UNIVERSIDAD SAN FRANCISCO DE QUITO USFQ

Colegio de Ciencias e Ingenierías

Applicability of the O'Connell & Budiansky model to a conventional concrete mixture

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Ingeniería Civil

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HOJA DE CALIFICACIÓN DE TRABAJO DE FIN DE CARRERA

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RESUMEN

El hormigón es un material usado a nivel mundial para todo tipo de estructuras, por lo cual el conocimiento del funcionamiento y posible modelado de sus propiedades es fundamental para el desarrollo de la industria de la construcción. Una propiedad fundamental del hormigón es el del módulo de elasticidad. O´Connell y Budiansky presentaron un modelo el cual describe la relación que existe entre el módulo de elasticidad y el contenido de fisuras de un material homogéneo e isotrópico. El hormigón, al ser una mezcla de varios componentes, no se comporta como tal. Sin embargo, el principal objetivo de esta investigación es comprobar si este modelo puede ser aplicado para este material. En este estudio se utilizó una mezcla de hormigón de resistencia de 44 Mpa. De los cuales 6 cilindros se utilizaron para obtener su módulo de elasticidad dinámico y el contenido de microfisuras. Posteriormente se compararon las mediciones realizadas con los valores calculados por medio del modelo de O´Connell y Budiansky. El comportamiento general sigue el comportamiento predicho por el modelo, sin embargo, el hormigón se comporta más como un fluido viscoso, que como un material sólido.

Palabras clave: Módulo de elasticidad, microfisuras, O´Connell & Budiansky.

ABSTRACT

Concrete is a material used worldwide for all types of structures. Therefore, it is important to understand its behaviour, and possible modeling its properties, which it is essential for the development of the construction industry. One of the most fundamental property of the concrete is the elasticity module. O'Connell and Budiansky presented a model which describes the relationship between elasticity module, and the connect of cracks for a homogeneous and isotropic material. Concrete, being a mixture of several components, does not behave as such. However, the main objective of this research is to check if this model could be applied for this material. In this research, a 44 Mpa strength concrete mix was used. Of which 6 cylinders were used to obtain its dynamic modulus of elasticity, and the density cracks parameter. The measurements were compared with the values obtained with the O'Connell and Budiansky model. The general behavior follows the behavior predicted by the model. However, concrete behaves more like a viscous fluid than a solid material.

Key words: elastic modulus, micro cracks, O´Connell & Budiansky, cracking.

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INTRODUCTION

In a seismic country, it is important to analyze and understand the behavior of the structural materials after a seismic event. Therefore, there are many variables that will alter its performance, whether they are cyclic loads, environmental conditions, constant loads or materials properties.

Concrete is a material that tends to crack in its natural state. The amount, size and density of cracks will generate changes in the response of concrete to loading, particularly due to changes on the modulus of elasticity. (Mehta P.K., Paulo J.M. Monteiro, 2006). Although, it will be interrelated with the amount of the cracks in the specimen, according if it is micro cracked or completely cracked.

The main purpose of this research is to measure the changes in dynamic modulus of elasticity of cylindrical test specimens from a conventional concrete mixture at different saturation ratios and compare to the moduli predicted by the O'Connell & Budiansky (1974) model for cracked media. Measurements of dynamic elastic modulus were obtained using non-destructive methods, when the samples are water-saturated, oven-dried, and at different saturation ratios corresponding to a single value of crack density parameter (ε).

LITERATURE REVIEW

Compressive strength (fc)

Compressive strength of concrete measures the ultimate stress that the material can withstand. It can be measured using test method ASTM C39. For structural concrete, ACI 318-19 (2019) states that the specified compressive strength should not be less than 17 MPa. The compressive strength of concrete depends on the properties of the aggregates, the water-to-cementitiousmaterials ratio, age of testing, and amount of curing. Figure 1 shows the increase in compressive strength with age and with the amount of curing.

Figure 1 Compressive strength (MPa) vs Age at test (days). (McDonald, 2012)

Dynamic modulus of elasticity.

The dynamic modulus of elasticity (Ed) corresponds to the Young's modulus of the material at small strains, usually determined using resonant methods or stress waves. The dynamic modulus of concrete cylinders can be estimated using test method ASTM C 215, by measuring the resonant frequency of cylindrical or prismatic specimens. Figure 2 shows the configuration used by this test method in transverse mode.

