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Vibration Modes and Frequencies of Pre-Hispanic Arsenic Bronze Bells

Proyecto de investigación

.

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Ingeniería Mecánica

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RESUMEN

Los cascabeles son instrumentos que se destacan en culturas prehispánicas en América Central y del Sur. Los materiales con los cuales éstos eran fabricados varían según la época y la región. Fueron fabricados principalmente de bronce y cobre-arsénico. Estudios previos han tenido lugar con respecto a las aleaciones que se utilizaron para fabricarlos. Este proyecto busca encontrar los modos de vibración y las frecuencias resonantes de los cascabeles a través de simulaciones computacionales utilizando el software ANSYS y la verificación de ABAQUS.

Palabras clave: Acústica, cascabeles, instrumentos prehispánicos, ingeniería de vibraciones

ABSTRACT

Bells are prominent examples of musical instruments in prehispanic cultures in Central and South America. The manufacturing materials vary greatly depending on geographical location and time period. However, Copper-Arsenic (arsenic bronze) alloys were widely used to manufacture bells and many other tools. The material properties of arsenic bronze have been determined in previous studies. The bells elemental composition will be analyzed with SEM-EDS for elemental composition and microstructure. This project aims to obtain the modes of vibration of the bells, along with the bells' fundamental resonant frequencies through simulations using ANSYS.

Key words: Acoustics, bells, prehispanic instruments, vibration engineering

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1. Introduction

1.1. Background

Metallurgy was used in many pre-Hispanic cultures in Central and South America. This resulted in the creation of multiple tools and traditional artifacts. Among these, bells stand out for their capability of producing sound. Many studies have taken place regarding the alloys that were used to manufacture them. Bronze as an alloy has a rich history throughout the world, with copper being its matrix, arsenic was used since early times to change the metal's properties. Researchers have proved that copper-arsenic (Cu-As) alloys have more desirable properties than pure copper. However, this hypothesis does not have any vibrational or acoustic engineering background. This project seeks to fill the gap regarding the acoustic properties of these specific type of bells through simulation with ANSYS in order to obtain the modes of vibration and the resonant frequency of the bells.

Archaeological and ethno-historical research has been conducted to evaluate the historical importance, manufacturing methods, and the composition of several artifacts, including bells. However, there is not much information about the mechanical properties and its effects on their functionality. The lack of this type of approach misses vital information for a better understanding of the function and properties of the bells. Moreover, it acknowledges the lack of fruitful multidisciplinary collaboration and exchange, which this study is trying to overcome by working with the National Institute for Cultural Heritage (Instituto Nacional de Patrimonio Cultural, INPC) and raising awareness on the importance of analyzing archaeological artifacts with the lens that vibrational or acoustic engineering provides. Moreover, this can be used in order to determine how desirable the bells' acoustic properties would be as seen from a musical standpoint.

Aside from enriching the knowledge we have of prehispanic societies, this type of study allows institutions to make better and well-informed decisions regarding the conservation of these valuable artifacts that belong in the Ecuadorian cultural heritage.

According to Meyer (2009), the field of Acoustics of Musical Instruments is of great importance for Acousticians, Audio Engineers, Musicians, Architects and Musical Instrument Makers and has diverse applications in musical performance and the study of musical instruments, from a vibration studies perspective. In this context, Damaske (2008) states that the study of physical sound waves, and out of these waves the subjective sound impressions that are created in the listener's head is key to develop a theory of how music is appreciated.

Hartmann (2013), in his text on Principles of Musical Acoustics focuses on the basic principles in the science and technology of music and how musical examples and specific musical instruments demonstrate the principles. The science of musical acoustics is possible thanks to the study of vibrations and waves, the basic physical properties of sound, and the human element, the physiological and psychological aspects of acoustical science. The perceptual (and often referred to as "subjective") aspects of sound include loudness, pitch, tone color, and localization of sound.

In relation to archaeo-acoustic studies and acoustics informed by ethnomusicology, Zalaquett and Espino (2018) suggest that the sounds produced by human beings, as well as those of the instruments they produce and the environment that surrounds them, are perceived and codified depending on their experiences as part of a cultural group. In this sense, when developing an acoustic study of a musical instrument it is very important to incorporate a perspective on how the instrument plays a role as part of communication inside a social group and among different groups. Particularly, Zalaquett and Espino present their study of two triple Mayan flutes that come from excavations at the archaeological sites. The authors make clear that archeoacoustic analysis are very new and they are important because they provide information about the variety of sounds produced, the process of manufacturing and the sonorous preferences for the case of pre-Hispanic societies.

Following McLachlan and Nigjeh (2003), musical bells have had limited application due to the presence of inharmonic partials in the lower part of their acoustic spectra. The use of methods such as finite-element analysis models to produce acoustic profiles and fine tuning using gradient projection method. In their work, a range of bell geometries and timbres are susceptible of being analyzed using psycho-acoustic models and is discussed in relation to European carillon bells. In this regard, Rossing (1984) underlines the necessity of the development of a field of study that refers to the acoustics of bells. According to this author, studying the vibrations of large and small bells is helpful to understand the sounds of one of the world's oldest musical instruments.

Palmer et al (2007) presents a chemical analysis of pre-Hispanic bells but lacks an archeoacoustic perspective. Sixteen of the analyzed bells contained minor and trace amounts of silver, antimony, arsenic, lead, and in some cases tin, strontium, and selenium, and nine bells contained little or no detectable amounts of these elements, with no further information on how the presence of this elements affects the sound produced by these instruments.

Throughout the world, there is a crescent need for archeoacoustic profile analysis. An example of this is the study developed by Jing (2003) around the acoustics of an ancient Chinese bell that was made 3000 years B. C. According to the author, an ancient Chinese bell is sometimes called a two-tone bell or a music bell because unlike a western church bell and an ancient Indian bell, the Chinese bell has interesting acoustics: two tones can be heard separately as the bell is struck at two special points as the interval between the two pitches is always a minor or major third, and the tones of the bell attenuate fast. Jing’s study uses a three-dimensional model to simulate the acoustics of the bell, so that the two tones of the ancient Chinese bell can be simulated by the vibration of a double-circular arch and the quick attenuation of tones can be simulated by acoustics of a cylinder with the lens-shaped cross section like a double-circular arch.

In a different context but with similar aims, Aldoshina et al (2003) speak of an analysis of peculiarities of Russian Bells acoustic parameters. According to these authors, the particularities of the tuning and acoustical characteristics of Russian bells were investigated by various scientists for a long time but they highlight the necessity renewing those conclusions through new research. Their methods include the digital recording of sounding of 16 to 20th century bells made in various monasteries and temples of Russia; the computer processing and restoration of obtained recordings; the spectral and statistical analysis of soundings and comparison of bells tuning with that of a conventional Dutch system; development of mathematical models of bells vibration; creation of the software for the analysis spectral frequencies and modes of vibration; and finally the synthesis of their geometric form to optimize the spectrum structure.

