 2\cancel{m_q^2} M_{H^\pm}^4 \Big] + \frac{m_q m_{q'}^2 C_{t2}}{2M_W^2} \Big[3\cancel{s}M_W^2 + \cancel{M_W^2} M_{H^\pm}^2 - \cancel{M_W^4}$$

$$- \cancel{s}M_W^2 - \cancel{s}^2 + s\cancel{M_{H^\pm}^2} + \cancel{M_W^2} M_{H^\pm}^2 + s\cancel{M_{H^\pm}^2} - \cancel{M_{H^\pm}^4} \Big] \Big\}$$

$$= \sum_{q'} \frac{g^4 m_q V_{q'q}}{4M_W^2} C^*_{A_b} \Big\{ \frac{m_q C_{t1}}{4M_W^2} \Big[2s m_q^2 M_W^2 - 4s M_{H^\pm}^2 M_W^2 + 2s\cancel{t} M_W^2 - 2m_q^2 M_W^4 + 4m_q^2 M_{H^\pm}^2 M_W^2 - 2\cancel{s^2}t + 2s m_q^2 M_{H^\pm}^2$$

$$+ 2s\cancel{t} M_{H^\pm}^2 - 2m_q^2 M_{H^\pm}^4 \Big] + \frac{m_q m_{q'}^2 C_{t2}}{2M_W^2} \Big[2s M_W^2 + 2M_W^2 M_{H^\pm}^2 + 2s M_{H^\pm}^2 - s^2 - M_W^4 - M_{H^\pm}^4 \Big] \Big\} \tag{111}$$

$$s + t + u = 2m_q^2 + M_{H^\pm}^2 + M_W^2 \quad \Rightarrow \quad 2s^2t + 2st^2 + 2sut = 4m_q^2 st + 2st M_{H^\pm}^2 + 2st M_W^2$$

$$= \sum_{q'} \frac{g^4 m_q V_{q'q}}{4M_W^2} C^*_{A_b} \Big\{ \frac{m_q C_{t1}}{4M_W^2} \Big[2s m_q^2 M_W^2 - 4s M_{H^\pm}^2 M_W^2 + 2st^2 + 2sut - 4m_q^2 st - 2m_q^2 M_W^4 + 4m_q^2 M_{H^\pm}^2 M_W^2$$

$$+ 2s m_q^2 M_{H^\pm}^2 - 2m_q^2 M_{H^\pm}^4 \Big] + \frac{m_q m_{q'}^2 C_{t2}}{2M_W^2} \Big[2s M_W^2 + 2M_W^2 M_{H^\pm}^2 + 2s M_{H^\pm}^2 - s^2 - M_W^4 - M_{H^\pm}^4 \Big] \Big\} \tag{112}$$

$$= \sum_{q'} \frac{g^4 m_q V_{q'q}}{4M_W^2} C^*_{A_b} \Big\{ \frac{m_q C_{t1}}{4M_W^2} \Big[2s\cancel{m_q^2} M_W^2 - 4s \cancel{M_{H^\pm}^2} M_W^2 + 2\cancel{s}t^2 + 2\cancel{s}m_q^4 - 2s\cancel{m_q^2} M_{H^\pm}^2 - 2s\cancel{m_q^2} M_W^2 + 2s\cancel{M_{H^\pm}^2} M_W^2$$

$$+ 2\cancel{m_q^2} M_{H^\pm}^4 + 2m_q^2 \cancel{M_W^4} - 4m_q^2 \cancel{M_{H^\pm}^2} M_W^2 + \frac{1}{2}\sin^2\cancel{\theta}\,\lambda (s - 4m_q^2) - 4m_q^2 \cancel{st} - 2\cancel{m_q^2} M_W^4 + 4m_q^2 \cancel{M_{H^\pm}^2} M_W^2$$

$$+ 2s m_q^2 \cancel{M_{H^\pm}^2} - 2m_q^2 \cancel{M_{H^\pm}^4} \Big] + \frac{m_q m_{q'}^2 C_{t2}}{2M_W^2} \Big[2s M_W^2 + 2M_W^2 M_{H^\pm}^2 + 2s M_{H^\pm}^2 - s^2 - M_W^4 - M_{H^\pm}^4 \Big] \Big\}$$

$$\boxed{\begin{aligned} M^{\dagger}_{3_b} M_4 = \sum_{q'} \frac{g^4 m_q^2 V_{q'q}}{16 M_W^4} C^*_{A_b} \Big\{ & C_{t1} \Big[-2s M_{H^\pm}^2 M_W^2 + 2st^2 + 2s m_q^4 + \frac{1}{2}\sin^2\theta\,\lambda (s - 4m_q^2) - 4m_q^2 st \Big] \\ & + 2 m_{q'}^2 C_{t2} \Big[2s M_W^2 + 2M_W^2 M_{H^\pm}^2 + 2s M_{H^\pm}^2 - s^2 - M_W^4 - M_{H^\pm}^4 \Big] \Big\} \end{aligned}} \tag{113}$$

For ~~m_q~~ small

$$\boxed{\begin{aligned} M^{\dagger}_{3_b} M_4 = \sum_{q'} \frac{g^4 m_q V_{q'q}}{16 M_W^4} C^*_{A_b} \Big\{ & m_q C_{t1} \Big[-2s M_{H^\pm}^2 M_W^2 + 2st^2 + \frac{1}{2}\sin^2\theta\,\lambda s \Big] + 2 m_q m_{q'}^2 C_{t2} \Big[2s M_W^2 + 2M_W^2 M_{H^\pm}^2 \\ & + 2s M_{H^\pm}^2 - s^2 - M_W^4 - M_{H^\pm}^4 \Big] \Big\} \end{aligned}} \tag{114}$$

$$M_{3_b}^{+} M_4 + M_4^{+} M_{3_b} = M_{3_b}^{+} M_4 + (M_{3_b}^{+} M_4)^{+}$$

$$\boxed{\begin{aligned} M_{3_b}^{+} M_4 + M_4^{+} M_{3_b} = {} & \frac{g^4 m_q}{16 M_W^4} \sum_{q'} \left(V_{q'q} C_{A_b}^{*} + V_{q'q}^{*} C_{A_b} \right) \Big\{ m_q C_{t1} s \left[-2 M_{H_S}^2 M_W^2 + 2t^2 + \tfrac{1}{2} \sin^2\theta\, \lambda \right] \\ & + 2 m_q m_{q'}^2 C_{t2} \left[2 s M_W^2 + 2 M_W^2 M_{H_S}^2 + 2 s M_{H_S}^2 - s^2 - M_W^4 - M_{H_S}^4 \right] \Big\} \end{aligned}} \qquad (115)$$

$$\left(\frac{d\sigma}{d\Omega} \right)_{CM} = \frac{1}{64 \pi^2 s} \left(\frac{P_f}{P_i} \right) \overline{|M|^2} \qquad (116)$$

$$d\Omega = 2\pi \, d\cos\theta$$

$$\frac{P_f}{P_i} = \frac{|\vec{P}_3|}{|\vec{P}_1|} = \overset{\wedge P_f}{\left(\frac{\lambda^{1/2}(s, M_W^2, M_{H_S}^2)}{2\sqrt{s}} \right)} \cdot \overset{\wedge P_i^{-1}}{\left(\frac{2}{(s - 4m_q^2)^{1/2}} \right)} \qquad (117)$$

$$dt = \frac{1}{2} \lambda^{1/2}(s, M_{H_S}^2, M_W^2) \frac{(s - 4m_q^2)^{1/2}}{s^{1/2}} \, d\cos\theta = \frac{1}{2} \lambda^{1/2} \frac{(s - 4m_q^2)^{1/2}}{s^{1/2}} \frac{d\Omega}{2\pi} \qquad (118)$$

$$\frac{d\sigma}{dt} = \frac{\cancel{2} \cancel{s^{1/2}} \, 2\pi}{\cancel{\lambda^{1/2}} (s - 4m_q^2)^{1/2}} \frac{1}{64 \pi^2 s} \frac{\cancel{\lambda^{1/2}}}{\cancel{2\sqrt{s}}} \frac{1}{P_i} \overline{|M|^2} = \frac{1}{64 \pi s} \frac{1}{P_i^2} \overline{|M|^2} \qquad (119)$$

$$\boxed{\frac{d\sigma}{dt} = \frac{1}{64\pi s}\cdot\frac{1}{p_i^2}\overline{|M|^2}} \qquad (120)$$

neglecting m_q

$$\frac{d\sigma}{dt} = \frac{1}{\underset{16}{\cancel{64}}\pi s}\cdot\frac{\overset{1}{\cancel{4}}}{s}\overline{|M|^2} = \frac{\overline{|M|^2}}{16\pi s^2}$$

$$\frac{d\sigma}{dt}(b\bar{b}\to H^-W^+) = \frac{G_F^2}{24\pi s}\Big\{\frac{1}{2}\lambda(s, M_W^2, M_{H^\pm}^2)\, m_b^2\,[|C_{H_b}|^2 + |C_{A_b}|^2] + m_b^2 C_{t_{1_t}}^2\left(\frac{\lambda}{2s}M_W^2\sin^2\theta + t^2\right)$$

$$+ m_t^4 C_{t_{2_t}}^2\left(2M_W^2 + \frac{\lambda}{4s}\sin^2\theta\right) - \frac{m_b^2}{2}C_{t_{1_t}}\left(-2t^2 + 2M_{H^\pm}^2 M_W^2 - \frac{1}{2}\lambda\sin^2\theta\right)\mathrm{Re}\,(C_{H_b} + C_{A_b})\Big\}$$

---

(121) OK.

Returning to (36)

I)

$$\boxed{M_4 = -\frac{g^2}{4M_W}\varepsilon^*_{\mu 4}\sum_i V_{iq}\,\bar{V}_2\gamma^\mu(1-\gamma^5)[m_q C_{t_{1_i}}\not{p}_i + m_i^2 C_{t_{2_i}}]U_1} \qquad (36a)$$

$q = d, s, b \qquad i = u, c, t$

For $q\bar{q}\to H^-W^+$

II) Returning to (40)

$$\boxed{M_{4_{II}} = -\frac{g^2}{4M_W}\varepsilon^*_{\mu 4}\sum_i V^*_{iq}\,\bar{V}_2[m_q C_{u_{1_i}}\not{p}_i + m_i^2 C_{u_{2_i}}]\gamma^\mu(1-\gamma^5)U_1} \qquad (40a)$$

For $q\bar{q}\to H^+W^-$ with $q = d, s, b$; $i = u, c, t$

Returning to (44)

III)

$$\boxed{M_{4_{III}} = -\frac{g^2}{4M_W}\varepsilon^*_{\mu 4}\sum_i V_{qi}\,\bar{V}_2[m_q C_{u_{2_i}}\not{p}_i + m_i^2 C_{u_{1_i}}]\gamma^\mu(1-\gamma^5)U_1} \qquad (44a)$$

For $q\bar{q}\to H^-W^+$ with $q: u, c$; $i = d, s, b$

IV) Returning to (48)

$$\boxed{M_{4_{IV}} = -\frac{g^2}{4M_W}\varepsilon^*_{\mu 4}\sum_i V^*_{qi}\,\bar{V}_2\gamma^\mu(1-\gamma^5)[m_q C_{t_{2_i}}\not{p}_i + m_i^2 C_{t_{1_i}}]U_1} \qquad (48a)$$

For $q\bar{q}\to H^+W^-$ with $q: u, c$; $i = d, s, b$

$$M_4^+ M_b^H + M_b^{H+} M_4 = -\frac{g^4 m_q^2}{16 M_W^4} \sum_i \left[C_{H_b}^* V_{iq} + C_{H_b} V_{iq}^* \right] \Big\{ C_{t1_i} \Big[-2st^2 + 2s m_q^4 - 4s m_q^2 M_W^2 + 2s M_{H^\pm}^2 M_W^2$$

$$+ 4 m_q^2 M_W^4 - 4 m_q^2 M_{H^\pm}^2 M_W^2 + 4 m_q^4 M_W^2 - 4 m_q^2 t M_W^2 - 4 m_q^4 M_{H^\pm}^2 + 4 m_q^2 t M_{H^\pm}^2 - \frac{1}{2} \sin^2\theta \lambda \cdot$$

$$\cdot (s - 4 m_q^2) \Big] + 2 m_i^2 C_{t2_i} \Big\{ M_W^4 - 4t M_W^2 - s^2 + 2s M_{H^\pm}^2 + 2s m_q^2 - 4t m_q^2 + 2t^2 - M_{H^\pm}^4 - 4 m_q^2 M_{H^\pm}^2$$

$$+ 2 m_q^4 + 2 M_{H^\pm}^2 M_W^2 + \frac{2}{s} \Big[m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2 m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4} \sin^2\theta \lambda (s - 4 m_q^2) \Big] \Big\} \Big\}$$

(103a) OK

$$C_{t1i} = \frac{\tan\beta}{\hat{t} - m_i^2} \; ; \quad C_{t2i} = \frac{\cot\beta}{\hat{t} - m_i^2} \; ; \quad C_{U1i} = \frac{\tan\beta}{\hat{U} - m_i^2} \; ; \quad C_{U2i} = \frac{\cot\beta}{\hat{U} - m_i^2}$$

$$M_{3_b}^+ M_4 + M_4^+ M_{3_b} = \frac{g^4 m_q^2}{16 M_W^4} \sum_i \left(V_{iq} C_{A_b}^* + V_{iq}^* C_{A_b} \right) \Big\{ C_{t1i} \Big[-2s M_{H^\pm}^2 M_W^2 + 2st^2 + 2s m_q^4 + \frac{1}{2} \sin^2\theta \lambda (s - 4 m_q^2)$$

$$- 4 m_q^2 st \Big] + 2 m_i^2 C_{t2i} \Big[2s M_W^2 + 2 M_W^2 M_{H^\pm}^2 + 2s M_{H^\pm}^2 - s^2 - M_W^4 - M_{H^\pm}^4 \Big] \Big\}$$

(113a) OK

where $q = d, s, b$; $i = U, c, t$

$0.002 < |V_{ub}| < 0.005$

$0.037 < |V_{cb}| < 0.043$

$V_{tb} = 1$

m_u : 1 to 5 MeV

m_d : 3 to 9 MeV

m_s : 75 to 170 MeV

m_c : 1.15 to 1.35 GeV

m_b : 4 to 4.4 GeV

m_t : 174.3 ± 5.1 GeV

$$|M_4|^2 = \frac{g^4}{16 M_W^2} \sum_\lambda \epsilon_{\mu 4}^* \epsilon_{\nu 4} \sum_{i,j} V_{iq} V_{jq}^* \sum_s \left[\bar{V}_2 \gamma^\nu (1-\gamma^5)(m_q C_{t1j} \not{p}_j + m_j^2 C_{t2j}) U_1 \right]^+ \cdot$$

$$\left[\bar{V}_2 \gamma^\mu (1-\gamma^5)(m_q C_{t1i} \not{p}_i + m_i^2 C_{t2i}) U_1 \right]$$

$$\Big[\underset{\overset{\vee}{V_2^+ \gamma^0}}{\bar{V}_2} \gamma^\nu (1-\gamma^5)(m_q C_{t1j} \overset{\gamma^\alpha p_{j\alpha}}{\widehat{\not{p}_j}} + m_j^2 C_{t2j}) U_1 \Big]^+ = \Big[U_1^+ (m_q C_{t1j} \gamma^{+\alpha} p_{j\alpha} + m_j^2 C_{t2j})(1-\gamma^5) \overset{\gamma^0\gamma^\nu}{\widehat{\gamma^{\nu+}\gamma^0}} V_2 \Big]$$

$$= \left[U_1^+ (m_q C_{t1j} \gamma^{+\alpha} p_{j\alpha} + m_j^2 C_{t2j}) \gamma^0 (1+\gamma^5) \gamma^\nu V_2 \right]$$

$$= \left[\bar{U}_1 (m_q C_{t1j} \not{p}_j + m_j^2 C_{t2j}) \gamma^\nu (1-\gamma^5) V_2 \right]$$

$$\Rightarrow |M_4|^2 = \frac{g^4}{16M_W^2}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)\sum_{i,j} V_{iq}V_{jq}^{*}\sum_{s}\left[\bar{U}_1\left(m_q C_{t1j}\not{p}_j + m_j^2 C_{t2j}\right)\gamma^\nu(1-\gamma^5)V_2\right].$$

$$\cdot\left[\bar{V}_2\gamma^\mu(1-\gamma^5)\left(m_q C_{t1i}\not{p}_i + m_i^2 C_{t2i}\right)U_1\right]$$

$$= \frac{g^4}{16M_W^2}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)\sum_{i,j} V_{iq}V_{jq}^{*}\,Tr\left[(\not{p}_1 + m_q)\left(m_q C_{t1j}\not{p}_j + m_j^2 C_{t2j}\right)\gamma^\nu(1-\gamma^5).\right.$$

$$\left.\cdot(\not{p}_2 - m_q)\gamma^\mu(1-\gamma^5)\left(m_q C_{t1i}\not{p}_i + m_i^2 C_{t2i}\right)\right]$$

$$\gamma^\nu(1-\gamma^5)(\not{p}_2 - m_q)\gamma^\mu(1-\gamma^5)$$

$$= \gamma^\nu(1-\gamma^5)(\not{p}_2 - m_q)(1+\gamma^5)\gamma^\mu$$

$$= \gamma^\nu(1-\gamma^5)\left[\not{p}_2(1+\gamma^5) - m_q(1+\gamma^5)\right]\gamma^\mu$$

$$= \left(\gamma^\nu(1-\gamma^5)(1-\gamma^5)\not{p}_2 - m_q\gamma^\nu\overset{0}{\cancel{(1-\gamma^5)(1+\gamma^5)}}\right)\gamma^\mu$$

$$= 2\gamma^\nu(1-\gamma^5)\not{p}_2\gamma^\mu$$

(*)

$$\Rightarrow |M_4|^2 = \frac{2g^4}{16M_W^2}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)\sum_{i,j} V_{iq}V_{jq}^{*}\,Tr\left[(\not{p}_1 + m_q)\left(m_q C_{t1j}\not{p}_j + m_j^2 C_{t2j}\right)\gamma^\nu(1-\gamma^5)\not{p}_2\gamma^\mu.\right.$$

$$\left.\cdot\left(m_q C_{t1i}\not{p}_i + m_i^2 C_{t2i}\right)\right]$$

$$= \frac{g^4}{8M_W^2}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)\sum_{i,j} V_{iq}V_{jq}^{*}\,Tr\left[\left(m_q C_{t1i}\not{p}_i + m_i^2 C_{t2i}\right)(\not{p}_1 + m_q)\left(m_q C_{t1j}\not{p}_j + m_j^2 C_{t2j}\right).\right.$$

$$\left.\cdot\underbrace{\gamma^\nu(1-\gamma^5)\not{p}_2\gamma^\mu}_{(\gamma^\nu\not{p}_2\gamma^\mu - \gamma^\nu\gamma^5\not{p}_2\gamma^\mu)}\right]$$

$$\left(m_q C_{t1i}\not{p}_i + m_i^2 C_{t2i}\right)(\not{p}_1 + m_q)\left(m_q C_{t1j}\not{p}_j + m_j^2 C_{t2j}\right) = \left(m_q C_{t1i}\not{p}_i + m_i^2 C_{t2i}\right)\left(m_q C_{t1j}\not{p}_1\not{p}_j\right.$$

$$\left. + m_j^2 C_{t2j}\not{p}_1 + m_q^2 C_{t1j}\not{p}_j + m_q m_j^2 C_{t2j}\right) = m_q^2 C_{t1j}C_{t1i}\not{p}_i\not{p}_1\not{p}_j + m_q m_j^2 C_{t1i}C_{t2j}\not{p}_i\not{p}_1$$

$$+ m_q^3 C_{t1i}C_{t1j}\not{p}_i\not{p}_j + m_q^2 m_j^2 C_{t1i}C_{t2j}\not{p}_i + m_i^2 m_q C_{t2i}C_{t1j}\not{p}_1\not{p}_j + m_i^2 m_j^2 C_{t2i}C_{t2j}\not{p}_1$$

$$+ m_i^2 m_q^2 C_{t2i}C_{t1j}\not{p}_j + m_i^2 m_q m_j^2 C_{t2i}C_{t2j}$$

$$|M_4|^2 = \frac{g^4}{8M_W^2}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)\sum_{i,j} V_{iq}V_{jq}^{*}\left\{Tr\underset{0}{\left[m_q^2 C_{t1i}C_{t1j}\not{p}_i\not{p}_1\not{p}_j\gamma^\nu\not{p}_2\gamma^\mu\right]} + m_q m_j^2 C_{t1i}C_{t2j}\,Tr\left[\not{p}_i\not{p}_1\right.\right.$$

$$\left.\gamma^\nu\overset{0}{\cancel{\not{p}_2\gamma^\mu}}\right] + m_q^3 C_{t1i}C_{t1j}\,Tr\left[\not{p}_i\not{p}_j\overset{0}{\cancel{\gamma^\nu\not{p}_2\gamma^\mu}}\right] + m_q^2 m_j^2 C_{t1i}C_{t2j}\,Tr\left[\not{p}_i\gamma^\nu\not{p}_2\gamma^\mu\right]$$

$$+ m_i^2 m_q C_{t2i}C_{t1j}\,Tr\left(\not{p}_1\not{p}_j\overset{0}{\cancel{\gamma^\nu\not{p}_2\gamma^\mu}}\right) + m_i^2 m_j^2 C_{t2i}C_{t2j}\,Tr\left(\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu\right)$$

$$+ m_i^2 m_q^2 C_{t2i}C_{t1j}\,Tr\left(\not{p}_j\gamma^\nu\not{p}_2\gamma^\mu\right) + m_i^2 m_q m_j^2 C_{t2i}C_{t2j}\,Tr\left(\gamma^\nu\overset{0}{\cancel{\not{p}_2}}\gamma^\mu\right)$$

$$- m_q^2 c_{t1i} c_{t1j} \, Tr(\not{p}_i \not{p}_1 \not{p}_j \gamma^\nu \gamma^5 \not{p}_2 \gamma^\mu) - m_q m_j^2 c_{t1i} c_{t2j} \, Tr(\not{p}_i \not{p}_1 \cancelto{0}{\gamma^\nu} \gamma^5 \not{p}_2 \gamma^\mu)$$

$$- m_q^3 c_{t1i} c_{t1j} \, Tr(\not{p}_i \not{p}_j \cancelto{0}{\gamma^\nu} \gamma^5 \not{p}_2 \gamma^\mu) - m_q^2 m_j^2 c_{t1i} c_{t2j} \, Tr(\not{p}_i \gamma^\nu \gamma^5 \not{p}_2 \gamma^\mu)$$

$$- m_i^2 m_q c_{t2i} c_{t1j} \, Tr(\not{p}_1 \not{p}_j \cancelto{0}{\gamma^\nu} \gamma^5 \not{p}_2 \gamma^\mu) - m_i^2 m_j^2 c_{t2i} c_{t2j} \, Tr(\not{p}_1 \gamma^\nu \gamma^5 \not{p}_2 \gamma^\mu)$$

$$- m_i^2 m_q^2 c_{t2i} c_{t1j} \, Tr(\not{p}_j \gamma^\nu \gamma^5 \not{p}_2 \gamma^\mu) - m_i^2 m_q m_j^2 c_{t2i} c_{t2j} \, Tr(\cancelto{0}{\gamma^\nu \gamma^5 \not{p}_2 \gamma^\mu}) \Big\}.$$

$$|M_4|^2 = \frac{g^4}{8 M_W^2} \left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) \sum_{i,j} V_{iq} V_{jq}^* \Big\{ m_q^2 c_{t1i} c_{t1j} \, Tr(\not{p}_i \not{p}_1 \not{p}_j \gamma^\nu \not{p}_2 \gamma^\mu) + m_q^2 m_j^2 c_{t1i} c_{t2j} \cdot$$

$$\cdot Tr(\gamma^\mu \not{p}_i \gamma^\nu \not{p}_2) + m_i^2 m_j^2 c_{t2i} c_{t2j} \, Tr(\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_2) + m_i^2 m_q^2 c_{t2i} c_{t1j} \, Tr(\gamma^\mu \not{p}_j \gamma^\nu \not{p}_2)$$

$$- m_q^2 c_{t1i} c_{t1j} \, Tr(\gamma^5 \not{p}_i \not{p}_1 \not{p}_j \gamma^\nu \not{p}_2 \gamma^\mu) - m_q^2 m_j^2 c_{t1i} c_{t2j} \, Tr(\gamma^5 \not{p}_i \gamma^\nu \not{p}_2 \gamma^\mu)$$

$$- m_i^2 m_j^2 c_{t2i} c_{t2j} \, Tr(\gamma^5 \not{p}_1 \gamma^\nu \not{p}_2 \gamma^\mu) - m_i^2 m_q^2 c_{t2i} c_{t1j} \, Tr(\gamma^5 \not{p}_j \gamma^\nu \not{p}_2 \gamma^\mu) \Big\}$$

$$|M_4|^2 = \frac{g^4}{8 M_W^2} \left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) \sum_{i,j} V_{iq} V_{jq}^* \Big\{ m_q^2 c_{t1i} c_{t1j} \, Tr(\not{p}_i \not{p}_1 \not{p}_j \gamma^\nu \not{p}_2 \gamma^\mu) + 4 m_q^2 m_j^2 c_{t1i} c_{t2j} \cdot$$

$$(P_i^\mu P_2^\nu + P_i^\nu P_2^\mu - (P_i \cdot P_2) g^{\mu\nu}) + 4 m_i^2 m_j^2 c_{t2i} c_{t2j} (P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - (P_1 \cdot P_2) g^{\mu\nu})$$

$$+ 4 m_i^2 m_q^2 c_{t2i} c_{t1j} (P_j^\mu P_2^\nu + P_j^\nu P_2^\mu - (P_j \cdot P_2) g^{\mu\nu}) - m_q^2 c_{t1i} c_{t1j} \, Tr(\gamma^5 \not{p}_i \not{p}_1 \not{p}_j \gamma^\nu \not{p}_2 \gamma^\mu)$$

$$+ 4 i \, m_q^2 m_j^2 c_{t1i} c_{t2j} \, \varepsilon^{\alpha\nu\beta\mu} P_{i\alpha} P_{2\beta} + 4 i \, m_i^2 m_j^2 c_{t2i} c_{t2j} \, \varepsilon^{\alpha\nu\beta\mu} P_{1\alpha} P_{2\beta}$$

$$+ 4 i \, m_i^2 m_q^2 c_{t2i} c_{t1j} \, \varepsilon^{\alpha\nu\beta\mu} P_{j\alpha} P_{2\beta} \Big\}$$

but :

$$\left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) \varepsilon^{\alpha\nu\beta\mu} = - g_{\mu\nu} \cancelto{0}{\varepsilon^{\alpha\nu\beta\mu}} + \frac{1}{M_W^2} P_{4\mu} \cancelto{0}{P_{4\nu} \varepsilon^{\alpha\nu\beta\mu}} = 0$$

$$\Rightarrow |M_4|^2 = \frac{g^4}{8 M_W^2} \left(-g_{\mu\nu} + \frac{P_{4\mu} P_{4\nu}}{M_W^2}\right) \sum_{i,j} V_{iq} V_{jq}^* \Big\{ m_q^2 c_{t1i} c_{t1j} \, Tr(\not{p}_i \not{p}_1 \not{p}_j \gamma^\nu \not{p}_2 \gamma^\mu) + 4 m_q^2 m_j^2 c_{t1i} c_{t2j} \cdot$$

$$(P_i^\mu P_2^\nu + P_i^\nu P_2^\mu - (P_i \cdot P_2) g^{\mu\nu}) + 4 m_i^2 m_j^2 c_{t2i} c_{t2j} (P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - (P_1 \cdot P_2) g^{\mu\nu})$$

$$+ 4 m_i^2 m_q^2 c_{t2i} c_{t1j} (P_j^\mu P_2^\nu + P_j^\nu P_2^\mu - (P_j \cdot P_2) g^{\mu\nu}) - m_q^2 c_{t1i} c_{t1j} \, Tr(\gamma^5 \not{p}_i \not{p}_1 \not{p}_j \gamma^\nu \not{p}_2 \gamma^\mu) \Big\}$$

$$|M_4|^2 = \frac{g^4}{8 M_W^2} \sum_{i,j} V_{iq} V_{jq}^* \Big\{ m_q^2 c_{t1i} c_{t1j} \Big[- Tr(\not{p}_i \not{p}_1 \big| \not{p}_j \overbrace{\gamma^\nu \not{p}_2 \gamma_\nu}^{-2\not{p}_2}) \Big] + \frac{m_q^2}{M_W^2} c_{t1i} c_{t1j} \cdot$$

$$\cdot Tr(\not{p}_i \not{p}_1 \not{p}_j \not{p}_4 \not{p}_2 \not{p}_4) \Big] + 4 m_q^2 m_j^2 c_{t1i} c_{t2j} \big[-2 (P_i \cdot P_2) + 4 (P_i \cdot P_2) \big] + 4 m_i^2 m_j^2 c_{t2i} c_{t2j} \cdot$$

$$\cdot \big[-2 (P_1 \cdot P_2) + 4 (P_1 \cdot P_2) \big] + 4 \frac{m_q^2 m_j^2}{M_W^2} c_{t1i} c_{t2j} \big[2 (P_i \cdot P_4)(P_2 \cdot P_4) - (P_i \cdot P_2) M_W^2 \big]$$

$$+ 4 \frac{m_i^2 m_j^2}{M_W^2} c_{t2i} c_{t2j} \big[2 (P_1 \cdot P_4)(P_2 \cdot P_4) - (P_1 \cdot P_2) M_W^2 \big] + 4 m_i^2 m_q^2 c_{t2i} c_{t1j} \big[-2 (P_2 \cdot P_j) + 4 (P_2 \cdot P_j) \big]$$

$$+ 4 \frac{m_i^2 m_q^2}{M_W^2} c_{t2i} c_{t1j} \big[2 (P_j \cdot P_4)(P_2 \cdot P_4) - (P_j \cdot P_2) M_W^2 \big] + m_q^2 c_{t1i} c_{t1j} \, Tr(\gamma^5 \big| \not{p}_i \not{p}_1 \not{p}_j \overbrace{\gamma^\nu \not{p}_2 \gamma_\nu}^{-2\not{p}_2})$$

$$- \frac{m_q^2}{M_W^2} c_{t1i} c_{t1j} \, Tr(\gamma^5 \not{p}_i \not{p}_1 \not{p}_j \not{p}_4 \not{p}_2 \not{p}_4) \Big\}$$

$$|M_4|^2 = \frac{g^4}{8M_W^2} \sum_{i,j} V_{iq} V_{j4}^* \Big\{ m_y^2 C_{t1i} C_{t1j} \left[2\,\mathrm{Tr}\left(\not{p}_i \not{p}_1 \not{p}_j \not{p}_2\right)\right] + \frac{m_q^2}{M_W^2} C_{t1i} C_{t1j} \cdot \mathrm{Tr}\Big(\not{p}_i \not{p}_1 \not{p}_j \not{p}_4 \cdot$$

$$\cdot \left(-\not{p}_4 \not{p}_2 + 2(p_2 \cdot p_4)\right)\Big) + 8 m_y^2 m_j^2 C_{t1i} C_{t2j} (p_i \cdot p_2) + 8 m_i^2 m_j^2 C_{t2i} C_{t2j} (p_1 \cdot p_2)$$

$$+ 8 m_i^2 m_y^2 C_{t2i} C_{t1j} (p_2 \cdot p_j) + 4 \frac{m_y^2 m_j^2}{M_W^2} C_{t1i} C_{t2j} \left[2 (p_i \cdot p_4)(p_2 \cdot p_4) - (p_i \cdot p_2) M_W^2 \right]$$

$$+ 4 \frac{m_i^2 m_j^2}{M_W^2} C_{t2i} C_{t2j} \left[2 (p_1 \cdot p_4)(p_2 \cdot p_4) - (p_1 \cdot p_2) M_W^2 \right] + \frac{4 m_i^2 m_y^2}{M_W^2} C_{t2i} C_{t1j} \Big[2 (p_j \cdot p_4)(p_2 \cdot p_4) -$$

$$- (p_j \cdot p_2) M_W^2 \Big] - 2 m_y^2 C_{t1i} C_{t1j}\, \mathrm{Tr}\left(\gamma^5 \not{p}_i \not{p}_1 \not{p}_j \not{p}_2\right) - \frac{m_y^2}{M_W^2} C_{t1i} C_{t1j}\, \mathrm{Tr}\Big(\gamma^5 \not{p}_i \not{p}_1 \not{p}_j \not{p}_4 \big(-\not{p}_4 \not{p}_2$$

$$+ 2 (p_2 \cdot p_4)\big)\Big) \Big\}$$

$$|M_4|^2 = \frac{g^4}{8M_W^2} \sum_{i,j} V_{iq} V_{j4}^* \Big\{ m_y^2 C_{t1i} C_{t1j}\, \mathrm{Tr}\left(\not{p}_i \not{p}_1 \not{p}_j \not{p}_2\right) + 2 \frac{m_y^2}{M_W^2} C_{t1i} C_{t1j} (p_2 \cdot p_4)\, \mathrm{Tr}\left(\not{p}_i \not{p}_1 \not{p}_j \not{p}_4\right)$$

$$+ 8 m_y^2 m_j^2 C_{t1i} C_{t2j} (p_i \cdot p_2) + 8 m_i^2 m_j^2 C_{t2i} C_{t2j} (p_1 \cdot p_2) + 8 m_i^2 m_y^2 C_{t2i} C_{t1j} (p_2 \cdot p_j)$$

$$+ 4 \frac{m_y^2 m_j^2}{M_W^2} C_{t1i} C_{t2j} \left[2 (p_i \cdot p_4)(p_2 \cdot p_4) - (p_i \cdot p_2) M_W^2 \right] + \frac{4 m_i^2 m_j^2}{M_W^2} C_{t2i} C_{t2j} \Big[2 (p_1 \cdot p_4)(p_2 \cdot p_4) -$$

$$(p_1 \cdot p_2) M_W^2 \Big] + \frac{4 m_i^2 m_y^2}{M_W^2} C_{t2i} C_{t1j} \left[2 (p_j \cdot p_4)(p_2 \cdot p_4) - (p_j \cdot p_2) M_W^2 \right] + 4 i\, m_y^2 C_{t1i} C_{t1j}\, \varepsilon^{\mu\nu\alpha\beta}$$

$$p_{i\mu}\, p_{1\nu}\, p_{j\alpha}\, p_{2\beta} - 2 \frac{m_y^2}{M_V^2} C_{t1i} C_{t1j} (p_2 \cdot p_4) \left(-4 i\, \varepsilon^{\mu\nu\alpha\beta}\right) p_{i\mu}\, p_{1\nu}\, p_{j\alpha}\, p_{4\beta} \Big\}$$

$$p_{i_\mu} = (p_1 - p_3)_\mu$$

$$p_{j_\alpha} = (p_1 - p_3)_\alpha$$

$$\begin{aligned}
\varepsilon^{\mu\nu\alpha\beta}\, p_{i\mu}\, p_{1\nu}\, p_{j\alpha} &= \varepsilon^{\mu\nu\alpha\beta} (p_{1\mu} - p_{3\mu})\, p_{1\nu}\, p_{j\alpha} \\
&= - \varepsilon^{\mu\nu\alpha\beta}\, p_{3\mu}\, p_{1\nu}\, p_{j\alpha} \\
&= - \varepsilon^{\mu\nu\alpha\beta}\, p_{3\mu}\, p_{1\nu} \left(p_{1\alpha} - p_{3\alpha}\right) \\
&= - \varepsilon^{\mu\nu\alpha\beta}\, p_{3\mu}\, p_{1\nu}\, p_{1\alpha} + \varepsilon^{\mu\nu\alpha\beta}\, p_{3\mu}\, p_{1\nu}\, p_{3\alpha} \quad = 0
\end{aligned}$$

$$\Rightarrow |M_4|^2 = \frac{g^4}{8M_W^2} \sum_{i,j} V_{iq} V_{j4}^* \Big\{ 4 m_y^2 C_{t1i} C_{t1j} \left[(p_i \cdot p_1)(p_j \cdot p_2) + (p_i \cdot p_2)(p_1 \cdot p_j) - (p_i \cdot p_j)(p_1 \cdot p_2) \right]$$

$$+ 8 \frac{m_y^2}{M_W^2} C_{t1i} C_{t1j} (p_2 \cdot p_4) \left[(p_i \cdot p_1)(p_j \cdot p_4) + (p_i \cdot p_4)(p_1 \cdot p_j) - (p_i \cdot p_j)(p_1 \cdot p_4) \right] + 8 m_y^2 m_j^2 C_{t1i} C_{t2j} (p_i \cdot p_2)$$

$$+ 8 m_i^2 m_j^2 C_{t2i} C_{t2j} (p_1 \cdot p_2) + 8 m_i^2 m_q^2 C_{t2i} C_{t1j} (p_2 \cdot p_j) + 8 \frac{m_y^2 m_j^2}{M_W^2} C_{t1i} C_{t2j} (p_i \cdot p_4)(p_2 \cdot p_4)$$

$$- 4 m_y^2 m_j^2 C_{t1i} C_{t2j} (p_i \cdot p_2) + 8 \frac{m_i^2 m_j^2}{M_W^2} C_{t2i} C_{t2j} (p_1 \cdot p_4)(p_2 \cdot p_4) - 4 m_i^2 m_j^2 C_{t2i} C_{t2j} (p_1 \cdot p_2)$$

$$+ 8 \frac{m_i^2 m_y^2}{M_W^2} C_{t2i} C_{t1j} (p_j \cdot p_4)(p_2 \cdot p_4) - 4 m_i^2 m_q^2 C_{t2i} C_{t1j} (p_j \cdot p_2) \Big\}$$

