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### **Flat slabs in eccentric punching shear: experimental database and analysis Artículo Académico**

.

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### **HOJA DE CALIFICACIÓN DE TRABAJO DE TITULACIÓN**

**Flat slabs in eccentric punching shear: experimental database and analysis**

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### **RESUMEN**

La falla por punzonamiento excéntrico puede ocurrir en conexiones losa-columna cuando la conexión está sometida a momentos desbalanceados. Normalmente, esta situación se produce en las columnas de borde y de esquina y, por lo tanto, es un caso práctico común. Sin embargo, la mayoría de los experimentos de punzonamiento disponibles en la literatura son experimentos de naturaleza concéntrica. Este documento presenta una base de datos de 66 experimentos sobre losas planas bajo falla por punzonamiento excéntrico, incluyendo un breve resumen del procedimiento de prueba de cada referencia y una descripción de las muestras de losa. Adicionalmente, se incluye un análisis de elementos finitos lineales de todas las muestras para determinar las fuerzas internas y los momentos relevantes. Por último, la capacidad medida de los experimentos de la base de datos se comparan con las capacidades por punzonamiento determinadas con el ACI 318-14, el Eurocode 2 NEN-EN 1992-1-1:2005, *fib* Model Code 2010 y el Critical Shear Crack Theory. El resultado de esta comparación muestra que el *fib* Model Code 2010 es el modelo más preciso con una relación prueba versus pronóstico promedio de 1,26 y un coeficiente de variación de 34%. Se puede concluir que este estudio representa las inconsistencias de los métodos actualmente utilizados y la falta de información experimental.

Palabras clave: base de datos; punzonamiento excéntrico; losas planas; concreto reforzado; cortante; experimentos

### **ABSTRACT**

Eccentric punching shear can occur in concrete slab-column connections when the connection is subjected to shear and unbalanced moments. Typically, this situation results at edge and corner columns, and is thus a common practical case. However, most punching experiments in the literature are concentric punching shear experiments. This paper presents a database of sixty-six experiments on flat slabs under eccentric punching shear, including a brief summary of the testing procedure of each reference and a description of the slab specimens. Additionally, a linear finite element analysis of all the specimens is included to determine the relevant sectional shear forces and moments. Finally, the ultimate shear stresses from the database experiments are compared to the shear capacities determined with the ACI 318-14, Eurocode 2 NEN-EN 1992-1-1:2005, Model Code 2010, and the Critical Shear Crack Theory. The result of this comparison shows that the Model Code 2010 is the most precise model with an average predicted shear of 1.26 and a coefficient of variation of 34%. It can be concluded that this study represents the inconsistencies of the currently used methods and the lack of experimental information.

Keywords: database; eccentric punching shear; experiments; flat slab; punching; reinforced concrete; shear; shear reinforcement

# **TABLA DE CONTENIDO**







# **Flat slabs in eccentric punching shear: experimental database and analysis**

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**Abstract:** Eccentric punching shear can occur in concrete slab-column connections when the connection is subjected to shear and unbalanced moments. Typically, this situation results at edge and corner columns, and is thus a common practical case. However, most punching experiments in the literature are concentric punching shear experiments. This paper presents a database of sixtysix experiments on flat slabs under eccentric punching shear, including a brief summary of the testing procedure of each reference and a description of the slab specimens. Additionally, a linear finite element analysis of all the specimens is included to determine the relevant sectional shear forces and moments. Finally, the ultimate shear stresses from the database experiments are compared to the shear capacities determined with the ACI 318-14, Eurocode 2 NEN-EN 1992-1- 1:2005, Model Code 2010, and the Critical Shear Crack Theory. The result of this comparison shows that the Model Code 2010 is the most precise model with an average predicted shear of 1.26 and a coefficient of variation of 34%. It can be concluded that this study represents the inconsistencies of the currently used methods and the lack of experimental information.

**Keywords:** database; eccentric punching shear; experiments; flat slab; punching; reinforced concrete; shear; shear reinforcement

#### <span id="page-6-0"></span>**1. Introduction**

Structural concrete flat slabs are an interesting solution for building design due to the simplicity of the construction process and the associated (economical) advantages. Nevertheless, a difficulty lies in the uncertainty of predicting slab-column connection behavior and capacity when lateral loads or unbalanced gravity loads cause a transfer of moments between the slab and the column [1]. Such moments can also be caused by asymmetrical spans, creep, and differential shrinkage between two continuous slabs [2].

An important number of collapses caused by punching failure have been reported throughout the years, which gained the attention of researchers and practitioners [3]. One example of the most representative cases is the collapse of the underground parking garage in Gretzenbach, Switzerland on November 2004 [4]. The collapsed structure had no shear reinforcement, only column capitals were provided as a shear enhancement resource. This collapse caused the dead of seven men.

Typically, the most critical slab-column connections are located on corners and edges as these connections are subjected to moment transfer and eccentric loading. However, these are the less studied in comparison with internal slab-column connections.

This work aims to present a wider view of the problem by compiling and analyzing information from different authors on eccentric punching shear. The analysis of the compiled experiments can be used to analyze the performance of the currently available building codes and identify which types of experiments would be a valuable contribution to the body of knowledge. Additional experiments could be used to refine and improve the existing models.

The first coherent studies on punching shear were made in the 1960s by Kinnunen and Nylander [5], but their mechanical models resulted in complicated expressions, which the codes found unpractical to use [6]. Instead, empirical expressions were developed. Given that there is a lack of experimental information on eccentric punching shear on large scale flat slabs, it becomes difficult to provide a satisfactory design expression [2]. To account for eccentric loading, the ACI 318-14 [7] and Eurocode 2 EN 1992-1-1:2005 [8] models use a factored shear stress on the critical perimeter. On the other hand, the *fib* Model Code 2010 [9] and the Critical Shear Crack Theory [10,11] use a reduction of the critical perimeter.

This article compiles 66 experiments on flat slabs in eccentric punching shear. Vertical, horizontal and combined loading setups are reported in the literature. Both slabs with and without shear reinforcement are included in the database. Internal forces of the slabs for the maximum applied load, i.e. at the onset of punching shear failure, and elementary design magnitudes are typically not available in the references. To complete the missing information, a linear finite element model of each specimen is made. The experimental shear capacities from the database are then compared to the strengths predicted by the design expressions found in ACI 318-14 [7], Eurocode 2 NEN-EN 1992-1-1:2005 [8], *fib* Model Code 2010 [9] and the Critical Shear Crack Theory [10,11].

