

UNIVERSIDAD SAN FRANCISCO DE QUITO USFQ

Colegio de Ciencias e Ingenierías

**Nonlinear finite element analysis of beam experiments for
stop criteria**
Proyecto de investigación

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Ingeniería Civil

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COLEGIO DE CIENCIAS E INGENIERÍAS

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Nonlinear finite element analysis of beam experiments for stop criteria

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RESUMEN

Las pruebas de carga son una técnica experimental siendo estudiada para evaluar la capacidad estructural de puentes existentes cuando no se dispone de información relevante sobre el sistema estructural. Los criterios de parada, derivados de mediciones realizadas durante las pruebas de carga, pretenden determinar si una prueba de carga debe detenerse antes de alcanzar la carga objetivo para mantener la integridad del sistema estructural bajo estudio. Se propone un modelo no lineal de elementos finitos para continuar con la investigación de criterios de parada durante las pruebas de carga. El espécimen modelado es una viga de hormigón armado con acero de refuerzo liso, que se asemeja a puentes existentes de losas macizas de hormigón armado. El objetivo es desarrollar un modelo de elementos finitos confiable, con modelos constitutivos de materiales adecuados, para analizar los criterios de parada disponibles en los códigos existentes. El análisis de elementos finitos se realiza utilizando un software de FE comercial (ABAQUS). Las principales limitaciones del modelo de elementos finitos son que no se modela el agrietamiento preexistente por flexión y se asume una unión perfecta entre el hormigón y el acero de refuerzo. Se selecciona un estudio experimental existente y se verifica en términos de deformaciones. Los criterios de parada de ACI 437.2M-13 y la guía alemana (DAfStB, Deutscher Ausschuss für Stahlbeton) se analizan para el modelo.

Palabras clave: pruebas de carga, criterios de parada, análisis por elementos finitos

ABSTRACT

Proof load testing is an experimental technique being studied to assess the structural capacity of existing bridges when relevant information about the structural system is unavailable. Stop criteria, derived from measurements taken during proof load tests, pursue to determine if a test should be stopped before reaching the target proof load in order to maintain the integrity of the structural system under study. A non-linear finite element model is proposed in order to further investigate stop criteria during proof load testing. The modeled specimen is a reinforced concrete beam with plain reinforcement which displays resemblance with existing reinforced concrete solid slab bridges. The goal is to develop a reliable finite element model with adequate material constitutive models in order to analyze available stop criteria from existing codes. The finite element analysis is performed using a commercial FE software (ABAQUS). The main limitations of the finite element model are that pre-existing bending cracks are not modeled and that perfect bond between concrete and reinforcement steel bars is assumed. An existing experimental study is selected and beam experiment is verified in terms of strains. Stop criteria from ACI 437.2M-13 and the German guideline (DAfStB, Deutscher Ausschuss für Stahlbeton) are analyzed for the beam model.

Key words: Proof load testing, stop criteria, finite element analysis

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INTRODUCTION

Background

During the post-war period, European countries faced new challenges such as reactivating the economy and supplying infrastructure. Construction works, in newly formed and growing urban and rural centers, proved to be a solution by injecting money into the economy and creating jobs for an increasing population. The road network, consisting of roads and reinforced concrete viaducts, was expanded during this period and is now approaching the end of its lifespan. Several existing bridges capacity may be insufficient according to current codes and to replace such structures or perform rehabilitation works is not economically viable. Consequently, the assessment of their current state to prove sufficient structural capacity is required and can be achieved by improving the conservative models used in the codes and carrying out proof load testing (Lantsoght, Yang, Tersteeg, van der Veen & de Boer, 2016).

Assessment of the structural capacity of existing bridges has proven to be challenging even when relevant information about the structural system is available (e. g. as-built plans) since it is not clear how to precisely quantify the effects of material degradation and the activation of additional load bearing mechanisms that are not included in calculations. In such cases proof load testing has proven to be a satisfactory method to determine the capacity and guarantee adequate performance of the structure (Koekkoek, Lantsoght, Yang, de Boer & Hordijk, 2016). In general, proof load testing can be classified into two categories: proof load tests pursue to prove that the structural capacity exceeds the design capacity during a short - term test with minimum preparations, and diagnostic load tests with loading levels below the maximum expected live load.

Proof load testing implies contradictory requirements as the maximum load should be as high as possible in order to prove sufficient structural capacity and to gather information about the behavior of the structural system, nevertheless is also limited so that irreversible damage or reduction of the capacity due to excessive load levels is prevented. Therefore, questions such as which maximum load demonstrates sufficient structural capacity and how irreversible damage or failure can be prevented during proof load testing, need to be answered and have been matters of research over the last few years. In-situ measurements of the structure during proof load testing allow to guarantee the integrity of the structural system under consideration by remaining within predefined limits which will be referred to onwards as stop criteria or acceptance criteria.

Proof load testing is not a cutting-edge technique to assess the capacity of existing structures by nondestructive testing and documentation on proof load testing and stop criteria has been included in codes and guidelines (Koekkoek, et al., 2016). The German guideline, published by DAfStB (Deutscher Ausschuss für Stahlbeton, 2000), and American Concrete Institute code ACI 437.2M-13 (ACI Committee 437, 2013) provide a detailed test method developed for reinforced concrete structures and contain a thorough description of stop criteria to be checked during proof load testing. The German guideline is applicable to cast in-situ concrete structures in plain or reinforced concrete and stop criteria are based on the deformation of the structure. ACI 437.2M-13 provides requirements for a load test on concrete structures, in addition to chapter 27 of ACI 318-14 (ACI Committee 318, 2014). Proof load testing according to ACI code ACI 437.2M-13 pursues to evaluate whether an existing building requires repair or rehabilitation, or to verify such works. Acceptance criteria are differentiated in accordance with the loading protocol, monotonic and cyclic loading protocols, and are based on residual deflections and load-deflection envelope.

Outline

The following subsections contain a brief description of the study contained throughout the present document. The aim and scope, essentially are to develop a reliable finite element model with adequate material constitutive models in order to further investigate stop criteria during proof load testing, also limitations such as simplifications for modelling and analysis are presented. The next section includes a brief introduction to proof load testing and stop criteria as well as a thorough description of stop criteria from ACI 437.2M-13 and the German guideline published by DAfStB. The third section describes the finite element modelling framework, material constitutive models, interaction constraints are defined, and geometry and boundary conditions are presented. The subsequent section contains validation of the model with experimental work and analysis of results with stop criteria from both documents described above. The following section provides future work insights and proposes recommendations. The last section reviews overall results and conclusions.

Aim and scope

The aim of this document is to develop a non-linear finite element model of a reinforced concrete beam with plain reinforcement in order to further investigate stop criteria. The model intends to capture the significant phenomena for analysis with the existing stop criteria from codes and guidelines without excessive computational time.

Limitations

Important simplifications regarding constraints and material behavior are made in order to simplify modelling and analysis. The application of loading is through a monotonic protocol only. Pre-existing bending cracks are not modelled since the beam experiment from the study selected for validation of the model has an un-cracked cross section and perfect bond interaction between the plain reinforcement steel and concrete is assumed.

LITERATURE REVIEW ON PROOF LOAD TESTING AND STOP CRITERIA

Proof load testing and stop criteria overview

Existing bridges are continuously aging, deterioration of materials is difficult to quantify and sometimes structural plans of these structures are lost. But bridges are not the only ones changing, traffic loads are increasing and new insights in the behavior of structures go hand in hand with the evolution of building codes and standards (Vos, 2016). Therefore, existing structures may not fulfill safety requirements of the codes in force but are too expensive to replace. Proof load testing is an experimental technique being studied to assess the structural capacity of existing bridges and demonstrate safety requirements when relevant information about the structural system is unavailable or when effects from material degradation or additional load bearing mechanisms are difficult to quantify precisely. The structural capacity of a structure is updated if requirements evaluated during proof load testing for certain load levels are fulfilled and the lifespan can be prolonged (Koekkoek, et al., 2016).

Proof load testing requires the use of high loads in order to guarantee a sufficient performance level of the structure but irreversible damage can occur before the target load has been reached if the load levels are excessive. Hence a question arises: How can failure and irreversible damage of the structural system be avoided during proof load testing? To prevent failure and irreversible damage a limit should be used based on in situ measurements during proof load testing. Such limits will be referred to onwards as stop criteria or acceptance criteria, the last being typically checked after testing. The limits are based on measurable parameters that indicate the condition of the structure while the test is executed. Stop criteria pursue to control these limits in order to determine if a test should be stopped before reaching the target proof load in order to maintain the integrity of the structural system under study.

Acceptance criteria ACI 437.2M-13

The American Concrete Institute code 437.2M-13 (ACI Committee 437, 2013) establishes requirements and acceptance criteria for load testing of concrete structures. The code is conceived for buildings rather than bridges and applies to reinforced concrete or prestressed concrete structures with normal strength concrete ($f'_c \leq 55$ MPa). Loading shall be applied without causing vibration and impact, measurements shall be taken where maximum responses are expected. The structure is to be visually inspected at each load level and shall not show evidence of failure. Licensed professional supervision is required for decision making during the load test. The code makes a distinction for the acceptance criteria according to the load applied for the proof load testing, which can be monotonic (Figure 1) and cyclic (Figure 2) loading protocols.

Monotonic loading protocol.

- Use at least 4 approximately equal load increments.
- Applied sustained load should be $\pm 5\%$ of the full applied test load.
- Stabilized deflections (difference between successive deflections no less than 2 minutes apart does not exceed 10% of the initial deflection) must be measured at each load level.
- Hold each load step for at least 2 minutes.
- Hold the full applied test load for at least 24 hours. Measurements must be taken at the beginning and end of the 24 hour period, and 24 hours after removing the full applied test load.