(174)

$$|M_4|^2 = \frac{g^4}{8M_w^2} \sum_{i,j} V_{iq} V_{jq}^* \Big\{ 4m_q^2 C_{t1i} C_{t1j} \big[(P_i \cdot P_1)(P_j \cdot P_2) + (P_i \cdot P_2)(P_1 \cdot P_j) - (P_i \cdot P_j)(P_1 \cdot P_2) \big]$$

$$+ 8\frac{m_q^2}{M_w^2} C_{t1i} C_{t1j} (P_2 \cdot P_4) \big[(P_1 \cdot P_i)(P_j \cdot P_4) + (P_i \cdot P_4)(P_1 \cdot P_j) - (P_i \cdot P_j)(P_1 \cdot P_4) \big]$$

$$+ 4m_q^2 m_j^2 C_{t1i} C_{t2j} (P_i \cdot P_2) + 4 m_i^2 m_j^2 C_{t2i} C_{t2j} (P_1 \cdot P_2) + 4 m_i^2 m_q^2 C_{t2i} C_{t1j} (P_2 \cdot P_j)$$

$$+ 8\frac{m_q^2 m_j^2}{M_w^2} C_{t1i} C_{t2j} (P_i \cdot P_4)(P_2 \cdot P_4) + \frac{8 m_i^2 m_j^2}{M_w^2} C_{t2i} C_{t2j} (P_1 \cdot P_4)(P_2 \cdot P_4) + \frac{8 m_i^2 m_q^2}{M_w^2} C_{t2i} C_{t1j} (P_j \cdot P_4)(P_2 \cdot P_4) \Big\}$$

$$P_i = P_j = K = P_{q'}$$

$$K^2 = t$$

$$|M_4|^2 = \frac{g^4}{8M_w^2} \sum_{i,j} V_{iq} V_{jq}^* \Big\{ 4m_q^2 C_{t1i} C_{t1j} \big[2(K \cdot P_1)(K \cdot P_2) - t(P_1 \cdot P_2) \big] + \frac{8m_q^2}{M_w^2} C_{t1i} C_{t1j} (P_2 \cdot P_4) \big[$$

$$2(K \cdot P_1)(K \cdot P_4) - t(P_1 \cdot P_4) \big] + 4m_q^2 m_j^2 C_{t1i} C_{t2j} (K \cdot P_2) + 4 m_i^2 m_j^2 C_{t2i} C_{t2j} (P_1 \cdot P_2)$$

$$+ 4 m_i^2 m_q^2 C_{t2i} C_{t1j} (K \cdot P_2) + \frac{8 m_q^2 m_j^2}{M_w^2} C_{t1i} C_{t2j} (K \cdot P_4)(P_2 \cdot P_4) + \frac{8 m_i^2 m_j^2}{M_w^2} C_{t2i} C_{t2j} (P_1 \cdot P_4)(P_2 \cdot P_4)$$

$$+ \frac{8 m_i^2 m_q^2}{M_w^2} C_{t2i} C_{t1j} (K \cdot P_4)(P_2 \cdot P_4) \Big\}$$

$$|M_4|^2 = \frac{g^4}{8M_w^2} \sum_{i,j} V_{iq} V_{jq}^* \Big\{ \frac{4m_q^2}{M_w^2} C_{t1i} C_{t1j} \Big[\frac{1}{2}(m_q^2 + t - M_{H^\pm}^2)(M_w^2 - m_q^2 - t) M_w^2 - t M_w^2 \left(\frac{s}{2} - m_q^2\right)$$

$$+ \cancel{2} \frac{1}{\cancel{2}} (M_w^2 + m_q^2 - t) \Big((m_q^2 + t - M_{H^\pm}^2) \frac{1}{2} (M_w^2 - m_q^2 + t) - t \frac{1}{2} (s + t - m_q^2 - M_{H^\pm}^2) \Big) \Big]$$

$$+ \frac{4 m_q^2 m_j^2}{M_w^2} C_{t1i} C_{t2j} \Big[\frac{1}{2}(M_w^2 - m_q^2 - t) M_w^2 + (M_w^2 - m_q^2 + t) \frac{1}{2} (M_w^2 + m_q^2 - t) \Big]$$

$$+ 4 \frac{m_i^2 m_j^2}{M_w^2} C_{t2i} C_{t2j} \Big[\left(\frac{s}{2} - m_q^2\right) M_w^2 + (s + t - m_q^2 - M_{H^\pm}^2) \frac{1}{2} (M_w^2 + m_q^2 - t) \Big] \swarrow$$

$$+ \frac{4 m_i^2 m_q^2}{M_w^2} C_{t2i} C_{t1j} \Big[\frac{1}{2} (M_w^2 - m_q^2 - t) M_w^2 + \frac{1}{2} (M_w^2 - m_q^2 + t)(M_w^2 + m_q^2 - t) \Big] \Big\}$$

$$|M_4|^2 = \frac{g^4}{8M_w^2} \sum_{i,j} V_{iq} V_{jq}^* \Big\{ \frac{4m_q^2}{M_w^2} C_{t1i} C_{t1j} \frac{1}{2} \Big[m_q^2 \cancel{M_w^4} - m_q^4 \cancel{M_w^2} - t\, \cancel{m_q^2} M_w^2 + t \cancel{M_w^4} - t \cancel{m_q^2} M_w^2 - t^2 \cancel{M_w^2}$$

$$- M_{H^\pm}^2 \cancel{M_w^4} + m_q^2 \cancel{M_w^2} M_{H^\pm}^2 + t \cancel{M_w^2} M_{H^\pm}^2 - s t \cancel{M_w^2} + 2 \cancel{m_q^2} t M_w^2$$

$$+ (M_w^2 + m_q^2 - t) \big(m_q^2 M_w^2 - m_q^4 + t \cancel{m_q^2} + t M_w^2 - t \cancel{m_q^2} + \cancel{t^2} - M_{H^\pm}^2 M_w^2 + m_q^2 M_{H^\pm}^2 - t \cancel{M_{H^\pm}^2}$$

$$- st - \cancel{t^2} + t m_q^2 + t \cancel{M_{H^\pm}^2} \big) \Big] + \frac{4 m_q^2 m_j^2}{M_w^2} C_{t1i} C_{t2j} \frac{1}{2} \Big[\cancel{M_w^4} - m_q^2 \cancel{M_w^2} - \cancel{t M_w^2} + \cancel{M_w^4} + m_q^2 \cancel{M_w^2}$$

$$- t \cancel{M_w^2} - m_q^2 \cancel{M_w^2} - \cancel{m_q^4} + t \cancel{m_q^2} + t \cancel{M_w^2} + t \cancel{m_q^2} - \cancel{t^2} \Big] + 4 \frac{m_i^2 m_j^2}{M_w^2} C_{t2i} C_{t2j} \frac{1}{2} \Big[s \cancel{M_w^2} - 2 \cancel{m_q^2} M_w^2$$

$$+ s \cancel{M_w^2} + s \cancel{m_q^2} - \cancel{st} + t \cancel{M_w^2} + t \cancel{m_q^2} - \cancel{t^2} - \cancel{m_q^2} M_w^2 - \cancel{m_q^4} + t \cancel{m_q^2} - M_{H^\pm}^2 \cancel{M_w^2} - M_{H^\pm}^2 \cancel{m_q^2} + t \cancel{M_{H^\pm}^2} \Big]$$

$$+ \frac{4 m_i^2 m_q^2}{M_W^2} C_{t2i} C_{t1j} \frac{1}{2}\left[\cancel{M_W^4} - \cancel{m_q^2 M_W^2} - \cancel{t M_W^2} + \cancel{M_W^4} + \cancel{m_q^2 M_W^2} - \cancel{t M_W^2} - \cancel{m_q^2 M_W^2} - \cancel{m_q^4} + \cancel{t m_q^2} + \cancel{t M_W^2} + \cancel{t m_q^2} - \cancel{t^2} \right]\Big\}$$

$$|M_4|^2 = \frac{g^4}{8 M_W^2} \sum_{i,j} V_{iq} V_{jq}^* \Big\{ \frac{2 m_q^2}{M_W^2} C_{t1i} C_{t1j} \Big[\cancel{m_q^2 M_W^4} - \cancel{m_q^4 M_W^2} + \cancel{m_q^2 t M_W^2} + \cancel{t M_W^4} - \cancel{t m_q^2 M_W^2} - \cancel{t^2 M_W^2} - \cancel{M_{H^\pm}^2 M_W^4}$$

$$+ \cancel{m_q^2 M_W^2 M_{H^\pm}^2} + \cancel{t M_W^2 M_{H^\pm}^2} - \cancel{st M_W^2} + \cancel{m_q^2 M_W^4} - \cancel{m_q^4 M_W^2} + \cancel{t M_W^4} - \cancel{M_{H^\pm}^2 M_W^4} + \cancel{m_q^2 M_{H^\pm}^2 M_W^2} - \cancel{st M_W^2} + \cancel{t m_q^2 M_W^2}$$

$$+ \cancel{m_q^4 M_W^2} - \cancel{m_q^6} + \cancel{t m_q^2 M_W^2} - \cancel{m_q^2 M_{H^\pm}^2 M_W^2} + \cancel{m_q^4 M_{H^\pm}^2} - \cancel{st m_q^2} + \cancel{t m_q^4} - \cancel{t m_q^2 M_W^2} + \cancel{t m_q^4} - \cancel{t^2 M_W^2}$$

$$+ \cancel{t M_{H^\pm}^2 M_W^2} - \cancel{t m_q^2 M_{H^\pm}^2} + \cancel{st^2} - \cancel{t^2 m_q^2} \Big] + \frac{2 m_q^2 m_j^2}{M_W^2} C_{t1i} C_{t2j} \left[2 M_W^4 - t M_W^2 - \cancel{m_q^2 M_W^2} - m_q^4 + 2 t m_q^2 - t^2 \right]$$

$$+ \frac{2 m_i^2 m_j^2}{M_W^2} C_{t2i} C_{t2j} \left[2s M_W^2 - 3 m_q^2 M_W^2 + s m_q^2 - st + \cancel{t M_W^2} + 2 t m_q^2 - t^2 - m_q^4 - M_{H^\pm}^2 M_W^2 - M_{H^\pm}^2 m_q^2 + t M_{H^\pm}^2 \right]$$

$$+ \frac{2 m_i^2 m_q^2}{M_W^2} C_{t2i} C_{t1j} \left[\cancel{2 M_W^4} - t M_W^2 - m_q^2 M_W^2 - m_q^4 + 2 t m_q^2 - t^2 \right]\Big\}$$

(**)

$$|M_4|^2 = \frac{g^4}{4 M_W^4} \sum_{i,j} V_{iq} V_{jq}^* \Big\{ m_q^2 C_{t1i} C_{t1j} \Big[2 m_q^2 M_W^4 - m_q^4 M_W^2 + 2 t M_W^4 - 2 t^2 M_W^2 - 2 M_{H^\pm}^2 M_W^4 + m_q^2 M_W^2 M_{H^\pm}^2 + 2 t M_W^2 M_{H^\pm}^2$$

$$- 2 st M_W^2 - m_q^6 + t m_q^2 M_W^2 + m_q^4 M_{H^\pm}^2 - st m_q^2 + 2 t m_q^4 - t m_q^2 M_{H^\pm}^2 + st^2 - t^2 m_q^2 \Big]$$

$$+ m_q^2 m_j^2 C_{t1i} C_{t2j} \left[2 M_W^4 - t M_W^2 - \cancel{m_q^2 M_W^2} - m_q^4 + 2 t m_q^2 - t^2 \right] + m_i^2 m_q^2 C_{t2i} C_{t1j} \Big[2 M_W^4 - \cancel{t M_W^2}$$

$$- m_q^2 M_W^2 - m_q^4 + 2 t m_q^2 - t^2 \Big] + m_i^2 m_j^2 C_{t2i} C_{t2j} \Big[2s M_W^2 - 3 m_q^2 M_W^2 + s m_q^2 - st + t M_W^2 + 2 t m_q^2$$

$$- t^2 - m_q^4 - M_{H^\pm}^2 M_W^2 - M_{H^\pm}^2 m_q^2 + t M_{H^\pm}^2 \Big] \Big\}$$

$$|M_4|^2 = \frac{g^4}{4 M_W^4} \sum_{i,j} V_{iq} V_{jq}^* \Big\{ m_q^2 C_{t1i} C_{t1j} \Big[\xcancel{2 m_q^2 M_W^4} - \xcancel{m_q^4 M_W^2} + \xcancel{2 t M_W^4} - \xcancel{2 t^2 M_W^2} - \xcancel{2 M_{H^\pm}^2 M_W^4} + \xcancel{m_q^2 M_W^2 M_{H^\pm}^2}$$

$$+ \xcancel{2 t M_W^2 M_{H^\pm}^2} - \xcancel{2 t M_W^4} - \xcancel{2 t M_W^2 M_{H^\pm}^2} - \xcancel{4 t m_q^2 M_W^2} + \xcancel{2 t M_W^2} + \xcancel{2 ut M_W^2} - \xcancel{m_q^6} + \xcancel{t m_q^2 M_W^2} + \xcancel{m_q^4 M_{H^\pm}^2} + \cancel{2 t m_q^4}$$

$$- \xcancel{t m_q^2 M_{H^\pm}^2} + \xcancel{st^2} - \xcancel{t^2 m_q^2} - \xcancel{t m_q^2 M_W^2} - \xcancel{t m_q^2 M_{H^\pm}^2} - \xcancel{2 t m_q^4} + \xcancel{t^2 m_q^2} + \xcancel{ut m_q^2} \Big]$$

$$+ m_q^2 \left[2 M_W^4 - t M_W^2 - m_q^2 M_W^2 - m_q^4 + 2 t m_q^2 - t^2 \right] \left[m_j^2 C_{t1i} C_{t2j} + m_i^2 C_{t2i} C_{t1j} \right]$$

$$+ m_i^2 m_j^2 C_{t2i} C_{t2j} \Big[\cancel{2s M_W^2} - \cancel{3 m_q^2 M_W^2} + \cancel{s m_q^2} - \cancel{t M_W^2} - \cancel{t M_{H^\pm}^2} - \cancel{2 t m_q^2} + \cancel{t^2} + \cancel{ut} + \cancel{t M_W^2}$$

$$+ \cancel{2 t m_q^2} - \cancel{t^2} - \cancel{m_q^4} - \cancel{M_{H^\pm}^2 M_W^2} - \cancel{M_{H^\pm}^2 m_q^2} + \cancel{t M_{H^\pm}^2} \Big] \Big\}$$

$$|M_4|^2 = \frac{g^4}{4 M_W^4} \sum_{i,j} V_{iq} V_{jq}^* \Big\{ m_q^2 C_{t1i} C_{t1j} \Big[2 m_q^2 M_W^4 - m_q^4 M_W^2 - 2 M_{H^\pm}^2 M_W^4 + m_q^2 M_W^2 M_{H^\pm}^2 - 4 t m_q^2 M_W^2$$

$$+ 2 ut M_W^2 - m_q^6 + m_q^4 M_{H^\pm}^2 - 2 t m_q^2 M_{H^\pm}^2 + st^2 + ut m_q^2 \Big] +$$

$$+ m_q^2 \left[2 M_W^4 - t M_W^2 - m_q^2 M_W^2 - m_q^4 + 2 t m_q^2 - t^2 \right] \left[m_j^2 C_{t1i} C_{t2j} + m_i^2 C_{t2i} C_{t1j} \right] + m_i^2 m_j^2 C_{t2i} C_{t2j} \Big[2s M_W^2$$

$$- 3 m_q^2 M_W^2 + s m_q^2 + ut - m_q^4 - M_{H^\pm}^2 M_W^2 - M_{H^\pm}^2 m_q^2 \Big] \Big\}$$

$$
\begin{aligned}
|M_4|^2 = \frac{g^4}{4M_W^4}\sum_{i,j} V_{iq}V_{jq}^{*} \Big\{ & m_q^2 C_{t_1 i} C_{t_1 j} \Big[\cancel{2m_q^2 M_W^4} \cancel{- m_q^4 M_W^2} \cancel{- 2M_{H^\pm}^2 M_W^4} \cancel{+ m_q^2 M_W^2 M_{H^\pm}^2} \cancel{- 4t\, m_q^2 M_W^2} \\
& + 2M_W^2 \big(\cancel{m_q^4} \cancel{- m_q^2 M_{H^\pm}^2} \cancel{- m_q^2 M_W^2} \cancel{+ M_{H^\pm}^2 M_W^2} + \tfrac{1}{5} (\cancel{m_q^2 M_{H^\pm}^4} \cancel{+ m_q^2 M_W^4} \cancel{- 2m_q^2 M_{H^\pm}^2 M_W^2} \cancel{+ \tfrac{1}{4} \sin^2\theta\, \lambda\, (s-4m_q^2)}) \big) \\
& \cancel{- m_q^6} \cancel{+ m_q^4 M_{H^\pm}^2} \cancel{- 2t\, m_q^2 M_{H^\pm}^2} \cancel{+ st^2} \cancel{+ m_q^6} \cancel{- m_q^4 M_{H^\pm}^2} \cancel{- m_q^4 M_W^2} \cancel{+ m_q^2 M_{H^\pm}^2 M_W^2} + \tfrac{1}{5} \big[\cancel{m_q^4 M_{H^\pm}^4} \cancel{+ m_q^4 M_W^4} \\
& \cancel{- 2m_q^4 M_{H^\pm}^2 M_W^2} \cancel{+ \tfrac{1}{4} m_q^2 \sin^2\theta\, \lambda\, (s-4m_q^2)} \big] \Big] + m_q^2 (2M_W^4 - tM_W^2 - m_q^2 M_W^2 - m_q^4 + 2t\, m_q^2 - t^2)(m_j^2 C_{t_1 i} C_{t_2 j} \\
& + m_i^2 C_{t_2 i} C_{t_1 j}) + m_i^2 m_j^2 C_{t_2 i} C_{t_2 j} \Big[2s \cancel{M_W^2} \cancel{- 3m_q^2 M_W^2} + s \cancel{m_q^2} \cancel{- m_q^4} \cancel{- M_{H^\pm}^2 M_W^2} \cancel{- M_{H^\pm}^2 m_q^2} \cancel{+ m_q^4} \cancel{- m_q^2 M_{H^\pm}^2} \\
& \cancel{- m_q^2 M_W^2} \cancel{+ M_{H^\pm}^2 M_W^2} + \tfrac{1}{5} (m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac{1}{4} \sin^2\theta\, \lambda\, (s-4m_q^2)) \Big] \Big\}
\end{aligned}
$$

$$
\begin{aligned}
|M_4|^2 = \frac{g^4}{4M_W^4}\sum_{i,j} V_{iq}V_{jq}^{*} \Big\{ & m_q^2 C_{t_1 i} C_{t_1 j} \Big[-4t\, m_q^2 M_W^2 - 2t\, m_q^2 M_{H^\pm}^2 + st^2 + 2\frac{M_W^2}{5} \big(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 \\
& - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac{1}{4} \sin^2\theta\, \lambda\, (s-4m_q^2) \big) + \tfrac{1}{5} \big(m_q^4 M_{H^\pm}^4 + m_q^4 M_W^4 - 2m_q^4 M_{H^\pm}^2 M_W^2 + \frac{m_q^2}{4} \sin^2\theta\, \lambda\, (s-4m_q^2) \big) \Big] \\
& + m_q^2 (2M_W^4 - tM_W^2 - m_q^2 M_W^2 - m_q^4 + 2t\, m_q^2 - t^2)(m_j^2 C_{t_1 i} C_{t_2 j} + m_i^2 C_{t_2 i} C_{t_1 j}) + m_i^2 m_j^2 C_{t_2 i} C_{t_2 j} \Big[2s M_W^2 \\
& - 4m_q^2 M_W^2 + s m_q^2 - 2 M_{H^\pm}^2 m_q^2 + \tfrac{1}{5} (m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac{1}{4} \sin^2\theta\, \lambda\, (s-4m_q^2)) \Big] \Big\}
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
|M_4|^2 = \frac{g^4}{4M_W^4}\sum_{i,j} V_{iq}V_{jq}^{*} \Big\{ & m_q^2 C_{t_1 i} C_{t_1 j} \Big[-2t\, m_q^2 (M_{H^\pm}^2 + 2M_W^2) + st^2 + 2\frac{M_W^2}{5} \big(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 \\
& - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac{1}{4} \sin^2\theta\, \lambda\, (s-4m_q^2) \big) + \frac{m_q^2}{5} \big(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac{1}{4} \sin^2\theta\, \lambda\, (s-4m_q^2) \big) \Big] \\
& + m_q^2 (2M_W^4 - tM_W^2 - m_q^2 M_W^2 - m_q^4 + 2t\, m_q^2 - t^2)(m_j^2 C_{t_1 i} C_{t_2 j} + m_i^2 C_{t_2 i} C_{t_1 j}) + m_i^2 m_j^2 C_{t_2 i} C_{t_2 j} \Big[2s M_W^2 \\
& - 4m_q^2 M_W^2 + s m_q^2 - 2m_q^2 M_{H^\pm}^2 + \tfrac{1}{5} (m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac{1}{4} \sin^2\theta\, \lambda\, (s-4m_q^2)) \Big] \Big\}
\end{aligned}
}
$$

(74a) OK

$$
\begin{aligned}
\overline{|M_I|^2} = \frac{g^4}{96 M_W^4} \Big\{ & m_q^2 |C_{H_b}|^2 \lambda(s, M_W^2, M_{H^\pm}^2)(s-4m_q^2) + m_q^2 |C_{A_b}|^2 s \lambda(s, M_W^2, M_{H^\pm}^2) + \\
& + 2 \sum_{i,j} V_{iq} V_{jq}^* \Big\{ m_q^2 C_{t1i} C_{t1j} \Big[-2t m_q^2 (M_{H^\pm}^2 + 2M_W^2) + st^2 + \frac{2M_W^2}{s} \big(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 \\
& + \tfrac{1}{4} \sin^2\theta \, \lambda (s-4m_q^2) \big) + \frac{m_q^2}{s} \big(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac{1}{4} \sin^2\theta \, \lambda (s-4m_q^2) \big) \Big] \\
& + (m_j^2 C_{t1i} C_{t2j} + m_i^2 C_{t2i} C_{t1j}) m_q^2 (2M_W^4 - t M_W^2 - m_q^2 M_W^2 - m_q^4 + 2t m_q^2 - t^2) + m_i^2 m_j^2 C_{t2i} C_{t2j} \cdot \\
& \cdot \Big[2s M_W^2 - 4m_q^2 M_W^2 + s m_q^2 - 2m_q^2 M_{H^\pm}^2 + \frac{1}{s} \big(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac{1}{4} \sin^2\theta \, \lambda (s-4m_q^2) \big) \Big] \Big\} \\
& - \frac{1}{2} m_q^2 \sum_i (C_{H_b}^* V_{iq} + C_{H_b} V_{iq}^*) \Big\{ C_{t1i} \Big[-2st^2 + 2s m_q^4 - 4s m_q^2 M_W^2 + 2s M_{H^\pm}^2 M_W^2 + 4m_q^2 M_W^4 - 4m_q^2 M_{H^\pm}^2 M_W^2 \\
& + 4m_q^4 M_W^2 - 4m_q^2 t M_W^2 - 4m_q^4 M_{H^\pm}^2 + 4m_q^2 t M_{H^\pm}^2 - \tfrac{1}{2} \sin^2\theta \, \lambda (s-4m_q^2) \Big] + 2m_i^2 C_{t2i} \Big\{ M_W^4 - 4t M_W^2 - s^2 \\
& + 2s M_{H^\pm}^2 + 2s m_q^2 - 4t m_q^2 + 2t^2 - M_{H^\pm}^4 - 4m_q^2 M_{H^\pm}^2 + 2m_q^4 + 2M_{H^\pm}^2 M_W^2 + \frac{2}{s} \Big[m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 \\
& + \tfrac{1}{4} \sin^2\theta \, \lambda (s-4m_q^2) \Big] \Big\} \Big\} + \frac{1}{2} m_q^2 \sum_i (V_{iq} C_{A_b}^* + V_{iq}^* C_{A_b}) \Big\{ C_{t1i} \Big[-2s M_{H^\pm}^2 M_W^2 + 2st^2 + 2s m_q^4 + \\
& + \tfrac{1}{2} \sin^2\theta \, \lambda (s-4m_q^2) - 4m_q^2 st \Big] + 2m_i^2 C_{t2i} \Big[2s M_W^2 + 2M_W^2 M_{H^\pm}^2 + 2s M_{H^\pm}^2 - s^2 - M_W^4 - M_{H^\pm}^4 \Big] \Big\} \Big\}
\end{aligned}
$$

(116a)

$$\boxed{\frac{d\sigma_I}{dt} = \frac{1}{64\pi s} \cdot \frac{4}{(s-4m_q^2)} \overline{|M_I|^2}}$$

or:

$$\boxed{\frac{d\sigma_I}{dt} = \frac{1}{16\pi s} \cdot \frac{1}{(s-4m_q^2)} \overline{|M_I|^2}} \quad (120a)$$

$$\boxed{\frac{d\sigma_I}{dt}(q\bar{q} \to H^- W^+) = \frac{G_F^2}{48\pi s} \cdot \frac{1}{(s-4m_q^2)} \overline{|M_I|^2}} \longrightarrow \text{only } \{\} \quad (120b)$$

with $q = d, s, b$; $i, j = u, c, t$

The only important contribution is $q = b$ and $i, j = t$

Then $\frac{d\sigma}{dt}$ is given by (121)

(IV)

$$\boxed{|M_t^H|^2 = \frac{g^4 m_q^2 |C_{Ht}|^2}{8M_W^4}\lambda(s, M_W^2, M_{H^\pm}^2)(s-4m_q^2)} \qquad (122)$$

$$\boxed{|M_{3_t}|^2 = \frac{g^4 m_q^2 |C_{At}|^2}{8M_W^4}\lambda(s, M_W^2, M_{H^\pm}^2)\, s} \qquad (123)$$

$$\boxed{M_t^{H+} M_{3_t} + M_{3_t}^{+} M_t^H = 0} \qquad (124)$$

$|M_{4_{IV}}|^2$ is obtained from (74a) changing $C_{t_{1i}} \leftrightarrow C_{t_{2i}}$ and $V_{iq} \leftrightarrow V^*_{qi}$

$$\Rightarrow \boxed{\begin{aligned}|M_{4_{IV}}|^2 = {}& \frac{g^4}{4M_W^4}\sum_{i,j} V^*_{qi}V_{qj}\Big\{ m_q^2 C_{t2i}C_{t2j}\Big[-2t m_q^2(M_{H^\pm}^2+2M_W^2) + st^2 + 2\frac{M_W^2}{s}(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 \\ & - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac14 \sin^2\theta\,\lambda\,(s-4m_q^2)) + \frac{m_q^2}{s}(m_q^2M_{H^\pm}^4 + m_q^2M_W^4 - 2m_q^2M_{H^\pm}^2M_W^2 + \tfrac14\sin^2\theta\,\lambda\,(s-4m_q^2))\Big] \\ & + m_q^2(2M_W^4 - tM_W^2 - m_q^2M_W^2 - m_q^4 + 2tm_q^2 - t^2)(m_j^2 C_{t2i}C_{t1j} + m_i^2 C_{t1i}C_{t2j}) + m_i^2m_j^2 C_{t1i}C_{t1j}\Big[\\ & 2sM_W^2 - 4m_q^2M_W^2 + sm_q^2 - 2m_q^2M_{H^\pm}^2 + \frac1s(m_q^2M_{H^\pm}^4 + m_q^2M_W^4 - 2m_q^2M_{H^\pm}^2M_W^2 + \tfrac14\sin^2\theta\,\lambda\,(s-4m_q^2))\Big]\Big\}\end{aligned}}$$

$C_{Hb} \leftrightarrow C_{Ht}$; $C_{Ab} \leftrightarrow C_{At}$ OK (125)

$$\boxed{\begin{aligned}M^+_{4_{IV}} M^H_t + M^{H+}_t M_{4_{IV}} = {}& -\frac{g^4 m_q^2}{16M_W^4}\sum_i \big[C^*_{H_t}V^*_{qi} + C_{H_t}V_{qi}\big]\Big\{C_{t2i}\big[-2st^2 + 2sm_q^4 - 4sm_q^2M_W^2 + 2sM_{H^\pm}^2M_W^2 \\ & + 4m_q^2M_W^4 - 4m_q^2M_{H^\pm}^2M_W^2 + 4m_q^4M_W^2 - 4m_q^2tM_W^2 - 4m_q^4M_{H^\pm}^2 + 4m_q^2tM_{H^\pm}^2 - \tfrac12\sin^2\theta\,\lambda\,\cdot \\ & \cdot(s-4m_q^2)\big] + 2m_i^2 C_{t1i}\Big\{M_W^4 - 4tM_W^2 - s^2 + 2sM_{H^\pm}^2 + 2sm_q^2 - 4tm_q^2 + 2t^2 - M_{H^\pm}^4 - 4m_q^2M_{H^\pm}^2 \\ & + 2m_q^4 + 2M_{H^\pm}^2M_W^2 + \frac2s\big[m_q^2M_{H^\pm}^4 + m_q^2M_W^4 - 2m_q^2M_{H^\pm}^2M_W^2 + \tfrac14\sin^2\theta\,\lambda\,(s-4m_q^2)\big]\Big\}\Big\}\end{aligned}}$$

(126)

$$\begin{aligned}M^+_{3_t}M_{4_{IV}} + M^+_{4_{IV}}M_{3_t} = {}& \frac{g^4m_q^2}{16M_W^4}\sum_i(V^*_{qi}C^*_{A_t} + V_{qi}C_{A_t})\Big\{C_{t2i}\big[-2sM_{H^\pm}^2M_W^2 + 2st^2 + 2sm_q^4 + \tfrac12\sin^2\theta\,\lambda\,(s-4m_q^2) \\ & - 2m_q^2 st\big] + 2m_i^2C_{t1i}\big[2sM_W^2 + 2M_W^2M_{H^\pm}^2 + 2sM_{H^\pm}^2 - s^2 - M_W^4 - M_{H^\pm}^4\big]\Big\}\end{aligned}$$

(127)

$$\begin{aligned}\overline{|M_{IV}|^2} = {}& \frac1{12}\Big[\,|M_t^H|^2 + |M_{3_t}|^2 + |M_{4_{IV}}|^2 - (M_t^{H+}M_{3_t} + M_{3_t}^+M_t^H) - (M_t^{H+}M_{4_{IV}} + M^+_{4_{IV}}M_t^H) \\ & + (M_{3_t}^+ M_{4_{IV}} + M^+_{4_{IV}} M_{3_t})\Big]\end{aligned}$$

(128)

For my small

$$\overline{|M_W|^2} = \frac{g^4}{96M_W^4}\Big\{ m_q^2(|C_{H_t}|^2 + |C_{A_t}|^2)\,\lambda(s, M_W^2, M_{H^\pm}^2)\,s + 2\sum_{i,j} V_{qi}^* V_{qj}\Big\{ m_q^2 C_{t1i} C_{t2j}\,[\,st^2 + \tfrac{\lambda}{2} M_W^2 \sin^2\theta\,]$$

$$+ m_q^2\,(2M_W^4 - tM_W^2 - t^2)\,(m_j^2 C_{t2i} C_{t1j} + m_i^2 C_{t1i} C_{t2j}) + m_i^2 m_j^2 C_{t1i} C_{t1j}\,(2sM_W^2 + \tfrac{1}{4}\lambda s \sin^2\theta)\Big\}$$

$$+ \tfrac{1}{2} m_q^2 \sum_i\,[\,C_{H_t}^* V_{qi}^* + C_{H_t} V_{qi}\,]\Big\{ C_{t2i}\,[\,-2st^2 + 2sM_{H^\pm}^2 M_W^2 - \tfrac{1}{2}\lambda s \sin^2\theta\,] + 2m_i^2 C_{t1i}\,[\,M_W^4 - 4tM_W^2$$

$$- s^2 + 2sM_{H^\pm}^2 + 2t^2 - M_{H^\pm}^4 + 2M_{H^\pm}^2 M_W^2 + \tfrac{1}{2}\lambda \sin^2\theta\,]\Big\} + \tfrac{1}{2} m_q^2 \sum_i\,[\,V_{qi}^* C_{A_t}^* + V_{qi} C_{A_t}\,]\Big\{ C_{t2i}\,[\,-2sM_{H^\pm}^2 M_W^2$$

$$+ 2st^2 + \tfrac{1}{2}\lambda s \sin^2\theta\,] + 2m_i^2 C_{t1i}\,[\,2sM_W^2 + 2M_W^2 M_{H^\pm}^2 + 2sM_{H^\pm}^2 - s^2 - M_W^4 - M_{H^\pm}^4\,]\Big\}\Big\}$$

$$= \frac{G_F^2}{3} s\Big\{ m_q^2(|C_{H_t}|^2 + |C_{A_t}|^2)\,\lambda(s, M_W^2, M_{H^\pm}^2) + 2\sum_{i,j} V_{qi}^* V_{qj}\Big\{ m_q^2 C_{t2i} C_{t2j}\,[\,t^2 + \frac{\lambda}{2s} M_W^2 \sin^2\theta\,]$$

$$+ \frac{m_q^2}{s}\,(2M_W^4 - tM_W^2 - t^2)\,(m_i^2 C_{t1i} C_{t2j} + m_j^2 C_{t2i} C_{t1j}) + m_i^2 m_j^2 C_{t1i} C_{t1j}\,(2M_W^2 + \frac{\lambda}{4s}\sin^2\theta)\Big\}$$

$$+ \frac{1}{2}\frac{m_q^2}{s} \sum_i\,[\,C_{H_t}^* V_{qi}^* + C_{H_t} V_{qi}\,]\Big\{ C_{t2i}\, s\,[\,-2t^2 + 2M_{H^\pm}^2 M_W^2 - \tfrac{1}{2}\lambda \sin^2\theta\,] + 2m_i^2 C_{t1i}\,[\,M_W^4 - 4tM_W^2$$

$$- s^2 + 2sM_{H^\pm}^2 + 2t^2 - M_{H^\pm}^4 + 2M_{H^\pm}^2 M_W^2 + \tfrac{1}{2}\lambda \sin^2\theta\,]\Big\} + \frac{1}{2}\frac{m_q^2}{s} \sum_i\,[\,V_{qi}^* C_{A_t}^* + V_{qi} C_{A_t}\,]\Big\{ C_{t2i}\,[\,-2sM_{H^\pm}^2 M_W^2$$

$$+ 2st^2 + \tfrac{1}{2}\lambda s \sin^2\theta\,] + 2m_i^2 C_{t1i}\,[\,2sM_W^2 + 2M_W^2 M_{H^\pm}^2 + 2sM_{H^\pm}^2 - s^2 - M_W^4 - M_{H^\pm}^4\,]\Big\}\Big\}$$

$\underset{\sim}{(129\,a)}$ ok

$$\overline{|M_{IV}|^2} = \frac{g^4}{96 M_W^4} \Big\{ m_q^2 |C_{H_t}|^2 \lambda(s, M_W^2, M_{H^\pm}^2)(s-4m_q^2) + m_q^2 |C_{A_t}|^2 \lambda(s, M_W^2, M_{H^\pm}^2)\, s$$

$$+ 2 \sum_{i,j} V^*_{qi} V_{qj} \Big\{ m_q^2 c_{t2i} c_{t2j} \Big[-2t m_q^2 (M_{H^\pm}^2 + 2M_W^2) + st^2 + \frac{2M_W^2}{s}(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4$$

$$-2 m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4} \sin^2\theta\, \lambda (s-4m_q^2)) + \frac{m_q^2}{s}(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4} \sin^2\theta\, \lambda(s-4m_q^2))\Big]$$

$$+ m_q^2(2M_W^4 - t M_W^2 - m_q^2 M_W^2 - m_q^4 + 2t m_q^2 - t^2)(m_j^2 c_{t2i} c_{t1j} + m_i^2 c_{t1i} c_{t2j}) + m_i^2 m_j^2 c_{t1i} c_{t1j} \Big[2s M_W^2$$

$$-4 m_q^2 M_W^2 + s m_q^2 - 2 m_q^2 M_{H^\pm}^2 + \frac{1}{s}(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2 m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4} \sin^2\theta\, \lambda(s-4m_q^2))\Big]\Big\}$$

$$+ \frac{1}{2} m_q^2 \sum_i [C^*_{H_t} V^*_{qi} + C_{H_t} V_{qi}] \Big\{ c_{t2i}\Big[-2st^2 + 2s m_q^4 - 4s m_q^2 M_W^2 + 2s M_{H^\pm}^2 M_W^2 + 4m_q^2 M_W^4 - 4m_q^2 M_{H^\pm}^2 M_W^2$$

$$+ 4 m_q^4 M_W^2 - 4m_q^2 t M_W^2 - 4m_q^4 M_{H^\pm}^2 + 4 m_q^2 t M_{H^\pm}^2 - \frac{1}{2} \sin^2\theta\, \lambda (s-4m_q^2)\Big] + 2m_i^2 c_{t1i}\Big\{ M_W^4 - 4t M_W^2 - s^2 + 2s M_{H^\pm}^2$$

$$+ 2s m_q^2 - 4t m_q^2 + 2t^2 - M_{H^\pm}^4 - 4m_q^2 M_{H^\pm}^2 + 2m_q^4 + 2M_{H^\pm}^2 M_W^2 + \frac{2}{s}\Big[m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 +$$

$$+ \frac{1}{4} \sin^2\theta\, \lambda(s-4m_q^2)\Big]\Big\}\Big\} + \frac{1}{2} m_q^2 \sum_i (V^*_{qi} C^*_{A_t} + V_{qi} C_{A_t})\Big\{ c_{t2i}\Big[-2s M_{H^\pm}^2 M_W^2 + 2st^2 + 2s m_q^4 + \frac{1}{2} \sin^2\theta\, \lambda \cdot$$

$$\cdot(s-4m_q^2) - 4m_q^2 st\Big] + 2m_i^2 c_{t1i}\Big[2s M_W^2 + 2M_W^2 M_{H^\pm}^2 + 2s M_{H^\pm}^2 - s^2 - M_W^4 - M_{H^\pm}^4\Big]\Big\}\Big\}$$

(129) OK

$$\boxed{\frac{d\sigma_{IV}}{dt}(q\bar q \to H^+W^-) = \frac{G_F^2}{48\pi s} \cdot \frac{1}{(s-4m_q^2)} \overline{|M_{IV}|^2}} \quad (130)$$

→ only { }

with $q = u, c$ and $i, j = d, s, b$

The contribution is negligible compared with (121)

The only important terms are:

$$\overline{|M_{IV}|^2} = \frac{G_F^2}{3} s \Big\{ m_q^2 \lambda(s, M_W^2, M_{H^\pm}^2)(|C_{H_t}|^2 + |C_{A_t}|^2) + 2\sum_{i,j} V^*_{qi} V_{qj}\Big[m_q^2 c_{t2i} c_{t2j}(t^2 + \frac{\lambda}{2s} M_W^2 \sin^2\theta)$$

$$+ m_i^2 m_j^2 c_{t1i} c_{t1j}(2M_W^2 + \frac{\lambda}{4s}\sin^2\theta)\Big] + m_q^2 \mathrm{Re}(C_{H_t} - C_{A_t})\sum_i V_{qi} c_{t2i}(-2t^2 + 2M_{H^\pm}^2 M_W^2 - \frac{1}{2}\lambda\sin^2\theta)$$

$$+ 2m_q^2 \mathrm{Re}\, C_{H_t}\sum_i V_{qi} m_i^2 c_{t1i}(-s + 2M_{H^\pm}^2 + \frac{\lambda}{2s}\sin^2\theta) + 2m_q^2 \mathrm{Re}\, C_{A_t}\sum_i V_{qi} m_i^2 c_{t1i}(2M_W^2 + 2M_{H^\pm}^2 - s)\Big\}$$

$$\frac{d\sigma_{IV}}{dt}(c\bar c \to H^+W^-) \simeq \frac{G_F^2}{48\pi s}\Big\{ m_c^2 \lambda(s, M_W^2, M_{H^\pm}^2)(|C_{H_t}|^2 + |C_{A_t}|^2) + 2\sum_{i,j} V^*_{ci} V_{cj}\, m_c^2 c_{t2i} c_{t2j} \times$$

$$\times(t^2 + \frac{\lambda}{2s} M_W^2 \sin^2\theta) + 2|V_{cb}|^2 m_b^4 c_{t1b}^2 (2M_W^2 + \frac{\lambda}{4s}\sin^2\theta) + m_c^2 \mathrm{Re}(C_{H_t} - C_{A_t})\sum_i V_{ci} c_{t2i}$$

$$(-2t^2 + 2M_{H^\pm}^2 M_W^2 - \frac{1}{2}\lambda\sin^2\theta) + 2m_c^2 \mathrm{Re}(C_{H_t} + C_{A_t}) V_{cb} m_b^2 c_{t1b}(2M_{H^\pm}^2 - s)$$

$$+ 4m_c^2 M_W^2 \mathrm{Re}\, C_{A_t} V_{cb} m_b^2 c_{t1b} + \frac{\lambda}{s}\sin^2\theta\, m_c^2 \mathrm{Re}\, C_{H_t} V_{cb} m_b^2 c_{t1b} \Big\} \quad (131)$$

