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**Effect of Torsion on Shear Capacity of Slabs**

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**Effect of Torsion on Shear Capacity of Slabs**

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# Effect of torsion on shear capacity of slabs

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**ABSTRACT:** A large number of existing bridges in Europe and North-America are reaching the end of their devised service life. Therefore, it is necessary to improve the methods of assessment for existing bridges. One method, suitable for existing reinforced concrete slab bridges, is the Modified Bond Model. This method, however, currently only takes the effect of torsion for loads close to the edge into account in a simplified manner. In this study, finite element models are created of a slabs with two supports, three concentrated (prestressing) loads and a distributed load, representing a truck wheel print. The load is varied along the longitudinal and transverse directions of the slab to find the bending moments ( $m_x$  and  $m_y$ ) and torsional moments ( $m_{xy}$ ). The results is an expression for the effect of torsion in slabs, which can be used with the Modified Bond Model for assessment and design of slab bridges.

## 1 INTRODUCTION

Currently, a large number of existing bridges in Europe and North-America are reaching the end of their originally devised service life (Lantsoght et al. 2013a, Johansson et al. 2014, Teworte et al. 2015). Since replacing or demolishing all these structures is economically not viable, a sharper method for assessment is necessary. In the Netherlands, a considerable subset of the ageing bridge stock consists of reinforced concrete slab bridges. In particular, the shear capacity of slab bridges is subject to discussion in the Netherlands. Therefore, over the last few years, the shear capacity of reinforced concrete slabs has been studied experimentally (Lantsoght et al. 2013c, Lantsoght et al. 2014b, Lantsoght et al. 2015b) as well as numerically (Falbr 2011, Voormeeren 2011, Doorgeest 2012, van Hemert 2012)

One analytical method that was proposed to assess the capacity of reinforced concrete slabs is the Modified Bond Model (Lantsoght et al. 2015a). However, for loads close to the edge, the effect of torsional moments reduces the capacity. Therefore, this effect needs to be quantified based on the loading position. Studying the relative effect of the torsional moments with respect to the bending moments is the topic of this paper.

## 2 LITERATURE REVIEW

Several methods had been developed to analyze slab behavior from using orthogonal beam systems to finite element methods to get more accurate stress fields and deflection values. For slab design purposes the Modified Bond Model currently takes into account the effect of torsion in a simplified manner.

The numerical technique applied to model and approach the deformations of the slabs is the finite element method based on dividing the slab in small symmetric areas and “the general method of analysis is to concentrate the load at the corners of the separated elements, and the restore continuity of slope and deflection at each node point..., so as to satisfy equilibrium and boundary condition requirements” (Park and Gamble 2000).

The results of the current analysis will be implemented in the Modified Bond Model (Lantsoght 2012, Lantsoght 2013a, Lantsoght 2013b, Lantsoght 2014, Lantsoght et al. 2014a, Lantsoght et al. 2015c). This model is an adaptation of the Bond Model for concentric punching shear (Alexander 1990, Alexander and Simmonds 1992, Afhami 1997), so that it can be used for slabs under concentrated loads in non-axis-symmetric situations. The model is based on the lower bound theorem of plasticity, such as the strip method for flexure (Hillerborg 1975). The geometry of the slab is taken into account by factors which reduce the capacity of the strip, which reaches its maximum, ideal capacity for the theoretical case of an infinitely large slab with a single concentrated load in the middle (i.e., the theoretical idealized case of concentric punching shear). Currently, the Modified Bond Model takes the effect of torsion into account in a simplified way.

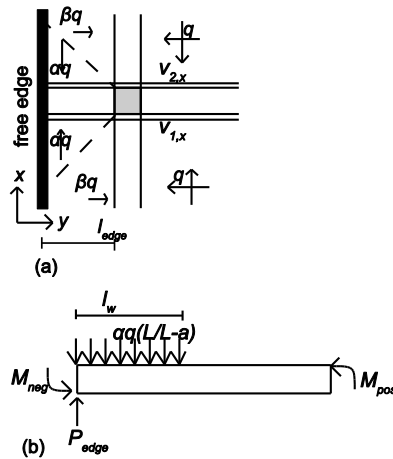


Figure 1. Modified Bond Model for loading close to a free edge: (a) resulting strips and quadrants, and loading in the quadrants; (b) loading on y-direction strip between load and free edge.

The applied maximum loading to determine the capacity of a strip is multiplied with a factor  $\beta = 1$  if the effect of torsion is not important (for example, when the load is placed in the middle of the width), and with  $\beta = 0$  when the effect of torsion is important and the capacity needs to be limited (for example, for loads close to the edge). This principle is illustrated in Figure 1.

### 3 DESCRIPTION OF FEM MODELS

Twelve linear elastic models of a 5000 mm x 2500 mm concrete slabs were created to evaluate the effect of torsion. The dimensions of the slab and the applied loading are based on experiment S1T1 (Lantsoght et al. 2013c) from a series of slabs tested in shear at the Stevin Laboratory of Delft University of Technology. S1T1 was used as a benchmark to develop the twelve models for this study.

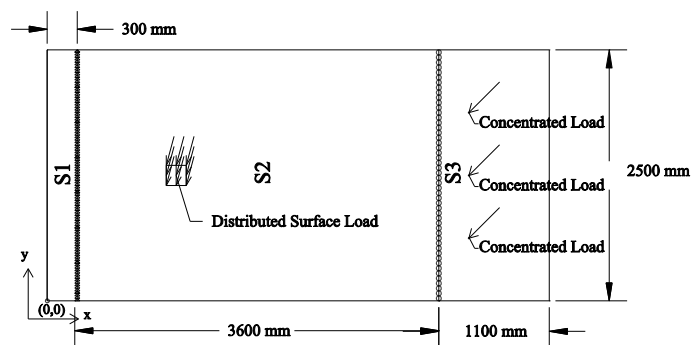


Figure 2: Slab Model Dimensions

Using SCIA Engineer 15.1 (Nemetschek-Scia 2015), a finite element software, the flexural and bending moments produced by a concentrated load over a modeled half-scale wheel print (as given by NEN-EN 1991-2:2003 (CEN 2003)) are analyzed, as well as the effect of the load close to the support and the free edge of the slab.

The structure was modeled using three 2D slab members consisting of shell elements with the dimensions as shown in Figure 1. The separate slab members are indicated as S1, S2 and S3 in Figure 1. The slab members are used to comply with the finite elements theory so each element is divided by a 100 x 100 mm mesh and the thickness of the shell elements is the total thickness of the slab, 300 mm.

The slab is supported by two line supports: the first is hinges, 300 mm from the edge of the slab to the center of the support and the second support consists of rollers, 3900 mm from the edge to the center of the support, to exemplify a simply supported structure. Three concentrated loads that represent prestressing bars creating a moment over the second support are placed 600 mm away from the second row of line supports, of magnitude 54.67 KN each.

Table 1: Models overview and load description.

Model	Magnitude (kN/m <sup>2</sup> )	<i>x</i> -coordinate (mm)	<i>y</i> -coordinate (mm)
L1	11950	1000	1250
L2	11950	1000	850
L3	11950	1000	438
L4	11950	700	438
L5	11950	700	850
L6	11950	700	1250
L7	11950	850	438
L8	11950	850	850
L9	11950	850	1250
L10	11950	2100	438
L11	11950	2100	850
L12	11950	2100	1250

In each of the twelve models the loading is applied by a distributed surface load over a wheel print. The location along the length and width of the slab of this load is varied to study the relative effect of torsion. Table 2 describes the different load positions applied in the models. The *x*-coordinate is taken from the origin of the slab, so that the distance *a*, the center-to-center distance between the load and the support, is the value of the *x*-coordinate minus 300 mm, see Figure 1.

## 4 RESULTS OF FEM ANALYSES

### 4.1 Averaging the observed ratios

The effective depth of the slab  $d$  is equal to 265 mm (the thickness is 300 mm minus a 20 mm of reinforcement bar diameter minus 25 mm of concrete cover). A distance  $2d$  from the position of the load is taken (Fig. 3) on all the twelve models in both directions (along the  $x$  and  $y$ ) to find the average of the moment's ratio  $m_{xy}/m_x$  along this distance. Also at a distance equal to  $4d$  this procedure is followed (Lantsoght et al. 2013b). This procedure ensures that the effect of peaks from the mesh is not affecting the results.

### 4.2 Results of finite element models

The developed finite element models are used to study the flexural and torsional moments. Then, to study the relative importance of torsion, the ratio of torsional moments to flexural moments are studied. The plots of the benchmark case S1T1, called model L1, are discussed in this paragraph. The results of the other models can be found in the background report (Valdivieso et al. 2015).

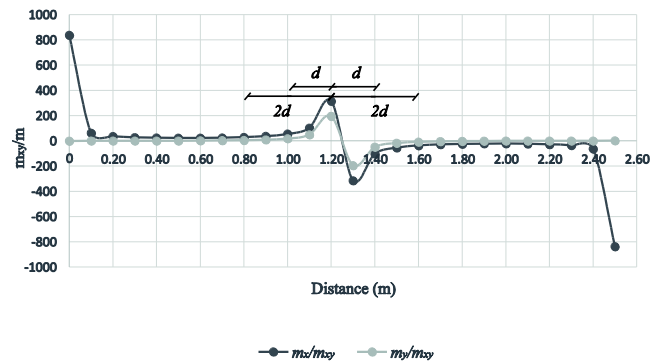


Figure 3. Averaging over a distance  $d$  from the center of the load in  $m_x/m_{xy}$  and  $m_y/m_{xy}$  ratios for case L1.

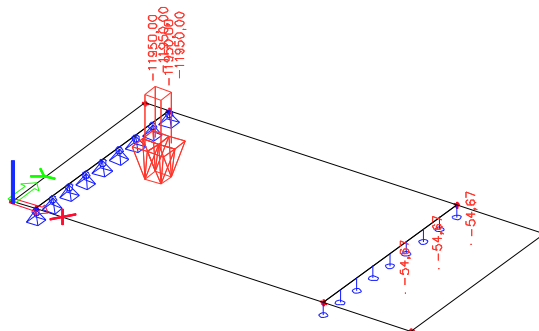


Figure 4. L1 Model with Distributed Surface Load representing wheel print with coordinates (1000 mm, 1250 mm). The concentrated loads beyond the second support create a moment over support 2.



First, the applied loading in the model, as discussed previously and given in Table 2, is shown in Figure 4. Then, the finite element software SCIA Engineer 15.1 (Nemetschek-Scia 2015), is used to determine the plot of the flexural moments in the span direction,  $m_x$ , and in the transverse direction,  $m_y$ . These plots are respectively shown in Figure 5 and Figure 6. In a next step, the plot of the torsional moments,  $m_{xy}$ , are determined, as shown in Figure 7. Figure 35, Figure 6 and Figure 7 are based on average magnitudes of the moments in the finite elements.

Once the plots of the moments in the slab are determined, the relative importance of torsion can be studied. This study is carried out by determining the ratios of the moments  $m_{xy}/m_x$  and  $m_{xy}/m_y$ . To visualize the effect of this moment ratio with positions relative from the point of application of the wheel print, the ratio is presented along a cut in the  $x$  and  $y$  direction. The cut in the  $x$  direction studies the longitudinal direction and has a constant  $y = 1250$  mm. The cut in the  $y$  direction studies the transverse direction, and has a constant  $x$  through the center of the applied load. For model L1, the constant value of  $x$  for the transverse cut is 1000 mm, see Table 2. The resulting ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  are then shown for the longitudinal cut in Figure 8 and for the transverse cut in Figure 9.

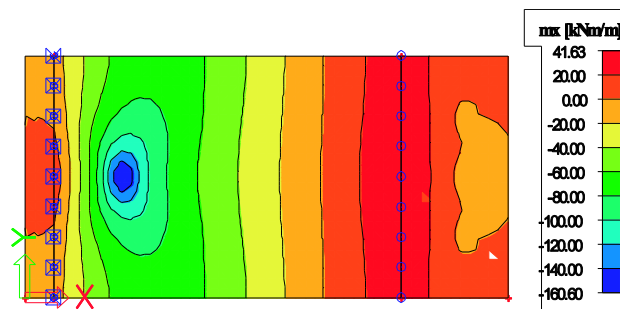


Figure 5.  $m_x$  plot of the L1 slab model analyzed using finite elements method.

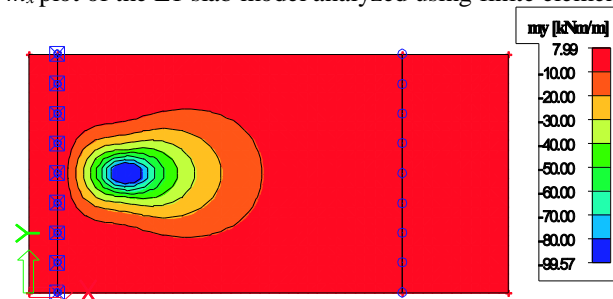


Figure 6.  $m_y$  plot of the L1 slab model analyzed using finite elements method.

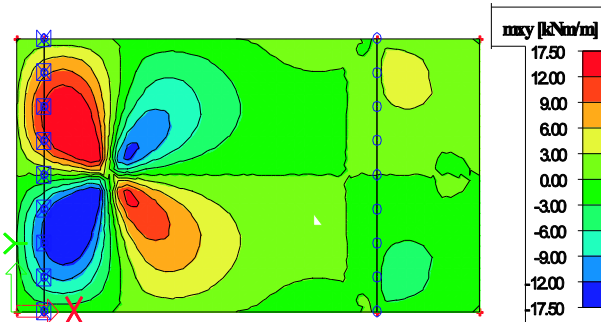


Figure 7.  $m_{xy}$  plot of the L1 slab model analyzed using finite elements method.

For the other cases (L1, L6, L9, L12) where the load is applied in the center of the slab in the  $y$  direction (along width) the  $m_x/m_{xy}$  and  $m_y/m_{xy}$  plots have a peak just where the distributed load is applied due to the large flexural moments and small (approximately zero) torsional moments. In the  $x$ - direction (along length) as the distributed area load gets far from the first row of continuous supports the plots are asymptotic to the ordinates which means that the flexural moments  $m_y$  are small close to the edges and decrease while the distance increases from the position of the load in both directions. For the cases where the load gets closer to the free edges the resultant torsional moments get larger and the flexural moments are also larger and the moment ratios are low, so the plots have the  $x$  axis as an asymptote but have a small peak in the position of the load because it creates small torsional moments

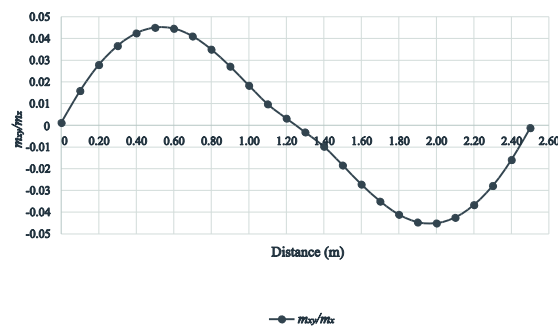


Figure 8. Torsional moment ratios  $m_{xy}/m_x$  along the width of the L1 model.

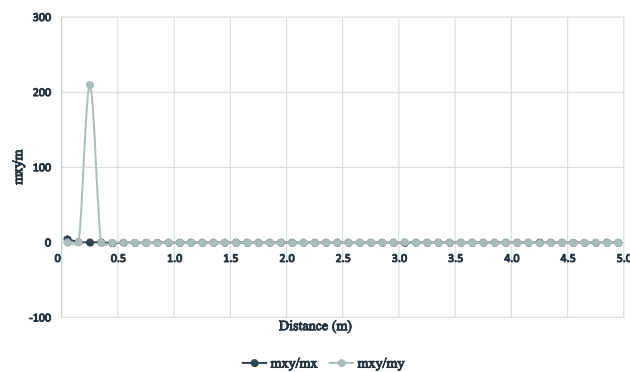


Figure 9. Flexural moment's ratio along the length of the L1 model.

## 5 DESCRIPTION OF THE EFFECT OF TORSION

### 5.1 Discussion of the observed flexural and torsional moments

An analysis of the dependence of the load position in the  $m_{xy}/m_x$  and  $m_{xy}/m_y$  ratios shows that as the load approaches the center of the slab (y-direction), the effect of torsion becomes less at the position of the load. If the load is closer to the first support, the torsional moments are larger but the flexural moments  $m_x$  and  $m_y$  are smaller. This result is expected, as the span is smaller, thus the effect of shear becomes more important relative to bending. As the load gets closer to the free edge (y-direction) the torsional moments are larger. This means that the moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  are the largest for a load close to the support and close to the free edge as shown in Figures 8 and 9.

As the distributed load moves away from the vicinity of the support into the span along the longitudinal direction, the torsional moments ( $m_{xy}$ ) become smaller; in the width direction, when the load is placed in the center, the torsional moments are approximately zero due to the symmetry, but as the position of the load moves closer to the edges the torsional moments increase.

### 5.2 Derivation of an expression for the effect of torsion to be used with the Modified Bond Model

In a next step, the results of the different models are brought together. Now, the relative effect of torsion is shown for different loading positions, see Figures 10 and 11. Figure 10 shows the effect of torsional moments as the load is placed at different positions in the longitudinal direction and Figure 11 shows the results with different transverse directions on the x-axis. In each of the plots of Figures 10 and 11 there is an approximately linear relation between the position of the load along the width and length of the slab models.

The effect of torsion will now be derived as a function of the position of the load in the  $x$ - and  $y$ -direction. To find a generally valid expression, the position of the load is expressed based on the distance  $a$ , the center-to-center distance between the load and the support, and  $b_r$  the distance from the free edge to the center of the load. These positions are indicated in Figure 12. As can be seen from Figure 12, the value of  $b_r$  lies between 0 and  $b/2$ ,  $0 \leq b_r \leq b/2$ , with  $b$  the total width of the slab, as a result of symmetry in the transverse direction.

The distances  $a$  and  $b_r$  describe the position of the load and are used to derive a new expression for the factor  $\beta$  that can be used in the Modified Bond Model. As shown in Figure 1, the bounds that need to be respected for this value are 0 (lower bound, case for which torsion is important and the capacity needs to be reduced) and 1 (upper bound, case for which torsion can be neglected and does not affect the capacity). To mathematically express the dependence of the reduction factor  $\beta$  on the  $x$  and  $y$  coordinates, the following expression is used:

$$\beta(x, y) \rightarrow \beta(a/d, b_r/b) \quad (1)$$

From the results in Figures 10 and 11, a linear relation between the moments ratios  $m_{xy}/m_x$  and the loading position can be observed. This observation is valid for a position of the load between  $a = (0 - 700 \text{ mm})$ . After this position, the effect of the vicinity of the support fades out. As such, the effect needs to be only considered for shear span to depth ratios  $a/d$  up to 2.5.

First, the effect of torsion for different positions of the load in the longitudinal direction is assessed based on the span-to-depth ratio  $a/d$ . Assuming a linear approximation for the moment ratios  $m_{xy}/m_x$  (as observed in Figures 10 and 11), the resulting expression is then:

$$\beta\left(\frac{a}{d}\right) = 0.4 \frac{a}{d} \text{ for } 0 \leq \frac{a}{d} \leq 2.5 \quad (2)$$

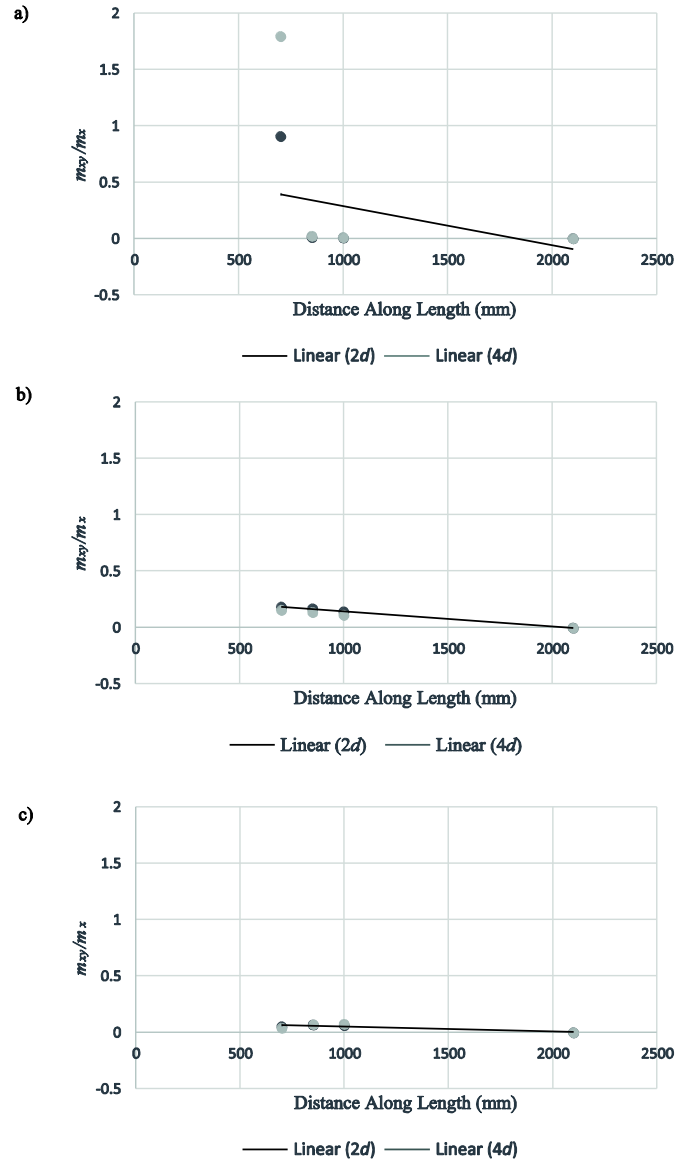


Figure 10. Moment ratios ( $m_{xy}/m_x$ ) for the twelve models at distances: a) 1250 mm, b) 438 mm and c) 850 mm along the width.

For loads placed along the  $y$  direction (Fig. 11) the values of the  $m_{xy}/m_x$  ratios of the slab models are approximated to a linear relation between those values and the position of the load along the length of the slab model.

In a next step of the derivation, the transverse direction is considered. The value of  $b_r$  goes from 0 to  $b/2$ , because of symmetry around the  $x$ -axis. The position of the load is thus taken from one free edge to the middle of the slab. Then, the expression becomes:

$$\beta\left(\frac{b_r}{b}\right) = 2\frac{b_r}{b} \text{ for } 0 \leq \frac{b_r}{b} \leq \frac{1}{2} \quad (3)$$

Finally, equations (2) and (3) are combined into the general type of expression from equation (1) to get the final expression for  $\beta$ . In this expression, the distance from the load to the center of the support  $a$  and the distance  $b_r$  from the free edge to the center of the load are used to express the loading position. The final equation becomes:

$$\beta = 0.8 \frac{a}{d} \frac{b_r}{b} \text{ for } 0 \leq \frac{a}{d} \leq 2.5 \text{ and } 0 \leq \frac{b_r}{b} \leq \frac{1}{2} \quad (4)$$

The upper and lower bounds that were previously already used in the Modified Bond Model are respected by this equation, as shown for the two extreme cases:

Case 1:  $\beta = 1$  for  $b_r/b = 1/2$  and  $a/d = 2.5$

Case 2:  $\beta = 0$  for  $b_r/b = 0$  and  $a/d = 0$

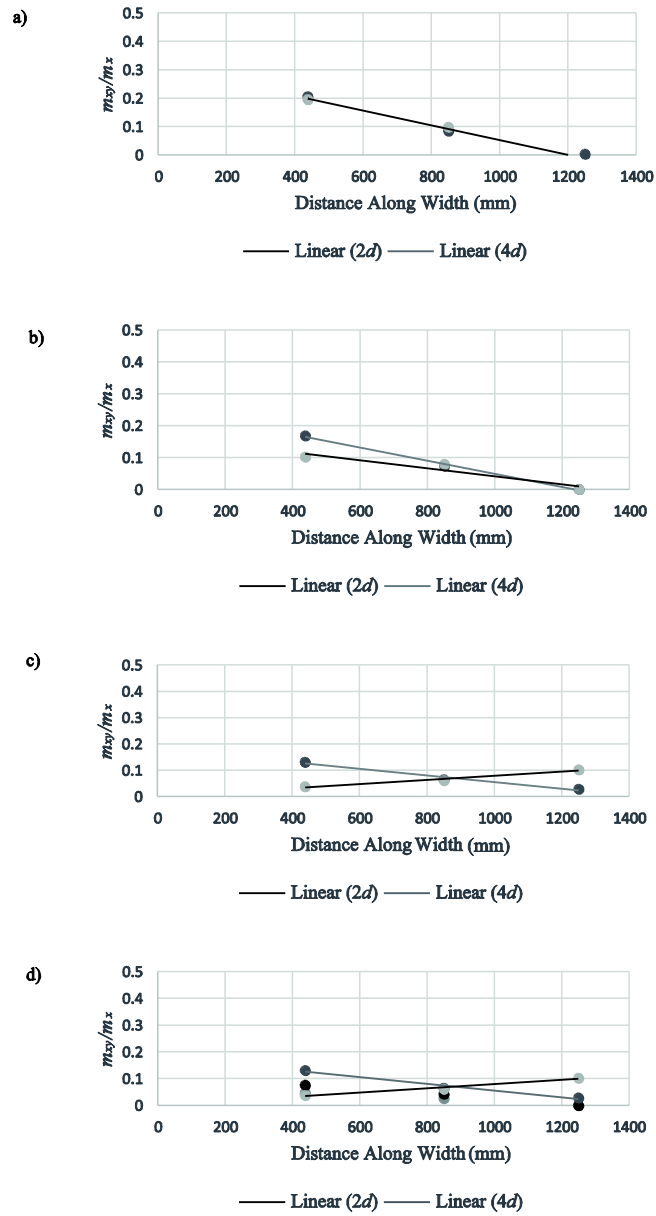


Figure 11. Moment ratios ( $m_x/m_y$ ) for the twelve models at distances: a) 700 mm, b) 850 mm, c) 1000 mm and d) 2100 mm along the length.

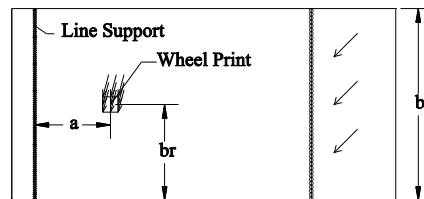


Figure 12. Distances  $a$  and  $b_r$  along the length and width.

## 6 SUMMARY AND CONCLUSIONS

In this paper, an expression for the effect of torsion on the shear and bending moment capacity of slabs is derived. The method used to quantify the ultimate capacity of slabs is the Modified Bond Model. This model needs a better expression for the effect of the torsional moments, which reduce the ultimate capacity. Therefore, the relative magnitude of the torsional moments with respect to the bending moments is studied by means of linear finite element models. In the models, different loading positions are considered. The relative effect of torsion is then linked to the distance along the width and length of the slab. The resulting expression for implementation in the Modified Bond Model is:

$$\beta\left(\frac{a}{d}, \frac{b_r}{b}\right) = 0.8 \frac{a}{d} \frac{b_r}{b} \text{ for } 0 \leq \frac{a}{d} \leq 2.5 \text{ and } 0 \leq \frac{b_r}{b} \leq \frac{1}{2}$$

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2.4 MODEL L4.....	
2.4.1 Description.....	
2.4.2 Moments along the Width.....	
2.4.3 $m_{xy}/m$ Plots Along the Width.....	
2.4.4 Moments along the Length.....	
2.4.5 $m_{xy}/m$ Plot along the Length.....	
2.5 MODEL L5.....	
2.5.1 Description.....	
2.5.2 Moments along the Width.....	
2.5.3 $m_{xy}/m$ Plots Along the Width.....	

2.5.4	Moments along the Length.....	
2.5.5	$m_{xy}/m$ Plot along the Length.....	
2.6	MODEL L6.....	
2.6.1	Description.....	
2.6.2	Moments along the Width.....	
2.6.3	$m_{xy}/m$ Plots Along the Width.....	
2.6.4	Moments along the Length.....	
2.6.5	$m_{xy}/m$ Plot along the Length.....	
2.7	MODEL L7.....	
2.7.1	Description.....	
2.7.2	Moments along the Width.....	
2.7.3	$m_{xy}/m$ Plots Along the Width.....	
2.7.4	Moments along the Length.....	
2.7.5	$m_{xy}/m$ Plot along the Length.....	
2.8	MODEL L8.....	
2.8.1	Description.....	
2.8.2	Moments along the Width.....	
2.8.3	$m_{xy}/m$ Plots Along the Width.....	
2.8.4	Moments along the Length.....	
2.8.5	$m_{xy}/m$ Plot along the Length.....	
2.9	MODEL L9.....	
2.9.1	Description.....	
2.9.2	Moments along the Width.....	
2.9.3	$m_{xy}/m$ Plots Along the Width.....	
2.9.4	Moments along the Length.....	
2.9.5	$m_{xy}/m$ Plot along the Length.....	
2.10	MODEL L10.....	
2.10.1	Description.....	
2.10.2	Moments along the Width.....	
2.10.3	$m_{xy}/m$ Plots Along the Width.....	
2.10.4	Moments along the Length.....	