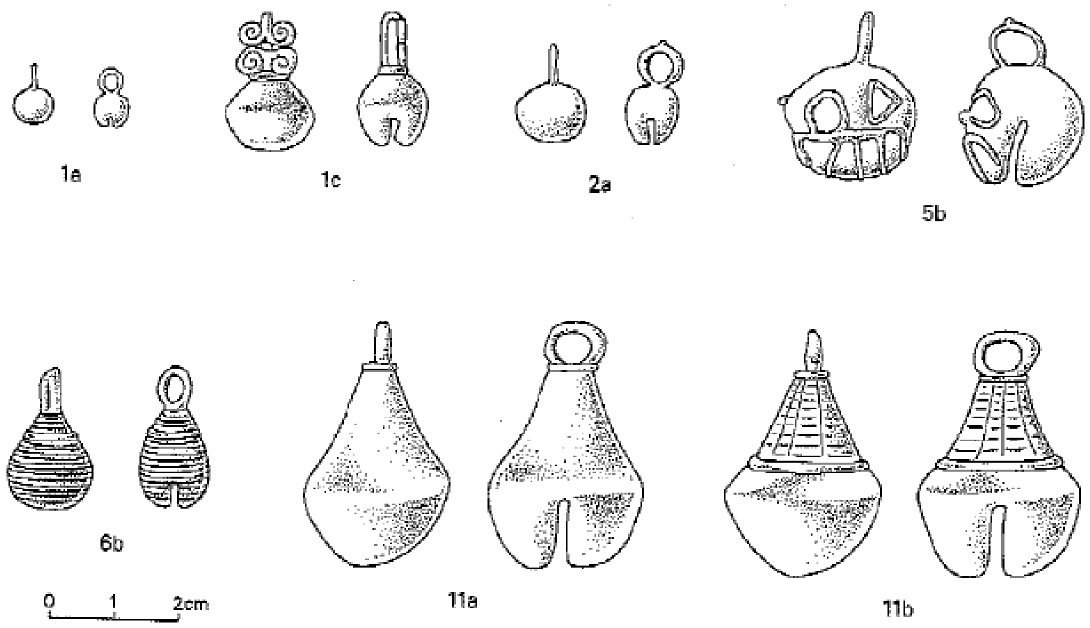


Figure 1: Different types of bells found in South and Central America

According to Roebben, et al (1997), solid objects have a characteristic set of mechanical resonant frequencies (f_r) which are related to the object's mass, dimensions, and elastic properties.

1.2. Phase diagrams

The interaction of the Copper-Arsenic alloy has been thoroughly studied by Lechtman (1996), who created the phase diagram shown in **Figure 2**.

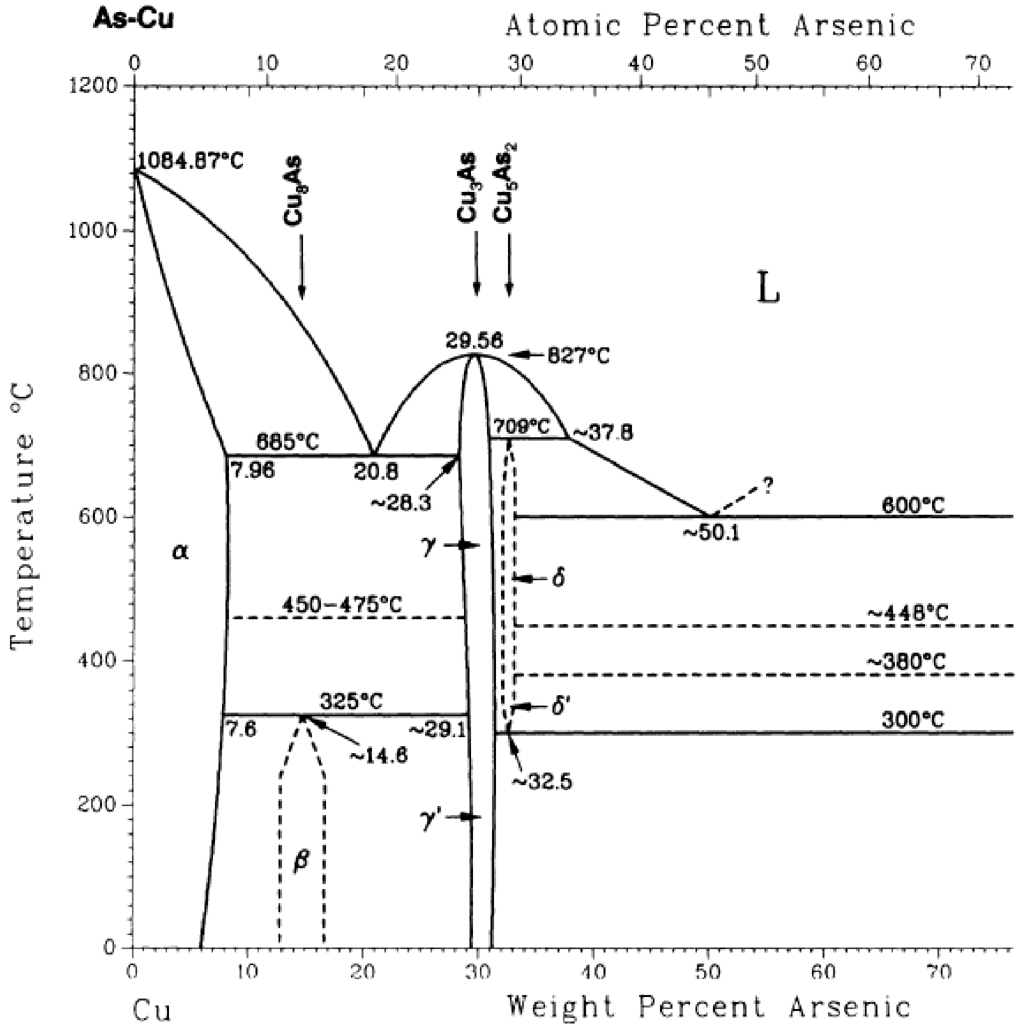


Figure 2: Cu-As phase diagram

The phase diagram shows a steep decrease in the melting temperature of the alloy as the weight percent of As until the eutectic point at 20.8% As.