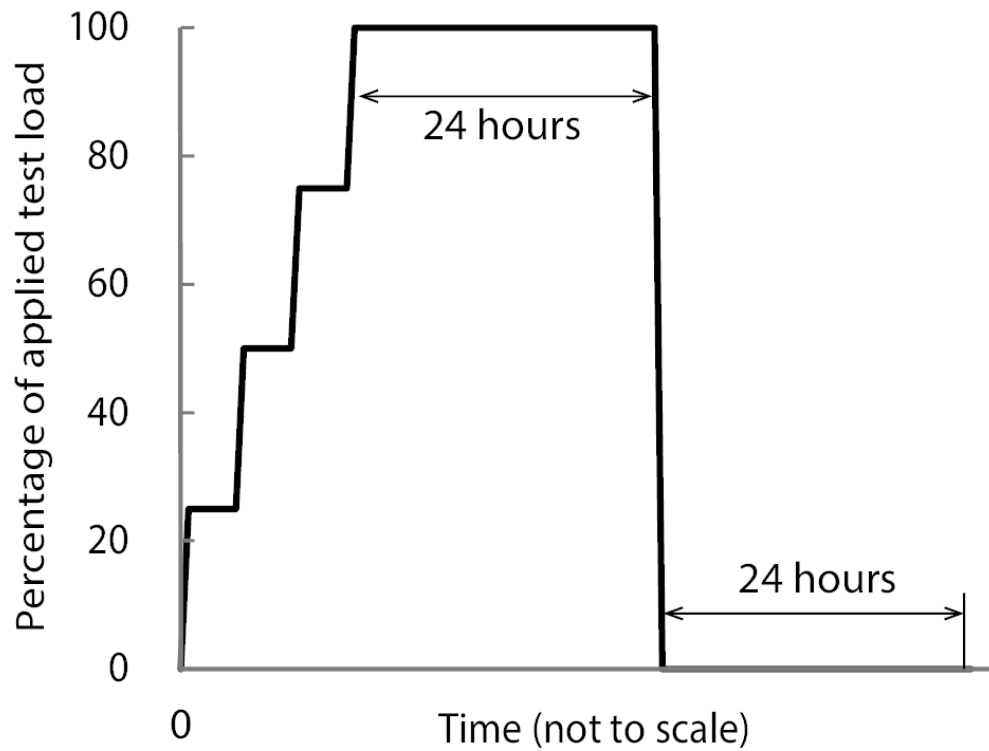


Figure 1: Loading protocol for monotonic load test procedure from ACI 437.2M-13 (ACI Committee 437, 2013).

Acceptance criteria:

Based on deflection limits, where Δ_r is the residual deflection, Δ_l is the maximum deflection and l_t is the span length.

$$\Delta_r \leq \frac{\Delta_l}{4} \quad (1)$$

$$\Delta_l \leq \frac{l_t}{180} \quad (2)$$

The residual deflection requirement is permitted to be waived if the maximum deflection is less than 1.3 mm or $l_t / 2000$.

Cyclic loading protocol.

- Use at least 6 repeated cycles. Cycles A and B: 50% of applied test load or service load level if serviceability is a criterion. Cycles C and D: halfway between load level of cycle A and full applied test load. Cycles E and F: full applied test load.
- Allow $\pm 5\%$ tolerance for the applied load for each load cycle.
- Stabilized deflections (as defined for monotonic loading) must be measured at each load step.
- Hold each load step for at least 2 minutes.

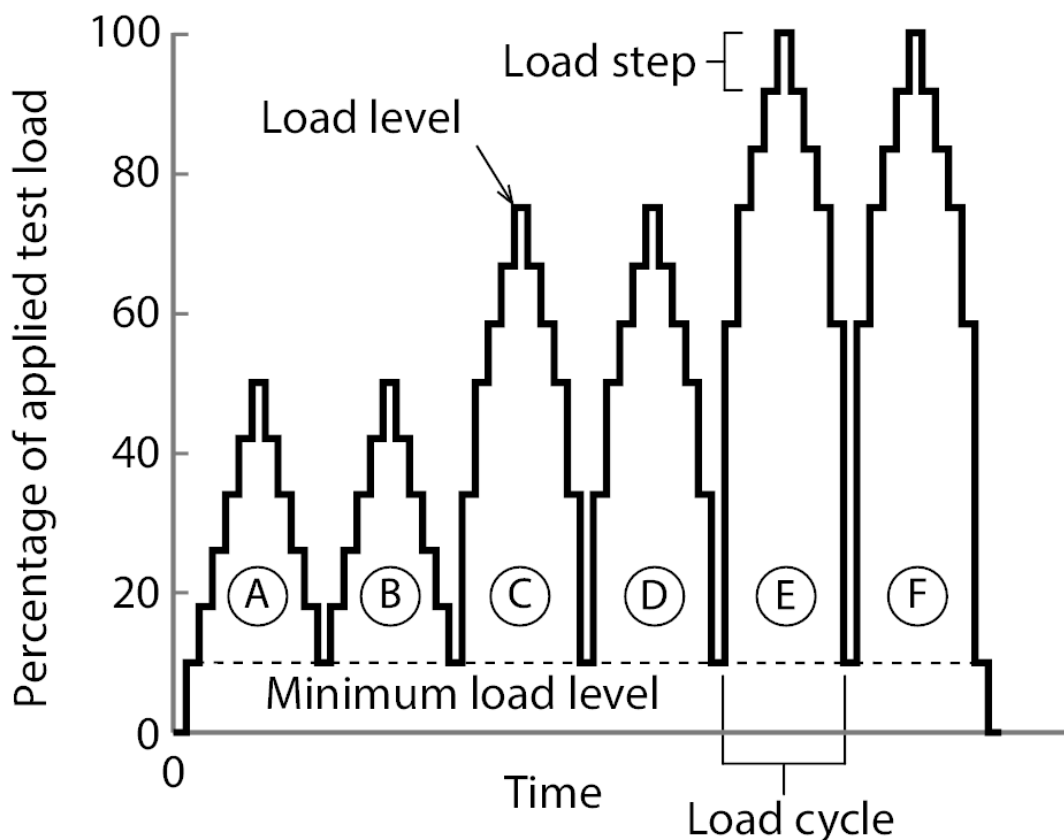


Figure 2: Loading protocol for cyclic load test procedure from ACI 437.2M-13 (ACI Committee 437, 2013).

Acceptance criteria:

The deviation from linearity index (I_{DL}) and permanency ratio (I_{pr}) shall be monitored during the execution of the proof load testing.

$$I_{DL} = 1 - \frac{\tan(\alpha_i)}{\tan(\alpha_{ref})} \leq 0.25 \quad (3)$$

where $\tan(\alpha_i)$ is the secant stiffness of any point i on the increasing loading portion of the load deflection envelope, and $\tan(\alpha_{ref})$ is the slope of the reference secant line for the load deflection envelope (Figure 3).

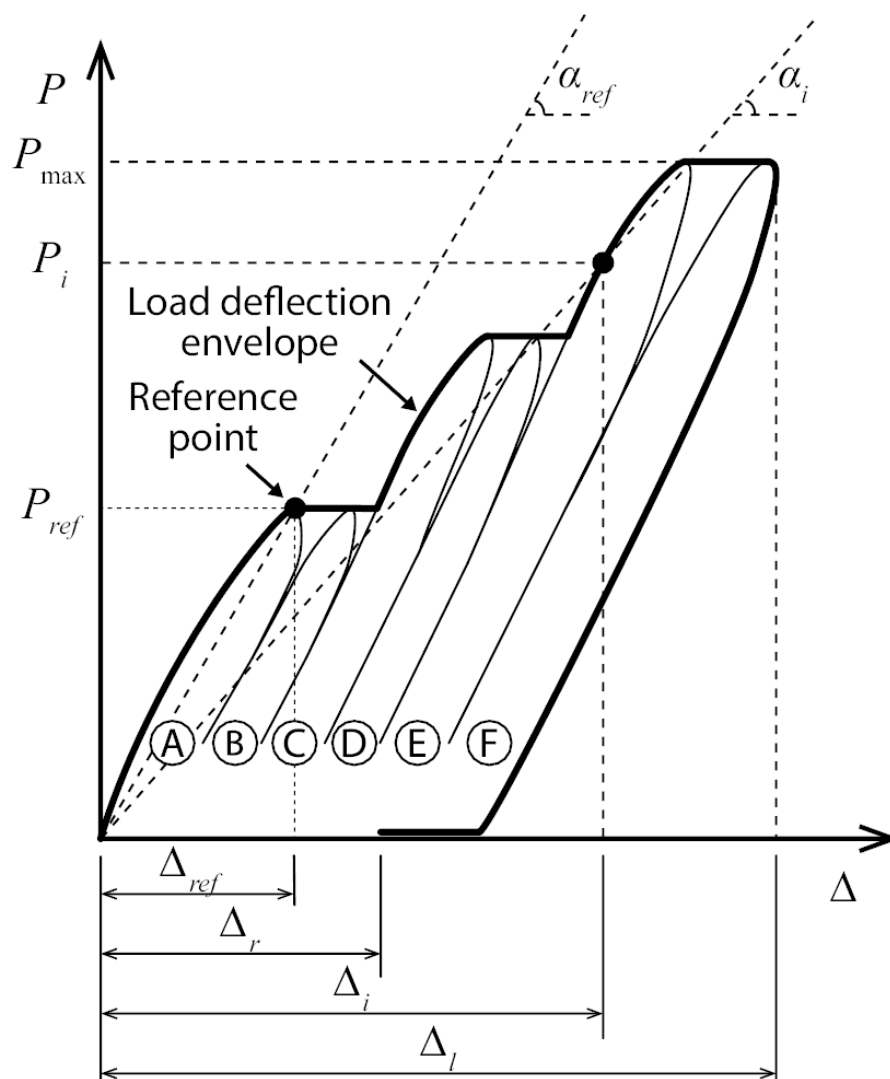


Figure 3: Schematic load-deflection curve for cyclic load test from ACI 437.2M-13 (ACI Committee 437, 2013).

$$I_{pr} = \frac{I_{p(i+1)}}{I_{pi}} \leq 0.5 \quad (4)$$

where I_{pi} and $I_{p(i+1)}$ are the permanency indices calculated for the i -th and $(1+i)$ -th load cycles, at the same load level and deflections as defined in Figure 4.