- 2.10.5  $m_{xy}/m$  Plot along the Length.....
  
- 2.11 MODEL L11.....
  - 2.11.1 Description.....
  - 2.11.2 Moments along the Width.....
  - 2.11.3  $m_{xy}/m$  Plots Along the Width.....
  - 2.11.4 Moments along the Length.....
  - 2.11.5  $m_{xy}/m$  Plot along the Length.....
  
- 2.12 MODEL L12.....
  - 2.12.1 Description.....
  - 2.12.2 Moments along the Width.....
  - 2.12.3  $m_{xy}/m$  Plots Along the Width.....
  - 2.12.4 Moments along the Length.....
  - 2.12.5  $m_{xy}/m$  Plot along the Length.....
  
- 3. Relation plots along the *y-direction*
- 4. Relation plots along the *x-direction*

## 1. Introduction

A theoretical expression to assess the effect of torsion over the first row of support is developed evaluating the results in 12 models of a concrete reinforced slab subjected to concentrated loads close to the supports and a distributed surface load which varies in position along the width and length of the slab. On each of the twelve models the geometry of the slab, the position of the supports and the magnitude of the concentrated and distribute loads are the same. An interpretation of the moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along both spans are used to evaluate the effect of torsion on the shear capacity of the slabs; expression which is sought to be used on the Modified Bond Model to determine the ultimate capacity of slabs.

## 2. Model Descriptions

All the twelve models have the geometry shown on Figure 1. The concentrated loads are stationary and represent the continuity of the slab. The first line of supports are hinged and the second line are rollers exemplifying a simple supported structure. Each of the three loads have a magnitude of 54.67 kN and are located 600 mm from the second row of supports. The distributed surface load has a magnitude of 11950 kN/m<sup>2</sup> applied on a 200 mm x 200 mm area. The slab was modeled using shell elements and a linear-elastic finite element analysis is used to determine the torsional moments ( $m_{xy}$ ) and the flexural moments ( $m_x$  and  $m_y$ ).

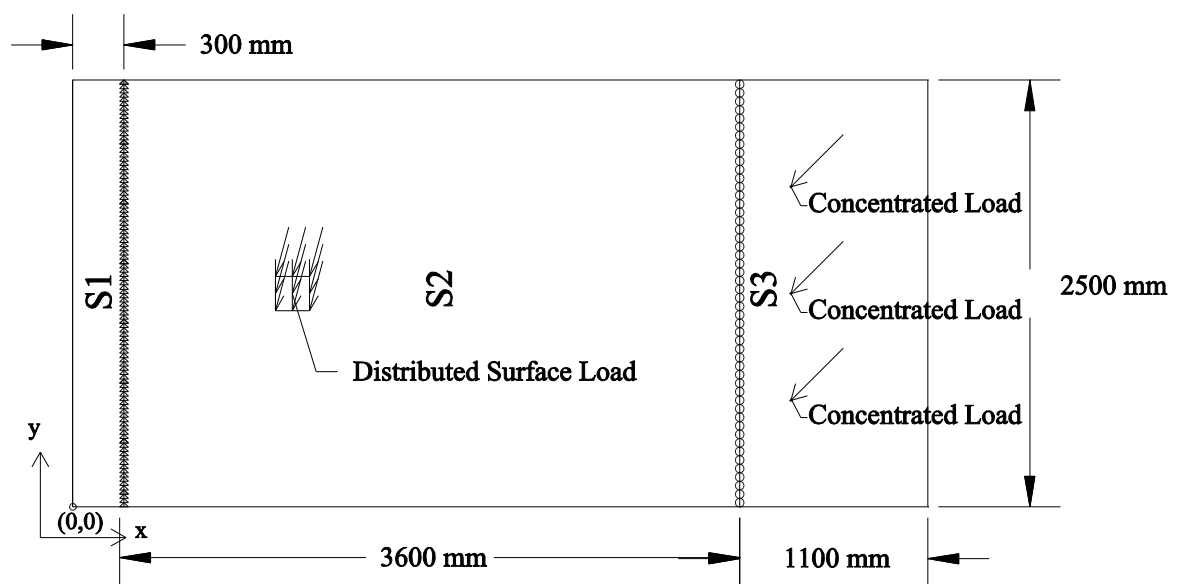


Figure 1: Slab Model Dimensions



## 2.1 Model L1

### 2.1.1 Description

The slabs was modeled using finite elements using a 100 x 100 mm mesh. The  $(x, y)$  position of the distributed surface load is  $(1000, 1250)$  mm.

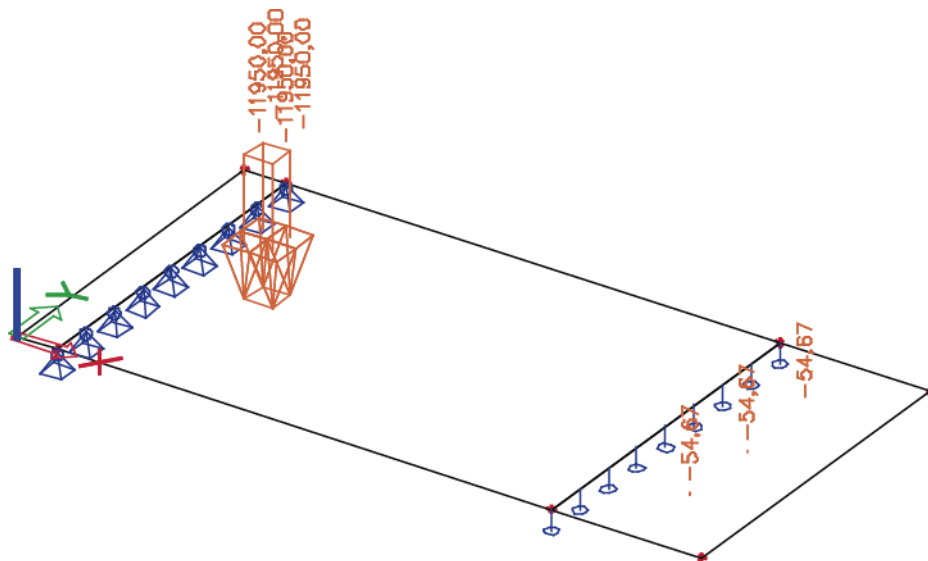


Figure 2. L1 Model with Distributed Surface Load representing wheel print with coordinates  $(1000, 1250)$  mm). The concentrated loads beyond the second support create a moment over support 2.

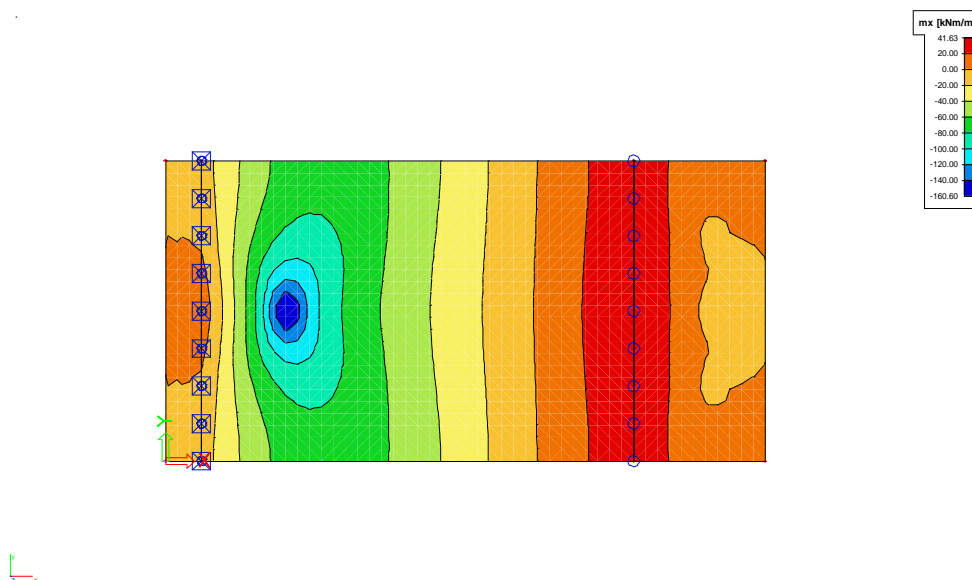


Figure 3.  $m_x$  plot of the L1 slab model analyzed using finite elements method.

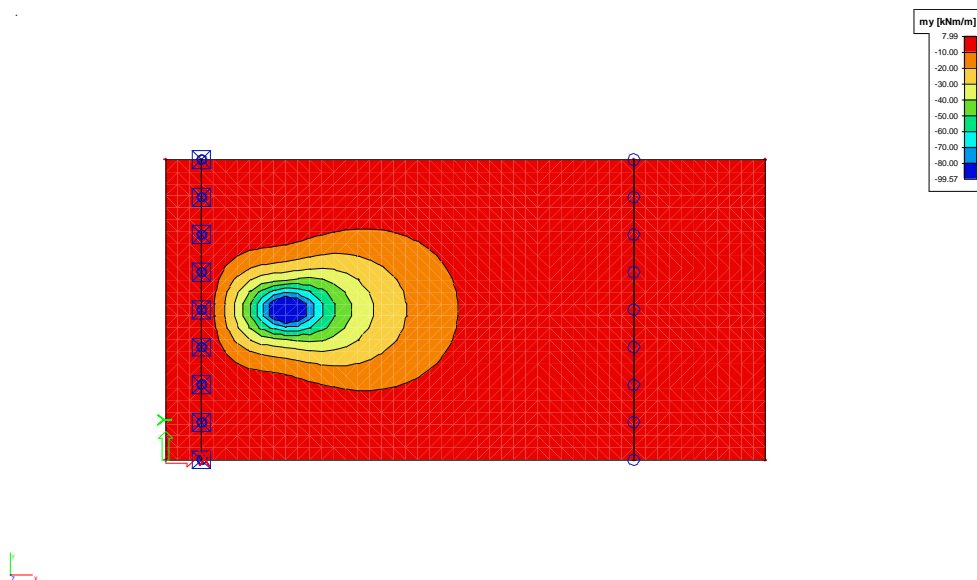


Figure 4.  $m_y$  plot of the L1 slab model analyzed using finite elements method.

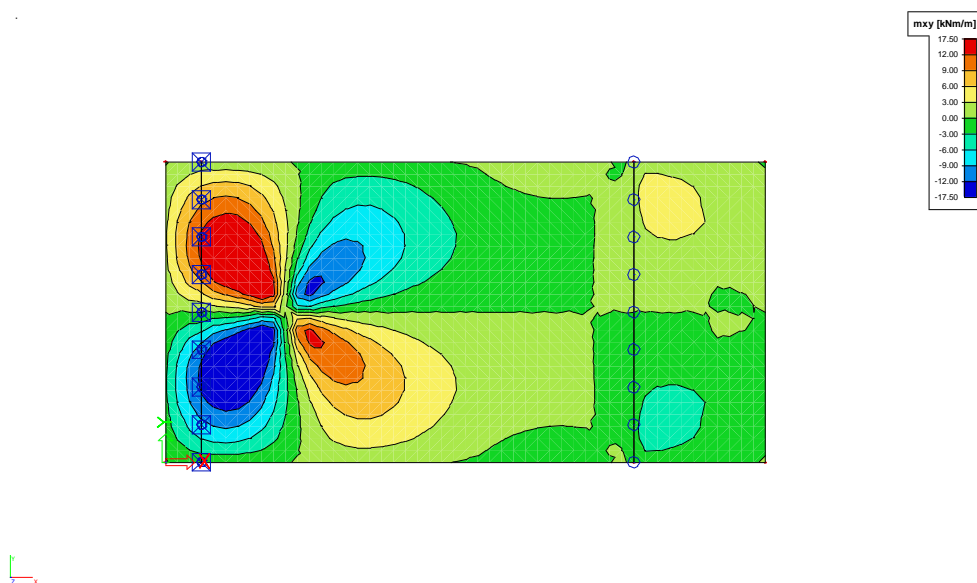


Figure 5.  $m_{xy}$  plot of the L1 slab model analyzed using finite elements method.

### 2.1.2 Moments Along the Width

Table 2: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<i>y direction</i> L1 model results					
Distance (m)	$m_{xx}$ (kN -m/m)	$m_{yy}$ (kN -m/m)	$m_{xy}$ (KN -m/m)	$m_{xy}/m_{xx}$	$m_{xy}/m_{yy}$
0,00	-66,94	0,15	-0,08	0,00	-0,53
0,10	-67,32	0,56	-1,07	0,02	-1,91
0,20	-68,50	0,86	-1,91	0,03	-2,22
0,30	-70,66	0,68	-2,59	0,04	-3,81

0,40	-73,70	-0,24	-3,13	0,04	13,04
0,50	-77,66	-2,07	-3,50	0,05	1,69
0,60	-82,70	-5,01	-3,69	0,04	0,74
0,70	-89,06	-9,38	-3,66	0,04	0,39
0,80	-97,13	-15,71	-3,40	0,04	0,22
0,90	-107,54	-24,89	-2,92	0,03	0,12
1,00	-121,91	-39,06	-2,24	0,02	0,06
1,10	-142,46	-68,20	-1,39	0,01	0,02
1,20	-160,60	-99,57	-0,51	0,00	0,01
1,30	-160,60	-99,57	0,51	0,00	-0,01
1,40	-142,46	-68,20	1,39	-0,01	-0,02
1,50	-121,91	-39,06	2,24	-0,02	-0,06
1,60	-107,40	-24,89	2,92	-0,03	-0,12
1,70	-97,13	-15,71	3,40	-0,04	-0,22
1,80	-89,06	-9,38	3,66	-0,04	-0,39
1,90	-82,70	-5,01	3,69	-0,04	-0,74
2,00	-77,66	-2,07	3,50	-0,05	-1,69
2,10	-73,70	-0,24	3,13	-0,04	-13,04
2,20	-70,66	0,68	2,59	-0,04	3,81
2,30	-68,50	0,86	1,91	-0,03	2,22
2,40	-67,32	0,56	1,07	-0,02	1,91
2,50	-66,94	0,15	0,08	0,00	0,53

### 2.1.3 $m_{xy}/m$ Plots Along the Width

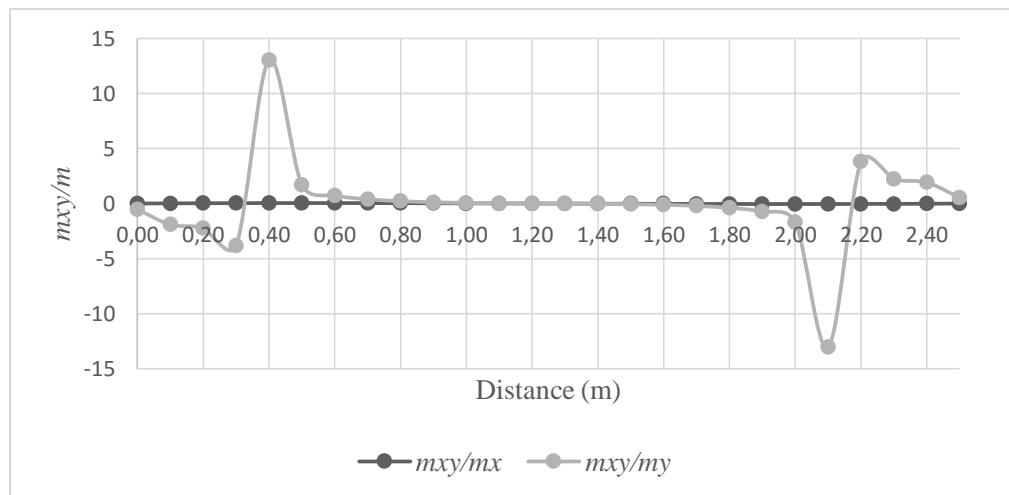


Figure 6. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L1 model.

## 2.1.4 Moments Along the Length

Table 2: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<i>x direction L1 model results</i>					
Distance (m)	$m_{xx}$ (kN -m/m)	$m_{yy}$ (kN -m/m)	$m_{xy}$ (kN -m/m)	$m_{xy}/m_{xx}$	$m_{xy}/m_{yy}$
0,1	0,2	3,9	0,9	4,4	0,2
0,2	2,5	2,7	1,6	0,6	0,6
0,3	7,8	0,0	2,1	0,3	210,0
0,4	15,3	-9,7	2,7	0,2	-0,3
0,5	-5,5	-20,9	3,3	-0,6	-0,2
0,6	-25,1	-33,5	4,1	-0,2	-0,1
0,7	-44,9	-48,3	5,2	-0,1	-0,1
0,8	-66,3	-66,9	7,2	-0,1	-0,1
0,9	-94,5	-87,8	6,7	-0,1	-0,1
1,0	-134,9	-99,6	0,5	0,0	0,0
1,1	-160,6	-92,1	-5,6	0,0	0,1
1,2	-144,7	-75,4	-6,1	0,0	0,1
1,3	-114,4	-61,3	-4,2	0,0	0,1
1,4	-96,5	-51,0	-3,1	0,0	0,1
1,5	-85,8	-43,0	-2,4	0,0	0,1
1,6	-77,6	-36,7	-1,9	0,0	0,1
1,7	-70,9	-31,5	-1,5	0,0	0,0
1,8	-64,9	-27,1	-1,3	0,0	0,0
1,9	-59,6	-23,4	-1,0	0,0	0,0
2,0	-54,5	-20,2	-0,9	0,0	0,0
2,1	-49,6	-17,4	-0,7	0,0	0,0
2,2	-44,9	-14,9	-0,6	0,0	0,0
2,3	-40,3	-12,7	-0,5	0,0	0,0
2,4	-35,6	-10,7	-0,4	0,0	0,0
2,5	-31,0	-8,9	-0,4	0,0	0,0
2,6	-26,5	-7,2	-0,3	0,0	0,0
2,7	-21,9	-5,6	-0,2	0,0	0,0
2,8	-17,2	-4,2	-0,2	0,0	0,1
2,9	-12,6	-2,8	-0,2	0,0	0,1
3,0	-7,9	-1,5	-0,2	0,0	0,1
3,1	-3,2	-0,2	-0,1	0,0	0,6
3,2	1,6	1,1	-0,1	-0,1	-0,1
3,3	6,4	2,3	-0,1	0,0	0,0
3,4	11,3	3,4	-0,1	0,0	0,0
3,5	16,4	4,5	0,0	0,0	0,0
3,6	21,5	5,6	0,0	0,0	0,0
3,7	26,8	6,6	0,1	0,0	0,0
3,8	32,3	7,6	0,1	0,0	0,0
3,9	38,1	8,5	0,2	0,0	0,0

4,0	44,0	6,4	0,2	0,0	0,0
4,1	35,9	4,1	0,3	0,0	0,1
4,2	28,0	1,7	0,5	0,0	0,3
4,3	20,3	-1,2	0,7	0,0	-0,6
4,4	12,6	-4,8	1,0	0,1	-0,2
4,5	3,3	-7,6	0,3	0,1	0,0
4,6	-3,9	-7,2	0,7	-0,2	-0,1
4,7	-3,7	-6,0	0,3	-0,1	-0,1
4,8	-1,3	-5,5	0,1	-0,1	0,0
4,9	-0,6	-5,3	0,0	0,0	0,0
5,0	-0,2	-5,3	0,0	-0,1	0,0

### 2.1.5 $m_{xy}/m$ Plots Along the Length

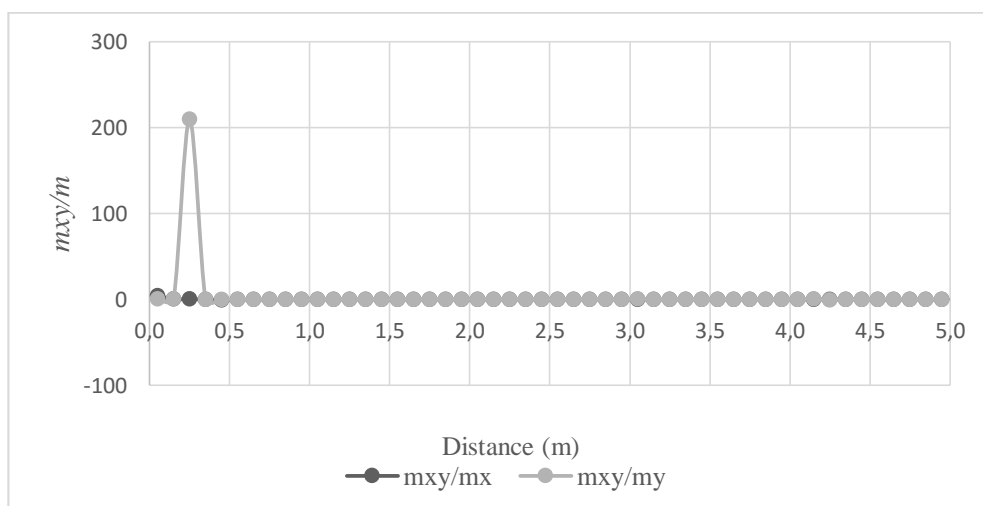


Figure 7. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L1 model.

## 2.2 Model L2

### 2.2.1 Description

The slabs was modeled using finite elements using a 100 x 100 mm mesh. The (x, y) position of the distributed surface load is (1000, 850) mm.

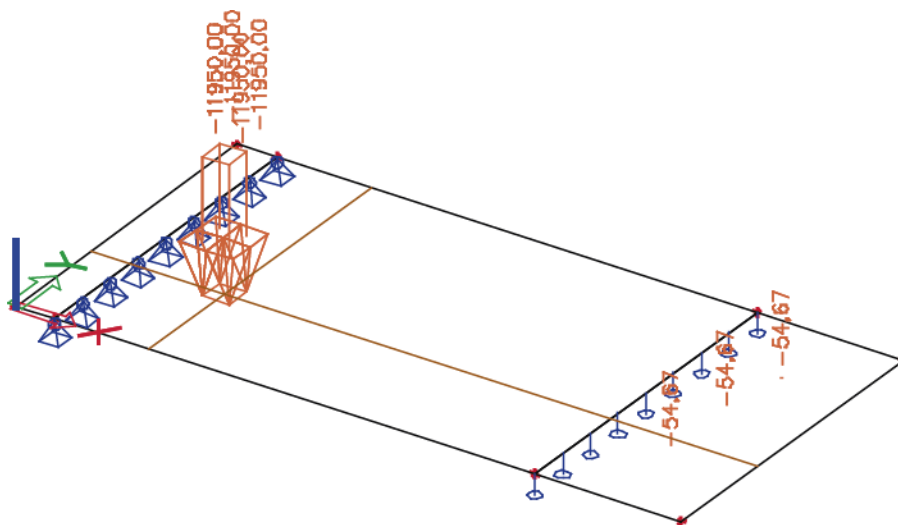


Figure 8. L2 Model with Distributed Surface Load representing wheel print with coordinates (1000 mm, 850 mm). The concentrated loads beyond the second support create a moment over support 2.

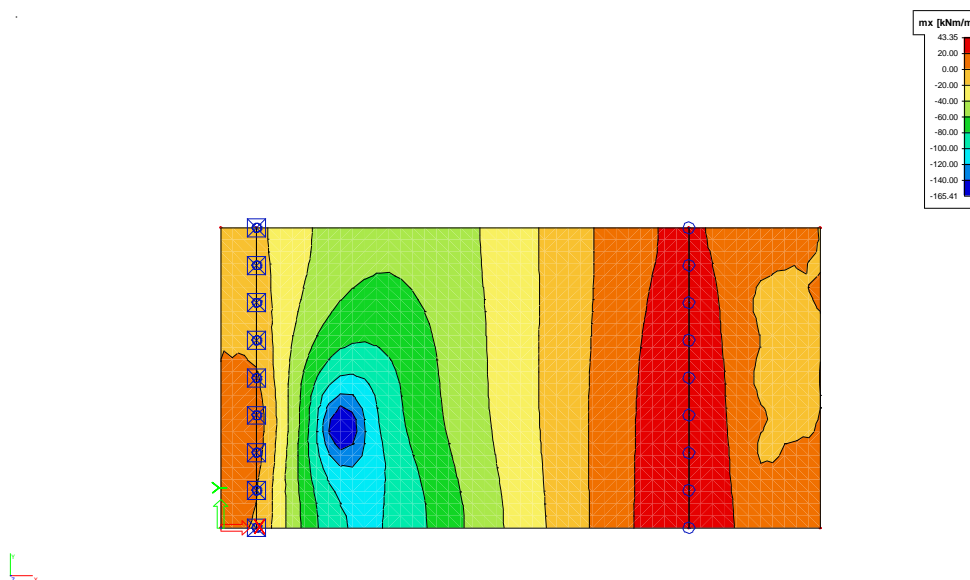


Figure 9.  $m_x$  plot of the L2 slab model analyzed using finite elements method.

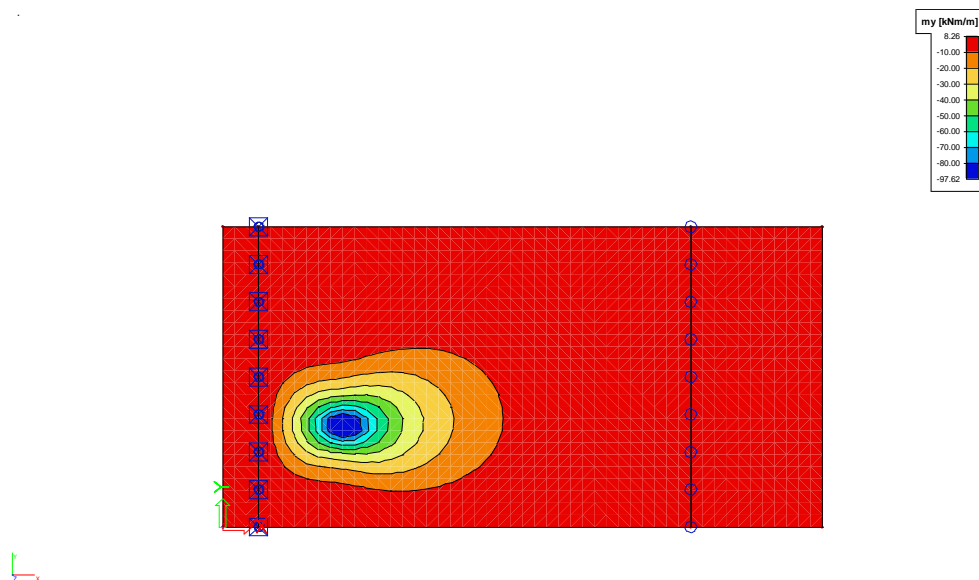


Figure 10.  $m_y$  plot of the L2 slab model analyzed using finite elements method.

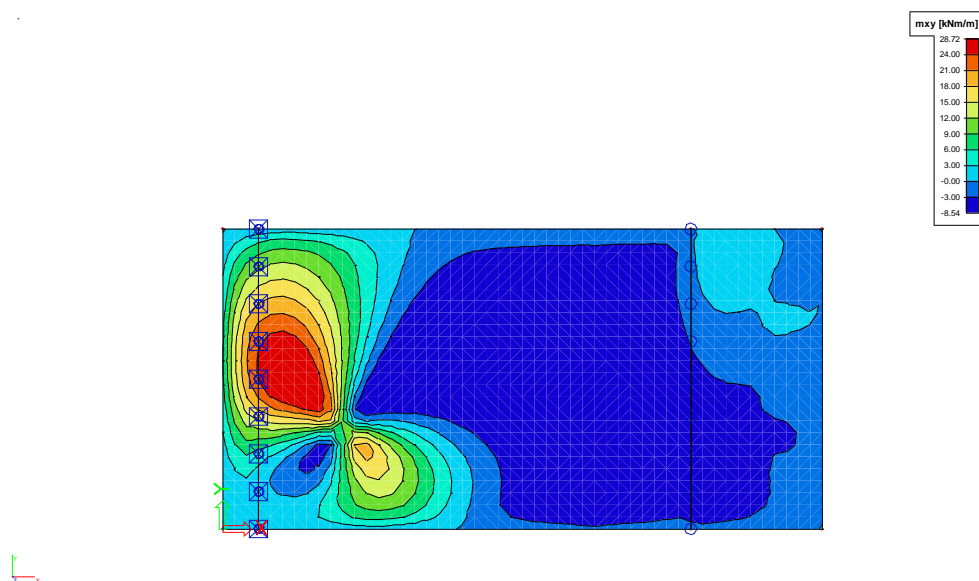


Figure 11.  $m_{xy}$  plot of the L2 slab model analyzed using finite elements method.