Figure 2 Scheme of the specimen. (ASTM C-215)

The dynamic modulus of elasticity is estimated using the following equation:

Dynamics modulus of elasticity = $C * M * n^2$

M= mass of the cylinder [kg] n=transverse frequency [Hz]

 $C= 1.6067$ [$L^{3}T/d^{4}$]

Research significance

In concrete structures, it is important to characterize the mechanical properties of all the materials. Although, it has microcracks which has been created by different exposures due to overload, cyclic loadings, or other natural or man-made exposures, may decrease the modulus of elasticity as well as the compressive strength. About the concrete, there are many causes why microcracks appear, as shown in Table 1. It is important to study this behaviour, because the element failure is associated with internal microcracks (Calixto, 2002). On the other hand, Hilal (Hilal, 2016) mentions that there are some forms of change in the microstructure, and development of the microcracks that are revealed when they are already propagated at the

macroscopic level. Although, the search has been undertaken to quantify the microstructure of the element, and to associate its mechanical properties both before deterioration and after deterioration. The heterogeneity of cement paste at the microscopic level is pronounced, because it is a mixture of different types of crystalline structures with different degrees of hydration that collectively forms an amorphous gel (F. O. Slate, K. C. Hover, 1984).

Table 1 Type of cracks (Technologies, 2019)

			SHRINKABLE AGGREGATES
		PHYSICAL	DRYING SHRINKAGE
			CRAZING
		CHEMICAL	CORROSION OF REINFORCEMENT
			ALKALI-AGGREGATE REACTIONS
			CEMENT CARBONATION SHRINKAGE
	AFTER HARDENING		
			FREEZE/THAWS CYCLES
		THERMAL	EXTERNAL SEASONAL
TYPES OF CRACKS			TEMPERATURE VARIATIONS
			EARLY THERMAL
			CONTRACTION (EXTERNAL AND INTERNAL)
			ACCIDENTAL OVERLOAD
		STRUCTURAL	CREEP
			DESIGN LOADS
			PLASTING SHRINKAGE
	BEFORE	PLASTIC	PLASTIC SETTLEMENT
	HARDENING		
		CONSTRUCTIONAL	FORMWORK MOVEMENT
		MOVEMENT	SUB-GRADE MOVEMENT

In the microstructure, it has seen that the micro-fissures have caused several problems in the concrete. Specially, in the application of load to the concrete element. It happens that the microcracks expand itself, causing more cracks.

Figure 3 Microcracking in the concrete. (Rosero, 2018)

According to (Recalde, 2009) an increase in the permeability of the concrete mix reduces the durability of the concrete. Therefore, it is important to study the permeability index with respect to the change of the mechanical properties in the concrete microstructure. Although, it is important to obtain the crack density parameter (ε), which was studied by (O'Connell & Budiansky , 1997). It considers that the shear modulus varies according with its saturation. It describes the changes of Young's modulus, shear modulus, compressibility modulus and Poisson ratio. The mode is used to predict the elastic modulus of solids (homogeneous and isotropic).

Figure 4 Elastic properties vs Saturation ratio. a) Total saturation b) Partial saturation (O'Connell & Budiansky , 1997)

The following equations are used for the saturated and dry states:

$$
\frac{E}{E_o} = 1 - \frac{16}{45} (1 - v^2) \left[3D + \frac{4}{(2 - v)} \right] \varepsilon \quad [1]
$$

$$
\varepsilon = \frac{45(v_o - v)(2 - v)}{16(1 - v^2) [D(1 + 3v_o)(2 - v) - 2(1 - 2v_o)]} \varepsilon \quad [2]
$$

v^o = Poisson modulus of the uncracked solid

 $G =$ effective shear modulus [Pa]

 $G0$ = shear modulus of the uncracked solid [Pa]

 $a =$ largest elliptical radius [m]

 $c =$ crack width [m]

 $KA =$ modulus of compressibility of the fluid $[Pa]$

 $D =$ depends on the degree of saturation

$$
\xi_{OB} = saturation level
$$

The degree of total or partial saturation, it will be obtained using the following equations:

$$
D_{sf} = 1 - \xi_{OB} \quad [3]
$$

METHODOLOGY

The materials used for the mixture was water, aggregate and white cement. It was used white cement to increase the f´c. The aggregate was subjected to various standards to verify that it was the most suitable for mixing. The standards used were C29-17 for bulk density and voids, C70-13 for surface moisture in fine aggregate, C128-15 for relative density and absorption on fine aggregate, C 127-15 15 for relative density and absorption on coarse aggregate, C121-14 for degradation of small-size coarse aggregate by abrasion and impact in the Los Angeles Machine. In the mixture of 0.55 W/C. The design of the mixture was guided with the ACI-211. It was melted 11 cylinders of 100x200mm by the ASTM-C192-16. It was calculated the bulk density and voids in aggregate with ASTM C29-17, and C566-13 for total evaporable moisture content of aggregate by drying. Later, the specimens were curing 48 hour later, and, exposed 14 days in 23 ± 2 Celsius degrees in water with Ca(OH)2, by accelerated curing ASTM-C1768-17. Of the 11 cylinders, two cylinders were used to obtain its f´c at 28 days, 3 cylinders were used to obtain the time at 85f´c, and 6 cylinders to obtain the CDP.