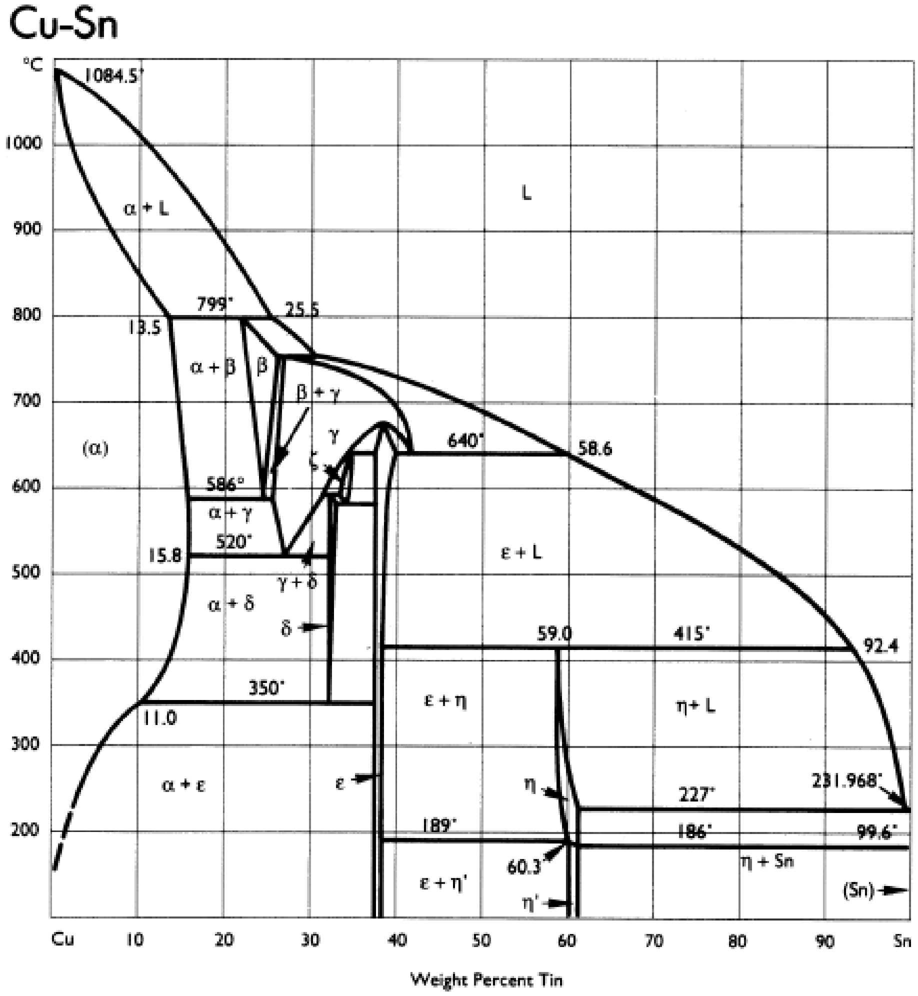


Figure 3: Cu-Sn phase diagram

Dr Lechtman (1996) confirmed in her study that Arsenic was added to copper intentionally. Her study found the effects of Arsenic on Copper similar to the ones of Tin.

As it can be seen in the Copper-Arsenic phase diagram (**Figure 2**), to maintain a temperature where the alloy is in a liquid state, it requires a more thorough control of the temperature due to the peak at 827 ºC; 29.56% w/w, which limits the alloy to a smaller percentage of the arsenic content and therefore, higher melting temperatures. Whereas, as shown in the copper-tin phase diagram **(Figure 3**), as the percentage of tin increases, the melting temperature decreases almost constantly. Therefore, for the technology of the time, replacing the arsenic with tin generated results with less variability because it did not need so much temperature control to maintain a liquid phase in the alloy.

1.3. Scanning electron microscopy

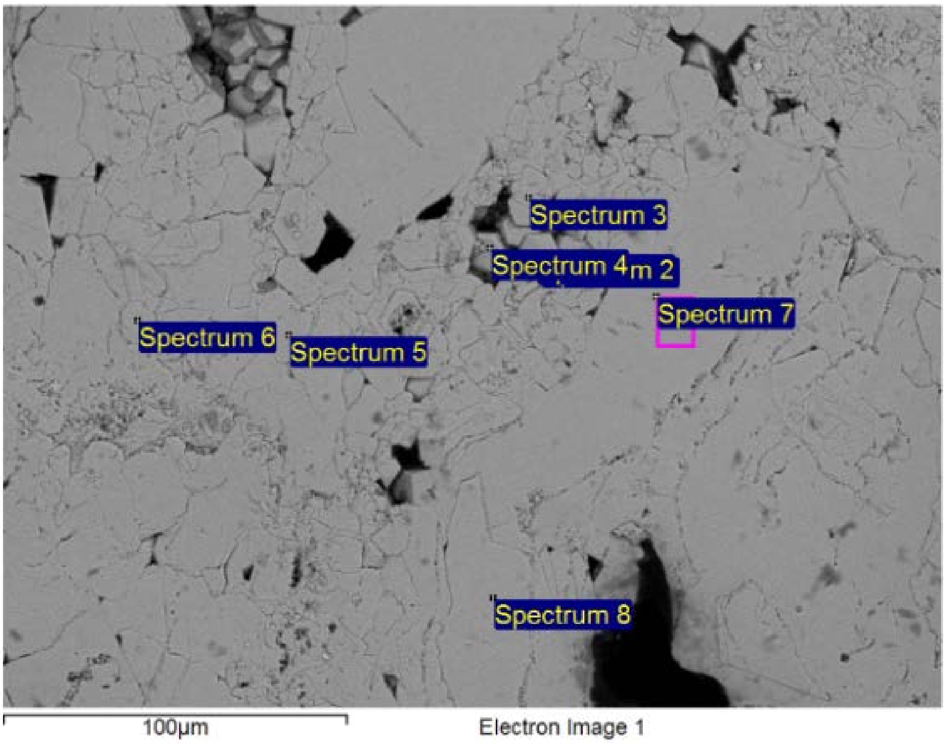


Figure 4: SEM for the bell sample 250X indicating various spectra that correspond to the EDS