### 2.2.2 Moments Along the Width

Table 3: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<i>y direction</i> L2 model results					
Distance (m)	$m_{xx}$ (kN -m/m)	$m_{yy}$ (kN -m/m)	$m_{xy}$ (kN -m/m)	$m_{xy}/m_{xx}$	$m_{xy}/m_{yy}$
0,00	-97,39	0,20	1,64	-0,02	8,20
0,10	-98,06	-0,26	5,21	-0,05	-20,04
0,20	-100,18	-2,27	6,79	-0,07	-2,99
0,30	-104,07	-6,10	7,05	-0,07	-1,16

0,40	-109,92	-12,26	7,06	-0,06	-0,58
0,50	-118,31	-21,51	7,16	-0,06	-0,33
0,60	-130,77	-35,96	7,43	-0,06	-0,21
0,70	-140,49	-65,52	7,67	-0,05	-0,12
0,80	-166,41	-97,62	8,64	-0,05	-0,09
0,90	-162,13	-94,50	9,32	-0,06	-0,10
1,00	-142,66	-63,41	9,66	-0,07	-0,15
1,10	-122,57	-38,09	10,51	-0,09	-0,28
1,20	-107,64	-23,73	11,12	-0,10	-0,47
1,30	-96,13	-14,64	11,50	-0,12	-0,79
1,40	-86,89	-8,39	11,68	-0,13	-1,39
1,50	-79,32	-4,95	11,67	-0,15	-2,36
1,60	-73,04	-1,07	11,48	-0,16	-10,73
1,70	-67,79	0,92	11,12	-0,16	12,09
1,80	-63,40	2,16	10,62	-0,17	4,92
1,90	-59,75	2,82	10,01	-0,17	3,55
2,00	-56,72	3,03	9,31	-0,16	3,07
2,10	-54,26	2,84	8,54	-0,16	3,01
2,20	-52,30	2,33	7,66	-0,15	3,29
2,30	-50,86	1,54	6,49	-0,13	4,21
2,40	-50,13	0,73	4,41	-0,09	6,04
2,50	-49,91	0,18	1,02	-0,02	5,67

### 2.2.3 $m_{xy}/m$ Plots Along the Width

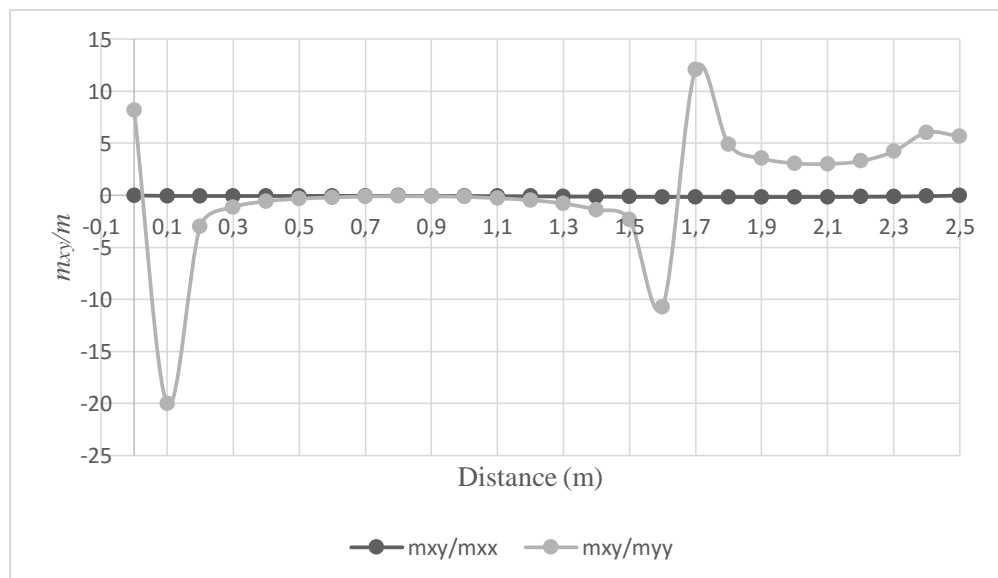


Figure 12. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L2 model.



## 2.2.4 Moments Along the Length

Table 4: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>x direction L2 model results</i></b>					
<b>Distance (m)</b>	<b><i>m<sub>xx</sub> (kN -m/m)</i></b>	<b><i>m<sub>yy</sub> (kN -m/m)</i></b>	<b><i>m<sub>xy</sub> (KN -m/m)</i></b>	<b><i>m<sub>xy</sub>/m<sub>xx</sub></i></b>	<b><i>m<sub>xy</sub>/m<sub>yy</sub></i></b>
0,00	0,18	3,91	3,29	18,28	0,84
0,10	2,24	3,82	8,45	3,77	2,21
0,20	8,05	3,32	11,93	1,48	3,59
0,30	10,13	-1,70	12,63	1,25	-7,43
0,40	-6,33	-8,51	13,39	-2,12	-1,57
0,50	-27,32	-19,98	13,13	-0,48	-0,66
0,60	-47,79	-32,17	12,76	-0,27	-0,40
0,70	-70,03	-46,51	12,15	-0,17	-0,26
0,80	-98,83	-64,76	11,86	-0,12	-0,18
0,90	-138,57	-85,08	11,17	-0,08	-0,13
1,00	-163,77	-96,06	8,98	-0,05	-0,09
1,10	-149,32	-88,70	6,89	-0,05	-0,08
1,20	-120,33	-72,17	5,97	-0,05	-0,08
1,30	-102,31	-58,02	4,72	-0,05	-0,08
1,40	-91,44	-47,79	3,44	-0,04	-0,07
1,50	-82,97	-39,90	2,16	-0,03	-0,05
1,60	-75,85	-33,62	0,93	-0,01	-0,03
1,70	-69,54	-28,51	-0,21	0,00	0,01
1,80	-63,72	-24,26	-1,25	0,02	0,05
1,90	-58,22	-20,68	-2,19	0,04	0,11
2,00	-52,92	-17,61	-3,02	0,06	0,17
2,10	-47,77	-14,97	-3,75	0,08	0,25
2,20	-42,71	-12,65	-4,37	0,10	0,35
2,30	-37,72	-10,61	-4,91	0,13	0,46
2,40	-32,77	-8,78	-5,35	0,16	0,61
2,50	-27,84	-7,14	-5,71	0,21	0,80
2,60	-22,94	-5,64	-5,99	0,26	1,06
2,70	-18,03	-4,25	-6,21	0,34	1,46
2,80	-13,13	-2,96	-6,35	0,48	2,15
2,90	-8,21	-1,74	-6,44	0,78	3,70
3,00	-3,26	-0,57	-6,47	1,98	11,35
3,10	1,72	0,56	-6,45	-3,75	-11,52
3,20	6,74	1,66	-6,40	-0,95	-3,86
3,30	11,84	2,75	-6,31	-0,53	-2,29
3,40	17,03	3,82	-6,20	-0,36	-1,62
3,50	22,33	4,87	-6,08	-0,27	-1,25
3,60	27,79	5,90	-5,94	-0,21	-1,01
3,70	33,43	6,91	-5,80	-0,17	-0,84
3,80	39,87	8,01	-5,70	-0,14	-0,71
3,90	43,03	8,24	-5,25	-0,12	-0,64

4,00	37,83	6,90	-5,20	-0,14	-0,75
4,10	29,29	4,76	-4,78	-0,16	-1,00
4,20	21,43	2,85	-4,31	-0,20	-1,51
4,30	13,65	1,16	-3,70	-0,27	-3,19
4,40	6,29	-0,17	-3,24	-0,52	19,06
4,50	0,77	-1,05	-3,49	-4,53	3,32
4,60	-1,05	-2,25	-3,68	3,50	1,64
4,70	-0,92	-3,19	-3,27	3,55	1,03
4,80	-0,49	-3,71	-2,65	5,41	0,71
4,90	-0,15	-3,96	-1,82	12,13	0,46
5,00	0,08	-4,10	-0,50	-6,25	0,12

### 2.2.5 $m_{xy}/m$ Plots Along the Length

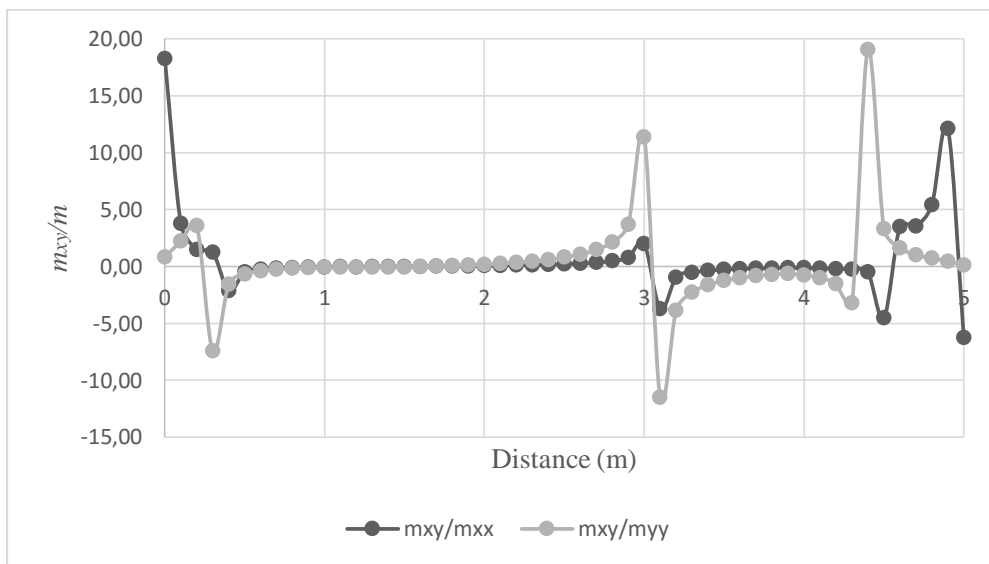


Figure 13. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L2 model.



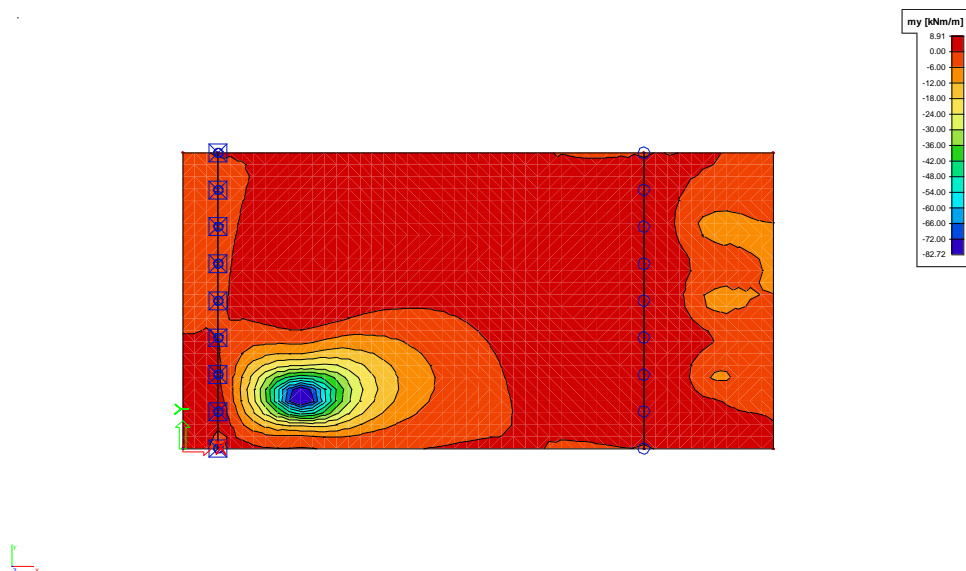


Figure 16.  $m_y$  plot of the L3 slab model analyzed using finite elements method.

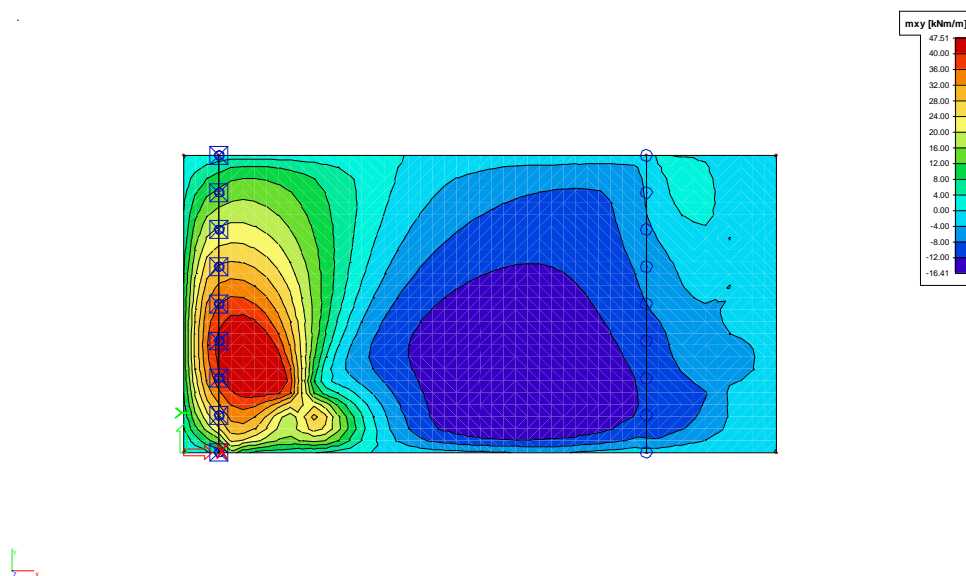


Figure 17.  $m_{xy}$  plot of the L3 slab model analyzed using finite elements method.

### 2.3.2 Moments Along the Width

Table 5: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<i>y direction L3 model results</i>					
Distance (m)	$m_{xx}$ (kN -m/m)	$m_{yy}$ (kN -m/m)	$m_{xy}$ (KN -m/m)	$m_{xy}/m_{xx}$	$m_{xy}/m_{yy}$
0,00	-159,42	0,31	4,71	-0,03	15,19
0,10	-161,33	-4,57	16,29	-0,10	-3,56
0,20	-168,48	-19,34	21,98	-0,13	-1,14
0,30	-182,06	-50,81	23,19	-0,13	-0,46
0,40	-191,79	-82,72	24,11	-0,13	-0,29

0,50	-182,39	-77,26	23,98	-0,13	-0,31
0,60	-158,57	-46,46	23,35	-0,15	-0,50
0,70	-135,75	-24,11	23,50	-0,17	-0,97
0,80	-118,74	-12,28	23,28	-0,20	-1,90
0,90	-105,33	-4,95	22,87	-0,22	-4,62
1,00	-94,38	-0,11	22,33	-0,24	-203,00
1,10	-85,26	3,08	21,55	-0,25	7,00
1,20	-77,56	5,14	20,85	-0,27	4,06
1,30	-71,01	6,40	19,95	-0,28	3,12
1,40	-65,40	7,08	18,97	-0,29	2,68
1,50	-60,58	7,34	17,94	-0,30	2,44
1,60	-56,45	7,28	16,87	-0,30	2,32
1,70	-52,91	6,98	15,80	-0,30	2,26
1,80	-49,88	6,47	14,74	-0,30	2,28
1,90	-47,30	5,80	13,71	-0,29	2,36
2,00	-45,12	4,98	12,72	-0,28	2,55
2,10	-43,30	4,03	11,74	-0,27	2,91
2,20	-41,82	2,95	10,70	-0,26	3,63
2,30	-40,70	1,78	9,28	-0,23	5,21
2,40	-40,21	0,81	6,51	-0,16	8,04
2,50	-40,09	0,25	1,62	-0,04	6,48

### 2.3.3 $m_{xy}/m$ Plots Along the Width

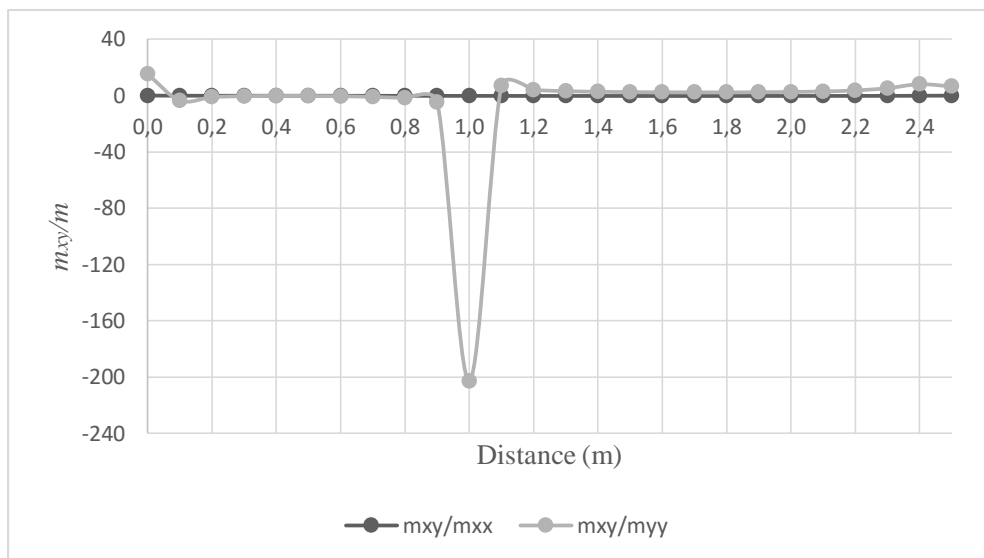


Figure 18. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L3 model.

## 2.3.4 Moments Along the Length

Table 6: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>x direction L3 model results</i></b>					
<b>Distance (m)</b>	<b><i>m<sub>xx</sub> (kN -m/m)</i></b>	<b><i>m<sub>yy</sub> (kN -m/m)</i></b>	<b><i>m<sub>xy</sub> (KN -m/m)</i></b>	<b><i>m<sub>xy</sub>/m<sub>xx</sub></i></b>	<b><i>m<sub>xy</sub>/m<sub>yy</sub></i></b>
0,00	0,27	7,02	7,35	27,22	1,05
0,10	3,67	7,25	18,96	5,17	2,62
0,20	12,16	7,04	27,84	2,29	3,95
0,30	15,56	2,19	33,21	2,13	15,16
0,40	-4,22	-3,09	37,38	-8,86	-12,10
0,50	-30,41	-12,80	38,90	-1,28	-3,04
0,60	-56,43	-22,82	38,50	-0,68	-1,69
0,70	-83,92	-35,02	36,50	-0,43	-1,04
0,80	-117,11	-51,59	33,78	-0,29	-0,65
0,90	-160,60	-70,56	29,66	-0,18	-0,42
1,00	-188,22	-80,65	24,06	-0,13	-0,30
1,10	-174,59	-72,46	18,69	-0,11	-0,26
1,20	-145,42	-55,31	13,73	-0,09	-0,25
1,30	-126,32	-41,26	8,74	-0,07	-0,21
1,40	-113,68	-31,58	4,35	-0,04	-0,14
1,50	-103,07	-24,50	0,49	0,00	-0,02
1,60	-93,67	-19,21	-2,82	0,03	0,15
1,70	-85,10	-15,18	-5,61	0,07	0,37
1,80	-77,15	-12,05	-7,94	0,10	0,66
1,90	-69,71	-9,58	-9,87	0,14	1,03
2,00	-62,66	-7,61	-11,40	0,18	1,50
2,10	-55,94	-6,01	-12,74	0,23	2,12
2,20	-49,50	-4,69	-13,78	0,28	2,94
2,30	-43,27	-3,58	-14,60	0,34	4,08
2,40	-37,23	-2,63	-15,24	0,41	5,79
2,50	-31,32	-1,81	-15,72	0,50	8,69
2,60	-25,52	-1,08	-16,04	0,63	14,85
2,70	-19,80	-0,41	-16,24	0,82	39,61
2,80	-14,10	0,21	-16,32	1,16	-77,71
2,90	-8,50	0,80	-16,29	1,92	-20,36
3,00	-2,88	1,38	-16,15	5,61	-11,70
3,10	2,76	1,97	-15,91	-5,76	-8,08
3,20	8,43	2,58	-15,58	-1,85	-6,04
3,30	14,14	3,24	-15,15	-1,07	-4,68
3,40	19,93	3,96	-14,65	-0,74	-3,70
3,50	25,82	4,79	-14,07	-0,54	-2,94
3,60	31,85	5,73	-13,45	-0,42	-2,35
3,70	38,12	6,82	-12,82	-0,34	-1,88
3,80	45,38	8,23	-12,32	-0,27	-1,50
3,90	49,36	8,54	-11,22	-0,23	-1,31

4,00	43,81	7,38	-10,90	-0,25	-1,48
4,10	34,94	5,14	-10,18	-0,29	-1,98
4,20	27,06	3,22	-9,59	-0,35	-2,98
4,30	19,29	1,49	-8,94	-0,46	-6,00
4,40	11,46	0,16	-7,91	-0,69	-49,44
4,50	5,28	-0,65	-5,86	-1,11	9,02
4,60	3,12	-1,52	-3,66	-1,17	2,41
4,70	2,25	-1,56	-2,88	-1,28	1,85
4,80	1,30	-1,39	-2,49	-1,92	1,79
4,90	0,42	-1,24	-1,86	-4,43	1,50
5,00	0,20	-1,41	-0,67	-3,35	0,48

### 2.3.5 $m_{xy}/m$ Plots Along the Length

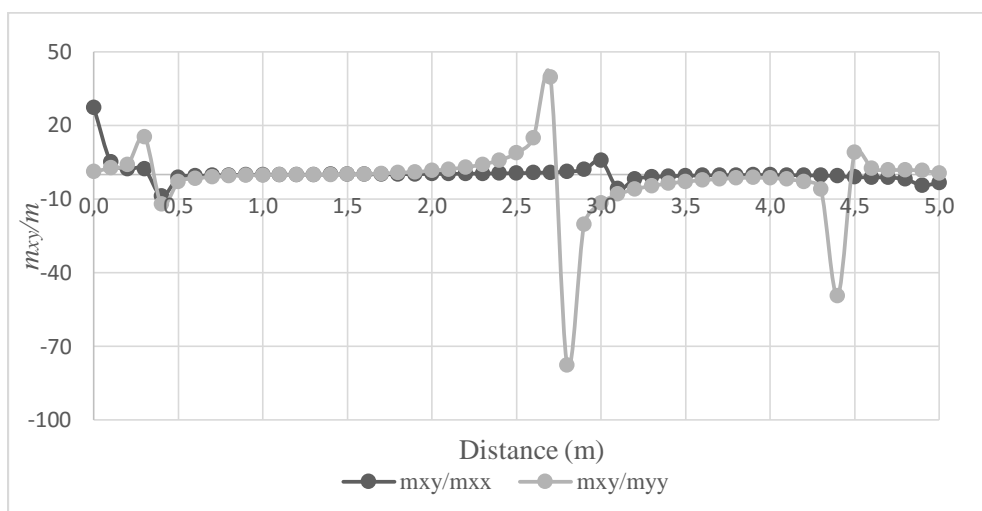


Figure 19. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L3 model.

## 2.4 Model L4

### 2.4.1 Description

The slabs was modeled using finite elements using a 100 x 100 mm mesh. The (x, y) position of the distributed surface load is (700, 438) mm.

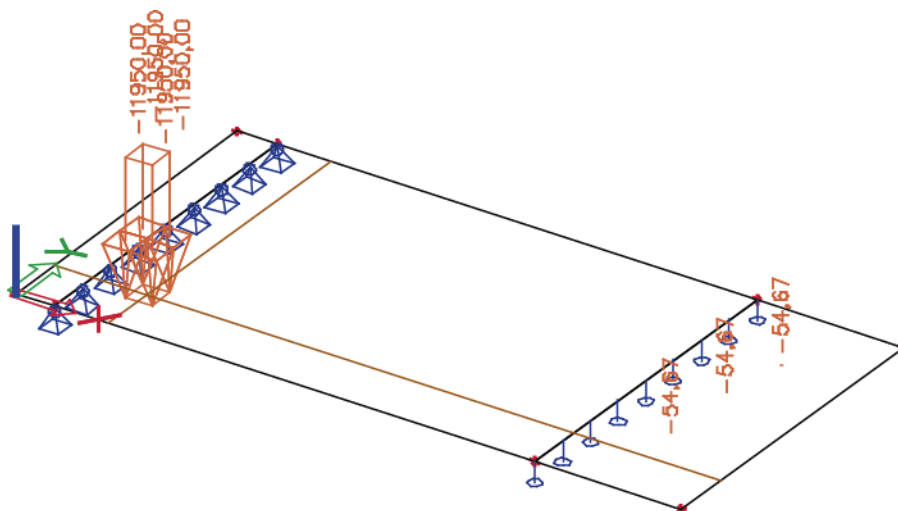


Figure 20. L4 Model with Distributed Surface Load representing wheel print with coordinates (700 mm, 438 mm). The concentrated loads beyond the second support create a moment over support 2.

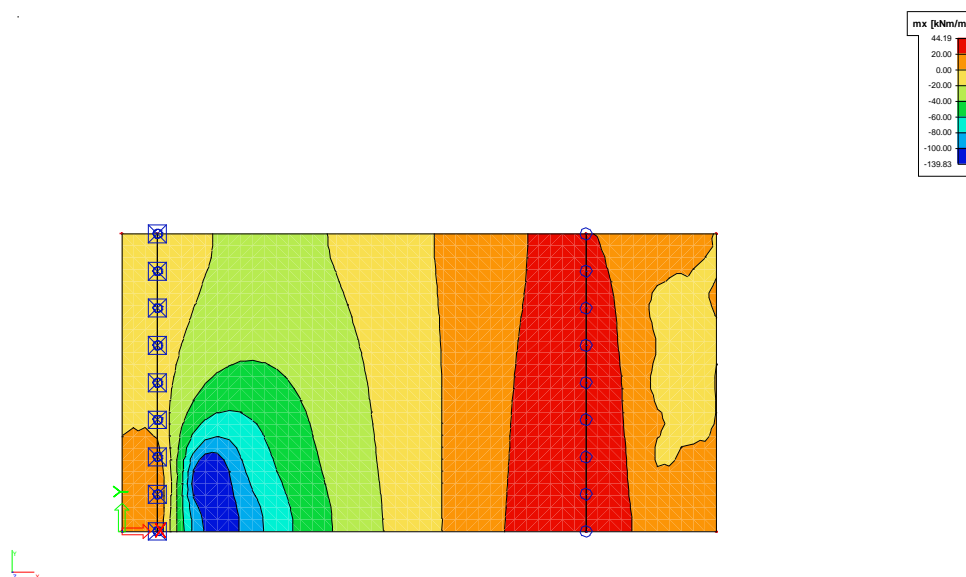


Figure 21.  $m_x$  plot of the L4 slab model analyzed using finite elements method.



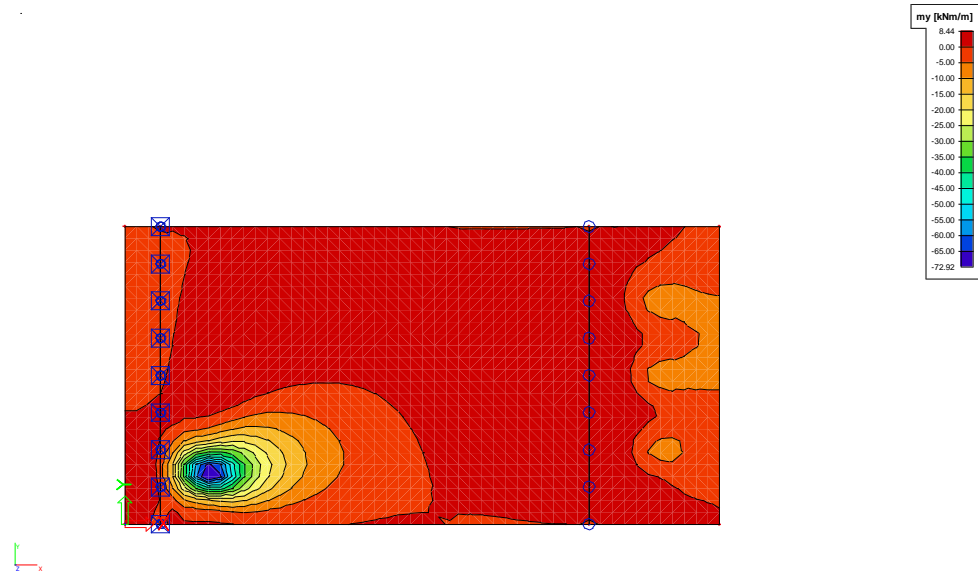


Figure 22.  $m_y$  plot of the L4 slab model analyzed using finite elements method.

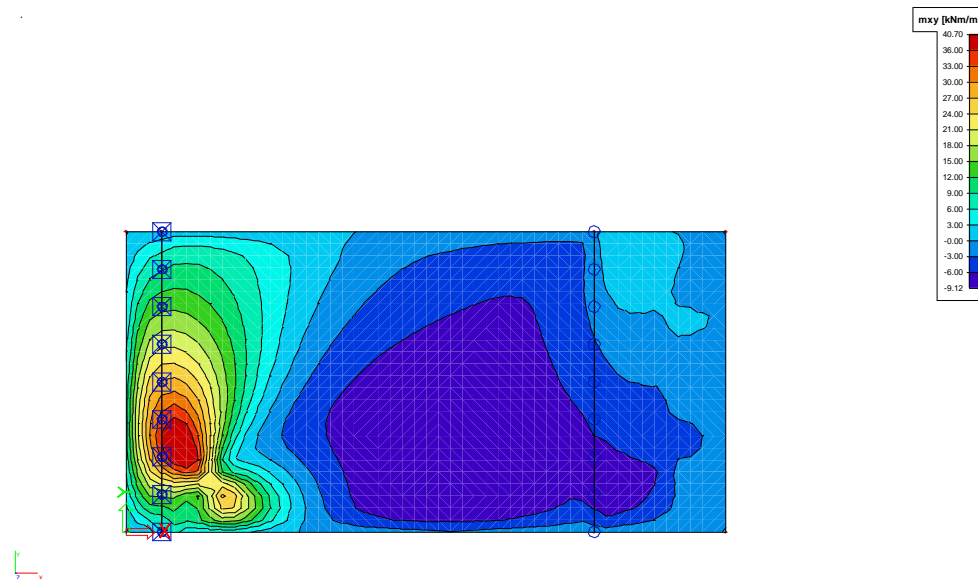


Figure 23.  $m_{xy}$  plot of the L4 slab model analyzed using finite elements method.

