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Verification of Flexural Stop Criteria for Proof Load Tests on Concrete Bridges Based on Beam Experiments

Proyecto de investigación

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RESUMEN

Los ensayos de prueba de carga permiten a los ingenieros determinar si una estructura aún es segura para su uso. Sin embargo al someter la estructura a este tipo de ensayos, esta puede sufrir daños irreparables. Para evitar esta situación, códigos de construcción y guías para ensayos de carga han establecido criterios para detener el ensayo antes de generar daño irreversible. Estos criterios a menudo se basan en datos recolectados a medida que el ensayo de carga se está llevando a cabo No obstante, estos criterios deben ser revisados a fin de mejorarlos de manera que los ensayos puedan ser llevados a cabo de manera segura. Además de los criterios existentes en los respectivos códigos de construcción se han presentado nuevas propuestas de criterios de parada a fin de mejor la seguridad con que se realiza los ensayos de prueba de carga. Este reporte analiza los resultados obtenidos de 4 experimentos en 2 vigas fabricadas y ensayadas en laboratorio y compara estos resultados con los valores límite obtenidos en base a los criterios de parada establecidos en el código ACI 437.2M-13 y la guía alemana establecida por DAfStB. Adicionalmente un nuevo criterio de parada, propuesto por Werner Vos de TU Delft en Holanda, también es comparado con los resultados experimentales. Esta investigación busca analizar en qué circunstancias es mejor aplicar un criterio de parada respectivo, que deficiencias tienen y que tan confiable y conservador es cada criterio para ser aplicado no solo en edificios sino también en puentes de hormigón armado. Se encontró que respecto a los criterios establecidos por ACI, el protocolo de carga del ensayo es imperativo para obtener resultados confiables adecuados para ensayos de prueba de carga. En cuanto a las otras propuestas, dependiendo del nivel de seguridad que se busca, se encontraron resultados consistentes y confiables. Se espera que con más investigación respecto a criterios de parad basados en flexión se puedan desarrollar mejores formas de determinar los valores máximos admisibles lo que permitirá aplicar de una forma más segura los ensayos de pruebas de carga.

Palabras clave: flexión, criterio de parada, ensayos de prueba de cargas, deflexión, ancho de agrita, deformación del hormigón, rigidez del hormigón dañado.

ABSTRACT

Proof load tests allow engineers to determine if a structure is still suitable for use. However, as the structure is subjected to this test it may suffer irreparable damage. To avoid this scenario, building codes have established stop criteria for proof load tests. These stop criteria often refer to data that is taken as the test is being carried out. However stop criteria need to be revised in order to be improved. Additionally, other proposals of stop criteria have been submitted to improve safety of proof load tests. This report analyses the results obtained from 4 experiments on 2 cast-in-laboratory beams, and compares them to the values obtained with the stop criteria established by the ACI 437.2M-13 and the German guidelines of the DAfStB. Additionally, a new proposal for stop criteria by Werner Vos from TU Delft in the Netherlands is also compared to the experimental results. This research aims to analyze under which circumstances it is better to apply a specific stop criterion, which are the flaws on the criteria from the codes and the new proposal, and how reliable they are to be applied not only on buildings but on concrete bridges. It was found for the ACI stop criteria, that the loading protocol is imperative to have consistent results and perform adequate proof load tests. As for the other proposals, depending on the margin of safety considered to avoid permanent damage, reliable results were found. Hopefully, further investigation in flexural stop criteria would help to develop better ways to calculate the maximum allowable values, which will lead to a better and safer application of proof load tests.

Key words: Flexure, Stop Criteria, Proof load test, deflection, crack width, concrete deformation, damaged concrete stiffness.

TABLE OF CONTENTS

TABLE INDEX

FIGURES INDEX

INTRODUCTION

Existing civil structures deteriorate with time due to the continuous loading and environmental conditions they are subjected to, or it can be that a structure has suffered severe damage due to any accident. This causes a loss of their initial properties and consequently large uncertainties on the structural behavior. Therefore analyses should be carried out in order to confirm that the structure is still safe for use. If there is background data about the structure to be tested, simulations and computer analyses can be done. However, the level of assessment of these analyses may not be as close to reality as needed since the level of damage and deterioration is not a hundred percent clear. An option for analyzing deteriorated or damaged structure with or without background data is load testing in which the actual structure is loaded and its behavior is measured. There are two types of load testing: the first one is diagnostic load testing, in which the structure is loaded in order to obtain its mechanical properties or to update analytical models. The second type is proof load testing, which is the subject of this research. Its purpose is to assure the safety of a structure by subjecting it to a specific maximum load known as target load. If the structure withstands the target proof load, it passes the test and is still suitable for use.

As a structure is heavily loaded during a proof load test, it can easily get damaged before reaching the target load since its approximate resistance is unknown due to the lack of background data and, the level of deterioration and damage. Therefore, parameters with threshold values must be defined in order to identify that further loading would induce permanent damage to the structure and the test must be stopped immediately. These parameters are the so-called “stop criteria”.

Since the stop criteria must be evaluated as the test is being carried out, parameters related to ductile failure, such as deformation, deflections or changes in stiffness, can easily be measured, thus, they are convenient when establishing stop criteria. On the other hand, brittle failure caused by shear which is instantaneous can be used for stop criteria as well but it is out of scope of this report.

Some building codes such as the German guidelines (Deutcher Ausschuss für Stahlbeton, 2007) and the American ACI 437.2M-13 code (ACI Committee 437, 2013), already establish stop criteria. Nonetheless, these criteria were developed to be applied to buildings, not bridges. Therefore, several studies and experiments have been carried out in order to improve existing stop criteria, such as the investigation made by Werner Vos (Vos, W., 2016) at TU Delft, which is the one analyzed in this report. The stop criterion proposed by Vos aims to establish theoretical threshold values prior to performing the test. The procedure to obtain this value is derived from Monnier's (Monnier. Th., 1970) investigation on the relation between bending moment and curvature.

Bridges represent previously loaded structures, already cracked and with a residual existing deformation. Since they are civil structures used by hundreds of people, old bridges must be tested with a level of assessment that can assure they are still suitable for use, or they must be repaired or replaced immediately. This level of assessment can be reached through proof load testing, always protecting the structure from permanent damage with its respective stop criterion. Since existing stop criteria has been developed for buildings, not bridges, it needs to be studied and reevaluated so that proof loading tests can be carried out in a safer way.

This investigation looks at revising the stop criteria mentioned above, and compares it with the stop criterion proposed by Vos. Results obtained from beams cast and tested in the laboratory are used to analyze the level of safety and accuracy of the existing stop criteria from the German guideline and ACI 437.2-M13 and Vos proposal as well.

LITERATURE REVIEW

In order to understand the parameters used to compare experimental results in this investigation, a brief explanation of what is proof loading, and what are the existing stop criteria is presented.

General Aspects of Proof Load Testing

A proof load test is a test carried out on both new and old structural elements in order to assure their safety. The objective of this test is to load a structure gradually until it reaches a maximum specific load, known as the target load, which proves that the structure is suitable for use.

Proof loading is a common practice when there is not enough background information to perform a structural analysis, after the structure has been subjected to loads it was not designed to withstand, or when it has suffered severe damage or material degradation. Therefore, checking if the structure is able to bear a specific load allows to determine if the structure must be repaired or replaced.

Three parameters must be established before the test is carried out: the loading protocol, the target load and the stop criteria.

The loading protocol establishes how the test is going to be performed. This includes how and where the loads will be applied. Loads can be in cycles, increasing the maximum applied load, or in a monotonic way, increasing the load continuously after established periods of time. A single test may include different load cases with their respective parameters. The position of the load aims to recreate the most unfavorable condition.

The stop criteria are parameters established to protect the integrity of the structure during the proof load test. As the structure is tested in order to reach the target load, stresses on the structure may increase to the point at which the structure suffers permanent damage, or in the worst case scenario, it collapses. To avoid this, stop criteria must be established, usually as parameters that will be measured on the structure while the test is being performed. Existing stop criteria, which will be discussed later in this report include parameters such as: maximum deflection, crack width, deviation of linearity index among others. If one of these parameters is exceeded during the test, it must be aborted immediately, whether the target load has been reached or not.

There are many ways for the load to be applied. Tanks continuously filled with water can be used for monotonic load protocol, of the BELFA truck from Germany designed to apply loads to the deck that can be easily controlled and monitored. However, BELFA truck has a maximum load that cannot be increased. (Koekkoek, R., 2015). A common practice in the Netherlands is a system of hydraulic jacks in which all the weight available is placed over a surface that transmits the load directly the supports before loading is started. During the tests the jacks gradually transfer the load to the superstructure as they push down the surface. (Lantsoght, E., 2016)

Sensors must be installed in order to obtain as much information as possible from the structure. First of all, the loading process must be controlled. LVDTs and lasers are placed at strategic points in order to measure displacements and deformations. Sometimes, acoustic emission signals are measured as well, in order to relate their results to cracking. These measurements allow the engineer who follows the measurements to identify if any stop criterion has been exceeded.

Existing Stop Criteria

German Guideline for Proof Load Testing DAfStB Richtlinie

The German guideline for proof load testing was established in 2000, and applies to both plain concrete and reinforced concrete structures. The protocols established rely on a ductile failure mode. Load testing of shear-critical structures or elements is not permitted.

For proof load tests, cyclic loading must be carried out with at least 3 steps of loading and unloading. The maximum load at which a stop criterion has been reached is defined as F_{lim}. (Deutcher Ausschuss für Stahlbeton, 2007)

Parameters established for stop criteria are:

- Concrete strain

$$\varepsilon_c < \varepsilon_{c\,lim} - \varepsilon_{c0} \tag{1}$$

Where ε_c is the measured strain during the proof load test, $\varepsilon_{c\,lim}$ is the limit value for concrete strain based on concrete characteristic compressive strength defined by the German guideline as 0.006%, which can be increased to 0.008% in concrete compressive strength is greater than 25MPa, and ε_{c0} is the short term strain caused in the concrete by the permanent loads, determined analytically.

- Strain in reinforcement steel

$$\varepsilon_{s2} < 0.7\frac{f_{ym}}{E_s} - \varepsilon_{s02} \tag{2}$$

$$\varepsilon_{s2} < 0.9\frac{f_{0.01m}}{E_s} - \varepsilon_{s02} \tag{3}$$

f_{ym} is the average yield strength of steel, $f_{0.01}$ is the average yield strength based on a strain of 0.01%, E_s is the modulus of elasticity of the steel, ε_{s2} is the steel strain during experiment, and ε_{s02} is the analytically determined strain caused by the permanent loads

The second equation (3) may be used when the stress-strain relationship of the steel is known completely.

- Crack width and increase in crack width

  The crack width of new cracks formed during and after the test is limited to:

  $w \leq 0.5mm$ during proof loading

  $\leq 0.3w$ after proof loading.

  For previously formed cracks, their increase in width is limited as follows:

  $\Delta w \leq 0.3mm$ during proof loading

  $\leq 0.2\Delta w$ after proof loading.

- Deflection

Test must be stopped if more than 10% permanent deformation occurs after removing the load, or if there is a clear increase of the nonlinear part of the deformation.

- Deformation in the shear span of beams with shear reinforcement

  The test must be stopped if 60% of the concrete strain ε_c is reached at the concrete compressive struts.

  The test must be stopped if 50% of ε_{s2} occurs in the shear reinforcement.

American Code ACI 437.2M-13

As stated in the code: ‘’The purpose of this code is to establish the minimum requirements for the test load magnitudes, load test procedures, and acceptance criteria applied to existing concrete structures as part of an evaluation of safety and serviceability to determine whether an existing structure requires repair and rehabilitation’’ (ACI Committee 437, 2013). Only the acceptance criteria will be discussed. For further information, refer to the code itself.

Chapter 6 of the code does not establish how to determine a value for stop criteria, but explicitly defines quantitative rules to determine if the structure passes the load test, known as acceptance criteria. Acceptance criteria describe the acceptable limits of performance indicators, and thus serve the same purpose as stop criteria. The codes defines qualitative requirements as well related to the observation of cracks that could indicate failure, but it is out of the scope of this research.

Parameters for acceptance criteria established in the code are:

- Monotonic load protocol, deflection limits:

As established on the equations 6.3.1 and 6.3.2 respectively

$$\Delta_r \leq \frac{\Delta_l}{4} \tag{4}$$

$$\Delta_l \leq \frac{L}{180} \tag{5}$$

Δ_r is the residual deflection measured 24 hours after the removal of the load and Δ_l represents the maximum deflection, and L is the span length

- Cyclic load protocol, deviation of linearity index I_{DL}:

Based on a hysteretic model, the deviation of linearity index analyses the variation of the slope in a load-deflection plot for every loading cycle, which is calculated as established on equation 6.4.1

$$I_{DL} = 1 - \frac{\tan(\alpha_i)}{\tan(\alpha_{ref})} < 0.25 \tag{6}$$

α_i is the secant stiffness of a point *i* in the loading section of the plot and α_{ref} is the slope of the secant of the load-deflection envelope, as shown in Figure 1(a)

- Cyclic load protocol, permanency ratio I_{pr}:

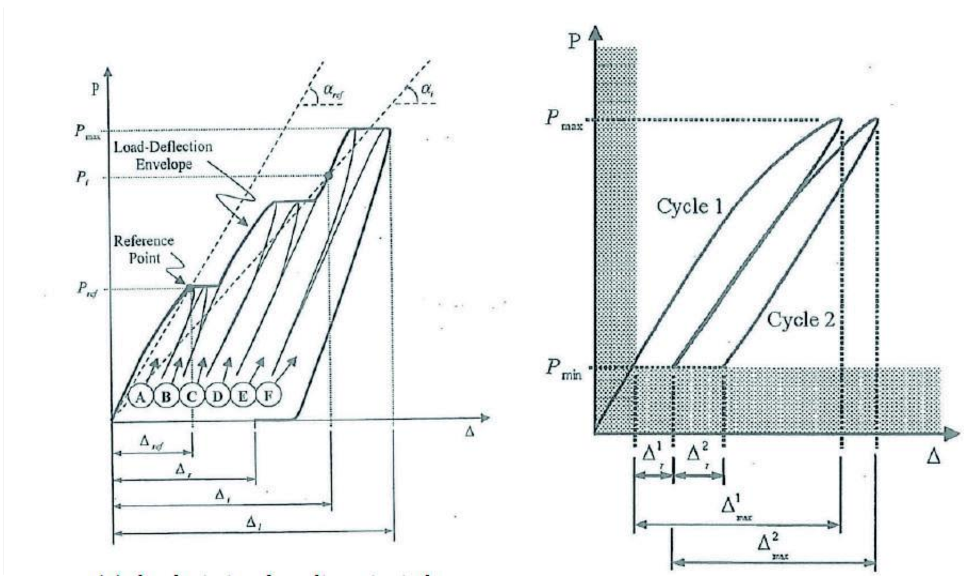
According to the set of equations 6.4.2

$$I_{pr} = \frac{I_{p(i+1)}}{I_{pi}} < 0.5 \tag{7}$$

$$I_{pi} = \frac{\Delta_r^i}{\Delta_{max}^i} \tag{7.1}$$

$$I_{p(i+1)} = \frac{\Delta_r^{i+1}}{\Delta_{max}^{i+1}} \tag{7.2}$$

Where i represents the number of the cycle, Δ_r^i is the deflection shown at minimum load P_{min} and Δ_{max}^i for P_{max} at the i th cycle of loading. The permanency ratio is acceptable if it does not exceed 0.5 for every pair of cycles. Data for determining I_{pr} is taken as seen on figure 1(b). ACI 437.2M-13 also defines a cyclic loading protocol shown in Figure 2.