$$I_{pi} = \frac{\Delta_r^i}{\Delta_{\max}^i} \quad (5)$$

$$I_{p(i+1)} = \frac{\Delta_r^{(i+1)}}{\Delta_{\max}^{(i+1)}} \quad (6)$$

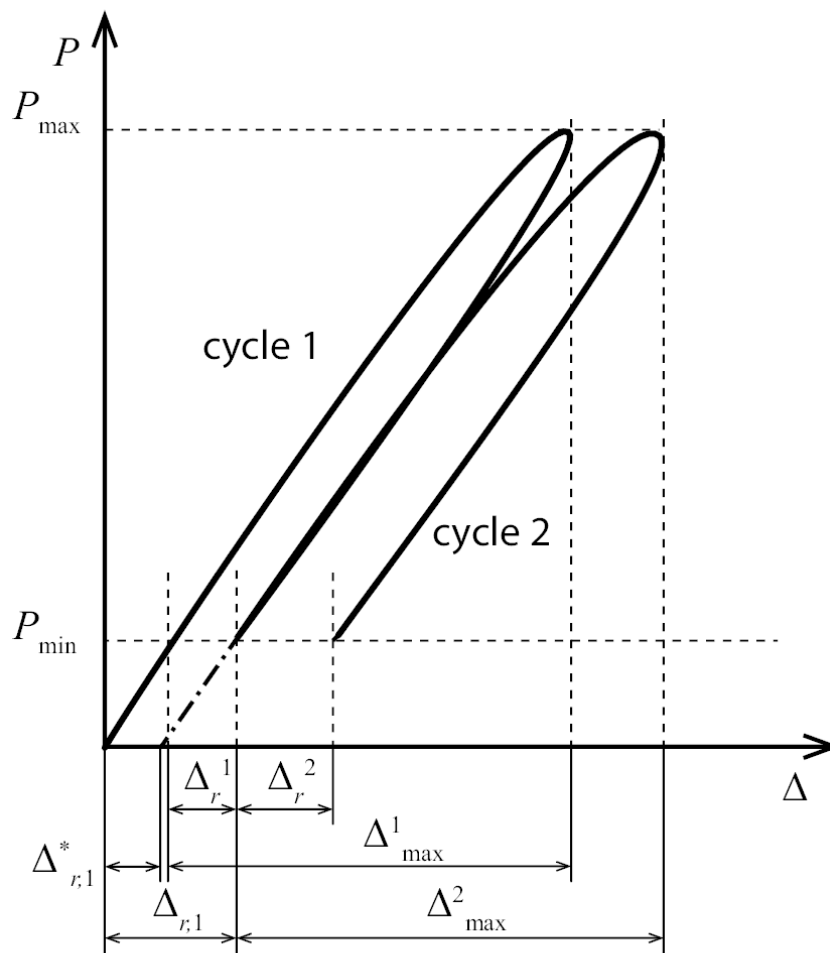


Figure 4: Schematic load-deflection curve for two cycles at same load level from ACI 437.2M-13 (ACI Committee 437, 2013).

The residual deflection (Δ_r), measured at least 24 hours after the removal of the load after the test, shall be considered adequate if Eqn. (1) is fulfilled.

Stop criteria DAfStB

The DAfStB guideline (Deutscher Ausschuss für Stahlbeton, 2000) establishes stop criteria for proof load testing of plain and reinforced concrete structures. Information of the structural system, geometry, material properties and structural capacity shall be available and if not shall be gathered through existing documentation or from measurements or testing of the structure. Execution of visual inspection and destructive and non-destructive tests is required prior the execution of the proof load testing. Qualified personnel are required for the preparation, execution and evaluation of the load test. “The preparation of the load test needs to identify the measurements expected during the test, the effect of changes to the state or system (uncracked/cracked section, effect of temperature), the expected stresses and strains for the actual load and the effect of the load test on the substructure” (Lantsoght, E., 2016, pp. 8). The position of the load shall be selected to cause the most unfavorable scenario and has to be applied in at least 3 steps. Unloading is required at least once after each step.

F_{lim} is the maximum load at which a stop criterion is reached. Further loading would cause permanent damage to the structural system. The concrete strain, reinforcement strain and non-linear deflection criteria aim to maintain the safety of the structure during proof load testing, meanwhile the crack width criterion intends to stop further damage once irreversible damage has occurred. The criteria regarding structural safety are stricter and more important. If any of these criteria are reached the test shall be aborted to prevent failure of the structural system. For the criteria regarding damage, additional loading shall be allowed in order to reach the target load if further damage is allowed by the owner and licensed professional, repairing costs should be considered. (Vos, 2016). Stop criteria from the DAfStB guideline are presented as follows.

Concrete strain.

$$\varepsilon_c < \varepsilon_{c,lim} - \varepsilon_{c0} \quad (7)$$

where ε_c is the strain measured during proof load testing, ε_{c0} is the analytically determined short-term strain due to permanent loads prior to proof load testing and $\varepsilon_{c,lim}$ is a limit value of the concrete strain established as $800 \mu\epsilon$ for $f'_c > 25 \text{ MPa}$.

Reinforcement strain.

$$\varepsilon_{s2} < 0.7 \frac{f_{ym}}{E_s} - \varepsilon_{s02} \quad (8)$$

If the stress-strain curve for the reinforcement steel is known, the following expression is allowed:

$$\varepsilon_{s2} < 0.9 \frac{f_{0.01m}}{E_s} - \varepsilon_{s02} \quad (9)$$

where ε_{s2} is the reinforcement strain during proof load testing, f_{ym} is the average tensile yield strength of the steel, E_s is the modulus of elasticity of the steel, ε_{s02} is the analytically determined strain (cracked condition) caused by permanent loads prior to proof load testing and $f_{0.01m}$ is the average yield strength as for the 0.01% elastic strain limit.

Crack width.

Limits are established for maximum allowable crack width, w , for newly formed cracks and for maximum allowable increase in crack width for existing cracks, Δw .

Table 1: Crack width stop criteria for new and existing cracks from DAfStB guideline (Deutscher Ausschuss für Stahlbeton, 2000)

	During proof load testing	After unloading
Existing cracks	$\Delta w \leq 0.3 \text{ mm}$	$\leq 0.2 \Delta w$
New cracks	$w \leq 0.5 \text{ mm}$	$\leq 0.3 w$

Nonlinear deflection.

Cracked concrete state. Clear increase of non-linear deformation or >10% residual deformation after unloading is not allowed.

FINITE ELEMENT MODELLING FRAMEWORK

The goal of this document is to develop a reliable finite element model with adequate material constitutive models in order to analyze available stop criteria from existing codes and guidelines. The model intends to capture the significant phenomena for analysis with existing stop criteria without excessive computational time. Constitutive models are selected according to Guidelines for Nonlinear Finite Element Analysis of Concrete Structures (Rijkswaterstaat Centre for Infrastructure, 2012) and need to be defined to resemble material behavior and interaction between the materials. Initially a finite element model of a concrete cylinder under uniaxial stress conditions is developed to validate material behavior and constitutive models. Afterwards, the beam finite element model is constructed with the same material constitutive models for concrete. Reinforcement steel properties and interaction between steel and concrete are defined, in addition to boundary conditions and a deflection controlled protocol. An existing experimental study (Lantsoght, Yang, van der Veen, & Bosman, 2016) is selected and beam experiment P804B is verified in terms of strains in order to validate the finite element model. In the following subsections all the ideas presented above are described thoroughly.

FEM Program

Non-linear finite element modelling is performed using a commercial FE package (ABAQUS). The software mentioned includes a large variety of material modelling capabilities including non-linear behavior. In particular, ABAQUS/CAE is used to develop a non-linear finite element model to further investigate stop criteria during proof load testing. Concrete damaged plasticity (CDP) is used to model concrete (Tao & Chen, 2015), the reinforcement steel is modelled as an elastoplastic material, and the interaction between concrete and steel is assumed to be perfect bond.

Material behavior and constitutive models

Concrete.

Concrete is the most widely used material for civil engineering projects and results by mixing cement, aggregate, water and admixtures. Concrete is strong in compression but weak in tension. The compressive stress-strain relationship of concrete is linear elastic until cracking, after which the behavior is non-linear. Once the ultimate compressive strength is reached, the stress decreases while the strain continues to increase. In uniaxial tension the behavior of concrete is linear elastic until the tensile strength is reached and cracking develops, beyond this point a stress-strain softening behavior is expected. The compressive and tensile stress-strain relationships are modelled, according to the Rijkswaterstaat Centre for Infrastructure (2012), by a parabolic stress-strain diagram with softening branch and an exponential softening diagram, respectively.

There are several available mathematical models to represent the physical behavior of concrete. The ABAQUS package includes the concrete damaged plasticity (CDP) to assess the response of concrete in both tension and compression. “The model is a continuum, plasticity-based, damage model for concrete. It assumes that the main two failure mechanisms are tensile cracking and compressive crushing of the concrete material” (Simulia, 2014). Concrete’s compressive and tensile response is represented by scalar (isotropic) stiffness degradation variables. The parameters required for the CDP model are density, plasticity variables, elastic properties (modulus of elasticity and Poisson’s ratio) and stress- inelastic strain relationships as tabular functions. Annex A contains MATLAB scripts, developed for the present work, which calculate material properties and constitutive models of concrete, as tabular functions, for input in ABAQUS/CAE.

Material properties.

The material properties shown below are determined according to the Guidelines for Nonlinear Finite Element Analysis of Concrete Structures (Rijkswaterstaat Centre for Infrastructure, 2012) and Model Code 2010 (fib, 2013). The density of concrete ($\rho = 2429.6 \text{ kg / m}^3$) and cube compressive strength at 28 days ($f_{c,cyl,m} = 63.51 \text{ MPa}$) are selected from the experimental study under consideration.

1. Cube compressive strength

$$f_{c,cyl,m} = 63.51 \text{ MPa}$$

2. Cylinder strength

$$f_{cm} = 0.82(f_{c,cyl,m})$$

$$f_{cm} = 0.82(63.51 \text{ MPa})$$

$$f_{cm} = 52 \text{ MPa}$$

3. Classification by strength

C40 (Normal strength concrete-NSC $f_{ck} \leq 50 \text{ MPa}$)

4. Classification by density

Normal weight concrete ($2000 \text{ kg / m}^3 < \rho < 2600 \text{ kg / m}^3$)

5. Compressive strength

a. Characteristic compressive strength

$$f_{ck} = 44 \text{ MPa}$$

b. Mean compressive strength

$$f_{cm} = f_{ck} + \Delta f$$

$$f_{cm} = 44 \text{ MPa} + 8 \text{ MPa}$$

$$f_{cm} = 52 \text{ MPa}$$

6. Tensile strength

a. Mean tensile strength

$$f_{ctm} = 0.3(f_{ck})^{2/3}$$

$$f_{ctm} = 0.3(44 \text{ MPa})^{2/3}$$

$$f_{ctm} = 3.74 \text{ MPa}$$

b. Lower bound of the characteristic tensile strength

$$f_{ctk.min} = 0.7 f_{ctm}$$

$$f_{ctk.min} = 0.7(3.74 \text{ MPa})$$

$$f_{ctk.min} = 2.62 \text{ MPa}$$

c. Upper bound of the characteristic tensile strength

$$f_{ctk.min} = 1.3 f_{ctm}$$

$$f_{ctk.min} = 1.3(3.74 \text{ MPa})$$

$$f_{ctk.min} = 4.86 \text{ MPa}$$

7. Fracture energy

$$G_F = 73(f_{cm})^{0.18}$$

$$G_F = 73(52 \text{ MPa})^{0.18}$$

$$G_F = 148.66 \text{ N / m}$$

8. Compressive fracture energy (Nakamura & Higai, 2001)

$$G_C = 250G_F$$

$$G_C = 250(148.66 \text{ N / m})$$

$$G_C = 37165.80 \text{ N / m}$$

9. Modulus of elasticity at 28 days

$$E_{ci} = E_{c0} \alpha_E \left(\frac{f_{cm}}{10} \right)^{1/3}$$

Where $E_{c0} = 21.5 \times 10^3$ MPa, and $\alpha_E = 1.0$ for quartzite aggregates

$$E_{ci} = 21.5 \times 10^3 \text{ MPa} \left(\frac{52 \text{ MPa}}{10} \right)^{1/3}$$

$$E_{ci} = 37248.28 \text{ MPa}$$

10. Poisson's ratio

$$\nu = 0.15$$

Compressive behavior.