#### 2.4.2 Moments Along the Width

Table 7: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b>y direction L4 model results</b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0,00	-102,17	0,66	4,23	-0,04	6,41
0,10	-104,31	-2,54	14,05	-0,13	-5,53
0,20	-112,45	-14,37	18,70	-0,17	-1,30
0,30	-128,12	-42,73	19,90	-0,16	-0,47
0,40	-139,83	-72,92	21,75	-0,16	-0,30

0,50	-132,55	-66,66	22,67	-0,17	-0,34
0,60	-111,06	-36,13	22,98	-0,21	-0,64
0,70	-90,12	-15,29	24,07	-0,27	-1,57
0,80	-75,00	-5,45	24,40	-0,33	-4,48
0,90	-63,68	-0,30	24,17	-0,38	-80,57
1,00	-54,75	2,40	23,58	-0,43	9,83
1,10	-47,65	3,71	22,58	-0,47	6,09
1,20	-41,94	4,22	21,28	-0,51	5,04
1,30	-37,30	4,31	19,92	-0,53	4,62
1,40	-33,50	4,16	18,51	-0,55	4,45
1,50	-30,36	3,88	17,10	-0,56	4,41
1,60	-27,78	3,55	15,72	-0,57	4,43
1,70	-25,64	3,17	14,40	-0,56	4,54
1,80	-23,88	2,77	13,17	-0,55	4,75
1,90	-22,44	2,35	12,01	-0,54	5,11
2,00	-21,28	1,91	10,93	-0,51	5,72
2,10	-20,36	1,46	9,92	-0,49	6,79
2,20	-19,64	1,02	8,90	-0,45	8,73
2,30	-19,13	0,58	7,63	-0,40	13,16
2,40	-18,93	0,30	5,37	-0,28	17,90
2,50	-18,97	0,22	1,38	-0,07	6,27

### 2.4.3 $m_{xy}/m$ Plots Along the Width

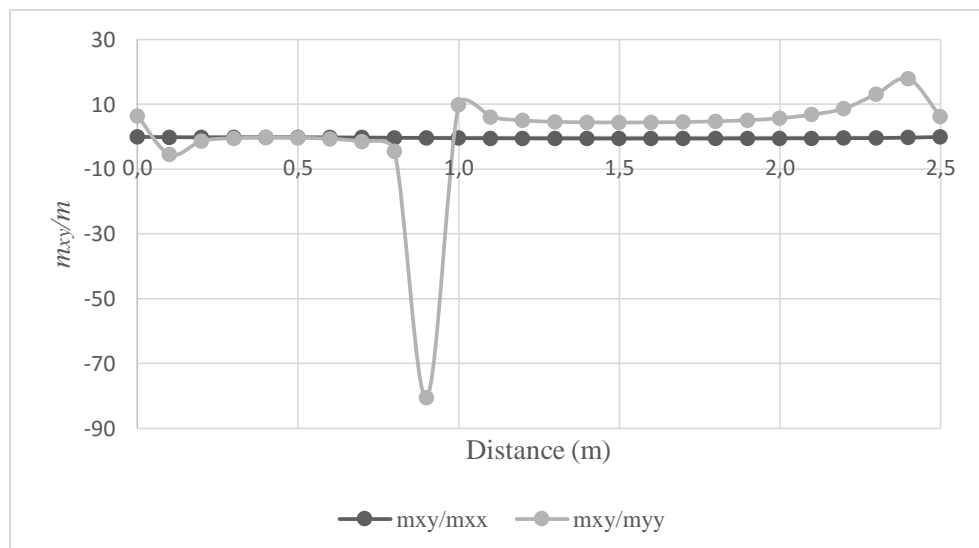


Figure 24. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L4 model.

## 2.4.4 Moments Along the Length

Table 8: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>x direction L4 model results</i></b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0,00	0,32	5,66	5,53	17,28	0,98
0,10	3,64	5,73	14,27	3,92	2,49
0,20	13,27	5,29	20,85	1,57	3,94
0,30	16,30	-2,86	23,70	1,45	-8,29
0,40	-13,46	-14,37	26,24	-1,95	-1,83
0,50	-55,04	-36,33	26,63	-0,48	-0,73
0,60	-104,56	-58,12	25,51	-0,24	-0,44
0,70	-137,10	-70,54	22,10	-0,16	-0,31
0,80	-127,54	-64,38	18,58	-0,15	-0,29
0,90	-101,95	-48,84	15,16	-0,15	-0,31
1,00	-86,27	-35,89	11,30	-0,13	-0,31
1,10	-76,62	-27,21	7,94	-0,10	-0,29
1,20	-68,77	-20,94	4,91	-0,07	-0,23
1,30	-61,95	-16,30	2,29	-0,04	-0,14
1,40	-55,81	-12,82	0,06	0,00	0,00
1,50	-50,16	-10,14	-1,81	0,04	0,18
1,60	-44,91	-8,06	-3,36	0,07	0,42
1,70	-39,08	-6,41	-4,65	0,12	0,73
1,80	-35,32	-5,09	-5,70	0,16	1,12
1,90	-30,87	-4,01	-6,57	0,21	1,64
2,00	-26,61	-3,12	-7,26	0,27	2,33
2,10	-22,50	-2,37	-7,82	0,35	3,30
2,20	-18,51	-1,73	-8,25	0,45	4,77
2,30	-14,61	-1,17	-8,57	0,59	7,32
2,40	-10,80	-0,67	-8,80	0,81	13,13
2,50	-7,06	-0,22	-8,95	1,27	40,68
2,60	-3,35	0,20	-9,01	2,69	-45,05
2,70	0,31	0,59	-9,01	-29,06	-15,27
2,80	3,95	0,98	-8,94	-2,26	-9,12
2,90	7,58	1,37	-8,81	-1,16	-6,43
3,00	11,20	1,76	-8,63	-0,77	-4,90
3,10	14,81	2,18	-8,39	-0,57	-3,85
3,20	18,44	2,63	-8,11	-0,44	-3,08
3,30	22,07	3,13	-7,79	-0,35	-2,49
3,40	25,72	3,71	-7,46	-0,29	-2,01
3,50	29,41	4,37	-7,14	-0,24	-1,63
3,60	33,16	5,15	-6,87	-0,21	-1,33
3,70	37,05	6,04	-6,74	-0,18	-1,12
3,80	41,79	7,15	-6,87	-0,16	-0,96
3,90	43,86	7,44	-6,86	-0,16	-0,92

4,00	38,60	6,26	-7,35	-0,19	-1,17
4,10	30,68	4,24	-7,36	-0,24	-1,74
4,20	23,58	2,42	-7,27	-0,31	-3,00
4,30	16,50	0,77	-7,01	-0,42	-9,10
4,40	9,26	-0,50	-6,28	-0,68	12,56
4,50	3,63	-1,30	-4,45	-1,23	3,42
4,60	1,94	-2,15	-2,39	-1,23	1,11
4,70	1,47	-2,19	-1,76	-1,20	0,80
4,80	0,89	-2,04	-1,55	-1,74	0,76
4,90	0,33	-1,93	-1,19	-3,61	0,62
5,00	0,13	-1,99	-0,39	-3,00	0,20

#### 2.4.5 $m_{xy}/m$ Plots Along the Length

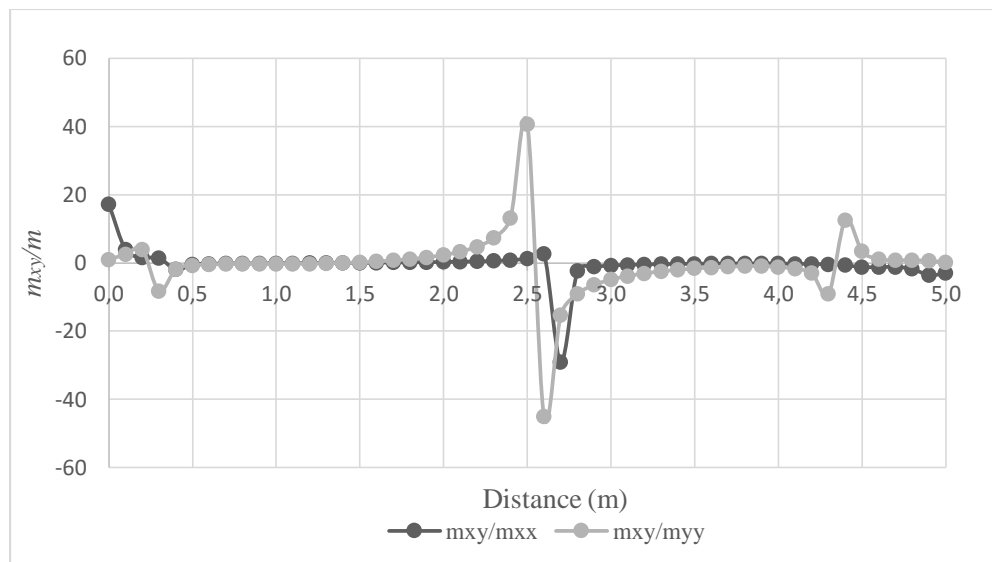


Figure 25. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L4 model.

## 2.5 Model L5

### 2.5.1 Description

The slabs was modeled using finite elements using a 100 x 100 mm mesh. The (x, y) position of the distributed surface load is (700, 850) mm.

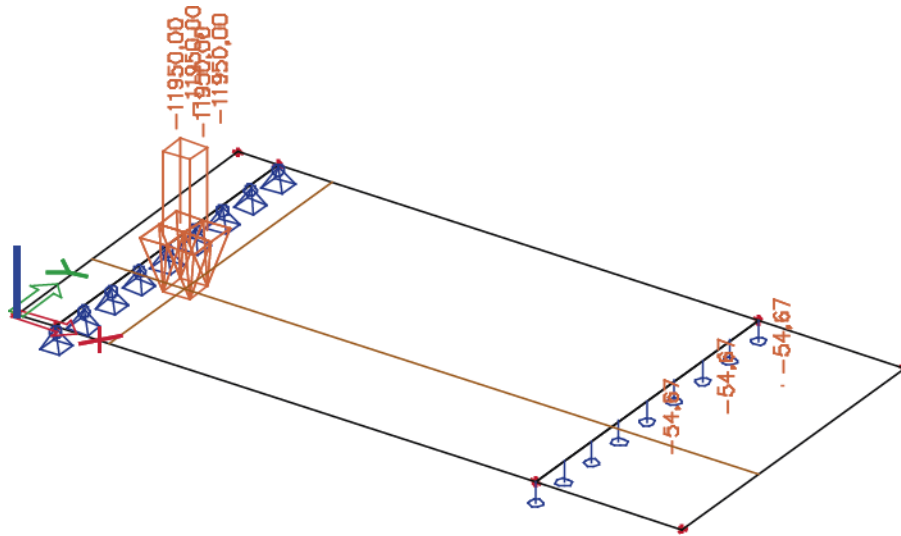


Figure 26. L5 Model with Distributed Surface Load representing wheel print with coordinates (700 mm, 850 mm). The concentrated loads beyond the second support create a moment over support 2.

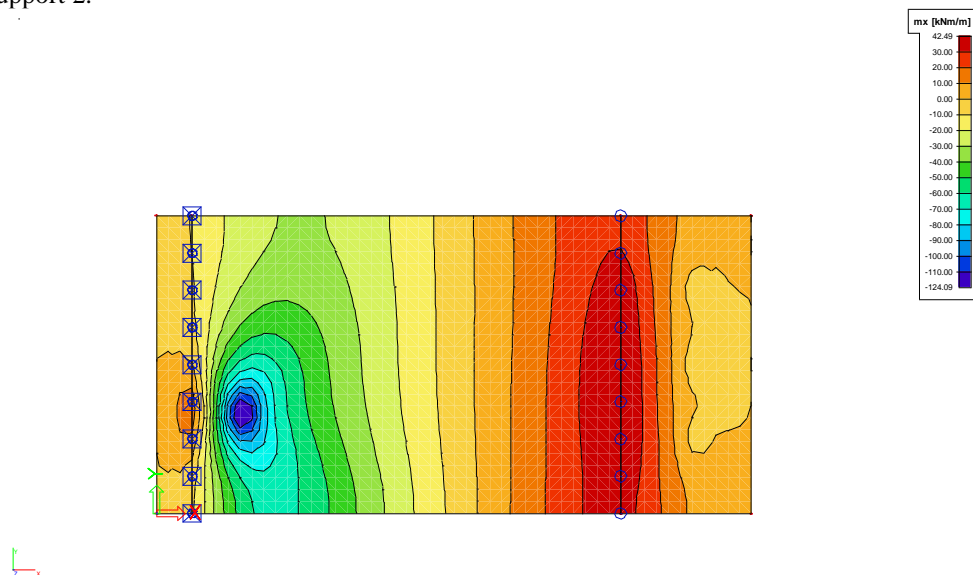


Figure 27.  $m_x$  plot of the L5 slab model analyzed using finite elements method.

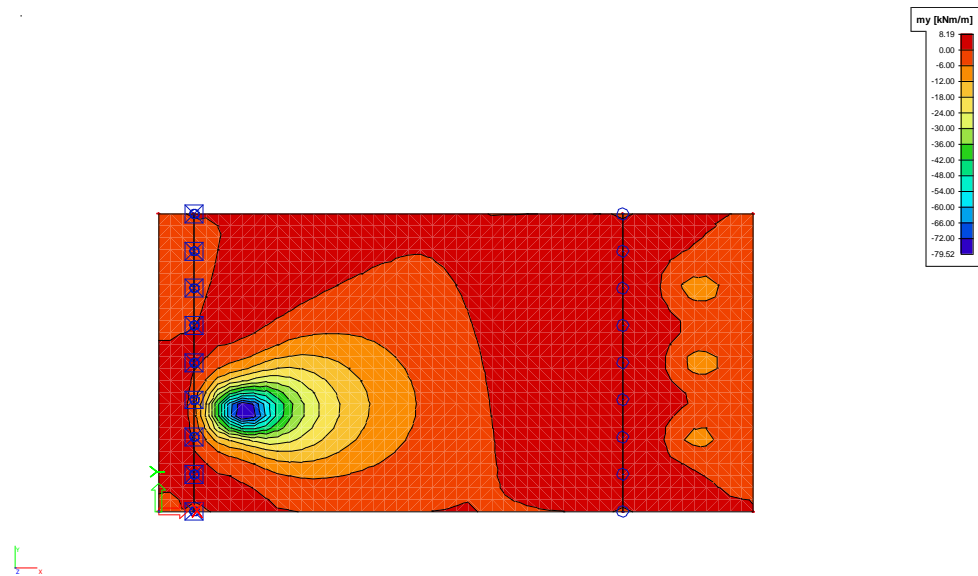


Figure 28.  $m_y$  plot of the L5 slab model analyzed using finite elements method.

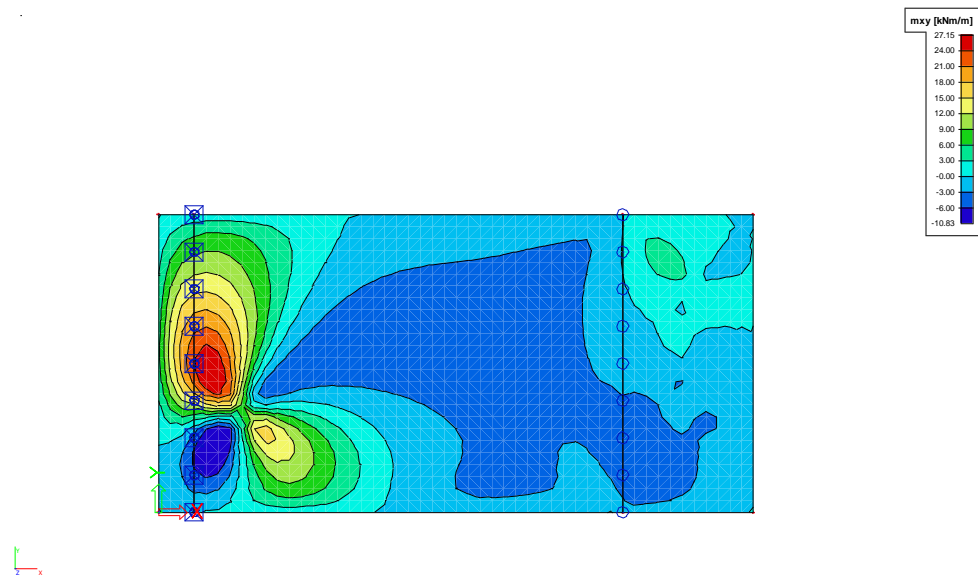


Figure 29.  $m_{xy}$  plot of the L5 slab model analyzed using finite elements method.

### 2.5.2 Moments Along the Width

Table 9: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<i>y direction L5 model results</i>					
Distance (m)	$m_{xx}$ (kN -m/m)	$m_{yy}$ (kN -m/m)	$m_{xy}$ (kN -m/m)	$m_{xy}/m_{xx}$	$m_{xy}/m_{yy}$
0,00	-52,91	0,25	0,73	-0,01	2,92
0,10	-53,60	0,67	1,46	-0,03	2,18
0,20	-55,81	0,57	1,40	-0,03	2,46

0,30	-59,98	-0,70	0,98	-0,02	-1,40
0,40	-66,19	-3,91	0,83	-0,01	-0,21
0,50	-75,02	-10,10	1,21	-0,02	-0,12
0,60	-88,05	-21,66	2,25	-0,03	-0,10
0,70	-107,46	-48,83	3,70	-0,03	-0,08
0,80	-124,09	-79,52	6,20	-0,05	-0,08
0,90	-121,32	-76,34	8,58	-0,07	-0,11
1,00	-102,64	-46,09	10,27	-0,10	-0,22
1,10	-83,19	-22,76	12,35	-0,15	-0,54
1,20	-69,09	-10,93	13,77	-0,20	-1,26
1,30	-58,58	-4,59	14,48	-0,25	-3,15
1,40	-50,49	-1,03	14,63	-0,29	-14,20
1,50	-44,17	0,89	14,33	-0,32	16,10
1,60	-39,20	1,86	13,69	-0,35	7,36
1,70	-35,27	2,25	12,84	-0,36	5,71
1,80	-32,16	2,31	11,86	-0,37	5,13
1,90	-29,70	2,18	10,81	-0,36	4,96
2,00	-27,76	1,91	9,75	-0,35	5,10
2,10	-26,25	1,57	8,70	-0,33	5,54
2,20	-25,11	1,17	7,63	-0,30	6,52
2,30	-24,31	0,73	6,37	-0,26	8,73
2,40	-23,94	0,37	4,34	-0,18	11,73
2,50	-23,86	0,16	1,03	-0,04	6,44

### 2.5.3 $m_{xy}/m$ Plots Along the Width

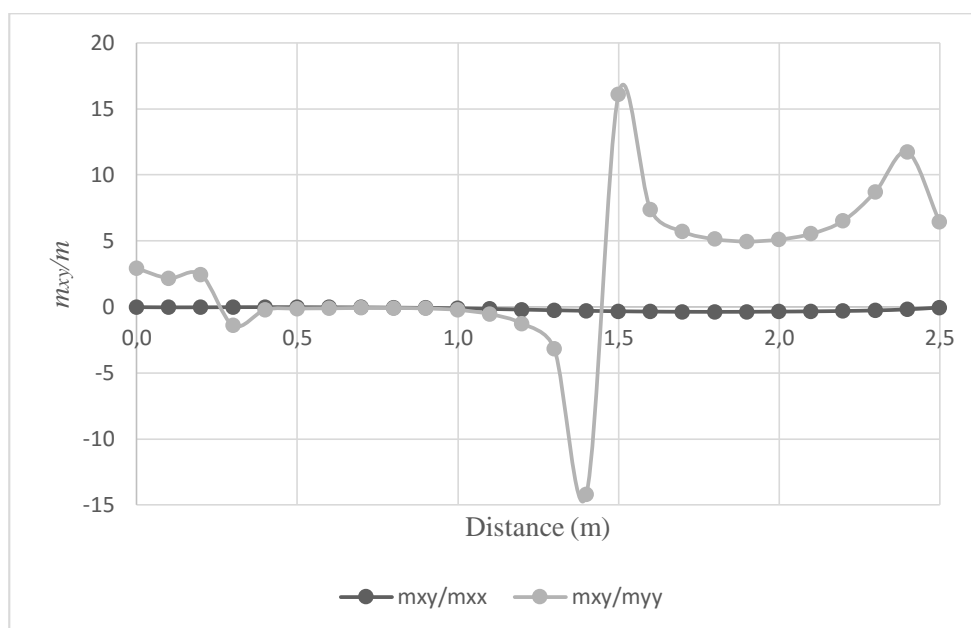


Figure 30. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_{yy}$  along the width of the L5 model.

## 2.5.4 Moments Along the Length

Table 30: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>x direction L5 model results</i></b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0,00	0,30	4,77	2,19	7,30	0,46
0,10	3,28	4,56	5,61	1,71	1,23
0,20	12,43	3,55	8,09	0,65	2,28
0,30	15,01	-4,22	7,90	0,53	-1,87
0,40	-10,30	-18,23	9,01	-0,87	-0,49
0,50	-48,11	-41,11	8,63	-0,18	-0,21
0,60	-93,63	-64,10	8,92	-0,10	-0,14
0,70	-122,70	-77,93	7,39	-0,06	-0,09
0,80	-111,32	-73,04	5,84	-0,05	-0,08
0,90	-84,77	-58,76	5,47	-0,06	-0,09
1,00	-69,10	-46,35	4,70	-0,07	-0,10
1,10	-60,29	-37,67	3,89	-0,06	-0,10
1,20	-53,74	-31,13	3,04	-0,06	-0,10
1,30	-48,44	-26,02	2,19	-0,05	-0,08
1,40	-43,89	-21,93	1,39	-0,03	-0,06
1,50	-39,78	-18,58	0,64	-0,02	-0,03
1,60	-35,96	-15,78	-0,05	0,00	0,00
1,70	-32,33	-13,41	-0,66	0,02	0,05
1,80	-8,82	-11,39	-1,21	0,14	0,11
1,90	-26,40	-9,64	-1,68	0,06	0,17
2,00	-22,04	-8,10	-2,09	0,09	0,26
2,10	-18,72	-6,74	-2,44	0,13	0,36
2,20	-15,42	-5,52	-2,73	0,18	0,49
2,30	-12,15	-4,42	-2,96	0,24	0,67
2,40	-8,89	-3,11	-3,14	0,35	1,01
2,50	-5,64	-2,47	-3,28	0,58	1,33
2,60	-2,39	-1,58	-3,38	1,41	2,14
2,70	0,86	-0,74	-3,44	-4,00	4,65
2,80	4,12	0,07	-3,46	-0,84	-49,43
2,90	7,40	0,86	-3,45	-0,47	-4,01
3,00	10,70	1,64	-3,42	-0,32	-2,09
3,10	14,03	2,42	-3,37	-0,24	-1,39
3,20	17,41	3,21	-3,31	-0,19	-1,03
3,30	20,85	4,00	-3,25	-0,16	-0,81
3,40	24,37	4,80	-3,21	-0,13	-0,67
3,50	28,02	5,60	-3,19	-0,11	-0,57
3,60	31,82	6,39	-3,21	-0,10	-0,50
3,70	35,82	7,16	-3,27	-0,09	-0,46
3,80	40,59	7,99	-3,39	-0,08	-0,42
3,90	42,45	8,08	-3,32	-0,08	-0,41



4,00	36,98	6,68	-3,48	-0,09	-0,52
4,10	28,55	4,58	-3,35	-0,12	-0,73
4,20	20,77	2,68	-3,09	-0,15	-1,15
4,30	13,08	1,00	-2,64	-0,20	-2,64
4,40	5,84	-0,33	-2,30	-0,39	6,97
4,50	0,42	-1,22	-2,67	-6,36	2,19
4,60	-1,31	-2,43	-2,94	2,24	1,21
4,70	-1,09	-3,37	-2,59	2,38	0,77
4,80	-0,58	-3,90	-2,05	3,53	0,53
4,90	-0,17	-4,16	-1,36	8,00	0,33
5,00	0,04	-4,25	-0,35	-8,75	0,08

### 2.5.5 $m_{xy}/m$ Plots Along the Length

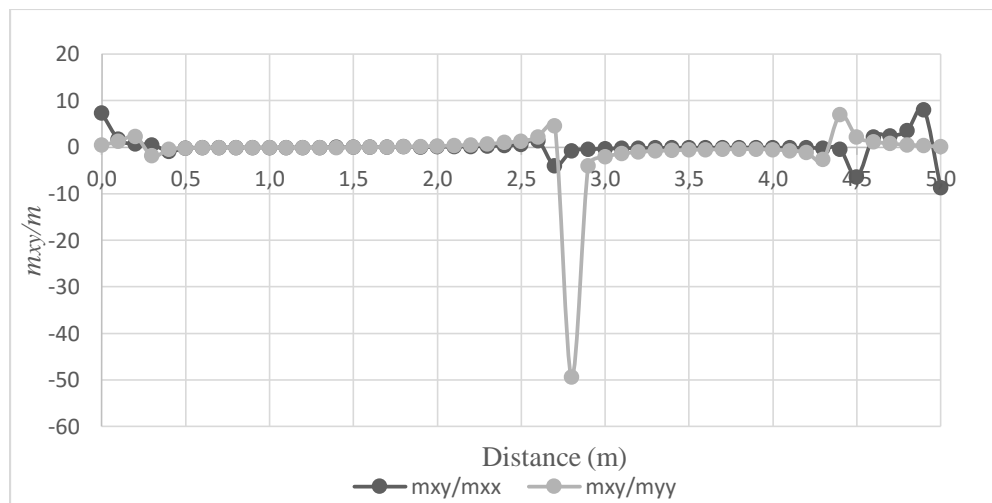


Figure 31. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L5 model.

## 2.6 Model L6

### 2.6.1 Description

The slabs was modeled using finite elements using a 100 x 100 mm mesh. The (x, y) position of the distributed surface load is (700, 1250) mm.

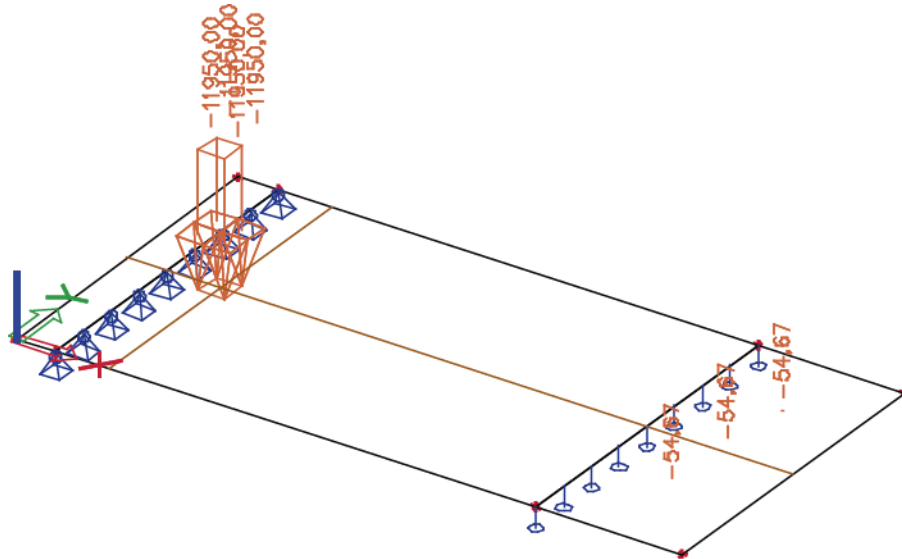


Figure 32. L6 Model with Distributed Surface Load representing wheel print with coordinates (700 mm, 1250 mm). The concentrated loads beyond the second support create a moment over support 2.