(a) (b)

Figure 1. ACI Cyclic loading protocol stop criteria: (a) Deviation from linearity index; (b) Permanency ratio. (ACI Committee 437, 2013)

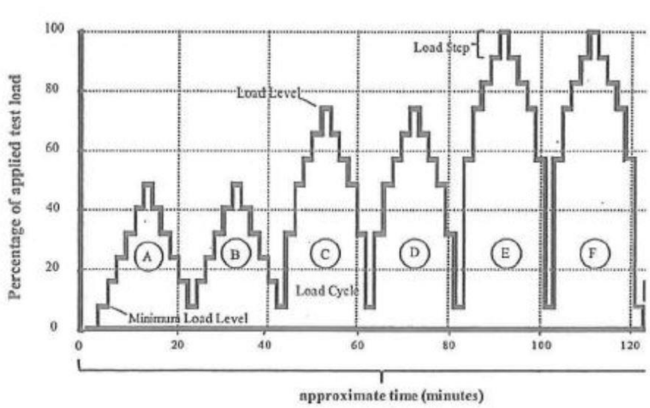


Figure 2. ACI Cyclic loading protocol. (ACI Committee 437, 2013)

Werner Vos proposal:

As a TU Delft investigation, Werner Vos proposed two stop criteria. The first one is based on the relation stiffness-deflection, developed through a theoretical approach based on the theoretical moment-curvature relation developed by Monnier (Monnier. Th., 1970). The second proposal is based on the relation between crack width and deformation as developed by Van Leeuwen. (Van Leeuwen. J., 1962)

- Stiffness-deflection proposal:

  This proposal starts from the moment-curvature diagram established by Monnier, which plots the bending moment at a section of the beam against its curvature. The slopes of the lines in the plot represent the stiffness of the element at different stages of the concrete: un-cracked and cracked. These values of the stiffness can be calculated using the element dimensions and the percentage of steel reinforcement. It must be mentioned that the plot is simplified to be semi linear, with straight lines with different slopes, in order to have constant stiffness in-between stages. In reality, the plot is curved since the stiffness continuously changes as concrete keeps cracking and the steel yields.

  Monnier established the moment-curvature diagram for both first time and alternate loading. The alternate loading model resembles the cyclic loads applied during a proof load test and; additionally it allows to find the maximum applied load in the history of the specimen and its residual deformation. Therefore, this model suits the circumstances under which a bridge is subjected to a proof load test: with existing cracks and with an existing residual deformation.

As concrete continues to crack due to bending, its stiffness decreases with every cycle in relation to the curvature of the beam. Therefore it is a convenient parameter to include in stop criteria. However, measuring curvature during a test requires more complex equipment compared to the usually measured parameters: deformation, deflection and cracks. Therefore a relation between load, stiffness, moment curvature and deflection is established:

$$k = \frac{d^2\delta(x)}{dx^2} \tag{8}$$

where *k* is the curvature, and δ(x) the deflection.

Vos defines a semi-linear model, as shown in Figure 3 in which the concrete element has two stiffness: the un-cracked stiffness, EI_o, and cracked stiffness from the retrograde branch EI_{te}, and a two-step calculation is done; one before the cracking moment is reached, and one afterwards, using its respective stiffness. With a semi-linear approach the relation for elastic materials can be applied:

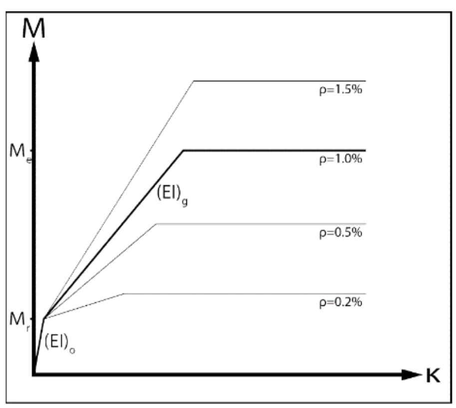


Figure 3. Moment curvature diagram, semi linear approach. (Vos, 2015)

Once the relations are established the stop criteria go as follows:

If a moment at a certain load level overpass the maximum deflection it means the beam is yielding and the process must be stopped.

If there is a residual deformation after a cycle, larger than existing deformations caused by self-weight and previous loads, once the cracking moment has been reached, it means yielding has occurred and the test must be stopped. However residual deformation for preloaded existing structures should be considered when checking for residual deformations during the test. This values if possible can be determined with the load history of the structure or can be measured prior to start the test

Additionally, building codes establish maximum allowable deflections based on the element span. If this deflection is reached the test must be stopped. However, these values are meant to be used not on bridges but buildings.

- The second proposal is based on the relation between crack width and deformation as identified by Van Leeuwen:

$$w_{max} = \beta \; 6.12 \; f_y \; s \; 10^{-6} \; mm \tag{9}$$

$$w_{res} = \beta \; 6.12 \; \sigma_{s1} \; s \; 10^{-6} \; mm \tag{10}$$

β is the ratio between the permanent or cyclic load and total load. In the worst case scenario this ratio will be one. Therefore according to the relation in the formula the maximum crack width can be found when β=1. σ_{s1} is the steel stress right at the crack, and s is the space between cracks whose equation must be in accordance to the type of bar: ribbed or plain.

Additionally, Vos includes minimum residual and maximum allowable crack width from the Eurocode, however this does not establish any difference between plain and ribbed bars. These values will be calculated in this report to have more results for comparison

$$w_{max,Eurocode} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\emptyset}{\rho_{eff}}\frac{f_y - k_t\frac{f_{ctm}}{\rho_{eff}}(1-\alpha_e\rho_{eff})}{E_s} \tag{11}$$

$$w_{res,Eurocode} = \frac{1}{2}\frac{f_{ctm}}{\tau_{bm}}\frac{\emptyset}{\rho_{eff}}\frac{\sigma_s - k_t\frac{f_{ct}^*}{\rho_{eff}}(1-\alpha_e\rho_{eff})}{E_s} \tag{12}$$

Where f_{ctm} is the concrete mean tensile strength, τ_{bm} is bond between reinforcement and concrete, Ø is the bar diameter, ρ_{eff} is effective reinforcement ratio calculated only with area of concrete under tension, k_t equals 0.4 or 0.6 depending if loads applied are long term or short term respectively, α_e is the ratio between steel and concrete Young's modulus and f_{ct}^* is a lowered value of concrete mean tensile strength used to calculate residual crack width. All values should be used in MPa to get crack width in millimeters.

Applying a safety factor of 10% for maximum cracks, the proposal is as follows

$$w_{max,measured} < 0.9\omega_{max}$$

$$w_{max,measured} < 0.9\omega_{max,Eurocode}$$

$$w_{SLS,measured} < 0.2mm$$

$$w_{res,measured} < 0.9\omega_{res}$$

$$w_{res,measured} < \omega_{res,Eurocode}$$

Where $\omega_{SLS,measured}$ is the maximum measured crack at total applied load at the load level that corresponds to the serviceability limit state during the load test.

INVESTIGATION DESIGN AND METHODOLOGY

Methodology of the investigation consists on submitting experimental results to the threshold values from the stop criterion defined in the previous section. This way, reliability, accuracy, level of safety and applicability to bridges of each stop criteria can be discussed Additionally, a step by step process on how to find Vos's proposal threshold values is explained.

Data was collected by submitting two beams cast in the laboratory to several tests and monitoring its behavior. By monitoring the beam, data needed to apply stop criterion was obtained with sensors.

DESCRIPTION OF EXPERIMENTS

Tests were carried at TU Delft in the Stevin II Laboratory on two beams cast in the laboratory. The tests for this research are fully explained in the associated analysis report. (Lantsoght. E., 2016).

Three experiments marked as P804 were carried out on a beam of 10 m long cast in the laboratory in order to evaluate the stop criteria used during proof load testing, with material properties designed to resemble concrete solid slab bridges. Additionally, one 8m beam marked as P502 was tested

- Beam Geometry

  The cross section of the specimen P804 is 800mm x 300mm with 6 plain bars of 20 mm each, with a total area of steel A_s = 1885mm^2. The effective depth is d_l=755mm and the reinforcement ratio ρ=0.83%. The additional experiment, P502A2, was carried out on a 500 x 300mm beam with 3 plain bars of 20mm instead of 6. (Lantsoght. E, 2016)

- Material properties

For concrete, the average compressive strength, obtained following the respective standards is 63.51 MPa at 28 days with a density of 2429.6 kg/m^3 tested at the age of 90 days. For the second beam, P502, the compressive strength was 71.47 MPa.

For the plain bars a yielding stress of 296.8 MPa and ultimate stress of 425.9MPa was measured. This properties resemble the existing ones in slab bridges built in the 60's in the Netherlands.

- Test Set up

The beam is simply supported as shown in Figure 4, subjected to a point load at a distance *a* away from the support, with the values varying depending on each experiment.

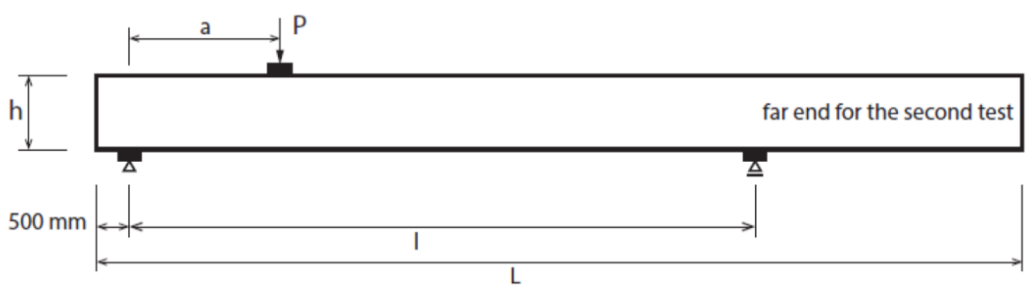


Figure 4. Beam experiments layout

The values from the figure for each experiment are shown in the Table Nr.1

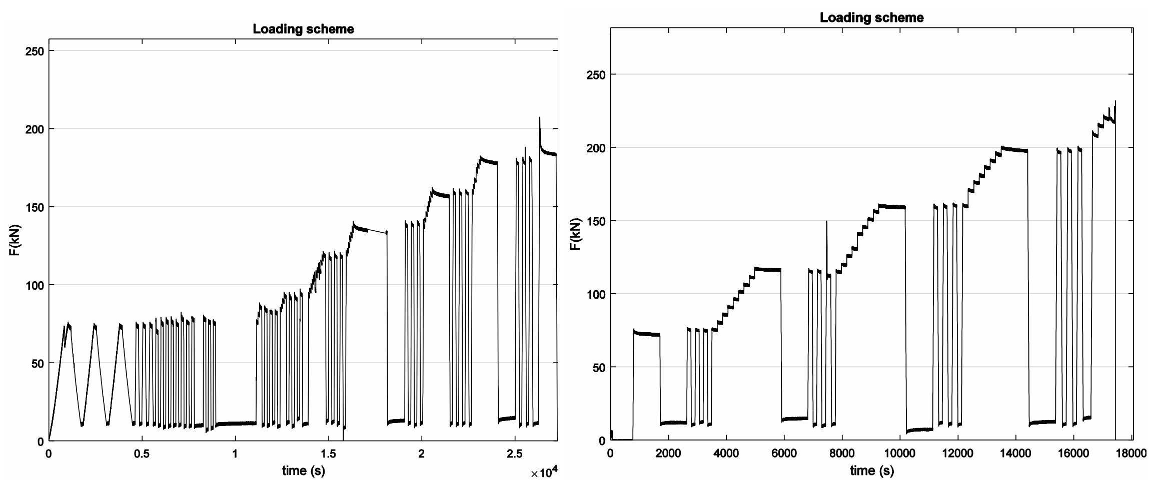
Table 1: Values of the experiment layout.

Experiment	a (mm)	h (mm)	I (mm)	L (mm)	F_{max} (kN)
P804A1	3000	800	8000	10000	207
P804A2	2500	800	8000	10000	231
P804B	2500	800	8000	10000	196
P502A2	1000	500	5000	8000	150

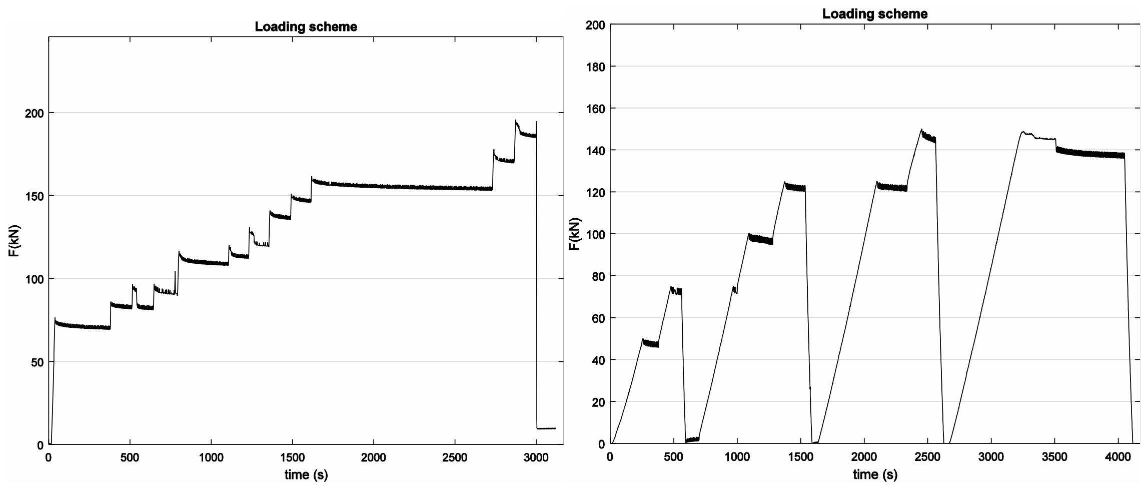
Crack opening, horizontal and vertical deformation, deflection at loading and supports, acoustic emission and strain were measured using LVDTs, laser distance finders, acoustic emission sensors and photogrammetric measurements.

- Loading procedure

Load scheme for every experiment is shown in the figure 5.