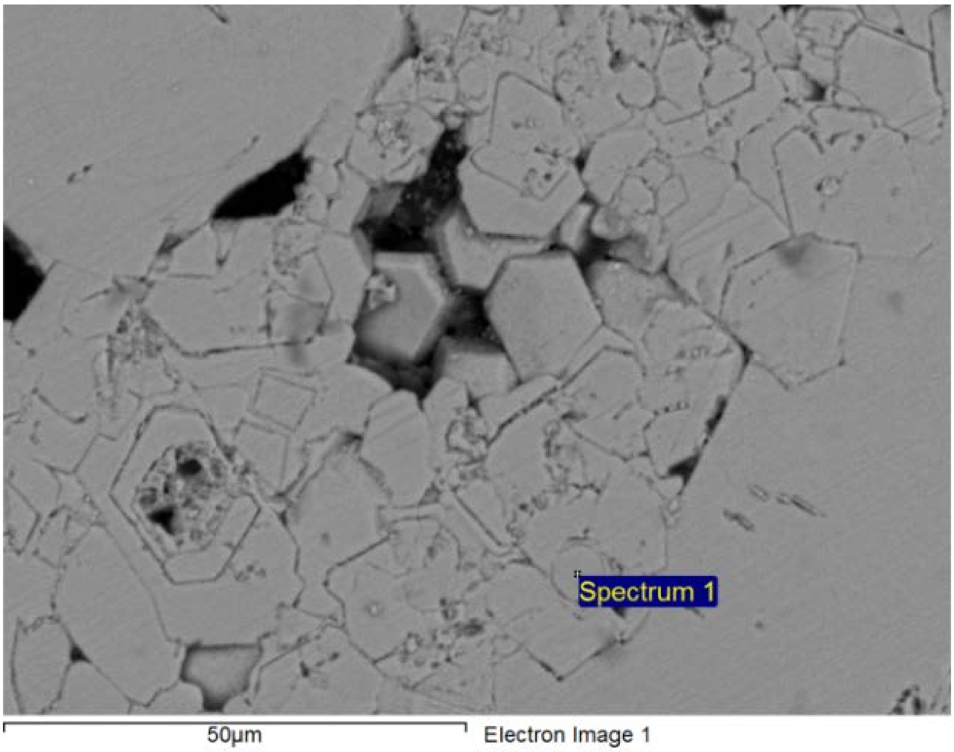


Figure 5: SEM for the bell sample 500X

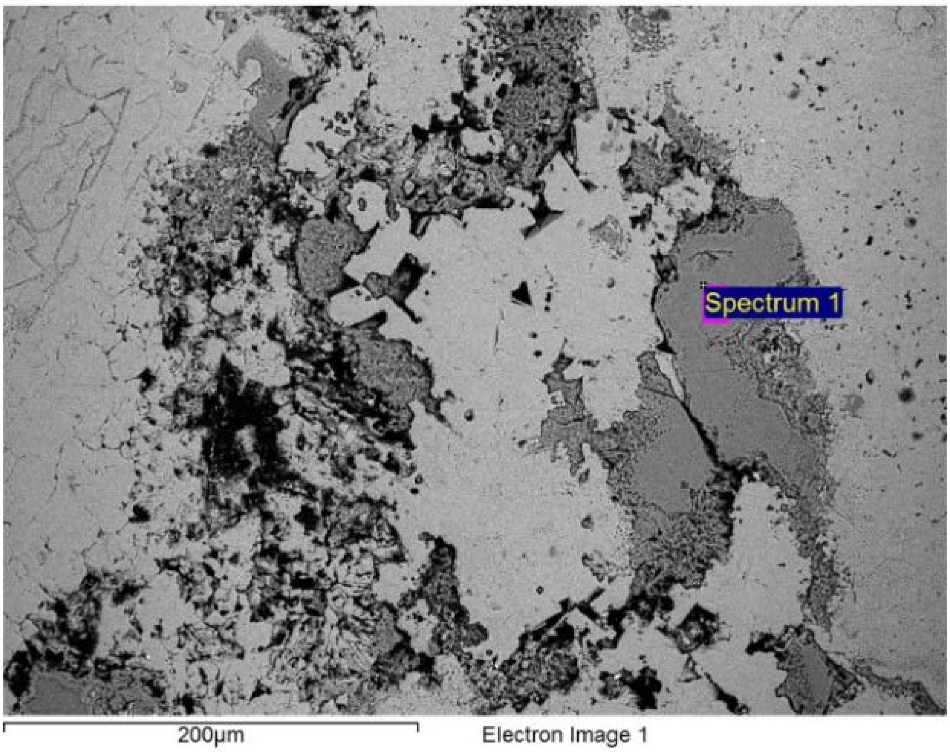


Figure 6: SEM for the bell sample 100X

In **Figure 5** octagonal cooper crystals can be seen with a segregation of Cu-As: Cu (65-67)%, As (32-33) %, and impurities of silicon, aluminum, and sulfur. **Figure 7** shows the EDS result that confirms the two most common elements to be Cu and As.

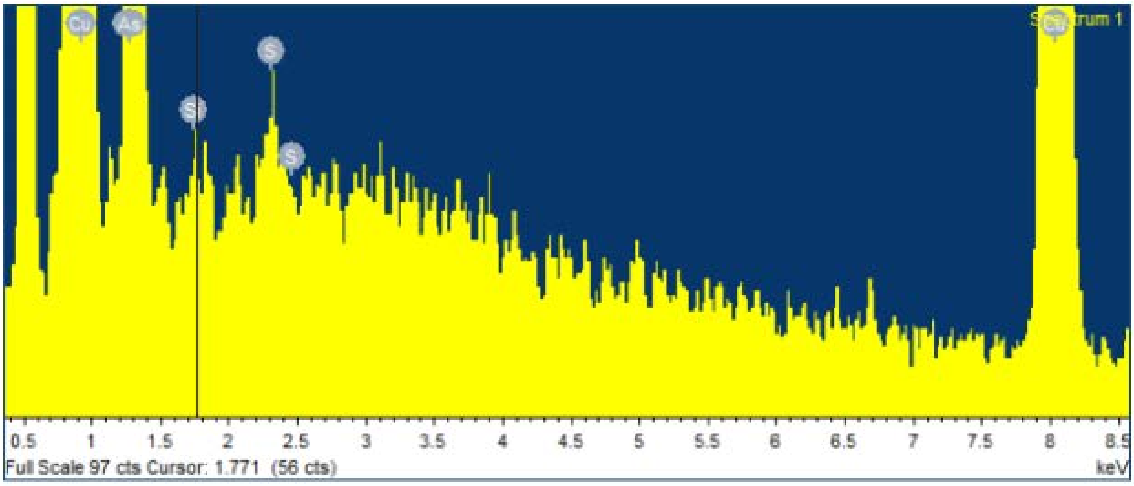
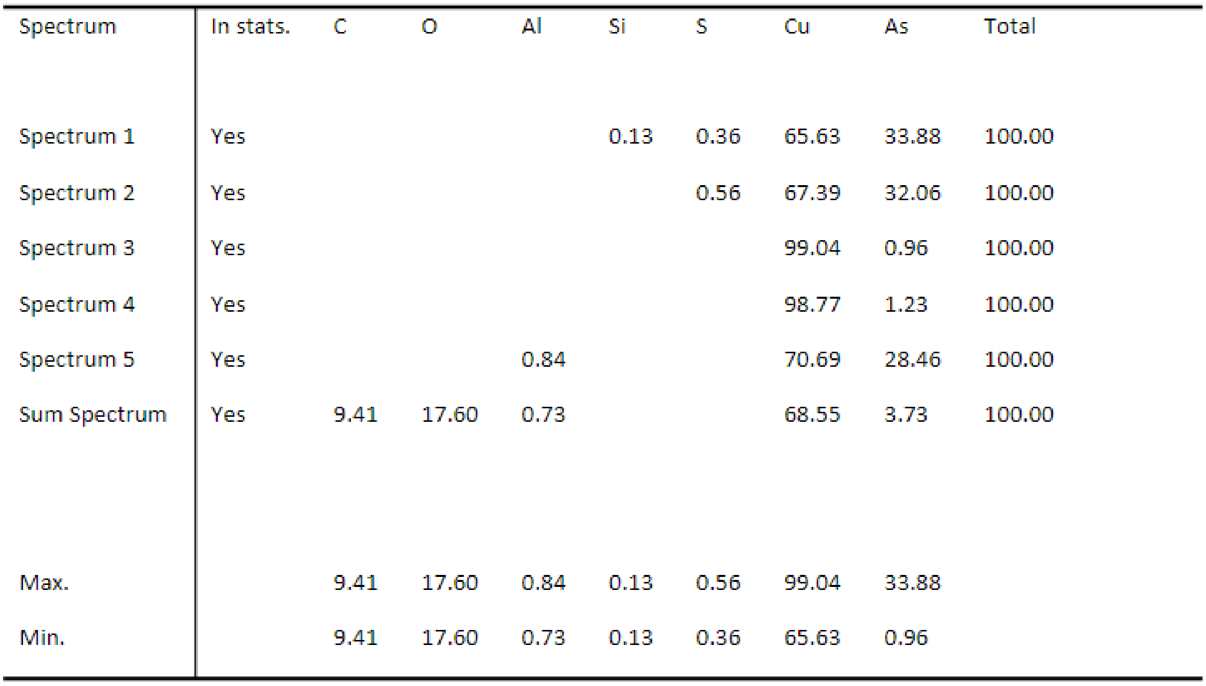


Figure 7: EDS Results for the bell sample.