#### <span id="page-7-0"></span>**2. Methods**

#### <span id="page-7-1"></span>*2.1 Overview of code provisions*

(a)

#### <span id="page-7-2"></span>2.1.1 ACI 318 – 14

The punching shear provisions from ACI 318-14 are empirical equations resulting from the work of Moe [12] and ACI-ASCE Committee 426 [13]. The ACI 318-14 method is based on the maximum shear stress  $v<sub>u</sub>$  on the critical perimeter  $b<sub>o</sub>$  of the slab, which is located at 0.5*d* from the face of the column, where  $d$  is the average slab effective depth. The maximum shear stress  $v<sub>u</sub>$  should not exceed the nominal shear strength of the slab  $v_n$ . The ACI 318-14  $\S 8.4.1.1$  expresses that it is necessary to consider unbalanced moments, but it doesn't prescribe how this should be done. Figure 1 is a sketch of the shear stress produced by axial load and moment transfer [1].





Figure 1. Shear stress produced by applied load and moment transfer, modified from [1]: (a) transfer of unbalanced moments to column; (b) shear stress caused by direct shear; (c) shear stress caused by unbalanced moments; (d) total shear stress: sum of (b) and (c).

MacGregor and Wight [1] define *v<sup>u</sup>* using the following equation:

$$
v_{u} = \frac{V_{u}}{b_{o}d} \pm \frac{\gamma_{v}M_{u1}C}{J_{c1}} \pm \frac{\gamma_{v}M_{u2}C}{J_{c2}}
$$
(1)

where  $V_u$  is the factored shear being transferred from the slab to the column acting on the centroid of the critical section; *c* is the distance from the centroid of the critical section to the point where the shear stress is calculated; *J<sup>c</sup>* is the polar moment of inertia of the critical section and *ϒvM<sup>u</sup>* is the fraction of moment transferred by eccentricity of shear, with *ϒ<sup>v</sup>* as follows:

$$
\gamma_v = 1 - \gamma_f \tag{2}
$$

where *ϒ<sup>f</sup>* is the fraction of moment transmitted by flexure:

$$
\gamma_f = \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}}
$$
\n(3)

where  $b_1$  is the total width of the critical section measured perpendicular to the axis about which the moment acts, and  $b_2$  is the total width parallel to the axis [1]. Figure 2 shows a sketch of the critical perimeter of an interior, edge, and corner slab-column connection.





**Figure 2.** Critical perimeter of an interior, edge and corner slab-column connections, modified from [1]: (a) interior slab-column connection; (b) edge slab-column connection; (c) corner slabcolumn connection.

The polar moment of inertia *J<sub>c</sub>* can be calculated as follows:

$$
J_c = 2\left(\frac{b_1 d^3}{12} + \frac{db_1^3}{12}\right) + 2(b_2 d) \left(\frac{b_1}{2}\right)^2
$$
 for an interior slab-column connection (4)

$$
J_c = 2 \left[ \frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + (b_1 d) \left( \frac{b_1}{2} - C_{AB} \right)^2 \right] + b_2 d C_{AB}^2 \text{ for an edge slab-column connection}
$$
 (5)

$$
J_c = \left[\frac{b_1 d^3}{12} + \frac{db_1^3}{12} + (b_1 d) \left(\frac{b_1}{2} - C_{AB}\right)^2\right] + b_2 dC_{AB}^2
$$
 for a corner slab-column connection (6)

where  $C_{AB}$  is the distance to the centroid of the critical perimeter:

$$
C_{AB} = \frac{(b_1 d)b_1}{2(b_1 d) + b_2 d}
$$
 for an edge slab-column connection (7)

$$
C_{AB} = \frac{(b_1 d)b_1 / 2}{b_1 d + b_2 d}
$$
 for a corner slab-column connection (8)

According to ACI 318-14 section 22.6.5.2 [7] in slabs without shear reinforcement, the shear stress shall not exceed the least of the following three expressions, with *fc'* in [MPa].

$$
v_c = 0.33 \lambda \sqrt{f_c} \tag{9}
$$

$$
v_c = 0.17 \left( 1 + \frac{2}{\beta} \right) \lambda \sqrt{f_c}
$$
 (10)

$$
v_c = 0.083 \left( 2 + \frac{\alpha_s d}{b_o} \right) \lambda \sqrt{f_c}
$$
 (11)

The value of *α<sup>s</sup>* is 40 for interior columns, 30 for edge columns, and 20 for corner columns; *λ* is the lightweight factor; and *β* is the ratio of long to short directions of the critical perimeter [7]. Section 22.6.6.1 [7] indicates that the value of *v<sup>c</sup>* for shear reinforced slabs shall not exceed the following:

$$
v_c = 0.17\lambda \sqrt{f_c} \tag{12}
$$

$$
v_c = 0.25\lambda \sqrt{f_c} \tag{13}
$$

Eq. (12) is used for stirrup reinforcement and Eq. (13) is used for headed shear stud reinforcement. When shear reinforcement is used, the critical perimeter  $b<sub>o</sub>$  shall be taken outside the reinforced section as illustrated in Figure 3 [7].





**Figure 3.** Critical perimeter for a shear reinforced interior, edge, and corner slab-column connection, modified from [7]: (a) interior slab-column connection; (b) edge slab-column connection; (c) corner slab-column connection.

The contribution of the shear reinforcement *v<sup>s</sup>* is determined as:

$$
v_s = \frac{A_v f_{yt}}{b_o s} \tag{14}
$$

where  $A_v$  is the sum of the area of all legs of reinforcement on the peripheral line that is geometrically like the perimeter of the column section, *fyt* is the yield strength of the transverse reinforcement and *s*is the spacing of transversal reinforcement [7]. The ultimate shear capacity *v<sup>n</sup>* is calculated as follows, with  $v<sub>u</sub>$  as determined by Eq. (1):

$$
v_n = v_c + v_s \ge v_u \tag{15}
$$

<span id="page-11-0"></span>2.1.2 NEN-EN 1992-1-1:2005

The punching shear provisions of NEN-EN 1992-1-1:2005 contain empirical equations for the concrete contribution to the two-way shear capacity. It is assumed that the concrete contribution to the shear capacity is equal for one-way shear (beam shear) and two-way shear (punching shear).