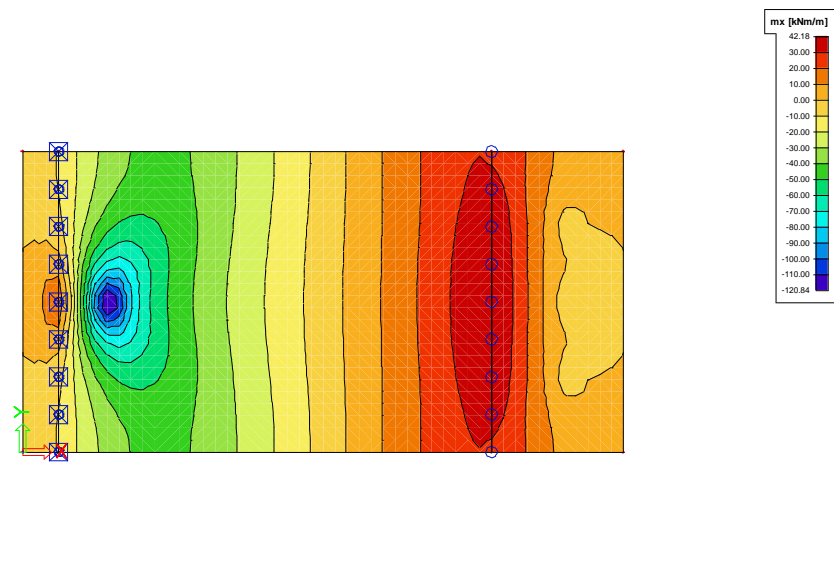


Figure 33.  $m_x$  plot of the L6 slab model analyzed using finite elements method.

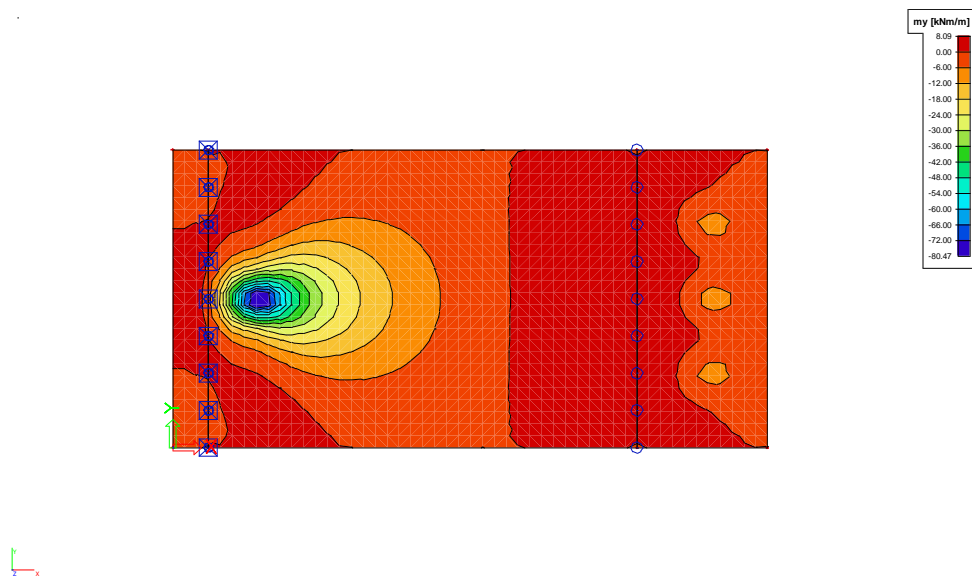


Figure 34.  $m_y$  plot of the L6 slab model analyzed using finite elements method.

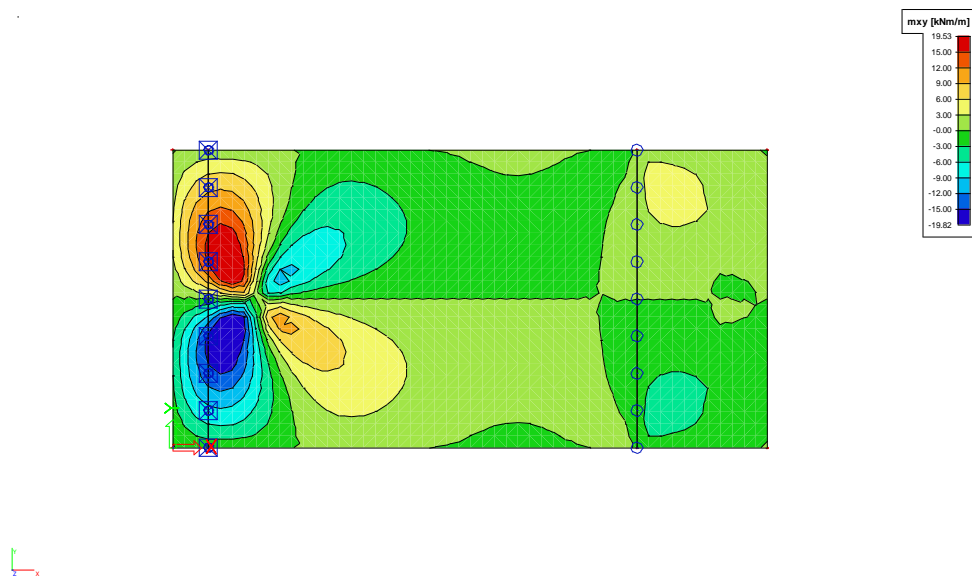


Figure 35.  $m_{xy}$  plot of the L6 slab model analyzed using finite elements method.

## 2.6.2 Moments Along the Width

Table 14: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<i>y direction L6 model results</i>					
Distance (m)	$m_{xx}$ (kN -m/m)	$m_{yy}$ (kN -m/m)	$m_{xy}$ (kN -m/m)	$m_{xy}/m_{xx}$	$m_{xy}/m_{yy}$
0,00	-33,37	0,14	-0,50	0,01	-3,57
0,10	-33,64	0,54	-2,70	0,08	-5,00
0,20	-34,47	1,06	-4,28	0,12	-4,04
0,30	-36,10	1,48	-5,45	0,15	-3,68
0,40	-38,47	1,62	-6,46	0,17	-3,99

0,50	-41,68	1,35	-7,32	0,18	-5,42
0,60	-45,96	0,46	-7,94	0,17	-17,26
0,70	-51,60	-1,44	-8,20	0,16	5,69
0,80	-59,05	-4,98	-7,99	0,14	1,60
0,90	-68,98	-11,29	-7,18	0,10	0,64
1,00	-83,01	-22,83	-5,73	0,07	0,25
1,10	-103,35	-49,91	-3,68	0,04	0,07
1,20	-120,84	-80,47	-1,42	0,01	0,02
1,30	-118,86	-77,15	1,51	-0,01	-0,02
1,40	-100,76	-46,95	3,61	-0,04	-0,08
1,50	-82,18	-23,46	5,70	-0,07	-0,24
1,60	-68,74	-11,51	7,19	-0,10	-0,62
1,70	-58,93	-5,10	7,99	-0,14	-1,57
1,80	-51,54	-1,51	8,20	-0,16	-5,43
1,90	-45,92	0,41	7,93	-0,17	19,34
2,00	-41,66	1,32	7,31	-0,18	5,54
2,10	-38,45	1,60	6,46	-0,17	4,04
2,20	-36,09	1,47	5,44	-0,15	3,70
2,30	-34,46	1,06	4,27	-0,12	4,03
2,40	-33,62	0,54	2,69	-0,08	4,98
2,50	-33,36	0,14	0,50	-0,01	3,57

### 2.6.3 $m_{xy}/m$ Plots Along the Width

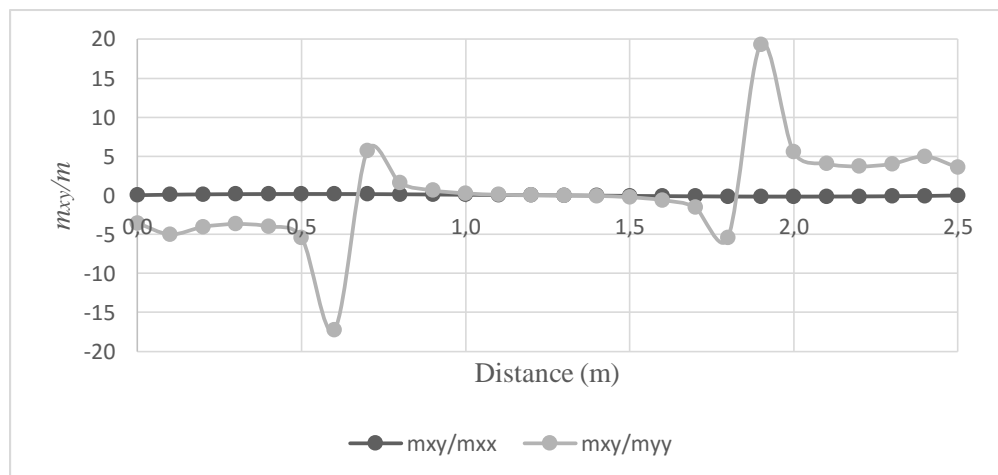


Figure 36. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L6 model.

## 2.6.4 Moments Along the Length

Table 52: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>x direction L6 model results</i></b>					
<b>Distance (m)</b>	<b><i>m<sub>xx</sub> (kN -m/m)</i></b>	<b><i>m<sub>yy</sub> (kN -m/m)</i></b>	<b><i>m<sub>xy</sub> (KN -m/m)</i></b>	<b><i>m<sub>xy</sub>/m<sub>xx</sub></i></b>	<b><i>m<sub>xy</sub>/m<sub>yy</sub></i></b>
0,00	0,30	5,10	0,01	0,03	0,00
0,10	3,46	4,88	0,02	0,01	0,00
0,20	13,26	3,57	0,04	0,00	0,01
0,30	15,63	-3,66	0,07	0,00	-0,02
0,40	-8,54	-18,60	0,18	-0,02	-0,01
0,50	-46,11	-41,52	0,40	-0,01	-0,01
0,60	-91,02	-64,92	0,79	-0,01	-0,01
0,70	-119,85	-78,81	0,04	0,00	0,00
0,80	-108,01	-74,25	-0,89	0,01	0,01
0,90	-81,24	-60,12	-0,43	0,01	0,01
1,00	-65,41	-47,86	-0,13	0,00	0,00
1,10	-56,58	-39,26	-0,06	0,00	0,00
1,20	-50,10	-32,77	-0,03	0,00	0,00
1,30	-44,55	-27,69	-0,02	0,00	0,00
1,40	-40,59	-23,59	-0,01	0,00	0,00
1,50	-36,24	-20,21	-0,01	0,00	0,00
1,60	-33,19	-17,37	-0,01	0,00	0,00
1,70	-29,05	-14,94	0,00	0,00	0,00
1,80	-26,63	-12,84	0,00	0,00	0,00
1,90	-23,48	-11,00	0,00	0,00	0,00
2,00	-20,39	-9,37	0,00	0,00	0,00
2,10	-17,31	-7,91	0,00	0,00	0,00
2,20	-14,25	-6,59	0,00	0,00	0,00
2,30	-11,18	-5,38	0,00	0,00	0,00
2,40	-8,12	-4,26	0,00	0,00	0,00
2,50	-5,04	-3,21	0,00	0,00	0,00
2,60	-1,94	-2,22	0,00	0,00	0,00
2,70	1,16	-1,26	0,00	0,00	0,00
2,80	4,30	-0,34	0,00	0,00	0,00
2,90	7,46	0,57	0,00	0,00	0,00
3,00	10,66	1,45	0,00	0,00	0,00
3,10	13,91	2,33	0,00	0,00	0,00
3,20	17,23	3,20	0,00	0,00	0,00
3,30	20,63	4,06	0,00	0,00	0,00
3,40	24,14	4,91	0,00	0,00	0,00
3,50	27,79	5,73	0,00	0,00	0,00
3,60	31,60	6,52	0,00	0,00	0,00
3,70	35,61	7,25	0,00	0,00	0,00
3,80	40,38	8,04	0,00	0,00	0,00
3,90	42,18	8,09	0,00	0,00	0,00

4,00	36,68	6,53	0,00	0,00	0,00
4,10	28,18	4,19	0,00	0,00	0,00
4,20	20,39	1,74	0,00	0,00	0,00
4,30	12,65	-1,07	0,00	0,00	0,00
4,40	3,32	-4,72	0,00	0,00	0,00
4,50	-3,84	-7,48	0,00	0,00	0,00
4,60	-3,62	-7,11	0,00	0,00	0,00
4,70	-1,26	-5,87	0,00	0,00	0,00
4,80	-0,63	-5,41	0,00	0,00	0,00
4,90	-0,21	-5,22	0,00	0,00	0,00
5,00	-0,01	-5,17	0,00	0,00	0,00

### 2.6.5 $m_{xy}/m$ Plots Along the Length

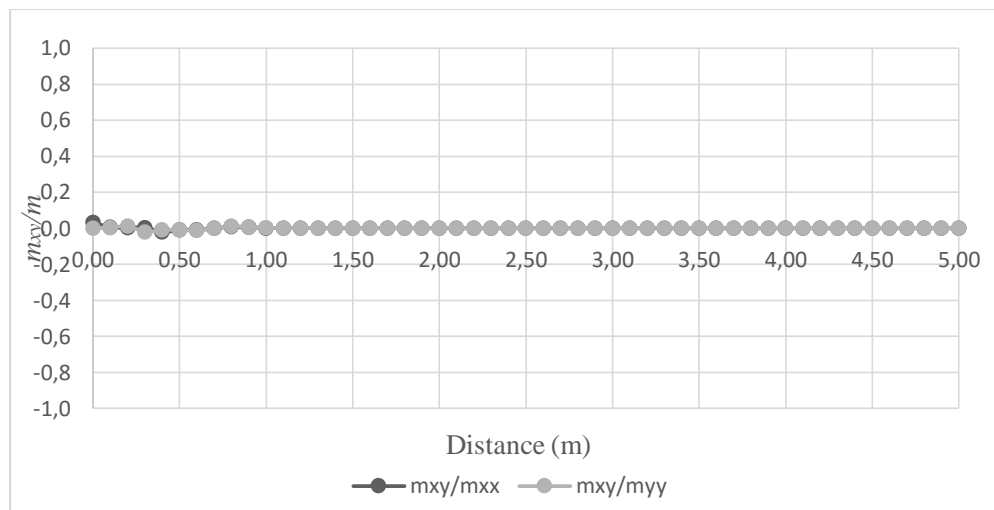


Figure 37. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L6 model.

## 2.7 Model L7

### 2.7.1 Description

The slabs was modeled using finite elements using a 100 x 100 mm mesh. The (x, y) position of the distributed surface load is (850, 438) mm.

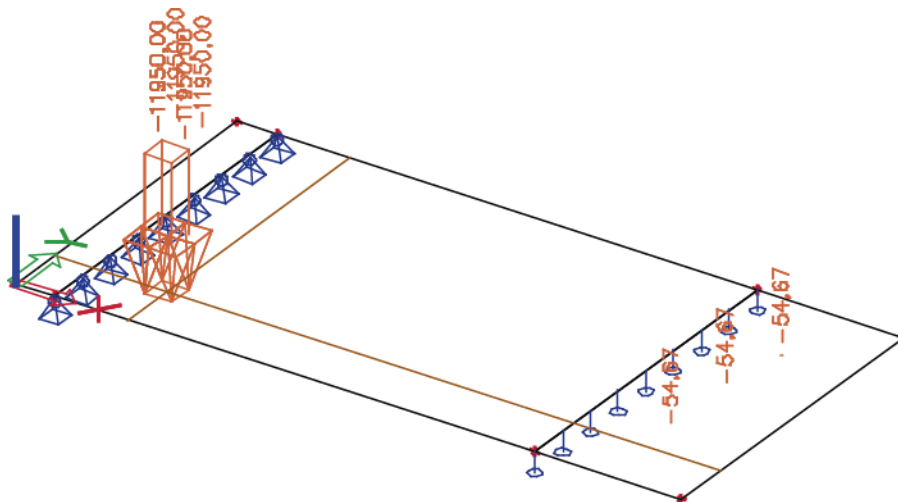


Figure 38. L7 Model with Distributed Surface Load representing wheel print with coordinates (850 mm, 438) mm). The concentrated loads beyond the second support create a moment over support 2.

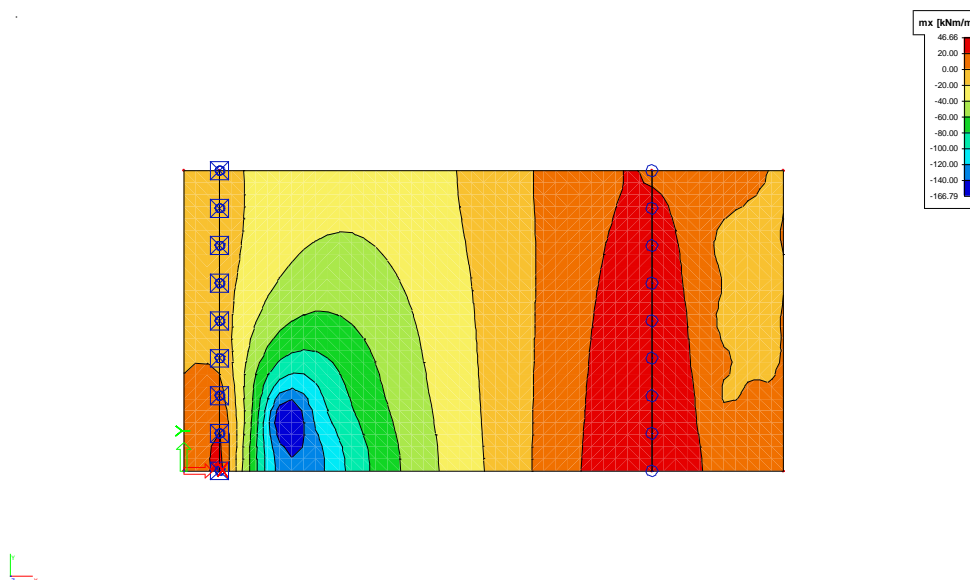


Figure 39.  $m_x$  plot of the L7 slab model analyzed using finite elements method.

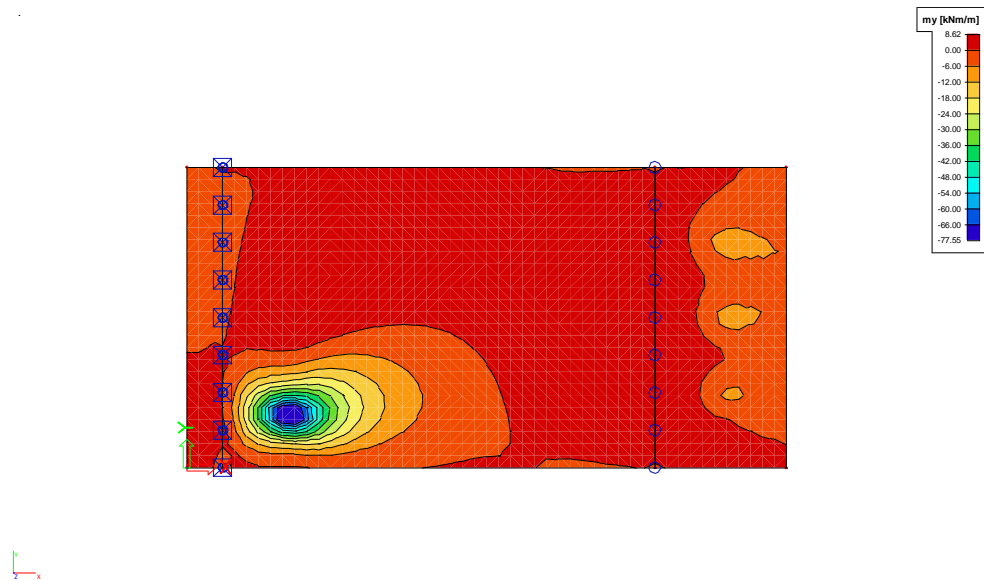


Figure 40.  $m_y$  plot of the L7 slab model analyzed using finite elements method.

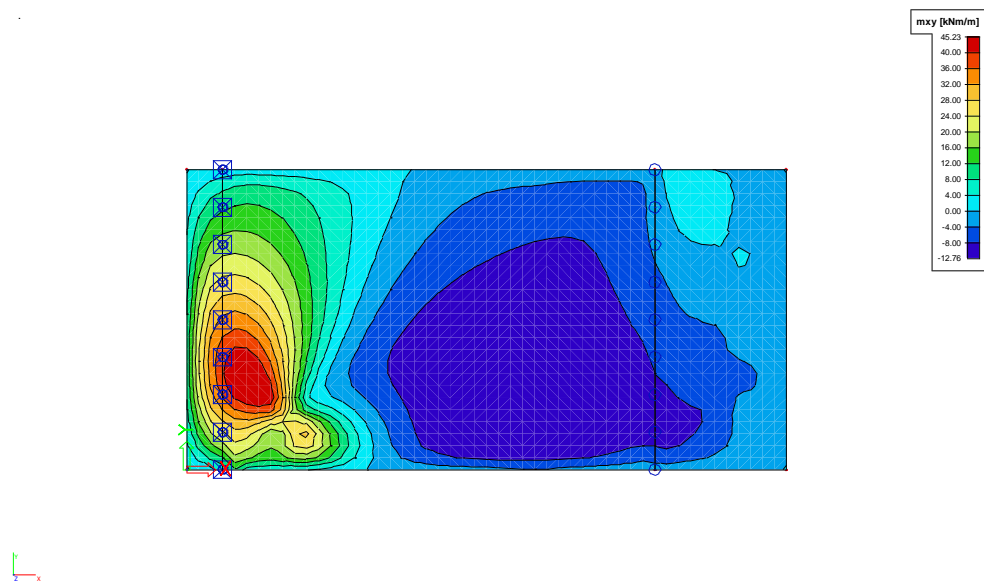


Figure 41.  $m_{xy}$  plot of the L7 slab model analyzed using finite elements method.

### 2.7.2 Moments Along the Width

Table 63: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b>y direction L7 model results</b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0,00	-132,62	0,42	4,75	-0,04	11,31
0,10	-134,45	-3,95	16,19	-0,12	-4,10
0,20	-141,07	-17,68	21,74	-0,15	-1,23
0,30	-153,23	-45,53	22,99	-0,15	-0,50



0,40	-162,55	-76,43	24,24	-0,15	-0,32
0,50	-155,09	-72,66	24,27	-0,16	-0,33
0,60	-134,05	-42,11	23,98	-0,18	-0,57
0,70	-113,43	-20,76	24,47	-0,22	-1,18
0,80	-97,85	-9,65	24,45	-0,25	-2,53
0,90	-85,53	-3,05	24,14	-0,28	-7,91
1,00	-75,63	1,01	23,59	-0,31	23,36
1,10	-67,29	3,47	22,82	-0,34	6,58
1,20	-60,41	4,88	21,86	-0,36	4,48
1,30	-54,65	5,61	20,76	-0,38	3,70
1,40	-49,79	5,88	19,57	-0,39	3,33
1,50	-45,69	5,85	18,33	-0,40	3,13
1,60	-42,22	5,62	17,08	-0,40	3,04
1,70	-39,29	5,24	15,84	-0,40	3,02
1,80	-36,83	4,75	14,63	-0,40	3,08
1,90	-34,77	4,17	13,48	-0,39	3,23
2,00	-33,06	3,51	12,39	-0,37	3,53
2,10	-31,66	2,79	11,34	-0,36	4,06
2,20	-30,54	2,02	10,26	-0,34	5,08
2,30	-29,71	1,20	8,85	-0,30	7,38
2,40	-29,36	0,57	6,21	-0,21	10,89
2,50	-29,33	0,25	1,56	-0,05	6,24

### 2.7.3 $m_{xy}/m$ Plots Along the Width

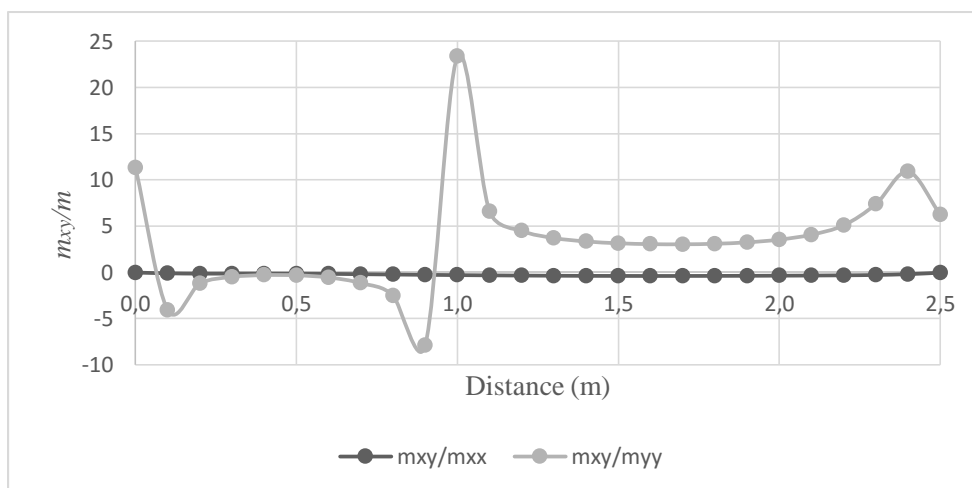


Figure 42. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L7 model.

## 2.7.4 Moments Along the Length

Table 74: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>x direction L7 model results</i></b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0,00	0,29	6,53	6,65	22,93	1,02
0,10	3,68	6,70	17,15	4,66	2,56
0,20	12,64	6,41	25,10	1,99	3,92
0,30	15,95	0,26	29,45	1,85	113,27
0,40	-7,68	-7,31	32,82	-4,27	-4,49
0,50	-38,65	-21,14	33,51	-0,87	-1,59
0,60	-70,77	-36,70	32,44	-0,46	-0,88
0,70	-112,50	-56,76	30,46	-0,27	-0,54
0,80	-155,26	-74,18	26,63	-0,17	-0,36
0,90	-164,18	-75,81	21,87	-0,13	-0,29
1,00	-139,26	-61,94	18,07	-0,13	-0,29
1,10	-115,70	-45,57	13,13	-0,11	-0,29
1,20	-102,36	-34,28	8,75	-0,09	-0,26
1,30	-92,20	-26,34	4,95	-0,05	-0,19
1,40	-83,50	-20,50	1,65	-0,02	-0,08
1,50	-75,71	-16,11	-1,17	0,02	0,07
1,60	-68,59	-12,75	-3,53	0,05	0,28
1,70	-61,97	-10,14	-5,50	0,09	0,54
1,80	-55,77	-8,08	-7,13	0,13	0,88
1,90	-49,90	-6,42	-8,46	0,17	1,32
2,00	-44,31	-5,08	-9,56	0,22	1,88
2,10	-38,95	-3,97	-10,44	0,27	2,63
2,20	-33,77	-3,03	-11,14	0,33	3,68
2,30	-28,75	-2,24	-11,69	0,41	5,22
2,40	-23,86	-1,54	-12,10	0,51	7,86
2,50	-19,05	-0,93	-12,39	0,65	13,32
2,60	-14,33	-0,37	-12,57	0,88	33,97
2,70	-9,65	0,15	-12,66	1,31	-84,40
2,80	-5,02	0,64	-12,65	2,52	-19,77
2,90	-0,40	1,12	-12,56	31,40	-11,21
3,00	4,21	1,60	-12,39	-2,94	-7,74
3,10	8,83	2,10	-12,15	-1,38	-5,79
3,20	13,46	2,63	-11,84	-0,88	-4,50
3,30	18,13	3,20	-11,46	-0,63	-3,58
3,40	22,84	3,84	-11,04	-0,48	-2,88
3,50	27,62	4,58	-10,59	-0,38	-2,31
3,60	32,50	5,44	-10,15	-0,31	-1,87
3,70	37,57	6,43	-9,77	-0,26	-1,52
3,80	43,57	7,68	-9,58	-0,22	-1,25

3,90	46,58	7,99	-9,02	-0,19	-1,13
4,00	41,18	6,82	-9,11	-0,22	-1,34
4,10	32,70	4,69	-8,76	-0,27	-1,87
4,20	25,31	2,81	-8,42	-0,33	-3,00
4,30	17,88	1,13	-7,07	-0,40	-6,26
4,40	10,35	-0,18	-7,09	-0,69	39,39
4,50	4,45	-0,98	-5,15	-1,16	5,26
4,60	2,52	-1,84	-3,02	-1,20	1,64
4,70	1,85	-1,88	-2,32	-1,25	1,23
4,80	1,10	-1,72	-2,02	-1,84	1,17
4,90	0,38	-1,59	-1,53	-4,03	0,96
5,00	0,17	-1,71	-0,52	-3,06	0,30

### 2.7.5 $m_{xy}/m$ Plots Along the Length

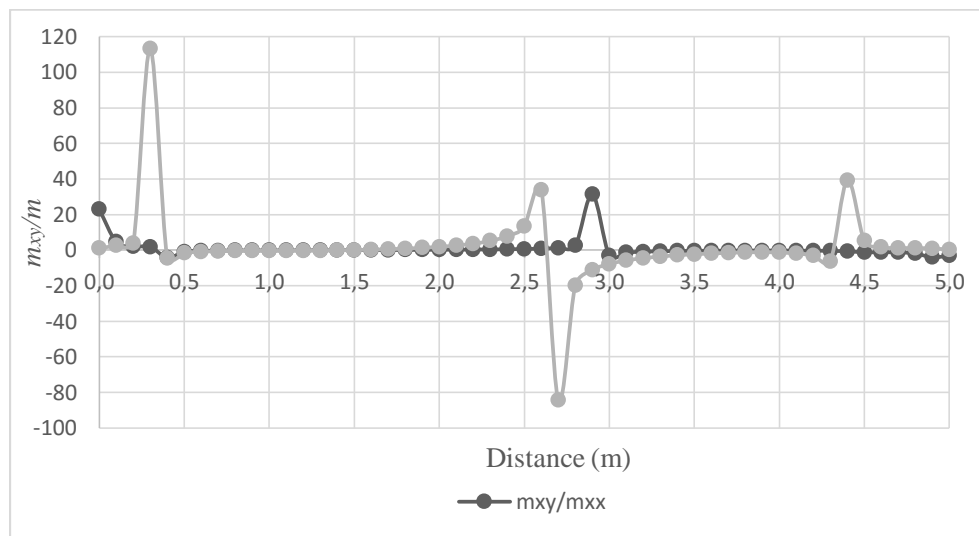


Figure 43. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_{yy}$  along the width of the L7 model.