Table 1: Normalized elements in %



Spectrum	In stats.	C	O	Al	Si	S	Cu	As	Total
Spectrum 1	Yes				0.13	0.36	65.63	33.88	100.00
Spectrum 2	Yes					0.56	67.39	32.06	100.00
Spectrum 3	Yes						99.04	0.96	100.00
Spectrum 4	Yes						98.77	1.23	100.00
Spectrum 5	Yes			0.84			70.69	28.46	100.00
Sum Spectrum	Yes	9.41	17.60	0.73			68.55	3.73	100.00
Max.		9.41	17.60	0.84	0.13	0.56	99.04	33.88	
Min.		9.41	17.60	0.73	0.13	0.36	65.63	0.96	

As seen on **Table 3** some results show that the alloy has up to 33% of arsenic, which can be explained considering the types of corrosion that affected the bells. Selective and intragranular corrosion could affect this kind of alloys when exposed to humid environments as it acts like a galvanic cell. In this case, it could be said that the alloy lost part of its copper mass because of selective corrosion at the grain boundaries. As a result of this, the proportional concentration of arsenic increases. Also, considering that the people who made these bells had limitations around temperature, heat control and insulation, and that the result is a non-homogeneous material as it can be seen on **Figure 6**.

2. Theoretical framework

2.1. Mathematical model

In order to understand the method used by ANSYS to obtain the modes of vibrations and frequency response of a 3D part, a mathematical model that yields the mode shapes of a rectangular vibrating plate is presented. The equation that represents the free vibration of a rectangular plate is presented below.

$$D\nabla^2\nabla^2 w(x,y,t) + \rho h \frac{\partial^2 w}{\partial t^2}(x,y,t) = 0$$

Given that the plates response is known, the following solution can be assumed: $w(x,y,t) = (Acos\omega t + Bsin\omega t)W(x,y)$ which is a separable equation that contains a two-dimension shape function $W(x,y)$ that represents the plate's modes of vibration, and a harmonic function of time. ω is the natural frequency of the plate, which is related to the vibration period T in $\omega = \frac{2\pi}{T}$. The displacement vector $w(x,y,t)$ can be expressed as a function of the unknown joint displacements in the form

$$w(x,y,t) = \sum_{i=1}^{n} N_i(x,y) w_i(t)$$

Where $w_i(t)$ represents each joint's displacement, n represents the number of unknown joint displacements, and $N_i(x,y)$ represents the shape function for the i_{th} mode. The meshing process creates a similar model with 25341 nodes and 13575 to perform the above calculations in order to obtain the deformed shapes of the vibrating bell.

2.1.1. Plate approximation

The walls of the bell can be modeled as plate sections with different boundary conditions to resemble the way plates are joined together. First, a rectangular plate is modeled. Then, a second plate with the top corners rounded is proposed as a more accurate approximation to the bell's wall shape. Both plates are shown in **Figure 8**. A mesh is generated under the “fine” setting in ANSYS to apply the modal analysis along with the boundary conditions explained below.

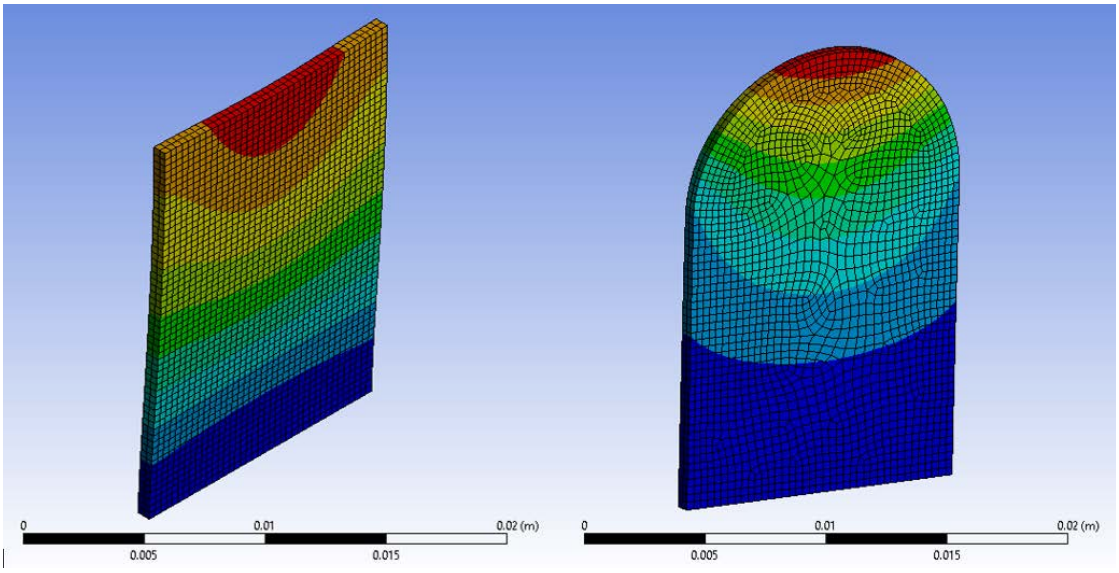


Figure 8: Rectangular and rounded plates with the displacement of the 1st mode of vibration

2.1.2. Plates’ boundary conditions

The boundary conditions on the plate are set in a way that resembles how individual segments are linked on the real bell. For this, a fixed condition is set on the bottom of the plate to resemble the bell portion connected to the handle. Additionally, an axis specific boundary condition is set to both lateral edges of the plate. Boundary conditions are shown on **Figure 9**.

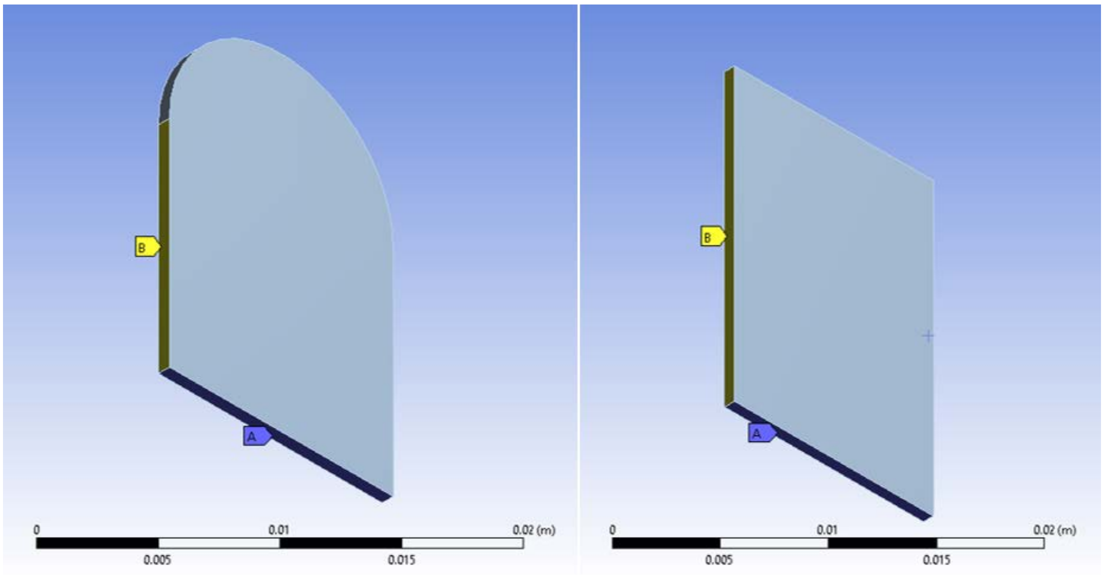


Figure 9: Boundary conditions for both plates

2.1.3. Damping

According to Pouladkhan (2011), damping effects are created by internal friction or by interaction with the surrounding medium. The effect that damping can have on the modes of vibrations and frequencies is negligible unless the system is overdamped, where the bell would not resonate enough to create an appreciable sound. This is the reason for this study to consider no damping as to obtain the undamped frequency.