OK

$\underline{II})$

(181)

$$M_{4II} = -\frac{g^2}{4M_W} \varepsilon^*_{\mu 4} \sum_i V^*_{iq} \bar{V}_2 [m_q CU_{1i} \not{p}_i + m_i^2 CU_{2i}] \gamma^\mu (1-\gamma^5) U_1$$

$$|M_{4II}|^2 = \frac{g^4}{16M_W^2} \sum_\lambda \varepsilon^*_{\mu 4} \varepsilon_{\nu 4} \sum_{i,j} V^*_{iq} V_{jq} \sum_s \{ \overset{V_2^+\gamma^0}{\bar{V}_2} [m_q CU_{1j} \not{p}_j + m_j^2 CU_{2j}] \gamma^\nu (1-\gamma^5) U_1 \}^+ \cdot$$
$$\cdot \{ \bar{V}_2 [m_q CU_{1i} \not{p}_i + m_i^2 CU_{2i}] \gamma^\mu (1-\gamma^5) U_1 \} \qquad (132)$$

$$|M_{4II}|^2 = \frac{g^4}{16M_W^2} \left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right) \sum_{i,j} V^*_{iq} V_{jq} \sum_s \{ U_1^+ (1-\gamma^5) \gamma^{\nu +} [m_q CU_{1j} \gamma^{\alpha +} P_{j\alpha} + m_j^2 CU_{2j}] \gamma^0 V_2 \}$$
$$\cdot \{ \bar{V}_2 [m_q CU_{1i} \not{p}_i + m_i^2 CU_{2i}] \gamma^\mu (1-\gamma^5) U_1 \}$$

$$|M_{4II}|^2 = \frac{g^4}{16M_W^2} \left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right) \sum_{i,j} V^*_{iq} V_{jq} \sum_s \{ U_1^+ (1-\gamma^5) \overset{\gamma^0\gamma^\nu}{\gamma^{\nu +} \gamma^0} [m_q CU_{1j} \not{p}_j + m_j^2 CU_{2j}] V_2 \}$$
$$\cdot \{ \bar{V}_2 [m_q CU_{1i} \not{p}_i + m_i^2 CU_{2i}] \gamma^\mu (1-\gamma^5) U_1 \}$$

$$|M_{4II}|^2 = \frac{g^4}{16M_W^2} \left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right) \sum_{i,j} V^*_{iq} V_{jq} \sum_s \{ \bar{U}_1 (1+\gamma^5) \gamma^\nu [m_q CU_{1j} \not{p}_j + m_j^2 CU_{2j}] V_2 \} \cdot$$
$$\cdot \{ \bar{V}_2 [m_q CU_{1i} \not{p}_i + m_i^2 CU_{2i}] \gamma^\mu (1-\gamma^5) U_1 \}$$

$$|M_{4II}|^2 = \frac{g^4}{16M_W^2} \left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right) \sum_{i,j} V^*_{iq} V_{jq} \sum_s \{ \bar{U}_1 \gamma^\nu (1-\gamma^5) [m_q CU_{1j} \not{p}_j + m_j^2 CU_{2j}] V_2 \} \cdot$$
$$\cdot \{ \bar{V}_2 [m_q CU_{1i} \not{p}_i + m_i^2 CU_{2i}] \gamma^\mu (1-\gamma^5) U_1 \}$$

$$|M_{4II}|^2 = \frac{g^4}{16M_W^2} \left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right) \sum_{i,j} V^*_{iq} V_{jq} \, Tr \{ (\not{p}_1 + m_q) \gamma^\nu (1-\gamma^5) [m_q CU_{1j} \not{p}_j + m_j^2 CU_{2j}] (\not{p}_2 - m_q)$$
$$[m_q CU_{1i} \not{p}_i + m_i^2 CU_{2i}] \gamma^\mu (1-\gamma^5) \}$$

$$= \frac{g^4}{16M_W^2} \left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right) \sum_{i,j} V^*_{iq} V_{jq} \, Tr \{ \gamma^\mu (1-\gamma^5) (\not{p}_1 + m_q) \gamma^\nu (1-\gamma^5) [m_q CU_{1j} \not{p}_j + m_j^2 CU_{2j}] (\not{p}_2 - m_q)$$
$$[m_q CU_{1i} \not{p}_i + m_i^2 CU_{2i}] \}$$

$$\gamma^\mu (1-\gamma^5)(\not{p}_1 + m_q) \gamma^\nu (1-\gamma^5) = \gamma^\mu (1-\gamma^5)(\not{p}_1 + m_q)(1+\gamma^5)\gamma^\nu$$

$$= \gamma^\mu (1-\gamma^5) [\not{p}_1 (1+\gamma^5) + m_q (1+\gamma^5)] \gamma^\nu$$

$$= \gamma^\mu (1-\gamma^5) [(1-\gamma^5)\not{p}_1 + m_q (1+\gamma^5)] \gamma^\nu$$

$$= (2\gamma^\mu (1-\gamma^5)\not{p}_1 + m_q \gamma^\mu \overset{0}{\cancel{(1-\gamma^5)(1+\gamma^5)}}) \gamma^\nu$$

$$= 2\gamma^\mu (1-\gamma^5) \not{p}_1 \gamma^\nu$$

$$\Rightarrow |M_{4II}|^2 = \frac{2g^4}{16M_W^2}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)\sum_{i,j} V_{iq}^* V_{jq}\, Tr\left\{\gamma^\mu(1-\gamma^5)\not{P}_1\gamma^\nu\left[m_q c_{u1j}\not{P}_j + m_j^2 c_{u2j}\right](\not{P}_2 - m_q)\right.$$
$$\left.\left[m_q c_{u1i}\not{P}_i + m_i^2 c_{u2i}\right]\right\}$$

$$= \frac{-g^4}{8M_W^2}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)\sum_{i,j} V_{iq}^* V_{jq}\, Tr\left\{\left[m_q c_{u1j}\not{P}_j + m_j^2 c_{u2j}\right](-\not{P}_2 + m_q)\left[m_q c_{u1i}\not{P}_i + m_i^2 c_{u2i}\right]\right.$$
$$\left.\gamma^\mu(1-\gamma^5)\not{P}_1\gamma^\nu\right\}$$

$i \leftrightarrow j$

$\mu \leftrightarrow \nu$

$$\boxed{\begin{aligned}\Rightarrow |M_{4II}|^2 = -\frac{g^4}{8M_W^2}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)\sum_{i,j} V_{iq} V_{jq}^*\, Tr\Big\{&\left[m_q c_{u1i}\not{P}_i + m_i^2 c_{u2i}\right)(-\not{P}_2 + m_q)\left[m_q c_{u1j}\not{P}_j + m_j^2 c_{u2j}\right]\\ &\gamma^\nu(1-\gamma^5)\not{P}_1\gamma^\mu\Big\}\end{aligned}} \qquad (133)$$

The trace is obtained from the trace in $|M_4|^2$ (*) replacing $c_{t1i} \to c_{u1i}$

$c_{t2i} \to c_{u2i}$; $P_1 \to -P_2$; $c_{t1j} \to c_{u1j}$; $c_{t2j} \to c_{u2j}$; $P_2 \to P_1$; $K^2 = U$

$$\therefore |M_{4II}|^2 = -\frac{g^4}{8M_W^2}\sum_{i,j} V_{iq} V_{jq}^*\Big\{4m_q^2 \cancel{c_{u1i}}\, c_{u1j}\left[-2(K\cdot P_2)(K\cdot P_1) + U(P_1\cdot P_2)\right] + 8\frac{m_q^2}{M_W^2}\cancel{c_{u1i}}\, c_{u1j}$$
$$(P_1\cdot P_4)\left[-2(K\cdot P_2)(K\cdot P_4) + U(P_2\cdot P_4)\right] + 4m_q^2 m_j^2 \cancel{c_{u1i}}\, c_{u2j}(K\cdot P_1) - 4m_i^2 m_j^2 \cancel{c_{u2i}}\, c_{u2j}(P_1\cdot P_2)$$
$$+ 4m_i^2 m_q^2 c_{u2i}\cancel{c_{u1j}}(K\cdot P_1) + 8\frac{m_q^2 m_j^2}{M_W^2}\cancel{c_{u1i}}\, c_{u2j}(K\cdot P_4)(P_1\cdot P_4) - 8\frac{m_i^2 m_j^2}{M_W^2}\cancel{c_{u2i}}\, c_{u2j}(P_2\cdot P_4)(P_1\cdot P_4)$$
$$+ 8\frac{m_i^2 m_q^2}{M_W^2}\cancel{c_{u2i}}\, c_{u1j}(K\cdot P_4)(P_1\cdot P_4)\Big\} \qquad (134)$$

$$\boxed{(P_1\cdot P_2) = \frac{s}{2} - m_q^2} \quad ✓ \qquad ((35)$$

$$\boxed{(P_1\cdot P_4) = \frac{m_q^2 + M_W^2 - U}{2}} \quad ✓ \qquad ((36)$$

$$\boxed{(P_2\cdot P_4) = \frac{s + U - m_q^2 - M_W^2}{2}} \quad ✓ \qquad ((37)$$

$K = P_1 - P_4$

$P_1 = K + P_4$

$m_q^2 = U + 2K\cdot P_4 + M_W^2$

$$\boxed{K\cdot P_4 = \frac{m_q^2 - U - M_W^2}{2}} \quad ✓ \qquad ((38)$$

$$K = P_1 - P_4 \Rightarrow P_4 = P_1 - K \Rightarrow M_W^2 = m_q^2 - 2P_1.K + U$$

$$\boxed{(P_1.K) = \frac{m_q^2 - M_W^2 + U}{2}} \qquad (139)$$

$$K = P_3 - P_2 \Rightarrow P_3 = K + P_2 \Rightarrow M_{H^\pm}^2 = U + m_q^2 + 2K.P_2$$

$$\boxed{(P_2.K) = \frac{M_{H^\pm}^2 - U - m_q^2}{2}} \qquad (140)$$

$$|M_{4II}|^2 = \frac{-g^4}{8M_W^2}\sum_{i,j} V_{iq}V_{jq}^* \Big\{ \frac{4m_q^2}{M_W^2} C_{U1i} C_{U1j} \Big\{ -(M_{H^\pm}^2 - U - m_q^2)\frac{1}{2}(m_q^2 - M_W^2 + U)M_W^2 + U\left(\frac{S}{2} - m_q^2\right)M_W^2$$

$$+ (m_q^2 + M_W^2 - U)\left[-(M_{H^\pm}^2 - U - m_q^2)\frac{1}{2}(m_q^2 - U - M_W^2) + U\frac{1}{2}(S + U - m_q^2 - M_{H^\pm}^2)\right]$$

$$+ \frac{4m_q^2 m_j^2}{M_W^2} C_{U1i} C_{U2j}\left[\frac{1}{2}(m_q^2 - M_W^2 + U)M_W^2 + \frac{1}{2}(m_q^2 - U - M_W^2)(m_q^2 + M_W^2 - U)\right]$$

$$+ \frac{4m_i^2 m_q^2}{M_W^2} C_{U2i} C_{U1j}\left[\frac{1}{2}(m_q^2 - M_W^2 + U)M_W^2 + \frac{1}{2}(m_q^2 - U - M_W^2)(m_q^2 + M_W^2 - U)\right]$$

$$- \frac{4m_i^2 m_j^2}{M_W^2} C_{U2i} C_{U2j}\left[\left(\frac{S}{2} - m_q^2\right)M_W^2 + (S + U - m_q^2 - M_{H^\pm}^2)\frac{1}{2}(m_q^2 + M_W^2 - U)\right]$$

$$|M_{4II}|^2 = -\frac{g^4}{4M_W^4}\sum_{i,j} V_{iq}V_{jq}^* \Big\{ m_q^2 C_{U1i} C_{U1j} \Big\{ -m_q^2 \cancel{M_{H^\pm}^2} M_W^2 + M_{\cancel{H^\pm}}^2 M_W^4 - U M_{H^\pm}^2 \cancel{M_W^2} + m_q^2 \cancel{U} M_W^2 - U\cancel{M_W^4} + U^2\cancel{M_W^2}$$

$$+ m_q^4 \cancel{M_W^2} - m_q^2\cancel{M_W^4} + U\cancel{m_q^2}M_W^2 + US\cancel{M_W^2} - 2m_q^2\cancel{U}M_W^2 + (m_q^2 + M_W^2 - U)\left[-\cancel{m_q^2}M_{H^\pm}^2 + U\cancel{M_{H^\pm}^2} + \cancel{M_W^2}M_{H^\pm}^2\right.$$

$$\left. + U\cancel{m_q^2} - \cancel{U^2} - U\cancel{M_W^2} + \cancel{m_q^4} - \cancel{m_q^2}U - m_q^2\cancel{M_W^2} + \cancel{U}S + \cancel{U^2} - U\cancel{m_q^2} - U\cancel{M_{H^\pm}^2}\right]\Big\} + m_q^2\left[m_j^2 C_{U1i} C_{U2j} + \right.$$

$$\left. + m_i^2 C_{U2i} C_{U1j}\right]\left[m_q^2\cancel{M_W^2} - \cancel{M_W^4} + U\cancel{M_W^2} + \cancel{m_q^4} + \cancel{m_q^2}M_W^2 - U\cancel{m_q^2} - \cancel{U}m_q^2 - U\cancel{M_W^2} + \cancel{U^2} - \cancel{m_q^2}M_W^2 - \cancel{M_W^4}\right.$$

$$\left. + \cancel{U}M_W^2\right] - m_i^2 m_j^2 C_{U2i} C_{U2j}\left[S\cancel{M_W^2} - 2\cancel{m_q^2}M_W^2 + S\cancel{m_q^2} + S\cancel{M_W^2} - \cancel{S}U + U\cancel{m_q^2} + U\cancel{M_W^2} - \cancel{U^2}\right.$$

$$\left. - \cancel{m_q^4} - m_q^2\cancel{M_W^2} + \cancel{m_q^2}U - m_q^2\cancel{M_{H^\pm}^2} - M_W^2\cancel{M_{H^\pm}^2} + U\cancel{M_{H^\pm}^2}\right]$$

$$|M_{4II}|^2 = -\frac{g^4}{4M_W^4}\sum_{i,j} V_{iq}V_{jq}^* \Big\{ m_q^2 C_{U1i} C_{U1j} \Big\{ -m_q^2 M_{H^\pm}^2 M_W^2 + M_{H^\pm}^2 M_W^4 - U M_{H^\pm}^2 M_W^2 - U M_W^4 + U^2 M_W^2 + m_q^4 M_W^2$$

$$- m_q^2 M_W^4 + US M_W^2 + (m_q^2 + M_W^2 - U)\left[-m_q^2 M_{H^\pm}^2 + M_W^2 M_{H^\pm}^2 - U M_W^2 + m_q^4 - m_q^2 U - m_q^2 M_W^2 + US\right]$$

$$+ m_q^2\left[m_j^2 C_{U1i} C_{U2j} + m_i^2 C_{U2i} C_{U1j}\right]\left[m_q^2 M_W^2 - 2M_W^4 + U M_W^2 + m_q^4 - 2U m_q^2 + U^2\right]$$

$$- m_i^2 m_j^2 C_{U2i} C_{U2j}\left[2S M_W^2 - 3m_q^2 M_W^2 + S m_q^2 - US + 2U m_q^2 + U M_W^2 - U^2 - m_q^4 - m_q^2 M_{H^\pm}^2\right.$$

$$\left. - M_W^2 M_{H^\pm}^2 + U M_{H^\pm}^2\right]\Big\}$$

$$|M_{4II}|^2 = -\frac{g^4}{4M_W^4}\sum_{i,j} V_{iq}V_{jq}^* \Big\{ m_q^2 C_{U1i}C_{U1j} \Big\{ -\cancel{m_q^2 M_{H^\pm}^2 M_W^2} + \cancel{M_{H^\pm}^2 M_W^4} - \cancel{u M_{H^\pm}^2 M_W^2} - \cancel{u M_W^4} + \cancel{u^2 M_W^2} + \cancel{m_q^4 M_W^2}$$

$$- \cancel{m_q^2 M_W^4} + \cancel{us M_W^2} - \cancel{m_q^4 M_{H^\pm}^2} + \cancel{m_q^2 M_W^2 M_{H^\pm}^2} - \cancel{u m_q^2 M_W^2} + \cancel{m_q^6} - \cancel{m_q^4 u} - \cancel{m_q^4 M_W^2} + \cancel{us m_q^2}$$

$$- \cancel{m_q^2 M_W^2 M_{H^\pm}^2} + \cancel{M_W^4 M_{H^\pm}^2} - \cancel{u M_W^4} + \cancel{m_q^4 M_W^2} - \cancel{m_q^2 u M_W^2} - \cancel{m_q^2 M_W^4} + \cancel{us M_W^2} + \cancel{m_q^2 u M_{H^\pm}^2} - \cancel{u M_W^2 M_{H^\pm}^2} + \cancel{u^2 M_W^2}$$

$$- \cancel{u m_q^4} + \cancel{u^2 m_q^2} + \cancel{u m_q^2 M_W^2} - \cancel{u^2 s} \Big\} + m_q^2 [m_j^2 C_{U1i}C_{U2j} + m_i^2 C_{U2i}C_{U1j}][m_q^2 M_W^2 - 2M_W^4 + uM_W^2 + m_q^4$$

$$-2u m_q^2 + u^2] - m_i^2 m_j^2 C_{U2i}C_{U2j}[2sM_W^2 - 3m_q^2 M_W^2 + s m_q^2 - us + 2u m_q^2 + u M_W^2 - u^2 - m_q^4 - m_q^2 M_{H^\pm}^2 - M_W^2 M_{H^\pm}^2$$

$$+ u M_{H^\pm}^2]\Big\}$$

$$|M_{4II}|^2 = -\frac{g^4}{4M_W^4}\sum_{i,j} V_{iq}V_{jq}^* \Big\{ m_q^2 C_{U1i}C_{U1j} \Big[-m_q^2 M_W^2 M_{H^\pm}^2 + 2M_{H^\pm}^2 M_W^4 - 2u M_W^2 M_{H^\pm}^2 - 2u M_W^4 + 2u^2 M_W^2$$

$$-2m_q^2 M_W^4 + 2us M_W^2 - m_q^4 M_{H^\pm}^2 - u m_q^2 M_W^2 + m_q^6 - 2m_q^4 u + us m_q^2 + m_q^4 M_W^2$$

$$+ m_q^2 u M_{H^\pm}^2 + u^2 m_q^2 - u^2 s \Big\} + m_q^2 [m_j^2 C_{U1i}C_{U2j} + m_i^2 C_{U2i}C_{U1j}][m_q^2 M_W^2 - 2M_W^4 + uM_W^2 + m_q^4$$

$$-2u m_q^2 + u^2] - m_i^2 m_j^2 C_{U2i}C_{U2j}[+2sM_W^2 - 3m_q^2 M_W^2 + s m_q^2 - us + 2u m_q^2 + uM_W^2 - u^2 - m_q^4 - m_q^2 M_{H^\pm}^2$$

$$- M_W^2 M_{H^\pm}^2 + u M_{H^\pm}^2]\Big\} \qquad (141)$$

That is (74a) replacing $t \leftrightarrow u$

then

$$|M_{4II}|^2 = \frac{g^4}{4M_W^4}\sum_{i,j} V_{iq}V_{jq}^* \Big\{ m_q^2 C_{U1i}C_{U1j} \Big[-2u m_q^2 (M_{H^\pm}^2 + 2M_W^2) + su^2 + 2\frac{M_W^2}{s}(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4$$

$$-2m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4}\sin^2\theta\,\lambda(s-4m_q^2)) + \frac{m_q^2}{s}(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4}\sin^2\theta\,\lambda(s-4m_q^2))\Big]$$

$$+ m_q^2 (2M_W^4 - uM_W^2 - m_q^2 M_W^2 - m_q^4 + 2u m_q^2 - u^2)(m_j^2 C_{U1i}C_{U2j} + m_i^2 C_{U2i}C_{U1j}) + m_i^2 m_j^2 C_{U2i}C_{U2j}$$

$$\Big[2sM_W^2 - 4m_q^2 M_W^2 + s m_q^2 - 2m_q^2 M_{H^\pm}^2 + \frac{1}{s}(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4}\sin^2\theta\,\lambda(s-4m_q^2))\Big]\Big\}$$

In fact the term containing $C_{U1i}C_{U1j}$ is: <u>OK</u> (142)

$$-m_q^2 M_W^2 M_{H^\pm}^2 + 2M_{H^\pm}^2 M_W^4 - 2u M_W^2 M_{H^\pm}^2 - 2u M_W^4 + 2u^2 M_W^2 - 2m_q^2 M_W^4 + 2u M_W^2 (2m_q^2 + M_W^2 + M_{H^\pm}^2$$

$$-u-t) - m_q^4 M_{H^\pm}^2 - u m_q^2 M_W^2 + m_q^6 - 2m_q^4 u + u m_q^2 (2m_q^2 + M_W^2 + M_{H^\pm}^2 - u - t) + m_q^4 M_W^2$$

$$+ m_q^2 u M_{H^\pm}^2 + u^2 m_q^2 - u^2 s$$

$$= -\cancel{m_q^2 M_W^2 M_{H^\pm}^2} + \cancel{2M_{H^\pm}^2 M_W^4} - \cancel{2u M_W^2 M_{H^\pm}^2} - \cancel{2u M_W^4} + \cancel{2u^2 M_W^2} - \cancel{2m_q^2 M_W^4} + \cancel{4u M_W^2 m_q^2} + \cancel{2u M_W^4} + \cancel{2u M_W^2 M_{H^\pm}^2}$$

$$-\cancel{2u^2 M_W^2} - \cancel{2ut M_W^2} - \cancel{m_q^4 M_{H^\pm}^2} - \cancel{u m_q^2 M_W^2} + \cancel{m_q^6} - \cancel{2m_q^4 u} + \cancel{2u m_q^4} + \cancel{u m_q^2 M_W^2} + \cancel{u m_q^2 M_{H^\pm}^2} - \cancel{u^2 m_q^2} - \cancel{ut m_q^2}$$

$$+ \cancel{m_q^4 M_W^2} + \cancel{m_q^2 u M_{H^\pm}^2} + \cancel{u^2 m_q^2} - \cancel{u^2 s}$$

$$= -m_y^2 M_W^2 M_{H^\pm}^2 + 2M_W^4 M_{H^\pm}^2 - 2m_y^2 M_W^4 + 4U M_W^2 m_y^2 - 2Ut M_W^2 - m_y^4 M_{H^\pm}^2 + m_y^6 + 2U m_y^2 M_{H^\pm}^2 - Ut m_y^2$$
$$+ m_y^4 M_W^2 - U^2 s$$

$$= -m_y^2 \cancel{M_W^2} M_{H^\pm}^2 + 2\cancel{M_W^4} M_{H^\pm}^2 - 2\cancel{m_y^2} M_W^4 + 4U \cancel{M_W^2} m_y^2 - 2\cancel{M_W^2} m_y^4 + 2M_W^2 \cancel{m_y^2} M_{H^\pm}^2 + 2\cancel{m_y^2} M_W^4 - 2\cancel{M_{H^\pm}^2} M_W^4$$
$$- \frac{M_W^2}{2s}\left[4m_y^2 M_{H^\pm}^4 + 4m_y^2 M_W^4 - 8m_y^2 M_{H^\pm}^2 M_W^2 + \sin^2\theta\, \lambda\, (s-4m_y^2)\right] - \cancel{m_y^4} M_{H^\pm}^2 + \cancel{m_y^6} + 2U\cancel{m_y^2} M_{H^\pm}^2$$
$$+ m_y^4 \cancel{M_W^2} - \cancel{U^2} s - \cancel{m_y^6} + m_y^4 \cancel{M_{H^\pm}^2} + m_y^4 \cancel{M_W^2} - m_y^2 \cancel{M_{H^\pm}^2} M_W^2 - \frac{m_y^2}{4s}\left[4m_y^2 M_{H^\pm}^4 + 4m_y^2 M_W^4 - 8m_y^2 M_{H^\pm}^2 M_W^2\right.$$
$$\left. + \sin^2\theta\, \lambda\, (s-4m_y^2)\right]$$

$$= -U^2 s + 4U M_W^2 m_y^2 + 2U m_y^2 M_{H^\pm}^2 - \frac{M_W^2}{2s}\left[4m_y^2 M_{H^\pm}^4 + 4m_y^2 M_W^4 - 8m_y^2 M_{H^\pm}^2 M_W^2 + \sin^2\theta\, \lambda\, (s-4m_y^2)\right]$$
$$- \frac{m_y^2}{4s}\left[4m_y^2 M_{H^\pm}^4 + 4m_y^2 M_W^4 - 8m_y^2 M_{H^\pm}^2 M_W^2 + \sin^2\theta\, \lambda\, (s-4m_y^2)\right] \qquad \text{OK.}$$

The term containing $C_{u2i}\, C_{u2j}$ is:

$$2s M_W^2 - 3m_y^2 M_W^2 + s m_y^2 - U(2m_y^2 + M_W^2 + M_{H^\pm}^2 - U - t) + 2U m_y^2 + U M_W^2 - U^2 - m_y^4 - m_y^2 M_{H^\pm}^2 - M_W^2 M_{H^\pm}^2 + U M_{H^\pm}^2$$
$$= 2s\cancel{M_W^2} - 3\cancel{m_y^2} M_W^2 + s\cancel{m_y^2} - 2U\cancel{m_y^2} - U\cancel{M_W^2} - U\cancel{M_{H^\pm}^2} + \cancel{U^2} + \cancel{m_y^4} - m_y^2\cancel{M_{H^\pm}^2} - \cancel{m_y^2} M_W^2 + \cancel{M_{H^\pm}^2} M_W^2$$
$$+ \frac{1}{s}\left[m_y^2 M_{H^\pm}^4 + m_y^2 M_W^4 - 2m_y^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4}\sin^2\theta\, \lambda\, (s-4m_y^2)\right] + 2U\cancel{m_y^2} + U\cancel{M_W^2} - \cancel{U^2} - \cancel{m_y^4} - \cancel{m_y^2} M_{H^\pm}^2 - \cancel{M_W^2} M_{H^\pm}^2$$
$$+ U\cancel{M_{H^\pm}^2}$$

$$= 2s M_W^2 - 4m_y^2 M_W^2 + s m_y^2 - 2m_y^2 M_{H^\pm}^2 + \frac{1}{s}\left[m_y^2 M_{H^\pm}^4 + m_y^2 M_W^4 - 2m_y^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4}\sin^2\theta\, \lambda\, (s-4m_y^2)\right]$$

OK.

$$M_b^{H^+} M_{4_{II}} = \frac{-g^4}{16M_W^2} m_q C^*_{H_b} \sum_\lambda \varepsilon^*_{\mu 4}\, \varepsilon_{\nu 4}\, (P_1+P_2+P_3)^\nu \sum_s (\bar{U}_1 V_2) \sum_i V^*_{iq}\, \bar{V}_2 \left[m_q Cv_{1i} \not{k} + m_i^2 Cv_{2i}\right] \gamma^\mu (1-\gamma^5) U_1$$

$$= \frac{-g^4}{16M_W^2} m_q C^*_{H_b}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)(P_1+P_2+P_3)^\nu \sum_i V^*_{iq} \sum_s \bar{U}_1 V_2 \bar{V}_2 \left[m_q Cv_{1i}\not{k} + m_i^2 Cv_{2i}\right]\gamma^\mu(1-\gamma^5) U_1$$

$$= \frac{-g^4}{16M_W^2} m_q C^*_{H_b}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)(P_1+P_2+P_3)^\nu \sum_i V^*_{iq}\, Tr\Big[(\not{P}_2 - m_q)\left[m_q Cv_{1i}\not{k} + m_i^2 Cv_{2i}\right]$$

$$\underbrace{\gamma^\mu(1-\gamma^5)}_{(\gamma^\mu - \gamma^\mu\gamma^5)}(\not{P}_1 + m_q)\Big] \qquad (143)$$

$\Rightarrow$

$$M_b^{H^+} M_{4_{II}} = -\frac{g^4}{16M_W^2} m_q C^*_{H_b}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)(P_1+P_2+P_3)^\nu \sum_i V^*_{iq}\, Tr\Big\{\big[m_q Cv_{1i}\not{P}_2\not{k} + m_i^2 Cv_{2i}\not{P}_2 - m_q^2 Cv_{1i}\not{k}$$

$$- m_q m_i^2 Cv_{2i}\big]\left[\gamma^\mu \not{P}_1 + m_q\gamma^\mu - \gamma^\mu\gamma^5\not{P}_1 - m_q\gamma^\mu\gamma^5\right]\Big\} \qquad (144)$$

$$M_b^{H^+} M_{4_{II}} = -\frac{g^4}{16M_W^2} m_q C^*_{H_b}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)(P_1+P_2+P_3)^\nu \sum_i V^*_{iq}\Big\{m_q Cv_{1i}\, Tr(\not{P}_2\not{k}\gamma^\mu\not{P}_1)$$

$$+ m_q^2 Cv_{1i}\, Tr(\overset{0}{\cancel{\not{P}_2\not{k}}}\gamma^\mu) - m_q Cv_{1i}\, Tr(\not{P}_2\not{k}\gamma^\mu\gamma^5\not{P}_1) - m_q^2 Cv_{1i}\, Tr(\not{P}_2\not{k}\overset{0}{\cancel{\gamma^\mu}}\gamma^5)$$

$$+ m_i^2 Cv_{2i}\, Tr(\not{P}_2\overset{0}{\cancel{\gamma^\mu}}\not{P}_1) + m_i^2 Cv_{2i} m_q\, Tr(\not{P}_2\gamma^\mu) - m_i^2 Cv_{2i}\, Tr(\not{P}_2\gamma^\mu\overset{0}{\cancel{\gamma^5}}\not{P}_1) - m_i^2 m_q Cv_{2i}\, Tr(\overset{0}{\cancel{\not{P}_2\gamma^\mu}}\gamma^5)$$

$$- m_q^2 Cv_{1i}\, Tr(\not{k}\overset{0}{\cancel{\gamma^\mu}}\not{P}_1) - m_q^3 Cv_{1i}\, Tr(\not{k}\gamma^\mu) + m_q^2 Cv_{1i}\, Tr(\not{k}\gamma^\mu\overset{0}{\cancel{\gamma^5}}\not{P}_1) + m_q^3 Cv_{1i}\, Tr(\not{k}\overset{0}{\cancel{\gamma^\mu}}\gamma^5)$$

$$- m_q m_i^2 Cv_{2i}\, Tr(\gamma^\mu\not{P}_1) - m_q^2 m_i^2 Cv_{2i}\, Tr(\overset{0}{\cancel{\gamma^\mu}}) + m_q m_i^2 Cv_{2i}\, Tr(\gamma^\mu\overset{0}{\cancel{\gamma^5}}\not{P}_1) + m_q^2 m_i^2 Cv_{2i}\, Tr(\gamma^\mu\overset{0}{\cancel{\gamma^5}})\Big\}$$

$$= -\frac{g^4}{16M_W^2} m_q C^*_{H_b}\left(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2}\right)(P_1+P_2+P_3)^\nu \sum_i V^*_{iq}\Big\{m_q Cv_{1i}\, Tr(\not{P}_2\not{k}\gamma^\mu\not{P}_1) + m_q Cv_{1i}\, Tr(\gamma^5\not{P}_2\not{k}\gamma^\mu\not{P}_1)$$

$$+ m_i^2 Cv_{2i} m_q\, Tr(\gamma^\mu\not{P}_2) - m_q^3 Cv_{1i}\, Tr(\gamma^\mu\not{k}) - m_q m_i^2 Cv_{2i}\, Tr(\gamma^\mu\not{P}_1)\Big\}$$

$$= -\frac{g^4}{16M_W^2} m_q C^*_{H_b}\left(-(P_1+P_2+P_3)_\mu + \frac{((P_1+P_2+P_3)\cdot P_4)}{M_W^2} P_{4\mu}\right)\sum_i V^*_{iq}\Big\{m_q Cv_{1i}\, Tr\left(-\not{k}\not{P}_2\gamma^\mu\not{P}_1 + 2(P_2\cdot k)\gamma^\mu\not{P}_1\right)$$

$$+ m_q Cv_{1i}\left(-4i\,\varepsilon^{\alpha\nu\mu\beta}\right)P_{2\alpha}k_\nu P_{1\beta} + m_i^2 Cv_{2i} m_q P_{2\alpha}\, 4g^{\mu\alpha} - m_q^3 Cv_{1i} k_\alpha\, 4g^{\mu\alpha}$$

$$- m_q m_i^2 Cv_{2i} P_{1\alpha}\, 4g^{\mu\alpha}\Big\}$$

$$= -\frac{g^4}{16M_W^2} m_q C^*_{H_b}\left(-(P_1+P_2+P_3)_\mu + \frac{((P_1+P_2+P_3)\cdot P_4)}{M_W^2} P_{4\mu}\right)\sum_i V^*_{iq}\Big\{-m_q Cv_{1i}\, Tr(\gamma^\mu\not{P}_1\gamma^\alpha\not{P}_2)k_\alpha$$

$$+ 8(P_2\cdot k) m_q Cv_{1i} P_1^\mu - 4i\, m_q Cv_{1i}\,\varepsilon^{\alpha\nu\mu\beta} P_{2\alpha}k_\nu P_{1\beta} + 4 m_i^2 Cv_{2i} m_q P_2^\mu - 4 m_q^3 Cv_{1i} k^\mu$$

$$- 4 m_q m_i^2 Cv_{2i} P_1^\mu\Big\}$$

$$(P_1+P_2+P_3)_\mu \; \varepsilon^{\alpha\nu\mu\beta} P_{2\alpha} K_\nu P_{1\beta}$$

$$= (P_1+P_2+P_3)_\mu \; \varepsilon^{\alpha\nu\mu\beta} P_{2\alpha} (P_{1\nu} - P_{4\nu}) P_{1\beta}$$

$$= P_{3\mu} \; \varepsilon^{\alpha\nu\mu\beta} P_{2\alpha} (P_{1\nu} - P_{4\nu}) P_{1\beta}$$

$$= - P_{3\mu} \; \varepsilon^{\alpha\nu\mu\beta} P_{2\alpha} P_{4\nu} P_{1\beta}$$

$$= - P_{3\mu} \; \varepsilon^{\alpha\nu\mu\beta} P_{2\alpha} (P_{1\nu} + P_{2\nu} - P_{3\nu}) P_{1\beta}$$

$$= 0$$

$$P_{4\mu} \; \varepsilon^{\alpha\nu\mu\beta} P_{2\alpha} (P_{1\nu} - P_{4\nu}) P_{1\beta} = 0$$

$$\Rightarrow M_b^{H^+} M_{4\,II} = -\frac{g^4}{16 M_W^2} m_q C^*_{H_b} \left(-(P_1+P_2+P_3)_\mu + \frac{((P_1+P_2+P_3).P_4)}{M_W^2} P_{4\mu} \right) \sum_i V^*_{iq} \Big\{ -4 m_q C_{U1i} (P_1^\mu P_2^\alpha + P_1^\alpha P_2^\mu$$

$$- (P_1.P_2) g^{\mu\alpha}) K_\alpha + 8 (P_2.K) m_q C_{U1i} P_1^\mu + 4 m_i^2 C_{U2i} m_q P_2^\mu - 4 m_q^3 C_{U1i} K^\mu - 4 m_q m_i^2 C_{U2i} P_1^\mu \Big\}$$

$$= -\frac{g^4}{16 M_W^2} m_q C^*_{H_b} \left(-(P_1+P_2+P_3)_\mu + \frac{((P_1+P_2+P_3).P_4)}{M_W^2} P_{4\mu} \right) \sum_i V^*_{iq} \Big\{ -\cancel{4} m_q C_{U1i} P_1^\mu (P_2.K)$$

$$- 4 m_q C_{U1i} P_2^\mu \cancel{(}P_1.K) + 4 m_q C_{U1i} (\cancel{P}_1.P_2) K^\mu + 8 (P_2.\cancel{K}) m_q C_{U1i} P_1^\mu + 4 m_i^2 C_{\cancel{U}2i} m_q P_2^\mu - 4 m_q^3 \cancel{C}_{U1i} K^\mu$$

$$- 4 m_q \cancel{m}_i^2 C_{U2i} P_1^\mu \Big\}$$

$$= -\frac{g^4}{4 M_W^2} m_q C^*_{H_b} \left(-(P_1+P_2+P_3)_\mu + \frac{((P_1+P_2+P_3).P_4)}{M_W^2} P_{4\mu} \right) \sum_i V^*_{iq} \Big\{ m_q C_{U1i} (P_2.K) P_1^\mu$$

$$- m_q C_{U1i} (P_1.K) P_2^\mu + m_q C_{U1i} (P_1.P_2) K^\mu + m_i^2 m_q C_{U2i} P_2^\mu - m_q^3 C_{U1i} K^\mu - m_q m_i^2 C_{U2i} P_1^\mu \Big\}$$

$$M_b^{H^+} M_{4\,II} = -\frac{g^4}{4 M_W^2} m_q C^*_{H_b} \left(-(P_1+P_2+P_3)_\nu + \frac{((P_1+P_2+P_3).P_4)}{M_W^2} P_{4\nu} \right) \sum_i V^*_{iq} \Big\{ m_q C_{U1i} \big[(P_2.K) P_1^\nu$$

$$- (P_1.K) P_2^\nu + (P_1.P_2) K^\nu \big] + m_i^2 m_q C_{U2i} P_2^\nu - m_q^3 C_{U1i} K^\nu - m_q m_i^2 C_{U2i} P_1^\nu \Big\} \qquad \underset{\sim}{(145)}$$

$$= \frac{-g^4}{4 M_W^2} m_q C^*_{H_b} \sum_i V^*_{iq} \Big\{ + m_q C_{U1i} \Big[-(P_2.K)(P_1.(P_1+P_2+P_3)) + \frac{1}{M_W^2} (P_2.K)(P_1.P_4)((P_1+P_2+P_3).P_4)$$

$$+ (P_1.K)(P_2.(P_1+P_2+P_3)) - \frac{(P_1.K)(P_2.P_4)}{M_W^2} ((P_1+P_2+P_3).P_4) - (P_1.P_2)((P_1+P_2+P_3).K)$$

$$+ \frac{1}{M_W^2} (P_1.P_2)(P_4.K)((P_1+P_2+P_3).P_4) + m_q^2 ((P_1+P_2+P_3).K) - \frac{m_q^2}{M_W^2} (P_4.K)((P_1+P_2+P_3).P_4) \Big]$$

$$+ m_i^2 m_q C_{U2i} \Big[-(P_2-P_1).(P_1+P_2+P_3) + \frac{1}{M_W^2} (P_4.(P_2-P_1))((P_1+P_2+P_3).P_4) \Big] \Big\} \qquad (146)$$

$$P_1 \cdot (P_1 + P_2 + P_3) = P_1 \cdot (2P_1 + 2P_2 - P_4) = 2m_q^2 + 2\left(\frac{S}{2} - m_q^2\right) - \frac{1}{2}(m_q^2 + M_W^2 - U)$$

$$P_1 \cdot (P_1 + P_2 + P_3) = \frac{\cancel{4m_q^2} + 2S - \cancel{4m_q^2} - m_q^2 - M_W^2 + U}{2}$$

$$\boxed{P_1 \cdot (P_1 + P_2 + P_3) = \frac{2S - m_q^2 - M_W^2 + U}{2}} \qquad (147)$$

$$P_2 \cdot (P_1 + P_2 + P_3) = P_2 \cdot (2P_1 + 2P_2 - P_4) = 2\left(\frac{S}{2} - m_q^2\right) + 2m_q^2 - \frac{1}{2}(S + U - m_q^2 - M_{H^\pm}^2)$$

$$= \frac{\cancel{2}S - \cancel{4m_q^2} + \cancel{4m_q^2} - \cancel{S} - U + m_q^2 + M_{H^\pm}^2}{2}$$

$$\boxed{P_2 \cdot (P_1 + P_2 + P_3) = \frac{S - U + m_q^2 + M_{H^\pm}^2}{2}} \qquad (148)$$

$$P_4 \cdot (P_1 + P_2 + P_3) = P_4 \cdot (2P_1 + 2P_2 - P_4)$$

$$= \cancel{2}\,\frac{1}{\cancel{2}}(\cancel{m_q^2} + M_W^2 - \cancel{U}) + \cancel{2}\,\frac{1}{\cancel{2}}(S + \cancel{U} - \cancel{m_q^2} - M_{H^\pm}^2) - \cancel{M_W^2}$$

$$\boxed{P_4 \cdot (P_1 + P_2 + P_3) = S - M_{H^\pm}^2} \qquad (149)$$

$$K \cdot (P_1 + P_2 + P_3) = K \cdot (2P_1 + 2P_2 - P_4)$$

$$= 2\,\frac{1}{2}(\cancel{m_q^2} - \cancel{M_W^2} + \cancel{U}) + 2\,\frac{1}{2}(M_{\cancel{H^\pm}}^2 - \cancel{U} - \cancel{m_q^2}) - \frac{1}{2}(\cancel{m_q^2} - \cancel{U} - \cancel{M_W^2})$$

$$\boxed{K \cdot (P_1 + P_2 + P_3) = \frac{-m_q^2 - M_W^2 + U + 2M_{H^\pm}^2}{2}} \qquad (150)$$

$$\begin{aligned}
M_b^{H^\pm} M_{4\,II} = {} & -\frac{g^4}{4M_W^2} m_q C_b^{H^*} \sum_i V_{iq}^* \Big\{ m_q C_{U_{1i}} \Big[-\frac{1}{2}(M_{H^\pm}^2 - U - m_q^2)\,\frac{1}{2}(2S - m_q^2 - M_W^2 + U) + \frac{1}{M_W^2}\,\frac{1}{2}(M_{H^\pm}^2 - U - m_q^2) \cdot \frac{1}{2} \\
& (m_q^2 + M_W^2 - U)(S - M_{H^\pm}^2) + \frac{1}{2}(m_q^2 - M_W^2 + U)\,\frac{1}{2}(S - U + m_q^2 + M_{H^\pm}^2) - \frac{1}{M_W^2}\,\frac{1}{2}(m_q^2 - M_W^2 + U)\,\frac{1}{2}(S + U - m_q^2 - M_{H^\pm}^2) \,. \\
& \cdot (S - M_{H^\pm}^2) - \left(\frac{S}{2} - m_q^2\right)\frac{1}{2}(-m_q^2 - M_W^2 + U + 2M_{H^\pm}^2) + \frac{1}{M_W^2}\left(\frac{S}{2} - m_q^2\right)\frac{1}{2}(m_q^2 - U - M_W^2)(S - M_{H^\pm}^2) \\
& + m_q^2\,\frac{1}{2}(-m_q^2 - M_W^2 + U + 2M_{H^\pm}^2) - \frac{m_q^2}{M_W^2}\,\frac{1}{2}(m_q^2 - U - M_W^2)(S - M_{H^\pm}^2) + m_i^2 m_q C_{U_{2i}} \Big[-\frac{1}{2}(\cancel{S} - \cancel{U} + \cancel{m_q^2} + \cancel{M_{H^\pm}^2}) \\
& + \frac{1}{2}(2\cancel{S} - \cancel{m_q^2} - \cancel{M_W^2} + \cancel{U}) + \frac{1}{M_W^2}(S - M_{H^\pm}^2)\Big[\frac{1}{2}(\cancel{S} + \cancel{U} - \cancel{m_q^2} - \cancel{M_{H^\pm}^2}) - \frac{1}{2}(\cancel{m_q^2} + \cancel{M_W^2} - \cancel{U}) \Big] \Big] \Big\}
\end{aligned} \qquad (151)$$