The frequencies of the specimens were obtained following the C125-14 standard, when they were dried at constant mass with 100 ± 10 degrees using the ASTM C566-13, when they were completely saturated, and in intervals of 20% of their total saturation.

The model of O'Connell and Budiansky was used to obtain the crack density parameter. The results were obtained of the specimens saturated and dried. For the calculation of the Crack Density Parameter, a data processor was used. The initial values were established for v_0 (Poisson modulus of concrete) of 0.25, which is usual in conventional concretes, Eo (modulus of elasticity of the specimen), v_{sat} (Poisson modulus of the saturated specimen), and v_{dry} (Poisson modulus of the dry specimen). Furthermore, with equation (2), an initial value of ε_{sat} and ε_{dry} was obtained. The equation (3) was used to obtain the moisture percentage of the

mixture. Also, with equation (1), the E / E o ratio was obtained. The value of E was obtained from the O'Connell and Budiansky model. Finally, the error between the value of E of the model and E measured was calculated, which were squared. Using the data processor, the value of the squared error was minimized, establishing E_o, v_{sat} and v_{dry} as variables. For the value of ε, it was considered that $ε_{sat} = ε_{dry}$, for which the error was minimal, in this case 0, is what is the value of ε of the specimen.

RESULTS

Table 2, shows the results obtained for the aggregate properties of aggregates used in the mixture.

Aggregate properties:

Evaporable Moisture	Coarse Agg.	0.215%	ASTM	
Content	Fine Agg.	0.64%	$C566-13$	
	Coarse Agg.	4.056%	ASTM $C127-15$	
Absortion	Fine Agg.	0.416%	ASTM- C ₁₂₈ -15	
Resistance Coarse to Agg. Degradation		45%	ASTM- $C131-14$	

Table 2: Results of the aggregate properties used in the mixture

Mixture design was developed following ACI 211 procedure. Mixture Design:

> $water = 268.19$ [Kg] $cement = 391.7 [Kg]$ Coarse aggregate = 879.68 [Kg] Fine aggregate = 810.43 [Kg] Table 3 and Table 4 show the value of the elastic modulus obtained for different water

content for specimens A-F and average results. The measurement and calculation was

developed following ASTM C-215 procedure and calculation.

Results of the experiments:

A		B		$\mathbf C$		D	
Water Content	Ed [GPa]	Water Content	Ed [GPa]	Water Content	Ed [GPa]	Water Content	Ed [GPa]
1.000	28.069	1.000	26.280	1.000	23.220	1.000	28.203
0.813	29.801	0.795	27.019	0.750	21.727	0.800	29.172
0.616	29.128	0.616	26.908	0.600	24.134	0.582	29.002
0.402	26.447	0.389	25.386	0.401	24.044	0.388	26.544
0.211	25.787	0.192	22.594	0.201	21.685	0.202	25.525
0.000	22.464	0.000	23.407	0.000	20.851	0.000	21.600

Table 3: Ed measured at different water content, specimens A-D

Table 4: Ed measured at different water content, specimens E-F, Average

E		F		AVERAGE	
Water Content	Ed [GPa]	Water Content	Ed [GPa]	Water Content	Ed [GPa]
1.000	33.870	1.000	30.121	1.000	28.294
0.813	31.551	0.789	28.388	0.800	27.943
0.600	30.120	0.611	23.923	0.600	27.203
0.383	27.941	0.410	24.825	0.400	25.864
0.204	24.051	0.220	24.498	0.200	24.023
0.000	20.634	0.000	19.442	0.000	21.400

Table 5 shows the CDP calculated for specimens A-H and average results.

Specimen	ε.
A	0.182
в	0.099
C	0.093
D	0.194
E	0.293
F	0.194
AVERAGE	0.188

Table 5: ε value calculated for each Specimen

The average calculated ε value was obtained by performing the same calculation procedure as for the other cylinders, taking the average E values of all the cylinders.