According to the provisions of NEN-EN 1992-1-1:2005 [8] punching shear is checked at the basic control perimeter *U1*. The basic control perimeter *U<sup>1</sup>* is located at *2d* from the loaded area, with *d* the average effective depth of the slab. Figure 4 shows the basic control perimeter for an interior, edge, and corner slab-column connection [8]. Note that rounded corners are used for the perimeter.



**Figure 4.** Basic control perimeter for an interior, edge, and corner slab-column connection, modified forms [8]: (a) interior slab-column connection; (b) edge slab-column connection; (c) corner slab-column connection.

Punching shear is evaluated based on the following stresses: *vRd,c* the design value of the punching shear resistance of a slab without punching shear reinforcement, *vRd,s* the value of the punching shear resistance of a slab with punching shear reinforcement, and *vEd* the maximum shear stress along the control section. If  $v_{Ed} \le v_{Rd,c}$  punching shear reinforcement is not necessary. If the support reaction is eccentric with respect to the control perimeter, the maximum shear stress is:

$$
v_{Ed} = \beta \frac{V_{Ed}}{U_1 d} \tag{16}
$$

$$
\beta = 1 + k_c \frac{M_{Ed}}{V_{Ed}} \frac{U_1}{W_1}
$$
\n(17)

where *W<sup>1</sup>* represents the shear distribution on the control perimeter, *VEd* is the design value of the applied shear force, *MEd* is the design value of the applied bending moment, and *k<sup>c</sup>* is a coefficient on the ratio between the column dimensions given by Table 6.1 of NEN-EN 1992-1-1:2005 [8]. A few values of  $k_c$  are 0.6 for a  $c_1/c_2$  ratio of 1.0 and 0.70 for a  $c_1/c_2$  ratio of 2.0. Where  $c_1$  and  $c_2$  are the dimensions of the critical perimeter, see Figure 5. *W<sup>1</sup>* is calculated as:

$$
W_1 = \int_0^{U_i} |e| dl \tag{18}
$$

where *U<sup>i</sup>* is the length of the control perimeter under consideration, *dl* is a length increment of the perimeter, and *e* is the distance of *dl* from the axis about which the moment *MEd* acts [8]. Figure 5 shows the shear distribution due to an unbalanced moment at a slab-column connection.



**Figure 5.** Shear distribution due to an unbalanced moment at a slab-column connection, modified from [8].

For an internal rectangular column where the loading is eccentric to both orthogonal axes, *β* shall be calculated as follows:

$$
\beta = 1 + 1.8 \sqrt{\left(\frac{e_y}{b_x}\right)^2 + \left(\frac{e_x}{b_y}\right)^2} \tag{19}
$$

Where  $e_y$  and  $e_x$  are the eccentricities  $M_{Ed}/V_{Ed}$  along the axes *y* and *x* respectively and  $b_x$  and  $b_y$  are the dimensions of the control perimeter. For edge slab-column connections, where the eccentricity is perpendicular to the slab edge is towards the interior and there is no eccentricity parallel to the edge, the control perimeter may be reduced to *U<sup>1</sup> \** as illustrated in Figure 6a. For corner slab-column connections, where the eccentricity is towards the interior of the slab, the control perimeter may be reduced to *U<sup>1</sup> \** as illustrated in Figure 6b [8].



**Figure 6.** Reduced basic control perimeter, modified from [8]: (a) edge slab-column connection; (b) corner slab-column connection.

For edge slab-column connections, if there are eccentricities in both orthogonal directions, *β* shall be calculated as:

$$
\beta = \frac{U_1}{U_1^*} + k_c \frac{U_1}{W_1} e_{\text{par}}
$$
\n(20)

where *epar* is the eccentricity parallel to the slab edge. For corner column connections, where the eccentricity is toward the interior of the slab, *β* shall be calculated as

$$
\beta = \frac{U_1}{U_1^*} \tag{21}
$$

If the eccentricity is towards the exterior, *β* shall be calculated using equation Eq. (19). For practical purposes, *W1* shall be calculated as:

$$
W_1 = \frac{c_1^2}{2} + c_1 c_2 + 4c_2 d + 16d^2 + 2\pi d c_1
$$
 for a rectangular interior column (22)

$$
W_1 = \frac{c_1^2}{4} + c_1 c_2 + 4c_1 d + 8d^2 + \pi d c_2
$$
 for a rectangular edge column (23)

$$
W_1 = \frac{c_1 c_2}{2} + 2c_1 d + \frac{c_2^2}{4} + 4d^2 + \frac{\pi d c_2}{2}
$$
 for a rectangular corner column (24)

The punching shear resistance of slabs without shear reinforcement is *vRd,c*:

$$
v_{Rd,c} = C_{Rd,c} k (100 \rho_1 f_{ck})^{\frac{1}{3}} \ge v_{\min}
$$
 (25)

with *C<sub>Rd, c</sub>* taken as 0.18/*γ<sub>c</sub>*, with *γ<sub>c</sub>* the material factor for concrete (*γ<sub>c</sub>* = 1.5), and *k* is the size effect factor

$$
k = 1 + \sqrt{\frac{200}{d}} \le 2 \text{ with } d \text{ in } [\text{mm}]
$$
 (26)

The reinforcement ratio is the geometric average of the reinforcement ratio in the *y* ( $\rho_{l}$ ) and *x* ( $\rho_{l}$ *x*) direction:

$$
\rho_l = \sqrt{\rho_{lx} \rho_{ly}}
$$
 (27)

The lower bound of the shear capacity is a nationally determined parameter, with a recommended expression for *vmin* as:

$$
v_{\min} = 0.035k^{3/2} f_{ck}^{1/2} \text{ with } f_{ck} \text{ in [MPa]}
$$
 (28)

The punching shear resistance of slabs with shear reinforcement is calculated as:<br> $\frac{1}{r} = 0.75r_1 + 1.5\left(\frac{d}{r}\right)A + \left(-\frac{1}{r}\right)\sin\alpha$ 

$$
v_{Rd,cs} = 0.75v_{Rd,c} + 1.5\left(\frac{d}{s_r}\right)A_{sw}f_{ywd,ef}\left(\frac{1}{U_1d}\right)\sin\alpha\tag{29}
$$

where *Asw* is the area of one perimeter of shear reinforcement around the column, *s<sup>r</sup>* is the radial spacing of perimeters of shear reinforcement, *fywd,ef* is the effective design strength of the punching shear reinforcement and  $\alpha$  is the angle between shear reinforcement and the horizontal plane of the slab.