(a) (b)



(c) (d)

Figure 5. Experiments Loading Scheme: (a) P804A1; (b) P804A2; (c) P804B; (d) P502A2

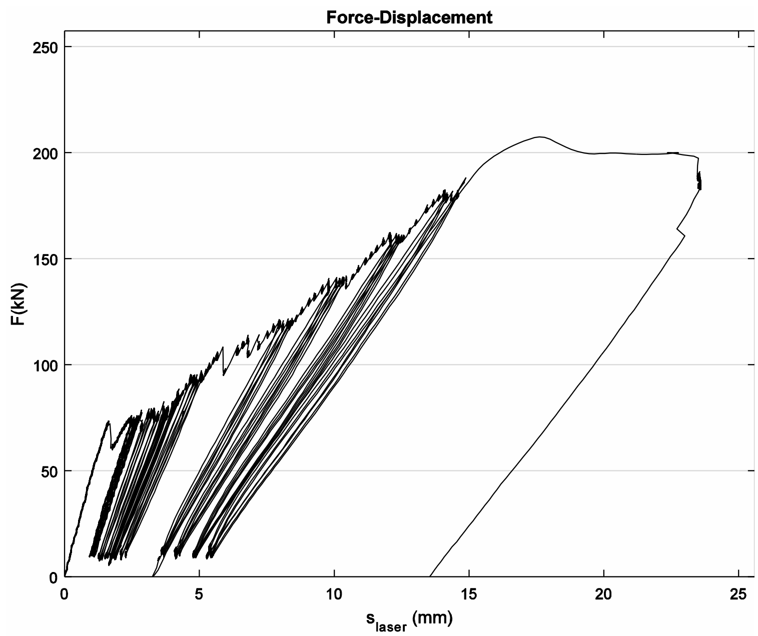
As can be seen, in none of the eperiments the loading procedure follows the one established by ACI 437.2M-13. As shown in Figure 2.

TEST RESULTS

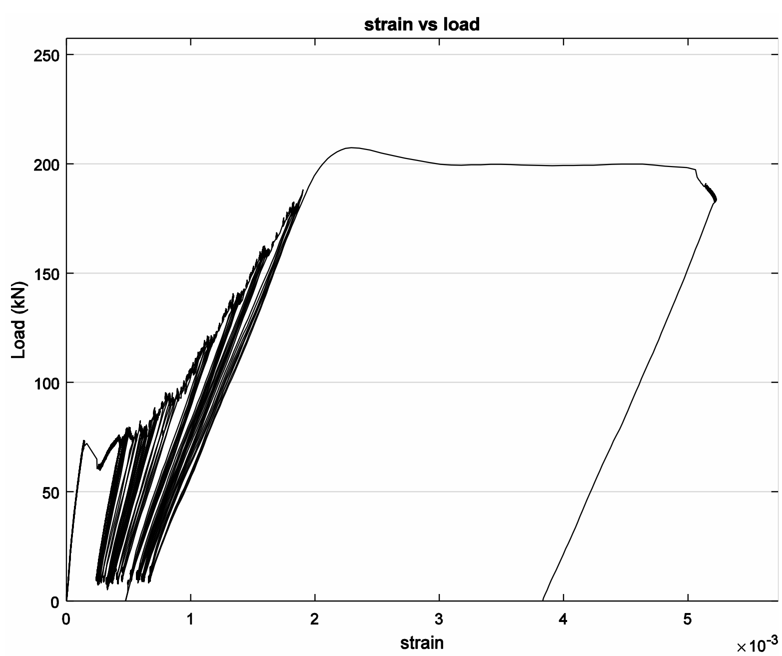
Results needed for the considered stop criterion are deflection, crack width and concrete strain. The steel strain needed for German guideline stop criterion was not measured. Given the amount of data registered, results are show in the plots found in Figures 6 to 9. The above mentioned parameters are plotted in terms of the force applied.

P804A1

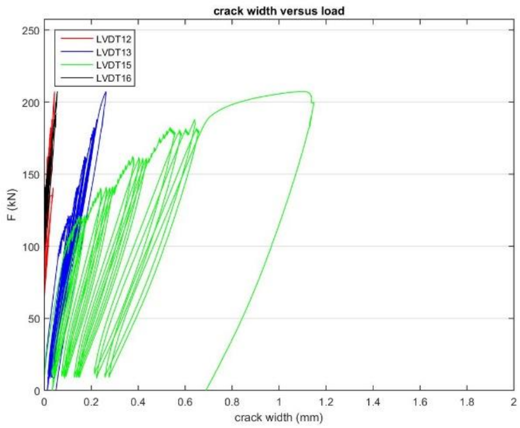
Figure 5 shows plots resuming results obtained for this experiment. Load displacement data can be seen on Figure 6(a). Data for displacement is the mean value of measurements from two lasers and corrected according to the displacement support obtained from other two lasers as well. The load displacement data is used for ACI deviation from linearity index and permanency ratio. The deflection data is used for German guideline residual deflection criteria and ACI maximum deflection and Vos's maximum deflection proposal. Figure 6(b) shows strain in terms of the load. This result is used to analyze concrete strain stop criteria. Figures 6(c) and 6(d) are used to analyze German guideline crack width criteria and Vos's crack width proposal.

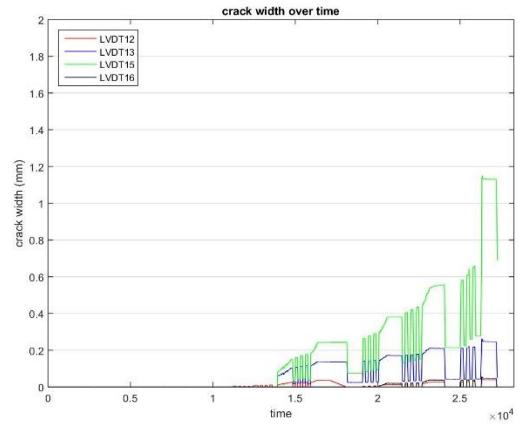


(a)



(b)



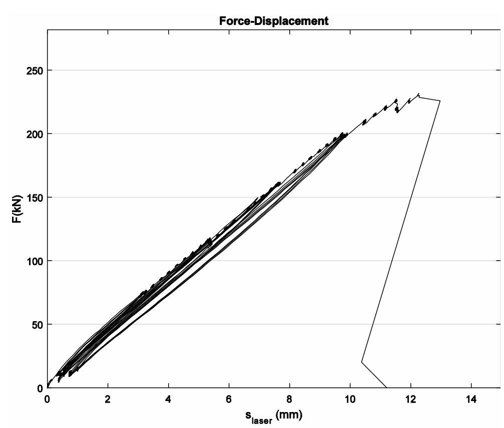


(c) (d)

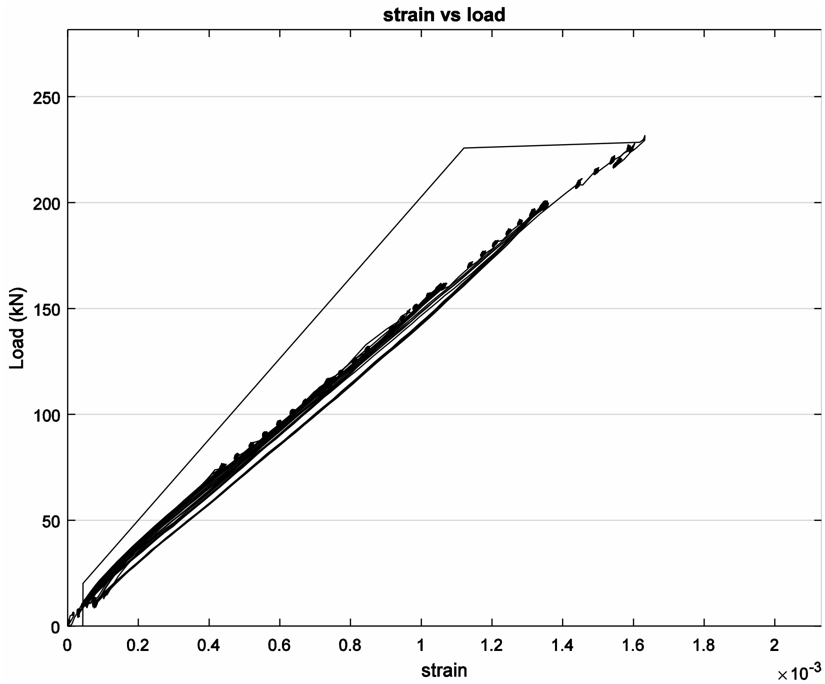
Figure 6. P804A1 experiment results: (a) load-displacement; (b) load-strain; (c) load-crack width; (d) time-crack width

P804A2

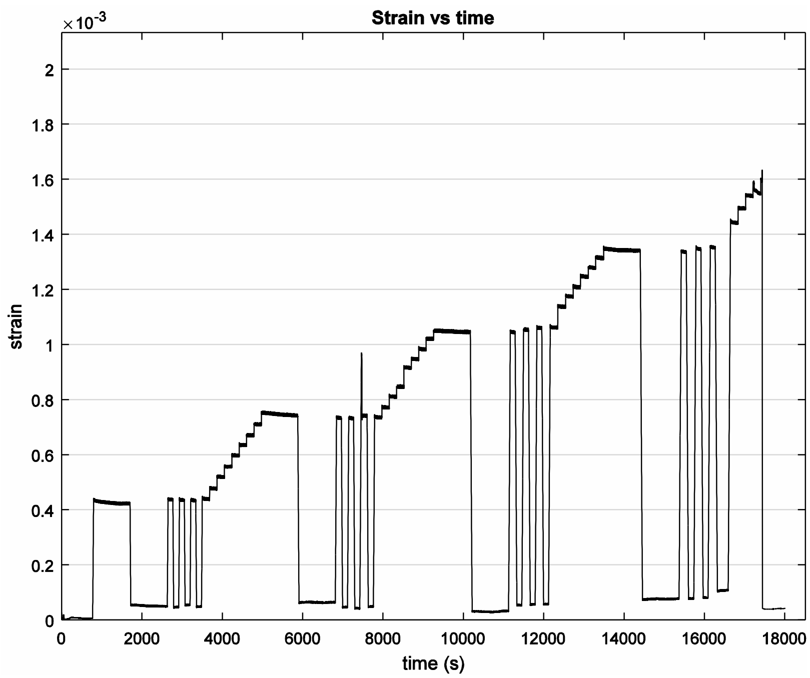
The load displacement data as seen on Figure 7(a) was obtained the same way as experiment P804A1. All plots from Figure 7 are analyzed in the same way as for the previous experiment. The load versus strain plot on figure 7(b) is difficult to read given that on every cycle measurements are very close to the previous one. To analyze the concrete strain data correctly, the time history of the strains and loading scheme on Figure 5(b) are compared to verify the load versus strain results. The same procedure was followed for the crack width data.



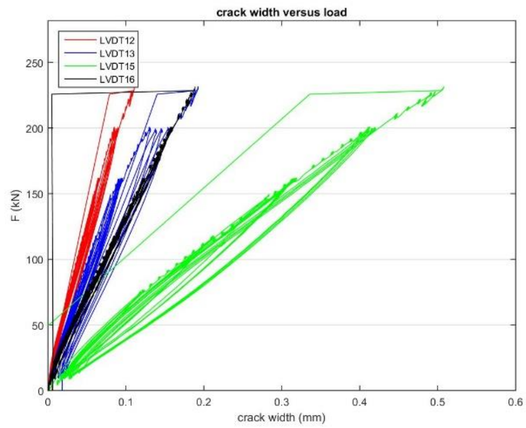
(a)

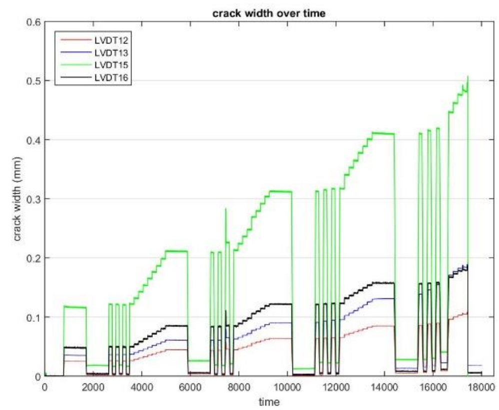

strain vs load
Load (kN)
0
50
100
150
200
250
strain
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
×10^-3

(b)


Strain vs time
×10^-3
strain
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
0
2000
4000
6000
8000
10000
12000
14000
16000
18000

(c)



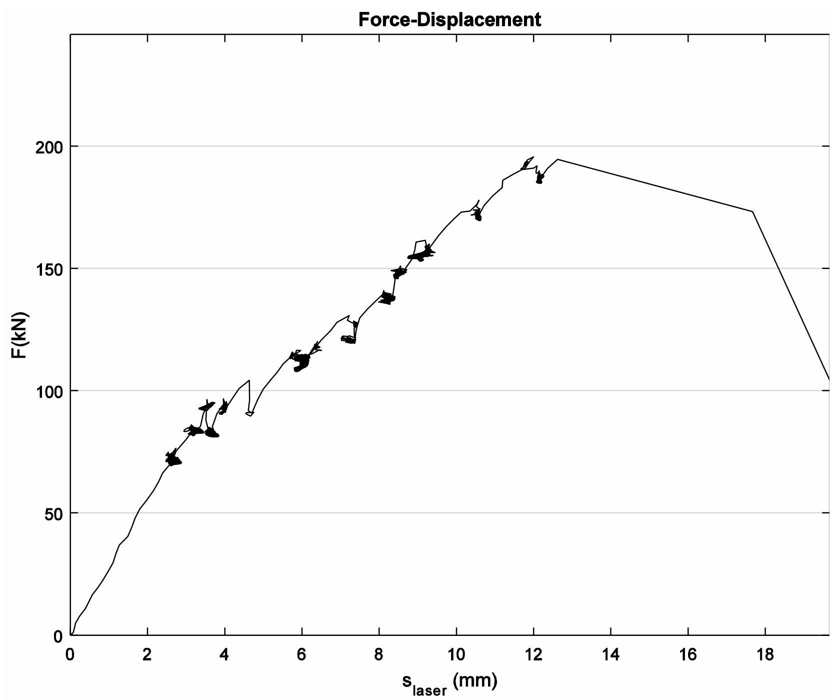


(d) (e)

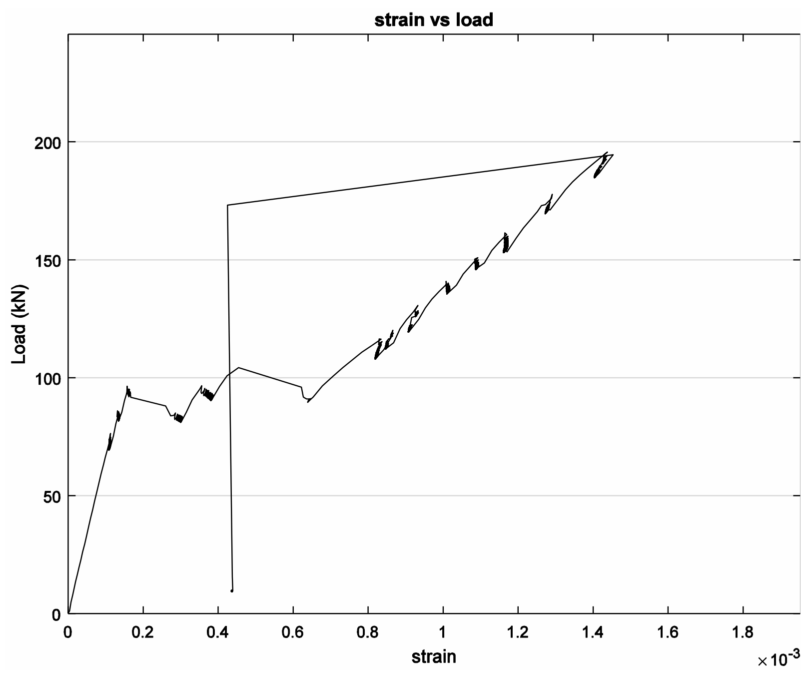
Figure 7. P804A2 experiment results: (a) load-displacement; (b) load-strain; (c) time-strain (d) load-crack width; (e) time-crack width

P804B

Experiment P804B was subjected to monotonic loading. As seen on Figure 8(a) no loading cycles were applied, however from the loading scheme on Figure 5(c) it can be seen that the load was held constant at certain magnitudes to take measurements. Without loading cycles no data of residual deformation or residual crack width is available, therefore this experiment is just analyzed using the ACI deflection criteria, and Vos's deflection proposal taken from displacement data in Figure 8(a), and the German guideline concrete strain stop criterion with data from Figure 8(b). Crack width data were erratic, so that no useful information could be taken to compare in terms of the stop criteria. Therefore the maximum crack width criterion from German guideline and Vos's proposal could not be analyzed. When maximum load was reached, the sensor was outside of its measurement range, as a result of the explosive nature of shear failure, this explains the shape of the plot at its maximum load.