## 2.8 Model L8

### 2.8.1 Description

The slabs was modeled using finite elements using a 100 x 100 mm mesh. The (x, y) position of the distributed surface load is (850, 850) mm.

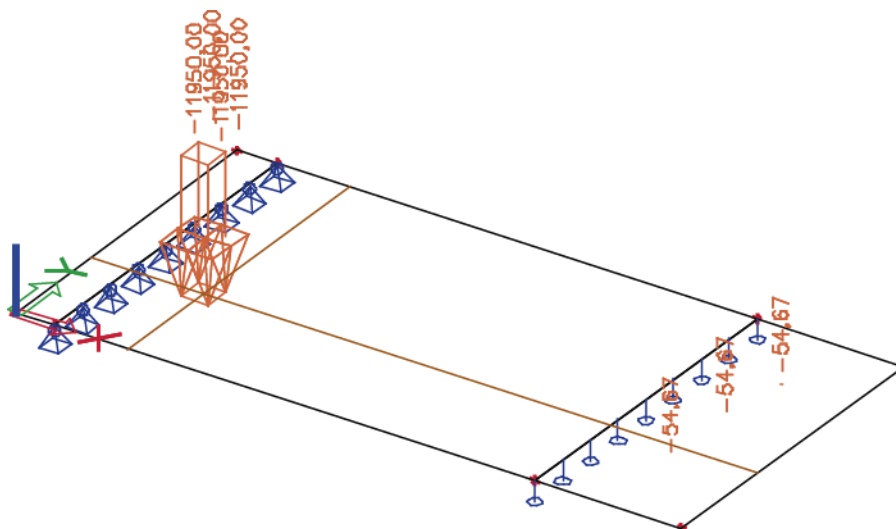


Figure 44. L8 Model with Distributed Surface Load representing wheel print with coordinates (850 mm, 850) mm). The concentrated loads beyond the second support create a moment over support 2.

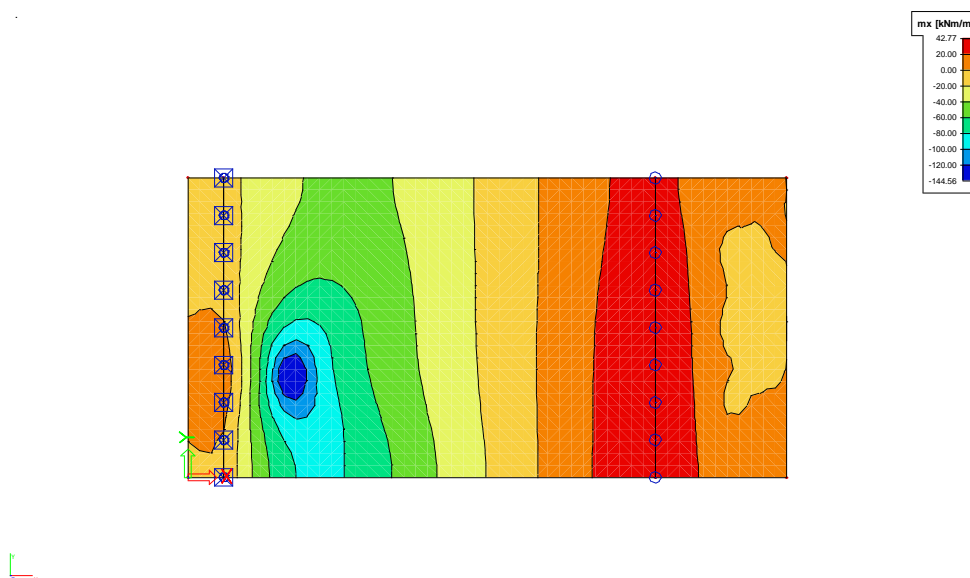


Figure 45.  $m_x$  plot of the L8 slab model analyzed using finite elements method.

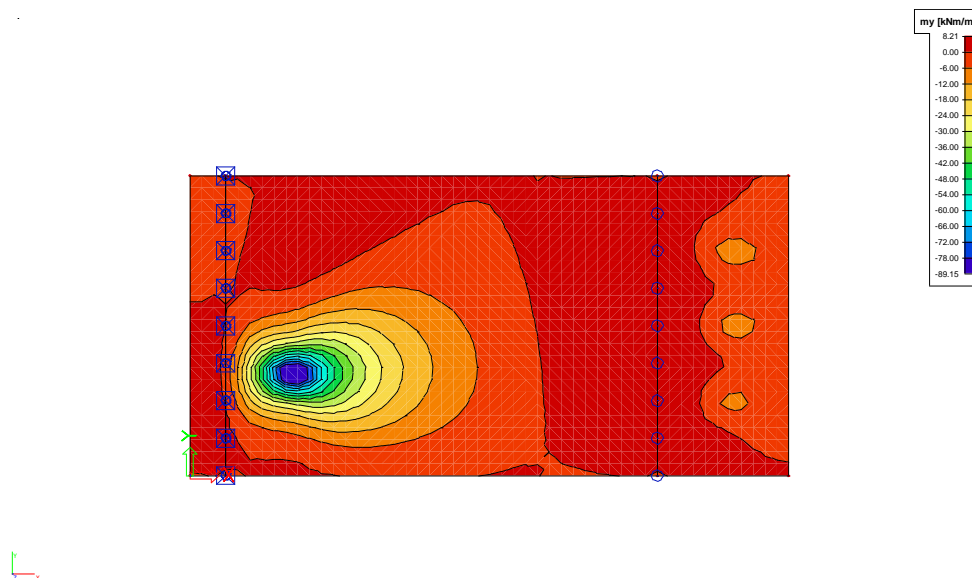


Figure 46.  $m_y$  plot of the L8 slab model analyzed using finite elements method.

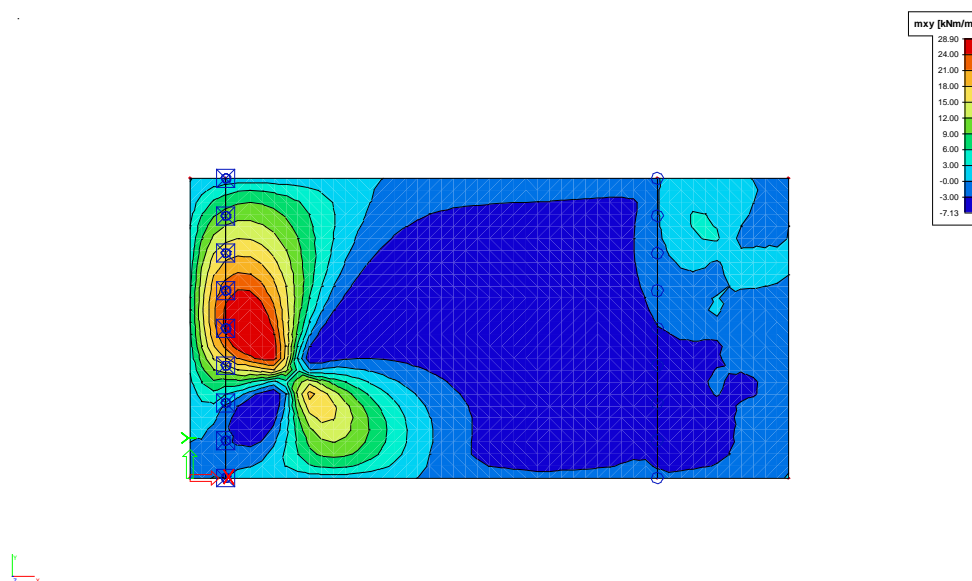


Figure 47.  $m_{xy}$  plot of the L8 slab model analyzed using finite elements method.

## 2.8.2 Moments Along the Width

Table 85: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>y</i> direction L8 model results</b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0,00	-76,19	0,24	1,30	-0,02	5,42
0,10	-76,88	0,19	3,76	-0,05	19,79
0,20	-79,08	-0,94	4,71	-0,06	-5,01
0,30	-83,14	-3,69	4,73	-0,06	-1,28
0,40	-89,11	-8,68	4,73	-0,05	-0,54

0,50	-97,47	-16,81	5,01	-0,05	-0,30
0,60	-109,30	-30,23	5,61	-0,05	-0,19
0,70	-126,16	-56,48	6,34	-0,05	-0,11
0,80	-141,56	-87,35	7,85	-0,06	-0,09
0,90	-140,32	-87,71	9,23	-0,07	-0,11
1,00	-122,86	-57,01	10,07	-0,08	-0,18
1,10	-103,57	-31,23	11,49	-0,11	-0,37
1,20	-89,29	-18,09	12,45	-0,14	-0,69
1,30	-78,37	-10,03	13,06	-0,17	-1,30
1,40	-69,65	-4,81	13,33	-0,19	-2,77
1,50	-62,57	-1,46	13,28	-0,21	-9,10
1,60	-56,78	0,65	12,96	-0,23	19,94
1,70	-52,02	1,91	12,42	-0,24	6,50
1,80	-48,12	2,67	11,72	-0,24	4,39
1,90	-44,92	2,81	10,90	-0,24	3,88
2,00	-42,33	2,73	10,01	-0,24	3,67
2,10	-40,26	2,42	9,07	-0,23	3,75
2,20	-38,64	1,90	8,05	-0,21	4,24
2,30	-37,47	1,23	6,78	-0,18	5,51
2,40	-36,90	0,59	4,61	-0,12	7,81
2,50	-36,74	0,17	1,07	-0,03	6,29

### 2.8.3 $m_{xy}/m$ Plots Along the Width

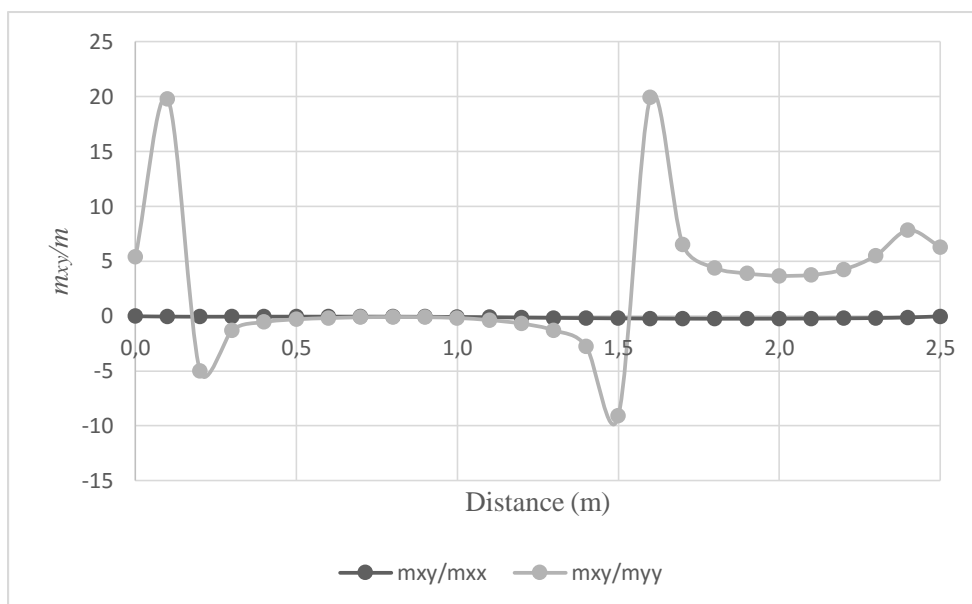


Figure 48. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L8 model.

## 2.8.4 Moments Along the Length

Table 96: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>x direction L8 model results</i></b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0,00	0,24	4,45	2,80	11,67	0,63
0,10	2,76	4,32	7,20	2,61	1,67
0,20	10,12	3,59	10,26	1,01	2,86
0,30	12,75	-2,62	10,45	0,82	-3,99
0,40	-7,05	-12,06	11,40	-1,62	-0,95
0,50	-32,95	-27,66	10,86	-0,33	-0,39
0,60	-59,99	-45,16	10,42	-0,17	-0,23
0,70	-97,31	-67,00	10,03	-0,10	-0,15
0,80	-137,66	-86,07	9,02	-0,07	-0,10
0,90	-144,21	-89,02	8,07	-0,06	-0,09
1,00	-117,40	-75,62	7,15	-0,06	-0,09
1,10	-93,94	-59,72	5,78	-0,06	-0,10
1,20	-81,71	-48,29	4,61	-0,06	-0,10
1,30	-73,18	-39,88	3,47	-0,05	-0,09
1,40	-66,39	-33,55	2,36	-0,04	-0,07
1,50	-60,60	-28,13	1,32	-0,02	-0,05
1,60	-55,41	-23,86	0,35	-0,01	-0,01
1,70	-50,61	-20,30	-0,53	0,01	0,03
1,80	-46,05	-17,30	-1,32	0,03	0,08
1,90	-41,65	-14,73	-2,02	0,05	0,14
2,00	-37,37	-12,51	-2,64	0,07	0,21
2,10	-33,16	-10,56	-3,16	0,10	0,30
2,20	-29,01	-8,84	-3,61	0,12	0,41
2,30	-24,89	-7,31	-3,99	0,16	0,55
2,40	-20,80	-5,92	-4,29	0,21	0,72
2,50	-16,72	-4,66	-4,54	0,27	0,97
2,60	-12,65	-3,49	-4,72	0,37	1,35
2,70	-8,57	-2,39	-4,85	0,57	2,03
2,80	-4,49	-1,36	-4,93	1,10	3,63
2,90	-0,40	-0,36	-4,96	12,40	13,78
3,00	3,72	-0,60	-4,96	-1,33	8,27
3,10	7,87	1,54	-4,92	-0,63	-3,19
3,20	12,08	2,48	-4,86	-0,40	-1,96
3,30	16,34	3,41	-4,79	-0,29	-1,40
3,40	20,70	4,34	-4,71	-0,23	-1,09
3,50	25,17	5,26	-4,64	-0,18	-0,88
3,60	29,79	6,16	-4,58	-0,15	-0,74
3,70	34,61	7,04	-4,53	-0,13	-0,64
3,80	40,21	8,00	-4,55	-0,11	-0,57
3,90	42,71	8,16	-4,29	-0,10	-0,53

4,00	37,39	6,78	-4,33	-0,12	-0,64
4,10	28,90	4,66	-4,06	-0,14	-0,87
4,20	21,09	2,76	-3,69	-0,17	-1,34
4,30	13,36	1,07	-3,17	-0,24	-2,96
4,40	6,05	-0,25	-2,77	-0,46	11,08
4,50	0,59	-1,14	-3,08	-5,22	2,70
4,60	-1,18	-2,35	-3,30	2,80	1,40
4,70	-1,00	-3,28	-2,92	2,92	0,89
4,80	-0,54	-3,81	-2,35	4,35	0,62
4,90	-0,16	-4,07	-1,59	9,94	0,39
5,00	0,06	-4,18	-0,42	-7,00	0,10

### 2.8.5 $m_{xy}/m$ Plots Along the Length

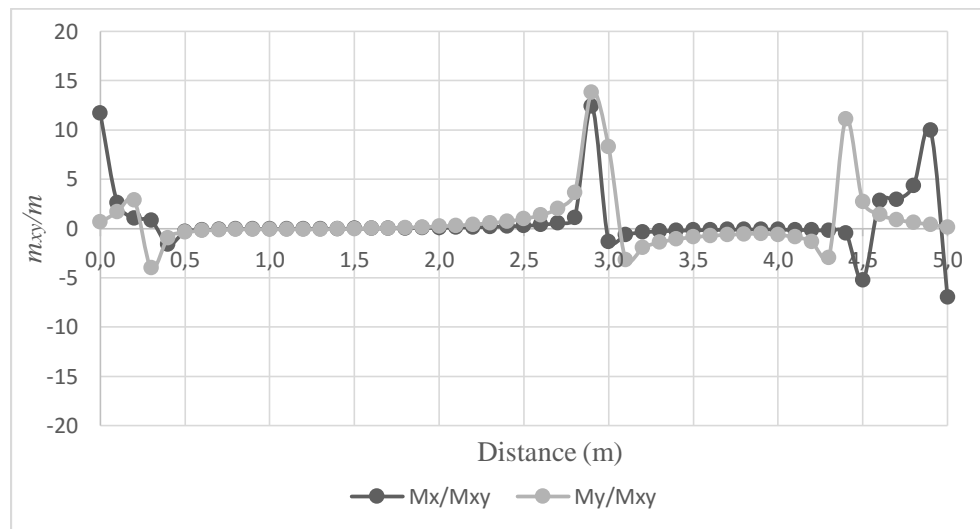


Figure 49. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L8 model.





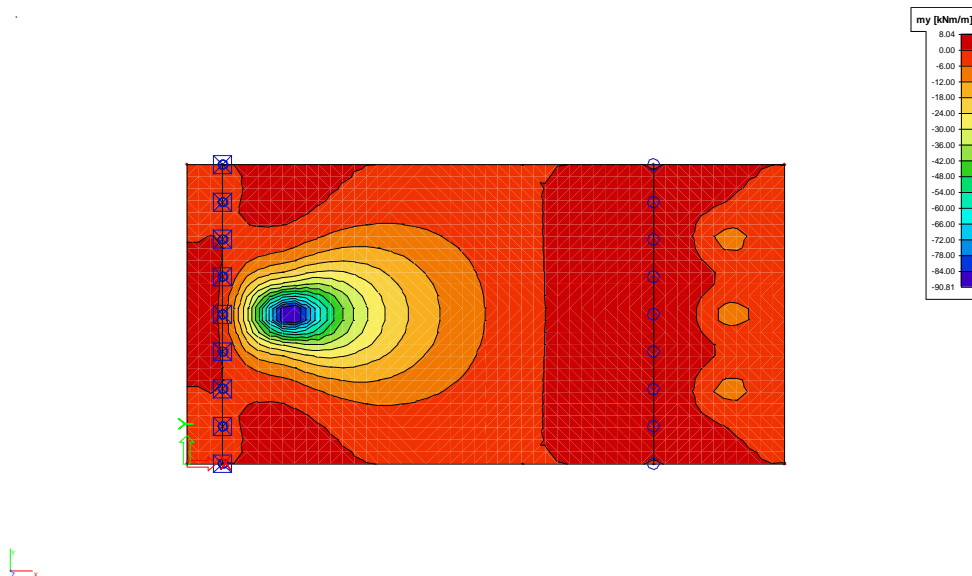


Figure 52.  $m_y$  plot of the L9 slab model analyzed using finite elements method.

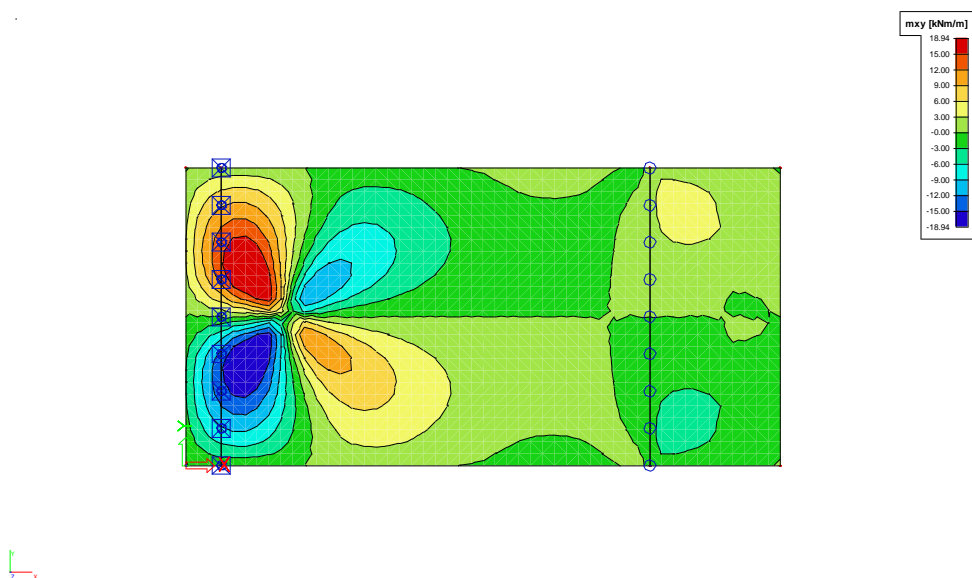


Figure 53.  $m_{xy}$  plot of the L9 slab model analyzed using finite elements method.

### 2.9.2 Moments Along the Width

Table 107: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>y direction L9 model results</i></b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0,00	-50,30	0,15	-0,30	0,01	-2,00
0,10	-50,65	0,62	-1,97	0,04	-3,18
0,20	-51,71	1,12	-3,22	0,06	-2,88
0,30	-53,72	1,30	-4,15	0,08	-3,19

0,40	-56,55	0,93	-4,39	0,08	-4,72
0,50	-60,30	-0,17	-5,44	0,09	32,00
0,60	-65,10	-2,23	-5,75	0,09	2,58
0,70	-71,19	-5,61	-5,76	0,08	1,03
0,80	-78,93	-10,91	-5,42	0,07	0,50
0,90	-88,86	-19,11	-4,71	0,05	0,25
1,00	-102,12	-32,43	-3,64	0,04	0,11
1,10	-120,30	-58,50	-2,25	0,02	0,04
1,20	-136,87	-89,23	-0,89	0,01	0,01
1,30	-136,87	-89,23	0,89	-0,01	-0,01
1,40	-120,30	-58,50	2,25	-0,02	-0,04
1,50	-102,12	-32,43	3,64	-0,04	-0,11
1,60	-88,86	-19,11	4,71	-0,05	-0,25
1,70	-78,93	-10,91	5,42	-0,07	-0,50
1,80	-71,19	-5,61	5,76	-0,08	-1,03
1,90	-65,10	-2,23	5,75	-0,09	-2,58
2,00	-60,30	-0,17	5,44	-0,09	-32,00
2,10	-56,55	0,93	4,89	-0,09	5,26
2,20	-53,72	1,30	4,15	-0,08	3,19
2,30	-51,71	1,12	3,22	-0,06	2,88
2,40	-50,65	0,62	1,97	-0,04	3,18
2,50	-50,30	0,15	0,30	-0,01	2,00

### 2.9.3 $m_{xy}/m$ Plots Along the Width

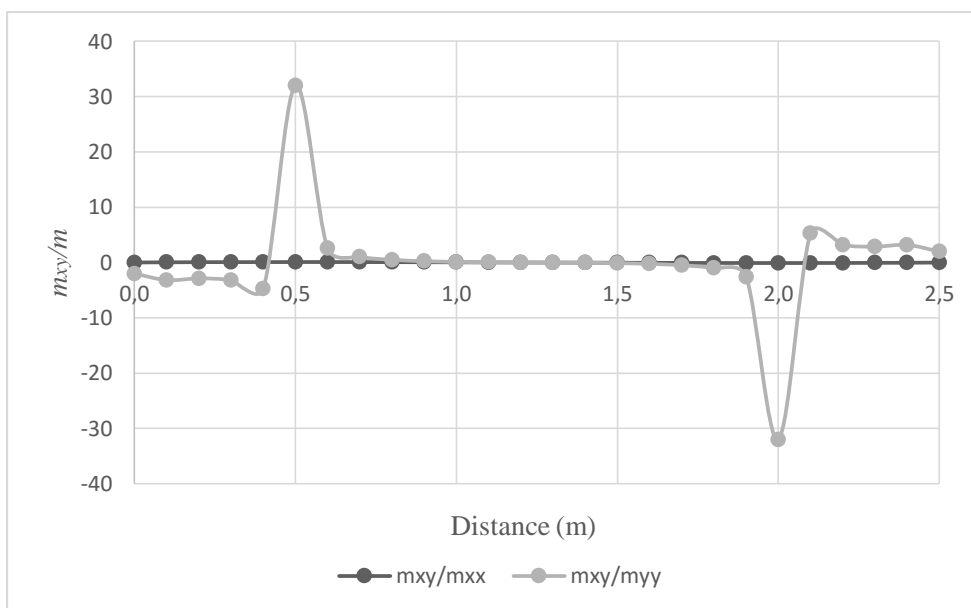


Figure 54. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L9 model.

## 2.9.4 Moments Along the Length

Table 118: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>x direction L9 model results</i></b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0	0,25	4,68	0	0	0
0,05	2,98	4,51	0	0	0
0,15	11,03	3,46	0	0	0
0,25	13,24	-1,97	0	0	0
0,35	-4,93	-12,81	0	0	0
0,45	-30,73	-28,25	0	0	0
0,55	-57,03	-46,3	0	0	0
0,65	-93,72	-68,51	0	0	0
0,75	-133,78	-87,65	0	0	0
0,85	-139,97	-90,81	0	0	0
0,95	-112,81	-77,68	0	0	0
1,05	-89,17	-61,96	0	0	0
1,15	-76,95	-50,6	0	0	0
1,25	-68,52	-42,25	0	0	0
1,35	-61,93	-35,73	0	0	0
1,45	-56,4	-30,48	0	0	0
1,55	-51,54	-26,16	0	0	0
1,65	-47	-22,53	0	0	0
1,75	-42,9	-19,43	0	0	0
1,85	-38,87	-16,75	0	0	0
1,95	-34,94	-14,41	0	0	0
2,05	-31,07	-12,34	0	0	0
2,15	-27,23	-10,49	0	0	0
2,25	-23,41	-8,82	0	0	0
2,35	-19,58	-7,29	0	0	0
2,45	-15,75	-5,88	0	0	0
2,55	-11,91	-4,57	0	0	0
2,65	-8,05	-3,34	0	0	0
2,75	-4,16	-2,16	0	0	0
2,85	-0,24	-1,03	0	0	0
2,95	3,72	0,07	0	0	0
3,05	7,73	1,13	0	0	0
3,15	11,8	2,17	0	0	0
3,25	15,96	3,2	0	0	0
3,35	20,23	4,19	0	0	0
3,45	24,63	5,16	0	0	0
3,55	29,2	6,08	0	0	0
3,65	33,95	6,96	0	0	0
3,75	39,49	7,88	0	0	0
3,85	41,88	8,04	0	0	0

3,95	36,57	6,5	0	0	0
4,05	28,07	4,14	0	0	0
4,15	20,31	1,69	0	0	0
4,25	12,59	-1,12	0	0	0
4,35	3,28	-4,77	0	0	0
4,45	-3,87	-7,52	0	0	0
4,55	-3,64	-7,16	0	0	0
4,65	-1,27	-5,91	0	0	0
4,75	-0,64	-5,46	0	0	0
4,85	-0,21	-5,25	0	0	0
4,95	-0,01	-5,22	0	0	0

### 2.9.5 $m_{xy}/m$ Plots Along the Length

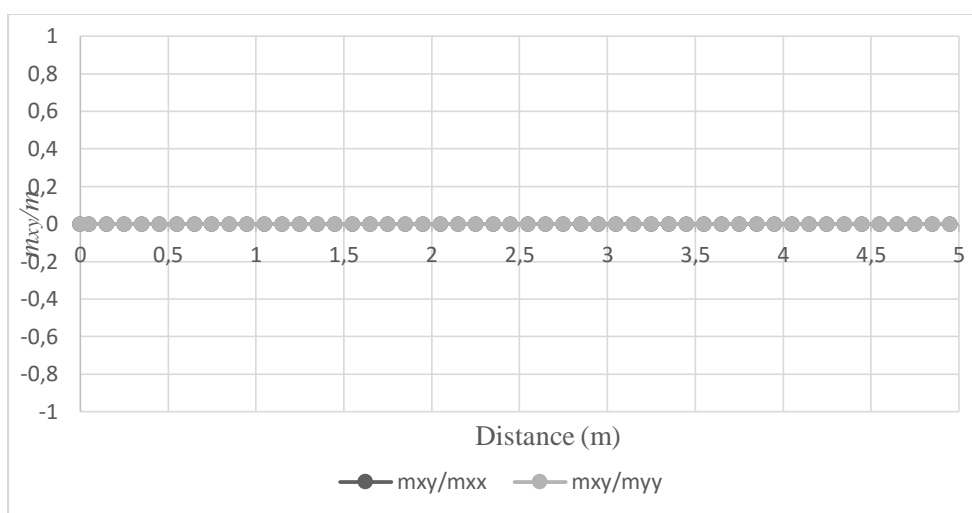


Figure 55. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L9 model.

## 2.10 Model L10

### 2.10.1 Description

The slabs was modeled using finite elements using a 100 x 100 mm mesh. The (x, y) position of the distributed surface load is (2100, 438) mm.