The term containing $C_{U_{2i}}$ is:

$$\frac{1}{2}(S - 2m_q^2 + 2U - M_W^2 - M_{H^\pm}^2) + \frac{1}{M_W^2}(S - M_{H^\pm}^2)\,\frac{1}{2}(S + 2U - 2m_q^2 - M_{H^\pm} - M_W^2)$$

$$= \frac{1}{2M_W^2}\Big[\cancel{sM_W^2} - \cancel{2m_q^2M_W^2} + \cancel{2uM_W^2} - \cancel{M_W^4} - \cancel{M_{H^\pm}^2M_W^2} + \cancel{s^2} + \cancel{2us} - \cancel{2sm_q^2} - \cancel{sM_{H^\pm}^2} - \cancel{sM_W^2} - \cancel{sM_{H^\pm}^2} - \cancel{2uM_{H^\pm}^2} + \cancel{2m_q^2M_{H^\pm}^2} + \cancel{M_{H^\pm}^4}$$
$$+ \cancel{M_{H^\pm}^2M_W^2}\Big]$$

$$= \frac{1}{2M_W^2}\Big[-2m_q^2M_W^2 + 2uM_W^2 - M_W^4 + s^2 + 2us - 2sm_q^2 - 2sM_{H^\pm}^2 - 2uM_{H^\pm}^2 + 2m_q^2M_{H^\pm}^2 + M_{H^\pm}^4\Big] \quad \text{OK.}$$

The same as (90) with opposite sign ~~and~~ $t \leftrightarrow u$ (152)

The terms with C_{ui} are:

$$-\frac12(M_{H^\pm}^2-u-m_q^2)\,\frac12(\cancel{2}s-m_q^2-M_W^2+u) + \frac{1}{M_W^2}\,\frac12(M_{H^\pm}^2-u-m_q^2)\,\frac12(m_q^2+M_W^2-u)(s-M_{H^\pm}^2)$$

$$+ \underset{\frac12(s-2m_q^2)}{\frac12(m_q^2-M_W^2+u)}\,\frac12(s-u+m_q^2+M_{H^\pm}^2) - \frac{1}{M_W^2}\,\frac12(m_q^2-M_W^2+u)\,\frac12(s+u-m_q^2-M_{H^\pm}^2)(s-M_{H^\pm}^2)$$

$$-\left(\frac s2-m_q^2\right)\frac12(-m_q^2-M_W^2+u+2M_{H^\pm}^2) + \frac{1}{M_W^2}\underset{\frac12(s-2m_q^2)}{\left(\frac s2-m_q^2\right)}\frac12(m_q^2-u-M_W^2)(s-M_{H^\pm}^2) + m_q^2\,\frac12(-\cancel{m_q^2}-M_W^2+u+2M_{H^\pm}^2)$$

$$-\frac{m_q^2}{M_W^4}\,\frac12(m_q^2-u-M_W^2)(s-M_{H^\pm}^2)$$

$$= -\frac14\Big(\cancel{2sM_{H^\pm}^2} - \cancel{m_q^2M_{H^\pm}^2} - \cancel{M_W^2M_{H^\pm}^2} + \cancel{uM_{H^\pm}^2} - \cancel{2us} + \cancel{um_q^2} + \cancel{uM_W^2} - \cancel{u^2} - \cancel{2sm_q^2} + \cancel{m_q^4} + \cancel{m_q^2M_W^2} - \cancel{m_q^2u}$$
$$- \cancel{m_q^2s} + \cancel{m_q^2u} - \cancel{m_q^4} - \cancel{m_q^2M_{H^\pm}^2} + \cancel{M_W^2s} - \cancel{uM_W^2} + \cancel{m_q^2M_W^2} + \cancel{M_W^2M_{H^\pm}^2} - \cancel{us} + \cancel{u^2} - \cancel{um_q^2} - \cancel{uM_{H^\pm}^2} - \cancel{sm_q^2} - \cancel{sM_W^2} + \cancel{us}$$
$$+ \cancel{2sM_{H^\pm}^2} + \cancel{2m_q^4} + \cancel{2m_q^2M_W^2} - \cancel{2m_q^2u} - \cancel{4m_q^2M_{H^\pm}^2} + \cancel{2m_q^4} + \cancel{2m_q^2M_W^2} - \cancel{2m_q^2u} - \cancel{4m_q^2M_{H^\pm}^2}\Big)$$
$$+ \frac{1}{4M_W^2}(s-M_{H^\pm}^2)\Big[\cancel{m_q^2M_{H^\pm}^2} + \cancel{M_W^2M_{H^\pm}^2} - \cancel{uM_{H^\pm}^2} - \cancel{um_q^2} - \cancel{uM_W^2} + \cancel{u^2} - \cancel{m_q^4} - \cancel{m_q^2M_W^2} + \cancel{m_q^2u} - \cancel{m_q^2s} - \cancel{m_q^2u}$$
$$+ \cancel{m_q^4} + \cancel{m_q^2M_{H^\pm}^2} + \cancel{M_W^2s} + \cancel{M_W^2u} - \cancel{m_q^2M_W^2} - \cancel{M_W^2M_{H^\pm}^2} - \cancel{us} - \cancel{u^2} + \cancel{um_q^2} + \cancel{uM_{H^\pm}^2} + \cancel{sm_q^2} - \cancel{us} - \cancel{M_W^2s}$$
$$- \cancel{2m_q^4} + \cancel{2m_q^2u} + \cancel{2m_q^2M_W^2} - \cancel{2m_q^4} + \cancel{2m_q^2u} + \cancel{2m_q^2M_W^2}\Big]$$

$$= -\frac14\left(4sM_{H^\pm}^2 - 10m_q^2M_{H^\pm}^2 - 2us - 4um_q^2 - 4sm_q^2 + 6m_q^2M_W^2 + 4m_q^4\right)$$
$$+ \frac{1}{4M_W^2}(s-M_{H^\pm}^2)\left(2m_q^2M_{H^\pm}^2 + 4um_q^2 - 4m_q^4 - 2us + 2m_q^2M_W^2\right)$$

$$= \frac{1}{4M_W^2}\Big[-\cancel{4sM_{H^\pm}^2M_W^2} + \cancel{10m_q^2M_{H^\pm}^2M_W^2} + \cancel{2usM_W^2} + \cancel{4um_q^2M_W^2} + \cancel{4sm_q^2M_W^2} - \cancel{6m_q^2M_W^4} - \cancel{4m_q^4M_W^2}$$
$$+ \cancel{2sm_q^2M_{H^\pm}^2} + \cancel{4usm_q^2} - \cancel{4sm_q^4} - \cancel{2us^2} + \cancel{2m_q^2M_W^2s} - \cancel{2m_q^2M_{H^\pm}^4} - \cancel{4um_q^2M_{H^\pm}^2} + \cancel{4m_q^4M_{H^\pm}^2} + \cancel{2usM_{H^\pm}^2}$$
$$- \cancel{2m_q^2M_W^2M_{H^\pm}^2}\Big]$$

$$= \frac{1}{4M_W^2}\Big[-4sM_{H^\pm}^2M_W^2 + 8m_q^2M_{H^\pm}^2M_W^2 + 2usM_W^2 + 4um_q^2M_W^2 + 6sm_q^2M_W^2 - 6m_q^2M_W^4 - 4m_q^4M_W^2$$
$$+ 2sm_q^2M_{H^\pm}^2 + 4usm_q^2 - 4sm_q^4 - 2us^2 - 2m_q^2M_{H^\pm}^4 - 4um_q^2M_{H^\pm}^2 + 4m_q^4M_{H^\pm}^2 + 2usM_{H^\pm}^2\Big] \quad \text{OK.}$$

(153)

that is the same that (88) with a - sign and interchanging $t \leftrightarrow U$

Again the term containing C_{U2i} is:

$$\frac{1}{2M_W^2}\Big[-2\cancel{m_q^2}M_W^2 + 2U\cancel{M_W^2} - \cancel{M_W^4} + \cancel{s^2} + 2U(2\cancel{m_q^2} + \cancel{M_W^2} + \cancel{M_H^2} - \cancel{s} - \cancel{t}) - 2s\cancel{m_q^2} - 2s\cancel{M_H^2} - 2U\cancel{M_H^2} + 2\cancel{m_q^2}M_H^2 + M_H^4\Big]$$

$$= \frac{1}{2M_W^2}\Big[-2m_q^2M_W^2 + 4UM_W^2 - M_W^4 + s^2 + 4Um_q^2 - 2U^2 - 2Ut - 2sm_q^2 - 2sM_H^2 + 2m_q^2M_H^2 + M_H^4\Big]$$

$$= \frac{1}{2M_W^2}\Big[-2\cancel{m_q^2}M_W^2 + 4U\cancel{M_W^2} - \cancel{M_W^4} + \cancel{s^2} + 4\cancel{U}m_q^2 - 2\cancel{U}^2 - 2\cancel{m_q^4} + 2\cancel{m_q^2}M_H^2 + 2m_q^2\cancel{M_W^2} - 2\cancel{M_H^2}M_W^2$$
$$- \frac{1}{2s}\Big[4m_q^2M_H^4 + 4m_q^2M_W^4 - 8m_q^2M_H^2M_W^2 + \sin^2\theta\,\lambda(s-4m_q^2)\Big] - 2s\cancel{m_q^2} - 2\cancel{s}M_H^2 + 2\cancel{m_q^2}M_H^2 + \cancel{M_H^4}\Big]$$

$$= \frac{1}{2M_W^2}\Big[4UM_W^2 - M_W^4 + s^2 + 4Um_q^2 - 2U^2 - 2m_q^4 + 4m_q^2M_H^2 - 2M_H^2M_W^2 - 2sm_q^2 - 2sM_H^2 + M_H^4$$
$$- \frac{1}{2s}\Big[4m_q^2M_H^4 + 4m_q^2M_W^4 - 8m_q^2M_H^2M_W^2 + \sin^2\theta\,\lambda(s-4m_q^2)\Big]\Big] \qquad (154)$$

Returning to (153)

$$s + t + U = 2m_q^2 + M_W^2 + M_H^2$$

$$2s^2U + 2sUt + 2sU^2 = 4m_q^2sU + 2sUM_W^2 + 2sUM_H^2$$

$\Rightarrow$ the term containing C_{U2i} is:

$$\frac{1}{4M_W^2}\Big[-4s\cancel{M_H^2}M_W^2 + 8m_q^2\cancel{M_H^2}M_W^2 + 2Us\cancel{M_W^2} + 4U\cancel{m_q^2}M_W^2 + 6s\cancel{m_q^2}M_W^2 - 6\cancel{m_q^2}M_W^4 - 4m_q^4\cancel{M_W^2}$$
$$+ 2sm_q^2\cancel{M_H^2} + 4Us\cancel{m_q^2} - 4s\cancel{m_q^4} - 4\cancel{m_q^2}Us - 2Us\cancel{M_W^2} - 2Us\cancel{M_H^2} + 2\cancel{s}Ut + 2\cancel{s}U^2 - 2\cancel{m_q^2}M_H^4 - 4U\cancel{m_q^2}M_H^2$$
$$+ 4\cancel{m_q^4}M_H^2 + 2Us\cancel{M_H^2}\Big]$$

$$= \frac{1}{4M_W^2}\Big[-4sM_H^2M_W^2 + 8m_q^2M_H^2M_W^2 + 4Um_q^2M_W^2 + 6sm_q^2M_W^2 - 6m_q^2M_W^4 - 4m_q^4M_W^2 + 2sm_q^2M_H^2 - 4sm_q^4$$
$$+ 2sUt + 2sU^2 - 2m_q^2M_H^4 - 4Um_q^2M_H^2 + 4m_q^4M_H^2\Big]$$

$$= \frac{1}{4M_W^2}\Big[-4s\cancel{M_H^2}M_W^2 + 8m_q^2\cancel{M_H^2}M_W^2 + 4U\cancel{m_q^2}M_W^2 + 6s\cancel{m_q^2}M_W^2 - 6\cancel{m_q^2}M_W^4 - 4\cancel{m_q^4}M_W^2 + 2s\cancel{m_q^2}M_H^2 - 4\cancel{s}m_q^4$$
$$+ 2s\cancel{U}^2 - 2m_q^2\cancel{M_H^4} - 4U\cancel{m_q^2}M_H^2 + 4\cancel{m_q^4}M_H^2 + 2s\cancel{m_q^4} - 2s\cancel{m_q^2}M_H^2 - 2s\cancel{m_q^2}M_W^2 + 2s\cancel{M_H^2}M_W^2$$
$$+ 2\cancel{m_q^2}M_H^4 + 2\cancel{m_q^2}M_W^4 - 4m_q^2\cancel{M_H^2}M_W^2 + \frac{1}{2}\sin^2\cancel{\theta}\,\lambda(s-4m_q^2)\Big]$$

$$= \frac{1}{4M_W^2}\Big[-2sM_H^2M_W^2 + 4m_q^2M_H^2M_W^2 + 4Um_q^2M_W^2 + 4sm_q^2M_W^2 - 4m_q^2M_W^4 - 4m_q^4M_W^2 - 2sm_q^4 + 2sU^2$$
$$- 4Um_q^2M_H^2 + 4m_q^4M_H^2 + \frac{1}{2}\sin^2\theta\,\lambda(s-4m_q^2)\Big] \qquad (155)$$

$$\Rightarrow M^{+}_{H_b} M_{4II} = \frac{-g^4}{4M_W^2}\, m_q\, C^{*}_{H_b} \sum_i V^{*}_{iq} \Big\{ \frac{m_q C_{U1i}}{4M_W^2} \Big[-2s M_{H^\pm}^2 M_W^2 + 4m_q^2 M_{H^\pm}^2 M_W^2 + 4u\, m_q^2 M_W^2 + 4s\, m_q^2 M_W^2$$

$$- 4m_q^2 M_W^4 - 4m_q^4 M_W^2 - 2s\, m_q^4 + 2s u^2 - 4u\, m_q^2 M_{H^\pm}^2 + 4m_q^4 M_{H^\pm}^2 + \frac{1}{2}\sin^2\theta\, \lambda\, (s-4m_q^2) \Big]$$

$$+ \frac{m_i^2 m_q C_{U2i}}{2M_W^2} \Big[4u M_W^2 - M_W^4 + s^2 + 4u\, m_q^2 - 2u^2 - 2m_q^4 + 4m_q^2 M_{H^\pm}^2 - 2M_{H^\pm}^2 M_W^2 - 2s\, m_q^2 - 2s M_{H^\pm}^2 + M_{H^\pm}^4$$

$$- \frac{1}{2s}\Big[4m_q^2 M_{H^\pm}^4 + 4m_q^2 M_W^4 - 8m_q^2 M_{H^\pm}^2 M_W^2 + \sin^2\theta\, \lambda\, (s-4m_q^2) \Big] \Big] \Big\}$$

$$M^{+}_{H_b} M_{4V} = \frac{g^4 m_q^2}{16M_W^4}\, C^{*}_{H_b} \sum_i V^{*}_{iq} \Big\{ C_{U1i} \Big[-2s u^2 + 2s\, m_q^4 - 4s\, m_q^2 M_W^2 + 2s M_{H^\pm}^2 M_W^2 + 4m_q^2 M_W^4 - 4m_q^2 M_{H^\pm}^2 M_W^2$$

$$+ 4m_q^4 M_W^2 - 4m_q^2 u M_W^2 - 4m_q^4 M_{H^\pm}^2 + 4u\, m_q^2 M_{H^\pm}^2 - \frac{1}{2}\sin^2\theta\, \lambda\, (s-4m_q^2) \Big]$$

$$+ 2m_i^2 C_{U2i} \Big[M_W^4 - 4u M_W^2 - s^2 + 2s M_{H^\pm}^2 + 2s\, m_q^2 - 4u\, m_q^2 + 2u^2 - M_{H^\pm}^4 - 4m_q^2 M_{H^\pm}^2 + 2m_q^4 + 2M_{H^\pm}^2 M_W^2$$

$$+ \frac{2}{s}\Big[m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4}\sin^2\theta\, \lambda\, (s-4m_q^2) \Big] \Big] \Big\} \qquad (156)$$

$$\boxed{\begin{aligned} &M_b^{H+} M_{4II} + M^{+}_{4II} M_b^{H} = \frac{g^4 m_q^2}{16M_W^4} \Big[C^{*}_{H_b} \sum_i V^{*}_{iq} + C_{H_b} \sum_i V_{iq} \Big] \Big\{ C_{U1i} \Big[-2s u^2 + 2s\, m_q^4 - 4s\, m_q^2 M_W^2 + 2s M_{H^\pm}^2 M_W^2 \\ &+ 4m_q^2 M_W^4 - 4m_q^2 M_{H^\pm}^2 M_W^2 + 4m_q^4 M_W^2 - 4m_q^2 u M_W^2 - 4m_q^4 M_{H^\pm}^2 + 4u\, m_q^2 M_{H^\pm}^2 - \frac{1}{2}\sin^2\theta\, \lambda\, (s-4m_q^2) \Big] \\ &+ 2m_i^2 C_{U2i} \Big[M_W^4 - 4u M_W^2 - s^2 + 2s M_{H^\pm}^2 + 2s\, m_q^2 - 4u\, m_q^2 + 2u^2 - M_{H^\pm}^4 - 4m_q^2 M_{H^\pm}^2 + 2m_q^4 + 2M_{H^\pm}^2 M_W^2 \\ &+ \frac{2}{s}\Big[m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4}\sin^2\theta\, \lambda\, (s-4m_q^2) \Big] \Big] \Big\} \qquad \underline{(157)} \end{aligned}}$$

<u>OK</u>.

(157) can be obtained from <u>(103a)</u> changing $t \leftrightarrow u$ and $V_{iq} \leftrightarrow V^{*}_{iq}$

$$M^{+}_{3_b} M_{4II} = \frac{g^4 m_q}{16M_W^2}\, C^{*}_{A_b} \sum_{\lambda} \varepsilon^{*}_{\mu 4} \varepsilon_{\nu 4} (P_1+P_2+P_3)^{\nu} \sum_i V^{*}_{iq} \sum_s (\bar{U}_1 \gamma^5 V_2)\, \bar{V}_2 \big[m_q C_{U1i} \not{k} + m_i^2 C_{U2i} \big] \gamma^{\mu}(1-\gamma^5) U_1 \qquad (158)$$

$$= \frac{g^4 m_q}{16M_W^2}\, C^{*}_{A_b} \Big(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2} \Big) (P_1+P_2+P_3)^{\nu} \sum_i V^{*}_{iq}\, Tr\big[(\not{P}_2 - m_q)(m_q C_{U1i} \not{k} + m_i^2 C_{U2i}) \gamma^{\mu}(1-\gamma^5) (\not{P}_1 + m_q)\gamma^5 \big]$$

$$\not{P}_1 \gamma^5 = -\gamma^5 \not{P}_1 \qquad (\not{P}_1 + m_q)\gamma^5 = -\gamma^5 \not{P}_1 + m_q \gamma^5 = \gamma^5(-\not{P}_1 + m_q)$$

$$M^{+}_{3_b} M_{4II} = -\frac{g^4 m_q}{16M_W^2}\, C^{*}_{A_b} \Big(-g_{\mu\nu} + \frac{P_{4\mu}P_{4\nu}}{M_W^2} \Big) (P_1+P_2+P_3)^{\nu} \sum_i V^{*}_{iq}\, Tr\big[(\not{P}_2 - m_q)(m_q C_{U1i} \not{k} + m_i^2 C_{U2i}) \gamma^{\mu}(1-\gamma^5) (-\not{P}_1 + m_q) \big] \qquad (159)$$

The trace is the same that appears in (143) if we change $P_1 \to -P_1$

Then:

$$M^{+}_{3_b} M_{4\,II} = -\frac{g^4 m_q}{16 M_W^2} C^{*}_{A_b}\left(-(P_1+P_2+P_3)_\mu + \frac{((P_1+P_2+P_3)\cdot P_4)}{M_W^2} P_{4\mu}\right)\sum_i V^{*}_{iq}\Big\{-4 m_q C_{U_1 i}\left(-P_1^{\mu} P_2^{\alpha} - P_1^{\alpha} P_2^{\mu} + (P_1\cdot P_2) g^{\mu\alpha}\right) K_\alpha$$

$$- 8 (P_2\cdot K) m_q C_{U_1 i} P_1^{\mu} + 4 m_i^2 C_{U_2 i} m_q P_2^{\mu} - 4 m_q^3 C_{U_1 i} K^{\mu} + 4 m_q m_i^2 C_{U_2 i} P_1^{\mu}\Big\} \qquad (160)$$

$$= -\frac{g^4}{4 M_W^2} m_q^2 C^{*}_{A_b} \sum_i V^{*}_{iq} \Big\{ -C_{U_1 i}\Big[-(P_1\cdot(P_1+P_2+P_3))(P_2\cdot K) + (P_2\cdot(P_1+P_2+P_3))(P_1\cdot K) - (K\cdot(P_1+P_2+P_3)) \times$$

$$(P_1\cdot P_2) + \frac{((P_1+P_2+P_3)\cdot P_4)}{M_W^2}\Big(+(P_1\cdot P_4)(K\cdot P_2) - (P_2\cdot P_4)(K\cdot P_1) + (P_4\cdot K)(P_1\cdot P_2)\Big)\Big)$$

$$+ m_q^2\Big(-((P_1+P_2+P_3)\cdot K) + \frac{1}{M_W^2}((P_1+P_2+P_3)\cdot P_4)(K\cdot P_4)\Big)\Big] + m_i^2 C_{U_2 i}\Big(-((P_1+P_2+P_3)\cdot P_2)$$

$$+ \frac{((P_1+P_2+P_3)\cdot P_4)}{M_W^2}(P_4\cdot P_2)\Big) + m_i^2 C_{U_2 i}\Big(-((P_1+P_2+P_3)\cdot P_1) + \frac{((P_1+P_2+P_3)\cdot P_4)}{M_W^2}(P_1\cdot P_4)\Big)\Big\} \qquad (161)$$

$$= -\frac{g^4}{4 M_W^2} m_q^2 C^{*}_{A_b} \sum_i V^{*}_{iq} \Big\{ -C_{U_1 i}\Big[-\frac{1}{2}(2S - m_q^2 - M_W^2 + U)\frac{1}{2}(M_{HI}^2 - U - m_q^2) + \frac{1}{2}(S - U + m_q^2 + M_{HI}^2)\cdot$$

$$\frac{1}{2}(m_q^2 - M_W^2 + U) - \frac{1}{2}(-m_q^2 - M_W^2 + U + 2 M_{HI}^2)\left(\frac{S - 2m_q^2}{2}\right) - m_q^2 \frac{1}{2}(-m_q^2 - M_W^2 + U + 2 M_{HI}^2)$$

$$+ \frac{(S - M_{HI}^2)}{M_W^2}\Big(\frac{1}{2}(m_q^2 + M_W^2 - U)\frac{1}{2}(M_{HI}^2 - U - m_q^2) - \frac{1}{2}(S + U - m_q^2 - M_{HI}^2)\frac{1}{2}(m_q^2 - M_W^2 + U) + \frac{1}{2}(m_q^2 - U - M_W^2)$$

$$\frac{1}{2}(S - 2m_q^2) + m_q^2 \frac{1}{2}(m_q^2 - U - M_W^2)\Big)\Big] + m_i^2 C_{U_2 i}\Big[-\frac{1}{2}(S - U + m_q^2 + M_{HI}^2) - \frac{1}{2}(2S - m_q^2 - M_W^2 + U)$$

$$+ \frac{(S - M_{HI}^2)}{M_W^2}\Big(\frac{1}{2}(S + U - m_q^2 - M_{HI}^2) + \frac{1}{2}(m_q^2 + M_W^2 - U)\Big)\Big]\Big\}$$

$$= -\frac{g^4}{16 M_W^2} m_q^2 C^{*}_{A_b} \sum_i V^{*}_{iq} \Big\{ -C_{U_1 i}\Big[\cancel{-2S M_{HI}^2} \cancel{+ 2US} \cancel{+ 2S m_q^2} \cancel{+ m_q^2 M_{HI}^2} \cancel{- m_q^2 U} \cancel{- m_q^4} \cancel{+ M_W^2 M_{HI}^2} \cancel{- M_W^2 U} \cancel{- m_q^2 M_W^2}$$

$$\cancel{- U M_{HI}^2} \cancel{+ U^2} \cancel{+ U m_q^2}$$

$$\cancel{+ S m_q^2} \cancel{- S M_W^2} \cancel{+ SU} \cancel{- U m_q^2} \cancel{+ U M_W^2} \cancel{- U^2} \cancel{+ m_q^4} \cancel{- m_q^2 M_W^2} \cancel{+ m_q^2 U} \cancel{+ m_q^2 M_{HI}^2} \cancel{- M_W^2 M_{HI}^2} \cancel{+ U M_{HI}^2} \cancel{+ S m_q^2} \cancel{- 2 m_q^4}$$

$$\cancel{+ S M_W^2} \cancel{- 2 m_q^2 M_W^2} \cancel{- US} \cancel{+ 2 m_q^2 U} \cancel{- 2S M_{HI}^2} \cancel{+ 4 m_q^2 M_{HI}^2} \cancel{+ 2 m_q^4} \cancel{+ 2 m_q^2 M_W^2} \cancel{- 2 m_q^2 U} \cancel{- 4 m_q^2 M_{HI}^2}$$

$$+ \frac{(S - M_{HI}^2)}{M_W^2}\Big(\cancel{m_q^2 M_{HI}^2} \cancel{- U m_q^2} \cancel{- m_q^4} \cancel{+ M_W^2 M_{HI}^2} \cancel{- U M_W^2} \cancel{- m_q^2 M_W^2} \cancel{- U M_{HI}^2} \cancel{+ U^2} \cancel{+ U m_q^2} \cancel{- S m_q^2} \cancel{+ S M_W^2} \cancel{- SU} \cancel{- U m_q^2}$$

$$\cancel{+ U M_W^2} \cancel{- U^2} \cancel{+ m_q^4} \cancel{- m_q^2 M_W^2} \cancel{+ m_q^2 U} \cancel{+ m_q^2 M_{HI}^2} \cancel{- M_W^2 M_{HI}^2} \cancel{+ U M_{HI}^2} \cancel{+ S m_q^2} \cancel{- 2 m_q^4} \cancel{- US} \cancel{+ 2U m_q^2} \cancel{- S M_W^2} \cancel{+ 2 m_q^2 M_W^2}$$

$$\cancel{+ 2 m_q^4} \cancel{- 2 m_q^2 U} \cancel{- 2 m_q^2 M_W^2}\Big)\Big] + 2 m_i^2 C_{U_2 i}\Big[\cancel{-S} \cancel{+ U} \cancel{- m_q^2} \cancel{- M_{HI}^2} \cancel{- 2S} \cancel{+ m_q^2} \cancel{+ M_W^2} \cancel{- U}$$

$$+ \frac{(S - M_{HI}^2)}{M_W^2}\Big(\cancel{S} \cancel{+ U} \cancel{- m_q^2} \cancel{- M_{HI}^2} \cancel{+ m_q^2} \cancel{+ M_W^2} \cancel{- U}\Big)\Big]\Big\}$$

$$= \frac{-g^4}{16M_W^2} m_q^2 C^*_{A_b}\sum_i V^*_{iq} \{ -C_{01i}[-4sM_{HI}^2 + 2us + 4sm_q^2 + 2m_q^2M_{HI}^2 - 2m_q^2M_W^2$$

$$+ \frac{(s-M_{HI}^2)}{M_W^2}(2m_q^2M_{HI}^2 - 2m_q^2M_W^2 - 2us)] + 2m_i^2C_{02i}[-3s + M_W^2 - M_{HI}^2 + \frac{(s-M_{HI}^2)}{M_W^2}(s+M_W^2-M_{HI}^2)]\}$$

$$= \frac{-g^4}{16M_W^4} m_q^2 C^*_{A_b}\sum_i V^*_{iq} \{ -C_{01i}[-4s\cancel{M_{HI}^2}M_W^2 + 2us\cancel{M_W^2} + 4s\cancel{m_q^2}M_W^2 + 2m_q^2\cancel{M_{HI}^2}M_W^2$$

$$-2\cancel{m_q^2}M_W^4 + 2s\cancel{m_q^2}M_{HI}^2 - 2s\cancel{m_q^2}M_W^2 - 2\cancel{u}s^2 - 2m_q^2\cancel{M_{HI}^4} + 2m_q^2\cancel{M_{HI}^2}M_W^2 + 2us\cancel{M_{HI}^2}]$$

$$+ 2m_i^2C_{02i}[-3\cancel{s}M_W^2 + \cancel{M_W^4} - M_{HI}^2\cancel{M_W^2} + \cancel{s^2} + s\cancel{M_W^2} - s\cancel{M_{HI}^2} - s\cancel{M_{HI}^2} - M_{HI}^2\cancel{M_W^2} + \cancel{M_{HI}^4}]\}$$

$$= -\frac{g^4}{16M_W^4} m_q^2 C^*_{A_b}\sum_i V^*_{iq} \{ -C_{01i}[-4sM_{HI}^2M_W^2 + 2us\cancel{M_W^2} + 2sm_q^2M_W^2 + 4m_q^2M_{HI}^2M_W^2$$

$$-2m_q^2M_W^4 + 2sm_q^2M_{HI}^2 - 2\cancel{u}s^2 - 2m_q^2M_{HI}^4 + 2us\cancel{M_{HI}^2}]$$

$$+ 2m_i^2C_{02i}[-2sM_W^2 + M_W^4 - 2M_{HI}^2M_W^2 + s^2 - 2sM_{HI}^2 + M_{HI}^4]\}$$

$$s+t+u = 2m_q^2 + M_{HI}^2 + M_W^2 \Rightarrow 2s^2u + 2sut + 2su^2 = 4sum_q^2 + 2suM_{HI}^2 + 2suM_W^2$$

$$= -\frac{g^4}{16M_W^4} m_q^2 C^*_{A_b}\sum_i V^*_{iq} \{ -C_{01i}[-4sM_{HI}^2\cancel{M_W^2} + 2s\cancel{u}t + 2\cancel{s}u^2 - 4s\cancel{u}m_q^2 + 2s\cancel{m_q^2}M_W^2 + 4m_q^2\cancel{M_{HI}^2}M_W^2$$

$$-2\cancel{m_q^2}M_W^4 + 2s\cancel{m_q^2}M_{HI}^2 - 2\cancel{m_q^2}M_{HI}^4] + 2m_i^2C_{02i}[-2sM_W^2 + M_W^4 - 2M_{HI}^2M_W^2 + s^2$$

$$-2sM_{HI}^2 + M_{HI}^4]\}$$

$$= -\frac{g^4}{16M_W^4} m_q^2 C^*_{A_b}\sum_i V^*_{iq} \{ -C_{01i}[-4s\cancel{M_{HI}^2}M_W^2 - 4s\cancel{u}m_q^2 + 2\cancel{s}u^2 + 2s\cancel{m_q^2}M_W^2 + 4m_q^2\cancel{M_{HI}^2}M_W^2 - 2m_q^2\cancel{M_W^4}$$

$$+2s\cancel{m_q^2}M_{HI}^2 - 2m_q^2\cancel{M_{HI}^4} + 2\cancel{s}m_q^4 - 2\cancel{s}m_q^2M_{HI}^2 - 2s\cancel{m_q^2}M_W^2 + 2s\cancel{M_{HI}^2}M_W^2 + 2\cancel{m_q^2}M_{HI}^4 + 2\cancel{m_q^2}M_W^4$$

$$-4m_q^2\cancel{M_{HI}^2}M_W^2 + \frac{1}{2}\sin^2\theta\,\lambda(s-4m_q^2)] + 2m_i^2C_{02i}[-2sM_W^2 + M_W^4 - 2M_{HI}^2M_W^2 + s^2 - 2sM_{HI}^2 + M_{HI}^4]\}$$

$$M_3^{+}M_{4II} = -\frac{g^4}{16M_W^4} m_q^2 C^*_{A_b}\sum_i V^*_{iq} \{ -C_{01i}[-2sM_{HI}^2M_W^2 - 4usm_q^2 + 2su^2 + 2sm_q^4 + \frac{1}{2}\sin^2\theta\,\lambda(s-4m_q^2)]$$

$$+ 2m_i^2C_{02i}[-2s\cancel{M_W^2} + \cancel{M_W^4} - 2M_{HI}^2\cancel{M_W^2} + \cancel{s^2} - 2s\cancel{M_{HI}^2} + M_{HI}^4]\}$$

$$\boxed{\begin{aligned} M_{3_b}^{+}M_{4II} = \frac{g^4}{16M_W^4} m_q^2 C^*_{A_b}\sum_i V^*_{iq} \{ C_{01i}[-2sM_{HI}^2M_W^2 - 4usm_q^2 + 2su^2 + 2sm_q^4 + \tfrac{1}{2}\sin^2\theta\,\lambda(s-4m_q^2)] \\ + 2m_i^2C_{02i}[2sM_W^2 - s^2 + 2sM_{HI}^2 + 2M_{HI}^2M_W^2 - M_W^4 - M_{HI}^4]\} \end{aligned}} \tag{162}$$

$$\boxed{\begin{aligned} M_{3_b}^{+}M_{4II} + M_{4II}^{+}M_{3_b} = \frac{g^4}{16M_W^4} m_q^2 [C^*_{A_b}\sum_i V^*_{iq} + C_{A_b}\sum_i V_{iq}]\{C_{01i}[-2sM_{HI}^2M_W^2 - 4usm_q^2 + 2su^2 + 2sm_q^4 + \tfrac{1}{2}\sin^2\theta\,\lambda(s-4m_q^2)] \\ + 2m_i^2C_{02i}[2sM_W^2 - s^2 + 2sM_{HI}^2 + 2M_{HI}^2M_W^2 - M_W^4 - M_{HI}^4]\} \end{aligned}} \tag{163}$$

$$\overline{|M_{II}|^2} = \frac{g^4}{96 M_W^4}\Big\{ m_q^2 |C_{H_b}|^2 \lambda(S, M_W^2, M_{H^\pm}^2)(S-4m_q^2) + m_q^2 |C_{A_b}|^2 S\,\lambda(S,M_W^2,M_{H^\pm}^2) + 2\sum_{i,j} V_{iq}V_{jq}^* \Big\{ m_q^2 C_{U1i} C_{U1j} \times$$

$$\times\Big[-2U m_q^2 (M_{H^\pm}^2 + 2M_W^2) + SU^2 + 2\frac{M_W^2}{S}\Big(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac14 \sin^2\theta\,\lambda\,(S-4m_q^2)\Big) +$$

$$+ \frac{m_q^2}{S}\Big(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac14\sin^2\theta\,\lambda\,(S-4m_q^2)\Big)\Big] + m_q^2\big(2M_W^4 - UM_W^2 - m_q^2 M_W^2 - m_q^4 + 2Um_q^2$$

$$- U^2\big)\big(m_j^2 C_{U1i}C_{U2j} + m_i^2 C_{U2i}C_{U1j}\big) + m_i^2 m_j^2 C_{U2i}C_{U2j}\Big[2SM_W^2 - 4m_q^2 M_W^2 + Sm_q^2 - 2m_q^2 M_{H^\pm}^2 +$$

$$+ \frac1S\Big(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac14 \sin^2\theta\,\lambda\,(S-4m_q^2)\Big)\Big]\Big\} - \frac12 m_q^2 \Big[C_{H_b}^* \sum_i V_{iq}^* + C_{H_b}\sum_i V_{iq}\Big]\Big\{C_{U1i}\Big[-2SU^2 +$$

$$+ 2Sm_q^4 - 4Sm_q^2M_W^2 + 2SM_{H^\pm}^2M_W^2 + 4m_q^2M_W^4 - 4m_q^2 M_{H^\pm}^2 M_W^2 + 4m_q^4 M_W^2 - 4m_q^2 U M_W^2 - 4m_q^4 M_{H^\pm}^2 + 4Um_q^2 M_{H^\pm}^2$$

$$- \frac12\sin^2\theta\,\lambda\,(S-4m_q^2)\Big] + 2m_i^2 C_{U2i}\Big[M_W^4 - 4UM_W^2 - S^2 + 2SM_{H^\pm}^2 + 2Sm_q^2 - 4Um_q^2 + 2U^2 - M_{H^\pm}^4 - 4m_q^2 M_{H^\pm}^2 + 2m_q^4$$

$$+ 2M_{H^\pm}^2 M_W^2 + \frac2S\Big[m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac14 \sin^2\theta\,\lambda\,(S-4m_q^2)\Big]\Big]\Big\} + \frac12 m_q^2\Big[C_{A_b}^*\sum_i V_{iq}^* + C_{A_b}\sum_i V_{iq}\Big]\times$$

$$\times\Big\{C_{U1i}\Big[-2SM_{H^\pm}^2M_W^2 - 4USm_q^2 + 2SU^2 + 2Sm_q^4 + \frac12\sin^2\theta\,\lambda\,(S-4m_q^2)\Big] + 2m_i^2 C_{U2i}\Big[2SM_W^2 - S^2 + 2SM_{H^\pm}^2 + 2M_{H^\pm}^2 M_W^2$$

$$- M_W^4 - M_{H^\pm}^4\Big]\Big\}\Big\} \tag{164}$$

$$\boxed{\frac{d\sigma_{II}}{dt}(q\bar q\to H^+W^-) = \frac{G_F^2}{48\pi S}\cdot\frac{1}{(S-4m_q^2)}\{\ \}} \tag{165}$$

with $q = d, s, b$; $i, j = u, c, t$

$$\boxed{\frac{g^4}{96M_W^4} = \frac{G_F^2}{3}}$$

For m_q small :

$$\overline{|M_{II}|^2} = \frac{2G_F^2}{3}S\Big\{\frac12\lambda(S,M_W^2,M_{H^\pm}^2)\,m_q^2\big(|C_{A_b}|^2+|C_{H_b}|^2\big) + \sum_{i,j}V_{iq}V_{jq}^*\Big\{m_q^2 C_{U1i}C_{U1j}\Big[U^2 + \frac{1}{2S}\lambda M_W^2\sin^2\theta\Big]$$

$$+ \big(m_j^2 C_{U1i}C_{U2j} + m_i^2 C_{U2i}C_{U1j}\big)\frac{m_q^2}{S}\big(2M_W^4 - UM_W^2 - U^2\big) + m_i^2 m_j^2 C_{U2i}C_{U2j}\Big[2M_W^2 + \frac{1}{4S}\sin^2\theta\,\lambda\Big]\Big\}$$

$$- \frac14\frac{m_q^2}{S}\Big[C_{H_b}^*\sum_i V_{iq}^* + C_{H_b}\sum_i V_{iq}\Big]\Big\{C_{U1i}\Big[-2SU^2 + 2SM_{H^\pm}^2 M_W^2 - \frac12\sin^2\theta\,\lambda S\Big] + 2m_i^2 C_{U2i}\Big[M_W^4$$

$$- 4UM_W^2 - S^2 + 2SM_{H^\pm}^2 + 2U^2 - M_{H^\pm}^4 + 2M_{H^\pm}^2M_W^2 + \frac12\sin^2\theta\,\lambda\Big]\Big\} + \frac14\frac{m_q^2}{S}\Big[C_{A_b}^*\sum_iV_{iq}^* + C_{A_b}\sum_i V_{iq}\Big]\times$$

$$\times\Big\{C_{U1i}\Big[-2SM_{H^\pm}^2M_W^2 + 2SU^2 + \frac12\sin^2\theta\,\lambda S\Big] + 2m_i^2C_{U2i}\Big[2SM_W^2 - S^2 + 2SM_{H^\pm}^2 + 2M_{H^\pm}^2M_W^2 - M_W^4$$

$$- M_{H^\pm}^4\Big]\Big\}\Big\} \tag{166}$$

ok

for $q = b$ and $i, j = t$

$$\overline{|M_{II}|^2} = \frac{2}{3} G_F^2 s \Big\{ \frac{1}{2} \lambda (s, M_W^2, M_{H^\pm}^2) m_b^2 (|C_{A_b}|^2 + |C_{H_b}|^2) + m_b^2 C_{V1t}^2 (U^2 + \frac{\lambda}{2s} M_W^2 \sin^2\theta)$$
$$+ m_t^4 C_{V2t}^2 (2 M_W^2 + \frac{\lambda}{4s} \sin^2\theta) - \frac{m_b^2}{2s} Re(C_{H_b}) \Big[C_{V1t} (-2sU^2 + 2s M_{H^\pm}^2 M_W^2 - \frac{1}{2} \lambda s \sin^2\theta)\Big]$$
$$+ \frac{m_b^2}{2s} Re(C_{A_b}) \Big[C_{V1t} (-2s M_{H^\pm}^2 M_W^2 + 2sU^2 + \frac{1}{2} \lambda s \sin^2\theta)\Big] \Big\}$$

$$\boxed{\begin{aligned}\overline{|M_{II}|^2} = \frac{2}{3} G_F^2 s \Big\{ &\frac{1}{2} \lambda (s, M_W^2, M_{H^\pm}^2) m_b^2 (|C_{A_b}|^2 + |C_{H_b}|^2) + m_b^2 C_{V1t}^2 (U^2 + \frac{\lambda}{2s} M_W^2 \sin^2\theta) \\ &+ m_t^4 C_{V2t}^2 (2 M_W^2 + \frac{\lambda}{4s} \sin^2\theta) - \frac{m_b^2}{2} C_{V1t} (-2U^2 + 2 M_{H^\pm}^2 M_W^2 - \frac{1}{2} \lambda \sin^2\theta) Re(C_{H_b} + C_{A_b}) \Big\}\end{aligned}}$$