(a)

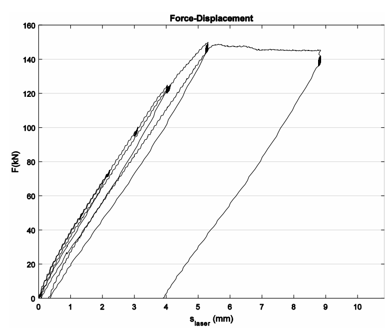


(b)

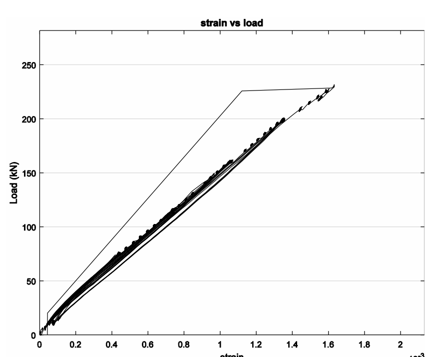
Figure 8. P804A1 experiment results: (a) load-displacement; (b) load-strain.

P502A2

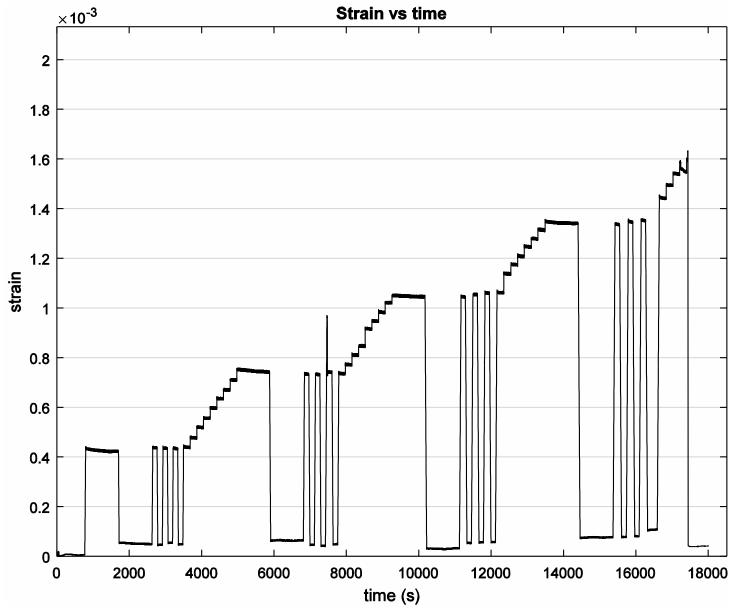
Plots for experiment P502A2 as shown in figure 9 are used in the same way as for experiment P804A1 to analyze the data and compare these to the corresponding stop criteria. In what concerns concrete strain, the plot in figure 9(b) is difficult to read and some data overlaps. Therefore time history of the strain n Figure 9(c) and load versus strain data from the loading scheme on Figure 5(d) are compared to verify load versus strain data. The same procedure was applied to verify the data from the load versus the crack width plot on Figure 9(d). On figure 9(d) it can be seen that during a part of the test, the largest crack width was measured by LVDT 6, but later on measurements from LVDT 7 become bigger as shown on Figure 9(e). Data from both LVDTs was used to analyze the corresponding stop criteria. However, by the end of the test, the measurements from LVDT 6 become much larger and different from the other measurements probably because of the opening of a crack after yielding.



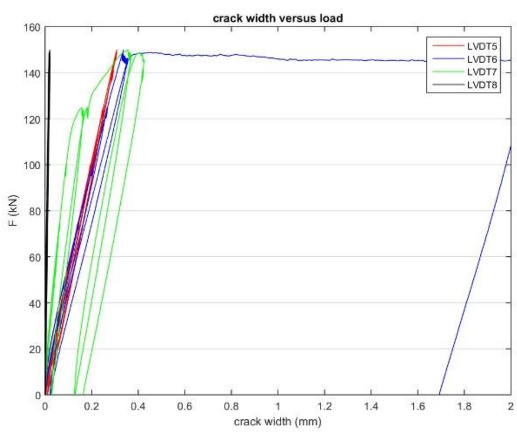
(a)

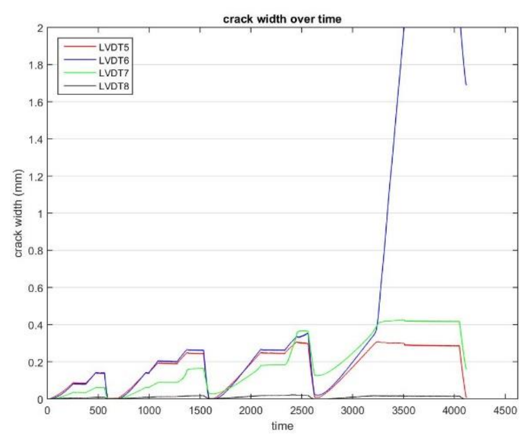


(b)



(c)





(d) (e)

Figure 9. P502A2 experiment results: (a) load-displacement; (b) load-strain; (c) time-strain (d) load-crack width; (e) time-crack width

For the crack width plots, since cracks appear all over the beam, four LVDT's were placed in different places of the specimen. Therefore, the crack width plot shows four values of the crack width in terms of the force applied. The values of each LVDT vary depending on its location.

CALULATION OF WERNER VOS STOP CRITERIA

- Deflection

The diagram from figure 10 is considered for the calculations.

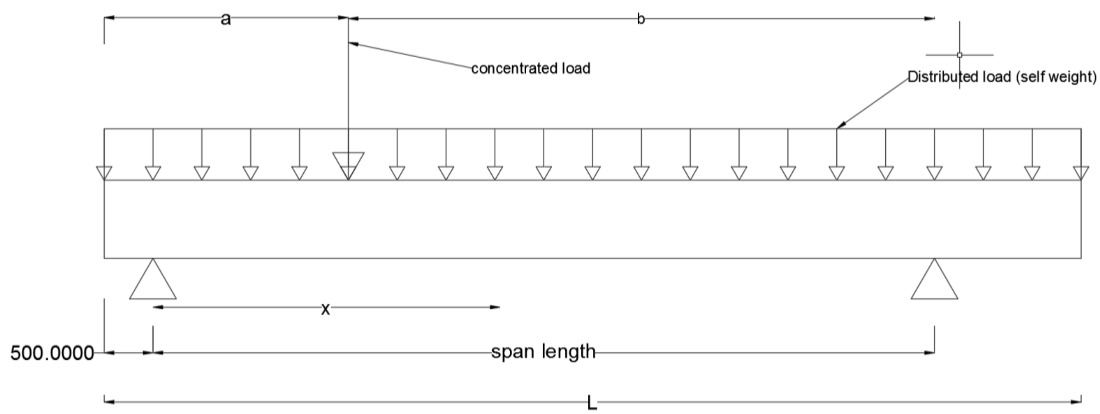


Figure 10. Experiment layout for Vos' proposal calulation

Equation (8) can be rewritten as

$$\delta(x) = \iint k\,dx \tag{13}$$

And, based on the semi-linear approach on the moment curvature diagram describing an elastic material, the deflection can be calculated according to

$$\delta(x) = \iint \frac{M(x)}{EI}\,dx \tag{14}$$

with its respective stiffness. Given the semi-linear assumption, the deflection only depends on the moment *M* which is a function of the distance from support *x*. Different loads can be separated with the superposition principle given the semi-linear assumption.

A concentrated load is used in all the experiments. Given this case, the equation for the deflection under a concentrated load is:

$$\delta(x) = \frac{P\,a\,(I_{span} - x)}{6\,(I_{span})EI^{*}}(x^2 + a^2 - 2(I_{span}\,x)) \tag{15}$$

Where x=0 at the support, P is the concentrated load at which the yielding moment occurs and EI^{*} correspond to the stiffness EI_o or EI_{te} depending on which deflection is being calculated. Equation (15) only applies when a < x <I_{span} , which is the region of the maximum moment and deflection. Therefore there is no need for establishing deflection equations in the other parts of the beam.

For the maximum allowable deflection, EI_{te} corresponding to the retrograde branch stiffness on the moment curvature diagram, must be used. A minimum deflection corresponds to the stiffness EI_o. To the value obtained, the deflection caused by the self-weight must be added. EI_{te} can be seen in Figure 11.

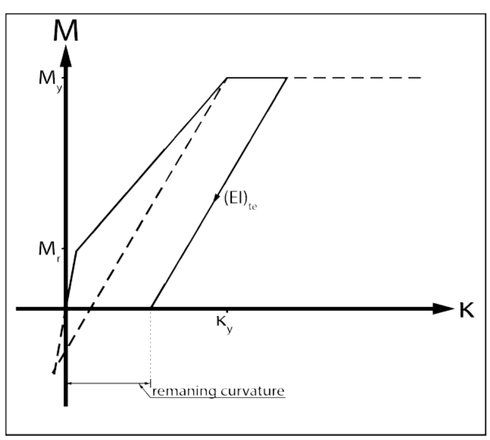


Figure 11. Stiffness from retrograde branch (Vos, 2015)

Given the load arrangement, the maximum moment occurs under the concentrated load, but this is not the same position for maximum deflection; in Vos's example (Vos. W., 2016) this does not occur. Therefore P for yielding must be in accordance of the maximum moment, which for this case is under the concentrated load, and deflection must be calculated where it is maximum.

EI_o is calculated with concrete modulus of elasticity and the moment of inertia of the cross section and the reinforcement combined.

EI_{te} is calculated as follows:

$$EI_{te} = \left(4.91\, {\rho_o}^2 + 17.66\, \rho_o + \frac{117.72}{7.274 * 10^{-4}\ f_y^2 +\ \rho_o + 4}\right) b\, d^3\, 10^{-7}\ \frac{kN}{m^2} \quad (14)$$

Where ρ_o is the reinforcement ratio in percentage, f_y is the steel yielding strength, and *b* and *d* are the width and effective depth of the beam respectively. Factors have been transformed to work with units MPa and mm.

For the maximum deflection, a residual deflection must be added. Nonetheless as Werner Vos states, the of residual the deformation cannot be done accurately with this method. Since it is calculated using the whole graph with the retrograde branch, the resulting value includes errors from all the branches since a semi-linear behavior was assumed. Therefore, it is chosen to take the residual deformation equal to zero, which will lead to conservative results as the maximum allowable deflection is smaller. The only deflection that must be added to *δ(x)* is the deflection from self-weight at the distance *x* where maximum deflection occurs.

The yielding moment M_y, must be found in order to establish the value of the concentrated load P. Vos, in his proposal uses the following formula:

$$M_y = A_s\, f_y\, (d - \frac{1}{3}\left(\sqrt{(\alpha_e\rho)^2 + 2\alpha_e\rho} - \alpha_e\rho\right) d) \tag{15}$$

Where α_e is the ration between the steel and the concrete Young's modulus and ρ is the reinforcement ratio.

In this report, the yielding moment was found using Thorenfeldt's theory according to the moment curvature diagram. These calculations can be found in appendix A. There is no significant difference in the values found.

Additionally, given that the superposition principle is being considered, and M_y is caused by both concentrated and distributed loads, the load P must cause a moment equal to $M_y - M_g$, where M_g is the moment due to beam's self weight, at a distance x where maximum moment occurs.

Vos suggests to change to work in terms of the load that causes yielding, and establishing a value of x so that the remaining equation for the deflection consists only of a factor multiplying the moment and the stiffnes. Calculations for this report were done in a more general way and can be found in appendix A.

- Crack Width

Equations (9) and (10) are based on Van Leeuwen's research about the influence of crack width on corrosion of the reinforcement. According to Van Leeuwen, the crack width can be found if the crack spacing s is known. Applying the correction from the old notation to Eurocode notation, and taking into account that the spacing in-between cracks is an average rather than a specific value, Vos states:

$$s = \left(c + \frac{1}{2}\emptyset + 0.3\,n\emptyset\right)\left(1 + \sqrt{\frac{1}{\rho n}}\right) \tag{16}$$

Where c is the smallest distance from a bar to a corner of the beam, $\emptyset$ is the bar diameter and *n* is the number of bars in tension.

In equations (9) and (10) it can be seen that the only difference between maximum and minimum crack width is the stress the beam is subjected to. For the maximum value, the yielding stress is used, while the minimum crack width is calculated using the stress in the steel caused by self-weight.

For equations (9) and (10), as stated before, β can be taken equal to 1. s is calculated with equation (16) and σ_{s1} is calculated according to Eurocode 2

Vos also assumes that for plain bars, the long term bond $\tau_{bm} = 0$. Therefore it does not appear on the equation. This produces more conservative results.