A			в		
E model [GPa]	E measured [GPa]	Error ² [kPa ²]	E model [GPa]	E measured [GPa]	Error ² [kPa ²]
28.690	28.690	0.00	26.280	26.280	0.00
27.563	29.800	$5.00E + 06$	25.703	27.019	$1.73E + 06$
26.353	29.130	$7.71E + 06$	25.194	26.908	$2.94E + 06$
25.018	26.450	$2.05E + 06$	24.542	25.386	$7.12E + 05$
23.808	25.790	$3.93E + 06$	23.970	22.465	$2.26E + 06$
22.460	22.460	0.00	23.407	23.407	0.00

Table 6: Model and measured values for E and error. Specimens A-B

Table 7:Model and measured values for E and error. Specimens C-D

C			D		
E model [GPa]	E measured [GPa]	Error ² [kPa ²]	E model [GPa]	E measured [GPa]	Error ² [kPa ²]
23.220	23.220	0.00	28.203	28.203	0.00
22.639	21.727	$8.33E + 05$	26.927	29.172	$5.04E + 06$
22.286	24.134	$3.41E + 06$	25.508	29.002	$1.22E + 07$
21.814	24.044	$4.97E + 06$	24.221	26.544	$5.40E + 06$
21.336	22.019	$4.66E + 05$	22.970	25.462	$6.21E + 06$
20.851	20.851	0.00	21.600	21.600	0.00

Table 8:Model and measured values for E and error. Specimens E-F

AVERAGE				
E model [GPa]	E measured [GPa]	Error ² [$kPa2$]		
28.078	28.078	0.000		
26.994	28.074	$1.2E + 06$		
25.669	28.049	$5.7E + 06$		
24.286	26.152	$3.5E + 06$		
21.834	24.173	$5.5E + 06$		
21.759	21.759	0.000		

Table 9:Model and measured values for E and error Average

Figure 5 shows the relationship between crack density parameter and E/Eo for all the specimens. Figure 6 show average CDP vs E/Eo in all the mixtures and average value. Finally Figure 7 shows the relationship between water content an elastic modulus in specimens A-F and average.

Figure 5: E/Eo vs CDP for specimens in mixture H, before damage

Figure 6: E/Eo vs Crack Density Parameter, Specimens A-F and Average Figure 7: E vs Water Content, Specimens A-F

ANALYSIS OF RESULTS

Figure 13 shows that the value of the elastic modulus (E) is decreasing when the sample is dried. This behaviour was expected, because when the voids are filled with water in the concrete microstructure, It provides resistance to deformation, for the reason of its incompressibility property. Also, this happens until the pressure of the water breaks the concrete microstructure and can generate new cracks, but in normal conditions this does not occur. Also, when the concrete is dry, the voids are filled with air, so there is no more resistance to deformation in the voids, causing that the concrete will be deformed easier. The average results obtained for the 6 samples, show that the reduction of E happens in smalls steps. It means that it is bigger when the sample are dry. For example, the reduction of E when the concrete is saturated (100% water content) and when lost 20% of the water (80% water content), is 0.00417 GPa. Also, when the concrete lost 40% of the water (60% water content), the difference between its 100% water content is 0.02833 GPa. This means that the concrete has a very similar behaviour when it is completely saturated, and when it has 60% of water content. After 60% of water content the E value starts to decrease in bigger steps, which changes the concrete behaviour considerably.

On the other hand, observing the results in Figures 5-11, the E values calculated according to the model and according to the CDP differ with a similar pattern. This is because the measured values of E are greater than the values calculated according to the model. Furthermore, figure 12 again shows the same behavior of E in the moisture content values of 100%, 80% and 60%. Similarly, the Crack Density Parameter is different in all cylinders, since the compaction of each specimen is different, however, an average of $\varepsilon = 0.182$ can be obtained. According to the model used the E / Eo ratio, when the concrete is completely saturated, it should be 0.83 and completely dry 0.68. Similarly, the average of the measurements under these conditions are 0.86 and 0.67 respectively (Figure 14).

However, observing the intermediate points, in the model these parameters decrease linearly, while the results shows that most of the values remains practically the same up to 60% of moisture content. Then they decay linearly, although they are higher values than those already given by the model. For example, in the case of 40% water content, the model expresses a value close to 0.70, while the average of the measurements has a value of 0.81.