The compressive stress-strain relationship, according to Rijkswaterstaat Centre for Infrastructure (2012), is modeled as a parabolic stress-strain diagram with softening branch (Figure 5) and calculated with Eqn. (10). The parameters are characteristic compressive strength (f_{ck}), modulus of elasticity (E_{ci}), compressive fracture energy G_C and equivalent length (h). The equivalent length for quadrilateral elements can be approached by $h = 2a$ where a is the element size. Initially it is assumed $h=100$ mm which has to be checked a posteriori when the mesh has been structured.

$$f(x) = \begin{cases} -f_{ck} \frac{1}{3} \frac{\alpha_j}{\alpha_{c/3}}, & \alpha_{c/3} < \alpha_j \leq 0 \\ -f_{ck} \frac{1}{3} \left[1 + 4 \left(\frac{\alpha_j - \alpha_{c/3}}{\alpha_c - \alpha_{c/3}} \right) - 2 \left(\frac{\alpha_j - \alpha_{c/3}}{\alpha_c - \alpha_{c/3}} \right)^2 \right], & \alpha_c < \alpha_j \leq \alpha_{c/3} \\ -f_{ck} \left[1 - \left(\frac{\alpha_j - \alpha_c}{\alpha_u - \alpha_c} \right)^2 \right], & \alpha_u < \alpha_j \leq \alpha_c \\ 0, & \alpha_j \leq \alpha_u \end{cases} \quad (10)$$

where f is the compressive stress, α_j is the compressive strain and $\alpha_{c/3}$, α_c , and α_u are given by the following expressions:

$$\alpha_{c/3} = -\frac{1}{3} \frac{f_{ck}}{E_{ci}}$$

$$\alpha_{c/3} = -\frac{1}{3} \frac{(44 \text{ MPa})}{(37248.28 \text{ MPa})}$$

$$\alpha_{c/3} = -3.94 \times 10^{-4}$$

$$\alpha_c = 5\alpha_{c/3}$$

$$\alpha_c = 5(-3.94 \times 10^{-4})$$

$$\alpha_c = -1.97 \times 10^{-3}$$

$$\alpha_u = \alpha_c - \frac{3 G_C}{2 h f_{ck}}$$

$$\alpha_u = -1.97 \times 10^{-3} - \frac{3 (37165.80 \text{ N/m})}{2 (0.2 \text{ m})(44 \text{ MPa})}$$

$$\alpha_u = -1.46 \times 10^{-2}$$

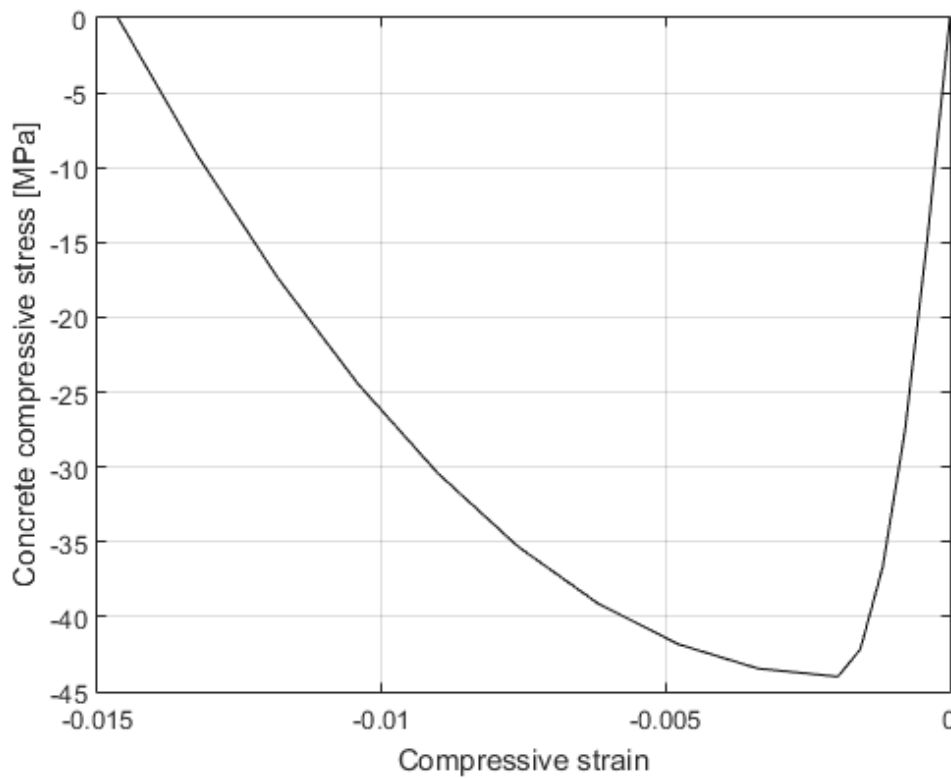


Figure 5: Compressive stress-strain relation of concrete (parabolic stress-strain diagram with softening branch).

Tensile behavior.

The tensile stress-inelastic strain relationship, according to Rijkswaterstaat Centre for Infrastructure (2012), is recommended to be modeled using Hordijk relation (1991), an exponential softening diagram (Figure 6). The parameters are mean tensile strength (f_{ctm}), critical crack width (w_c) and constants $C_1 = 3.0$ and $C_2 = 6.93$.

$$\sigma = \begin{cases} f_{ctm} \left(1 + \left(C_1 \frac{w}{w_c} \right)^3 \right) e^{-C_2 \frac{w}{w_c}} - \frac{w}{w_c} (1 + C_1^3) e^{-C_2}, & 0 \leq w \leq w_c \\ 0, & w > w_c \end{cases} \quad (11)$$

The critical crack width (w_c) depends on the fracture energy (G_F) and the mean tensile strength (f_{ctm}), and is given by the following expression:

$$w_c = 5.14 \frac{G_F}{f_{ctm}} \quad (12)$$

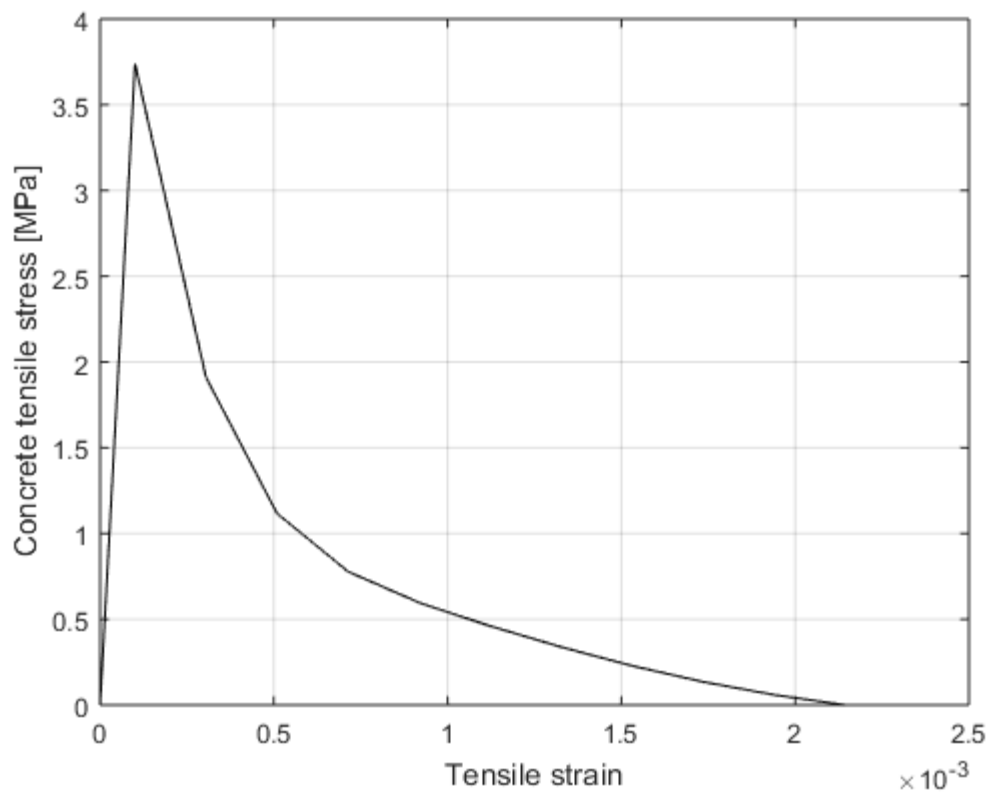


Figure 6: Tensile stress-strain relation of concrete (exponential softening diagram-Hordijk relation).

Plasticity variables.

The plasticity parameters are dilation angle, flow potential eccentricity, ratio of initial equibiaxial compressive yield stress to initial uniaxial compressive stress (f_{b0}/f_{c0}), the ratio of the second stress invariant on the tensile meridian to that on the compressive meridian (K), and viscosity parameter. The dilation angle is set to 56° and ABAQUS documentation suggests default values for the last four parameters and were set to 0.1, 1.16, 0.667, and 0.0001 (Simulia, 2014).

Reinforcement steel.

In reinforced concrete the reinforcement steel carries tensile stresses transferred from concrete, significantly increasing the tensile capacity of a given cross-section. The physical behavior of the stress-strain relationship is linear until the yield stress is reached. From this point onwards the behavior is nonlinear, at ultimate stress the reinforcement begins to neck reducing the cross-section and the load bearing capacity while the strain continues to increase until ultimate strain is reached and load bearing capacity is lost. The reinforcement steel used for the experimental study are plain soft steel bars with a yield strength of $f_y = 296.8$ MPa and ultimate stress of $f_u = 425.9$ MPa. An elastic-plastic constitutive relationship is considered for modelling the reinforcement steel (Figure 7). The required parameters are the modulus of elasticity ($E_s = 200$ GPa), Poisson's ratio ($\nu = 0.3$), and yield strength.