(167)

$$\boxed{\begin{aligned}\frac{d\sigma_{II}}{dt}(b\bar{b} \to H^+ W^-) = \frac{G_F^2}{24\pi s} \Big\{ &\frac{1}{2} \lambda (s, M_W^2, M_{H^\pm}^2) m_b^2 (|C_{A_b}|^2 + |C_{H_b}|^2) + m_b^2 C_{V1t}^2 (U^2 + \frac{\lambda}{2s} M_W^2 \sin^2\theta) \\ &+ m_t^4 C_{V1t}^2 (2 M_W^2 + \frac{\lambda}{4s} \sin^2\theta) - \frac{m_b^2}{2} C_{V1t} (-2U^2 + 2 M_{H^\pm}^2 M_W^2 - \frac{1}{2} \lambda \sin^2\theta) Re(C_{H_b} + C_{A_b}) \Big\}\end{aligned}}$$

(168)

ok

(168) can also be obtained replacing $t \to U$ in the rightside of ((21)

III)

$$\boxed{|M_t^H|^2 = \frac{g^4 m_q^2 |C_{H_t}|^2}{8 M_W^4} \lambda(s, M_W^2, M_{H^\pm}^2)(s-4m_q^2)} \qquad (169)$$

$$\boxed{|M_{3_t}|^2 = \frac{g^4 m_q^2 |C_{A_t}|^2 s}{8 M_W^4} \lambda(s, M_W^2, M_{H^\pm}^2)} \qquad (170)$$

$$\boxed{M_t^{H+} M_{3_t} + M_{3_t}^+ M_t^H = 0} \qquad (171)$$

$$M_{4III} = -\frac{g^2}{4M_W} \varepsilon^*_{\mu 4} \sum_i V_{qi} \bar{v}_2 \left[m_q C_{U_2 i} \not{P}_i + m_i^2 C_{U_1 i} \right] \gamma^\mu (1-\gamma^5) v_1$$

that is the same expression (40a) for M_{4II} if we change $v_1 \leftrightarrow v_2$ and $V_{iq}^* \to V_{qi}$
($V_{iq} \to V_{qi}^*$)

$$\Rightarrow |M_{4III}|^2 = \frac{g^4}{4M_W^4} \sum_{i,j} V_{qi}^* V_{qj} \Big\{ m_q^2 C_{U_2 i} C_{U_2 j} \Big[-2U m_q^2 (M_{H^\pm}^2 + 2M_W^2) + sU^2 + \frac{2M_W^2}{s} (m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4$$
$$- 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4} \sin^2\theta \, \lambda (s-4m_q^2)) + \frac{m_q^2}{s} (m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4} \sin^2\theta \, \lambda (s-4m_q^2)) \Big]$$
$$+ m_q^2 (m_j^2 C_{U_2 i} C_{U_1 j} + m_i^2 C_{U_1 i} C_{U_2 j}) (2M_W^4 - UM_W^2 - m_q^2 M_W^2 - m_q^4 + 2U m_q^2 - U^2) + m_i^2 m_j^2 C_{U_1 i} C_{U_1 j}$$
$$\Big[2s M_W^2 - 4m_q^2 M_W^2 + s m_q^2 - 2m_q^2 M_{H^\pm}^2 + \frac{1}{s} (m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4} \sin^2\theta \, \lambda (s-4m_q^2)) \Big] \Big\}$$

OK.

$$M_t^{H+} M_{4III} + M_{4III} M_t^{H+} = \frac{g^4 m_q^2}{16 M_W^4} \Big[C_{H_t}^* \sum_i V_{qi} + C_{H_t} \sum_i V_{qi}^* \Big] \Big\{ C_{U_2 i} \Big[-2sU^2 + 2s m_q^4 - 4s m_q^2 M_W^2 + 2s M_{H^\pm}^2 M_W^2 \qquad (172)$$
$$+ 4m_q^2 M_W^4 - 4m_q^2 M_{H^\pm}^2 M_W^2 + 4m_q^4 M_W^2 - 4m_q^2 U M_W^2 - 4m_q^4 M_{H^\pm}^2 + 4U m_q^2 M_{H^\pm}^2 - \frac{1}{2} \sin^2\theta \, \lambda (s-4m_q^2) \Big]$$
$$+ 2m_i^2 C_{U_1 i} \Big[M_W^4 - 4U M_W^2 - s^2 + 2s M_{H^\pm}^2 + 2s m_q^2 - 4U m_q^2 + 2U^2 - M_{H^\pm}^4 - 4m_q^2 M_{H^\pm}^2 + 2m_q^4 + 2M_{H^\pm}^2 M_W^2$$
$$+ \frac{2}{s} \Big[m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \frac{1}{4} \sin^2\theta \, \lambda (s-4m_q^2) \Big] \Big] \Big\}$$

OK (173)

$$\boxed{\begin{aligned} M_{3_t}^+ M_{4III} + M_{4III}^+ M_{3_t} = \frac{g^4}{16 M_W^4} m_q^2 \Big[C_{A_t}^* \sum_i V_{qi} + C_{A_t} \sum_i V_{qi}^* \Big] \Big\{ C_{U_2 i} \Big[-2s M_{H^\pm}^2 M_W^2 - 4Us m_q^2 + 2sU^2 + 2s m_q^4 \\ + \frac{1}{2} \sin^2\theta \, \lambda (s-4m_q^2) \Big] + 2m_i^2 C_{U_1 i} \Big[2s M_W^2 - s^2 + 2s M_{H^\pm}^2 + 2M_{H^\pm}^2 M_W^2 - M_W^4 - M_{H^\pm}^4 \Big] \Big\} \end{aligned}}$$

OK (174)

$$\overline{|M_{III}|^2} = \frac{1}{12}\Big[|M_t^H|^2 + |M_{3_t}|^2 + |M_{4III}|^2 + (M_t^{H+} M_{3_t} + M_{3_t}^{+} M_t^H) + (M_t^{H+} M_{4III} + M_{4III}^{+} M_t^H) + (M_{3_t}^{+} M_{4III} + M_{4III}^{+} M_{3_t})\Big] \qquad (175)$$

Implies :

$$\begin{aligned}
\overline{|M_{III}|^2} = \frac{g^4}{96 M_W^4} \Big\{ & m_q^2 \lambda(s, M_W^2, M_{H^\pm}^2)\, |C_{H_t}|^2 (s-4m_q^2) + m_q^2 |C_{A_t}|^2 s\, \lambda(s, M_W^2, M_{H^\pm}^2) + 2\sum_{i,j} V_{qi}^* V_{qj} \Big\{ m_q^2 \times \\
& \times C_{U2i} C_{U2j} \Big[-2u m_q^2 (M_{H^\pm}^2 + 2M_W^2) + su^2 + 2\frac{M_W^2}{s}\big(m_q^2 M_{H^\pm} + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac14 \sin^2\theta\, \lambda (s-4m_q^2)\big) \\
& + \frac{m_q^2}{s}\big(m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac14 \sin^2\theta\, \lambda (s-4m_q^2)\big)\Big] + m_q^2 (m_j^2 C_{U2i} C_{U1j} + m_i^2 C_{U1i} C_{U2j})(2M_W^4 \\
& - uM_W^2 - m_q^2 M_W^2 - m_q^4 + 2u m_q^2 - u^2) + m_i^2 m_j^2 C_{U1i} C_{U1j} \Big[2sM_W^2 - 4m_q^2 M_W^2 + s m_q^2 - 2m_q^2 M_{H^\pm}^2 + \frac1s (m_q^2 M_{H^\pm}^4 \\
& + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac14 \sin^2\theta\, \lambda (s-4m_q^2))\Big]\Big\} + \frac12 m_q^2 \Big[C_{H_t}^* \sum_i V_{qi} + C_{H_t} \sum_i V_{qi}^* \Big] \Big\{ C_{U2i} \Big[-2su^2 + 2s m_q^4 \\
& - 4s m_q^2 M_W^2 + 2s M_{H^\pm}^2 M_W^2 + 4m_q^2 M_W^4 - 4m_q^2 M_{H^\pm}^2 M_W^2 + 4m_q^4 M_W^2 - 4m_q^2 u M_W^2 - 4m_q^4 M_{H^\pm}^2 + 4u m_q^2 M_{H^\pm}^2 \\
& - \tfrac12 \sin^2\theta\, \lambda (s-4m_q^2)\Big] + 2m_i^2 C_{U1i} \Big[M_W^4 - 4uM_W^2 - s^2 + 2sM_{H^\pm}^2 + 2sm_q^2 - 4um_q^2 + 2u^2 - M_{H^\pm}^4 - 4m_q^2 M_{H^\pm}^2 + 2m_q^4 \\
& + 2M_{H^\pm}^2 M_W^2 + \frac2s \Big[m_q^2 M_{H^\pm}^4 + m_q^2 M_W^4 - 2m_q^2 M_{H^\pm}^2 M_W^2 + \tfrac14 \sin^2\theta\, \lambda (s-4m_q^2)\Big]\Big]\Big\} + \frac12 m_q^2 \Big[C_{A_t}^* \sum_i V_{qi} + C_{A_t} \sum_i V_{qi}^* \Big] \times \\
& \times \Big\{ C_{U2i} \Big[-2sM_{H^\pm}^2 M_W^2 - 4us m_q^2 + 2su^2 + 2s m_q^4 + \tfrac12 \sin^2\theta\, \lambda (s-4m_q^2)\Big] + 2m_i^2 C_{U1i} \Big[2sM_W^2 - s^2 + 2sM_{H^\pm}^2 \\
& + 2M_{H^\pm}^2 M_W^2 - M_W^4 - M_{H^\pm}^4 \Big]\Big\}\Big\}
\end{aligned}$$

$$(176)$$

$$\boxed{\frac{d\sigma_{III}}{dt}(q\bar{q} \to H^- W^+) = \frac{G_F^2}{48\pi s} \cdot \frac{1}{(s-4m_q^2)} \overset{\uparrow}{\{\ \}}} \qquad (177)$$

with $q = u, c$; $i, j = d, s, b$.

$$\frac{g^4}{96 M_W^4} = \frac{G_F^2}{3}$$

For m_q small:

$$\boxed{\begin{aligned}
\overline{|M_{III}|^2} = \frac{2G_F^2}{3} s \Big\{ & \frac12 m_q^2 \lambda\, (|C_{H_t}|^2 + |C_{A_t}|^2) + \sum_{i,j} V_{qi}^* V_{qj} \Big\{ m_q^2 C_{U2i} C_{U2j} \Big[u^2 + \frac{1}{2s} M_W^2 \sin^2\theta\, \lambda \Big] \\
& + \frac{m_q^2}{s} (m_j^2 C_{U2i} C_{U1j} + m_i^2 C_{U1i} C_{U2j})(2M_W^4 - uM_W^2 - u^2) + m_i^2 m_j^2 C_{U1i} C_{U1j} \big(2M_W^2 + \tfrac{1}{4s} \lambda \sin^2\theta\big)\Big\} \\
& + \frac14 \frac{m_q^2}{s} \Big[C_{H_t}^* \sum_i V_{qi} + C_{H_t} \sum_i V_{qi}^* \Big] \Big\{ C_{U2i} \Big[-2su^2 + 2sM_{H^\pm}^2 M_W^2 - \tfrac12 \sin^2\theta\, \lambda s \Big] + 2m_i^2 C_{U1i} \Big[M_W^4 - 4uM_W^2 - s^2 \\
& + 2sM_{H^\pm}^2 + 2u^2 - M_{H^\pm}^4 + 2M_{H^\pm}^2 M_W^2 + \tfrac12 \lambda \sin^2\theta \Big]\Big\} + \frac{m_q^2}{4s} \Big[C_{A_t}^* \sum_i V_{qi} + C_{A_t} \sum_i V_{qi}^* \Big] \Big\{ C_{U2i} \Big[-2sM_{H^\pm}^2 M_W^2 + 2su^2 \\
& + \tfrac12 \lambda s \sin^2\theta \Big] + 2m_i^2 C_{U1i} \Big[2sM_W^2 - s^2 + 2sM_{H^\pm}^2 + 2M_{H^\pm}^2 M_W^2 - M_W^4 - M_{H^\pm}^4 \Big]\Big\}\Big\}
\end{aligned}}$$

OK (178)

The only important terms give us:

$$\begin{aligned}|\overline{M_{III}}|^2 = {} & \frac{2}{3} G_F^2 s \{ \frac{1}{2} m_q^2 \lambda (|C_{H_t}|^2 + |C_{A_t}|^2) + \sum_{i,j} V_{qi}^* V_{qj} [m_q^2 C_{U_{2i}} C_{U_{2j}} (U^2 + \frac{\lambda}{2s} M_W^2 \sin^2\theta) \\ & + m_i^2 m_j^2 C_{U_{1i}} C_{U_{1j}} (2M_W^2 + \frac{\lambda}{4s} \sin^2\theta)] + \frac{m_c^2}{2} \mathrm{Re}\,(C_{H_t} - C_{A_t}) \sum_i V_{qi} C_{U_{2i}} (-2U^2 + 2M_{H^\pm}^2 M_W^2 - \frac{1}{2}\sin^2\theta\,\lambda) \\ & + m_c^2 \mathrm{Re}\, C_{H_t} \sum_i V_{qi} m_i^2 C_{U_{1i}} (-s + 2M_{H^\pm}^2 + \frac{\lambda}{2s}\sin^2\theta) + m_c^2 \mathrm{Re}\, C_{A_t} \sum_i V_{qi} m_i^2 C_{U_{1i}} (2M_W^2 - s + 2M_{H^\pm}^2)\end{aligned}$$

$$\begin{aligned}\frac{d\sigma_{III}}{dt}(c\bar{c} \to H^- W^+) = {} & \frac{G_F^2}{48\pi s} \{ m_c^2 \lambda (|C_{H_t}|^2 + |C_{A_t}|^2) + 2 \sum_{i,j} V_{ci}^* V_{cj}^* [m_c^2 C_{U_{2i}} C_{U_{2j}} (U^2 + \frac{\lambda}{2s} M_W^2 \sin^2\theta) \\ & + m_i^2 m_j^2 C_{U_{1i}} C_{U_{1j}} (2M_W^2 + \frac{\lambda}{4s}\sin^2\theta)] + m_c^2 \mathrm{Re}\,(C_{H_t} - C_{A_t})(-2U^2 + 2M_{H^\pm}^2 M_W^2 - \frac{1}{2}\lambda\sin^2\theta) \\ & \sum_i V_{qi} C_{U_{2i}} + 2m_c^2 \mathrm{Re}\,(C_{H_t} + C_{A_t}) \sum_i V_{qi} m_i^2 C_{U_{1i}} (-s + 2M_{H^\pm}^2) + 4M_W^2 m_c^2 \mathrm{Re}\, C_{A_t} \sum_i V_{qi} m_i^2 C_{U_{1i}} \\ & + 2\frac{\lambda}{2s}\sin^2\theta\, m_c^2 \mathrm{Re}\, C_{H_t} \sum_i V_{ci} m_i^2 C_{U_{1i}} \}\end{aligned}$$

(179) could be obtained from (131) replacing t ⟷ U

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{\lambda^{1/2}}{4\pi}\left(\frac{d\sigma}{dt}\right)$$

$$\boxed{\begin{aligned}\frac{d\sigma_{III}}{dt}(c\bar{c} \to H^- W^+) = {} & \frac{G_F^2}{48\pi s} \{ m_c^2 \lambda (|C_{H_t}|^2 + |C_{A_t}|^2) + 2m_c^2 \sum_{i,j} V_{ci}^* V_{cj} C_{U_{2i}} C_{U_{2j}} (U^2 + \frac{\lambda}{2s} M_W^2 \sin^2\theta) \\ & + 2|V_{cb}|^2 m_b^4 C_{U_{1b}}^2 (2M_W^2 + \frac{\lambda}{4s}\sin^2\theta)] + m_c^2 \mathrm{Re}\,(C_{H_t} - C_{A_t})(-2U^2 + 2M_{H^\pm}^2 M_W^2 - \frac{1}{2}\lambda\sin^2\theta \\ & \times \sum_i V_{ci} C_{U_{2i}} + 2m_c^2 \mathrm{Re}\,(C_{H_t} + C_{A_t})(-s + 2M_{H^\pm}^2) V_{cb} m_b^2 C_{U_{1b}} + 4m_c^2 M_W^2 \mathrm{Re}\, C_{A_t} \cdot \\ & V_{cb} m_b^2 C_{U_{1b}} + \frac{\lambda}{s}\sin^2\theta\, m_c^2 \mathrm{Re}\, C_{H_t} V_{cb} m_b^2 C_{U_{1b}} \}\end{aligned}}$$

OK (179)

$q\bar q \to H^\mp W^\pm$

$$\frac{d\sigma_I}{d\hat t}(q\bar q \to H^- W^+) = \frac{G_F^2}{48\pi \hat s^2} \Big\{ m_q^2 |C_{Hb}|^2 \Lambda(\hat s, M_W^2, \cancel{M_{H^\pm}^2}) \hat s + m_q^2 |C_{\cancel{A}b}|^2 \hat s \lambda(\hat s, M_W^2, M_{H^\pm}^2) +$$

$$+ 2\sum_{i,j} V_{iq} V_{jq}^* \Big\{ m_q^2 C_{t_1 i} C_{t_1 j} \Big[\hat s \hat t^2 + \frac{\cancel{2} M_W^2}{\cancel{\hat s}} \frac{1}{\cancel{4}\,2} \sin^2\theta \lambda \cancel{\hat s} \Big] + (m_j^2 C_{t_1 i} C_{t_2 j} +$$

$$+ m_i^2 C_{t_2 i} C_{t_1 j}) m_q^2 (2M_W^4 - \hat t M_W^2 - \hat t^2) + m_i^2 m_j^2 C_{t_2 i} C_{t_2 j} \Big[2\hat s M_W^2 - 4\cancel{m_q^2} M_W^2 + \hat s \cancel{m_q^2}$$

$$- 2\cancel{m_q^2 M_{H^\pm}^2} + \frac{1}{3}(\cancel{m_q^2 M_{H^\pm}^4} + \cancel{m_q^2 M_W^4} - 2m_q^2 \cancel{M_{H^\pm}^2} M_W^2 + \frac{1}{4}\sin^2\theta \lambda \hat s) \Big] \Big\}$$

$$- \frac{1}{2} m_q^2 \sum_i (C_{Hb}^* V_{iq} + C_{Hb} V_{iq}^*) \Big\{ C_{t_1 i} \Big[-2\hat s \hat t^2 + 2\hat s M_{H^\pm}^2 M_W^2 - \frac{1}{2}\sin^2\theta \lambda \hat s \Big]$$

$$+ 2\frac{m_i^2}{\hat s} \hat s C_{t_2 i} \Big\{ M_W^4 - 4\hat t M_W^2 - \hat s^2 + 2\hat s M_{H^\pm}^2 + 2\hat t^2 - M_{H^\pm}^4 + 2M_{H^\pm}^2 M_W^2 + \frac{2}{\hat s}\frac{1}{4}\sin^2\theta \lambda \hat s \Big\}\Big\}$$

$$+ \frac{1}{2} m_q^2 \sum_i (V_{iq} C_{Ab}^* + V_{iq}^* C_{Ab}) \Big\{ C_{t_1 i} \Big[-2\hat s M_{H^\pm}^2 M_W^2 + 2\hat s \hat t^2 + \frac{1}{2}\sin^2\theta \lambda \hat s \Big]$$

$$+ 2\frac{m_i^2}{\hat s} C_{t_2 i} \Big[2\hat s M_W^2 + \cancel{2M_W^2 M_{H^\pm}^2} + 2\hat s M_{H^\pm}^2 - \hat s^2 - M_W^4 - M_{H^\pm}^4 \Big] \Big\}$$

for q : d, s, b ; $i,j = u, c, t$

$$\frac{d\sigma_I}{d\hat t}(q\bar q \to H^- W^+) = \frac{G_F^2}{48\pi \hat s^2} \Big\{ m_q^2 \hat s \lambda(\hat s, M_W^2, M_{H^\pm}^2) \big[|C_{Hb}|^2 + |C_{Ab}|^2 \big] + 2\sum_{i,j} V_{iq} V_{jq}^* \Big\{$$

$$m_q^2 C_{t_1 i} C_{t_1 j} \hat s \Big[\hat t^2 + \frac{\lambda \sin^2\theta M_W^2}{2\hat s} \Big] + \hat s (m_j^2 C_{t_1 i} C_{t_2 j} + m_i^2 C_{t_2 i} C_{t_1 j}) \frac{m_q^2}{\hat s} (2M_W^4 - \hat t M_W^2$$

$$- \hat t^2) + m_i^2 m_j^2 C_{t_2 i} C_{t_2 j} \Big[2\hat s M_W^2 + \frac{1}{4}\lambda \sin^2\theta \Big] \Big\} - \frac{1}{2} m_q^2 \sum_i (C_{Hb}^* V_{iq} + C_{Hb} V_{iq}^*)$$

$$C_{t_1 i} \Big[-2\hat s \hat t^2 + 2\hat s M_{H^\pm}^2 M_W^2 - \frac{1}{2}\sin^2\theta \lambda \hat s \Big] + \frac{1}{2} m_q^2 \sum_i (V_{iq} C_{Ab}^* + V_{iq}^* C_{Ab}) C_{t_1 i} \cdot$$

$$\Big[-2\hat s M_{H^\pm}^2 M_W^2 + 2\hat s \hat t^2 + \frac{1}{2}\sin^2\theta \lambda \hat s \Big] \Big\}$$

$$\frac{d\sigma_I}{d\hat t}(q\bar q \to H^- W^+) = \frac{G_F^2}{48\pi \hat s} \Big\{ m_q^2 \lambda(\hat s, M_W^2, M_{H^\pm}^2) \big[|C_{Hb}|^2 + |C_{Ab}|^2 \big] + 2\sum_{i,j} V_{iq} V_{jq}^* \cdot$$

$$\cdot \Big\{ m_q^2 C_{t_1 i} C_{t_1 j} \Big(\hat t^2 + \frac{\lambda \sin^2\theta M_W^2}{2\hat s} \Big) + m_i^2 m_j^2 C_{t_2 i} C_{t_2 j} \Big(2M_W^2 + \frac{\lambda \sin^2\theta}{4\hat s} \Big) \Big\}$$

$$+ \frac{1}{2} m_q^2 \Big[-2M_{H^\pm}^2 M_W^2 + 2\hat t^2 + \frac{1}{2}\lambda \sin^2\theta \Big] \Big[\sum_i (\underbrace{V_{iq} C_{Ab}^* + V_{iq}^* C_{Ab}}_{2 C_{Ab} \mathrm{Re} V_{iq}}) C_{t_1 i}$$

$$+ \sum_i (\underbrace{C_{Hb}^* V_{iq} + C_{Hb} V_{iq}^*}_{2 C_{Hb} \mathrm{Re} V_{iq}}) C_{t_1 i} \Big] \Big\}$$

$$\boxed{\begin{aligned}\frac{d\sigma_I}{d\hat{t}}\,(q\bar{q}\to H^-W^+) = \frac{G_F^2}{48\pi\hat{s}}\Big\{ & m_q^2\,\lambda(\hat{s}, M_W^2, M_{H^\pm}^2)\,[(C_{Hb})^2+(C_{Ab})^2] + 2\sum_{\substack{i,j\\ u,c,t}} V_{iq}V_{jq}^* \,\cdot \\ & \cdot\Big[m_q^2\,C_{t_1 i}C_{t_1 j}\Big(\hat{t}^2+\frac{\lambda\sin^2\theta\,M_W^2}{2\hat{s}}\Big) + m_i^2 m_j^2\,C_{t_2 i}C_{t_2 j}\Big(2M_W^2+\frac{\lambda\sin^2\theta}{4\hat{s}}\Big)\Big] \\ & + m_q^2\Big[-2M_{H^\pm}^2M_W^2+2\hat{t}^2+\frac{1}{2}\lambda\sin^2\theta\Big](C_{Hb}+C_{Ab})\sum_i(\mathrm{Re}\,V_{iq})\,C_{t_1 i}\Big\}\end{aligned}}$$

$\uparrow$ <u>final</u>

$q = d, s, b$

$i; j = u, c, t$

$$\boxed{\begin{aligned}\frac{d\sigma_I}{d\hat{t}}\,(q\bar{q}\to H^-W^+) = \frac{G_F^2}{48\pi\hat{s}}\Big\{ & m_q^2\,\lambda(\hat{s}, M_W^2, M_{H^\pm}^2)\,[(C_{Hb})^2+(C_{Ab})^2] + 2\,|V_{tq}|^2 \\ & \cdot\Big[m_q^2\,C_{t_1 t}^2\Big(\hat{t}^2+\frac{\lambda\sin^2\theta\,M_W^2}{2\hat{s}}\Big) + m_t^2\,C_{t_2 t}^2\Big(2M_W^2+\frac{\lambda\sin^2\theta}{4\hat{s}}\Big)\Big] \\ & + m_q^2\Big[-2M_{H^\pm}^2M_W^2+2\hat{t}^2+\frac{1}{2}\lambda\sin^2\theta\Big](C_{Hb}+C_{Ab})\,\mathrm{Re}(V_{tq})\,C_{t_1 t}\Big\}\end{aligned}}$$

$C_{Hb} = \hat{C}_H = C_H$ replacing $s\to\hat{s}$

$C_{Ab} = \hat{C}_A = C_A$ " "

For $q = u, c$; $i, j = d, s, b$

(In the paper I have written $\frac{d\sigma_{II}}{d\hat t}$)

$$\frac{d\sigma_{III}}{d\hat t}(q\bar q \to H^- W^+) = \frac{G_F^2}{48\pi \hat s}\Big\{ m_q^2 \lambda \,(C_{Ht}^2 + C_{At}^2) + 2\sum_{i,j} V_{qi}^* V_{qj} \{ m_q^2 C_{U2i} C_{U2j} \cdot$$

$$\cdot [\hat u^2 + \frac{1}{2\hat s} M_W^2 \sin^2\theta \lambda] + m_i^2 m_j^2 C_{U1i} C_{U1j} (2M_W^2 + \frac{1}{4\hat s}\lambda \sin^2\theta)\}$$

$$+ \frac{1}{\cancel{2}} m_q^2 [\, C_{Ht} \cancel{2} \sum_i Re(V_{qi}) C_{U2i} (-2\hat u^2 + 2M_{H^\pm}^2 M_W^2 - \frac{1}{2}\lambda \sin^2\theta)]$$

$$+ \frac{1}{\cancel{2}} m_q^2 [\, C_{At} \cancel{2} \sum_i Re(V_{qi}) C_{U2i} (2\hat u^2 - 2M_{H^\pm}^2 M_W^2 + \frac{1}{2}\lambda \sin^2\theta)]\Big\}$$

$$\frac{d\sigma_{III}}{d\hat t}(q\bar q \to H^- W^+) = \frac{G_F^2}{48\pi \hat s}\Big\{ m_q^2 \lambda (C_{Ht}^2 + C_{At}^2) + 2\sum_{i,j} V_{qi}^* V_{qj} \{ m_q^2 C_{U2i} C_{U2j} \cdot$$

$$\cdot [\hat u^2 + \frac{1}{2\hat s} M_W^2 \sin^2\theta \lambda] + m_i^2 m_j^2 C_{U1i} C_{U1j} (2M_W^2 + \frac{1}{4\hat s}\lambda \sin^2\theta)\}$$

$$+ m_q^2 (\sum_i Re(V_{qi}) C_{U2i}) (C_{At} - C_{Ht}) (2\hat u^2 - 2M_{H^\pm}^2 M_W^2 + \frac{1}{2}\lambda \sin^2\theta)\Big\}$$

$(\lambda = \lambda(\hat s, M_W^2, M_{H^\pm}^2))$

$$\frac{d\sigma_{IV}}{d\hat t}(q\bar q \to H^+ W^-) = \frac{G_F^2}{48\pi \hat s^2}\Big\{ m_q^2 (C_{Ht})^2 \lambda \hat s + m_q^2 (C_{At})^2 \lambda \hat s + 2\sum_{i,j} V_{qi}^* V_{qj} \{ m_q^2$$

$$C_{t2i} C_{t2j} [\hat s \hat t^2 + \frac{M_W^2}{2}\lambda \sin^2\theta \frac{\hat s}{\hat s}] + m_q^2 (2M_W^4 - \hat t M_W^2 - \hat t^2)(m_j^2 C_{t2i} C_{t1j} + m_i^2 C_{t1i} C_{t2j})$$

$$+ m_i^2 m_j^2 C_{t1i} C_{t1j} [2\hat s M_W^2 + \frac{1}{4}\lambda \sin^2\theta]\} + \frac{1}{\cancel{2}} m_q^2 C_{Ht} \sum_i \cancel{2} Re(V_{qi}) \{ C_{t2i} [-2\hat s \hat t^2$$

$$+ 2\hat s M_{H^\pm}^2 M_W^2 - \frac{1}{2}\lambda \sin^2\theta \hat s] + 2m_i^2 C_{t1i} [M_W^4 - 4\hat t M_W^2 - \hat s^2 \cancel{+ 2\hat s M_{H^\pm}^2} + 2\hat t^2 - M_{H^\pm}^4 + 2M_{H^\pm}^2 M_W^2$$

$$+ \frac{1}{2}\lambda \sin^2\theta] + \frac{1}{\cancel{2}} m_q^2 C_{At} \sum_i \cancel{2} Re(V_{qi}) \{ C_{t2i} [-2\hat s M_{H^\pm}^2 M_W^2 + 2\hat s \hat t^2 + \frac{1}{2}\lambda \sin^2\theta \hat s]$$

$$+ 2m_i^2 C_{t1i} [2\hat s M_W^2 + 2M_W^2 \cancel{M_{H^\pm}^2} + 2\hat s M_{H^\pm}^2 - \hat s^2 - M_W^4 - M_{H^\pm}^4]\}\Big\}$$

$$\frac{d\sigma_{IV}}{d\hat t}(q\bar q \to H^+ W^-) = \frac{G_F^2}{48\pi \hat s}\Big\{ m_q^2 \lambda (C_{Ht}^2 + C_{At}^2) + 2\sum_{i,j} V_{qi}^* V_{qj} \{ m_q^2 C_{t2i} C_{t2j} \cdot$$

$$\cdot [\hat t^2 + \frac{M_W^2 \lambda \sin^2\theta}{2\hat s}] + m_i^2 m_j^2 C_{t1i} C_{t1j} [2M_W^2 + \frac{1}{4}\frac{\lambda \sin^2\theta}{\hat s}] + m_q^2 (\sum_i Re(V_{qi}) C_{t2i})$$

$$(C_{At} - C_{Ht}) [2\hat t^2 - 2M_{H^\pm}^2 M_W^2 + \frac{1}{2}\lambda \sin^2\theta]\Big\}$$

---

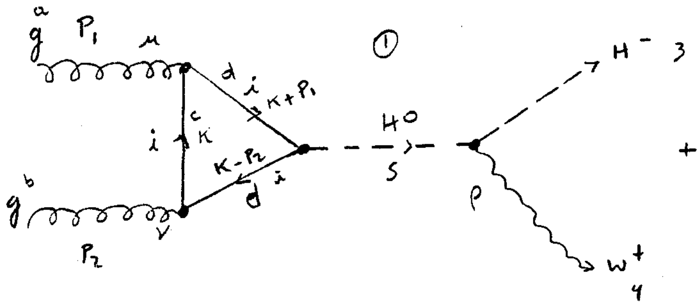
for $q = u, c$; $i, j = d, s, b$

$\frac{d\sigma_{IV}}{d\hat t}(q\bar q \to H^+ W^-)$ is obtained from $\frac{d\sigma_{III}}{d\hat t}(q\bar q \to H^- W^+)$ replacing in the last expression

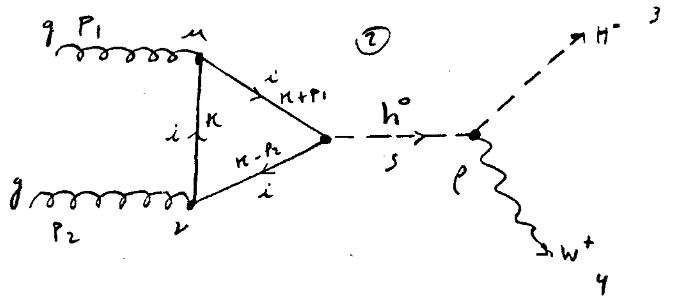
$\hat u \to \hat t$

$gg \to H^- W^+$ $\qquad$ $i = u, c, t, d, s, b$

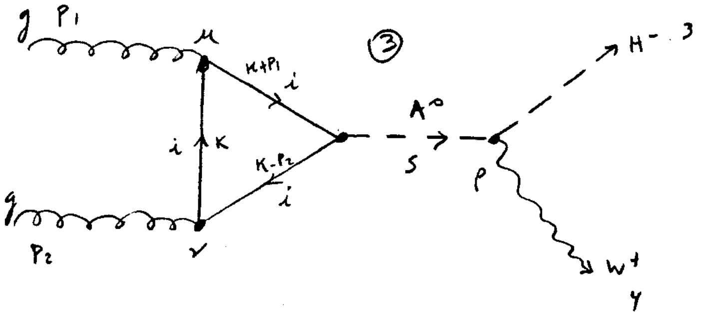
$\longrightarrow$ time



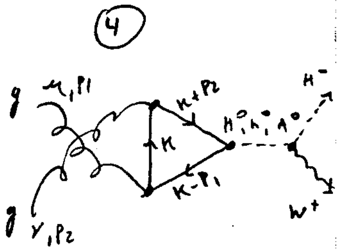
+



+



+



$-i(M_{4a} + M_{5a} + M_{6a})$

For $i = d, s, b$:

$$-iM_{1a} = +\varepsilon^*_{\rho 4}\frac{ig}{2}\sin(\alpha-\beta)\,(P_1+P_2+P_3)^\rho\,\frac{i}{s - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}}\left(-\frac{ig\cos\alpha}{2M_W\cos\beta}\right)\sum_{i=d,s,b} m_i\left\{\int\frac{d^dk}{(2\pi)^d}\mathrm{Tr}\Big(\big(-ig_s\gamma^\mu T^a_{dc}\big)\right.$$

$$\left.\frac{i}{\not{k}+\not{p}_1-m_i}\;\frac{i}{\not{k}-\not{p}_2-m_i}\big(-ig_s\gamma^\nu T^b_{cd}\big)\frac{i}{\not{k}-m_i}\Big)\right\}\varepsilon_{\mu_1}\varepsilon_{\nu_2}\,\mu_1^{4-d}\,\mu_2^{(4-d)\frac{1}{2}} \qquad (226)$$

$\Lambda^d = \Lambda^\delta\,\Lambda^{d-1}\,\Lambda^{\frac{d}{2}-1} \to\ \succ\!\text{-}^{H^0}$