For The Eurocode equations (11) and (12), the terms needed are calculated as follows:

To find the steel stress under self-weight in a cracked section the following formula is used:

$$\sigma_s = \frac{Mg}{A_s\,z} \tag{17}$$

$$z = d - \frac{1}{3}x \tag{17.1}$$

$$x = (\sqrt{(\alpha_e\rho)^2 + 2\alpha_e\rho} - \alpha_e\rho)d \tag{17.2}$$

Where *z* is the lever arm.

$$f_{ctm\ Eurocode} = 2.12\ Ln\left(1 + \frac{f'c}{10}\right)\text{MPa} \tag{18}$$

$$f_{ctm\ ACI} = \frac{7.5}{12}\sqrt{f'c}\text{MPa} \tag{19}$$

$$\tau_{bm} = \alpha f_{ctm} \tag{20}$$

$$f_{ct}^{*} = \frac{M_g}{M_r}\ f_{ctm} \tag{21}$$

Where α varies between 1.8 and 2. However Vos states that for plain bars this value is around 1

$$\rho_{eff} = \frac{A_s}{A_{c\ eff}} \tag{22}$$

Where $A_{c\ eff}$ is the effective area of concrete under 1tension with a height $h_{c\ eff}$ calculated as follows:

$$h_{c\ eff} = min\begin{cases} 2.5(h-d) \\ \dfrac{(h-x)}{3} \\ \dfrac{h}{2} \end{cases} \tag{22.1}$$

For the calculation of Werner Vos' stop criteria, the following parameters according to the cross section must be found:

- Moment of inertia of the compound section for the stiffness EI_o using the equivalent concrete and steel cross section.
- Stiffness from the retrograde branch on the moment curvature diagram EI_{te} from equation (14)
- Cracking momentM_r, using concrete tensile from equation (18) or (19) and a linear elastic analysis, strength from yielding moment M_y, with equation (15) or Thorenfeldt's theory as

shown on appendix A, and moment by self weight M_g using a linear elastic analysis since no cracking occurs due to self-weight.

- Steel stress and strain under self-weight σ_s, ε_s from a linear elastic analysis
- Crack spacing s with equation (16).

A comparison among theoretically determined values used for Vos's calculation and measured values can be seen in Table 2.

Table 2: comparison on calculated and real ultimate load

Experiment	Theoretical M_y (kNm)	Corresponding load to M_y(kN)	Failure load (kN)
P804A1	375	177	207
P804A2	375	196	231
P804B	375	196	195
P502A2	116	139	150

ANALYSIS OF RESULTS

In this section,the experimental results are compared to the threshold values obtained from the codes and Werner Vos' proposal. The results from the four experiments are discussed based on the previous comparison

The following measured values are the ones closest to the threshold that were taken while the load was being held constant as can be seen on the loading scheme.

ε_{c0} for this report was calculated with a linear elastic analysis since moment caused by self-weight was less than the cracking moment. The calculations can be found on appendix A.

Comparison with Code Values:

- **804A1**

  Concrete Strain

  Threshold Value: $\varepsilon_{c\,lim} - \varepsilon_{c0} = 0.0008 - 0.000031 = 0.000769$ or 769 micro strain

  ε_{c0} was found based on the self-weigh moment M_g. Since cracking moment has not occurred, a linear elastic analysis is used to find concrete existing strain:

$$\sigma_{c0} = \frac{M_g\, y}{I}$$

Table 3: concrete strain measured on P804A1 at every load step

Load (kN)	ε_c (με)
75	570
83	707
90	820
118	1200
136	1400
158	1600
179	1800

After 90kN which represents 43% of the ultimate load of 207 kN, the limit for the concrete strain was reached.

DASfB Deflection:

Threshold: Increase of 10% in residual deflection.

The stop Criterion was exceeded in load level number 2 for a load of 90kN which is 34% of the failure load. After the load increase from 90kN to 120kN a residual deflection increase of 36% was measured, exceeding the threshold value of 10%.

DAfStB crack width:

Threshold: 0.5mm for the width of the maximum crack, 30% of maximum crack for residual crack width.

Table 4: P804A1 crack width. From maximum and minimum among all LVDTs

Load (KN)	w_{max} (mm)	w_{res} (mm)	0.3 w_{max} (mm)
75	0.00	0.00	0.00
84	0.00	0.00	0.00
119	0.17	0.04	0.05
139	0.28	0.08	0.08
158	0.42	0.14	0.13
178	0.55	0.22	0.16

Since this was the first experiment carried out on the beam, threshold values are considered for newly caused cracks. As can be seen on Figure 6(c), only LVDT 13 and 15 performed correctly. Values from other sensors are erratic. Cracks smaller than 0.05mm should not be considered (Lantsoght, E., 2016). The resuls in Table 4 come from LVDT 15 which showed the largest values. It can be seen that the threshold value for the maximum crack width was reached on the increment to the 175kN step, close to the ultimate load. As for maximum residual crack width, it was reached one load step earlier on the cycles of 160 kN.

ACI Deviation from linearity index

Threshold: 0.25

Table 5: P804A1 Linearity index

tan α_{ref}	19.91	I_{DL}
tan α_1	12.57	0.37
tan α_2	11.68	0.41
tan α_3	10.69	0.46
tan α_4	10.05	0.49
tan α_5	9.88	0.50

Points from load-deflection plot to obtain α were taken at the maximum load on the first cycle of each increment where the load was held constant. During the load increment from 75kN to 85kN after ten cycles of 75kN load, a value for I_{DL} of 0.37 which surpasses the threshold value of 0.25 was found. A load of 75kN represents 36% of the ultimate load of 207KN. For every other value of α taken from the plot, the threshold was also exceeded.

ACI Permnency ratio

Threshold: *Ipr* < 0.50

Table 6: P804A1 permanency ratio

Load (kN)	Δ_r (mm)	Δ_{max} (mm)	I	$I\ pr$
75	0.16	1.76	0.09	0.47
	0.08	1.81	0.05	1.95
	0.18	1.95	0.09	
120	0.09	4.49	0.02	0.063
	0.01	4.60	0.00	14.96
	0.09	4.67	0.02	

Given the variation of the results and the fact that the threshold was exeeded in the first cycle of 75kN, no more values were taken from the plot for the calculation of permanency ratio.

ACI maximum deflection

Based on the span length the threshold value is:

$$\Delta_l \leq \frac{L}{180} = \frac{8000}{180} = 44.4\ mm$$

Table 7: P804A1, deflection measured at every step

Load (kN)	Δ_l (mm)
75	3.06
85	5.12
120	8.31
140	10.46
158	12.51
179	14.18

On the last load increment before yielding occurred, the deflection of the beam is far from reaching the threshold value. In Figure 6(a) it can be seen that even after yielding and removing the load, the maximum deflection was around 24mm, which is much less than 44mm. For the residual deflection, ACI 437.2M-13 states that it must be less that ¼ of the maximum deflection. However this value must be measured 24 hours after the test, which was not the case for this experiment.

- **P804A2**

  Concrete Strain

  Threshold: 0.000771

Table 8: concrete strain measured on P804A2 at every load step

Load (kN)	ε_c (με)
74	434
116	0.741
158	1050
197	1300
215	1490
231	1600

The concrete strain limit value was reached at 120kN, which is 51.7% of the ultimate load.

DAfStB deflection

At the second level of loading and increae of 18% of rpermanent deformation was measured. The stop criterion was exceeded for a load of 120 kN, which is 51% of the maximum load.

DAfStB crack width

Threshold: 0.3mm for the width of the maximum crack, 20% of maximum crack for residual crack width.

Table 9: P804A2 crack width. From maximum and minimum among all LVDTs

Load (KN)	w_{max} (mm)	w_{res} (mm)	0.2 w_{max} (mm)
75	0.12	0.00	0.024
115	0.21	0.0182	0.042
159	0.31	0.0186	0.062
192	0.41	0.0222	0.082
232	0.50	0.0036	0.1

Figure 7(d), shows the crack width of every LVDT against the applied load. However it is difficula to read the data. Nonetheless when comparing the plots of crack width versus time and load versus time in figures7(d) and 7(e) respectively, it can be seen that LVDT 15 measured the largest values. These values are used for Table 9. Since

P804A2 was the second test on the beam, threshold values were taken for existing cracks. When increasing the load to 160kN, the maximum crack width of 0.3mm was reached. The stop criterion for the residual crack width was not exceeded until the ultimate load of 231kN was applied, even when residual values where measured under a load of 10kN.

ACI Deviation from Linearity Index

Threshold: 0.25

Table 10: P804A2 Linearity index

tan α_{ref}	23.41	1-tan α /tan α_{ref}
tan α_1	21.84	0.07
tan α_2	21.26	0.09
tan α_3	20.16	0.14
tan α_4	19.08	0.19

The value of tan(α) in Table 10 represents the slope at a point of the load-displacement plot. As seen in Figure 8(a) these slopes are very similar to each other in every cycle, which suggests that deviation from linearity index is small. Indeed, as seen in table 10, the threshold value of 0.25 is not reached even in the last load step when the beam failed

ACI Permanency Ratio

Threshold: $I_{pr} < 0.5$

Table 11: P804A2 permanency ratio

Load (kN)	Δ_r (mm)	Δ_{max} (mm)	I	I_{pr}
75	0.038	2.882	0.013	-0.227
	-0.008	2.828	-0.003	
115	0.046	4.855	0.009	0.414
	0.019	4.907	0.004	
160	0.159	7.141	0.022	-0.050
	-0.008	7.063	-0.001	
197	0.058	9.042	0.006	3.647
	0.212	9.081	0.023	

The values of the residual and maxium deformation needed for calculations were taken by comparing load versus time data and deflection versus time, since the load-deflection plot was hard to read and only considers values on the same load level. As it can be seen in table 11, the values of permanecy ratio vary from much smaller to much larger the threshold value of 0.5, and even negative values were found. This negative values occurs when residual deformation was smaller than in the previous cycle. Therefore it can’t be determined when this stop criterion was exceeded.

ACI maximum deflection

Threshold value $44.4\ mm$

Table 12: Deflection P804A2 at every load step

Load (kN)	Δ_l (mm)
75	3.20
115	5.32
159	7.56
96	9.58
225	12.97

As in experiment P804A1, this acceptance criterion was never exceeded. The maximum deflection value after shear failure was around 13mm.

- **P804B Monotonic Load**

Stop criteria not considered for this experiment are omitted

Concrete Strain

Threshold: 0.000771 or 771 microstrain

Table 13: concrete strain measured on P804B at every load step

Load (kN)	ε_c ($\mu\varepsilon$)
72	106
82	138
92	351
110	821
120	912
15	1200
186	1400
195	1453

At a load of 105KN or 53% of the ultimate load, the stop criterion for the concrete strain is exceeded.

Crack Width

Since this experiment was carried following a monotonic loading protocol, there are no values of the residual crack width to compare with the stop criterion. As for the maximum crack width, the collected data were erratic and could not be analized.

ACI Maximum and Minimum Deflection

Threshold: max=44.1mm; min:25% of maximum measured

Table 14: deflection measured on P804B at every load step

Load (kN)	Δ_l (mm)
70	2.57
83	3.51
112	5.93
120	7.27
149	8.65
158	9.23
171	10.58
192	11.84
9	22.89

Even after shear failure, the maximum deflection from the acceptance criterion was not reached. The residual minimum deflection at was masured at the very end of the test, not 24 hours later as stated in the code. Moreover the residual deformation large value is an effect of shear crack developed at failure. Therefore ACI 437.2M-13 residual deflection stop criterion can't be compared in this experiment.

- **P502A2**

Concrete Strain

Threshold= Threshold Value: $\varepsilon_{c\ lim} - \varepsilon_{c0} = 0.0008 - 0.0000089 = 0.000791$

Table 15: concrete strain measured on P502A2 at every load step

Load (kN)	ε_c (με)
48	342
73	471
97	831
121	1000
121	1100
146	1200
138	3300

At a load of 91kN the stop criterion was exceeded. This load representes 61% of the ultimate load of 150kN

DAfStB defletion

In this experiment the scpecimen is unloaded to 0kN on every cycle, therefore measured residual deflections are very small and small changes would imply an increase grater then 10% from the original premanent deflection> however at all the unloading steps down to 0kN cthe stop criterion was never exceeded.

DAfStB crack width

Threshold: 0.3mm for maximum crack, 20% of maximum crack width for residual cracks.

Since maximum values for crack width vary between two LVDTs, both values are presented.

Table 16: P502A2 crack width. Maximum and residual cracks as measured by LVDTs

Load (KN)	w_{max} (mm) LVDT 6	w_{max} (mm)LVDT 7	w_{res} (mm)LVDT 6	w_{res} (mm)LVDT 7	0.2 w_{max} (mm)LVDT6	0.2 w_{max} (mm)LVDT7
47	0.079	0.034				
72	0.142	0.061	0.0003	0.0002	0.028	0.012
97	0.212	0.089				
122	0.265	0.168	-0.001	0.027	0.053	0.033
122	0.261	0.184				
146	0.340	0.364	0.020	0.126	0.068	0.073
139	2.063	0.416	-	0.161	0.412	0.083

As in experminet P804A2, the crack width versus load plot in Figure 9(d) is hard to read, but from Figure 9(e) that shows the crack width versus time, it can be seen that at different stages of the loading, different LVDTs measured the maximum values. Table

16 shows the measurements from LVDTs 6 and 7. However from Figure 8(e) it can be seen that on the last step, LVDT 6 shows larger values due to the opening of a yielding crack as explained before. Cosidering these factors, the threshold of 0.3mm for the maximum crack width of existing cracks was reached in LVDTs 6 and 7 between 120kN and 150 kN, at more than 80% of the ultimate load durig the last step before reaching this maximum load. On the same step, the maximum residual crack width value was exceeded but only by the cracks measured by LVDT 7.

ACI Deviation from linearity index

Table 17: P502A2 Linearity index

$\tan \alpha_{ref}$	33.19	$1-\tan \alpha / \tan \alpha_{ref}$
$\tan \alpha_1$	30.09	0.09
$\tan \alpha_2$	27.40	0.17
$\tan \alpha_3$	15.86	0.52

The value of tan a_3 was taken on the last load increase before yielding occurred at 150kN. It was not until this phase that the threshold value was surpassed.

ACI permanency Ratio

Threshold: $I_{pr} < 0.5$

Table 18:P502A2 permanency ratio

Cycle	Δ_r	Δ_{max}	I	I_{pr}
1; 75kN	0.05	2.18	0.03	0.29
2; 120kN	0.03	3.94	0.01	5.80
3; 147kN	0.22	5.19	0.04	

Based on the results from Table 18, it can be concluded that the permanency ratio fluctuates and that this acceptance criterion cannot be used for concrete bridges.

ACI Maximum deflection

Threshold value $27.7\ mm$

Table 19: deflection measured on P502A2 at every load step

Load (kN)	Δ_l (mm)
73	2.21
97	3.04
122	4.05
147	5.29
139	8.77

The threshold value for this experiment is lower than the others because the span length was smaller. Still, stop criterion was never exceeded. Maximum deformation after yielding and maintaining the load was close to 9mm.