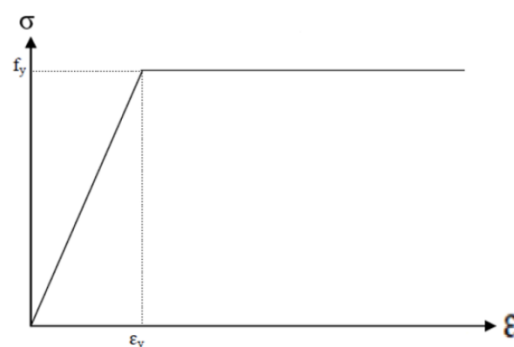


Figure 7: Idealized stress-strain relationship of reinforcement steel.

Interface between concrete and reinforcement steel.

Perfect bond between concrete and steel is assumed. Reinforcement steel is modelled as an embedded region inside the concrete.

Geometry and setup

Figure 8 displays the beam cross-section and reinforcement layout. The beam cross-section is 800 mm x 300 mm with 2 layers of 3 ϕ 20 mm @ 115 mm o.c as tensile reinforcement and 25 mm as cover. The area of steel is $A_s = 1885 \text{ mm}^2$ and effective depth is $d = 755 \text{ mm}$, so the reinforcement ratio is $\rho = 0.83\%$.

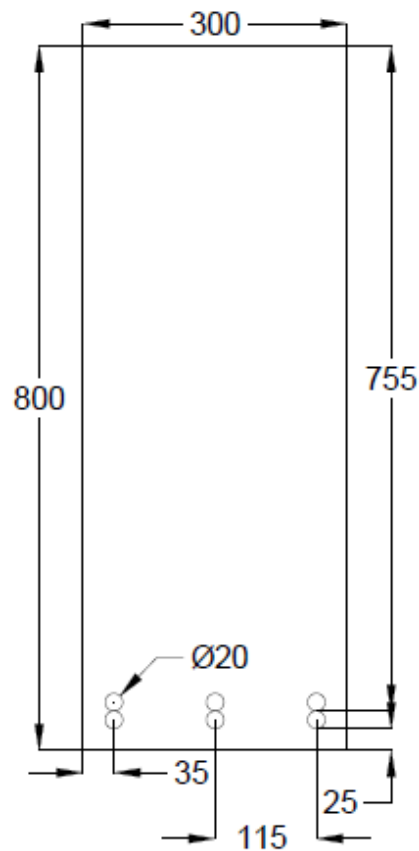


Figure 8: Beam cross-section and reinforcement layout (mm).

A general overview of the test setup is shown in Figure 9. The total length of the beam is 10000 mm and the clear span length is 8000 mm. A concentrated load is applied at a distance a from the support. The support and loading plates are 100 mm wide. The value of a for beam experiment P804B is 2500 mm.

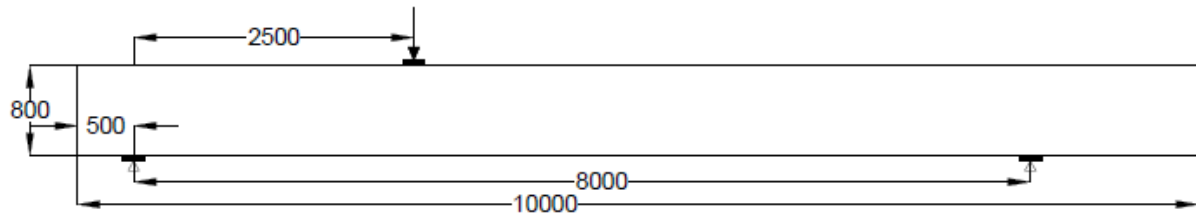


Figure 9: Test setup (mm).

Structural supports are represented by boundary conditions by specifying the values of displacement and rotation at the corresponding nodes. Pinned support boundary conditions ($U_1=0$, $U_2=0$ and $U_3=0$) are applied to the support plates. Tied contact is specified as the interaction between the mesh of the support plates and the mesh of the concrete beam. This interaction allows that the meshes on both surfaces have the same displacement. Figure 10 exhibits the setup of the overall assembly of the model on ABAQUS/CAE; boundary conditions, applied loading and partitions for meshing purposes are displayed as well.

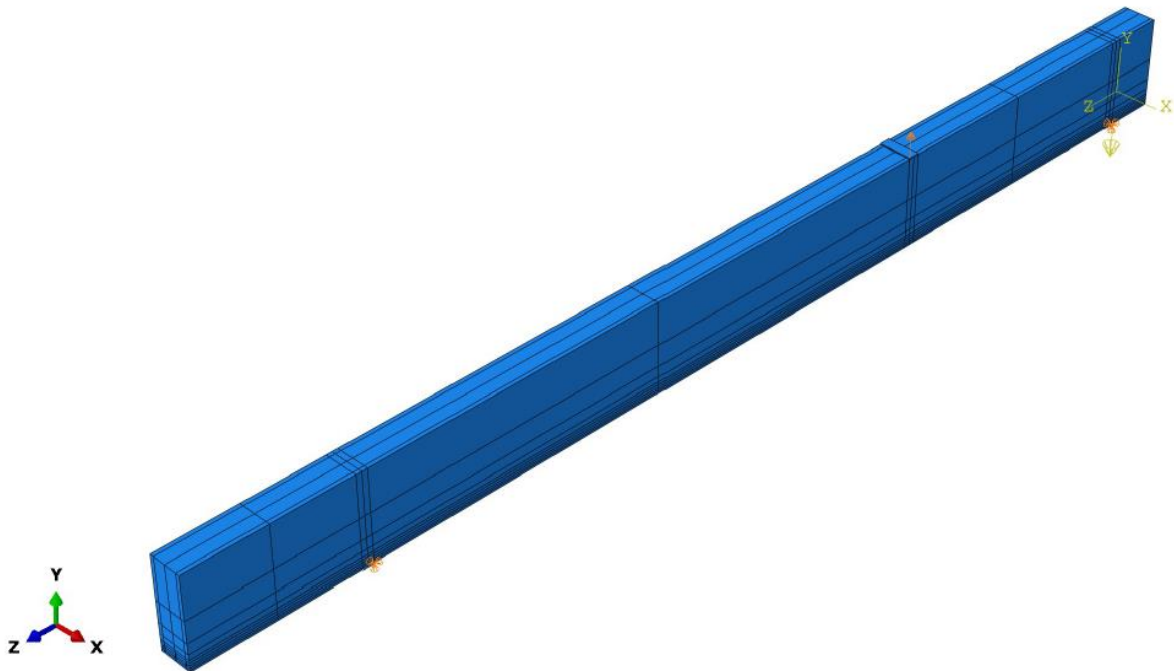


Figure 10: FEM assembly and boundary conditions

Mesh controls are assigned in order to verify the suitability and detect warnings. C3D8R elements are selected for the mesh of the model, which are general purpose brick elements with 1 integration point (MIT, 2014).

Deflection controlled protocol

Deflection is applied to the beam along the loading plate. The interaction between the loading plate and the concrete beam is tied contact as well. The deflection applied to the model corresponds to the deflection measured for beam experiment P804B (Lantsoght, et al., 2016) which is shown in Figure 11.

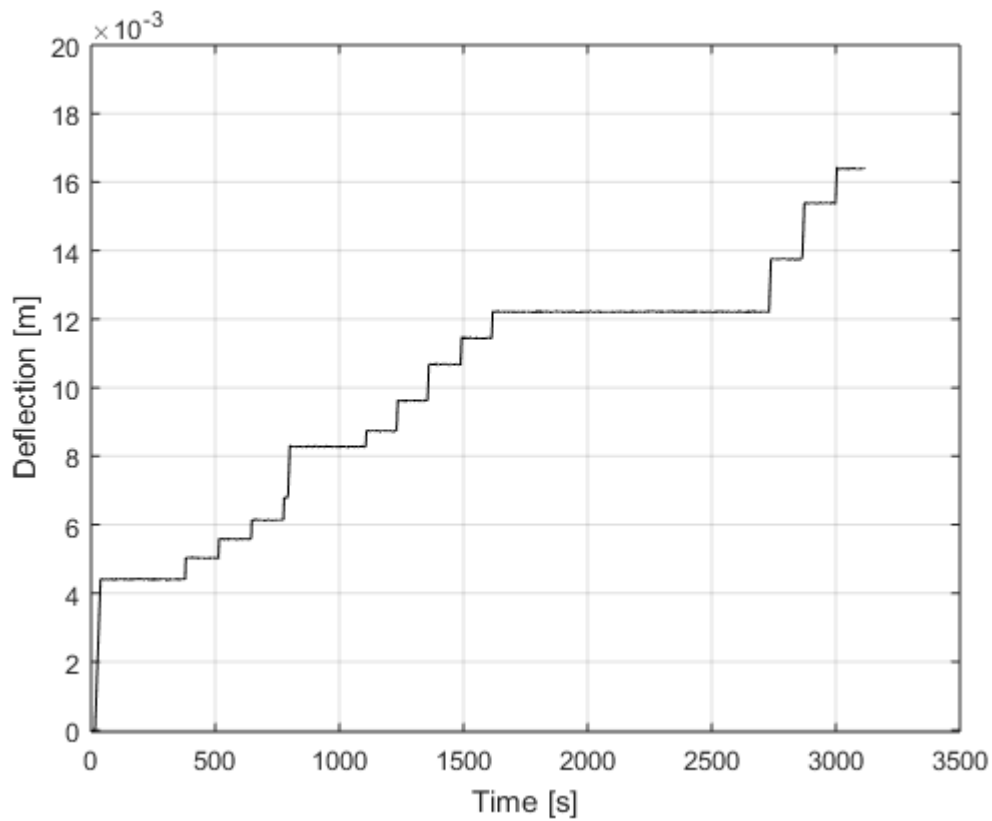


Figure 11: Deflection controlled protocol as measured by Lantsoght, Yang, van der Veen, & Bosman (2016).

ANALYSIS AND RESULTS

Validation of material constitutive models

A 6"x12" concrete cylinder is modelled in ABAQUS/CAE in order to verify material constitutive models. The dimensions of the cylinder are selected according to ASTM C31-12 (ASTM International, 2012). Material properties and constitutive models, described in the previous section, are imported directly from .txt files generated by MATLAB scripts shown in Annex A. Uniform displacement is gradually applied on the top and bottom of the concrete cylinder, as shown in Figure 12. Figure 13 displays the tensile stress-strain results exported from ABAQUS/CAE, meanwhile Figure 14 provides insight on the adequacy of the model by displaying both the tensile constitutive model and FEM results. Similarly, Figure 15 and Figure 16, allow to present the same deduction about the adequacy of the cylinder model regarding the compressive behavior of concrete.