#### <span id="page-14-0"></span>2.1.3 Model Code 2010

The *fib* Model Code 2010 punching shear provisions are based on the Critical Shear Crack Theory [10,11]. The design shear demand  $V_{Ed}$  acts on the basic control perimeter  $b_{1,MC}$ , at 0.5 $d_v$  from the supported area, where *d<sup>v</sup>* is the effective depth of the slab. Figure 7 illustrates the basic control perimeter for different supported areas.



**Figure 7.** Basic control perimeter, modified from [9]: (a) interior column; (b) edge slab-column connection.

Then, for calculating the punching shear resistance of the slab, a shear-resisting control perimeter *b<sup>0</sup>* is used. This perimeter accounts for the non-uniform distribution of shear forces along *b1, MC*, which can be caused by concentrations of the shear forces due to moment transfer between the slab and the supported area as a result of eccentricities in the load application [9**].** Figure 8 illustrates the eccentricity of the resultants [9].



**Figure 8.** Resultant of shear forces, modified from [9]

The control perimeter *b<sup>0</sup>* is determined as:

$$
b_0 = k_e b_{1,MC} \tag{30}
$$

*k<sup>e</sup>* represents the coefficient of eccentricity:

$$
k_e = \frac{1}{1 + \frac{e_u}{b_u}}
$$
\n<sup>(31)</sup>

where *e<sup>u</sup>* is the eccentricity of the resultant shear forces with respect to the centroid of *b1,MC*, and *b<sup>u</sup>* is the diameter of a circle with the same area as the region inside *b1, MC*. The punching shear resistance *VRd* is calculated as:

$$
V_{Rd} = V_{Rd,c} + V_{Rd,s} \ge V_{Ed}
$$
\n(32)

The design shear resistance attributed to the concrete is calculated as:

$$
V_{Rd,c} = k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \text{ with } f_{ck} \text{ in [MPa]}
$$
 (33)

 $k\psi$  is a parameter that depends on the rotations of the slab and shall be calculated as:

$$
k_{\psi} = \frac{1}{1.5 + 0.9k_{dg}\Psi d} \le 0.6
$$
\n(34)

where *d* is the mean value of the effective depth of the slab for *x* and *y* directions. If the maximum aggregate size *d<sup>g</sup>* is greater than 16 mm, then *kdg =* 1. If not, *kdg* shall be calculated as follows:

$$
k_{d_g} = \frac{32}{16 + d_g} \ge 0.75 \text{ with } d_g \text{ in [MPa]}
$$
 (35)

The design shear resistance attributed to the shear reinforcement is calculated as:

$$
V_{Rd,s} = \sum A_{sw} k_e \sigma_{swd} \sin \alpha \tag{36}
$$

where ∑*Asw* is the sum of the area of all the shear reinforcement acting on the zone between 0.*35d<sup>v</sup>* and *d*v, which has a length of 0.65*dv,* see Figure 9 [9].



**Figure 9.** Shear reinforcement acting at failure, based on [9].

The stress *σswd* is calculated as:

$$
\sigma_{\text{swd}} = \frac{E_{\text{s}} \Psi}{6} (\sin \alpha + \cos \alpha) \left( \sin \alpha + \frac{f_{bd}}{f_{\text{good}}} \frac{d}{\phi_w} \right) \le f_{\text{good}} \tag{37}
$$

where *ϕ<sup>w</sup>* represents the diameter of the shear reinforcement and *fywd* its yield strength. The bond strength *fbd* is assumed to be equal to 3 MPa.

The load rotation behavior of the slab is calculated as follows:

$$
\psi = 1.5 \frac{r_s}{d} \frac{f_{yd}}{E_s} \left( \frac{m_{sd}}{m_{rd}} \right)^{1.5}
$$
 (38)

where  $r<sub>s</sub>$  is the distance from the column axis to the line of contraflexure of the radial bending moments; *fyd* is the yield strength of the flexural reinforcement, *E<sup>s</sup>* the modulus of elasticity of the flexural steel, *msd* the average moment per unit length for calculating flexural reinforcement in the support strip, and  $m$ *Rd* is the average flexural strength per unit length in the support strip [9]. The values of the mechanical parameters in the formula can be calculated with different levels of approximation (LoA), considering that every level of approximation represent a different grade of precision [6].

LoA I assumes that  $m_{sd} = m_{Rd}$ , which implies that the strength of the slab will be governed by its bending moment capacity. For regular slabs with a long over short side ratio 0.5 *≤ Lx/Ly ≤* 2.0, *r<sup>s</sup>* can be estimated as follows:

$$
r_{sx} = 0.22L_x
$$
  
\n
$$
r_{sy} = 0.22L_y
$$
\n(39)

Figure 10 illustrates  $L_x$  and  $L_y$  [9].



**Figure 10.** Slab dimensions, modified from [9].

LoA II includes a simplified of estimation *msd* 

$$
m_{sd} = V_{Ed} \left( \frac{1}{8} + \frac{e_u}{2b_s} \right) \tag{40}
$$

where  $2b<sub>s</sub>$  is the width where the transferred moment acts. Considering that half of the moment acts on each side of the column [6], *bsr* is the width of the support strip for corner and edge slabs. LoA II considers a significant bending moment redistribution in design [9]. This LoA is recommended for irregular slabs where *Lx/L<sup>y</sup>* is not between 0.5 and 2.0 [9].

LoA III takes the coefficient 1.5 in Eq. (38) and replace it by 1.2 if *r<sup>s</sup>* and *msd* are calculated using a linear elastic model. LoA IV is based on a nonlinear analysis of the structure and, it considers cracking, tension-stiffening effects, yielding of the reinforcement, and any other relevant nonlinear effects [9].