The vibrational amplitude A_0 for each mode decays exponentially as follows:

$$y(t) = A_0 e^{-\epsilon\omega(t)} sin(\omega(t) + \phi)$$

Where ϵ is the modal damping, represented as a fraction of the critical damping. ($\epsilon = 1$ means critically damped), and is dimensionless. Experimental and analytical data was found by van Houten, Schoofs and van Campen (1997) for damping values of a handbell. Their study determined damping values for different frequencies on a handbell and found an average value for the audible frequency spectrum of 0.2×10^{-3}, their results are shown on **Figure 10.** This diagram shows values on the 10^{-4} order of magnitude and no value exceeds 3×10^{-4}.

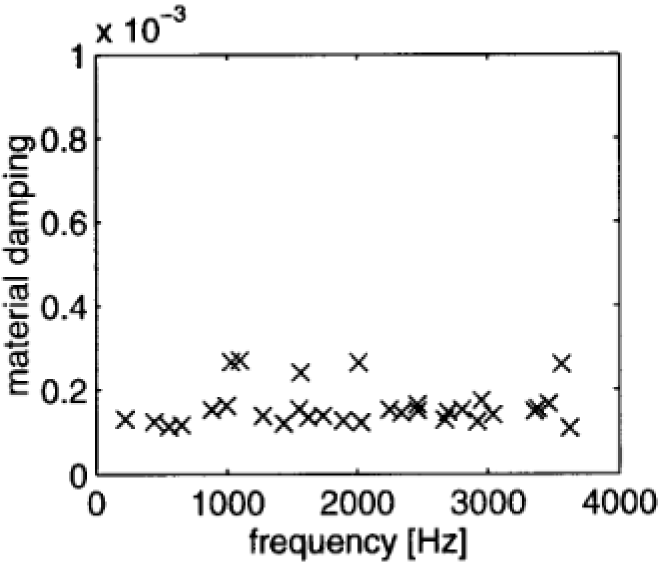


Figure 10: Experimental damping values for bronze handbells

3. Materials and methodology

3.1. Materials and equipment

The core materials for this project were the Arsenic bronze bells. Analysis of the elements and phases involved in the bells were obtained by using a Scanning electron microscope Jeol Model IT300 provided by the INPC.

3.1.1. Scanning electron microscope (SEM) Jeol IT300

Scanning electron microscopes utilize a tungsten filament which is charged to release a beam of electrons onto a sample surface. Sensors record the electrons that diffract from the surface, thus piecing together an image of the surface of the sample. This permits imaging analysis at a nanometric scale with very high resolution. Additional to this system, Energy Dispersive Spectroscopy (EDS) was also utilized. This tool emits an X-Ray when the electron beam shot by the SEM displaces an inner shell electron. Since elements have distinct energy differences between shells, this can be used to effectively determine the elemental composition of the substance that is being analyzed.

3.2. Methodology

3.2.1. Bell geometry measurement

The bells are thoroughly described by Hosler, (2005) and some characteristic shapes are found in **Figure 1**. There are many different designs that have been used throughout South and Central America. The samples that this study focuses on come from the northern highlands of Ecuador, in the province of Carchi.

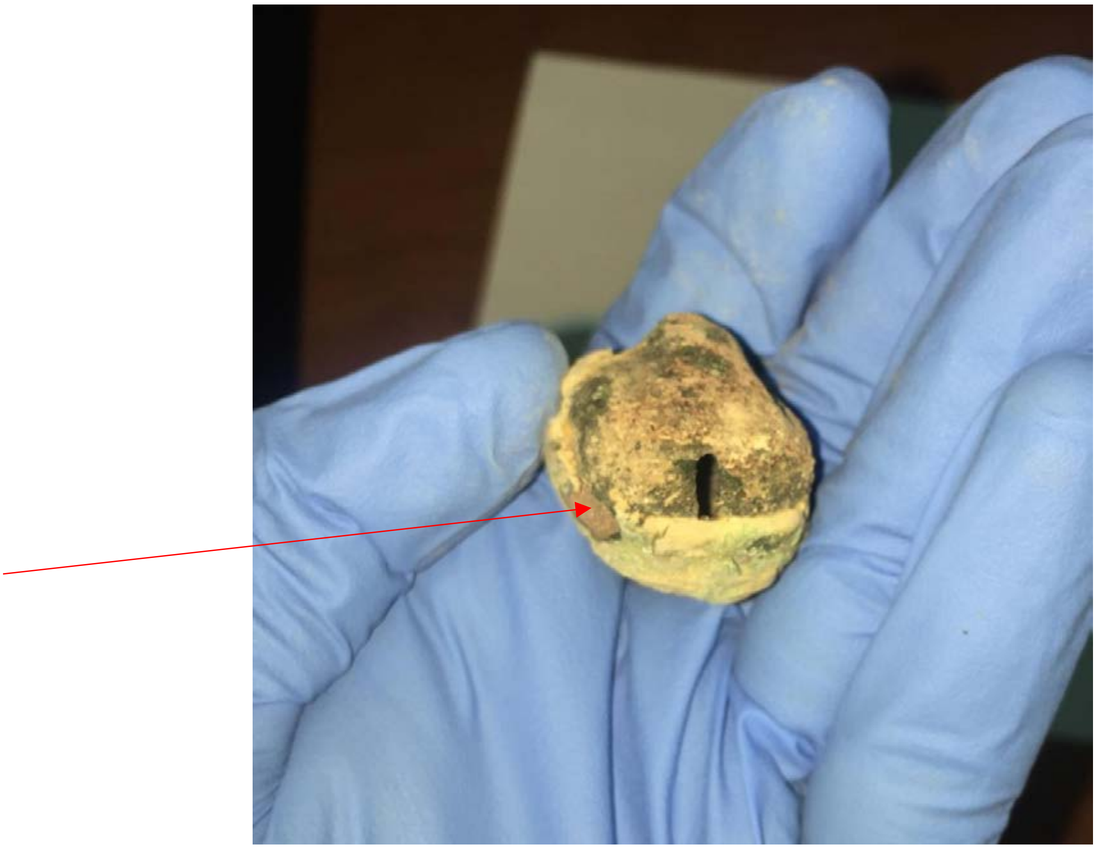


Figure 11: Bell showing its corrosion layer. Arsenic bronze is visible near the arrow.

Figure 11 shows the bell that has been conserved best. Measurements were performed with calipers and proved using Solidworks image scaling with the calipers shown in **Figure 12**. If a rectangular prism box were to be drawn around the bell, the dimensions would be 28.8 *mm* wide, 32.7 *mm* tall, and 28.64 *mm* deep.

Figure 12: Bell with calipers to acquire dimensions.

The bells are mostly elliptical spheres, they have an irregular surface and a shell thickness of 1.3 *mm*. Given that the bells are thought to be from around the 11th century (Periodo de Integración), corrosion has made it pointless to perform acoustic studies on the bells themselves. Moreover, the bells have been classified as cultural heritage, which means the studies are limited and must be non-destructive. A small piece from an incomplete bell was used to create the sample for the SEM analysis.