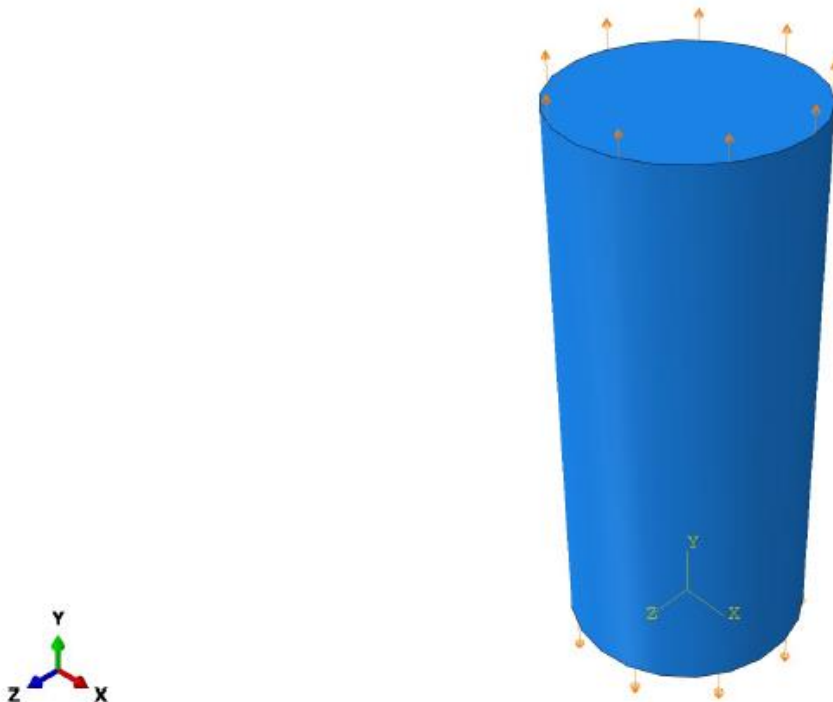


Figure 12: Concrete cylinder model setup.

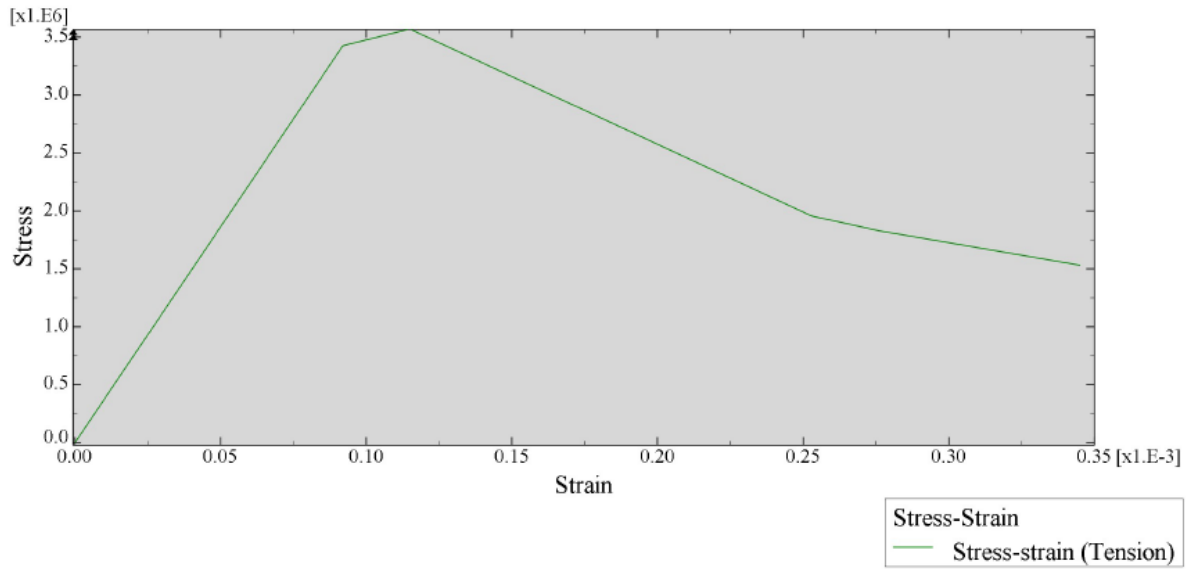


Figure 13: Tensile stress-strain diagram for concrete cylinder model.

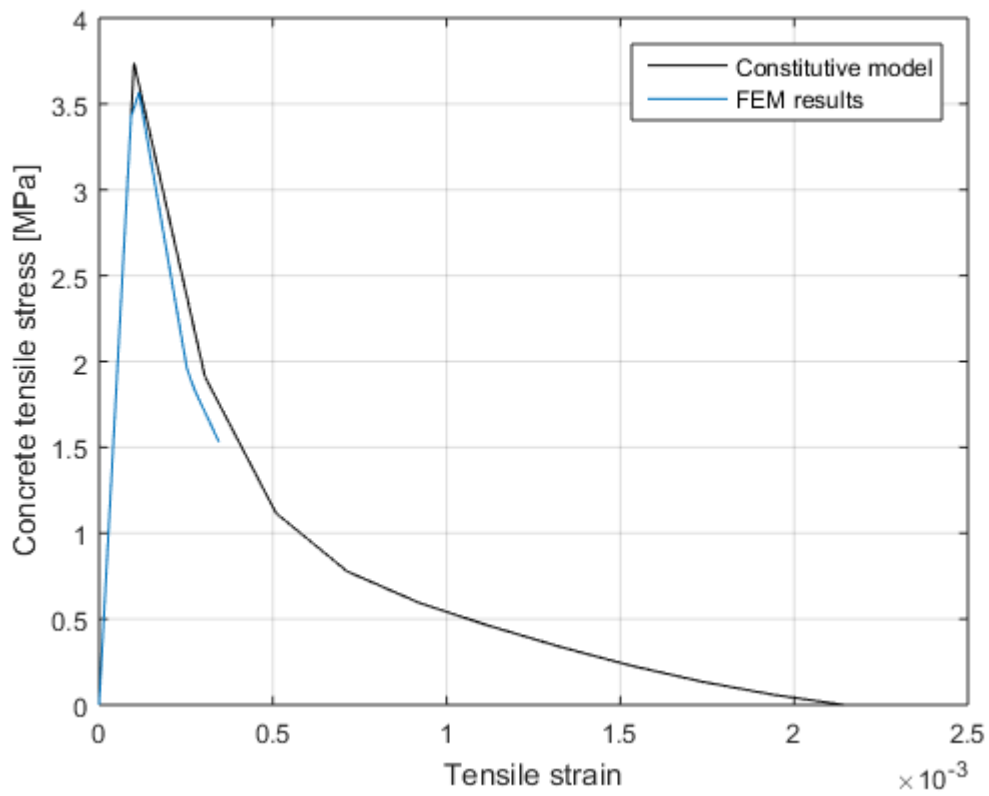


Figure 14: Constitutive models vs FEM results (Tensile stress-strain diagrams).

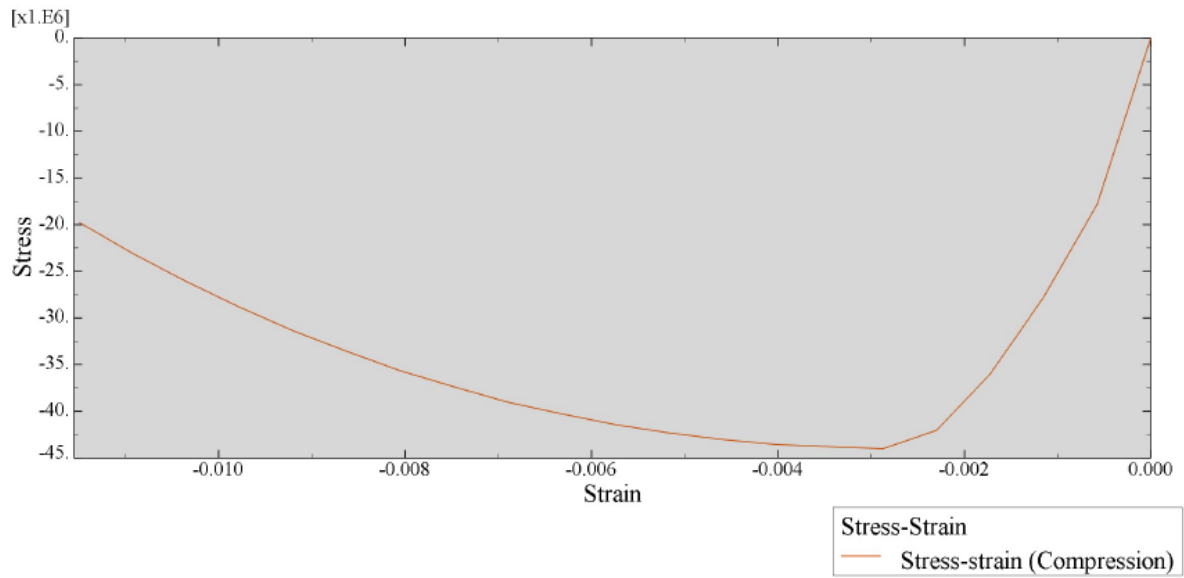


Figure 15: Compressive stress-strain diagram for concrete cylinder model.

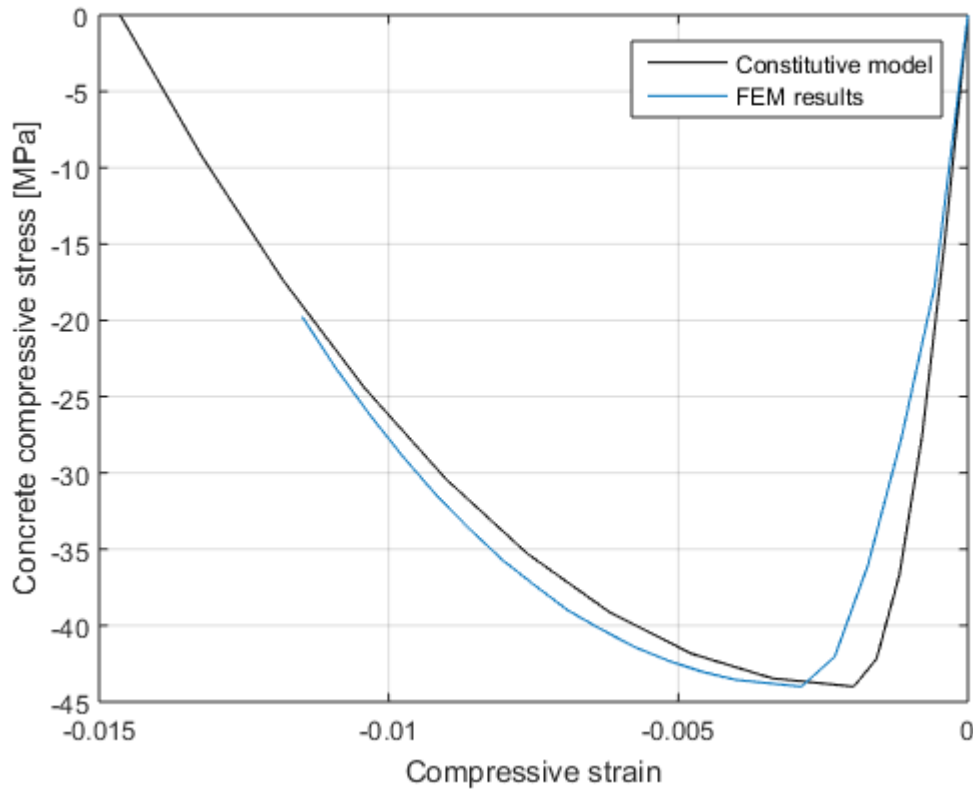


Figure 16: Constitutive models vs FEM results (Compressive stress-strain diagrams).