#### <span id="page-17-0"></span>2.1.4 Critical Shear Crack Theory (CSCT)

For reinforced slabs, a simplified code-like formulation can be used to calculate punching shear strength within the shear-reinforced zone is as follows [10]:

$$
V_{Rd} = V_{cd} + V_{sd} \tag{41}
$$

The shear force carried by the concrete  $V_{cd}$  is calculated as:

$$
V_{cd} = \frac{1}{\gamma_c} \frac{2}{3} \frac{b_{0,\text{int}} d\sqrt{f_{ck}}}{1 + 20 \frac{\psi d}{d_{g0} + d_g}} \text{ with } f_{ck} \text{ in [MPa]}
$$
(42)

where *b0,int* is the perimeter of the critical section inside the shear reinforced zone, *d* the effective depth of the slab, *fck* the compressive strength of the concrete, *ϒ<sup>c</sup>* the partial safety factor of the concrete *γ<sup>c</sup>* = The shear force carried by the shear reinforcement equals:

$$
V_{sd} = \frac{E_s \Psi}{6} A_{sw} \le f_{ywd} A_{sw}
$$
\n(43)

where *A<sup>s</sup>* is the amount of shear reinforcement within a perimeter at a distance *d* from the edge of the support region, *fywd* is the design yield strength of the shear reinforcement, and *E<sup>S</sup>* is modulus of elasticity.

For non-reinforced slabs, Eq. (41) is modified, making  $V_{sd}$  equal to 0, and Eq. (42) is modified, changing *b0,int* for the control perimeter *b<sup>0</sup>* defined in Figure 7.

#### <span id="page-18-0"></span>*2.2 Database of experiments*

#### <span id="page-18-1"></span>2.2.1 Development of database

The database developed for this study contains 66 experiments of eccentric punching shear on flat slabs with longitudinal reinforcement and with or without transverse shear reinforcement reported in the literature. The consulted references are Krüger [2], Albuquerque et al [14], Hammill and Ghali [15], Narashimhan [16], Zaghlool [17], Anis [18] and Tankut [19]. Tables A1-A3 present the database developed for this study. The full spreadsheet is available in the public domain in .xlsx format [20]. The notations used in this database are given in the "List of notations". Figure 11 illustrates the different slab geometries and slab-column connections found in the literature [2,16,17].



**Figure 11.** Slab geometries and test slab-column connection: (a) square interior slab-column connection [2]; (b) rectangular edge slab-column connection [16]; (c) square corner slab-column connection [17].

Reference [14] does not present the dimensions of the hard rubber pads that transmit the force from the hydraulic jacks to the slab; square pads of  $100$  mm  $\times$   $100$  mm are assumed based on the figures presented in the original reference. Reference [17] doesn't report the width of the slab support; a 100 mm width is estimated based on the provided drawings. The specimens in reference [19] are

not supported like the other slabs. Since the author tested two continuous slabs representing an actual building floor, the width of the support is taken as 0 mm for this reference, as only the slab-column connection is evaluated for this work.

The age of the specimens at testing is not given by the references [2,14,15]; 28 days is assumed. The tensile strength of the concrete  $f<sub>ct</sub>$  is calculated for the references that don't present this information [2,15,18,19] from the average compressive strength *fc*with the expression developed from Sarveghadi [21]:

$$
f_{ct} = 0.76\sqrt{f_c} \text{ with } f_c \text{ in } [\text{MPa}]
$$
 (44)

For references [2,15,19], the maximum aggregate size is assumed to be 9.5 mm.

For references [14,15,18,19], the spacing of the longitudinal reinforcement is taken from the technical drawings in the papers, or as the average spacing of the bars when the reinforcement layout is too complex. The number of bars for flexural reinforcement of the reference [18] was estimated from the drawings provided by the author. For references [2,15] the modulus of elasticity of the reinforcing steel *E<sup>s</sup>* is assumed to be 200 GPa. References [2,14,15,16] present slabs with transverse shear reinforcement: stirrups, shear hats (see Figure 12), and studs were the shear reinforcement types found in these references. Table A4 presents the database for shear reinforced specimens.



**Figure 12.** Shear hat setup, as used in [16].

The results are reported as the ultimate load applied to the slab-column connection and its moment caused by the eccentricity. References [2,14,16] don't give the moments transmitted to the slab by the columns. For these references, the bending moments are calculated using the applied load and the reported eccentricity. References [15,17] use a diagonal moment on the x-y axis on the square corner slabs. Figure 13 illustrates this type of loading.



**Figure 13.** Diagonal loading setup.

For the database in Table A3, the diagonally applied moment was divided into its components in the *x*- and *y*-directions. Reference [14] presents the sectional shear force *V<sup>u</sup>* caused by the applied load, neglecting the contribution of the self-weight of the slabs. All reported values for the sectional shear force at failure  $V_u$  in the database include the contribution of the self-weight when testing occurred in the gravity direction (i.e. self-weight increases sectional shear). For the references [2,14,19] the self-weight of the slabs was included in the analysis. Where the internal forces at failure were not found in the original reference, the linear finite element program SCIA Engineer [22] was employed to obtain sectional shear forces and moments, and elementary design magnitudes, see Figure 14. Special care on the type of support used in the model slabs was taken, as normal forces were not desired in the results. The models were as similar to the reported experiments as possible. The results presented in the Table A5 are the maximum internal forces of the slab at failure.

Most of the entries in the database failed in brittle punching shear failure. Nevertheless, references [2,14,18] present a few specimens that failed by flexure-induced punching shear.

All values in the database are presented in SI units. The information from [17,18,19] was converted from U.S customary units to SI units.



**Figure 14.** Example of Finite Element Analysis on an Albuquerque [14] specimen: (a) Applied loads in the model; (b) Internal force  $V_x$  calculated from the applied load and the self-weight of the specimen.

#### <span id="page-20-0"></span>2.2.2 Parameter ranges in the database

In this section, an evaluation of the distribution of the values of the parameters in the database is made. Table 1 gives the ranges of the most important parameters in the database.