μ^δ = mass parameter

$\cancel{d} = \delta + \cancel{d} - 2 + \frac{d}{2}$

$\delta = 2 - \frac{d}{2} = \frac{4-d}{2}$

$$-iM_{2a} = \varepsilon^*_{\rho 4}\frac{ig}{2}\cos(\beta-\alpha)\,(P_1+P_2+P_3)^\rho\,\frac{i}{s - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}}\left(\frac{ig\sin\alpha}{2M_W\cos\beta}\right)\sum_{i=d,s,b} m_i\left\{\int\frac{d^dk}{(2\pi)^d}\mathrm{Tr}\Big(\big(-ig_s\gamma^\mu T^a_{dc}\big)\right.$$

$$\left.\frac{i}{\not{k}+\not{p}_1-m_i}\;\frac{i}{\not{k}-\not{p}_2-m_i}\big(-ig_s\gamma^\nu T^b_{cd}\big)\frac{i}{\not{k}-m_i}\Big)\right\}\varepsilon_{\mu_1}\varepsilon_{\nu_2}\,\mu_1^{(4-d)}\,\mu_2^{\frac{(4-d)}{2}} \qquad (227)$$

$$-iM_{3a} = \varepsilon^*_{\rho 4}\frac{g}{2}(P_1+P_2+P_3)^\rho\,\frac{i}{s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0}}\left(-\frac{g\tan\beta}{2M_W}\right)\sum_{i=d,s,b} m_i\left\{\int\frac{d^dk}{(2\pi)^d}\mathrm{Tr}\Big(\big(-ig_s\gamma^\mu T^a_{dc}\big)\right.$$

$$\left.\frac{i}{\not{k}+\not{p}_1-m_i}\gamma^5\frac{i}{\not{k}-\not{p}_2-m_i}\big(-ig_s\gamma^\nu T^b_{cd}\big)\frac{i}{\not{k}-m_i}\Big)\right\}\varepsilon_{\mu_1}\varepsilon_{\nu_2}\,\mu_1^{(4-d)}\,\mu_2^{\frac{(4-d)}{2}} \qquad (228)$$

$$-iM_{1_a} = -\frac{g^2 g_s^2}{4}\,\frac{\sin(\alpha-\beta)\cos\alpha}{M_W\cos\beta}\,\frac{(p_1+p_2+p_3)^\rho}{s-m_{H^0}^2+im_{H^0}\Gamma_{H^0}}\,\varepsilon^{\kappa}_{p_4}\,\varepsilon_{\mu_1}\,\varepsilon_{\nu_i}\,\mu^{\frac{3}{2}(4-d)}\sum_i m_i\left\{\int\frac{d^dk}{(2\pi)^d}\,\mathrm{Tr}\left(\gamma^\mu(\not{k}+\not{p}_1+m_i)\right.\right.$$

$$\left.\left.(\not{k}-\not{p}_2+m_i)\gamma^\nu(\not{k}+m_i)\right)\right)\cdot\frac{1}{(k+p_1)^2-m_i^2}\cdot\frac{1}{(k-p_2)^2-m_i^2}\cdot\frac{1}{k^2-m_i^2}\,(T^a_{dc}\,T^b_{cd}) \qquad (229)$$

$$\vee$$

$$\mathrm{Tr}(T^aT^b)=\frac{1}{2}\delta^{ab}$$

$$\mathrm{Tr}\left(\gamma^\mu(\not{k}+\not{p}_1+m_i)(\not{k}-\not{p}_2+m_i)\gamma^\nu(\not{k}+m_i)\right)$$

$$=\mathrm{Tr}\left((\gamma^\mu\not{k}+\gamma^\mu\not{p}_1+m_i\gamma^\mu)(\not{k}\gamma^\nu\not{k}+m_i\not{k}\gamma^\nu-\not{p}_2\gamma^\nu\not{k}-m_i\not{p}_2\gamma^\nu+m_i\gamma^\nu\not{k}+m_i^2\gamma^\nu)\right)$$

$$=\mathrm{Tr}(\gamma^\mu\cancel{\not{k}\not{k}}\overset{0}{\gamma^\nu}\not{k})+m_i\mathrm{Tr}(\gamma^\mu\not{k}\not{k}\gamma^\nu)-\mathrm{Tr}(\gamma^\mu\cancel{\not{k}\not{p}_2}\overset{0}{\gamma^\nu}\not{k})-m_i\mathrm{Tr}(\gamma^\mu\not{k}\not{p}_2\gamma^\nu)+m_i\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{k})$$

$$+m_i^2\mathrm{Tr}(\gamma^\mu\cancel{\not{k}}\overset{0}{\gamma^\nu})+m_i\mathrm{Tr}(\gamma^\mu\not{p}_1\not{k}\gamma^\nu)-\mathrm{Tr}(\gamma^\mu\not{p}_1\cancel{\not{p}_2}\overset{0}{\gamma^\nu}\not{k})-m_i\mathrm{Tr}(\gamma^\mu\not{p}_1\not{p}_2\gamma^\nu)+m_i\mathrm{Tr}(\gamma^\mu\not{p}_1\gamma^\nu\not{k})$$

$$+m_i^2\mathrm{Tr}(\gamma^\mu\cancel{\not{p}_1}\overset{0}{\gamma^\nu})+m_i\mathrm{Tr}(\gamma^\mu\not{k}\gamma^\nu\not{k})+m_i^2\mathrm{Tr}(\gamma^\mu\cancel{\not{k}}\overset{0}{\gamma^\nu})-m_i\mathrm{Tr}(\gamma^\mu\not{p}_2\gamma^\nu\not{k})-m_i^2\mathrm{Tr}(\gamma^\mu\cancel{\not{p}_2}\overset{0}{\gamma^\nu})$$

$$+m_i^2\mathrm{Tr}(\gamma^\mu\cancel{\overset{0}{\gamma^\nu}}\not{k})+m_i^3\mathrm{Tr}(\gamma^\mu\gamma^\nu)+\mathrm{Tr}(\gamma^\mu\not{p}_1\cancel{\not{k}\overset{0}{\gamma^\nu}}\not{k})$$

$$=m_i k^2\,4\eta^{\mu\nu}-m_i\mathrm{Tr}\left[\gamma^\mu\not{k}(\gamma^\alpha p_{2\alpha}\gamma^\nu)\right]+8m_i\left[2k^\mu k^\nu-k^2\eta^{\mu\nu}\right]$$

$$+m_i\mathrm{Tr}\left[\gamma^\mu\not{p}_1(\gamma^\alpha k_\alpha\gamma^\nu)\right]-m_i\mathrm{Tr}\left[\gamma^\mu\not{p}_1(\gamma^\alpha p_{2\alpha}\gamma^\nu)\right]$$

$$+m_i4\left[p_1^\mu k^\nu+p_1^\nu k^\mu-(p_1\cdot k)\eta^{\mu\nu}\right]-4m_i\left[p_2^\mu k^\nu+p_2^\nu k^\mu-(p_2\cdot k)\eta^{\mu\nu}\right]$$

$$+4m_i^3\eta^{\mu\nu}$$

$$=4m_i\cancel{k}^2\eta^{\mu\nu}-m_i\mathrm{Tr}\left[\gamma^\mu\not{k}\,p_{2\alpha}(2\eta^{\nu\alpha}-\gamma^\nu\gamma^\alpha)\right]+16m_i k^\mu k^\nu$$

$$-8\cancel{m_i}k^2\eta^{\mu\nu}+m_i\mathrm{Tr}\left[\gamma^\mu\not{p}_1\,k_\alpha(2\eta^{\nu\alpha}-\gamma^\nu\gamma^\alpha)\right]-m_i\mathrm{Tr}\left[\gamma^\mu\not{p}_1\,p_{2\alpha}(2\eta^{\nu\alpha}\right.$$

$$\left.-\gamma^\nu\gamma^\alpha)\right]+4m_i\left[p_1^\mu k^\nu+p_1^\nu k^\mu-(p_1\cdot k)\eta^{\mu\nu}\right]-4m_i\left[p_2^\mu k^\nu+p_2^\nu k^\mu\right.$$

$$\left.-(p_2\cdot k)\eta^{\mu\nu}\right]+4m_i^3\eta^{\mu\nu}$$

$$=-4m_ik^2\eta^{\mu\nu}-2m_ip_2^\nu\mathrm{Tr}(\gamma^\mu\not{k})+16m_ik^\mu k^\nu+m_i\mathrm{Tr}\left[\gamma^\mu\not{k}\gamma^\nu\not{p}_2\right]$$

$$+2m_ik^\nu\mathrm{Tr}(\gamma^\mu\not{p}_1)-m_i\mathrm{Tr}\left[\gamma^\mu\not{p}_1\gamma^\nu\not{k}\right]-2m_ip_2^\nu\mathrm{Tr}\left[\gamma^\mu\not{p}_1\right]$$

$$+m_i\mathrm{Tr}\left[\gamma^\mu\not{p}_1\gamma^\nu\not{p}_2\right]+4m_i\left[p_1^\mu k^\nu+p_1^\nu k^\mu-(p_1\cdot k)\eta^{\mu\nu}\right]-4m_i\left[p_2^\mu k^\nu\right.$$

$$\left.+p_2^\nu k^\mu-(p_2\cdot k)\eta^{\mu\nu}\right]+4m_i^3\eta^{\mu\nu}$$

$$=-4m_i\cancel{k^2}\eta^{\mu\nu}-8m_i\cancel{p_2^\nu}k^\mu+16m_i\cancel{k^\mu}k^\nu+4m_i\left[\cancel{k^\mu}p_2^\nu+k^\nu p_2^\mu-(k\cdot p_2)\eta^{\mu\nu}\right]$$

$$+8m_i\cancel{k^\nu}p_1^\mu-4m_i\left[p_1^\mu k^\nu\cancel{+p_1^\nu}k^\mu-(p_1\cdot k)\eta^{\mu\nu}\right]-8m_i\cancel{p_2^\nu}p_1^\mu$$

$$+4m_i\left[p_1^\mu p_2^\nu+p_1^\nu p_2^\mu-(p_1\cdot p_2)\eta^{\mu\nu}\right]+4m_i\left[\cancel{p_1^\mu k^\nu}+p_1^\nu k^\mu-(p_1\cdot k)\eta^{\mu\nu}\right]$$

$$-4m_i\left[P_2^\mu K^\nu + \cancel{P_2^\nu K^\mu} - (P_2\cdot K)\eta^{\mu\nu}\right] + 4\cancel{m_i}^3\eta^{\mu\nu}$$

$$T_r = -4m_i K^2\eta^{\mu\nu} - 8m_i P_2^\nu K^\mu + 16m_i K^\mu K^\nu + 8m_i P_1^\mu K^\nu - 4m_i P_1^\mu P_2^\nu + 4m_i P_1^\nu P_2^\mu$$
$$-4m_i(P_1\cdot P_2)\eta^{\mu\nu} + 4m_i^3\eta^{\mu\nu} \qquad (230)$$

$$\frac{1}{abc} = 2\int_0^1 dx\int_0^{1-x}\frac{dy}{[a(1-x-y)+bx+cy]^3} \qquad (231)$$

$$\frac{1}{\underset{b}{[(K+P_1)^2-m_i^2]}\,\underset{c}{[(K-P_2)^2-m_i^2]}\,\underset{a}{[K^2-m_i^2]}} = 2\int_0^1 dx\int_0^{1-x}\frac{dy}{[\qquad]^3}$$

where:

$$[\qquad]^3 = \left[(K^2-m_i^2)(1-x-y) + [(K+P_1)^2-m_i^2]x + [(K-P_2)^2-m_i^2]y\right]^3$$

$$= [K^2 - K^2x - K^2y - m_i^2 + \cancel{m_i^2}x + \cancel{m_i^2}y + (K+P_1)^2x - \cancel{m_i^2}x + (K-P_2)^2y$$
$$- \cancel{m_i^2}y]^3$$

$$= [\cancel{K^2} - \cancel{K^2x} - \cancel{K^2y} - \cancel{m_i^2} + \cancel{K^2x} + 2(K\cdot P_1)x + \underset{\uparrow}{\overset{0}{\cancel{P_1^2}}}x + \cancel{K^2y} - 2(K\cdot P_2)y + \overset{0}{\cancel{P_2^2}}y]^3$$

$$= [K^2 - m_i^2 + 2(K\cdot P_1)x - 2(K\cdot P_2)y]^3$$

$$= [K^2 + 2K\cdot(P_1x - P_2y) - m_i^2]^3 \qquad (232)$$

$$I = \int\frac{d^dK}{(2\pi)^d}\,T_r(\qquad)\cdot\frac{1}{(\,)}\cdot\frac{1}{(\,)}\cdot\frac{1}{(\,)} \qquad (233)$$

$$= 8m_i\int_0^1 dx\int_0^{1-x}dy\int\frac{d^dK}{(2\pi)^d}\left[-K^2\eta^{\mu\nu} - 2P_2^\nu K^\mu + 4K^\mu K^\nu + 2P_1^\mu K^\nu\right.$$
$$\left.-P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - (P_1\cdot P_2)\eta^{\mu\nu} + m_i^2\eta^{\mu\nu}\right]\cdot\left[K^2 + 2K\cdot(P_1x - P_2y) - m_i^2\right]^3$$

$$K' = K + (P_1x - P_2y)$$
$$K'^2 = K^2 + 2K\cdot(P_1x - P_2y) + (P_1x - P_2y)^2$$
$$d^dK' = d^dK$$

$$\Rightarrow I = 8m_i\int_0^1 dx\int_0^{1-x}dy\int\frac{d^dK'}{(2\pi)^d}\left[-\eta^{\mu\nu}[K' - (P_1x - P_2y)]^2 - 2P_2^\nu[K'\right.$$
$$- (P_1x - P_2y)]^\mu + 4[K' - (P_1x - P_2y)]^\mu[K' - (P_1x - P_2y)]^\nu + 2P_1^\mu[K'$$
$$\left.- (P_1x - P_2y)]^\nu - P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - (P_1\cdot P_2)\eta^{\mu\nu} + m_i^2\eta^{\mu\nu}\right]\cdot$$
$$\cdot\left[K'^2 - (P_1x - P_2y)^2 - m_i^2\right]^{-3} \qquad (234)$$

$$\int \frac{d^d k' \, k'^{\mu}}{[k'^2 + 2k'\cdot q - m^2]^{\alpha}} = - I_0 q^{\mu} \qquad (235)$$

$$\Rightarrow \int \frac{d^d k' \, k'^{\mu}}{[k'^2 - (m_i^2 + (P_1 x - P_2 y)^2)]^3} = 0 \qquad (236)$$

$$I = 8 m_i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d k'}{(2\pi)^d} \Big[- \eta^{\mu\nu} k'^2 - (P_1 x - P_2 y)^2 \eta^{\mu\nu} + 2 P_2^{\nu} (P_1 x - P_2 y)^{\mu}$$

$$+ 4 k'^{\mu} k'^{\nu} + 4 (P_1 x - P_2 y)^{\mu} (P_1 x - P_2 y)^{\nu} - 2 P_1^{\mu} (P_1 x - P_2 y)^{\nu} - P_1^{\mu} P_2^{\nu}$$

$$+ P_1^{\nu} P_2^{\mu} - (P_1 \cdot P_2) \eta^{\mu\nu} + m_i^2 \eta^{\mu\nu} \Big] \cdot \frac{1}{[k'^2 - (P_1 x - P_2 y)^2 - m_i^2]^3} \qquad (237)$$

$$\int \frac{d^d k'}{[k'^2 - (m_i^2 + (P_1 x - P_2 y)^2)]^3} = \frac{i (-\pi)^{d/2} \Gamma\left(3 - \frac{d}{2}\right)}{\Gamma(3) \left[- m_i^2 - (P_1 x - P_2 y)^2 \right]^{3 - \frac{d}{2}}} = I_0$$

$$= \frac{i \pi^2}{2 \left[- m_i^2 - (P_1 x - P_2 y)^2 \right]} \qquad (238)$$

$$\int \frac{d^d k' \, k'^{\mu} k'^{\nu}}{[k'^2 - (m_i^2 + (P_1 x - P_2 y)^2)]^3} = - I_0 \left[m_i^2 + (P_1 x - P_2 y)^2 \right] \frac{\eta^{\mu\nu}}{2} \cdot \frac{1}{\left(2 - \frac{d}{2}\right)}$$

$$\int \frac{d^d k' \, k'^2}{[k'^2 - (m_i^2 + (P_1 x - P_2 y)^2]^3} = - \frac{d \, I_0}{2} \left[m_i^2 + (P_1 x - P_2 y)^2 \right] \cdot \frac{1}{\left(2 - \frac{d}{2}\right)}$$

$$\int d^d k' \, \frac{[- \eta^{\mu\nu} k'^2 + 4 k'^{\mu} k'^{\nu}]}{[k'^2 - (m_i^2 + (P_1 x - P_2 y)^2]^3} = \frac{d \, I_0}{2} \eta^{\mu\nu} \frac{[m_i^2 + (P_1 x - P_2 y)^2]}{\left(2 - \frac{d}{2}\right)} -$$

$$- \frac{2 I_0 \eta^{\mu\nu} [m_i^2 + (P_1 x - P_2 y)^2]}{\left(2 - \frac{d}{2}\right)} = I_0 \eta^{\mu\nu} [m_i^2 + (P_1 x - P_2 y)^2] \lim_{d \to 4} \overset{-1}{\cancel{\frac{\left[\frac{d}{2} - 2\right]}{\left[2 - \frac{d}{2}\right]}}}$$

$$= - I_0 \eta^{\mu\nu} [m_i^2 + (P_1 x - P_2 y)^2] \qquad (239)$$

$$\Rightarrow I = \frac{8 m_i}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \Big[-2 (P_1 x - P_2 y)^2 \eta^{\mu\nu} + 2 P_2^{\nu} (P_1 x - P_2 y)^{\mu}$$

$$+ 4 (P_1 x - P_2 y)^{\mu} (P_1 x - P_2 y)^{\nu} - 2 P_1^{\mu} (P_1 x - P_2 y)^{\nu} - P_1^{\mu} P_2^{\nu} + P_1^{\nu} P_2^{\mu} - (P_1 \cdot P_2) \eta^{\mu\nu}$$

$$\cancel{- m_i^2 \eta^{\mu\nu}} + \cancel{m_i^2 \eta^{\mu\nu}} \Big] \frac{i \pi^2}{2 \left[- m_i^2 - (P_1 x - P_2 y)^2 \right]} \qquad (240)$$

$$I = -\frac{4m_i}{(2\pi)^4}\, i\pi^2 \int_0^1 dx \int_0^{1-x} dy\, \Big[4(P_1\cdot P_2)\, x y\, \eta^{\mu\nu} + 2x \underset{\downarrow}{P_2^\nu} P_1^\mu - 2y\, P_2^\nu P_2^\mu$$

$$+ 4x^2 P_1^\mu P_1^\nu - 4xy\, P_1^\mu P_2^\nu - 4xy\, \underset{\downarrow}{P_2^\mu} P_1^\nu + 4y^2 P_2^\mu P_2^\nu$$

$$- 2x\, P_1^\mu P_1^\nu + 2y\, P_1^\mu P_2^\nu - P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - (P_1\cdot P_2)\eta^{\mu\nu}\Big]$$

$$\cdot \frac{1}{m_i^2 - 2xy(P_1\cdot P_2)} \qquad (241)$$

$a = m_i^2$; $b = -2x(P_1\cdot P_2)$.

$$\int_0^{1-x} \frac{dy}{a+by} = \int_0^{1-x} \frac{dy}{b(y+\frac{a}{b})} = \frac{1}{b}\ln\left|y+\frac{a}{b}\right| \Big|_0^{1-x}$$

$$= \frac{1}{b}\ln\left|1-x+\frac{a}{b}\right| - \frac{1}{b}\ln\left|\frac{a}{b}\right|$$

$$= \frac{1}{-2x(P_1\cdot P_2)}\ln\left|1-x-\frac{m_i^2}{2x(P_1\cdot P_2)}\right| + \frac{1}{2x(P_1\cdot P_2)}\ln\left|\frac{m_i^2}{2x(P_1\cdot P_2)}\right|$$

$$\int_0^{1-x}\frac{y\,dy}{a+by} = \int_a^{a+b(1-x)} \frac{(v-a)}{b^2 v}\,dv = \frac{1}{b^2}\Big[v - a\ln|v|\Big]\Big|_a^{a+b(1-x)}$$

$a + by = v$; $dv = b\,dy$

$$\rightarrow \int_0^{1-x}\frac{y\,dy}{a+by} = \frac{1}{b^2}\Big[a+b(1-x) - a\ln|a+b(1-x)|\Big] - \frac{1}{b^2}\Big[a - a\ln a\Big]$$

$$= \frac{1}{4x^2(P_1\cdot P_2)^2}\Big[\cancel{m_i^2} - 2x(P_1\cdot P_2)(1-x) - m_i^2\ln|m_i^2 - 2x(P_1\cdot P_2)(1-x)|$$

$$- \frac{\cancel{m_i^2}}{4x^2(P_1\cdot P_2)^2}\Big[1 - \ln m_i^2\Big] = \boxed{-\frac{(1-x)}{2x(P_1\cdot P_2)} - \frac{m_i^2}{4x^2(P_1\cdot P_2)^2}\ln\left|\frac{m_i^2-2x(1-x)(P_1\cdot P_2)}{m_i^2}\right|}$$

$$(242)$$

$$\rightarrow \int_0^{1-x}\frac{y^2\,dy}{a+by} = \int_a^{a+b(1-x)}\frac{(v-a)^2}{b^3 v}\,dv = \frac{1}{b^3}\Big[\frac{v^2}{2} - 2av + a^2\ln|v|\Big]\Big|_a^{a+b(1-x)}$$

$$= \frac{1}{b^3}\Big[\frac{1}{2}[a+b(1-x)]^2 - 2a[a+b(1-x)] + a^2\ln|a+b(1-x)| - \frac{a^2}{2}$$

$$+ 2a^2 - a^2\ln a\Big]$$

$$= \frac{-1}{8x^3(P_1\cdot P_2)^3}\Big[\frac{1}{2}\big[\cancel{m_i^2} - 2x(1-x)(P_1\cdot P_2)\big]^2 - 2m_i^2\big[\cancel{m_i^2} - 2x(1-x)(P_1\cdot P_2)\big]$$

$$+ m_i^4\ln|m_i^2 - 2x(1-x)(P_1\cdot P_2)| + \frac{3}{2}m_i^4 - m_i^4\ln m_i^2\Big]$$

$$= \frac{-1}{8x^3 (P_1\cdot P_2)^3} \Big[-2x(1-\cancel{x)}\, m_i^2 (P_1\cdot P_2) + 2x^2(1-x)^2 (P_1\cdot P_2)^2 + 4\cancel{m_i^2}\, x(1-x)(P_1\cdot P_2) + m_i^4 \ln\left| \frac{m_i^2 - 2x(1-x)(P_1\cdot P_2)}{m_i^2} \right|$$

$$\int_0^{1-x} \frac{y^2\, dy}{a+by} = -\frac{1}{8x^3 (P_1\cdot P_2)^3} \Big[2x(1-x)\, m_i^2 (P_1\cdot P_2) + 2x^2(1-x)^2 (P_1\cdot P_2)^2 + m_i^4 \ln\left| \frac{m_i^2 - 2x(1-x)(P_1\cdot P_2)}{m_i^2} \right| \qquad (243)$$

$$\rightarrow \boxed{\int_0^{1-x} \frac{dy}{a+by} = -\frac{1}{2x(P_1\cdot P_2)} \ln\left| \frac{m_i^2 - 2x(1-x)(P_1\cdot P_2)}{m_i^2} \right|} \qquad (244)$$

but

$$\varepsilon_{\mu_1} P_1^{\mu} = 0 \; ; \; \varepsilon_{\nu_2} P_2^{\nu} = 0 \; ; \; \varepsilon_{\rho_4} P_4^{\rho} = 0$$

$\Rightarrow$

$$-iM_{1_a} = -\frac{g^2 g_s^2}{4} \frac{\sin(\alpha-\beta)\cos\alpha}{M_W \cos\beta} \frac{(P_1+P_2+P_3)^{\rho}}{s - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} \varepsilon_{\rho_4}^{\alpha}\, \varepsilon_{\mu_1} \varepsilon_{\nu_2} \sum_i m_i^2 \left(\frac{-4i\pi^2}{(2\pi)^4} \right) \cdot$$

$$\cdot \int_0^1 dx \int_0^{1-x} dy \left[4(P_1\cdot P_2)\cancel{x}\, y\, \eta^{\mu\nu} - 4x\cancel{y}\, P_2^{\mu} P_1^{\nu} + P_1^{\nu} \cancel{P_2^{\mu}} - (P_1\cdot \cancel{P_2})\eta^{\mu\nu} \right]$$

$$\cdot \frac{1}{m_i^2 - 2xy(P_1\cdot P_2)} \; \mathrm{Tr}(T^a T^b) \qquad \underline{\text{OK}}$$

$$= -\frac{g^2 g_s^2}{4} \frac{\sin(\alpha-\beta)\cos\alpha}{M_W \cos\beta} \frac{(P_1+P_2+P_3)^{\rho}}{s - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} \varepsilon_{\rho_4}^{\alpha}\, \varepsilon_{\mu_1} \varepsilon_{\nu_2} \left(-\frac{i}{4\pi^2} \right) \cdot \sum_i m_i^2$$

$$\cdot \int_0^1 dx \Bigg[(P_1\cdot P_2)\eta^{\mu\nu} \Big[\frac{-4(1-\cancel{x})}{2(\cancel{P_1}\cdot P_2)} - 4\frac{m_i^2}{4x} \cdot \frac{1}{(P_1\cdot P_2)^2} \ln\left| \frac{m_i^2 - 2x(1-x)(P_1\cdot P_2)}{m_i^2} \right|$$

$$+ \frac{1}{2x(P_1\cdot \cancel{P_2})} \ln\left| \frac{m_i^2 - 2x(1-x)(P_1\cdot P_2)}{m_i^2} \right| \Big] + P_1^{\nu} P_2^{\mu} \Big[\frac{-1}{2x(P_1\cdot P_2)} \cdot$$

$$\cdot \ln\left| \frac{m_i^2 - 2x(1-x)(P_1\cdot P_2)}{m_i^2} \right| + \Big[\frac{2(1-x)}{(\cancel{P_1}\cdot P_2)} + \frac{m_i^2}{x(P_1\cdot \cancel{P_2})^2} \ln\left| \frac{m_i^2 - 2x(1-x)(P_1\cdot P_2)}{m_i^2} \right| \Big] \Big] \Bigg] .$$

$$\cdot \mathrm{Tr}(T^a T^b) \qquad (245)$$

<u>OK.</u>

$$-iM_{1_a} = \frac{-g^2 g_s^2}{4} \frac{\sin(\alpha-\beta)\cos\alpha}{M_W \cos\beta} \frac{(p_1+p_2+p_3)^\rho}{s-m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} \epsilon^{*}_{p_4} \epsilon_{\mu_1} \epsilon_{\nu_2} \left(\frac{-i}{4\pi^2}\right) \cdot \sum_i m_i^2$$

$$\cdot \int_0^1 dx \left\{ \left(p_1^\nu p_2^\mu - (p_1 \cdot p_2) \eta^{\mu\nu} \right) \left[\ln \left| \frac{m_i^2 - 2x(1-x)(p_1\cdot p_2)}{m_i^2} \right| \frac{1}{x(p_1\cdot p_2)} \left(\frac{m_i^2}{p_1\cdot p_2} - \frac{1}{2} \right. \right. \right.$$

$$\left. \left. + \frac{2(1-x)}{p_1 \cdot p_2} \right] \right\} Tr(T^a T^b)$$

OK

$$\hat{s} = (p_1+p_2)^2 = 2p_1\cdot p_2 \quad \Rightarrow \quad (p_1\cdot p_2) = \frac{\hat{s}}{2}$$

$$\Rightarrow -iM_{1_a} = -\frac{g^2 g_s^2}{4} \frac{\sin(\alpha-\beta)\cos\alpha}{M_W\cos\beta} \frac{(p_1+p_2+p_3)^\rho}{s-m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} \epsilon^{*}_{p_4} \epsilon_{\mu_1} \epsilon_{\nu_2} \left(\frac{-i}{4\pi^2}\right) \cdot \sum_i m_i^2$$

$$\cdot \int_0^1 dx \left\{ \left(p_1^\nu p_2^\mu - \frac{\hat{s}}{2}\eta^{\mu\nu} \right) \left[\ln \left| 1 - x(1-x)\frac{\hat{s}}{m_i^2} \right| \frac{2m_i^2}{x\,\hat{s}\,m_i^2} \left(\frac{2m_i^2}{\hat{s}} - \frac{1}{2} \right. \right. \right.$$

$$\left. \left. + \frac{4(1-x)}{\hat{s}} \right] \right\} Tr(T^a T^b)$$

$$\tau_i \equiv \frac{4m_i^2}{\hat{s}}$$

$$\Rightarrow -iM_{1_a} = \frac{-g^2 g_s^2}{4} \frac{\sin(\alpha-\beta)\cos\alpha}{M_W\cos\beta} \frac{(p_1+p_2+p_3)^\rho}{s-M_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} \epsilon^{*}_{p_4} \epsilon_{\mu_1} \epsilon_{\nu_2} \left(\frac{-i}{4\pi^2}\right) \cdot \sum_i m_i^2$$

$$\int_0^1 dx \left\{ \left(p_1^\nu p_2^\mu - \frac{\hat{s}}{2}\eta^{\mu\nu} \right) \left[\ln \left| 1 - \frac{4x(1-x)}{\tau_i} \right| \frac{\tau_i}{4x\,m_i^2} (\tau_i - 1) \right. \right.$$

$$\left. \left. + \frac{\tau_i}{m_i^2}(1-x) \right] \right\} Tr(T^a T^b) \qquad (246)$$

¡OK!

Now $\int \frac{\tau_i}{x} \ln \left| \frac{\tau_i - 4x + 4x^2}{\tau_i} \right| dx = ?$

$$\int \frac{\ln(ax+b)}{x} dx = \ln b \ln\left(\frac{ax}{b}\right) - \text{dilog}\left(\frac{ax+b}{b}\right)$$

In fact: $\boxed{\text{dilog}(x) \equiv \int_1^x \frac{\ln t}{1-t} dt}$ / OK.

$$\text{dilog}\left(\frac{a}{b}x+1\right) = \int_1^{\frac{a}{b}x+1} \frac{\ln t}{1-t} dt$$

$$\frac{d}{dx} \text{dilog}\left(\frac{a}{b}x+1\right) = \ln\left(\frac{a}{b}x+1\right) \cdot \frac{1}{-\cancel{\left(\frac{a}{b}\right)}x} \cdot \cancel{\left(\frac{a}{b}\right)} = -\frac{\ln(ax+b)}{x} + \frac{\ln b}{x}$$

$$\frac{d}{dx}\left[\ln b\,\ln\left(\frac{ax}{b}\right) - \mathrm{dilog}\left(\frac{ax+b}{b}\right)\right]$$

$$= \cancel{\left(\frac{b}{ax}\right)\left(\frac{a}{b}\right)\ln b} + \frac{\ln(ax+b)}{x} - \cancel{\frac{\ln b}{x}}$$

$$= \frac{\ln(ax+b)}{x}$$

$$\int_0^1 \frac{T_i}{x}\ln\left|\frac{4x^2-4x+T_i}{T_i}\right|dx$$

$$= \int_0^1 \frac{T_i}{x}\ln\left|\frac{x^2-x+\frac{T_i}{4}}{T_i/4}\right|dx$$

$$= T_i\, f(T_i)$$

where

$$\boxed{f(T_i) = \begin{cases} -2\left(\arcsin\left(\frac{1}{T_i^{1/2}}\right)\right)^2 & T_i > 1 \\ \frac{1}{2}\left[\ln\left(\frac{1+(1-T_i)^{1/2}}{1-(1-T_i)^{1/2}}\right) - i\pi\right]^2 & T_i \leq 1 \end{cases}} \quad \text{¡OK!} \qquad (247)$$

$$-iM_{1_a} = \frac{-g^2 g_s^2}{4}\,\frac{\sin(\alpha-\beta)\cos\alpha}{M_W\cos\beta}\,\frac{(P_1+P_2+P_3)^\rho}{(s-m_{H^0}^2+im_{H^0}\Gamma_{H^0})}\,\varepsilon_{P_4}^{\alpha}\,\varepsilon_{\mu_1}\,\varepsilon_{\nu_2}\left(\frac{-i}{4\pi^2}\right)\cdot\sum_i \cancel{m_i^2}\,.$$

$$\int_0^1 dx\left(P_1^\nu P_2^\mu - \frac{\hat{s}}{2}n^{\mu\nu}\right)\left[\ln\left|\frac{x^2-x+T_i/4}{T_i/4}\right|\frac{T_i}{x}\frac{1}{4\cancel{m_i^2}}(T_i-1) + \frac{T_i}{\cancel{m_i^2}}(1-x)\right]$$

$$\times \mathrm{Tr}(T^a T^b)$$

$$= \frac{i g^2 g_s^2}{8\pi^2}\,\frac{\sin(\alpha-\beta)\cos\alpha}{M_W\cos\beta}\,\frac{(P_1+P_2)^\rho\,\varepsilon_{P_4}^{\alpha}\,\varepsilon_{\mu_1}\,\varepsilon_{\nu_2}}{(s-m_{H^0}^2+im_{H^0}\Gamma_{H^0})}\sum_i\left\{\int_0^1 dx\,T_i(1-x)\left(P_1^\nu P_2^\mu - \frac{\hat{s}}{2}n^{\mu\nu}\right)\right.$$

$$\left. + \left[\frac{T_i(T_i-1)}{4}\left(P_1^\nu P_2^\mu - \frac{\hat{s}}{2}n^{\mu\nu}\right)\right] f(T_i)\right\}\mathrm{Tr}(T^a T^b)$$

$$-iM_{1_a} = \frac{i g^2 g_s^2}{8\pi^2}\,\frac{\sin(\alpha-\beta)\cos\alpha}{M_W\cos\beta}\,\frac{(P_1+P_2)^\rho\,\varepsilon_{P_4}^{\alpha}\,\varepsilon_{\mu_1}\,\varepsilon_{\nu_2}}{(s-m_{H^0}^2+im_{H^0}\Gamma_{A^0})}\sum_i\left\{\frac{1}{2}T_i\left(P_1^\nu P_2^\mu - \frac{\hat{s}}{2}n^{\mu\nu}\right)\right.$$

$$\left. + \frac{1}{4}T_i(T_i-1)\left(P_1^\nu P_2^\mu - \frac{\hat{s}}{2}n^{\mu\nu}\right)f(T_i)\right\}\underset{\frac{1}{2}\delta^{ab}}{\underset{\downarrow}{\mathrm{Tr}(T^a T^b)}} \qquad (248)$$

Let's evaluate:

$$\mathrm{Tr}\left(\gamma^\mu \quad (\not k + \not p_1 + m_i)\gamma^5(\not k - \not p_2 + m_i)\gamma^\nu(\not k + m_i)\right)$$

$$(\gamma^\mu\gamma^5 + \gamma^5\gamma^\mu = 0)$$

$$= +\mathrm{Tr}\left(\gamma^5\gamma^\mu(\not k + \not p_1 - m_i)(\not k - \not p_2 + m_i)(\gamma^\nu \not k + m_i\gamma^\nu)\right)$$

$$= +\mathrm{Tr}\Big((\gamma^5\gamma^\mu\not k + \gamma^5\gamma^\mu\not p_1 - m_i\gamma^5\gamma^\mu)(\not k\gamma^\nu\not k + m_i\not k\gamma^\nu - \not p_2\gamma^\nu\not k - m_i\not p_2\gamma^\nu + m_i\gamma^\nu\not k$$
$$+ m_i^2\gamma^\nu\Big)$$

$$= +\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\mu \overset{k^2}{\boxed{\not k\not k}}\gamma^\nu\not k]}} + m_i\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\mu\overset{k^2}{\not k\not k}\gamma^\nu]}} - \overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\mu\not k\not p_2\gamma^\nu\not k]}}$$
$$- m_i\mathrm{Tr}[\gamma^5\gamma^\mu\not k\not p_2\gamma^\nu] + m_i\mathrm{Tr}[\gamma^5\gamma^\mu\not k\gamma^\nu\not k] + m_i^2\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\mu\not k\gamma^\nu]}}$$
$$+ \overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\mu\not p_1\not k\gamma^\nu\not k]}} + m_i\mathrm{Tr}[\gamma^5\gamma^\mu\not p_1\not k\gamma^\nu] - \overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\mu\not p_1\not p_2\gamma^\nu\not k]}}$$
$$- m_i\mathrm{Tr}[\gamma^5\gamma^\mu\not p_1\not p_2\gamma^\nu] + m_i\mathrm{Tr}[\gamma^5\gamma^\mu\not p_1\gamma^\nu\not k] + m_i^2\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\mu\not p_1\gamma^\nu]}}$$
$$- m_i\mathrm{Tr}[\gamma^5\gamma^\mu\not k\gamma^\nu\not k] - m_i^2\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\mu\not k\gamma^\nu]}} + m_i\mathrm{Tr}[\gamma^5\gamma^\mu\not p_2\gamma^\nu\not k]$$
$$+ m_i^2\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\mu\not p_2\gamma^\nu]}} - m_i^2\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\not k]}} - m_i^3\overset{0}{\cancel{\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu]}}$$

$$= -m_i\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\alpha\gamma^\beta\gamma^\nu]k_\alpha p_{2\beta}$$
$$+ m_i\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\alpha\gamma^\beta\gamma^\nu]p_{1\alpha}k_\beta - m_i\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\alpha\gamma^\beta\gamma^\nu]p_{1\alpha}p_{2\beta}$$
$$+ m_i\mathrm{Tr}[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta]p_{1\alpha}k_\beta + m_i\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta]p_{2\alpha}k_\beta$$

$\Rightarrow$

$$\mathrm{Tr} = 4m_i i\,\epsilon^{\mu\alpha\beta\nu}k_\alpha p_{2\beta} - 4im_i\epsilon^{\mu\alpha\beta\nu}p_{1\alpha}k_\beta$$
$$+ 4im_i\epsilon^{\mu\alpha\beta\nu}p_{1\alpha}p_{2\beta} - 4im_i\epsilon^{\mu\alpha\nu\beta}p_{1\alpha}k_\beta - 4im_i\epsilon^{\mu\alpha\nu\beta}p_{2\alpha}k_\beta \qquad (249)$$

$$\int\frac{d^dk'\,k'^\mu}{[k'^2 - (m_i^2 + (p_1x - p_2y)^2]^3} = 0$$

$$\Rightarrow \quad -iM_{3_a} = 2\frac{g^2g_s^2}{4M_W}\tan\beta\frac{\epsilon^*_{p_4}\epsilon_{\mu_1}\epsilon_{\nu_2}(T^a_{dc}T^b_{cd})}{(s - m_{A^0}^2 + im_{A^0}\Gamma_{A^0})}(p_1 + p_2 + p_3)^\rho\sum_i m_i\int_0^1 dx\int_0^{1-x}dy$$

$$\int\frac{d^dk'}{(2\pi)^d}\frac{4im_i\epsilon^{\mu\alpha\beta\nu}p_{1\alpha}p_{2\beta}}{[k'^2 - (p_1x - p_2y)^2 - m_i^2]^3}$$

$$-iM_{3_a} = -\frac{\cancel{2}g^2g_s^2}{\cancel{4}M_W}\tan\beta\frac{\epsilon^*_{p_4}\epsilon_{\mu_1}\epsilon_{\nu_2}\mathrm{Tr}(T^aT^b)(p_1 + p_2 + p_3)^\rho}{(s - m_{A^0}^2 + im_{A^0}\Gamma_{A^0})}\sum_i m_i\int_0^1 dx\int_0^{1-x}dy\cdot\frac{1}{(2\pi)^4}$$

$$\cdot\ \cancel{4} i m_i\, \varepsilon^{\mu\alpha\rho\nu} p_{1\alpha} p_{2\beta} \frac{i\pi^2}{\cancel{2}} \left[+ m_i^2 + (p_1 x - p_2 y)^2\right]^{-1} \qquad (250)$$

$$\int_0^{1-x} \frac{dy}{\left[m_i^2 + (p_1 x - p_2 y)^2\right]} = \int_0^{1-x} \frac{dy}{\left[m_i^2 - 2xy(p_1 \cdot p_2)\right]}$$

$$= -\frac{1}{2x(p_1\cdot p_2)} \ln \left| \frac{m_i^2 - 2x(1-x)(p_1\cdot p_2)}{m_i^2} \right|$$

$$\Rightarrow \quad -iM_{3_a} = \frac{-g^2 g_s^2}{M_W} \tan\beta \frac{\mathrm{Tr}(T^a T^b)(p_1+p_2+p_3)^\rho \varepsilon^*_{p_4} \varepsilon_{\mu_1} \varepsilon_{\nu_2}}{(s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0})} \frac{1}{(2\pi)^4} \varepsilon^{\mu\alpha\rho\nu} p_{1\alpha} p_{2\beta} \pi^2$$

$$\sum_i m_i^2 \int_0^1 \frac{dx}{2x(p_1\cdot p_2)} \ln \left| \frac{m_i^2 - 2x(1-x)(p_1\cdot p_2)}{m_i^2} \right|$$

$$-iM_{3_a} = -\frac{g^2 g_s^2}{M_W} \tan\beta \frac{\mathrm{Tr}(T^a T^b)(p_1+p_2+p_3)^\rho \varepsilon^*_{p_4} \varepsilon_{\mu_1} \varepsilon_{\nu_2}}{(s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0})} \frac{1}{16\pi^2} \varepsilon^{\mu\alpha\beta\nu} p_{1\alpha} p_{2\beta}$$

$$\sum_i \frac{T_i}{4} \int_0^1 \frac{dx}{x} \ln \left| \frac{4x^2 - 4x + T_i}{T_i} \right|$$

$$-iM_{3_a} = \frac{-g^2 g_s^2}{16\pi^2 M_W} \tan\beta \frac{\mathrm{Tr}(T^a T^b)(p_1+p_2+p_3)^\rho \varepsilon^*_{p_4} \varepsilon_{\mu_1} \varepsilon_{\nu_2}}{(s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0})} \varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta}$$

$$\cdot \frac{1}{4} \sum_i T_i f(T_i)$$

$$(p_1+p_2+p_3)^\rho \varepsilon^*_{p_4} = (2(p_1+p_2) - p_4)^\rho \varepsilon^*_{p_4} = 2(p_1+p_2)^\rho \varepsilon^*_{p_4} - 2 \overset{0}{\cancel{p_4^\rho \varepsilon^*_{p_4}}}$$

$$\Rightarrow (p_1+p_2+p_3)^\rho \varepsilon^*_{p_4} = 2(p_1+p_2)^\rho \varepsilon^*_{p_4}$$

$$-iM_{3_a} = \frac{-g^2 g_s^2}{8\pi^2 M_W} \tan\beta \frac{\mathrm{Tr}(T^a T^b)(p_1+p_2)^\rho \varepsilon^*_{p_4} \varepsilon_{\mu_1} \varepsilon_{\nu_2} \varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta}}{(s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0})} \cdot \frac{1}{4} \sum_i T_i f(T_i)$$