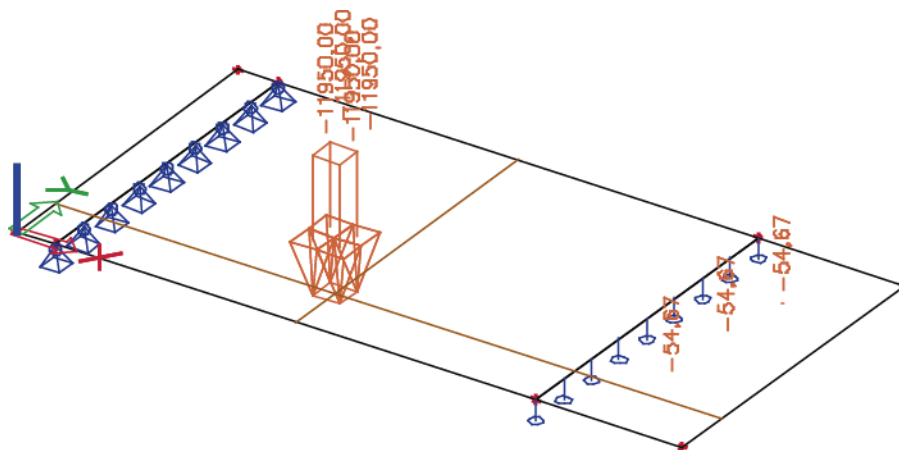


Figure 56. L10 Model with Distributed Surface Load representing wheel print with coordinates (2100 mm, 438 mm). The concentrated loads beyond the second support create a moment over support 2.

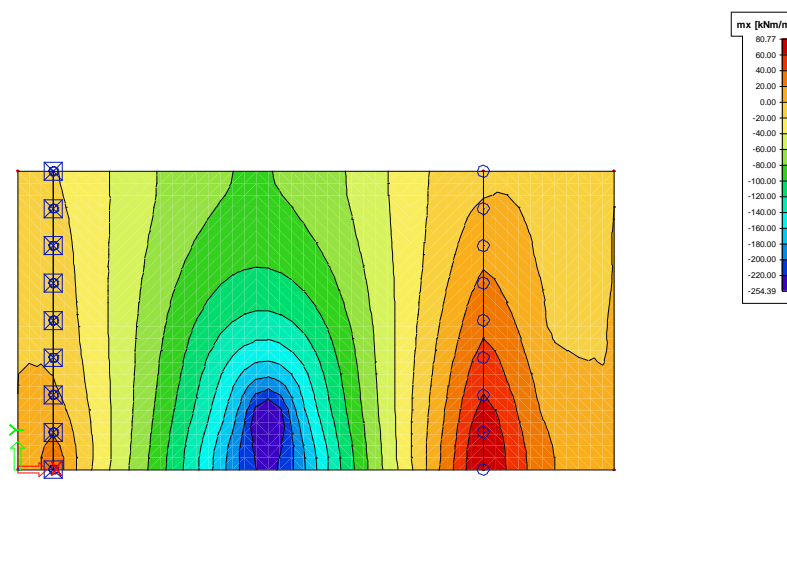


Figure 57.  $m_x$  plot of the L10 slab model analyzed using finite elements method.

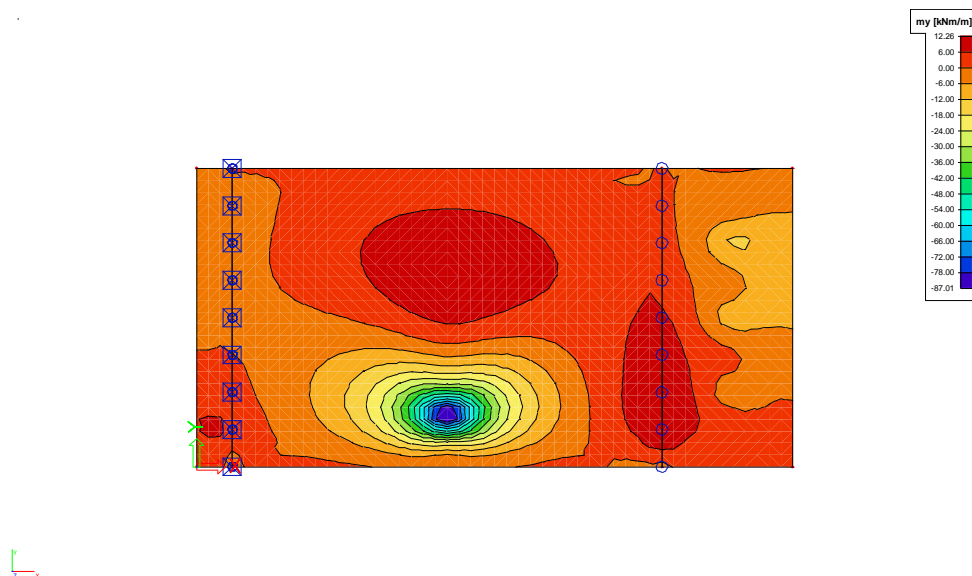


Figure 58.  $m_y$  plot of the L10 slab model analyzed using finite elements method.

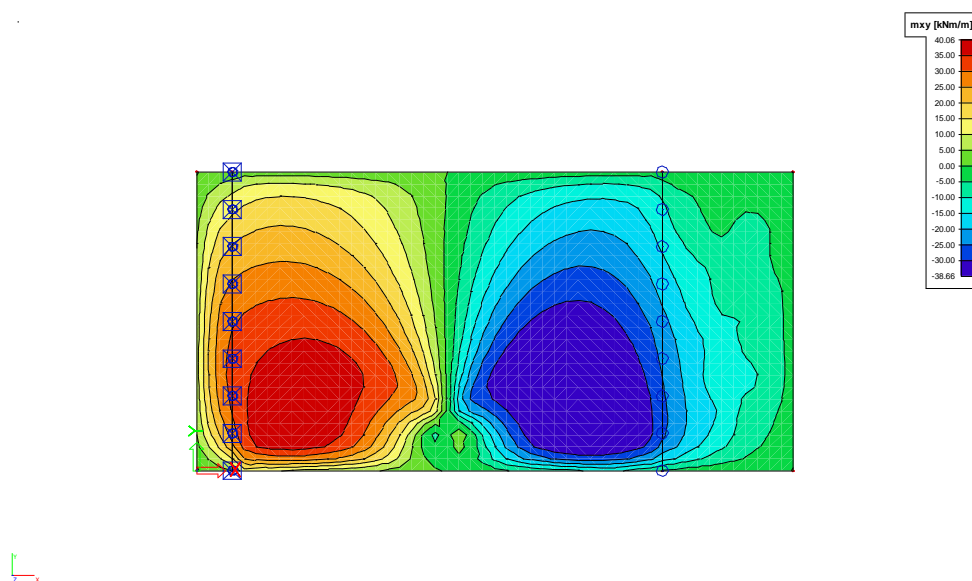


Figure 59.  $m_{xy}$  plot of the L10 slab model analyzed using finite elements method.

## 2.10.2 Moments Along the Width

Table 129: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>y direction L10 model results</i></b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0,00	-224,86	-0,10	-0,37	0,00	3,70
0,10	-226,65	-5,71	-1,18	0,01	0,21
0,20	-233,39	-21,50	-1,53	0,01	0,07
0,30	-245,71	-54,49	-1,54	0,01	0,03

0,40	-254,39	-87,01	-1,51	0,01	0,02
0,50	-243,60	-81,96	-1,43	0,01	0,02
0,60	-218,26	-51,25	-1,35	0,01	0,03
0,70	-194,32	-28,24	-1,32	0,01	0,05
0,80	-176,04	-15,62	-1,29	0,01	0,08
0,90	-161,36	-7,30	-1,27	0,01	0,17
1,00	-149,15	-1,38	-1,25	0,01	0,91
1,10	-138,79	2,92	-1,23	0,01	-0,42
1,20	-129,86	6,04	-1,21	0,01	-0,20
1,30	-122,00	8,27	-1,18	0,01	-0,14
1,40	-115,28	9,78	-1,14	0,01	-0,12
1,50	-109,28	10,69	-1,09	0,01	-0,10
1,60	-104,00	11,10	-1,03	0,01	-0,09
1,70	-99,33	11,06	-0,95	0,01	-0,09
1,80	-95,23	10,61	-0,85	0,01	-0,08
1,90	-91,61	9,79	-0,74	0,01	-0,08
2,00	-88,46	8,62	-0,62	0,01	-0,07
2,10	-85,73	7,10	-0,49	0,01	-0,07
2,20	-83,43	5,28	-0,35	0,00	-0,07
2,30	-81,66	3,23	-0,21	0,00	-0,07
2,40	-80,74	1,33	-0,09	0,00	-0,07
2,50	-80,44	0,20	0,02	0,00	0,10

### 2.10.3 $m_{xy}/m$ Plots Along the Width

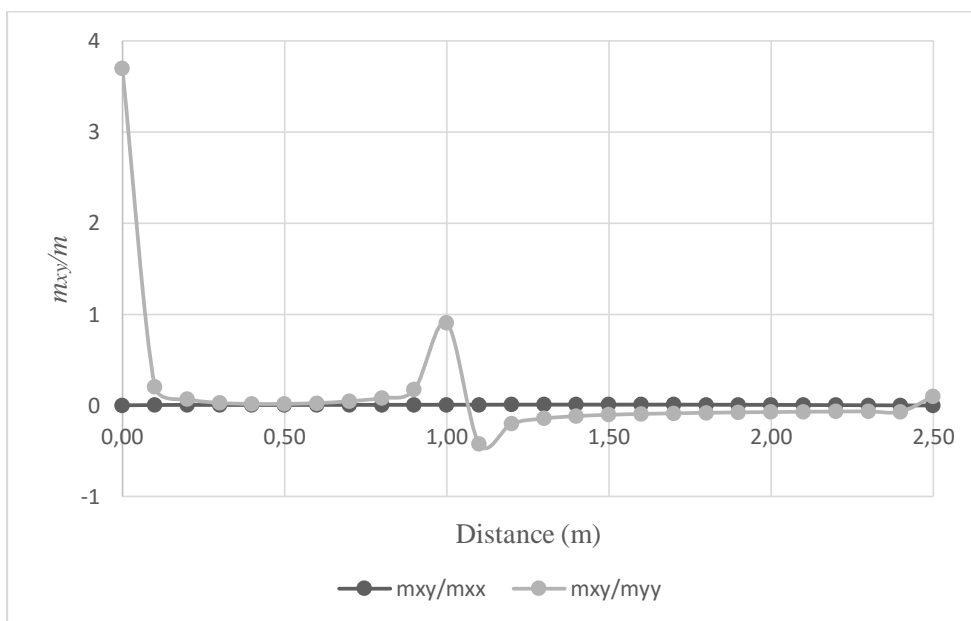


Figure 60. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L10 model.



## 2.10.4 Moments Along the Length

Table 20: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>x direction L10 model results</i></b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0,00	0,16	5,64	6,34	0,03	0,89
0,10	2,64	5,89	16,35	0,16	0,36
0,20	8,17	5,76	24,21	0,34	0,24
0,30	11,10	4,00	29,70	0,37	0,13
0,40	3,35	2,97	34,20	0,10	0,09
0,50	-7,88	0,97	37,18	-0,21	0,03
0,60	-19,19	-0,38	38,87	-0,49	-0,01
0,70	-30,52	-1,64	39,71	-0,77	-0,04
0,80	-41,81	-2,93	39,95	-1,05	-0,07
0,90	-53,08	-4,34	39,72	-1,34	-0,11
1,00	-64,43	-5,93	39,08	-1,65	-0,15
1,10	-75,92	-7,79	38,03	-2,00	-0,20
1,20	-87,66	-10,01	36,56	-2,40	-0,27
1,30	-99,75	-12,74	34,64	-2,88	-0,37
1,40	-112,32	-16,14	32,22	-3,49	-0,50
1,50	-125,53	-20,48	29,23	-4,29	-0,70
1,60	-139,62	-26,12	25,63	-5,45	-1,02
1,70	-155,00	-33,59	21,36	-7,26	-1,57
1,80	-172,54	-43,73	16,43	-10,50	-2,66
1,90	-196,77	-58,24	11,11	-17,71	-5,24
2,00	-231,21	-76,04	5,23	-44,21	-14,54
2,10	-250,20	-85,03	-1,48	169,05	57,45
2,20	-228,71	-75,78	-8,18	27,96	9,26
2,30	-191,75	-57,72	-14,00	13,70	4,12
2,40	-165,00	-42,92	-19,16	8,61	2,24
2,50	-144,91	-32,49	-23,91	6,06	1,36
2,60	-126,94	-24,71	-27,94	4,54	0,88
2,70	-110,22	-18,75	-31,25	3,53	0,60
2,80	-94,32	-14,06	-33,88	2,78	0,41
2,90	-78,99	-10,27	-35,88	2,20	0,29
3,00	-64,06	-7,13	-37,31	1,72	0,19
3,10	-49,38	-4,44	-38,22	1,29	0,12
3,20	-34,84	-2,07	-38,64	0,90	0,05
3,30	-20,30	0,11	-38,50	0,53	0,00
3,40	-5,66	2,19	-38,08	0,15	-0,06
3,50	9,22	4,25	-37,09	-0,25	-0,11
3,60	24,45	6,37	-35,60	-0,69	-0,18
3,70	40,17	8,64	-33,53	-1,20	-0,26
3,80	57,45	11,62	-30,97	-1,86	-0,38

3,90	69,12	12,24	-26,38	-2,62	-0,46
4,00	62,59	11,36	-23,34	-2,68	-0,49
4,10	50,23	8,37	-20,14	-2,49	-0,42
4,20	39,52	6,09	-17,79	-2,22	-0,34
4,30	29,28	4,09	-15,82	-1,85	-0,26
4,40	19,29	2,50	-13,77	-1,40	-0,18
4,50	11,28	1,65	-10,93	-1,03	-0,15
4,60	7,37	0,75	-8,14	-0,91	-0,09
4,70	5,02	0,72	-6,81	-0,74	-0,11
4,80	2,78	0,97	-5,82	-0,48	-0,17
4,90	0,87	1,16	-4,12	-0,21	-0,28
5,00	0,12	1,02	-1,66	-0,07	-0,61

### 2.10.5 $m_{xy}/m$ Plots Along the Length

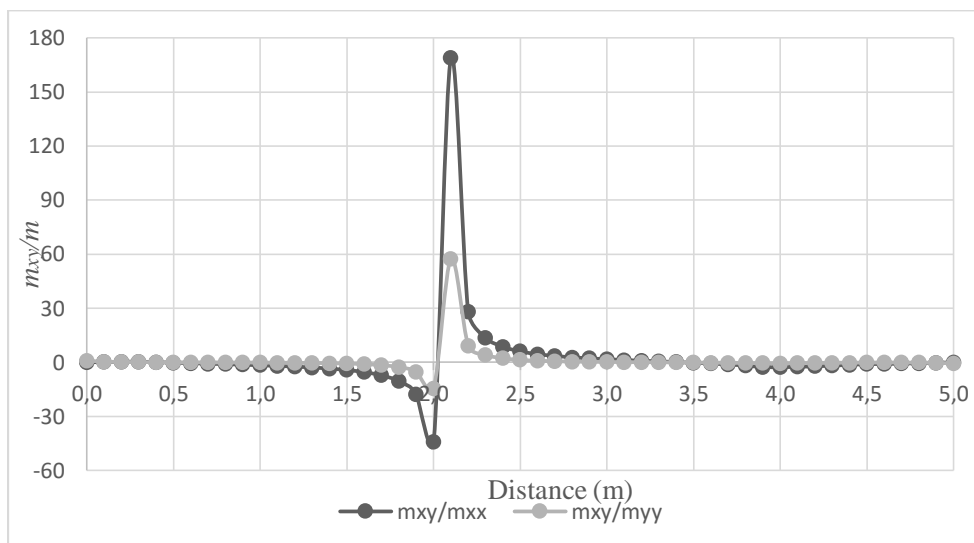


Figure 61. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_{yy}$  along the width of the L10 model.

## 2.11 Model L11

### 2.11.1 Description

The slabs was modeled using finite elements using a 100 x 100 mm mesh. The (x, y) position of the distributed surface load is (2100, 850) mm.

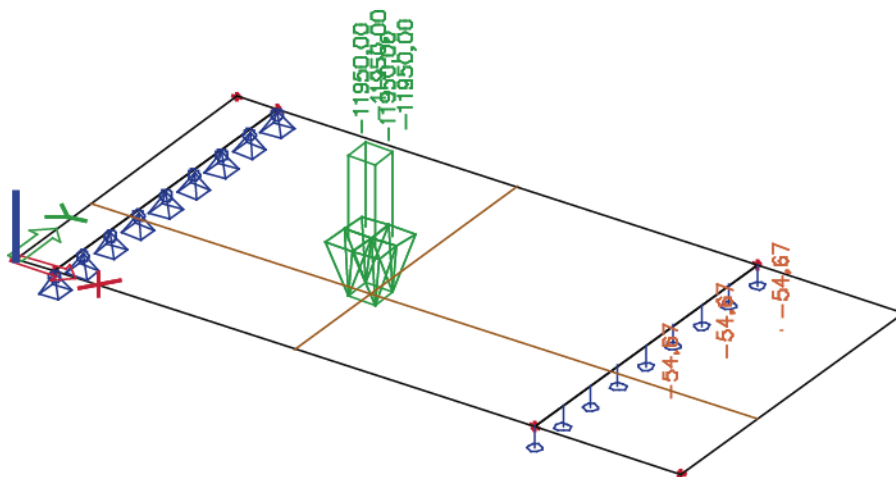


Figure 62. L11 Model with Distributed Surface Load representing wheel print with coordinates (2100 mm, 850mm). The concentrated loads beyond the second support create a moment over support 2.

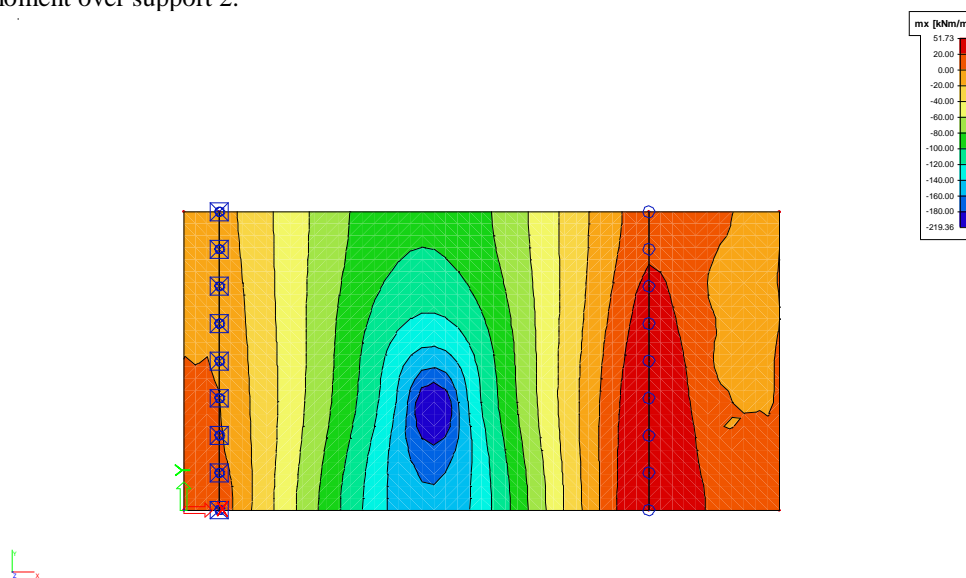


Figure 63.  $m_x$  plot of the L11 slab model analyzed using finite elements method.

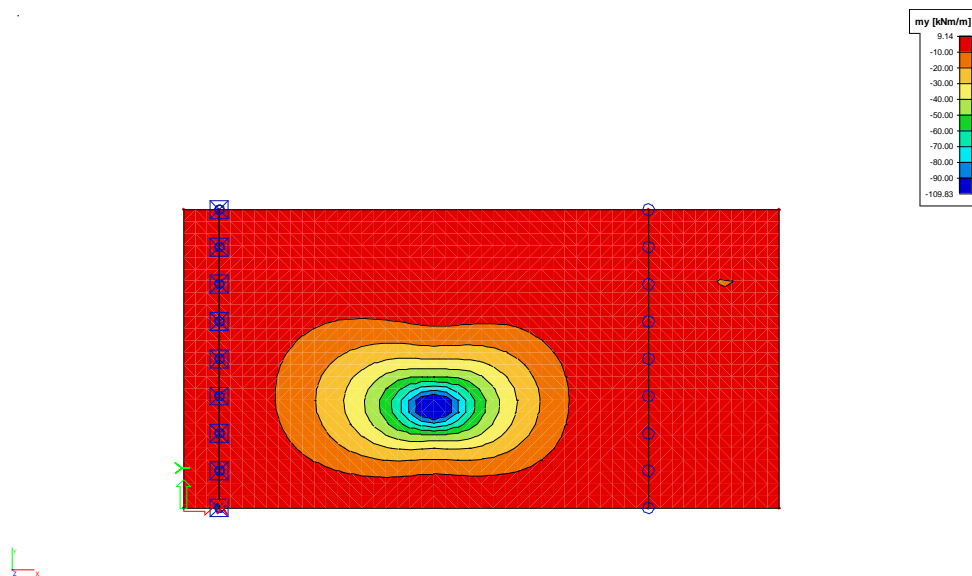


Figure 64.  $m_y$  plot of the L11 slab model analyzed using finite elements method.

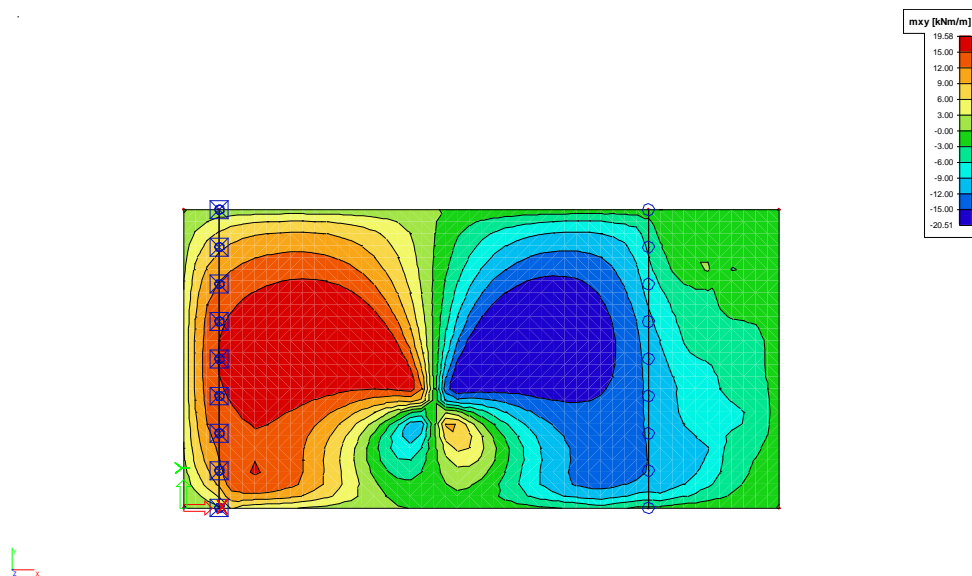


Figure 65.  $m_{xy}$  plot of the L11 slab model analyzed using finite elements method.

### 2.11.2 Moments Along the Width

Table 213: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<i>y direction</i> L11 model results					
Distance (m)	$m_{xx}$ (kN -m/m)	$m_{yy}$ (kN -m/m)	$m_{xy}$ (KN -m/m)	$m_{xy}/m_{xx}$	$m_{xy}/m_{yy}$
0,00	-157,40	0,01	-0,27	0,00	-27,00
0,10	-157,95	-1,37	-0,82	0,01	0,60
0,20	-159,71	-5,09	-1,04	0,01	0,20
0,30	-162,77	-10,97	-1,02	0,01	0,09
0,40	-167,66	-19,13	-0,94	0,01	0,05

0,50	-175,07	-30,18	-0,86	0,00	0,03
0,60	-186,65	-45,18	-0,78	0,00	0,02
0,70	-204,24	-77,01	-0,71	0,00	0,01
0,80	-219,36	-109,83	-0,69	0,00	0,01
0,90	-215,35	-106,99	-0,66	0,00	0,01
1,00	-195,17	-75,81	-0,62	0,00	0,01
1,10	-174,61	-49,88	-0,62	0,00	0,01
1,20	-159,17	-34,64	-0,61	0,00	0,02
1,30	-147,19	-24,42	-0,59	0,00	0,02
1,40	-137,52	-16,86	-0,57	0,00	0,03
1,50	-129,53	-11,12	-0,53	0,00	0,05
1,60	-122,83	-6,73	-0,49	0,00	0,07
1,70	-117,16	-3,39	-0,43	0,00	0,13
1,80	-112,35	-0,93	-0,36	0,00	0,39
1,90	-108,27	0,79	-0,27	0,00	-0,34
2,00	-104,83	1,84	-0,17	0,00	-0,09
2,10	-101,97	2,29	-0,06	0,00	-0,03
2,20	-99,66	2,18	0,06	0,00	0,03
2,30	-97,94	1,58	0,17	0,00	0,11
2,40	-97,01	0,76	0,19	0,00	0,25
2,50	-96,71	0,15	0,10	0,00	0,67

### 2.11.3 $m_{xy}/m$ Plots Along the Width

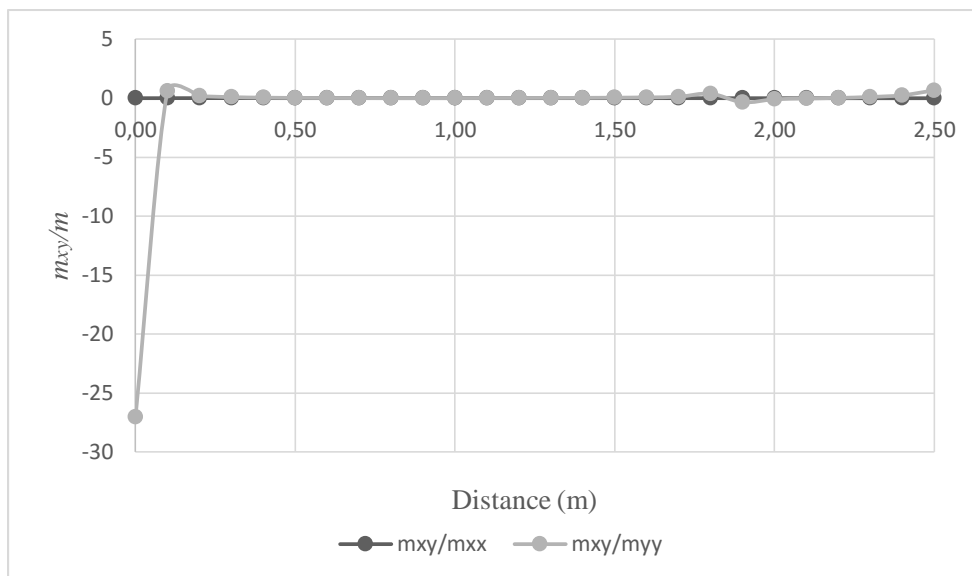


Figure 66. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L11 model.