Parameter	Min	Max
$h$ (mm)	102	180
$d$ (mm)	76	153
$L_x$ (mm)	762	3000
$L_y$ (mm)	762	3000
$a$ (mm)	400	1375
$a_v$ (mm)	200	1100
$\rho$ (%)	$0.72\%$	2.40%
$f_c(MPa)$	26	59
$a/d$ (-)	5.25	8.99
$a_v/d$ (-)	2.62	7.20

**Table 1.** Ranges of parameters in database

Figure 15 shows the distribution of the most important parameters in the database. Figure 15a shows that the majority of slabs are made of normal strength concrete. The developed database cannot be used to gain insight in the eccentric punching shear capacity of high strength concrete slabcolumn connections. Figure 15b shows that a tensile reinforcement ratio close to 1.25% was commonly used in the tested slabs. Typical slab designs use reinforcement ratios of 0.6% - 0.8%. Only 5 of the experiments in the database use these practical values. Most slabs were over-reinforced in flexure to achieve a punching shear failure. The distribution of the average effective depth of the slab is presented in Figure 15c. This plot shows that most experiments had an effective depth *d* in the range from 100 mm to 125 mm. The reported specimens are small-scale specimens which do not give us insights regarding the size effect for eccentric punching shear. Figure 15d shows the ratio between the shear span and the average effective depth *a/d*. The range of *a/d* in the experiments covers only situations in which no direct load transfer can occur.







**Figure 15.** Distribution of the most important parameters in the database: (a) concrete compressive strength  $f_c$ ; (b) tensile reinforcement ratio  $\rho$ ; (c) effective depth d; (d) shear span to average effective depth ratio *a/d*.

#### <span id="page-22-0"></span>**3. Results**

#### <span id="page-22-1"></span>3.1 Parameter studies

The raw data from the database are used to analyze the effect of different experimental parameters on the sectional shear stress at failure as a result of the applied load. ACI 318-14 expression is used for determining *vu.* Normalized shear stresses are used to discard the influence of the concrete compressive strength *fc*. An analysis of the normalized shear stress to the square root and to the cube root is made. Figure 16 shows the relation between the normalized shear strength and *fc.* From this figure, it can be seen that, for the studied experimental results, normalizing the shear

strength to the square root is to be preferred. A similar observation was made for the shear capacity of steel fiber reinforced concrete beams [23].



**Figure 16.** Normalized shear stresses to the concrete compressive strength: (a) normalized to the square root; (b) normalized to the cube root.

The influence of different parameters will be studied as a function of the shear stress normalized to the square root of *fc*. Figure 17 shows the influence of the most important parameters on the shear stress normalized to the square root of *fc*. Figure 17a shows the influence of the effective depth on the normalized shear stress. For the specimens in the compiled database, the effective depth has very little influence on the normalized shear stress. However, experiments on slabs with a larger effective depth are not available, so that this database cannot give insights regarding the size effect in eccentric punching shear. Figure 17b shows the influence of the reinforcement ratio *ρ*. Larger reinforcement ratios result in larger shear capacities, as expected. As more tension reinforcement is provided, the contribution of dowel action to the shear capacity increases, as reflected by the results from the database. Figure 17c shows the influence of the shear span to effective depth  $a/d$ . The shear capacity tends to decrease as *a/d* increases. This observation can be explained by the fact that for lower *a/d* ratios, direct load transfer between the point of application of the load and the support occurs. When direct load transfer occurs, the shear capacity is increased as a result of the shear-carrying mechanism of arching action.



**Figure 17.** Parameter studies based on the normalized shear stress at failure of all entries in the database: (a) effective depth  $d$ ; (b) longitudinal reinforcement ratio  $\rho$ ; (c) shear span to depth ratio  $a/d$ .

#### <span id="page-24-0"></span>3.2 Comparison to code predictions

The measured shear capacities from the database are then compared to the shear capacities predicted by four different models: ACI 318-14 [7], NEN-EN 1992-1-1:2005 [8], *fib* Model Code 2010 [9], and the Critical Shear Crack Theory [10,11]. Figure 18 shows the comparison between tested and predicted results, with the statistical properties of *Vtest/Vpred* in Table 2. Results for all the entries of the database are presented in Table A6. Some entries do not present direct load applied to the slabs, only moment transferred from the column. In these references NEN-EN 1992-1-1:2005 [8], *fib* Model Code 2010 [9], and the Critical Shear Crack Theory [10,11] models were not evaluated. The validation of

the spreadsheet used for calculating code predictions is available in the public domain [24]. Table 3 shows the statistical properties of *Vtest/Vpred* only for slabs with shear reinforcement and Table 4 only for slabs without shear reinforcement. All references used in this database made code comparisons. Typically, the codes used were ACI 318-14 [7] and NEN-EN 1992-1-1:2005 [8] showing results similar to the ones presented in this study.



**Figure 18.** Comparison between experimental *Vtest* and predicted shear capacities *Vpred* for 4 design methods from existing codes.

Model	AVG	<b>STD</b>	COV	mın	max
<b>ACI</b> [7]	1.22	0.54	44%	0.15	2.37
EC2 [8]	1.04	0.4	41%	0.25	1.97
MC2010 [9]	1.25	0.39	34%	0.41	2.33
CSCT [10,11]	1.21	0.42	38%	0.36	2.37

**Table 2.** Statistical properties of *Vtest/Vpred* for all 70 datapoints

**Table 3.** Statistical properties of *Vtest/Vpred* for slabs with shear reinforcement

Model	AVG.	STD	COV	mın	max
ACI [7]	0.57	0.35	61%	0.15	1.56
EC2 [8]	0.74	0.39	53%	0.25	1.53
MC2010 [9]	1.11	0.45	41%	0.41	2.32
CSCT [10,11]	0.86	0.41	48%	0.36	1.92

**Table 4.** Statistical properties of *Vtest/Vpred* for slabs without shear reinforcement



#### <span id="page-26-0"></span>**4. Discussion**

None of the codes presented highly conservative results, with average tested to predicted ratios between 1.04 and 1.25. The tested to predicted values using NEN-EN 1992-1-1:2005 [8] show the lower maximum value (for one entry) and the lowest average value, see Table 2. For the information in this database NEN-EN 1992-1-1:2005 [8] is the model that best predicts the capacity, although with a relatively large scatter (COV= 41%). The tested to predicted values using ACI 318-14 [7] show the lowest value for one entry, 0.15. Only using ACI 318-14 [7] the lowest value is this experiment. Whereas the other models consider only a part of the shear reinforcement, ACI 318-14 [7] considers all the reinforcement displaced on the peripheral line that is geometrically like the perimeter of the column section. As a result, this cause that the predicted shear resistance according to ACI 318-14 is significantly larger than the capacity predicted with the other methods.