(251)

OK

$$-iM_{1_a} = i\frac{\alpha_s G_F M_W}{2\sqrt{2}\pi} \frac{\sin(\alpha-\beta)\cos\alpha}{\cos\beta} \frac{(p_1+p_2)^\rho \, \varepsilon^{*}_{p_4} \, \varepsilon_{\mu_1} \varepsilon_{\nu_2}}{(\hat{s}-m_{H^0}^2 + i m_{H^0} \Gamma_{H^0})} \sum_{i=d,s,b} \left\{ (p_1^\nu p_2^\mu - \frac{3}{2} n^{\mu\nu}) \cdot \right.$$

$$\left. \cdot \left[T_i (T_i - 1) f(T_i) + 2 T_i \right] \right\} \delta^{ab} \qquad (252)$$

$$-iM_{3_a} = -\frac{\alpha_s G_F M_W}{2\sqrt{2}\pi} \tan\beta \frac{\delta^{ab} (p_1+p_2)^\rho \, \varepsilon^{*}_{p_4} \, \varepsilon_{\mu_1} \varepsilon_{\nu_2} \varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta}}{(\hat{s}-m_{A^0}^2 + i m_{A^0} \Gamma_{A^0})} \sum_{i=d,s,b} T_i f(T_i) \qquad (253)$$

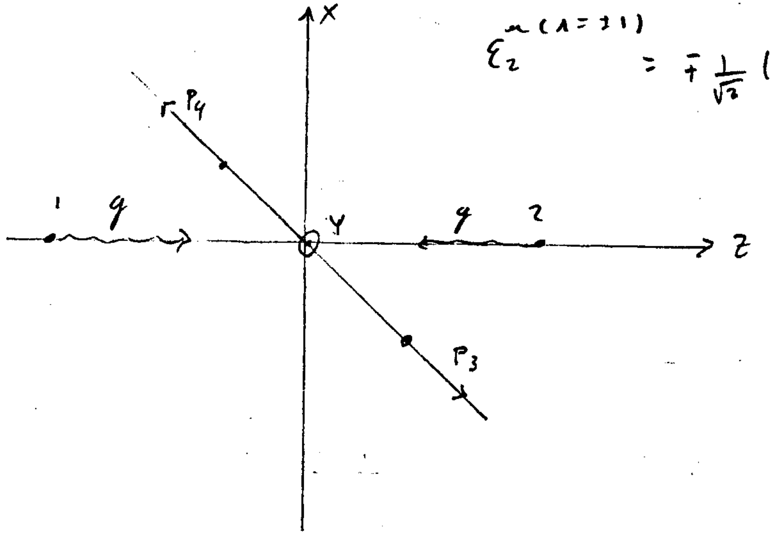
$$-iM_{2_a} = -\frac{i\alpha_s G_F M_W}{2\sqrt{2}\pi} \frac{\cos(\beta-\alpha)\sin\alpha}{\cos\beta} \frac{(p_1+p_2)^\rho \, \varepsilon^{*}_{p_4} \, \varepsilon_{\mu_1} \varepsilon_{\nu_2}}{(\hat{s}-m_{h^0}^2 + i m_{h^0} \Gamma_{h^0})} \sum_{i=d,s,b} \left\{ (p_1^\nu p_2^\mu - \frac{3}{2} n^{\mu\nu}) \cdot \right.$$

$$\left. \cdot \left[T_i (T_i - 1) f(T_i) + 2 T_i \right] \right\} \delta^{ab} \qquad (254)$$

Where $\varepsilon^a_\mu = \varepsilon^a_\mu(p_1)$; $\varepsilon^b_\nu = \varepsilon^b_\nu(p_2)$

$$\varepsilon_1^{\mu(\lambda=\pm1)} = \mp(0,1,\pm i,0)/\sqrt{2}$$ incoming gluon

$$\varepsilon_2^{\mu(\lambda=\pm1)} = \mp\frac{1}{\sqrt{2}}(0,-1,\pm i,0)$$



$$p_1^\mu = (k^0, 0, 0, k^0) \; ; \; p_2^\mu = (k^0, 0, 0, -k^0)$$

$$\Rightarrow p_{1\mu} \varepsilon^\mu_{1,2} = 0 \; ; \; p_{2\mu} \varepsilon^\mu_{1,2} = 0 \qquad (255)$$

$$\Rightarrow -iM_{1_a} = -\frac{i\alpha_s G_F M_W}{4\sqrt{2}\pi} \frac{\sin(\alpha-\beta)\cos\alpha}{\cos\beta} \frac{(p_1+p_2)^\rho \, \varepsilon^{*}_{p_4}}{(\hat{s}-m_{H^0}^2 + i m_{H^0} \Gamma_{H^0})} \varepsilon_{\mu_1} \; \varepsilon^\mu{}_2 \; \delta^{ab} \Big\} \cdot$$

$$\cdot \sum_i \left[2T_i + T_i(T_i-1) f(T_i) \right] \qquad (256)$$

$$-iM_{2_a} = \frac{i\alpha_s G_F M_W}{4\sqrt{2}\pi} \frac{\cos(\beta-\alpha)\ \sin\alpha}{\cos\beta} \frac{(P_1+P_2)^\rho\, \varepsilon^*_{\rho 4}\, \varepsilon_\mu(P_1)\, \varepsilon^\mu}{(\hat{s}-m_{h^0}^2 + i m_{h^0}\Gamma_{h^0})} (P_2)\, \delta^{ab}\, \hat{s} \cdot \sum_{i=d,s,b}$$

$$\cdot\ [2T_i + (T_i-1)T_i f(T_i)] \qquad (257)$$

$$M_{3_a} = \frac{-i\alpha_s G_F M_W}{2\sqrt{2}\pi} \tan\beta\ \delta^{ab} \frac{(P_1+P_2)^\rho\, \varepsilon^*_{\rho 4}\ \varepsilon_\mu(P_1)\, \varepsilon_\nu(P_2)\ \varepsilon^{\mu\nu\alpha\beta}\ P_{1\alpha} P_{2\beta}}{(\hat{s}-m_{A^0}^2 + i m_{A^0}\Gamma_{A^0})} \sum_{i=d,s,b} T_i f(T_i)$$

$$(258)$$

$$M_{12_a} = (M_{1_a} + M_{2_a}) = \frac{-\alpha_s G_F M_W}{4\sqrt{2}\pi} \cdot \frac{1}{\cos\beta} \left[\frac{\sin(\beta-\alpha)\ \cos\alpha}{\hat{s}-m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} + \frac{\cos(\beta-\alpha)\ \sin\alpha}{\hat{s}-m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}} \right] \cdot$$

$$\cdot\ (P_1+P_2)^\rho\, \varepsilon^*_{\rho 4}\ \varepsilon_\mu(P_1)\ \varepsilon^\mu\ (P_2)\ \delta^{ab}\ \hat{s} \cdot \sum_{i=d,s,b} [2T_i + T_i(T_i-1)\, f(T_i)] \qquad (259)$$

The Total amplitude is :

$$M = 2(M_{12} + M_3) \qquad (260)$$

$$|M|^2 = 4(M_{12}+M_3)(M_{12}^* + M_3^*) = 4[|M_{12}|^2 + M_{12}M_3^* + M_3 M_{12}^* + |M_3|^2] \qquad (261)$$

Using: $\delta^{ab}\delta^{ab} = 8$

$$|M_{12}|^2 = \frac{\alpha_s^2 G_F^2 M_W^2}{32\pi^2}\, \hat{s}^2 \left| \sum_{i=b,t} \{ C_{Hi} [2T_i + T_i(T_i-1) f(T_i)] \} \right|^2 8$$

$$(P_1+P_2)^\rho (P_1+P_2)^\sigma \sum_\lambda \varepsilon^*_{\rho 4}\ \varepsilon_{\sigma 4}\ \varepsilon_\mu \underbrace{(P_1)\ \varepsilon^\mu\ (P_2)\ \varepsilon^*_\nu}_{-g_{\mu\nu}} (P_1)\ \overset{-g^{\mu\nu}}{\varepsilon^\nu}\ (P_2).$$

$$(g_{\mu\nu} g^{\mu\nu} = 4)$$

$$= 4\frac{\alpha_s^2 G_F^2 M_W^2}{32\pi^2}\, \hat{s}^2 \left| \sum_{i=b,t} \{ C_{Hi} [2T_i + T_i(T_i-1) f(T_i)] \} \right|^2 \cdot 8$$

$$(P_1+P_2)^\rho (P_1+P_2)^\sigma \left(-g_{\rho\sigma} + \frac{P_{4\rho} P_{4\sigma}}{M_W^2} \right) \qquad (262)$$

$$L = (P_1+P_2)^\rho (P_1+P_2)^\sigma \left(-g_{\rho\sigma} + \frac{P_{4\rho} P_{4\sigma}}{M_W^2} \right) = \left(-(P_1+P_2)^2 + \frac{((P_1+P_2)\cdot P_4)^2}{M_W^2} \right)$$

$$= \left(-\hat{s}^2 + \frac{((P_1+P_2)\cdot(P_1+P_2-P_3))^2}{M_W^2} \right)$$

$$= \left(-\hat{s}^2 + \frac{(\hat{s} - (P_1+P_2)\cdot P_3)^2}{M_W^2} \right)$$

$$\hat{t} = (P_1 - P_3)^2 = M_{H^\pm}^2 - 2\, P_1 \cdot P_3$$

$$\Rightarrow \quad P_1 \cdot P_3 = \frac{M_{H^\pm}^2 - \hat{t}}{2}$$

$$\hat{U} = (P_1 - P_4)^2 = (P_3 - P_2)^2 = M_{H^\pm}^2 - 2\, P_2 \cdot P_3$$

$$\Rightarrow \quad P_2 \cdot P_3 = \frac{M_{H^\pm}^2 - \hat{U}}{2}$$

$$\Rightarrow \quad L = \left(-\hat{s}^2 + \frac{\left(\hat{s} - M_{H^\pm}^2 + \frac{(\hat{U} + \hat{t})}{2}\right)^2}{M_W^2} \right)$$

but $\hat{s} + \hat{t} + \hat{U} = M_{H^\pm}^2 + M_W^2$

$$\Rightarrow \quad L = \left(-\hat{s}^2 + \frac{\left(\hat{s} - M_{H^\pm}^2 + \frac{(M_{H^\pm}^2 + M_W^2 - \hat{s})}{2}\right)^2}{M_W^2} \right)$$

$$L = \frac{1}{4M_W^2}\left[-4\hat{s}^2 M_W^2 + \left(\cancel{2}\hat{s} - 2\cancel{M_{H^\pm}^2} + \cancel{M_{H^\pm}^2} + M_W^2 - \cancel{\hat{s}}\right)^2 \right]$$

$$L = \frac{1}{4M_W^2}\left[-4\hat{s}^2 M_W^2 + \left(\hat{s} - M_{H^\pm}^2 + M_W^2\right)^2 \right]$$

$$L = \frac{1}{4M_W^2}\left[-4\cancel{\hat{s}^2} M_W^2 + \cancel{\hat{s}}^2 + \cancel{M_{H^\pm}^4} + M_W^4 - 2\hat{s}\cancel{M_{H^\pm}^2} + 2\cancel{\hat{s}} M_W^2 - \cancel{2} M_W^2 M_{H^\pm}^2 \right]$$

$$L = \frac{1}{4M_W^2}\left[\hat{s}^2 + M_{H^\pm}^4 + M_W^4 - 2\hat{s} M_{H^\pm}^2 - 2\hat{s} M_W^2 - 2 M_W^2 M_{H^\pm}^2 \right]$$

$$L = \frac{1}{4M_W^2}\, \lambda(\hat{s}, M_{H^\pm}^2, M_W^2)$$

$$\Rightarrow \quad \boxed{|M_{12}|^2 = \frac{\alpha_S^2\, G_F^2}{4 \times 8\pi^2}\, \hat{s}^2 \left| \sum_{i=b,t} \left\{ C_{Hi} \left[2T_i + T_i (T_i - 1) f(T_i) \right] \right\} \right|^2 \lambda(\hat{s}, M_{H^\pm}^2, M_W^2) \cdot 8}$$

(263)

see next pages.

¡ok!

The amplitude corresponding to diagrams ④ is obtained from (226), (227) and (228) with the substitution: $\mu \leftrightarrow \nu$, $P_1 \leftrightarrow P_2$ in the trace. ε_μ, ε_ν are the same. Then $M_{4a} = M_{1a}$; $M_{5a} = M_{2a}$; $M_{6a} = M_{3a}$; $M_{12a} = M_{45a}$. Then we get (261).

For M_{3a}

$$\varepsilon^{\mu\nu\alpha\beta} P_{1\alpha} P_{2\beta} \longleftrightarrow \varepsilon^{\nu\mu\alpha\beta} P_{2\alpha} P_{1\beta} = \varepsilon^{\nu\mu\beta\alpha} P_{2\beta} P_{1\alpha} = \varepsilon^{\mu\nu\alpha\beta} P_{1\alpha} P_{2\beta} .$$

For $i = u,c,t$

$$-iM_{1b} = \epsilon^{*}_{p_4} \frac{ig}{2} \sin(\alpha-\beta) (p_1+p_2+p_3)^\rho \frac{i}{s - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} \left(\frac{-ig\sin\alpha}{2M_W\sin\beta} \right) \sum_{i=u,c,t} m_i \left\{ \int \frac{d^dk}{(2\pi)^d} \cdot \right.$$

$$\left. \cdot \mathrm{Tr}\left((-ig_s\gamma^\mu T^a_{dc}) \frac{i}{\not{k}+\not{p}_1-m_i} \frac{i}{\not{k}-\not{p}_2-m_i} (-ig_s\gamma^\nu T^b_{cd}) \frac{i}{\not{k}-m_i} \right) \right\} \epsilon_{\mu_1} \epsilon_{\nu_2} \mu^{\frac{3}{2}(4-d)} \qquad (264)$$

$$-iM_{2b} = \epsilon^{*}_{p_4} \frac{ig}{2} \cos(\beta-\alpha) (p_1+p_2+p_3)^\rho \frac{i}{s - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}} \left(\frac{-ig\cos\alpha}{2M_W\sin\beta} \right) \sum_{i=u,c,t} m_i \left\{ \int \frac{d^dk}{(2\pi)^d} \cdot \right.$$

$$\left. \cdot \mathrm{Tr}\left((-ig_s\gamma^\mu T^a_{dc}) \frac{i}{\not{k}+\not{p}_1-m_i} \cdot \frac{i}{\not{k}-\not{p}_2-m_i} (-ig_s\gamma^\nu T^b_{cd}) \frac{i}{\not{k}-m_i} \right) \right\} \epsilon_{\mu_1} \epsilon_{\nu_2} \mu^{\frac{3}{2}(4-d)} \qquad (265)$$

$$-iM_{3b} = \epsilon^{*}_{p_4} \frac{g}{2} (p_1+p_2+p_3)^\rho \frac{i}{s - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0}} \left(\frac{-g\cot\beta}{2M_W} \right) \sum_{i=u,c,t} \left\{ \int \frac{d^dk}{(2\pi)^d} \mathrm{Tr}\left((-ig_s\gamma^\mu T^a_{dc}) \right. \right.$$

$$\left. \left. \frac{i}{\not{k}+\not{p}_1-m_i} \gamma^5 \frac{i}{\not{k}-\not{p}_2-m_i} (-ig_s\gamma^\nu T^b_{cd}) \frac{i}{\not{k}-m_i} \right) \right\} \epsilon_{\mu_1} \epsilon_{\nu_2} \mu^{\frac{3}{2}(4-d)} \qquad (266)$$

$$-iM_{1b} = \frac{i\alpha_s G_F M_W}{2\sqrt{2}\pi} \frac{\sin(\alpha-\beta)\sin\alpha}{\sin\beta} \frac{(p_1+p_2)^\rho \epsilon^{*}_{p_4} \epsilon_{\mu_1} \epsilon_{\nu_2}}{(\hat{s} - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0})} \sum_{i=u,c,t} \left\{ (p_1^\nu p_2^\mu - \frac{\hat{s}}{2} g^{\mu\nu}) \cdot \right.$$

$$\left. \cdot \left[\tau_i(\tau_i-1) f(\tau_i) + 2\tau_i \right] \right\} \delta^{ab} \qquad (267)$$

$$-iM_{2b} = \frac{+i\alpha_s G_F M_W}{2\sqrt{2}\pi} \frac{\cos(\beta-\alpha)\cos\alpha}{\sin\beta} \frac{(p_1+p_2)^\rho \epsilon^{*}_{p_4} \epsilon_{\mu_1} \epsilon_{\nu_2}}{(\hat{s} - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0})} \sum_{i=u,c,t} \left\{ (p_1^\nu p_2^\mu - \frac{\hat{s}}{2} g^{\mu\nu}) \cdot \right.$$

$$\left. \cdot \left[\tau_i(\tau_i-1) f(\tau_i) + 2\tau_i \right] \right\} \delta^{ab} \qquad (268)$$

$$-iM_{3b} = -\frac{\alpha_s G_F M_W}{2\sqrt{2}\pi} \frac{\cot\beta\, \delta^{ab} (p_1+p_2)^\rho \epsilon^{*}_{p_4} \epsilon_{\mu_1} \epsilon_{\nu_2} \epsilon^{\mu\nu\alpha\rho} p_{1\alpha} p_{2\rho}}{(\hat{s} - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0})} \sum_{i=u,c,t} \tau_i f(\tau_i) \qquad (269)$$

$$M_{12b} = (M_{1b} + M_{2b}) = +\frac{\alpha_s G_F M_W}{4\sqrt{2}\pi} \frac{1}{\sin\beta} \left[-\frac{\sin(\beta-\alpha)\sin\alpha}{(\hat{s} - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0})} + \frac{\cos(\beta-\alpha)\cos\alpha}{(\hat{s} - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0})} \right] \cdot$$

$$(p_1+p_2)^\rho \epsilon^{*}_{p_4} \epsilon_\mu(p_1) \epsilon^\mu(p_2)\, \hat{s}\, \delta^{ab} \sum_{i=u,c,t} \left[\tau_i(\tau_i-1) f(\tau_i) + 2\tau_i \right] \qquad (270)$$

(218)

$$M_{3b} = -\frac{i \alpha_s G_F M_W}{2\sqrt{2}\,\pi} \cot\beta \, \frac{\delta^{ab} (p_1+p_2)^\rho \, \varepsilon^{*}_{p_4} \, \varepsilon_\mu(p_1) \, \varepsilon_\nu(p_2) \, \varepsilon^{\mu\nu\alpha\beta} \, p_{1\alpha} \, p_{2\beta}}{(\hat{s} - m_{A^0}^2 + i\, m_{A^0}\Gamma_{A^0})} \sum_{i=u,c,t} \tau_i f(\tau_i) \qquad (271)$$

$$M_{12} = M_{12a} + M_{12b} \qquad (272)$$

$$M_3 = M_{3a} + M_{3b} \qquad (273)$$

Defining:

$$\hat{C}_H = C_{H_b} \equiv \frac{1}{\cos\beta}\left[\frac{\sin\alpha \cos(\alpha-\beta)}{\hat{s} - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}} - \frac{\cos\alpha \sin(\alpha-\beta)}{\hat{s} - m_{H^0}^2 + i m_{H^0}\Gamma_{H^0}} \right] \qquad (274)$$

$$C_{H_t} \equiv \frac{-1}{\sin\beta}\left[\frac{\sin\alpha \sin(\alpha-\beta)}{\hat{s} - m_{H^0}^2 + i\, m_{H^0}\Gamma_{H^0}} + \frac{\cos\alpha \cos(\alpha-\beta)}{\hat{s} - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}} \right] \qquad (275)$$

$$\Rightarrow M_{12} = -\frac{\alpha_s G_F M_W}{4\sqrt{2}\,\pi} C_{H_b} \, (p_1+p_2)^\rho \, \varepsilon^{\alpha}_{p_4} \, \varepsilon_\mu(p_1) \, \varepsilon^\mu(p_2) \, \delta^{ab} \, \hat{s} \, [2\tau_b + \tau_b(\tau_b - 1) f(\tau_b)]$$

$$- \frac{\alpha_s G_F M_W}{2\sqrt{2}\,\pi} C_{H_t} \, (p_1+p_2)^\rho \, \varepsilon^{\alpha}_{p_4} \, \varepsilon_\mu(p_1) \, \varepsilon^\mu(p_2) \, \delta^{ab} \, \hat{s} \, [2\tau_t + \tau_t(\tau_t - 1) f(\tau_t)] \qquad (276)$$

$$M_{12} = -\frac{\alpha_s G_F M_W}{4\sqrt{2}\,\pi} (p_1+p_2)^\rho \, \varepsilon^{\alpha}_{p_4} \, \varepsilon_\mu(p_1) \, \varepsilon^\mu(p_2) \, \delta^{ab} \, \hat{s} \, \Big\{ C_{H_b} [2\tau_b + \tau_b(\tau_b - 1) f(\tau_b)]$$

$$+ C_{H_t} [2\tau_t + \tau_t(\tau_t - 1) f(\tau_t)] \Big\} \qquad (277)$$

$$\boxed{M_{12} = -\frac{\alpha_s G_F M_W}{4\sqrt{2}\,\pi} (p_1+p_2)^\rho \, \varepsilon^{\nu}_{p_4} \, \varepsilon_\mu(p_1) \, \varepsilon^\mu(p_2) \, \delta^{ab} \, \hat{s} \sum_{i=b,t} \{ C_{H_i} [2\tau_i + \tau_i(\tau_i - 1) f(\tau_i)] \qquad (278)}$$

Defining:

$$C_A = C_{A_b} \equiv \frac{\tan\beta}{\hat{s} - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0}} \qquad (279)$$

and

$$C_{A_t} \equiv \frac{\cot\beta}{\hat{s} - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0}} \qquad (280)$$

$$M_3 = -\frac{i\alpha_s G_F M_W}{2\sqrt{2}\pi}\,\delta^{ab} C_{Ab}\,(P_1+P_2)^\rho\,\varepsilon^*_{\rho 4}\,\varepsilon_\mu(P_1)\,\varepsilon_\nu(P_2)\,\varepsilon^{\mu\nu\alpha\beta} P_{1\alpha}P_{2\beta}\,\tau_b f(\tau_b)$$

$$-\frac{i\alpha_s G_F M_W}{2\sqrt{2}\pi}\,\delta^{ab} C_{At}\,(P_1+P_2)^\rho\,\varepsilon^*_{\rho 4}\,\varepsilon_\mu(P_1)\,\varepsilon_\nu(P_2)\,\varepsilon^{\mu\nu\alpha\beta} P_{1\alpha}P_{2\beta}\,\tau_t f(\tau_t)$$

$$M_3 = -\frac{i\alpha_s G_F M_W}{2\sqrt{2}\pi}\,\delta^{ab}(P_1+P_2)^\rho\,\varepsilon^*_{\rho 4}\,\varepsilon_\mu(P_1)\,\varepsilon_\nu(P_2)\,\varepsilon^{\mu\nu\alpha\beta} P_{1\alpha}P_{2\beta}\,[\,C_{Ab}\tau_b f(\tau_b)$$
$$+\,C_{At}\tau_t f(\tau_t)\,]$$

$$\boxed{M_3 = -\frac{i\alpha_s G_F M_W}{2\sqrt{2}\pi}\,\delta^{ab}(P_1+P_2)^\rho\,\varepsilon^*_{\rho 4}\,\varepsilon_\mu(P_1)\,\varepsilon_\nu(P_2)\,\varepsilon^{\mu\nu\alpha\beta} P_{1\alpha}P_{2\beta}\sum_{i=b,t}[\,C_{Ai}\tau_i f(\tau_i)]}$$

$$\delta^{ab}\delta^{ab} = 8 \qquad (281)$$

$$|M_3|^2 = \frac{\alpha_s^2 G_F^2 M_W^2}{4\times 2\pi^2}(P_1+P_2)^\rho(P_1+P_2)^\sigma\sum_i \varepsilon^*_{\rho 4}\,\varepsilon_{\sigma 4}\,\varepsilon_\mu(P_1)\,\varepsilon^*_{\mu'}(P_1)\,\varepsilon_\nu(P_2)\,\varepsilon^*_{\nu'}(P_2)$$
$$\cdot\,\varepsilon^{\mu\nu\alpha\beta}P_{1\alpha}P_{2\beta}\,\varepsilon^{\mu'\nu'\alpha'\beta'}P_{1\alpha'}P_{2\beta'}\left|\sum_i C_{Ai}\tau_i f(\tau_i)\right|^2\cdot 8$$

$$|M_3|^2 = \frac{\alpha_s^2 G_F^2 M_W^2}{4\times 2\pi^2}(P_1+P_2)^\rho(P_1+P_2)^\sigma\left(-g_{\rho\sigma}+\frac{P_{4\rho}P_{4\sigma}}{M_W^2}\right)(-g_{\mu\mu'})(-g_{\nu\nu'})$$
$$\varepsilon^{\mu\nu\alpha\beta}\,\varepsilon^{\mu'\nu'\alpha'\beta'}P_{1\alpha}P_{2\beta}P_{1\alpha'}P_{2\beta'}\left|\sum_i C_{Ai}\tau_i f(\tau_i)\right|^2\cdot 8$$

$$|M_3|^2 = \frac{\alpha_s^2 G_F^2 M_W^2}{4\times 2\pi^2}(P_1+P_2)^\rho(P_1+P_2)^\sigma\left(-g_{\rho\sigma}+\frac{P_{4\rho}P_{4\sigma}}{M_W^2}\right)\varepsilon^{\mu\nu\alpha\beta}\,\varepsilon_{\mu\nu}{}^{\alpha'\beta'}P_{1\alpha}P_{2\beta}$$
$$P_{1\alpha'}P_{2\beta'}\left|\sum_i C_{Ai}\tau_i f(\tau_i)\right|^2\cdot 8$$

but : $\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\mu\nu}{}^{\alpha'\beta'} = -2\,(g^{\alpha\alpha'}g^{\beta\beta'} - g^{\alpha\beta'}g^{\beta\alpha'})$ (282)

$$\Rightarrow \quad \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\mu\nu}{}^{\alpha'\beta'}P_{1\alpha}P_{2\beta}P_{1\alpha'}P_{2\beta'} = -2\,(g^{\alpha\alpha'}g^{\beta\beta'} - g^{\alpha\beta'}g^{\alpha'\beta})P_{1\alpha}P_{2\beta}P_{1\alpha'}P_{2\beta'}$$
$$= -2\,(P_{1\alpha}P_{2\beta}P_1^\alpha P_2^\beta - P_{1\alpha}P_{2\beta}P_1^\beta P_2^\alpha)$$

$$\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\mu\nu}{}^{\alpha'\beta'}P_{1\alpha}P_{2\beta}P_{1\alpha'}P_{2\beta'} = -2\,(\overset{0}{\cancel{P_1^2}}\,\overset{0}{\cancel{P_2^2}} - (P_1\cdot P_2)^2) = 2\,(P_1\cdot P_2)^2 = \frac{\hat{s}^2}{2}.$$

$$\Rightarrow \quad |M_3|^2 = \frac{\alpha_s^2 G_F^2 \cancel{M_W^2}}{4\times 2\pi^2}\,\frac{1}{4\cancel{M_W^2}}\,\lambda(\hat{s}, M_H^2, M_W^2)\,\frac{\hat{s}^2}{2}\left|\sum_i C_{Ai}\tau_i f(\tau_i)\right|^2\cdot 8$$

$$\boxed{|M_3|^2 = 8\cdot\frac{\alpha_s^2 G_F^2}{4\times 16\pi^2}\,\hat{s}^2\,\lambda(\hat{s}, M_H^2, M_W^2)\left|\sum_{i=b,t} C_{Ai}\tau_i f(\tau_i)\right|^2} \qquad (283)$$

¡ OK!

$$M_{12} M_3^{*} = \frac{i\alpha_s^2 G_F^2 M_W^2}{4\times 4\pi^2} (P_1+P_2)^{\rho} (P_1+P_2)^{\sigma} \sum_{\lambda} \varepsilon^{*}_{\rho 4}\, \varepsilon_{\sigma 4}\, \overbrace{\varepsilon_{\mu}(P_1)\, \varepsilon^{*}_{\mu'}(P_1)}^{-g_{\mu\mu'}}\, \overbrace{\varepsilon^{\mu}(P_2)\, \varepsilon^{*}_{\nu}(P_2)}^{-\delta^{\mu}{}_{\nu}}$$

$$\varepsilon^{\mu'\nu\alpha\beta} P_{1\alpha} P_{2\beta}\, \hat{s} \left\{ \sum_{i=b,t} \left[C_{Hi} \left(T_i + T_i (T_i - 1) f(T_i) \right) \right] \right\} \cdot$$

$$\left\{ \sum_{i=b,t} C^{*}_{Ai}\, T_i\, f^{*}(T_i) \right\} \qquad (284)$$

$$M_{12} M_3^{*} = \frac{i\alpha_s^2 G_F^2 M_W^2}{4\times 4\pi^2} (P_1+P_2)^{\rho} (P_1+P_2)^{\sigma} \left(-g_{\rho\sigma} + \frac{P_{4\rho} P_{4\sigma}}{M_W^2} \right) \underbrace{g_{\nu\mu'}\, \varepsilon^{\mu'\nu\alpha\beta}}_{=0} P_{1\alpha} P_{2\beta}\, \hat{s}$$

$$\{ \quad \} \{ \quad \}$$

$$\Rightarrow \boxed{M_{12} M_3^{*} = 0} \qquad (285)$$

$$\Rightarrow \boxed{M_3 M_{12}^{*} = 0} \qquad (286)$$

$$\frac{d\sigma_{\Delta}}{dt} = \frac{1}{64\pi \hat{s}\, p_i^2} \overline{|M|^2} \qquad (287)$$

color factor (gluons)

$$= \frac{4\times \cancel{4}}{64\pi \hat{s}^2} \cdot \frac{1_{8}}{\cancel{4}}\Big)^2 \left\{ \left(\frac{\alpha_s^2 G_F^2}{8\times 8\pi^2 \times 4} \left| \sum_{i=b,t} \left\{ C_{Hi} \left[2T_i + T_i (T_i - 1) f(T_i) \right] \right\} \right|^2 \cdot \lambda(\hat{s}, M_{H^{\pm}}^2, M_W^2) \right. \right.$$

$$\left. + \frac{\alpha_s^2 G_F^2}{8\times 8\; 2\pi^2 \times 4} \cdot \lambda \cdot \left| \sum_{i=b,t} C_{Ai} T_i f(T_i) \right|^2 \right\}$$

$$= \frac{\cancel{4}\times \alpha_s^2 G_F^2}{\cancel{4}\times 4096\pi^3} \lambda(\hat{s}, M_{H^{\pm}}^2, M_W^2) \left\{ \left| \sum_{i=b,t} \left\{ C_{Hi} \left[T_i + T_i (T_i - 1) f(T_i) \right] \right\} \right|^2 \right.$$

$$\left. + \frac{1}{2} \left| \sum_{i=b,t} C_{Ai} T_i f(T_i) \right|^2 \right\}$$

$$\boxed{ \left. \frac{d\sigma_{\Delta}}{dt} \right|_{gg\to H^- W^+} = \frac{\alpha_s^2 G_F^2}{4096\pi^3} \lambda(\hat{s}, M_{H^{\pm}}^2, M_W^2) \left\{ \left| \sum_{i=b,t} \left\{ C_{Hi} \left[2T_i + T_i (T_i - 1) f(T_i) \right] \right\} \right|^2 + \frac{1}{2} \left| \sum_{i=b,t} C_{Ai} T_i f(T_i) \right|^2 \right\} } \qquad (288)$$

where

$$f(\tau_i) = \begin{cases} -2\left(\arcsin\left(\frac{1}{\tau_i^{1/2}}\right)\right)^2 & \tau_i > 1 \\ \frac{1}{2}\left[\ln\left(\frac{1+(1-\tau_i)^{1/2}}{1-(1-\tau_i)^{1/2}}\right) - i\pi\right]^2 & \tau_i \leq 1 \end{cases}$$

and $\tau_i = \frac{4m_i^2}{\hat{s}}$

$$\boxed{\frac{d\sigma_\Delta}{dt}(gg \to H^+W^-) = \frac{d\sigma_\Delta}{dt}(gg \to H^-W^+)}$$ because of charge-conjugation invariance.

(289)

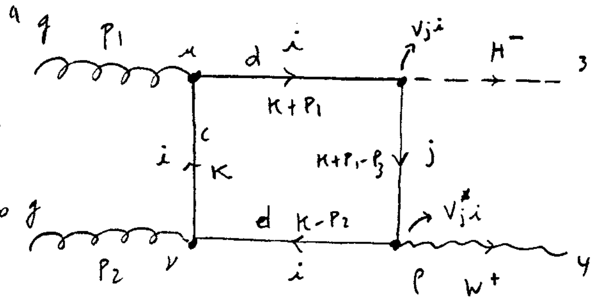
$$\therefore \boxed{\frac{d\sigma_\Delta}{dt}(gg \to H^\pm W^\mp) = \frac{\alpha_s^2 G_F^2}{2048\pi^3}\lambda(\hat{s}, M_{H^\pm}^2, M_W^2)\left\{\left|\sum_{i=b,t}\{C_{Hi}[2\tau_i + \tau_i(\tau_i - 1)f(\tau_i)]]\}\right|^2 + \frac{1}{2}\left|\sum_{i=b,t} C_{Ai}\tau_i f(\tau_i)\right|^2\right\}}$$

(290)

¡ok!