## 2.11.4 Moments Along the Length

Table 22: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>x direction</i> L11 model results</b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0,00	0,01	1,07	3,54	354,00	3,31
0,10	0,36	1,04	9,08	25,22	8,73
0,20	1,27	1,09	12,71	10,01	11,66
0,30	0,58	-0,06	14,19	24,47	-236,50
0,40	-6,35	-1,33	14,92	-2,35	-11,22
0,50	-15,03	-3,69	15,38	-1,02	-4,17
0,60	-23,62	-5,88	15,62	-0,66	-2,66
0,70	-32,30	-8,17	15,69	-0,49	-1,92
0,80	-41,06	-10,61	15,58	-0,38	-1,47
0,90	-49,88	-13,28	15,29	-0,31	-1,15
1,00	-58,76	-16,22	14,83	-0,25	-0,91
1,10	-67,74	-19,51	14,19	-0,21	-0,73
1,20	-76,84	-23,26	13,38	-0,17	-0,58
1,30	-86,15	-27,56	12,40	-0,14	-0,45
1,40	-95,78	-32,59	11,24	-0,12	-0,34
1,50	-105,91	-38,54	9,92	-0,09	-0,26
1,60	-116,86	-45,74	8,45	-0,07	-0,18
1,70	-129,18	-54,65	6,85	-0,05	-0,13
1,80	-143,95	-66,00	5,17	-0,04	-0,08
1,90	-165,95	-81,36	3,68	-0,02	-0,05
2,00	-198,81	-99,44	2,15	-0,01	-0,02
2,10	-217,36	-108,41	-0,68	0,00	0,01
2,20	-196,57	-99,07	-3,51	0,02	0,04
2,30	-161,47	-80,61	-5,00	0,03	0,06
2,40	-137,24	-64,86	-6,40	0,05	0,10
2,50	-120,23	-53,10	-8,00	0,07	0,15
2,60	-105,64	-43,78	-9,47	0,09	0,22
2,70	-92,42	-36,14	-10,80	0,12	0,30
2,80	-79,98	-29,72	-11,94	0,15	0,40
2,90	-68,01	-24,20	-12,88	0,19	0,53
3,00	-56,31	-19,36	-13,64	0,24	0,70
3,10	-44,76	-15,06	-14,10	0,32	0,94
3,20	-33,26	-11,17	-14,55	0,44	1,30
3,30	-21,75	-7,60	-14,71	0,68	1,94
3,40	-10,16	-4,28	-14,68	1,44	3,43
3,50	1,56	-1,17	-14,47	-9,28	12,37
3,60	13,47	1,77	-14,08	-1,05	-7,95
3,70	25,63	4,56	-13,50	-0,53	-2,96
3,80	38,88	7,51	-12,89	-0,33	-1,72
3,90	47,41	8,77	-11,33	-0,24	-1,29

4,00	42,68	8,06	-10,77	-0,25	-1,34
4,10	33,12	5,81	-9,54	-0,29	-1,64
4,20	24,58	3,90	-8,46	-0,34	-2,17
4,30	16,18	2,21	-7,35	-0,45	-3,33
4,40	8,26	0,91	-6,51	-0,79	-7,15
4,50	2,26	0,04	-6,41	-2,84	-160,25
4,60	0,03	-1,19	-6,34	-211,33	5,33
4,70	-0,23	-2,08	-5,72	24,87	2,75
4,80	-0,13	-2,57	-4,82	37,08	1,88
4,90	-0,09	-2,75	-3,40	37,78	1,24
5,00	0,20	-3,06	-1,16	-5,80	0,38

### 2.11.5 $m_{xy}/m$ Plots Along the Length

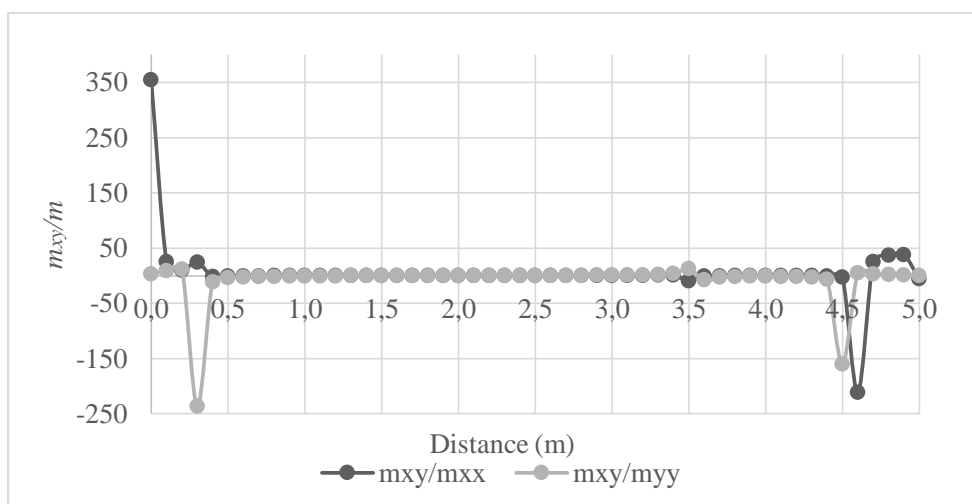


Figure 67. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L11 model.

## 2.12 Model L12

### 2.12.1 Description

The slabs was modeled using finite elements using a 100 x 100 mm mesh. The (x, y) position of the distributed surface load is (2100, 1250) mm.

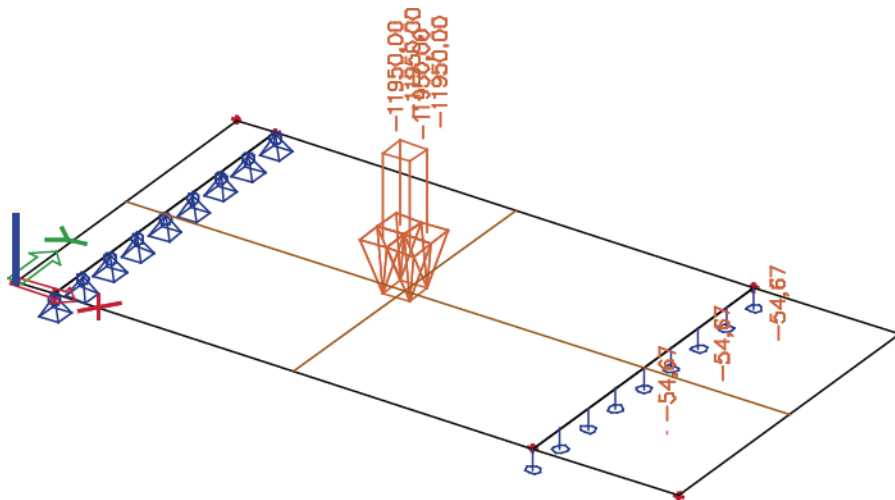


Figure 68. L12 Model with Distributed Surface Load representing wheel print with coordinates (2100 mm, 1250 mm). The concentrated loads beyond the second support create a moment over support 2.

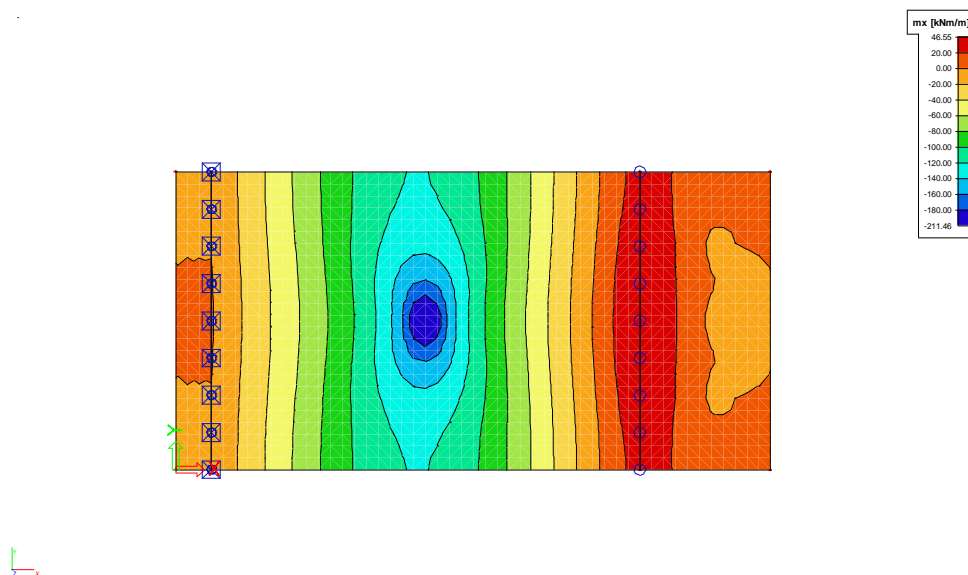


Figure 69.  $m_x$  plot of the L12 slab model analyzed using finite elements method.



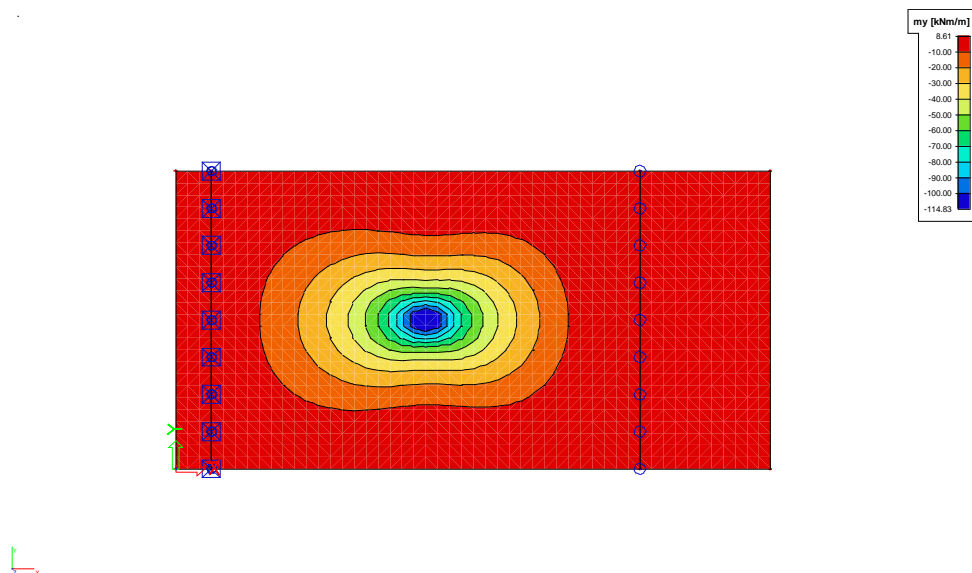


Figure 70.  $m_y$  plot of the L12 slab model analyzed using finite elements method.

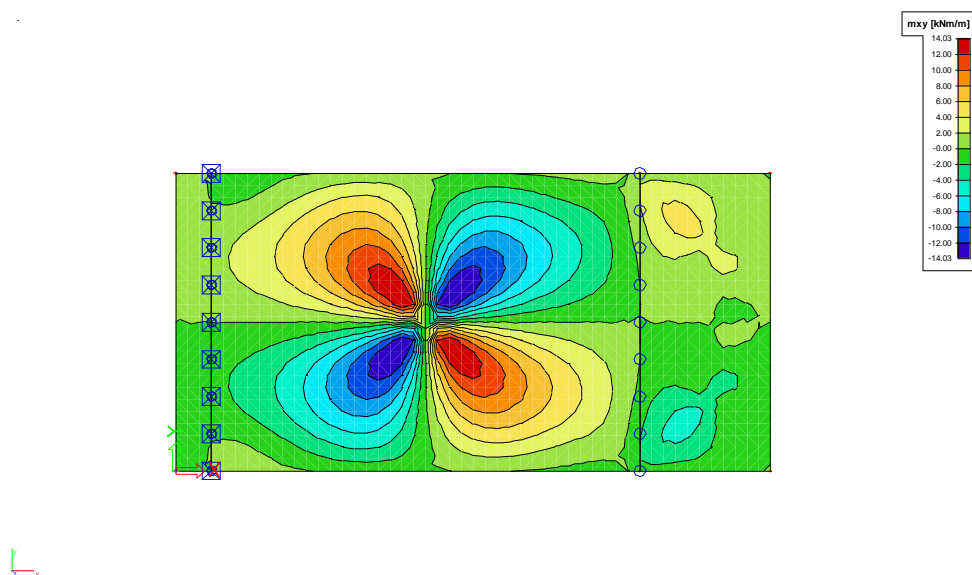


Figure 71.  $m_{xy}$  plot of the L12 slab model analyzed using finite elements method.

### 2.12.2 Moments Along the Width

Table 23: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<i>y direction</i> L12 model results					
Distance (m)	$m_{xx}$ (kN -m/m)	$m_{yy}$ (kN -m/m)	$m_{xy}$ (kN -m/m)	$m_{xy}/m_{xx}$	$m_{xy}/m_{yy}$
0,00	-120,49	0,09	-0,18	0,00	-2,00
0,10	-120,84	0,00	-0,49	0,00	-
0,20	-121,92	-0,71	-0,58	0,00	0,82
0,30	-123,84	-2,23	-0,52	0,00	0,23
0,40	-126,59	-4,69	-0,42	0,00	0,09
0,50	-130,25	-8,17	-0,32	0,00	0,04

0,60	-134,99	-12,84	-0,24	0,00	0,02
0,70	-141,09	-18,94	-0,17	0,00	0,01
0,80	-148,82	-26,91	-0,11	0,00	0,00
0,90	-158,98	-37,54	-0,07	0,00	0,00
1,00	-173,11	-52,93	-0,04	0,00	0,00
1,10	-193,43	-82,99	-0,02	0,00	0,00
1,20	-211,46	-114,83	-0,01	0,00	0,00
1,30	-211,46	-114,83	0,01	0,00	0,00
1,40	-193,43	-82,99	0,02	0,00	0,00
1,50	-173,11	-52,93	0,04	0,00	0,00
1,60	-158,98	-37,54	0,07	0,00	0,00
1,70	-148,82	-26,91	0,11	0,00	0,00
1,80	-141,09	-18,94	0,17	0,00	-0,01
1,90	-134,99	-12,84	0,24	0,00	-0,02
2,00	-130,25	-8,17	0,32	0,00	-0,04
2,10	-126,59	-4,69	0,42	0,00	-0,09
2,20	-123,84	-2,23	0,52	0,00	-0,23
2,30	-121,92	-0,71	0,58	0,00	-0,82
2,40	-120,84	0,00	0,49	0,00	-
2,50	-120,49	0,09	0,18	0,00	2,00

### 2.12.3 $m_{xy}/m$ Plots Along the Width

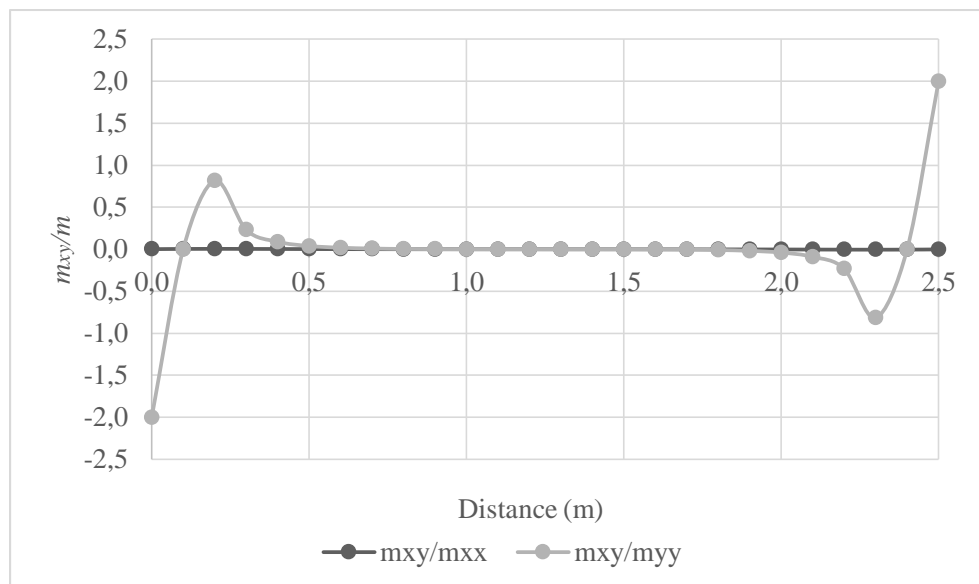


Figure 72. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L12 model.

## 2.12.4 Moments Along the Length

Table 24: Flexural moments ( $m_{xx}$  and  $m_{yy}$ ) and torsional moments ( $m_{xy}$ ) and moments ratios ( $m_{xy}/m_{xx}$  and  $m_{xy}/m_{yy}$ )

<b><i>x direction L12 model results</i></b>					
<b>Distance (m)</b>	<b><math>m_{xx}</math> (kN -m/m)</b>	<b><math>m_{yy}</math> (kN -m/m)</b>	<b><math>m_{xy}</math> (KN -m/m)</b>	<b><math>m_{xy}/m_{xx}</math></b>	<b><math>m_{xy}/m_{yy}</math></b>
0,00	0,02	0,41	0,00	0,00	0,00
0,10	0,26	0,40	0,00	0,00	0,00
0,20	1,17	0,40	0,00	0,00	0,00
0,30	-0,04	-0,24	0,00	0,00	0,00
0,40	-6,63	-2,14	0,00	0,00	0,00
0,50	-15,42	-4,65	0,00	0,00	0,00
0,60	-23,73	-7,16	0,00	0,00	0,00
0,70	-31,07	-9,80	0,00	0,00	0,00
0,80	-40,17	-12,62	0,00	0,00	0,00
0,90	-48,36	-15,67	0,00	0,00	0,00
1,00	-56,57	-18,99	0,00	0,00	0,00
1,10	-64,84	-22,66	0,00	0,00	0,00
1,20	-73,42	-26,76	0,00	0,00	0,00
1,30	-81,82	-31,42	0,00	0,00	0,00
1,40	-90,73	-36,78	0,00	0,00	0,00
1,50	-100,18	-43,05	0,00	0,00	0,00
1,60	-110,48	-50,55	0,00	0,00	0,00
1,70	-122,22	-59,73	0,00	0,00	0,00
1,80	-136,17	-71,36	0,00	0,00	0,00
1,90	-158,03	-86,96	0,00	0,00	0,00
2,00	-191,83	-105,47	0,00	0,00	0,00
2,10	-211,46	-114,83	0,00	0,00	0,00
2,20	-189,70	-105,08	0,00	0,00	0,00
2,30	-153,73	-86,18	0,00	0,00	0,00
2,40	-130,05	-70,17	0,00	0,00	0,00
2,50	-113,65	-58,12	0,00	0,00	0,00
2,60	-99,76	-48,50	0,00	0,00	0,00
2,70	-87,29	-40,55	0,00	0,00	0,00
2,80	-75,57	-33,81	0,00	0,00	0,00
2,90	-64,54	-27,95	0,00	0,00	0,00
3,00	-53,71	-22,70	0,00	0,00	0,00
3,10	-43,03	-18,13	0,00	0,00	0,00
3,20	-32,40	-13,90	0,00	0,00	0,00
3,30	-21,74	-10,01	0,00	0,00	0,00
3,40	-10,99	-6,39	0,00	0,00	0,00
3,50	-0,07	-3,01	0,00	0,00	0,00
3,60	11,05	0,18	0,00	0,00	0,00
3,70	22,48	3,18	0,00	0,00	0,00
3,80	35,19	6,17	0,00	0,00	0,00

3,90	42,05	7,88	0,00	0,00	0,00
4,00	38,58	6,86	0,00	0,00	0,00
4,10	29,21	4,53	0,00	0,00	0,00
4,20	21,09	2,12	0,00	0,00	0,00
4,30	13,12	-0,68	0,00	0,00	0,00
4,40	3,62	-4,33	0,00	0,00	0,00
4,50	-3,65	-7,10	0,00	0,00	0,00
4,60	-3,51	-6,75	0,00	0,00	0,00
4,70	-1,20	-5,51	0,00	0,00	0,00
4,80	-0,60	-5,07	0,00	0,00	0,00
4,90	-0,20	-4,88	0,00	0,00	0,00
5,00	-0,01	-4,83	0,00	0,00	0,00

### 2.12.5 $m_{xy}/m$ Plots Along the Length

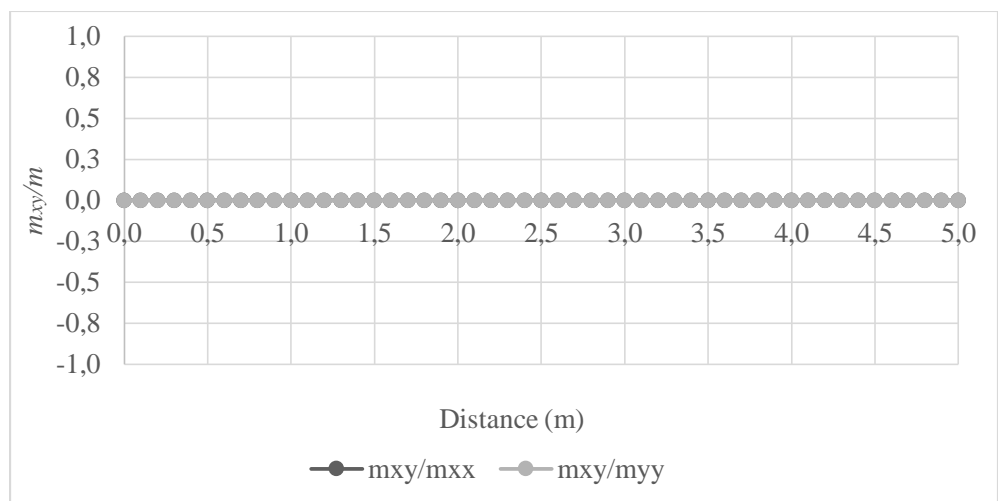


Figure 73. Torsional moment ratios  $m_{xy}/m_x$  and  $m_{xy}/m_y$  along the width of the L12 model.

### 3. Relation plots along the *y-direction*

Table 25: Averaging over a distance  $d$  from the center of the load in  $m_x/m_{xy}$

$m_{xy}/m_{xx}$	Load at 1250 mm		Load at 438 mm		Load at 850 mm	
<i>x-distance</i> (mm)	<b>2d</b>	<b>4d</b>	<b>2d</b>	<b>4d</b>	<b>2d</b>	<b>4d</b>
700	0,91	1,79	0,18	0,15	0,05	0,04
850	0,01	0,02	0,16	0,13	0,06	0,07
1000	0,01	0,01	0,14	0,11	0,06	0,07
2100	0,00	0,00	-0,01	-0,01	0,00	0,00

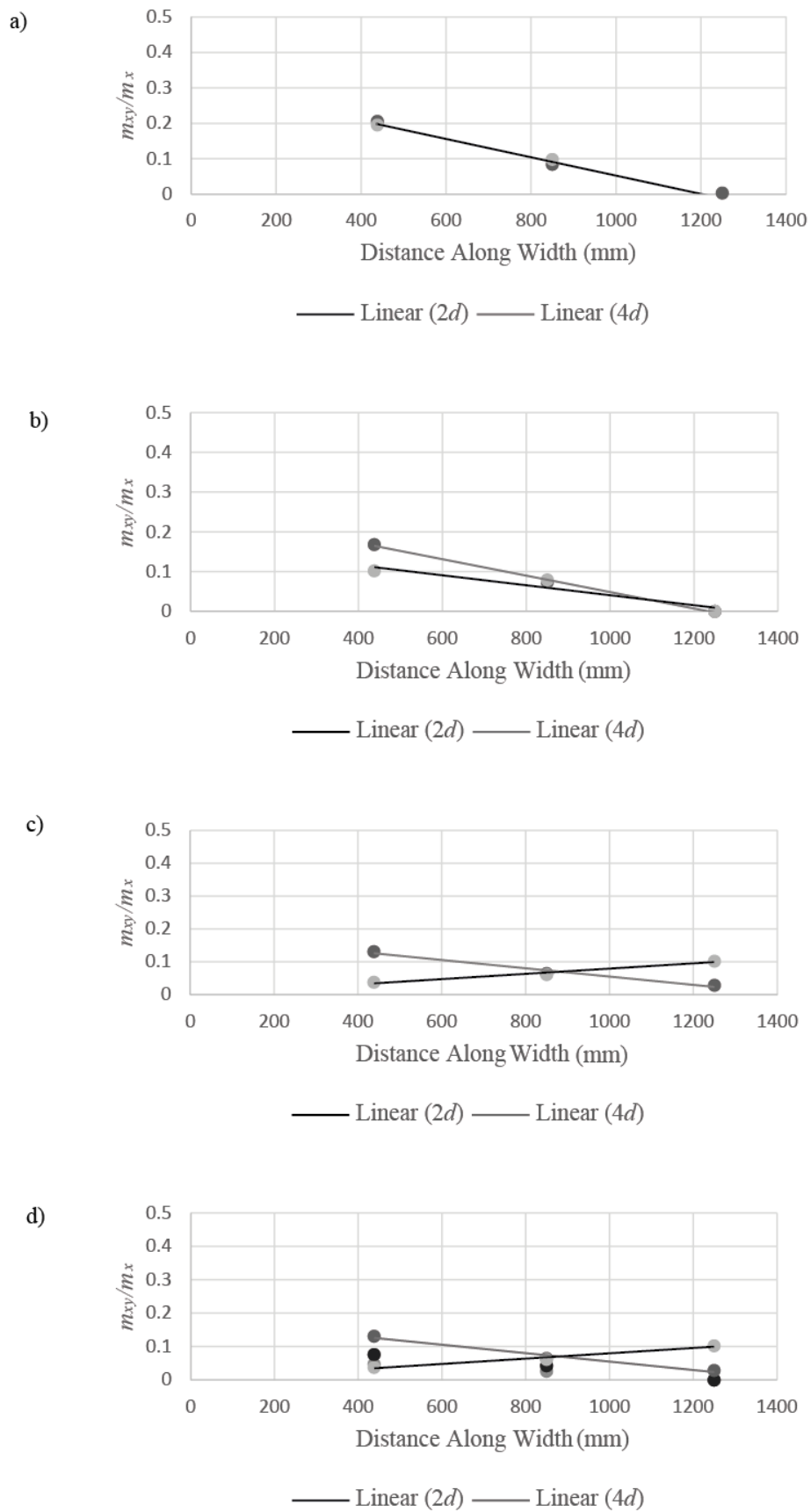


Figure 74. Moment ratios ( $m_{xy}/m_x$ ) for the twelve models at distances: a) 1250 mm, b) 438 mm and c) 850 mm along the width

#### 4. Relation plots along the $x$ -direction

Table 26: Averaging over a distance  $d$  from the center of the load in  $m_x/m_{xy}$

$m_{xy}/m_{xx}$	Load at 700 mm		Load at 850 mm		Load at 1000 mm		Load at 2100 mm	
$y$ -distance (mm)	2d	4d	2d	4d	2d	4d	2d	4d
438	0,21	0,19	0,17	0,10	0,13	0,04	0,05	0,08
850	0,08	0,10	0,07	0,08	0,06	0,06	0,03	0,04
1250	0,00	-0,02	-	-	0,03	0,10	-	-

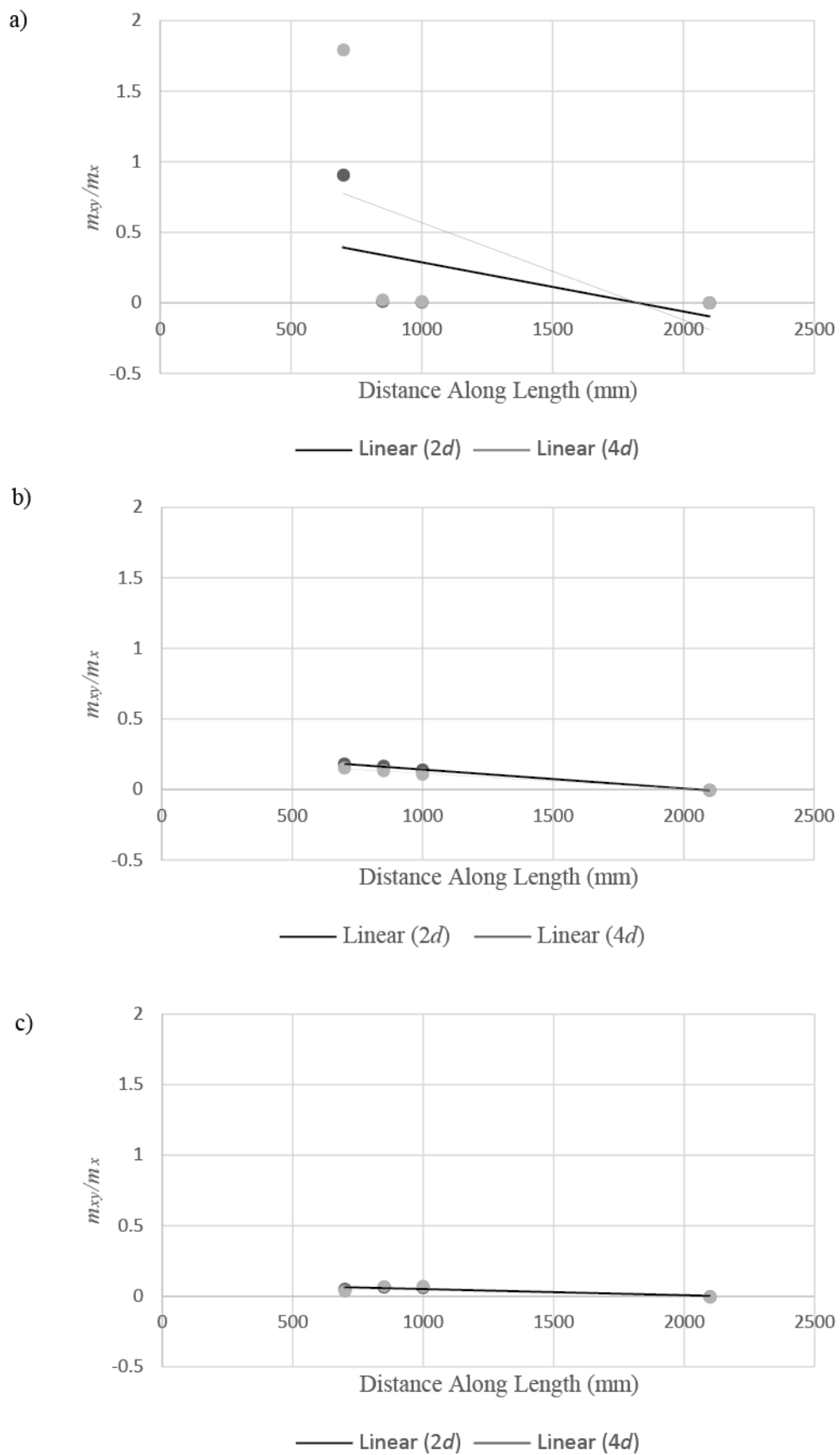


Figure 75. Moment ratios ( $m_{xy}/m_x$ ) for the twelve models at distances: a) 1250 mm, b) 438 mm and c) 850 mm along the width