Validation of beam model with experimental work

Figure 17 displays the strain results on the bottom of the cross-section at the point of application of loading exported from ABAQUS/CAE. In the experiment, the measurements are zeroed at the start of the test. Meaning that the strain measurements do not include the strain caused by permanent loads, while the results from the beam model do include this strain. An additional analysis of the beam model considering only gravity loads allows to determine the strain caused by permanent loads, being $39 \mu\epsilon$. In order to compare the strain results from the beam model with the strain measurements from the experimental work, the strain caused by proof load testing for the beam model is obtained by subtracting the strain due to permanent loads from the strain results. Figure 18 shows the strain caused by proof load testing and the strain measurements from the experimental study by Lantsoght, et al. (2016). The comparison of both allows to determine that the beam model yields adequate results. In fact, the resemblance for the first load level is evident.

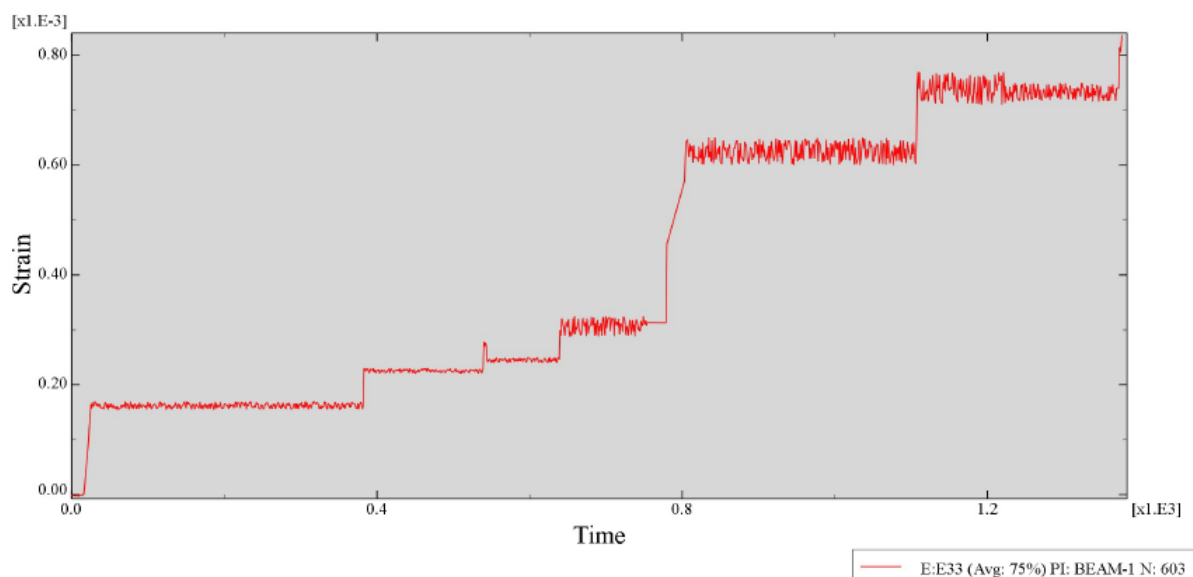


Figure 17: Strain results from beam model

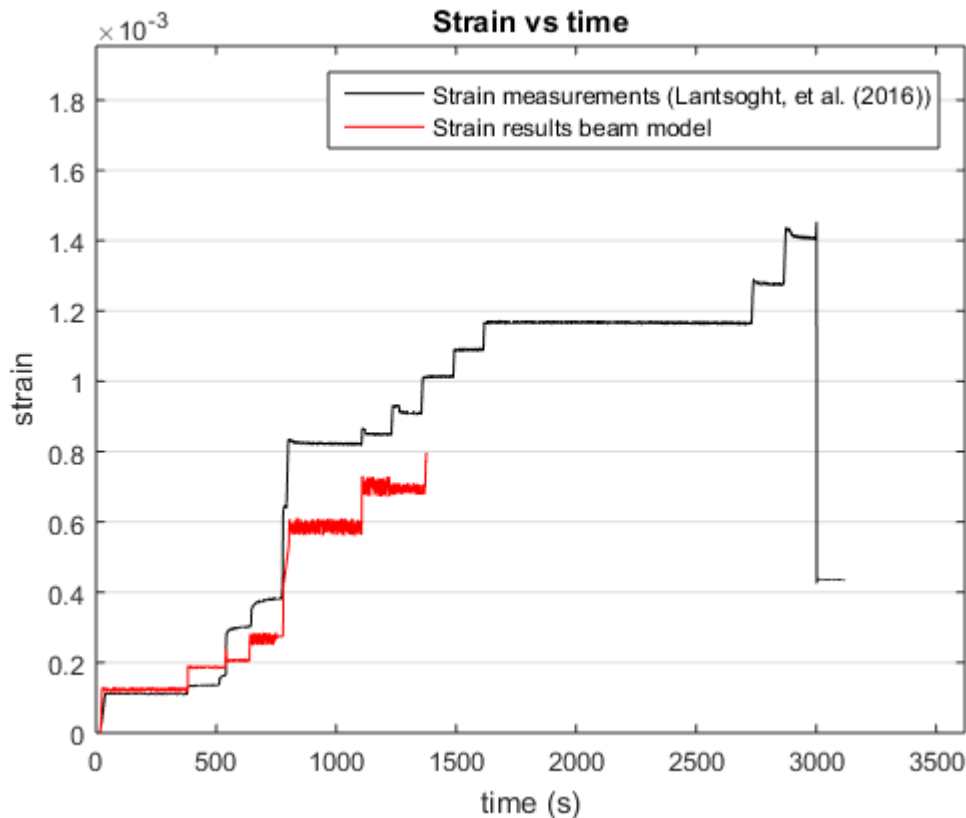


Figure 18: Strains on beam experiment P804B as measured by Lantsoght, Yang, van der Veen, & Bosman (2016) and strains caused by proof load testing.

Stop criteria analysis

ACI 437.2M-13 acceptance criteria for monotonic loading protocols is based uniquely on residual deflection, which can only be assessed on an experiment. Further work on beam models with cyclic loading protocol is suggested in order to analyze additional criteria from ACI 437.2M-13. Similarly, for the DAfStB guideline, the nonlinear deflection criteria are not taken into consideration for the same reason explained above. The beam model developed in ABAQUS/CAE software does not yield crack width results directly and crack width criteria provide serviceability information only; therefore, stop criteria regarding this variable are beyond the scope of the present document. It is necessary to remove the concrete cover in order to measure reinforcement strain in a proof load test, consequently the reinforcement strain criterion is not considered because in engineering practice such damage may not be allowed for the structural system under consideration (Lantsoght, et al., 2016).

The DAfStB maximum allowable concrete strain criterion is analyzed based on the strain caused by proof load testing, i.e. the strain results minus the strain caused by permanent loads for the beam model. Eqn. (7) allows to calculate the maximum allowable concrete strain according to the DAfStB guideline. The concrete strain due to self-weight obtained from the finite element model is $39 \mu\epsilon$, whereas the value calculated by Lantsoght, et al. (2017) is $33 \mu\epsilon$. Consequently, the maximum allowable concrete strains are $761 \mu\epsilon$ and $767 \mu\epsilon$, for the beam model and experiment, respectively. Table 2 displays the concrete strain criterion limits in parentheses, the strains and corresponding loads for which the criterion was exceeded and the maximum loads and concrete strains registered at failure, for the beam model and experiment.

Table 2: Concrete strain criterion results

	DAfStB concrete strain criterion			ϵ_{\max} [$\mu\epsilon$]	Max load [kN]
	ϵ_{c0} [$\mu\epsilon$]	ϵ_c [$\mu\epsilon$]	Load [kN]		
Beam model	39	775 (761)	138.5	797	139.7
Experiment P804B	33	784 (767)	111	1453	196

The maximum allowable concrete strain for the beam model is surpassed at 1373 seconds. Figure 19 shows the loading protocol of experiment P804B as measured by Lantsoght, et al. (2017), the corresponding load to the mentioned time step is 138.5 kN, The results from beam experiment P804B show that the concrete strain criterion was exceeded by a 111 kN load (Lantsoght, et al., 2016), yielding a 25% difference with the load obtained from the beam model results. The strain at failure obtained from the beam model is $797 \mu\epsilon$, whereas the maximum strain measured by Lantsoght, et al. (2017) is much greater at $1453 \mu\epsilon$; the difference between the maximum loads is large as well. Regardless, the maximum strain ($797 \mu\epsilon$) is almost equal to the limit value of the concrete strain ($800 \mu\epsilon$). Additionally, the maximum strain and load from the beam model results allows to determine that the limit value from the DAfStB guideline is adequate to prevent irreversible damage despite the fact that the experiment results show that the beam was able to bear substantial additional loading.

Furthermore, Benítez (2017) established a maximum allowable concrete strain of $863 \mu\epsilon$ for beam experiment P804B which yields only an 8% difference with the maximum strain obtained from the beam model.