3.2.2. CAD modeling

A CAD model was created based on the geometry described on the previous section with a handle based on the most commonly available bells from **Figure 1.** A few views of the model are depicted on **Figure 13**.

Figure 13: CAD model with handle

3.2.3. ANSYS simulation parameters

ANSYS geometry, modal, and harmonic response modules were used in order to create the vibro-acoustic profile of the bells. First, the geometry was imported as a Parasolid file, which was then assigned the copper alloy material. A mesh was generated after this with the “Fine” setting. This setting yields 25341 nodes and 13575 elements. The modal simulation was conducted with the parameters shown in **Tables 1** and harmonic response simulation parameters shown in **Table 2**.

The impact force required for the harmonic simulations was found based on a 1g clapper that strikes an inner wall of the bell. This force is an impulse function and was found to have a magnitude of 0.01N using $F = ma$.

As seen in the SEM images discussed in section 1.3, the material is not homogeneous, meaning that the density is not constant throughout the material. Therefore, an average value for the density found in Dr Lechtman's study (1996) allows us to have an accurate representation of the real material used, and feed this value into ANSYS.

As discussed in section 2.1.3, the damping value to be used is the average value found experimentally in the study mentioned above.

The following parameters were introduced in the software:

Table 2: *Modal simulation Parameters.*

Cu Alloy density	8300 kg/m^3 (Lechtman, 1996)
Modes of vibration	9
Lower limit	0 Hz
Upper limit	12000 Hz

Table 3*: Harmonic response simulation parameters*

Impact force	0.01 N
Constant damping ratio	0.2×10^{-3}
Solution intervals	250
Lower limit	0 Hz
Upper limit	12000 Hz
Young's module	1.1E+11
Poissons's ratio	0.34
Bulk module	1.146E+11
Shear module	4.104E+10

3.2.4. Boundary conditions and impact location

The boundary condition for both the modal and harmonic simulation consists on a fixed support located on the handle as shown on **Figure 14.**

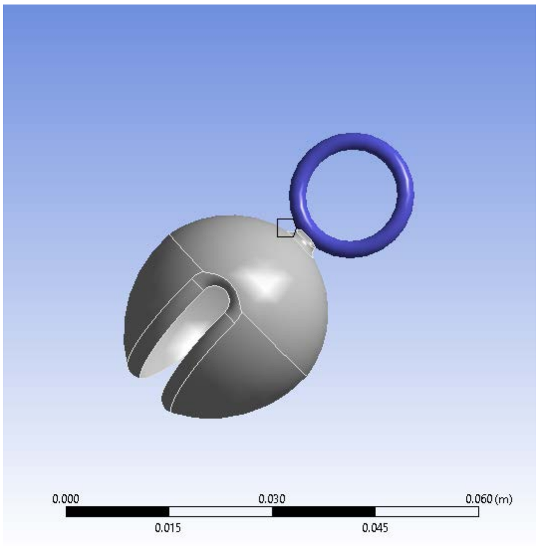


Figure 14: Location of fixed support

The impact force location is shown in **Figure 15.**

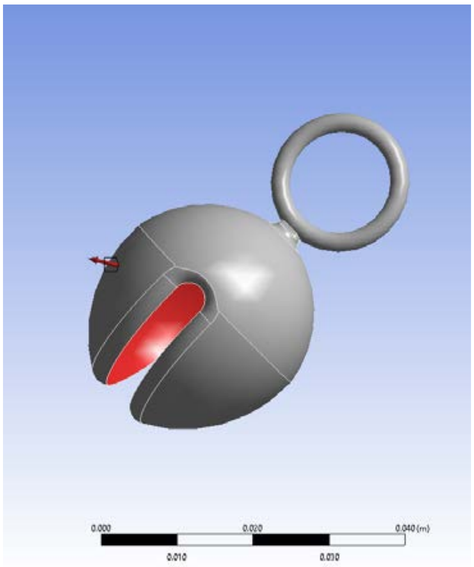


Figure 15: Location of clapper impact force (on the red internal surface, outwards direction)

The finite element method allows us to see the repeating shapes in which the bell deforms while creating sound by agitating the air molecules around it. The plate approximation shows a similar behavior with its given boundary conditions. The nodes and elements created by the meshing process are analyzed with a matrix system like the one shown in section 2.1. On the other hand, the harmonic response simulation yields the frequency response, which represents the actual sound produced by the bell. The audible frequency spectrum was divided in 250 segments to determine the resonant frequencies that follow the fundamental frequency.

The program solves the transient response for all the spectrum, where the 0.01N force imposed by the striker creates the excitation that makes the bell vibrate and produce sound according to its mechanical properties.

3.2.5. SEM-EDS

The metallic piece was prepared by making a transversal cut with a precision diamond saw. Afterwards, a chemical attack was done on the surface with a HNO3 and H2O solution in a 1:10 ratio. The sample was then analyzed in the SEM with the EDS probe in different locations labeled as Spectra in section 1.3.

4. Results and discussion

4.1. Modal simulation

The first thing to highlight regarding the modal simulations is that the modes that involve a total movement of the bell are not counted given that these modes do not create any sound. This study was conducted this way because these modes involve translational vibrations or rotations of the entire bell as opposed to the vibration that results in the deformation of different sections of the bell relative to the undeformed state. In consequence, the frequencies that result from the total movement of the bell described before are not considered in this study.

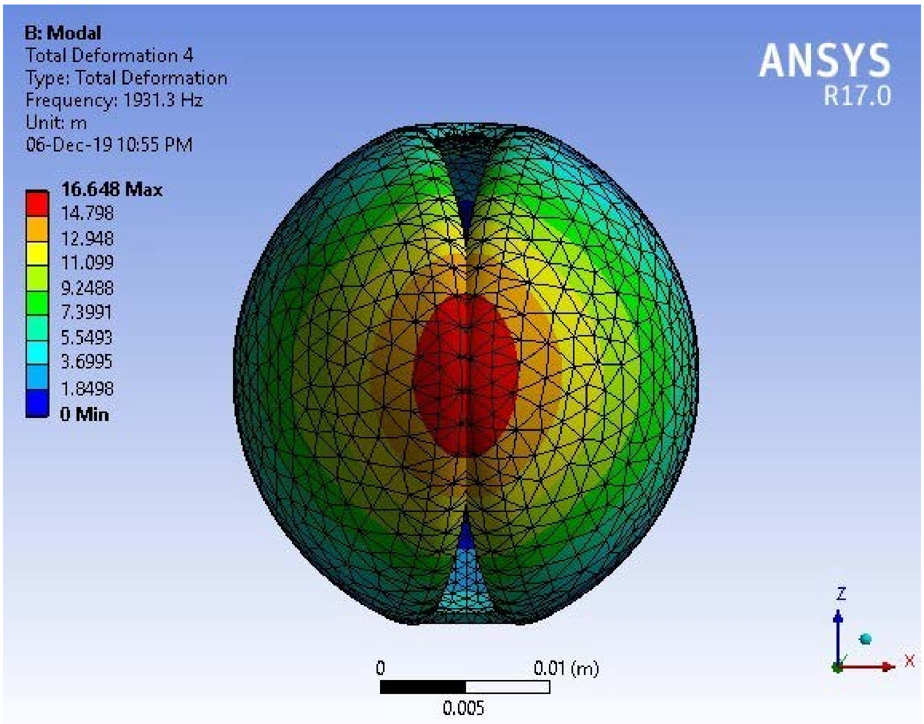


Figure 16: Modal simulation of the bell. Type: Total deformation. Frequency: 1931.3 Hz