Comparison with Vos's Proposal

Deflection

The values of the deflection were taken at the distance *x* from the support where deflection is maximum.

Table 20: Comparison of deflection values from Vos's proposal

	Werner Vos Deflection		Closest measured		% of ultimate load
	Δ_{min} (mm)	Δ_{max}(mm)	Δ(mm)	Load (kN)	
804A1	3.83	10.86	10.46	140	67%
804A2	3.81	10.79	9.58	195	84%
804B	3.81	10.79	10.58	171	87%
502A2	1.69	6.16	5.29	150	100%

Experiment P804A1 reached maximum deflection at a lower percentage of the ultimate load, while P502A2 reached right after yielding occurred. P804A2 and P804B presented good results where the maximum allowable deflection was not too conservative and a considerable amount of the ultimate load was applied. The difference between these two experiments and

P804A1, is that for A1 the beam was new, with no previous loads or cracks, had never been subjected to yielding and the failure mode was different. Since the slope of the retrograde branch EI_{te} represents stiffness after the yielding moment M_y has occurred which was not the case for P804A1 since it was a new beam, this may explain why the stop criterion was exceeded long before failure was reached.

As for P502A2, concentrated load was 1 m away from the support at a 5 m span, this would cause less moments and deflection than a load located at mid-span. This reduction on deflection due to the position of the load may have led to reaching maximum allowable deflection after yielding. Even though the load was close to the support, on this test flexural failure was achieved, perhaps due to the effect of previous cracking

Crack Width

In table 21, the values of maximum and residual crack width closest to the ones obtained with Vos's proposal are shown

Table 21: Comparison of crack width values from Vos's proposal

	closest measured			WERNER VOS LIMITS			
	Load (kN)	w_{max} (mm)	w_{res} (mm)	$0.9w_{max,Eurocode}$ (mm)	$w_{res,Eurocode}$ (mm)	$0.9w_{max,Leeuwen}$ (mm)	$w_{res,Leeuwen}$ (mm)
P804A1	118	0.17	0.043	0.1829	0.00707	0.1467	0.0694
P804A2	115	0.21	0.018	0.1829	0.013	0.1467	0.0694
P804B	-	-	-	0.1829	0.013	0.1467	0.0694
P502A2	122	0.265	0.02	0.234	0.00669	0.171	0.0075

Results are not as consistent as the ones obtained with the German guideline crack width. For P804A1, the maximum crack width was reached at a 57% of the maximum load for the code threshold and it already surpassed Van Leuween's threshold. In P804A2 the stop criterion was exceeded at 49% of the maximum load, while for P502A2 the stop criterion was exceeded at 81% of the ultimate load both for the maximum crack width of the Eurocode and Van Leeuwen's crack width. The stop for the residual crack width on the other hand was surpassed only for the value obtained from the Eurocode on experiments P804A1, 804A2 and P502A1. Therefore results are not consistent and a clear relation between conditions of the specimen and the results can be done. On what concerns failure mode, a relation cannot be established either. Experiments P804A1 and P52A2 presented flexural failure but the fraction of the ultimate applied load at which the threshold value was exceeded is considerably different probably because of the presence of cracks on experiment P502A2. As for P804A2 which failed under shear, percentage of the ultimate load applied at the moment the criterion was surpassed is even less than in the other two experiments

DISCUSSION

In this section, comparisons made in the previous section will be analized for every stop criterion:

- Concrete strain

Table 22 summarizes the results on every experiment for this criterion

Table 22: Comparison of concrete strain on every experiment

		MEASURED	DAfStB	
	Load KN	$\mu\varepsilon_c$ ε_c ($\mu\varepsilon$)	$\varepsilon_{c\ lim}$ ($\mu\varepsilon$)	% of ultimate load
804A1	90	790	769	43%
804A2	120	770	767	51%
804B	105	790	767	53%
502A2	91	810	791	61%

As can be seen, on all experiments, stop criteria was reached between 40% and 60% of the ultimate load. P804A1, in which the stop criterion was exceeded at the lowest fraction of the ultimate load was the only test where the beam was un-cracked besides P804B where the un-cracked part was tested. As for P804A2 and P804B, both reached the stop criterion at about half the ultimate load. Between these two experiments differences where the loading protocol, and that for P804B part of the beam was un-cracked which was the part that was tested. Differences between these two experiments and P804A1 was a variation of 500mm of the position of the load and the conditions of the beam prior the test. In an 8m span, one would expect that a 0.5m change in position of the load may not have a big influence. However, this small variation in the position of the load was large enough to change the failure mode from flexure to shear, which consequently influenced the results at which stop criteria was reached . P502A2 got the closest to the ultimate load before exceeding the stop criterion. This was a cracked beam, but the span length was smaller and the position of the load was closer to the support.

Additionally, when finding $\varepsilon_{c\,lim}$, ε_{c0} which is existing strain from permanent loads must be considered, but the limit value does not consider conditions of the beam or load history that may change the value of ε_{c0}. For example, in bridges, continuous traffic which is not a permanent and will not be present during the test may have an influence on existing strain, but it is not considered in calculations, thus, ε_{c0} is miscalculated, setting a wrong threshold value for this stop criteria. Overall, this stop criterion shows consistent results. However, even a though 60% of the ultimate load is considered a good fraction of the load to be applied, 40% which was the case for experiment P804A1 may be too conservative to stop the test since stop criteria was exceeded. Nonetheless, P804A1 does not representent the actiual conditions of beams on a bridge. Therefore, overall concrete strain seems a reliable stop criteria to apply on bridges

- DAfStB Deflection

One would expect to reach limit for the residual deflection as the structure gets closer to failure, as for the case of 804A2 However for P804A1 and P502A2 the increment occurred more than once and it occurred in the first cycles at which the structure is far from suffering permanent damage, and stopping the test at that point would not provide any relevant information.

Additionally, on the first cycles, the residual deformation is really small, especially if load is decreased until 0kN as in experiment P502A2. Therefore, small variations would represent a 10% increment, which may be the reason of the +10% increment in the first steps. Perhaps, loading protocol should not decrease the load to 0kN, or residual deformation should be measured at an established load before reaching 0kN. Also, generally, when proof load tests are performed, a baseline load level is maintained to keep sensors ad jacks activated during the test

Comparing experiments, results are not consistent since in some cases stop criteria was exceeded and in some it was not.

- Crack Width

Table 23 shows the summarized results of crack width on every experiment

Table 23: Comparison of maximum crack width on every experiment

	DASfB limits		Closest measured		% of ultimate load
	w_{max} (mm)	w_{res} (mm)	Load (kN)	w_{max} (mm)	
804A1	0.50	0.17	178	0.56	86%
804A2	0.30	0.09	159	0.31	69%
804B	0.30	-	-	-	
502A2	0.30	0.05	123	0.26	81%

Table 24: Comparison of minimum crack width on every experiment

	DASfB limits		Closest measured			% of ultimate load
	w_{max} (mm)	w_{res} (mm)	Load (kN)	w_{max} (mm)	w_{res} (mm)	
804A1	0.50	0.083	139	0.28	0.082	67%
804A2	0.30	0.06	159	0.31	0.02	69%
804B	0.50	-	-	-	-	
502A2	0.30	0.07	146	0.364	0.12	97%

In what concerns the maximum crack width, for experiments P804A1 and P502A2, the maximum crack width was reached at around 85% of the ultimate load, whereas for P804A2 it was around 70%. For the three cases, a considerable amount of the maximum load had already been applied, and considering a safety margin to avoid permanent damage, threshold values seem adequate for the stop criteria. Also, the threshold value, considers if cracks are new or already existing. Therefore, maximum crack width provides consistent results and it is not as conservative as the results obtained for concrete strain, allowing the test apply greater loads on the structure so that relevant information can be obtained

As for residual crack width, experiment P804A1 surpassed the threshold value but it happened on a cycle before reaching the maximum crack width as shown on Tables 24 and 24 the maximum crack width values was surpassed at a load close to 180kN while the value of maximum residual crack width was reached after applying a load of 140kN. For P502A2 on the other hand, the residual crack width stop criterion was exceeded one cycle before failure really when the maximum crack width threshold had already been exceeded. As for P804A2 this threshold was not surpassed before failure occurred. Table 24 shows the closest measured value to the threshold from the stop criteria, however, this is still far from being reached.

When analyzing residual crack width it should be considered that aggregate that spalls as concrete breaks, could get stuck inside cracks and avoid its closure while load is being removed causing a residual crack width larger than the allowable. For this case, the larger reading does not signal irreversible damage to the structure. Residual crack width also presented problems when collecting the data. As can be seen on the Figure 7(d), some measured values of residual crack width are negative, mostly because these are very small values and measurements combine elastic deformation and crack width which can’t be isolated. However, it should be noted that residual cracks smaller then 0.05mm can be neglected since it is considered a microcrack which is not structural (Lantsoght, E., 2016).

Even though the stop criterion related to the residual crack width was full-filled in two of the three experiments, the issues when measuring, and possible causes than prevent cracks from closing should be considered when using this stop criteria.

- ACI Deviation from linearity Index and permanency ratio

In what concerns the deviation from linearity index, P804A1 did not remained under the threshold value in any of the load steps. Meanwhile P502A2 surpassed the threshold at the third load step, and P805A2 remained always under the maximum value. This can easily been seen on Figure 7(a) for experiment 804A2 which shows the load-deflection plot, since the slopes are similar on every load step contrary to P804A1 and P502A2 force displacement plots, as shown in figure 6(a) and 9(a).

As for permanency ratio, for all three experiments the results are erratic. For one set of cycles I_{pr} is much less than the threshold value and for the next set of cycles it is much larger. Therefore this stop criterion should not be recommended for the use with proof load testing.

These two criteria are based on taking points from the load-deflection diagram obtained from the experiments and thus are influenced by the loading protocol that is followed. ACI 437.2M-13 defines a loading protocol in order to apply this acceptance criteria, shown in Figure 2. This protocol was not followed on any of the experiments as can be seen on the loading schemes on Figure 5. Therefore, this criterion does not provide any relevant information to this report.

The values for α_i and $I_{pr}$are sensitive to small changes on the values taken from the load displacement plot for calculations. Since this plot depends on the loading protocol, following ACI 437.2M-13 would provide better data to apply this stop criteria, Otherwise, calculated values are erratic and are not useful to compare with the threshold value. Additionally, to follow this loading protocol and it would be necessary to use force controlled loading instead of displacement controlled loading, but displacement controlled loading is safer when testing for bridges because once yielding is reched no more force is applied.

Finally, the deviation from linearity index and permanency ratio are not exactly stop criteria but acceptance criteria. This means that if the structures remains under the threshold values it passes the test, but this does not explicitly means that there has not occurred permanent damage. In experiment P804A2 for example, at all the cycles linearity index always remained below the limit which means it passed; but it still remained under the limit even in the last increment prior to failure which means the beam passed the acceptance criteria but was really close to reaching permanent damage. On the other hand, in experiment P804A1 threshold was already surpassed in the first load cycle, which means it did not pass the proof load test under ACI acceptance criteria. However, it was far from reaching failure.

- ACI deflection

ACI 437.2M-13 defines a monotonic loading protocol in accordance to deflection acceptance criteria which was not followed for any of the experiments, which may lead to incongruent results. One of the main differences in the loading protocol is that the one in ACI 437.2M-13 takes much more time: around two days to complete the test. In this case the whole loading process was completed in less than two hours.

As can be seen in the results, the maximum allowable deflections for beams are much larger than the maximum deflection measured prior to yielding. Even after yielding, the maximum allowable values were not reached. Under this circumstances 804B beam passed the proof load test under the maximum deflection acceptance criteria. However, it not only was subjected to permanent damage, it actually failed, which is what is trying to be avoided. Same happened with experiments under the cyclic loading protocol. Even after failure occurred, the threshold value was not reached. After failing the beam did not fullfill the criterion according to the residual deflection. However, residual deflection value was taken at the very end of the test after permanent damage occurred from the shear failure, so this stop criterion can't be compared with the measured value. Additionally, even if the beam had not failed ACI437.2M-13 establishes that residual deformation must be measured 24 hour after the load has been removed, not right after the load is removed. On a real life scenario a building can easily be closed for 24 hours until all the measurements are completed, but this is not practical for bridges. Setting up the equipment, performing the test and waiting 24 hour after the test is completed to measure residual deflection implies closing part of a road or highway long enough to cause problems with traffic.

- Werner Vos's Deflection

In the three P804 experiments, the maximum deflection was reached before failure. However in P804A1, the stop criterion was exceeded within a wide range before reaching the yielding point at about 67% of the ultimate load, whereas on the other two cases it was reached at a 87% of the ultimate load which may be considered as too close to the yielding point or not. This difference among experiments in the fraction of the ultimate load at which the threshold was reached may have to do with the failure mode On the other hand, for P502A2 the maximum allowable deflection occurred after yielding. Based on this, the semi-linear assumption on the moment curvature diagram and its respective stiffness from the retrograde branch seem to be quite a good approximation that considers the effect of cracking on the change of stiffness, it may not be too conservative but still it occurred before permanent damage in three of the four experiments. Additionally, the maximum values found are much smaller than the maximum deflection found based on ACI acceptance criteria given that stop criteria and acceptance criteria are have a slightly different function as explained before.

According to the results, there is no explicit relation in the load at which threshold was reached and the conditions of the beam before the test. Four cases can be considered based on conditions of the specimen and failure mode: flexural failure un-cracked like P804A1, shear failure cracked like P802A2, shear failure un-cracked like P804B and flexural failure cracked like P502A2. P804A1 reached the limit at a 67% of the maximum load, which is a good fraction of the ultimate load, but does no resemble conditions of a bridge. P804A2 was cracked, P804B was un-cracked and both failed in shear but did it around 85% of the maximum load. However, P502A2 which was cracked as well, reached the limit right after yielding. Based on this, a preloaded cracked condition of the beam cannot be directly related on how it affects deflection.

Additionally, it is interesting to compare P804A2 and P804B since these two experiments had the same load arrangement and span length therefore, calculations for the threshold values gave the same numbers. The only difference besides loading protocol was that about 1.5 m of the beam on P084B had not been tested since it was flipped for this tests. However, results are pretty much the same between both which means this partially un-cracked part of the beam, and the loading protocol had nothing to do with the magnitude at which the threshold value was reached. A cyclic load may cause fatigue and affect the stiffness in a different way a monotonic load does; and the loading scheme has no influence when calculating EI_{te} However, given the results obtained between P804A2 and P804B, EI_{te} was accurate enough to provide a maximum value for this proposal of stop criteria.

- Werner Vos' Crack width

As can be seen, the values for the maximum crack width obtained from the Eurocode and the Van Leuween formulas are smaller than those established in the German code, which means Vos's crack width limits in all experiments were reached long before failure. For P804A1 and P804A2 it was around 57% and 49% of the ultimate load respectively. Nonetheless for P504A2 it was around 80% of the ultimate load, which may bbe n quite large and not conservative enough . This experiment under the German guideline stop criteria reached the maximum value right at yielding therefore Vos's proposal on maximum crack width presented better results just for experiment P502A2, but overall, German guideline maximum crack width stop criteria was more consistent and showed more conservative results.