$gg \rightarrow H^- W^+$

①

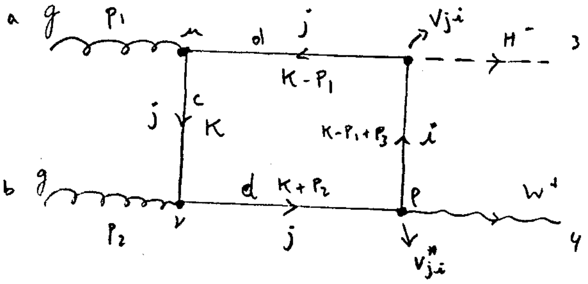


$i = d, s, b$
$j = u, c, t$

c, d are color indices

\+

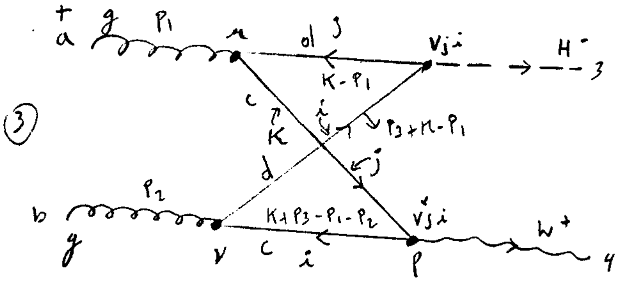
②



$P_1 + P_2 = P_3 + P_4$

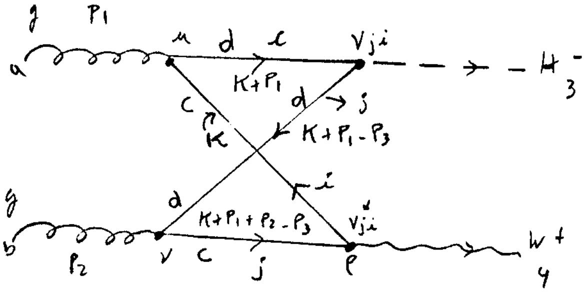
\+

③



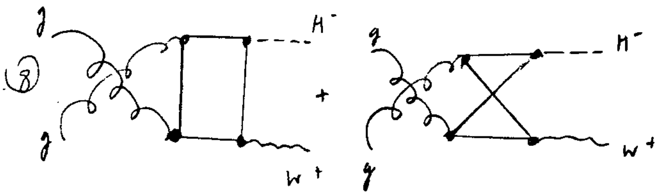
\+

④



\+ Crossed :

⑤, ... ⑧

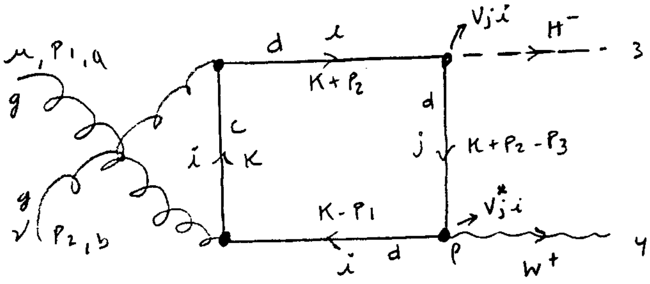


The crossed diagrams are:

i = d, s, b
j = u, c, t

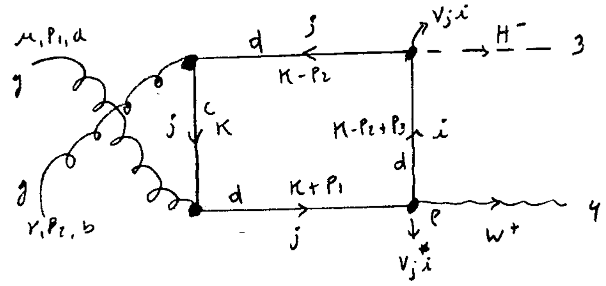
c/d are color indices

⑤



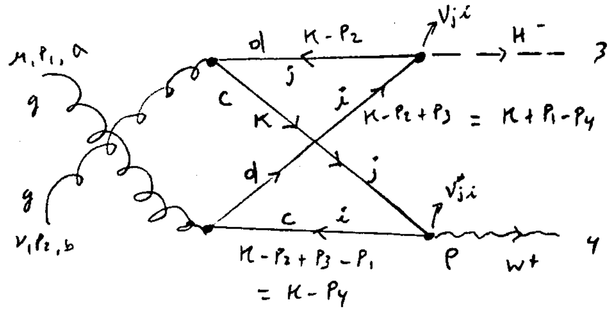
+

⑥



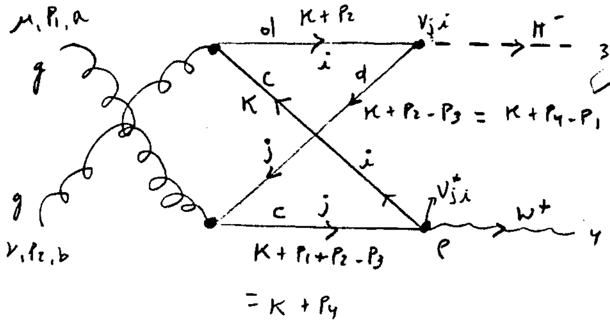
+

⑦

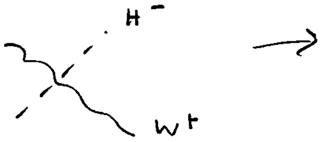


+

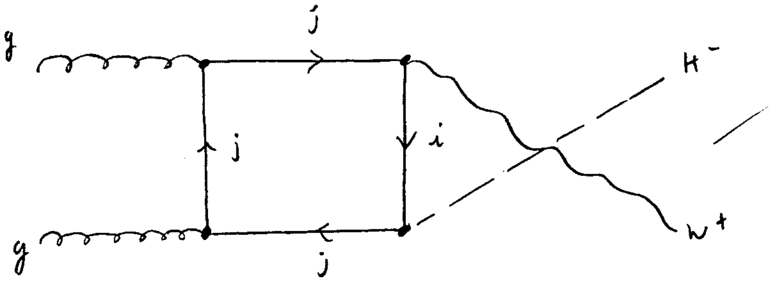
⑧



+ Crossed

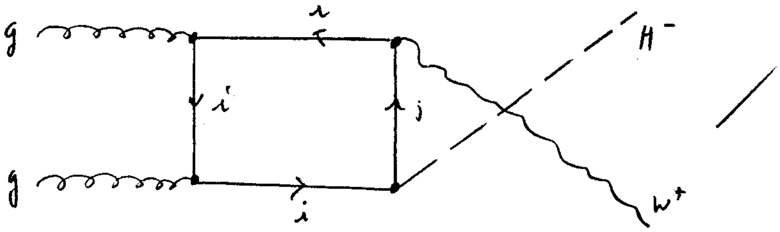


(Total = 16)

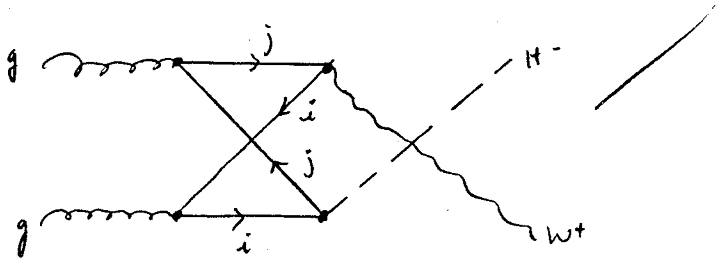


$j = u, c, t$
$i = d, s, b$

+

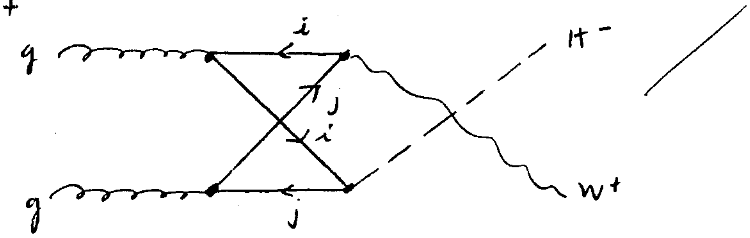


+

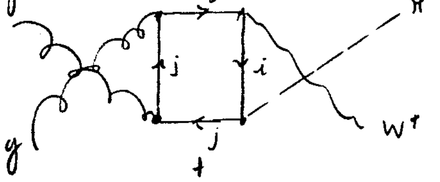


(in W^+ from $j \to i$)

+

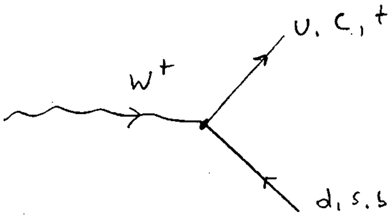


+



+

etc



$$\Lambda^d = \Lambda^{d-1}\Lambda^\gamma \quad \Lambda^{\frac{d}{2}-1}$$

$$\gamma : [\mu]$$

$$\gamma = 2 - \frac{d}{2} = \frac{4-d}{2}$$

$$-iM_{\Box 1} = \varepsilon^*_{4\rho}\sum_{i,j}\int\frac{d^dk}{(2\pi)^d}\, Tr\left\{ -\frac{ig}{\sqrt{2}}\gamma^\rho \frac{1}{2}(1-\gamma^5) \quad V^*_{ji}\frac{i}{(\not{k}-\not{p}_2-m_i)} \quad (-ig_s\gamma^\nu T^b_{cd})\frac{i}{(\not{k}-m_i)} \right.$$

$$(-ig_s\gamma^\mu T^a_{dc})\frac{i}{(\not{k}+\not{p}_1)-m_i} \quad \frac{ig}{2\sqrt{2}M_W} V_{ji}\{m_i \tan\beta(1+\gamma^5) + m_j\cot\beta(1-\gamma^5)\}\cdot$$

$$\left.\cdot\frac{i}{(\not{k}+\not{p}_1-\not{p}_3-m_j)}\right\} \quad \mu^* \quad \varepsilon_{1\mu}\,\varepsilon_{2\nu} \qquad (291)$$

$$\text{where: } \mu^* = \underset{\downarrow g}{\mu_1}^{(4-d)}\ \underset{\downarrow \nu}{\mu_2}^{(\frac{4-d}{2})}\ \underset{\downarrow H^-}{\mu_3}^{(\frac{4-d}{2})}$$

$$-iM_{\Box 1} = -\varepsilon^*_{4\rho}\frac{g^2 g_s^2}{8M_W}\sum_{i,j}|V_{ji}|^2\int\frac{d^dk}{(2\pi)^d}\, Tr\left\{\gamma^\rho(1-\gamma^5)(\not{k}-\not{p}_2+m_i)\gamma^\nu(\not{k}+m_i)\gamma^\mu(\not{k}+\not{p}_1+m_i)\right.$$

$$\left.[m_i\tan\beta(1+\gamma^5)+m_j\cot\beta(1-\gamma^5)](\not{k}+\not{p}_1-\not{p}_3+m_j)\right\}\cdot\varepsilon_{1\mu}\varepsilon_{2\nu}\mu^*$$

$$\cdot\frac{1}{[(K-P_2)^2-m_i^2][K^2-m_i^2][(K+P_1)^2-m_i^2][(K+P_1-P_3)^2-m_j^2]}Tr(T^aT^b) \qquad (292)$$

$$-iM_{\Box 2} = \varepsilon^*_{4\rho}\sum_{i,j}\int\frac{d^dk}{(2\pi)^d}\, Tr\left\{-\frac{ig}{\sqrt{2}}\gamma^\rho\frac{1}{2}(1-\gamma^5)V^*_{ji}\frac{i}{(\not{k}+\not{p}_2-m_j)}(-ig_s\gamma^\nu T^b_{dc})\frac{i}{(\not{k}-m_j)}\cdot\right.$$

$$\left.(-ig_s\gamma^\mu T^a_{cd})\frac{i}{\not{k}-\not{p}_1-m_j}V_{ji}\frac{ig}{2\sqrt{2}M_W}\{m_i\tan\beta(1+\gamma^5)+m_j\cot\beta(1-\gamma^5)\}\cdot\frac{i}{(\not{k}-\not{p}_1+\not{p}_3-m_i)}\right\}$$

$$\mu^*\ \varepsilon_{1\mu}\varepsilon_{2\nu} \qquad (293)$$

$$-iM_{\Box 2} = -\varepsilon^*_{4\rho}\frac{g^2g_s^2}{8M_W}\sum_{i,j}|V_{ji}|^2\int\frac{d^dk}{(2\pi)^d}\, Tr\left\{\gamma^\rho(1-\gamma^5)(\not{k}+\not{p}_2+m_j)\gamma^\nu(\not{k}+m_j)\gamma^\mu(\not{k}-\not{p}_1+m_j)\{m_i\tan\beta(1+\gamma^5)\right.$$

$$\left.+m_j\cot\beta(1-\gamma^5)\}(\not{k}-\not{p}_1+\not{p}_3+m_i)\right\}\varepsilon_{1\mu}\varepsilon_{2\nu}\mu^* \quad Tr(T^bT^a).$$

$$\cdot\frac{1}{[(K+P_2)^2-m_j^2][K^2-m_j^2][(K-P_1)^2-m_j^2][(K-P_1+P_3)^2-m_i^2]} \qquad (294)$$

$$-i\mathcal{M}_{\Box_3} = \varepsilon_{4\rho}^{*} \sum_{i,j} \int \frac{d^d k}{(2\pi)^d} \, Tr \Big\{ -\frac{ig}{\sqrt{2}} \gamma^\rho \frac{1}{2}(1-\gamma^5) V_{ji}^{*} \frac{i}{(\not k + \not p_3 - \not p_1 - \not p_2 - m_i)} \left(-ig_s \gamma^\nu T^b_{dc}\right) \frac{i}{\not k + \not p_3 - \not p_1 - m_i}$$

$$\frac{ig}{2\sqrt{2} M_W} V_{ji} \left\{ m_i \tan\beta \, (1+\gamma^5) + m_j \cot\beta \, (1-\gamma^5) \right\} \frac{i}{\not k - \not p_1 - m_j} \left(-ig_s \gamma^\mu T^a_{cd}\right) \frac{i}{\not k - m_j} \Big\}$$

$$\mathcal{M}^{*} \qquad \varepsilon_{1\mu} \varepsilon_{2\nu} \qquad (295)$$

$$-i\mathcal{M}_{\Box_3} = -\frac{\varepsilon_{4\rho}^{*} g^2 g_s^2}{8 M_W} \sum_{i,j} |V_{ji}|^2 \int \frac{d^d k}{(2\pi)^d} \, Tr \Big\{ \gamma^\rho (1-\gamma^5) (\not k + \not p_3 - \not p_1 - \not p_2 + m_i) \gamma^\nu (\not k + \not p_3 - \not p_1 + m_i)$$

$$\left\{ m_i \tan\beta \, (1+\gamma^5) + m_j \cot\beta \, (1-\gamma^5) \right\} (\not k - \not p_1 + m_j) \gamma^\mu (\not k + m_j) \Big\} \varepsilon_{1\mu} \varepsilon_{2\nu} \, \mathcal{M}^{*} \qquad Tr(T^a T^b).$$

$$\cdot \frac{1}{[(K + P_3 - P_1 - P_2)^2 - m_i^2][(K + P_3 - P_1)^2 - m_i^2][(K - P_1)^2 - m_j^2][K^2 - m_j^2]}$$

$$(296)$$

note that: $K + P_3 - P_1 - P_2 = K - P_4$

$P_3 - P_1 = P_2 - P_4$

$$-i\mathcal{M}_{\Box_4} = \varepsilon_{4\rho}^{*} \sum_{i,j} \int \frac{d^d K}{(2\pi)^d} \, Tr \Big\{ -\frac{ig}{\sqrt{2}} \gamma^\rho \frac{1}{2}(1-\gamma^5) V_{ji}^{*} \frac{i}{(\not K - m_i)} \left(-ig_s \gamma^\mu T^a_{dc}\right) \frac{i}{(\not K + \not p_1 - m_i)} \cdot$$

$$\cdot \frac{ig}{2\sqrt{2} M_W} V_{ji} \left\{ m_i \tan\beta \, (1+\gamma^5) + m_j \cot\beta \, (1-\gamma^5) \right\} \frac{i}{(\not K + \not p_1 - \not p_3 - m_j)} \left(-ig_s \gamma^\nu T^b_{cd}\right) \frac{i}{(\not K + \not p_1 + \not p_2 - \not p_3 - m_j}$$

$$\varepsilon_{1\mu} \varepsilon_{2\nu} \, \mathcal{M}^{*} \qquad (297)$$

$$-i\mathcal{M}_{\Box_4} = -\frac{\varepsilon_{4\rho}^{*} g^2 g_s^2}{8 M_W} \sum_{i,j} |V_{ji}|^2 \int \frac{d^d k}{(2\pi)^d} \, Tr \Big\{ \gamma^\rho (1-\gamma^5) (\not k + m_i) \gamma^\mu (\not k + \not p_1 + m_i) \left\{ m_i \tan\beta \, (1+\gamma^5) + \right.$$

$$\left. + m_j \cot\beta \, (1-\gamma^5) \right\} (\not k + \not p_1 - \not p_3 + m_j) \gamma^\nu (\not k + \not p_1 + \not p_2 - \not p_3 + m_j) \Big\} \varepsilon_{1\mu} \varepsilon_{2\nu} \, \mathcal{M}^{*} \quad . Tr(T^a T^b).$$

$$\cdot \frac{1}{[K^2 - m_i^2][(K + P_1)^2 - m_i^2][(K + P_1 - P_3)^2 - m_j^2][(K + P_1 + P_2 - P_3)^2 - m_j^2]}$$

$$(298)$$

note that: $P_1 - P_3 = P_4 - P_2$

$K + P_1 + P_2 - P_3 = K + P_4$

$$-iM_{\Box 5} = \varepsilon^{*}_{4\rho} \sum_{i,j} \int \frac{d^d k}{(2\pi)^d} \, \mathrm{Tr} \left\{ -\frac{ig}{\sqrt{2}} \gamma^{\rho} \frac{1}{2} (1-\gamma^5) V^{*}_{ji} \frac{i}{(\not{k} - \not{p}_1 - m_i)} \left(-ig_s \gamma^{\mu} T^{a}_{cd} \right) \frac{i}{(\not{k} - m_i)} \right.$$

$$\left(-ig_s \gamma^{\nu} T^{b}_{dc} \right) \frac{i}{(\not{k} + \not{p}_2 - m_i)} \, V_{ji} \, \frac{ig}{2\sqrt{2} M_W} \left\{ m_i \tan\beta (1+\gamma^5) + m_j \cot\beta (1-\gamma^5) \right\} .$$

$$\left. \cdot \frac{i}{(\not{k} + \not{p}_2 - \not{p}_3 - m_j)} \right\} \mu^{*} \quad \varepsilon_{1\mu} \varepsilon_{2\nu} \qquad (299)$$

$$-iM_{\Box 5} = -\frac{\varepsilon^{*}_{4\rho} \, g^2 g_s^2}{8 M_W} \sum_{i,j} |V_{ji}|^2 \int \frac{d^d k}{(2\pi)^d} \, \mathrm{Tr} \left\{ \gamma^{\rho} (1-\gamma^5) (\not{k} - \not{p}_1 + m_i) \gamma^{\mu} (\not{k} + m_i) \gamma^{\nu} (\not{k} + \not{p}_2 + m_i) \right.$$

$$\left. [m_i \tan\beta (1+\gamma^5) + m_j \cot\beta (1-\gamma^5)] (\not{k} + \not{p}_2 - \not{p}_3 + m_j) \right\} \varepsilon_{1\mu} \varepsilon_{2\nu} \mu^{*} \quad . \, \mathrm{Tr}(T^a T^b)$$

$$\cdot \frac{1}{[(k-p_1)^2 - m_i^2] [k^2 - m_i^2] [(k+p_2)^2 - m_i^2] [(k+p_2-p_3)^2 - m_j^2]} \qquad (300)$$

$$-iM_{\Box 6} = \varepsilon^{*}_{4\rho} \sum_{i,j} \int \frac{d^d k}{(2\pi)^d} \, \mathrm{Tr} \left\{ -\frac{ig}{\sqrt{2}} \gamma^{\rho} \frac{1}{2} (1-\gamma^5) V^{*}_{ji} \frac{i}{(\not{k} + \not{p}_1 - m_j)} \left(-ig_s \gamma^{\mu} T^{a}_{dc} \right) \frac{i}{(\not{k} - m_j)} \right.$$

$$\left(-ig_s \gamma^{\nu} T^{b}_{cd} \right) \frac{i}{(\not{k} - \not{p}_2 - m_j)} \, V_{ji} \, \frac{ig}{2\sqrt{2} M_W} [m_i \tan\beta (1+\gamma^5) + m_j \cot\beta (1-\gamma^5)] .$$

$$\left. \cdot \frac{i}{(\not{k} - \not{p}_2 + \not{p}_3 - m_i)} \right\} \mu^{*} \quad \varepsilon_{1\mu} \varepsilon_{2\nu} \qquad (301)$$

$$-iM_{\Box 6} = -\frac{\varepsilon^{*}_{4\rho} \, g^2 g_s^2}{8 M_W} \sum_{i,j} |V_{ji}|^2 \int \frac{d^d k}{(2\pi)^d} \, \mathrm{Tr} \left\{ \gamma^{\rho} (1-\gamma^5) (\not{k} + \not{p}_1 + m_j) \gamma^{\mu} (\not{k} + m_j) \gamma^{\nu} . \right.$$

$$\left. . \, (\not{k} - \not{p}_2 + m_j) [m_i \tan\beta (1+\gamma^5) + m_j \cot\beta (1-\gamma^5)] (\not{k} - \not{p}_2 + \not{p}_3 + m_i) \right\} \varepsilon_{1\mu} \varepsilon_{2\nu} \mu^{*} \, .$$

$$\mathrm{Tr}(T^a T^b) \cdot \frac{1}{[(k+p_1)^2 - m_j^2] [k^2 - m_j^2] [(k-p_2)^2 - m_j^2] [(k-p_2+p_3)^2 - m_i^2]} \qquad (302)$$

$$-iM_{\Box_7} = \varepsilon^{*}_{4\rho} \sum_{i,j} \int \frac{d^d k}{(2\pi)^d} \mathrm{Tr}\Big\{ -\frac{ig}{\sqrt{2}} \gamma^\rho \frac{1}{2}(1-\gamma^5) V^{*}_{ji} \frac{i}{(\not{k} - \not{p}_2 + \not{p}_3 - \not{p}_1 - m_i)} (-ig_s \gamma^\mu T^a_{dc}) \cdot$$

$$\cdot \frac{i}{(\not{k} - \not{p}_2 + \not{p}_3 - m_i)} V_{ji} \frac{ig}{2\sqrt{2} M_W} [m_i \tan\beta (1+\gamma^5) + m_j \cot\beta (1-\gamma^5)] \frac{i}{(\not{k} - \not{p}_2 - m_j)} (-ig_s \gamma^\nu T^b_{cd})$$

$$\cdot \frac{i}{(\not{k} - m_j)} \Big\} \mu^{*} \quad \varepsilon_{1\mu} \varepsilon_{2\nu} \qquad (303)$$

$$-iM_{\Box 7} = -\frac{\varepsilon^{*}_{4\rho} g^2 g_s^2}{8 M_W} \sum_{i,j} |V_{ji}|^2 \int \frac{d^d k}{(2\pi)^d} \mathrm{Tr}\Big\{ \gamma^\rho (1-\gamma^5) (\not{k} + \not{p}_3 - \not{p}_1 - \not{p}_2 + m_i) \gamma^\mu \cdot$$

$$\cdot (\not{k} - \not{p}_2 + \not{p}_3 + m_i) [m_i \tan\beta (1+\gamma^5) + m_j \cot\beta (1-\gamma^5)] (\not{k} - \not{p}_2 + m_j) \gamma^\nu (\not{k} + m_j) \Big\} \cdot$$

$$\mathrm{Tr}(T^a T^b) \cdot \mu^{*} \quad \varepsilon_{1\mu} \varepsilon_{2\nu} \cdot \frac{1}{[(K+P_3-P_1-P_2)^2 - m_i^2][(K+P_3-P_2)^2 - m_i^2][(K-P_2)^2 - m_j^2][K^2 - m_j^2]} \qquad (304)$$

Where $K - P_2 + P_3 = K + P_1 - P_4$

and $K + P_3 - P_1 - P_2 = K - P_4$

$$-iM_{\Box 8} = \varepsilon^{*}_{4\rho} \sum_{i,j} \int \frac{d^d k}{(2\pi)^d} \mathrm{Tr}\Big\{ -\frac{ig}{\sqrt{2}} \gamma^\rho \frac{1}{2}(1-\gamma^5) V^{*}_{ji} \frac{i}{(\not{k} - m_i)} (-ig_s \gamma^\nu T^b_{dc}) \frac{i}{(\not{k} + \not{p}_2 - m_i)}$$

$$\frac{ig}{2\sqrt{2} M_W} V_{ji} [m_i \tan\beta (1+\gamma^5) + m_j \cot\beta (1-\gamma^5)] \frac{i}{(\not{k} + \not{p}_2 - \not{p}_3 - m_j)} (-ig_s \gamma^\mu T^a_{cd}) \cdot$$

$$\cdot \frac{i}{(\not{k} + \not{p}_1 + \not{p}_2 - \not{p}_3 - m_j)} \Big\} \mu^{*} \quad \varepsilon_{1\mu} \varepsilon_{2\nu} \qquad (305)$$

$$-iM_{\Box 8} = -\frac{\varepsilon^{*}_{4\rho} g^2 g_s^2}{8 M_W} \sum_{i,j} |V_{ji}|^2 \int \frac{d^d k}{(2\pi)^d} \mathrm{Tr}\Big\{ \gamma^\rho (1-\gamma^5) (\not{k} + m_i) \gamma^\nu (\not{k} + \not{p}_2 + m_i) \cdot [m_i \tan\beta (1+\gamma^5) +$$

$$+ m_j \cot\beta (1-\gamma^5)] (\not{k} + \not{p}_2 - \not{p}_3 + m_j) \gamma^\mu (\not{k} + \not{p}_1 + \not{p}_2 - \not{p}_3 + m_j) \Big\} \mu^{*} \quad \varepsilon_{1\mu} \varepsilon_{2\nu} \mathrm{Tr}(T^a T^b) .$$

$$\cdot \frac{1}{[K^2 - m_i^2][(K+P_2)^2 - m_i^2][(K+P_2-P_3)^2 - m_j^2][(K+P_1+P_2-P_3)^2 - m_j^2]} \qquad (306)$$

Where $K + P_2 - P_3 = K + P_4 - P_1$

and $K + P_1 + P_2 - P_3 = K + P_4$

Rapidity of a particle:

$$y \equiv \frac{1}{2} \ln\left(\frac{E+P_z}{E-P_z}\right)$$ ①

$$\underset{\substack{\downarrow \\ \text{transverse mass}}}{m_T^2} = m^2 + p_x^2 + p_y^2 = m^2 + P_T^2$$ ②

$$\cosh y = \cosh \ln\left(\frac{E+P_z}{E-P_z}\right)^{1/2}$$

$$= \frac{e^{\ln\left(\frac{E+P_z}{E-P_z}\right)^{1/2}} + e^{\ln\left(\frac{E+P_z}{E-P_z}\right)^{-1/2}}}{2}$$

$$= \frac{\left(\frac{E+P_z}{E-P_z}\right)^{1/2} + \left(\frac{E-P_z}{E+P_z}\right)^{1/2}}{2}$$

$$= \frac{E+\cancel{P_z}+E-\cancel{P_z}}{2\,(E^2-P_z^2)^{1/2}} = \frac{E}{m_T}$$

$$\Rightarrow \boxed{E = m_T \cosh y}$$ ③

$$\sinh y = \sinh \ln\left(\frac{E+P_z}{E-P_z}\right)^{1/2} = \frac{e^{\ln\left(\frac{E+P_z}{E-P_z}\right)^{1/2}} - e^{\ln\left(\frac{E+P_z}{E-P_z}\right)^{-1/2}}}{2}$$

$$= \frac{1}{2}\left\{\left(\frac{E+P_z}{E-P_z}\right)^{1/2} - \left(\frac{E-P_z}{E+P_z}\right)^{1/2}\right\}$$

$$= \frac{1}{2} \; \frac{\cancel{E}+P_z-\cancel{E}+P_z}{(E^2-P_z^2)^{1/2}} = \frac{P_z}{m_T}$$

$$\Rightarrow \boxed{P_z = m_T \sinh y}$$ ④

$$\Rightarrow \boxed{p^\mu = (E, \vec{p}) = (m_T \cosh y,\ p_x,\ p_y,\ m_T \sinh y)}$$ ⑤

$$y = \ln\left(\frac{E+P_z}{E-P_z}\right)^{1/2} = \ln \frac{(E+P_z)}{(E+P_z)^{1/2}\,(E-P_z)^{1/2}} = \ln\left(\frac{E+P_z}{m_T}\right)$$

$$\boxed{y = \ln\left(\frac{E+P_z}{m_T}\right)}$$ ⑥

$$\tanh \ln\left(\frac{E+P_z}{m_T}\right) = \frac{e^{\ln\frac{(E+P_z)}{m_T}} - e^{\ln\frac{m_T}{(E+P_z)}}}{e^{\ln\frac{(E+P_z)}{m_T}} + e^{\ln\frac{m_T}{(E+P_z)}}}$$

$$= \frac{\frac{(E+P_z)}{m_T} - \frac{m_T}{(E+P_z)}}{\frac{(E+P_z)}{m_T} + \frac{m_T}{E+P_z}}$$

$$= \frac{(E+P_z)^2 - m_T^2}{(E+P_z)^2 + m_T^2}$$

$$= \frac{E^2 + P_z^2 - m_T^2 + 2EP_z}{E^2 + P_z^2 + 2EP_z + m_T^2}$$

$$= \frac{\cancel{m^2} + \cancel{P_x^2} + \cancel{P_y^2} + P_z^2 + P_z^2 - \cancel{m^2} - \cancel{P_x^2} - \cancel{P_y^2} + 2EP_z}{E^2 + \cancel{P_z^2} + 2E\cdot P_z + E^2 - \cancel{P_z^2}}$$

$$= \frac{\cancel{2}P_z\,\cancel{(E+P_z)}}{\cancel{2}E\,\cancel{(E+P_z)}} = \frac{P_z}{E}$$

$$\Rightarrow \boxed{\ln\left(\frac{E+P_z}{m_T}\right) = Y = \tanh^{-1}\left(\frac{P_z}{E}\right)} \quad ⑦$$

$$Y = \frac{1}{2}\ln\left(\frac{E+P_z}{E-P_z}\right) = \frac{1}{2}\ln\left(\frac{E^2 + 2EP_z + P_z^2}{E^2 - P_z^2}\right)$$

$$Y = \frac{1}{2}\ln\left(\frac{m^2 + p^2 + 2(m^2+p^2)^{1/2}P_z + P_z^2}{m^2 + p^2 - P_z^2}\right)$$

Defining $\cos\theta = \frac{P_z}{P}$

$$Y = \frac{1}{2}\ln\left(\frac{1 + \frac{m^2}{p^2} + 2\frac{P_z}{p}\left(1 + \frac{m^2}{p^2}\right)^{1/2} + \frac{P_z^2}{p^2}}{1 + \frac{m^2}{p^2} - \frac{P_z^2}{p^2}}\right)$$

If $P \gg m$

$$Y \simeq \frac{1}{2}\ln\left(\frac{1 + \frac{m^2}{p^2} + 2\cos\theta\left(1 + \frac{1}{2}\frac{m^2}{p^2}\right) + \cos^2\theta}{1 + \frac{m^2}{p^2} - \cos^2\theta}\right)$$

$$Y \simeq \frac{1}{2}\ln\left(\frac{(1+\cos\theta)^2 + \frac{m^2}{p^2}(1+\cos\theta)}{\sin^2\theta + \frac{m^2}{p^2}}\right)$$

$$\cos\theta = 2\cos^2\frac{\theta}{2} - 1$$

$$\Rightarrow (1+\cos\theta) = 2\cos^2\frac{\theta}{2}$$

$$\sin^2\theta = 4\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}$$

$$\Rightarrow y \simeq \frac{1}{2}\ln\left(\frac{4\cos^4\frac{\theta}{2} + 2\frac{m^2}{p^2}\cos^2\frac{\theta}{2}}{4\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2} + \frac{m^2}{p^2}}\right)$$

$$y \simeq \frac{1}{2}\ln\left(\frac{\cos^2\frac{\theta}{2} + \frac{m^2}{2p^2}}{\sin^2\frac{\theta}{2} + \frac{m^2}{4p^2\cos^2\frac{\theta}{2}}}\right)$$

$$y \simeq -\frac{1}{2}\ln\left(\tan^2\frac{\theta}{2}\right)$$

$$\boxed{y \simeq -\ln\left(\tan\frac{\theta}{2}\right) = \eta} : \text{pseudorapidity} \qquad (8)$$

$$\sinh\eta = \frac{e^{-\ln(\tan\frac{\theta}{2})} - e^{\ln(\tan\frac{\theta}{2})}}{2}$$

$$\sinh\eta = \frac{\frac{1}{\tan\frac{\theta}{2}} - \tan\frac{\theta}{2}}{2}$$

$$\sinh\eta = \left(\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} - \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)\frac{1}{2}$$

$$\sinh\eta = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

$$\Rightarrow \boxed{\sinh\eta = \cot\theta} \qquad (9)$$

$$\cosh\eta = \frac{e^{-\ln(\tan\frac{\theta}{2})} + e^{\ln(\tan\frac{\theta}{2})}}{2} = \frac{\frac{1}{\tan\frac{\theta}{2}} + \tan\frac{\theta}{2}}{2}$$

$$\cosh\eta = \frac{\sec^2\frac{\theta}{2}}{2\tan\frac{\theta}{2}} = \frac{\cancel{\cos}\frac{\theta}{2}}{2\cos^{\cancel{2}}\frac{\theta}{2}\,\sin\frac{\theta}{2}} = \frac{1}{\sin\theta}$$

$$\boxed{\cosh \eta = \frac{1}{\sin\theta}}$$ ⑩

$$\tanh \eta = \cot\theta \cdot \sin\theta = \cos\theta$$

$$\boxed{\tanh\eta = \cos\theta}$$ ⑪

$$dP_z = m_T \cosh y \, dy$$

$$E = m_T \cosh y$$

$$\Rightarrow dP_z = E \, dy$$

or $$\boxed{\frac{dy}{dP_z} = \frac{1}{E}}$$ ⑫

$$P_T^2 = P_x^2 + P_y^2 = P^2 - P_z^2 = P^2(1-\cos^2\theta) = P^2\sin^2\theta$$

$$P_T^2 = P^2 \sin^2\theta$$

$$P = \frac{\lambda^{1/2}}{2\sqrt{\hat{s}}}$$

$$\Rightarrow \boxed{P_T^2 = \frac{\lambda}{4\hat{s}}\sin^2\theta}$$ ⑬

$$\lambda = \lambda(\hat{s}, M_{H^\pm}^2, M_W^2)$$

in $q\bar{q} \to H^- W^+$

$$e^y = \left(\frac{E+P_z}{E-P_z}\right)^{1/2}$$

$$e^y = \frac{(E+P_z)}{E^2-P_z^2} = \frac{E+P_z}{(m_T^2)^{1/2}}$$

$$\Rightarrow \boxed{E+P_z = m_T e^y}$$ ⑭

$$e^{-y} = \left(\frac{E-P_z}{E+P_z}\right)^{1/2}$$

$$e^{-y} = \frac{(E-P_z)}{\underset{\substack{\downarrow \\ m_T}}{(E^2-P_z^2)^{1/2}}}$$

$$\Rightarrow \boxed{E-P_z = m_T e^{-y}}$$ ⑮

$$\Rightarrow X = \frac{\cancel{2}E\sqrt{s} + [\cancel{4}E^2 s + \cancel{4}s(M_{H^\pm}^2 - M_W^2)]^{1/2}}{\cancel{2}s}$$

$$\boxed{X = \frac{\sqrt{s}}{s}\left\{E + [E^2 + (M_{H^\pm}^2 - M_W^2)]^{1/2}\right\}} \tag{21}$$

$$E = M_W\left(\frac{\lambda(\hat{s}, M_H^2, M_W^2)}{4\hat{s}M_W^2} + 1\right)^{1/2}$$

So at least $E = M_W$ and then

$$\boxed{X_{min} = \frac{(M_W + M_H)}{\sqrt{s}}} \tag{22} \quad \Rightarrow \hat{s}_{min} = \frac{(M_W+M_H)^2}{\cancel{s}}\cancel{s} = (M_W+M_H)^2$$

$$\therefore \sqrt{\hat{s}}_{min.} = M_W + M_H$$

$X = 1$ when

$$\sqrt{s} = E + [E^2 + (M_H^2 - M_W^2)]^{1/2}$$

$$(\sqrt{s} - E)^2 = E^2 + (M_H^2 - M_W^2)$$

$$(\sqrt{s})^2 - 2E\sqrt{s} + \cancel{E}^2 = \cancel{E}^2 + M_H^2 - M_W^2$$

$$E = \frac{s - M_H^2 + M_W^2}{2\sqrt{s}}$$

$$\Rightarrow$$

$$\boxed{M_W \leq E \leq \frac{s - M_H^2 + M_W^2}{2\sqrt{s}}} \tag{23}$$

For $M_H = 120$ GeV and because $M_W = 80.423$, if $\sqrt{s} = 2$ TeV

we have $X_{min} = 0.1 = 10\%$. If $M_H > 120$ GeV, X_{min} will be bigger

So we will take conservatively, $X = 0.25 \Rightarrow \sqrt{\hat{s}} = 500$ GeV. If $\sqrt{s} = 14$ TeV

$X_{min} = 0{,}014316.$ //

→ In the parton model the differential cross section for PP or $P\bar{P} \to H^\pm W^\mp X$ is

$$\frac{d^2\sigma}{dy\, dP_T^2}(AB \to HW + X) = \sum_{a,b}\int dx_a\, dx_b\, F_{a/A}(x_a, M_a^2)\, F_{b/B}(x_b, M_b^2)\, \hat{s}\frac{d\sigma}{d\hat{t}}(ab \to HW)$$

$$\times\, \delta(\hat{s} + \hat{t} + \hat{u} - M_W^2 - M_{H^\pm}^2) \tag{24}$$

where $F_{a/A}(x_a, M_a^2)$ is the parton density function and M_a^2 is the factorization

Scale, The same $F_{b/B}(x_b, M_b^2)$

$$M_a^2 = M_b^2 = \hat{s} = \mu^2$$
$\downarrow$
($H^\pm W^\mp$ invariant mass)

From ⑲

$$x_a = \frac{x_b \sqrt{s}\, m_T e^{y} + M_{H^\pm}^2 - M_W^2}{x_b s - \sqrt{s}\, m_T e^{-y}} = \frac{M_{H^\pm}^2 - \hat{t}}{x_b s - \sqrt{s}\, m_T e^{-y}} \Rightarrow x_a x_b s - x_a \sqrt{s}\, m_T e^{-y} = M_{H^\pm}^2 - \hat{t}$$

$$\underset{(x_b = 1)}{x_{a\,min}} = \left(\frac{\sqrt{s}\, m_T e^{y} + M_{H^\pm}^2 - M_W^2}{s - \sqrt{s}\, m_T e^{-y}} \right)$$

$$\delta\left(\hat{s} + \hat{t} + \hat{u} - M_W^2 - M_{H^\pm}^2\right) = \delta\left[x_b \left(x_a s - \sqrt{s}\, m_T e^{y}\right) - \left(x_a \sqrt{s}\, m_T e^{-y} + M_{H^\pm}^2 - M_W^2\right)\right]$$

$$= \frac{1}{x_a s - \sqrt{s}\, m_T e^{y}}\, \delta\left[x_b - \frac{\left(x_a \sqrt{s}\, m_T e^{-y} + M_{H^\pm}^2 - M_W^2\right)}{\left(x_a s - \sqrt{s}\, m_T e^{y}\right)}\right]$$

$\Rightarrow$

$$= \frac{1}{x_a s + \frac{(\hat{t} - M_W^2)}{x_b}}\, \delta\left[x_b - \frac{\left(x_a \sqrt{s}\, m_T e^{-y} + M_{H^\pm}^2 - M_W^2\right)}{\left(x_a s - \sqrt{s}\, m_T e^{y}\right)}\right]$$

$$= \frac{x_b}{x_a x_b s + \hat{t} - M_W^2}\, \delta\left[x_b - \frac{\left(x_a \sqrt{s}\, m_T e^{-y} + M_{H^\pm}^2 - M_W^2\right)}{\left(x_a s - \sqrt{s}\, m_T e^{y}\right)}\right]$$

$$= \frac{x_b}{M_{H^\pm}^2 + x_a \sqrt{s}\, m_T e^{-y} - M_W^2}\, \delta[\quad]$$

$$= \frac{x_b}{M_{H^\pm}^2 - \hat{u}}\, \delta[\quad]$$

$$\Rightarrow \frac{d^2\sigma}{dy\, dp_T^2}(AB \to HW + X) = \sum_{a,b} \int_{x_{a\,min}}^{1} dx_a \int_0^1 dx_b\, F_{a/A}(x_a, M_a^2)\, F_{b/B}(x_b, M_b^2)\, \hat{s} \cdot$$
$$\cdot \frac{d\sigma}{d\hat{t}}(ab \to HW)\, \frac{x_b}{M_{H^\pm}^2 - \hat{u}}\, \delta\left[x_b - \frac{\left(x_a \sqrt{s}\, m_T e^{-y} + M_{H^\pm}^2 - M_W^2\right)}{\left(x_a s - \sqrt{s}\, m_T e^{y}\right)}\right]$$

$$\therefore \boxed{\frac{d^2\sigma}{dy\, dp_T^2}(AB \to HW + X) = \sum_{a,b} \int_{x_{a\,min}}^{1} dx_a\, F_{a/A}(x_a, M_a^2)\, F_{b/B}(x_b, M_b^2)\, \frac{x_b \hat{s}}{M_{H^\pm}^2 - \hat{u}}\, \frac{d\sigma}{d\hat{t}}(ab \to HW)} \quad (25)$$

with $$x_{a\,min} = \frac{\sqrt{s}\, m_T\, e^{y} + M_{H^\pm}^2 - M_W^2}{s - \sqrt{s}\, m_T\, e^{-y}}$$

$$m_T = (M_W^2 + p_T^2)^{1/2}$$

$$\hat{u}\hat{t} = M_{H^\pm}^2 M_W^2 + \frac{\lambda \sin^2\theta}{4}$$

$$\hat{u}\hat{t} = M_{H^\pm}^2 M_W^2 + \hat{s}\, p_T^2$$

$$\Rightarrow \boxed{\hat{s}\, p_T^2 = \hat{u}\hat{t} - M_W^2 M_{H^\pm}^2} \quad (26)$$

$$x_b = \frac{x_a \sqrt{s}\, m_T\, e^{-y} + M_{H^\pm}^2 - M_W^2}{x_a s - \sqrt{s}\, m_T\, e^{y}}$$

$$\hat{s} = x_a x_b s \, .$$

y is the rapidity, p_T is the transverse momentum of $W^\pm$.

$$p_T^2 = \frac{\lambda(\hat{s}, M_W^2, M_{H^\pm}^2) \sin^2\theta}{4\hat{s}}$$

$$\hat{t} = \frac{M_W^2 + M_{H^\pm}^2 - \hat{s}}{2} + \frac{1}{2}\left(\overbrace{1 - \frac{4\hat{s}\, p_T^2}{\lambda}}^{\cos\theta}\right)^{1/2} \lambda^{1/2}(\hat{s}, M_W^2, M_{H^\pm}^2)$$

$$\hat{u} = \frac{M_W^2 + M_{H^\pm}^2 - \hat{s}}{2} - \frac{1}{2}\left(\underbrace{1 - \frac{4\hat{s}\, p_T^2}{\lambda}}_{\cos\theta}\right)^{1/2} \lambda^{1/2}(\hat{s}, M_W^2, M_{H^\pm}^2)$$

$$\cos\theta = \left(1 - \frac{4\hat{s}\, p_T^2}{\lambda}\right)^{1/2}$$

$$\frac{d^2\sigma}{dy\, dp_T^2}(p\bar{p} \to H^\mp W^\pm X) = \sum_{(q\,or\,g)} \int_{x_{a\,min}}^{1} dx_a\, F_q(x_a, M_a^2)\, F_q(x_b, M_b^2)\, \frac{x_b \hat{s}}{M_{H^\pm}^2 - \hat{u}} \, .$$

$$\cdot \frac{d\sigma}{d\hat{t}}\left(\begin{matrix} q\bar{q} \\ gg \end{matrix} \to H^\mp W^\pm\right)$$

$F_q(x, M^2)$ is the parton (or gluon) density function. M^2 is the factorization scale

$M^2 = \hat{s}$ (invariant mass of the pair $H^\mp W^\pm$)