Figure 8: Model values for ε=0.182 (O'Connell & Budiansky , 1997)

CONCLUSIONS

In the research, it shows that the moisture content when the specimen is saturated between 60% -100% are quite similar. Therefore, in this humidity ratios, the behavior of the concrete could be assumed to be the same. In other words, the concrete has the same resistance to small deformations. Even though, it is not completely saturated since its dynamic modulus of elasticity is practically the same. On the other hand, the relationship between the Crack Density Parameter and the E / Eo fraction shows that when the specimen is complete saturated and complete dry, the mixture behaves like the model, but the intermediate values are not behaving as how it was expected with the model. In the 80% and 60% moisture content values, the fraction is practically the same as fully saturated, while the 40% and 20% moisture content values are far from each other, and from 0%. Turning out that the concrete has a behavior similar with the O´Connell & Budiansky soft fluid model. Figure 4 (b).

RECOMMENDATIONS

This study was performed only for one mixture in its natural state. There was any load applied on it. However, it is recommended that the specimens are subjected to a compression load to understand its behaviour once the concrete has been damaged. Also, given the behavior that the concrete had in this research, it is recommended to use the model for a soft fluid, and verify if the material behaves in this way. Finally, a second mixture can be made with a different water-cement ratio to compare the behavior of the material.

REFERENCES

- O'Connell , R. J., & Budiansky , B. (1997). Viscoelastic properties of fluid‐saturated cracked solids. *Journal of Geophysical Research (1896-1977)*, Volume 82, Issue 36.
- Calixto, F. (2002). Microcracking of High Performance Concrete Subjected to Biaxial Tension - Compression Stresses. . *Materials Research. 5. 10.1590/S1516- 14392002000300013.* .
- Designing Buildings Ltd. (2020, February 22). Retrieved from Designing Building Wiki: https://www.designingbuildings.co.uk/wiki/Concrete
- F. O. Slate, K. C. Hover. (1984). *Microcaking in concrete Volume 3.* ISBN : 978-94-009- 6151-7.
- Hilal, A. (2016). Microstructure of Concrete. *10.5772/64574*.
- McDonald, D. (2012, September). *Omafra*. Retrieved from http://www.omafra.gov.on.ca/english/engineer/facts/12-047.htm
- Mehta P.K., Paulo J.M. Monteiro. (2006). *Concrete: Microstructure, Properties, and Materials.* New York: McGraw-Hill.
- Recalde, J. J. (2009, December 18). *Estimating Crack Growth in Temperature Damaged Concrete.* Retrieved from http://www.lib.ncsu.edu/resolver/1840.16/3794
- Rosero, O. (2018, Mayo 11). Retrieved from
	- http://repositorio.usfq.edu.ec/bitstream/23000/7213/1/137615.pdf
- Technologies, B. (2019, August 17). *GIATEC.* Retrieved from https://www.giatecscientific.com/education/cracking-in-concrete-procedures/

ANNEX A: GRANULOMETRY

Figure Annex 1 Sieve analysis of fine aggregate

Figure Annex 2 Sieve analysis of coarse aggregate

ANNEX B: MIXTURE DESIGN ACI-211

ACI 211.1

- $Slump = 75 mm [5in]$
- *Water* = $205[Kg]$
- Water/Cement = 0.6
- *Cement* = $205[Kg]/0.6$ = 341.7 $[Kg]$

AGREGGATE:

$$
\frac{VAG}{VI} = 0.62
$$

\n
$$
VAG = 0.62m^{3}
$$

\n
$$
MAG = 0.62m^{3} \left(1415.8 \frac{Kg}{m^{3}} \right) = 877.796 \left[Kg \right] * 913.37 \left[Kg \right]
$$

\n
$$
DM = 2270 \frac{Kg}{m^{3}}
$$

First mixture design:

Humidity correction

- *CHAG* = 0.215%
- **A AG** = 4.053 %

Coarse aggregate:

 AG $dry = 906.1$ $[Kg] \rightarrow 877.796$

 $AGSSD = 942.8 [Kg] \rightarrow 913.37$ $AG AD = 877.796 [Kg] (1 + 0.00215) = 879.68 [Kg]$ $\Delta W\% = A - C.H = 4.063\% - 0.216\% = 3.838\%$ ΔW AG = 877.796 [Kg] (0.03838) = 33.69 [Kg] water

Fine aggregate: $C. HAF = 0.637\%$ $AAF = 43\% (aprox.)$ $AF = 839.93$ [Kg] / 1.043 = 805.302 [Kg] $AFAD = 805.302 [Kg] (1.00637) = 810.43 [Kg] (A.D)$ $\Delta W\% = (4.3 - 0.637) = 3.663\%$ $\Delta WAF = 805.302$ [Kg] * 0.03663 = 29.498 [Kg] water

Mixture design corrected:

 $water = 268.19$ [Kg] $cement = 391.7$ [Kg] Coarse aggregate = 879.68 [Kg] Fine aggregate = 810.43 [Kg]