It should be noted that residual crack from the Eurocode for all experiments are smaller than 0.05mm which was the magnitude at which crack width can be neglected. These equations for calculating these values were not taken directly from their respective documents, but from Vos's work. In his proposal, the description on how to apply this stop criteria proposal, or how to calculate threshold values is not clear. For some formulas, the terms included are not defined clearly and in some cases input units do not match units of the results, therefore transformation factors were included for calculations in this report. Finally, Equation (16) for crack spacing *s* was not used given that just by using the formula without any further explanation values around 400mm and 500m were found which are too large for crack spacing. *s* was calculated using a graph found on Figure 12, established by Van Leuween (Van Leuween. J., 1962) in which reinforcement ratio is related to crack spacing.

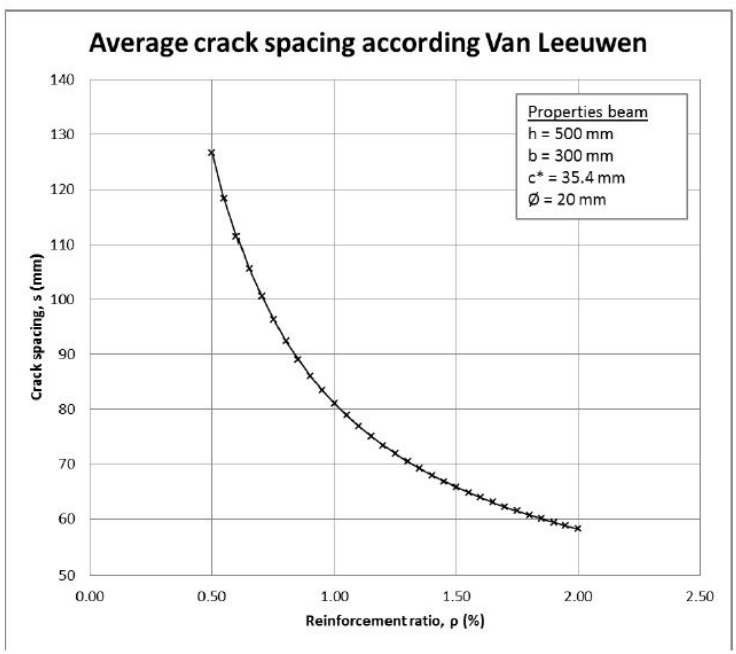


Figure 12: Average crack spacing according to Van Leeuwen (1962)

CONCLUSIONS

By analyzing all the theoretical and experimental data, and compare it to the stop criterion, it can be concluded that:

The ACI 437.2M-13 cyclic loading acceptance criteria must be used only if the ACI loading protocol is applied, otherwise the deviation form linearity index and permanency ratio values obtained will be erratic and can’t be compared to threshold values established by the acceptance criterion.

The ACI 437.2M-13 maximum deflection is too permissive. All specimens failed before reaching the threshold value, and even after failure, the limit was not exceeded. The ACI 437.2M-13 residual deflection is not considered since measurements were not done 24 hours after the test as the code requires and this procedure is not suitable for bridges.

The German guideline concrete strain stop criterion is suitable for cracked and non-cracked beams. Its results are consistent. On cracked beams, which resemble the conditions of beam on bridges, the stop criterion almost at 60% of the ultimate load which is a considerable amount of the load but not too permissive. Therefore German guideline concrete strain seems to be a good criterion to be applied on bridges.

The German guideline residual deflection stop criterion does not seem to be a suitable criterion for both cracked and non-cracked beams. Results are not consistent and in two experiments the threshold value was surpassed during the first cycles. This would lead to cancelation of a proof load test before obtaining any relevant information from the structure, and it would require closing of a structure that is still suitable for use.

The German guideline crack width stop criteria provided good results in what concerns the stop criterion for the maximum crack width, where about 80% of the maximum load was applied. This criterion was less conservative than the concrete strain stop criterion and does distinguish its threshold values between cracked and non-cracked specimens. The stop criterion seems to be a good criterion to apply regardless of the loading protocol that is being followed. The residual crack width on the other hand, did not provide results as consistent as the maximum crack width. When applying this stop criterion, it should be considered the aggregate preventing cracks from closing, and cracks smaller than 0.05mm should be neglected. Finally, positioning of the sensors should be done carefully in order to take measurements correctly, especially when dealing with non-cracked beams, where location of formation of cracks is not known prior to the test.

Vos's deflection proposal showed that a semi linear approach is an accurate approximation for the moment-curvature diagram. Three of the four experiments reached the threshold value at a considerable percentage of the final load being applied, all above 60% of the ultimate load. To apply this criteria on bridges further investigation should be done in what concerns stiffness from the retrograde branch so that it can take into account conditions of the structure prior to the test. This way, the possibility of fatigue from cyclic loads from traffic or a load larger than what was consider in design can be applied.

Vos’s crack width proposal showed more conservative threshold values than the German guideline values in what concerns the maximum crack width. However, description on how to apply this proposal and how to find threshold values should be improved in order to consider this proposal as a stop criterion.

Finally, as a question for further investigation: What percentage of the maximum allowable load must be applied when stop criterion is exceeded, so that it is considered conservative or not? Limits too conservative will cause the proof load tests to be cancelled long before target load could be reached and won’t provide any relevant information, and closing of a structure that is fine. On the other hand threshold values at which the ultimate load is almost reached might be too risky to perform on real structures and irreparable damage might be caused.

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APPENDIX INDEX

APPENDIX A: CALCULATIONS TO DETERMINE STOP CRITERIA THRESHOLD VALUES

In this appendix, calculations of deflection, moments, strain and stop criteria threshold values are presented. Calculations where done using Mathematica. Calculations for experiments 804A2 and 804B are the same given that the beam, the span length and position of the load was the same.

Experiment P804A1

```
(*EXPERIMENT P804A1*)
Clear["Global`*"]
fc1 = 63.51;
fy = 296;
Es = 200 000;
Astop = 0;
dtop = 45;
ϕ = 20;
η = 6;
As = 1885;
b = 300;
h = 800;
d = 755;
```

$$Ec = 22000 \left(\frac{fc1}{10}\right)^{0.3} (*MPa*)$$

38 306.8

```
k = 1;
```

$$\rho = \frac{As}{b\, d}$$

$$\frac{377}{45300}$$

```
ρo = 100 ρ;
(*Cracking Moment*)
```

$$n1 = \frac{Es}{Ec}$$

5.22101

$$cent = \frac{\left(b * h^2 / 2 + (n1 - 1)\ As * d + (n1 - 1)\ Astop * dtop\right)}{\left(b\, h + (n1 - 1) * As + (n1 - 1) * Astop\right)}$$

411.391

$$Io = b * cent^3 / 3 + b\ (h - cent)^3 / 3 + (n1 - 1)\ As\ (d - cent)^2 + (n1 - 1)\ Astop * (c - dtop)^2\ (*mm^4*)$$

1.37706×10^{10}

fr = 7.5 $\sqrt{0.00689\ fc1}$

4.96126

Mr = fr * Io / (h - cent) / 1 000 000

175.805

$\varepsilon t = \dfrac{fr}{Ec}$

0.000129514

$\phi cr = \dfrac{\varepsilon t}{(h - cent)}$

3.33276×10^{-7}

(*MOMENT at position of load and DISPLACEMENT BY SELF WEIGHT*)

concretesw = 25;

L = 10;

span = 8;

$w = concretesw * \dfrac{b}{1000} * \dfrac{h}{1000}$

6

$R1 = \dfrac{span - \left(\frac{L}{2} - 0.5\right)}{span} * w * L$

26.25

$M = \dfrac{-w\, x^2}{2} + R1\, (x - 0.5)$

$26.25\ (-0.5 + x) - 3\, x^2$

θ = Integrate[(M), x]

$-3.\left(4.375\, x - 4.375\, x^2 + \dfrac{x^3}{3}\right)$

δ = Integrate[θ, x]

$-6.5625\, x^2 + 4.375\, x^3 - 0.25\, x^4$

x = 0.5;

δ0 = δ + c1 x + c2

-1.10938 + 0.5 c1 + c2

x = span + 0.5;

```
δ1 = δ + c1 x + c2
907.641 + 8.5 c1 + c2

Solve[δ0 == 0 && δ1 == 0, {c1, c2}]
{{c1 → -113.594, c2 → 57.9063}}

x = 4.5;
```

$$\delta g = \frac{(\delta - 113.594\,x + 59.90)}{Ec\ Io} * 1000^3$$

```
-0.00054598

x = 3.5; (*at point of max M*)

Mg = M(*self weight moment*)
42.

Clear[x];

(*STRAIN BY SELF WEIGHT* linear elastic)
```

$$\sigma = \frac{Mg\,10^6\,\frac{h}{2}}{Io}\ (*MPa*)$$

```
1.21999
```

$$\varepsilon co = \frac{\sigma}{Ec}$$

```
0.000031848

(*YIELDING* Thorenfeldt)
```

$$\varepsilon y = \frac{fy}{Es}$$

$$\frac{37}{25\,000}$$

$$n = 0.8 + \left(fc1\,\frac{145.038}{2500}\right)$$

```
4.48455
```

$$\varepsilon o = \left(\frac{fc1}{Ec}\right) * \left(\frac{n}{n-1}\right)$$

```
0.00213373

c = 243.78;
```

$$\varepsilon c = \left(\frac{c}{d - c}\right) * \varepsilon y$$

```
0.000705752
```

```
ratio = (εc/εo)
0.33076

β1 = (Log[1 + ratio^2])/ratio
0.313887

fc = (0.9 fc1 * n * ratio)/(n - 1 + (ratio) ^ (n * k))
24.2828

fcavg = β1 * fc
7.62204

Cc = fcavg * b * c/1000
557.43

T = As * fy * 0.001
557.96

k2 = 1 - ((2 * (ratio - ArcTan[ratio]))/(ratio^2 * β1))
0.340288

My = T * (d - k2 * c) * 0.001
374.974

(*WERNER VOS DEFORMATION*)

x1 = 4;(*measured from support*)

a = 3;(*position of load from support*)

(*M caused by Py must be My-Mg*)

R1* = (My - Mg)/(a)
110.991

Solve[R1* == P (span - a)/span, P]
{{P → 177.586}}

Py = 177.58;
```

$$\text{dmin} = \frac{\text{Py a (span} - \text{x1)}}{6\ \text{span Ec}\ \frac{\text{Io}}{1000^3}}\left(\text{x1}^2 + \text{a}^2 - 2\ \text{span x1}\right) + \delta\text{g} \ (*\text{m}*)$$

-0.00382823

$$\text{EI}_{\text{te}} = \left(-4.91\ \rho\text{o}^2 + 17.66\ \rho\text{o} + \frac{117.72}{0.0007274\ \text{fy}^2 + \rho\text{o} + 4}\right)\ \text{b}\ \text{d}^3 / 10\,000\,000$$

168 017.

$$\text{dmax} = \frac{\text{Py a (span} - \text{x1)}}{6\ \text{span}\ \text{EI}_{\text{te}}}\left(\text{x1}^2 + \text{a}^2 - 2\ \text{span x1}\right) + \delta\text{g} \ (*\text{m}*)$$

-0.0108509

(*CRACK WIDTH*)

$\beta\text{c} = 1;$

$\tau\text{bm} = 1\ \text{fr};$

$$\alpha\text{eff} = \frac{\text{Es}}{\text{Ec}}$$

5.22101

$$\text{x} = \left(\sqrt{(\alpha\text{eff}\ \rho)^2 + 2\ \alpha\text{eff}\ \rho} - \alpha\text{eff}\ \rho\right)\ \text{d}\ (*\text{mm}*)$$

192.166

$$\text{z} = \text{d} - \frac{1}{3}\text{x}\ (*\text{mm}*)$$

690.945

$$\sigma\text{s} = \frac{\text{Mg} * 10^6}{\text{As} * \text{z}}\ (*\text{MPa}*)$$

32.2474

$\text{c} = 35;$

$$\text{s} = \left(\text{c} + \frac{1}{2}\phi + 0.3\ \eta\ \phi\right) * \left(1 + \sqrt{\frac{1}{\rho\ \eta}}\right)\ (*\text{mm}*)$$

443.483

s = 90; (*from Vos's figure base on ρ(%)*)

$\text{hc} = 2.5\ (\text{h} - \text{d})\ (*\text{mm}*)$

112.5

$$hc = \frac{(h - x)}{3}$$

202.611

$$hc = h/2$$

400

$$Aceff = b * hc\ (*mm^2*)$$

33750.

$$\rho eff = \frac{As}{Aceff}$$

0.0558519

$$kt = 0.6;$$

$$fct = \frac{Mg}{Mr} * fr$$

1.18525

$$\omega maxc = \frac{1}{2} * \frac{fr}{\tau bm} * \frac{\phi}{\rho eff} * \frac{\left(fy - kt * \frac{fr}{\rho eff} * (1 + \alpha eff * \rho eff)\right)}{Es}$$

0.20336

$$\omega resc = \frac{1}{2} * \frac{fr}{\tau bm} * \frac{\phi}{\rho eff} * \frac{\left(\sigma s - kt * \frac{fct}{\rho eff} * (1 + \alpha eff * \rho eff)\right)}{Es}$$

0.0141461

$$\omega maxl = 6.12\ \beta c * fy * s * 10^{-6}$$

0.163037

$$\omega resl = 6.12\ \beta c * \sigma s * s * 10^{-6}$$

0.0177619

Experiment P804A2-P804B

```
(*EXPERIMENTS P804A2, P804B*)

Clear["Global`*"]

fc1 = 63.51;

fy = 296;

Es = 200 000;

Astop = 0;

dtop = 45;

ϕ = 20;

η = 6;

As = 1885;

b = 300;

h = 800;

d = 755;
```

$$Ec = 22\,000 \left(\frac{fc1}{10}\right)^{0.3} (*MPa*)$$

38 306.8

`k = 1;`

$$\rho = \frac{As}{b\,d}$$

$$\frac{377}{45\,300}$$

`ρo = 100 ρ;`

`(*Cracking Moment*)`

$$n1 = \frac{Es}{Ec}$$

5.22101

$$cent = \frac{\left(b * h^2 / 2 + (n1 - 1)\ As * d + (n1 - 1)\ Astop * dtop\right)}{\left(b\,h + (n1 - 1) * As + (n1 - 1) * Astop\right)}$$

411.391

$$Io = b * cent^3 / 3 + b\ (h - cent)^3 / 3 + (n1 - 1)\ As\ (d - cent)^2 + (n1 - 1)\ Astop * (c - dtop)^2\ (*mm^4*)$$