Tables 3 and 4 show that the coefficient of variation for experiments without shear reinforcement are smaller than the ones with shear reinforcement. Models based on the Critical Shear Crack Theory don't present a big change, only empirical models such as ACI 318-14 [7] and NEN-EN 1992-1-1:2005 [8] show significantly larger scatters when analyzing only slabs with shear reinforcement.

High strength concrete slabs are not included in the study due to the lack of experiments on high-strength concrete slab-column connections. It would be interesting to investigate the behavior of this type of slabs in comparison with the ones considered for this study. Reference [25] presents a study on concentric experiments in high strength concrete slabs without shear reinforcement and concluded that the use of high strength concrete improves punching shear resistance.

In most cases, the results found in the literature indicate that there is an important reduction of the punching capacity when unbalanced moments occur in the slab column connection. Nevertheless, most databases, experiments and research focus on concentric punching shear. Models developed empirical equations that include the effect eccentricities by different methods such as critical perimeter reduction or increase of the applied shear stress, but there is not a mechanics-based model that is practical enough to be implemented in the construction codes. Mechanics-based models such as the Critical Shear Crack Theory are developed for the case of concentric punching shear and use simplified assumption for the extension to eccentric punching shear. For this database, the empirical methods showed large scatter on the results of the tested to predicted capacities, represented by the high coefficients of variation. This observation may be explained by the fact that all methods under consideration were originally developed for concentric punching shear, and validated with concentric punching sear tests, and then extended to the use of eccentric punching shear.

Realistic size slabs experiments in punching shear are not commonly found in the literature. None of the entries in this database is considered as a realistic size slab as none of these has an effective depth over 200 mm. Making this kind of experiments would represent the behavior of actual structures with a more realistic approach.

#### <span id="page-26-1"></span>**5. Conclusions**

The lack of understanding regarding eccentric punching shear presents a practical problem because local forces typically control slab design. The transfer of unbalanced moments from the slab to the column cause an increase of the resulting shear stress. When this effect is not well-understood, as has happened in practice, it may lead to a punching failure of the slab-column connection and a possible collapse of the building. This study evaluates the available code provisions against 66 experimental results reported in the literature.

Analyzing the available experimental results from the database resulted in the following conclusions:

- All experiments are carried out in slabs under 200 mm depth. As such, the experiments cannot be used to evaluate the size effect in shear.
- There is a lack of experiments in eccentric punching shear.
- Most specimens have large reinforcement ratios to avoid a flexural failure before reaching the punching shear capacity of the slab.

• All specimens are cast using normal strength concrete.

The parameter studies led to the following observations:

- An analysis of the data showed that the shear stresses should be normalized to the square root of the concrete compressive strength. This ratio shows a smaller relation to concrete compressive strength than when shear stress is normalized to the cube root of the concrete compressive strength.
- The normalized shear strength increased as the reinforcement ratio increased. This influence was expected because a larger amount of reinforcement results in more dowel action, and thus a larger shear capacity.

From the comparison between the experimental shear capacities and the capacities predicted by the available codes, the following conclusion result:

- The average value of tested to predicted shear capacity is obtained with the Eurocode provisions. For the Eurocode provisions, however, the associated coefficient of variation is rather large (41%).
- The coefficient of variation of the tested to predicted shear capacities is lower for the expressions based on the Critical Shear Crack Theory than for the empirical expressions from Eurocode 2 and ACI 318.
- The coefficient of variation of the tested to predicted shear capacities is lower for the experiments without shear reinforcement than for the experiments with shear reinforcement.

A better understanding of eccentric punching shear and further experiments on deeper slabs and slabs with high-strength concrete are necessary to obtain save designs, optimize the design of building floors, and develop better tools for the assessment of existing building slabs.

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<span id="page-27-0"></span>**Conflicts of Interest:** The authors declare no conflict of interest.

#### **List of notations**







- *mxD* model design moment on the *x-* axis.
- *myD* model design moment on the *y-* axis
- *s<sup>r</sup>* radial spacing of the reinforcement



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### <span id="page-31-0"></span>**Appendix A**















CS for Cylinder Split and S for using the Sarveghadi formula Eq. (44) for obtaining *fct*





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Reference		<b>Shear reinforcement</b>						
	Id	Shear	$\rho_v$		$\phi$	$S_v$	fywd	$A_{sw}$
		Reinforcement	$(\%)$	<i>Mhars</i>	mm)	(mm)	'MPa)	mm2)
	L10	Studs	$0.08\%$	28	8	150	587	1407
<b>ALBUQUERQUE [14]</b> KRÜGER <sup>[2]</sup>	L13	Stirrups	$0.17\%$	24	8	60	587	1206
	PP <sub>16</sub> B	Stirrups	$0.36\%$	48	10	120	480	7540
HAMMILL & GHALI [15]	NH <sub>3</sub>	Studs	$0.08\%$	12	10	57	440	942
	NH5	Studs	$0.09\%$	20	10	85	440	1571
	L <sub>3</sub>	Shear hat	$0.18\%$	24	9.5	52	309	1710
	L4	Shear hat	$0.18\%$	24	6.5	52	238	1710
	L5	Shear hat	$0.31\%$ 24		13	52	355	3040
<b>NARASHIMAN</b> [16]	L6	Shear hat	$0.12\%$ 24		8	52	366	1190
	L10	Shear hat	$0.12\%$	24	8	52	355	1190
	ES <sub>3</sub>	Shear hat	$0.05\%$	16	6.5	52	238	517
	ES <sub>4</sub>	Shear hat	$0.12\%$	-16	9.8	52	309	1140
	ES <sub>7</sub>	Shear hat	$0.05\%$	16	6.5	52	238	517