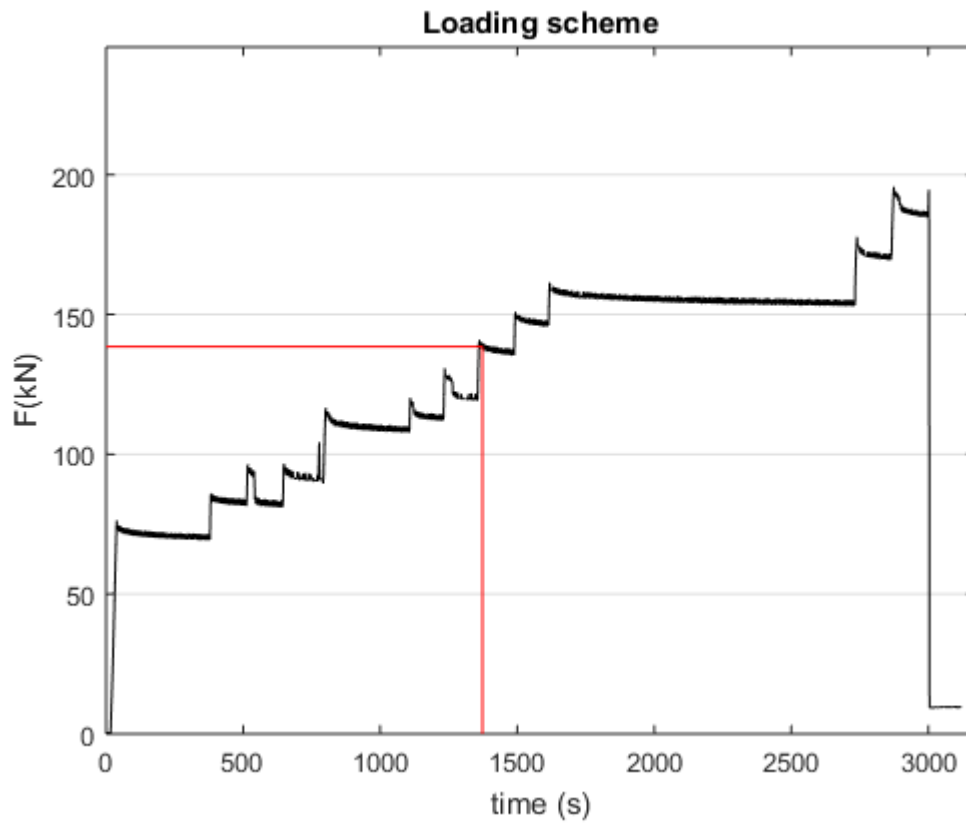


Figure 19: Loading protocol P804B as measured by Lantsoght, Yang, van der Veen, & Bosman (2016).

FURTHER WORK

Further research is recommended on cyclic protocol acceptance criteria from ACI 437.2M-13. The beam model presented above combined with a cyclic protocol can be developed for analyzing the deviation from linearity index and permanency ratio. It is important to be aware that cyclic loading protocols usually span over a larger period of time than monotonic loading protocols, therefore the computational time for such model would increase as well. Also the load-deflection envelope needs to be determined from the beam model results in order to analyze the acceptance criteria mentioned above. Regarding the DAfStB guideline, the reinforcement strain criterion has proven to be difficult to apply. Even though reinforcement strain results from the beam model can be obtained but experimental data regarding this variable is problematic because the removal of the concrete cover may not be allowed. Nonlinear deflection and residual deflection, from the DAfStB guideline and ACI 437.2M-13, respectively, need to be assessed experimentally.

The beam model represents a starting point for finite element analysis for stop criteria. The calculation of concrete properties and constitutive models for any concrete compressive strength can be rapidly obtained with the MATLAB scripts presented on Annex A. Material properties, cross-section dimensions, reinforcement quantities and boundary conditions can be modified and adjusted in order to run additional numerical experiments to analyze structural systems under different conditions. Model refinement is needed for the actual assessment of existing bridges instead of beam experiments. The addition of more detailed interaction models between concrete and reinforcement is suggested in order to get closer to the actual behavior of the structure. Existing flexural cracks could be included as well to capture the real condition of the cross-section under study.

SUMMARY AND CONCLUSIONS

Proof load testing is an experimental technique to evaluate existing reinforced concrete structures that are close to reach their lifespan and the capacity cannot be determined precisely due to several factors such as material degradation, activation of additional load bearing mechanisms or lack of relevant information about the structural system. Proof load testing allows to increase the lifespan and demonstrate capacity of a structure. The target load should be as high as possible to gain insight but should not be too high as the integrity of the structure may be compromised. Measurements are recorded during testing and are compared to defined limits, called stop criteria, in order to avoid permanent damage to the structure during proof load testing. Currently ACI 437.2M-13 and the DAfStB guideline provide the most detailed test methods and stop criteria parameters.

Material constitutive models were proven to be adequate and results demonstrate that the assumption of perfect bond between concrete and steel reinforcement was appropriate for the structural system under consideration. Additional criteria from the guidelines are not analyzed for the reasons stated on the preceding section, but further research is recommended on cyclic loading protocols to evaluate the deviation from linearity index and permanency ratio from ACI 437.2M-13. Also numerous further adjustments are needed for the actual assessment of existing bridges instead of beam experiments. Existing cracks should be modeled and in-depth interaction models between concrete and reinforcement should be included. Nonlinear deflection and residual deflection, from the DAfStB guideline and ACI 437.2M-13, respectively, need to be assessed experimentally. Nevertheless, the goal was reached, which was to provide adequate results for the analysis of existing criteria, without excessive computational time, which was around 3 hours.

The finite element model contained in the present document pursues to analyze stop criteria from the aforementioned guidelines and numerical results are verified with experimental work. The results obtained from the maximum allowable strain criterion prove the adequacy of the finite element model presented. The maximum strain value at failure ($797 \mu\epsilon$) and the maximum allowable concrete strain determined from the guideline ($800 \mu\epsilon$) are almost the same, which reveals that failure of the beam model occurred at a strain value that could have been expected but still experimental results show that additional load was carried by the beam after this strain was reached. The conclusion reached, by analyzing the maximum allowable concrete strain criterion from the DAfStB guideline, is that the limit value is adequate and the criterion has proven to be a worthy indicator in order to avoid irreversible damage during proof load testing.

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ANNEX A: MATLAB SCRIPTS FOR CONCRETE PROPERTIES AND CONSTITUTIVE MODELS

```

% Concrete Properties Model Code 2010

% Jose Eduardo Paredes
% Universidad San Francisco de Quito
% Trabajo de Titulacion
% Nonlinear FEM analysis of beam experiments for stop criteria

% Calculates concrete mechanical properties according to Model Code 2010

% fccylm cube compressive strength [MPa]
% deltaf (8 MPa)
% Ec0 (21500 MPa)
% alphaE (1.0 for quartzite aggregates)
% fck characteristic compressive strength [MPa]
% fcm mean compressive strength [MPa]
% fctm mean tensile strength [MPa]
% fctkmin lower bound of the characteristic tensile strength [MPa]
% fctkmax upper bound of the characteristic tensile strength [MPa]
% GF fracture energy [N/m]
% GC compressive fracture energy [N/m]
% Eci modulus of elasticity at 28 days [MPa]

function [fck,fctm,GF,GC,Eci] = concreteProperties(fccylm)

deltaf = 8;
Ec0 = 21.5E3;
alphaE = 1.0;
fck = round(0.82*(fccylm)) - deltaf;
fcm = fck + deltaf;
fctm = 0.3*fck^(2/3);
fctkmin = 0.7*fctm;
fctkmax = 1.3*fctm;
GF = 73*fcm^0.18;
GC = 250*GF;
Eci = Ec0*alphaE*(fcm/10)^(1/3);

```

```

% Compressive behavior Constitutive Model

% Jose Eduardo Paredes
% Universidad San Francisco de Quito
% Trabajo de Titulacion
% Nonlinear FEM analysis of beam experiments for stop criteria

% Exports .txt file (compressive stress-inelastic strain relation) for
% input in Abaqus CAE model.
% .txt file units: stress [Pa]

% fck      characteristic compressive strength [MPa]
% f        concrete compressive stress [MPa]
% Eci      modulus of elasticity [MPa]
% GC       compressive fracture energy [N/m]
% alpha    compressive strain (negative)
% heq      finite element equivalent length (2a) [m]

function [B] = concreteCompressive(fck,Eci,GC,heq)

% Constitutive compressive behavior
alphac3 = -fck/(3*Eci);
alphac = 5*alphac3;
alphau = alphac - 3*GC/(2*heq*fck*1E+6);
alphaj1 = [0 alphac3];
alphaj2 = alphac3 + (alphac-alphac3)/4:(alphac-alphac3)/4:alphac;
alphaj3 = alphac + (alphau-alphac)/9:(alphau-alphac)/9:alphau;

for i = 1:1:size(alphaj1,2)
    f1(i) = - fck*alphaj1(i)/(3*alphac3);
end
for j = 1:1:size(alphaj2,2)
    f2(j) = - fck*(1+4*(alphaj2(j)-alphac3)/(alphac-alphac3)...
        -2*((alphaj2(j)-alphac3)/(alphac-alphac3))^2)/3;
end
for k = 1:1:size(alphaj3,2)
    f3(k) = -fck*(1-((alphaj3(k)-alphac)/(alphau-alphac))^2);
end
alphaj = [alphaj1';alphaj2';alphaj3'];
f = [f1';f2';f3'];

% Stress-strain Plot
plot(alphaj,f,'k')
grid on
box on
xlabel('Compressive strain')
ylabel('Concrete compressive stress [MPa]')

% Write .txt file
f = [f1(1,2);f2';f3'];
alphaj = [0;alphaj2' - alphaj1(1,2);alphaj3' - alphaj1(1,2)];
B = [-f*1E+6,-alphaj];
dlmwrite('compressive.txt',B,'delimiter','\t','precision',10)

```

```

% Tensile behavior Constitutive Model

% Jose Eduardo Paredes
% Universidad San Francisco de Quito
% Trabajo de Titulacion
% Nonlinear FEM analysis of beam experiments for stop criteria

% Exports .txt file (tensile stress-inelastic strain relation) for input in
% Abaqus CAE model.
% .txt file units: crack width [m], stress [Pa]

% fctm mean tensile strength of concrete [MPa]
% E tensile strain
% Eu ultimate strain parameter
% sigma concrete tensile stress [MPa]
% heq finite element equivalent length (2a) [m]

function [A] = hordijk(fctm,GF,Eci,heq)

% Constitutive tensile behavior
ft = fctm;
c1 = 3;
c2 = 6.93;
Eu = 5.14*GF/(fctm*1e+6*heq);
E = 0:Eu/10:Eu;
sigma = ft*((1 + (c1*E/Eu).^3).*exp(-c2*E/Eu) - E*(1 + c1^3)*exp(-c2)/Eu);

% Hordijk relation plot
Eplot = [0 E + fctm/Eci];
sigmaplot = [0 sigma];
plot(Eplot,sigmaplot,'k')
grid on
box on
xlabel('Tensile strain')
ylabel('Concrete tensile stress [MPa]')

% Write .txt file
A = transpose([sigma*10^6;E]);
dlmwrite('hordijk.txt',A,'delimiter','\t','precision',10)

```