1.37706×10^{10}

$fr = 7.5\sqrt{0.00689\ fc1}$

4.96126

$Mr = fr * Io / (h - cent) / 1000000$

175.805

$\varepsilon t = \frac{fr}{Ec}$

0.000129514

$\phi cr = \frac{\varepsilon t}{(h - cent)}$

3.33276×10^{-7}

(*MOMENT at posotion of load and DISPLACEMENT BY SELF WEIGHT*)

concretesw = 25;

L = 10;

span = 8;

$w = concretesw * \frac{b}{1000} * \frac{h}{1000}$

6

$R1 = \frac{span - \left(\frac{L}{2} - 0.5\right)}{span} * w * L$

26.25

$M = \frac{-w\,x^2}{2} + R1\,(x - 0.5)$

$26.25\,(-0.5 + x) - 3\,x^2$

θ = Integrate[(M), x]

$-3.\left(4.375\,x - 4.375\,x^2 + \frac{x^3}{3}\right)$

δ = Integrate[θ, x]

$-6.5625\,x^2 + 4.375\,x^3 - 0.25\,x^4$

x = 0.5;

$\delta 0 = \delta + c1\,x + c2$

$-1.10938 + 0.5\,c1 + c2$

x = span + 0.5;

$\delta 1 = \delta + c1\, x + c2$

$907.641 + 8.5\, c1 + c2$

```
Solve[δ0 == 0 && δ1 == 0, {c1, c2}]
```

$\{\{c1 \to -113.594,\ c2 \to 57.9063\}\}$

```
x = 4.5; (*position of maximum deflection*)
```

$$\delta g = \frac{(\delta - 113.594\, x + 59.90)}{Ec\, Io} * 1000^3$$

-0.00054598

```
x = 3; (*at point of max M, of all loads combination*)
Mg = M(*self weight moment*)
```

38.625

```
(*STRAIN BY SELF WEIGHT* linear elastic)
```

$$\sigma = \frac{Mg\, 10^6\, \frac{h}{2}}{Io} (*MPa*)$$

1.12196

$$\varepsilon co = \frac{\sigma}{Ec}$$

0.0000292888

```
(*YIELDING* Thorenfeldt)
```

$$\varepsilon y = \frac{fy}{Es}$$

$$\frac{37}{25\,000}$$

$$n = 0.8 + \left(fc1\, \frac{145.038}{2500}\right)$$

4.48455

$$\varepsilon o = \left(\frac{fc1}{Ec}\right) * \left(\frac{n}{n-1}\right)$$

0.00213373

```
c = 243.78;
```

$$\varepsilon c = \left(\frac{c}{d - c}\right) * \varepsilon y$$

0.000705752

```
ratio = (εc / εo)
0.33076

β1 = (Log[1 + ratio^2]) / ratio
0.313887

fc = (0.9 fc1 * n * ratio) / (n - 1 + (ratio) ^ (n * k))
24.2828

fcavg = β1 * fc
7.62204

Cc = fcavg * b * c / 1000
557.43

T = As * fy * 0.001
557.96

k2 = 1 - (2 * (ratio - ArcTan[ratio]) / (ratio^2 * β1))
0.340288

My = T * (d - k2 * c) * 0.001
374.974

(*WERNER VOS DEFORMATION*)

x1 = 3.5; (*measured from support*)

a = 2.5; (*position of load from support*)

(*M caused by Py must be My-Mg*)

R1* = (My - Mg) / (a)
134.54

Solve[R1* == P (span - a) / span, P]
{{P → 195.694}}

Py = 195.64;
```

$$dmin = \frac{Py\ a\ (span - x1)}{6\ span\ Ec\ \frac{Io}{1000^3}}\ (x1^2 + a^2 - 2\ span\ x1) + \delta g\ (*m*)$$

-0.00380565

$$EI_{te} = \left(-4.91\ \rho o^2 + 17.66\ \rho o + \frac{117.72}{0.0007274\ fy^2 + \rho o + 4}\right)\ b\ d^3 / 10\,000\,000$$

168 017.

$$dmax = \frac{Py\ a\ (span - x1)}{6\ span\ EI_{te}}\ (x1^2 + a^2 - 2\ span\ x1) + \delta g\ (*m*)$$

-0.01078

(*CRACK WIDTH*)

$\beta c = 1;$

$\tau bm = 1\ fr;$

$$\alpha eff = \frac{Es}{Ec}$$

5.22101

$$x = \left(\sqrt{(\alpha eff\ \rho)^2 + 2\ \alpha eff\ \rho} - \alpha eff\ \rho\right)\ d\ (*mm*)$$

192.166

$$z = d - \frac{1}{3}\ x\ (*mm*)$$

690.945

$$\sigma s = \frac{Mg * 10^6}{As * z}\ (*MPa*)$$

29.6561

$c = 35;$

$$s = \left(c + \frac{1}{2}\ \phi + 0.3\ \eta\ \phi\right) * \left(1 + \sqrt{\frac{1}{\rho\ \eta}}\right)\ (*mm*)$$

s = 90; (*from Vos's figure base on ρ(%)*)

$hc = 2.5\ (h - d)\ (*mm*)$

112.5

$$hc = \frac{(h - x)}{3}$$

202.611

$$hc = h/2$$

400

$$Aceff = b * hc \ (*mm^2*)$$

33750.

$$\rho eff = \frac{As}{Aceff}$$

0.0558519

$$kt = 0.6;$$

$$fct = \frac{Mg}{Mr} * fr$$

1.09001

$$\omega maxc = \frac{1}{2} * \frac{fr}{\tau bm} * \frac{\phi}{\rho eff} * \frac{\left(fy - kt * \frac{fr}{\rho eff} * (1 + \alpha eff * \rho eff)\right)}{Es}$$

0.20336

$$\omega resc = \frac{1}{2} * \frac{fr}{\tau bm} * \frac{\phi}{\rho eff} * \frac{\left(\sigma s - kt * \frac{fct}{\rho eff} * (1 + \alpha eff * \rho eff)\right)}{Es}$$

0.0130093

$$\omega maxl = 6.12\, \beta c * fy * s * 10^{-6}$$

0.163037

$$\omega resl = 6.12\, \beta c * \sigma s * s * 10^{-6}$$

0.0163346

Experiment P502A2

```
(*EXPERIMENT 502A2*)
Clear["Global`*"]
fc1 = 71.47; (*MPa*)
fy = 296; (*MPa*)
Es = 200000; (*MPa*)
Astop = 0; (*mm^2*)
dtop = 45; (*mm*)
ϕ = 20; (*mm*)
η = 3;
```

$$As = \frac{\phi^2}{4} 3.1416 * \eta$$

942.48

```
b = 300; (*mm*)
h = 500; (*mm*)
d = 465; (*mm*)
```

$$Ec = 22\,000 \left(\frac{fc1}{10}\right)^{0.3} (*MPa*)$$

39 688.1

```
k = 1;
```

$$\rho = \frac{As}{b\,d}$$

0.00675613

```
ρo = 100 ρ;
(*Cracking Moment*)
```

$$n1 = \frac{Es}{Ec}$$

5.0393

$$cent = \frac{\left(b * h^2 / 2 + \left(n1 - 1\right) As * d + \left(n1 - 1\right) Astop * dtop\right)}{\left(b\,h + \left(n1 - 1\right) * As + \left(n1 - 1\right) * Astop\right)}$$

255.322

$$Io = b * cent^3 / 3 + b\,(h - cent)^3 / 3 + \left(n1 - 1\right) As\,(d - cent)^2 + \left(n1 - 1\right) Astop * (c - dtop)^2 (*mm^4*)$$

3.29662×10^9

$$fr = \frac{7.5}{12}\sqrt{fc1}\ (*MPa*)$$

5.28375

$$Mr = fr * \frac{Io}{(h - cent)\ 1\,000\,000}\ (*KN\ m*)$$

71.1894

$$\varepsilon t = \frac{fr}{Ec}$$

0.000133132

$$\phi cr = \frac{\varepsilon t}{(h - cent)}$$

5.4411×10^{-7}

(*concrete tensile strenght Eurocode*)

$$f_{ctm} = 2.12\ Log\left[1 + \frac{fc1}{10}\right]$$

4.44702

(*MOMENT at posotion of load and DISPLACEMENT BY SELF WEIGHT*)

concretesw = 25;

L = 8;

span = 5;

$$w = concretesw * \frac{b}{1000} * \frac{h}{1000}$$

$$\frac{15}{4}$$

$$R1 = \frac{span - \left(\frac{L}{2} - 0.5\right)}{span} * w * L$$

9.

$$M = \frac{-w\,x^2}{2} + R1\,(x - 0.5)\ (*at\ position\ of\ concentrated\ load*)$$

$$9.\ (-0.5 + x) - \frac{15\,x^2}{8}$$

θ = Integrate[(M), x]

$$-1.875\left(2.4\,x - 2.4\,x^2 + \frac{x^3}{3}\right)$$

```
δ = Integrate[θ, x]
```
$-2.25\,x^2 + 1.5\,x^3 - 0.15625\,x^4$

```
x = 0.5;
δ0 = δ + c1 x + c2
```
$-0.384766 + 0.5\,c1 + c2$

```
x = span + 0.5;
δ1 = δ + c1 x + c2
```
$38.5215 + 5.5\,c1 + c2$

```
Solve[δ0 == 0 && δ1 == 0, {c1, c2}] (*border conditions*)
```
{{c1 → -7.78125, c2 → 4.27539}}

```
x = 2.5; (*position of max deflection from edge of beam*)
```
$$\delta g = \frac{(\delta - 7.78125\,x + 4.27539)}{Ec\,Io} * 1000^3$$
-0.000091001

```
x = 1.5; (*position of maximum moment from edge of the beam*)
Mg = M
```
4.78125

```
(*STRAIN BY SELF WEIGHT at point of max moment, linear elastic*)
```
$$\sigma = \frac{Mg\,10^6\,(h - cent)}{Io}\ (*MPa*)$$
0.354869

$$\varepsilon co = \frac{\sigma}{Ec}$$
8.94146×10^{-6}

```
(*YIELDING* Thorenfeldt*)
```
$$\varepsilon y = \frac{fy}{Es}$$
$$\frac{37}{25000}$$

$$n = 0.8 + \left(\frac{fc1\,145.038}{2500\,(*psi*)}\right)$$
4.94635

$$\varepsilon o = \left(\frac{fc1}{Ec}\right) * \left(\frac{n}{n-1}\right)$$

0.00225711

c = 142.68;

$$\varepsilon c = \left(\frac{c}{d-c}\right) * \varepsilon y$$

0.000655145

$$ratio = \left(\frac{\varepsilon c}{\varepsilon o}\right)$$

0.290258

$$\beta 1 = \frac{(Log[1 + ratio\verb|^|2])}{ratio}$$

0.278677

$$fc = \frac{(0.9\, fc1 * n * ratio)}{(n - 1 + (ratio)\verb|^|(n * k))}$$

23.3882

$fcavg = \beta 1 * fc$

6.51777

$Cc = fcavg * b * c / 1000$

278.987

$T = As * fy * 0.001$

278.974

$$k2 = 1 - \left(\frac{2 * (ratio - ArcTan[ratio])}{ratio^2 * \beta 1}\right)$$

0.338746

$My = T * (d - k2 * c) * 0.001$

116.239

(*WERNER VOS DEFORMATION*)

x1 = 2;(*point of max deflection from support, m*)

a = 1;(*Position of load from support*)

(*M caused by Py must be My-Mg*)

$$R1^* = \frac{(My - Mg)}{(a)}$$

111.458

$$\text{Solve}\left[R1^* == P\,\frac{(\text{span}-a)}{\text{span}},\ P\right]$$

{{P → 139.323}}

Py = 139.323;

$$\text{dmin} = \frac{\text{Py}\ a\ (\text{span}-\text{x1})}{6\ \text{span}\ \text{Ec}\ \frac{\text{Io}}{1000^3}}\left(\text{x1}^2 + a^2 - 2\ \text{span}\ \text{x1}\right) + \delta g\ (*m*)$$

-0.0016883

$$\text{EI}_{\text{te}} = \left(-4.91\,\rho o^2 + 17.66\,\rho o + \frac{117.72}{0.0007274\ \text{fy}^2 + \rho o + 4}\right)\ b\ d^3 / 10\,000\,000$$

34 419.5

$$\text{dmax} = \frac{\text{Py}\ a\ (\text{span}-\text{x1})}{6\ \text{span}\ \text{EI}_{\text{te}}}\left(\text{x1}^2 + a^2 - 2\ \text{span}\ \text{x1}\right) + \delta g\ (*m*)$$

-0.0061627

(*CRACK WIDTH*)

$\beta c = 1;$

$\tau bm = 1\ fr;$

$$\alpha\text{eff} = \frac{\text{Es}}{\text{Ec}}$$

5.0393

$$x = \left(\sqrt{(\alpha\text{eff}\ \rho)^2 + 2\ \alpha\text{eff}\ \rho} - \alpha\text{eff}\ \rho\right)\ d\ (*mm*)$$

106.536

$$z = d - \frac{1}{3}\,x\ (*mm*)$$

429.488

$$\sigma s = \frac{\text{Mg} * 10^6}{\text{As} * z}\ (*MPa*)$$

11.8119

c = 35;

(*CRACK SPACING*)

(*empirical formula Leeuwen*)

$$\text{s1} = \left(c + 0.1\,\eta\,3.14\,d\left(1 + \frac{\sqrt{b\,h}}{3.14\,\eta\,d}\right)\right)$$

511.76

$$s = \left(c + \frac{1}{2}\phi + 0.3\,\eta\,\phi\right) * \left(1 + \sqrt{\frac{1}{\rho\,\eta}}\right) \text{(*mm*)}$$

505.518

s = 105; (*from Vos's figure based on ρ(%)*)

hc = 2.5 (h - d) (*mm*)

87.5

$$hc = \frac{(h - x)}{3}$$

131.155

hc = h / 2

250

Aceff = b * hc (*mm^2*)

26 250.

$$\rho eff = \frac{As}{Aceff}$$

0.035904

kt = 0.6;

$$fct = \frac{Mg}{Mr} * fr$$

0.354869

$$fct2 = \frac{Mg}{Mr} * f_{ctm}$$

0.298672

$$\omega maxc = \frac{1}{2} * \frac{fr}{\tau bm} * \frac{\phi}{\rho eff} * \frac{\left(fy - kt * \frac{fr}{\rho eff} * \left(1 + \alpha eff * \rho eff\right)\right)}{Es}$$

0.266999

$$\omega resc = \frac{1}{2} * \frac{fr}{\tau bm} * \frac{\phi}{\rho eff} * \frac{\left(\sigma s - kt * \frac{fct}{\rho eff} * \left(1 + \alpha eff * \rho eff\right)\right)}{Es}$$

0.00669646

$\omega maxl = 6.12\ \beta c * fy * s * 10^{-6}$

0.19021

$\omega resl = 6.12\ \beta c * \sigma s * s * 10^{-6}$

0.0075903