**Table A4.** Shear reinforcement.

Reference	Id	$M_{\text{mux}}$	$M_{mu}$ $(kNm)$ $(kNm)$	$V_{\text{max}}$ (kN)	$V_{mu}$	MxD $(kN)$ $(kNm)$	MyD (kNm)
	L1	152	136	637	267	173	157
	L <sub>5</sub>	341	145	1069	460	430	234
	L <sub>6</sub>	193	221	913	387	261	290
ALBUQUERQUE	L10	487	194	1496	657	572	279
$[14]$	L11	212	195	898	379	243	227
	L12	333	147	1055	453	432	246
	L13	339	165	1109	475	461	287
	P <sub>16</sub> A	26	27	140	134	33	31
KRÜGER <sub>[2]</sub>	P <sub>30</sub> A	22	24	127	116	27	28
	PP16B	34	35	182	173	42	40
	NH <sub>1</sub>	85	85	264	264	113	113
	NH <sub>2</sub>	80	80	251	251	106	106
HAMMILL & GHALI [15]	NH <sub>3</sub>	84	84	266	266	110	110
	NH <sub>4</sub>	30	30	114	114	44	44
	NH <sub>5</sub>	105	105	219	219	141	141
	L1	65	69	530	349	63	76
	L <sub>3</sub>	79	86	663	436	79	95
	L4	90	98	754	497	90	108
	L <sub>5</sub>	79	86	663	436	79	95
	L <sub>6</sub>	80	103	659	523	80	108
<b>NARASHIMAN</b> [16]	L10	85	110	703	558	85	116
	ES <sub>2</sub>	63	134	204	630	83	191
	ES <sub>3</sub>	100	212	325	1001	131	304
	ES4	91	193	296	911	119	276
	ES <sub>5</sub>	105	399	134	1776	219	526
	ES7	155	590	198	2627	324	779
	$Z-I(1)$	45	45	149	149	57	57
	$Z-II(1)$	78	78	248	248	102	102
	$Z-II(2)$	102	102	312	312	136	136
	$Z-II(3)$	104	104	305	305	142	142
	$Z-II(4)$	26	26	99	99	38	38
	$Z-II(6)$	52	52	120	120	80	80
	$Z-II(7)$	$26\,$	26	102	102	39	39
	$Z-II(8)$	84	84	226	226	120	120
ZAGHLOOL [17]	$Z-III(1)$	94	94	287	287	125	125
	$Z$ -IV $(1)$	32	$30\,$	232	136	32	34
	$Z-V(1)$	53	$50\,$	359	233	53	50
	$Z-V(2)$	59	57	425	262	59	63
	$Z-V(3)$	65	62	454	287	65	67
	$Z-V(4)$	59	69	309	109	72	69
	$Z-V(6)$	74	156	300	301	115	156
	$Z-VI(1)$	61	58	376	280	61	58
	B.1	$\mathbf{1}$	5	117	79	16	18
ANNIS <sup>[18]</sup>	B.3	32	32	285	256	41	34
	B.4	26	25	230	187	33	29

**Table A5.** SCIA Model results.



		EC <sub>2</sub>	<b>ACI</b>	<b>MC2010</b>	<b>CSCT</b>
Reference	Slab			Vtest/Vpred Vtest/Vpred Vtest/Vpred Vtest/Vpred	
	L1	0.5	0.2	1.9	2.0
	L <sub>5</sub>	1.2	1.2	1.3	1.4
	L <sub>6</sub>	1.2	1.2	1.4	1.4
<b>ALBUQUERQUE [14]</b>	L10	1.5	0.7	1.1	1.0
	L11	1.3	1.5	1.7	1.7
	L12	1.2	1.3	1.4	1.5
	L13	1.2	0.5	1.4	1.3
	P <sub>16</sub> A	1.0	1.2	1.5	1.6
KRÜGER <sup>[2]</sup>	P <sub>30</sub> A	1.1	1.3	1.6	1.6
	PP16B	0.4	0.4	0.4	0.4
	NH <sub>1</sub>	1.3	1.0	1.7	1.8
	NH <sub>2</sub>	1.3	0.9	1.6	1.6
HAMMILL & GHALI [15]	NH <sub>3</sub>	0.4	0.1	0.8	0.5
	NH <sub>4</sub>	$\overline{\phantom{a}}$	1.4	$\blacksquare$	$\overline{\phantom{a}}$
	NH <sub>5</sub>	0.6	0.2	1.3	0.8
	L1	1.2	1.6	1.3	1.3
	L <sub>3</sub>	0.5	0.5	0.9	0.6
	L4	0.6	0.7	1.0	0.7
	L5	0.3	0.3	0.6	0.4
	L6	0.5	0.6	0.9	0.7
NARASHIMAN <sup>[16]</sup>	L10	0.7	0.6	1.1	$0.8\,$
	ES <sub>2</sub>	1.5	1.9	1.4	1.4
	ES <sub>3</sub>	1.1	1.6	1.5	1.3
	ES4	0.6	0.7	1.0	0.8
	ES <sub>5</sub>	1.8	0.6	2.3	2.3
	ES7	1.3	0.5	2.3	1.9
	$Z-I(1)$	0.8	0.7	1.2	1.2
	$Z-II(1)$	1.3	0.9	1.4	1.4
	$Z-II(2)$	1.5	1.2	1.9	1.9
	$Z-II(3)$	1.4	1.3	2.3	2.4
	$Z-II(4)$	$\overline{\phantom{a}}$ $0.8\,$	1.1	$\overline{\phantom{a}}$	$\equiv$
	$Z-II(6)$ $Z-II(7)$		0.5 $1.0\,$	1.1	1.1
	$Z-II(8)$	$\bar{\phantom{a}}$		1.3	$\overline{\phantom{0}}$
ZAGHLOOL [17]	$Z-III(1)$	1.2 1.5	0.9 0.9	1.3	1.4 1.3
	$Z$ -IV $(1)$	1.2	1.9	1.6	1.6
	$Z-V(1)$	1.7	2.0	1.6	1.6
	$Z-V(2)$	1.7	2.0	1.6	1.7
	$Z-V(3)$	1.8	2.4	1.9	1.9
	$Z-V(4)$	$\pm$	1.0	$\pm$	$\frac{1}{\sqrt{2}}$
	$Z-V(6)$	1.3	1.6	1.3	1.3
	$Z-VI(1)$	2.0	2.0	1.6	1.6
	B.1		1.7		$\overline{\phantom{a}}$
ANNIS <sup>[18]</sup>	B.3	1.1	1.5	1.1	1.2
	B.4	$1.0\,$	1.5	$1.0\,$	1.1

**Table A6.** Code comparisons.