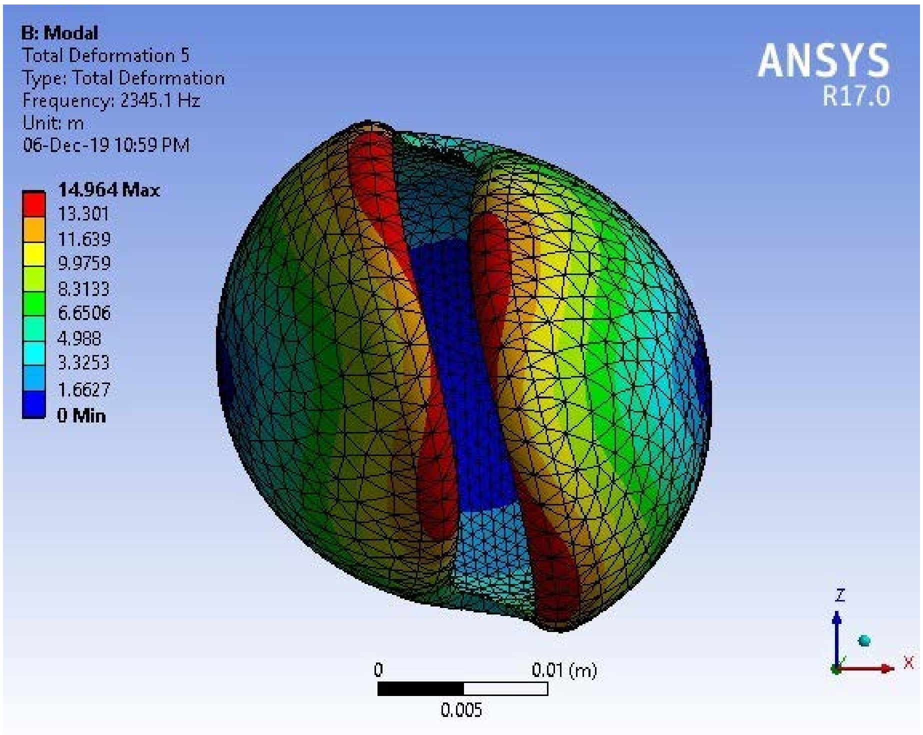


Figure 17: Modal simulation of the bell. Type: Total deformation. Frequency: 2345.1 Hz

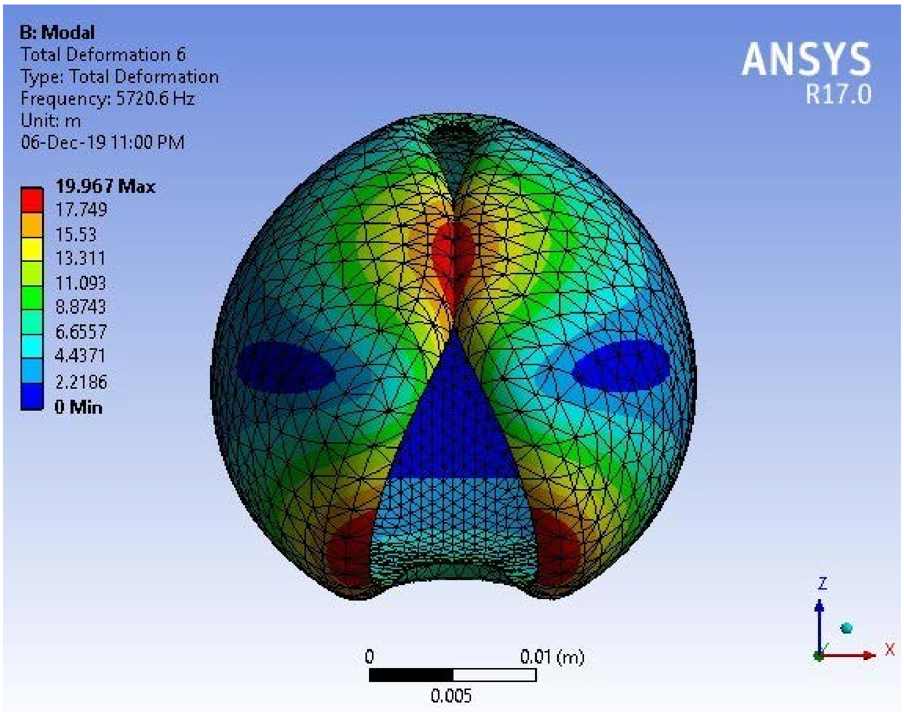


Figure 18: Modal simulation of the bell. Type: Total deformation. Frequency: 5720.6 Hz

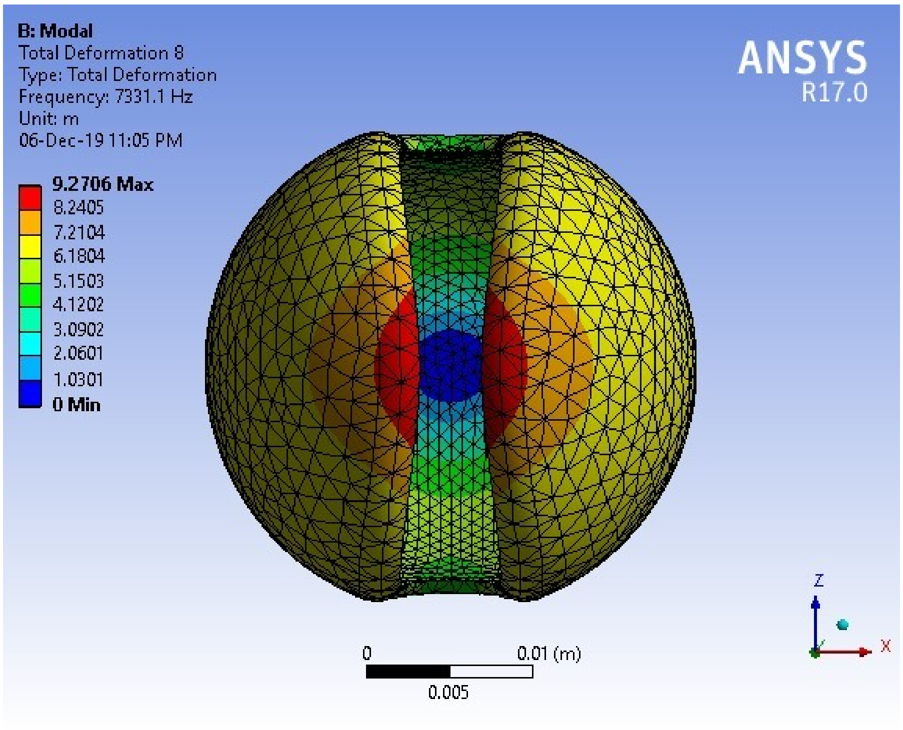


Figure 19: Modal simulation of the bell. Type: Total deformation. Frequency: 7331.1 Hz

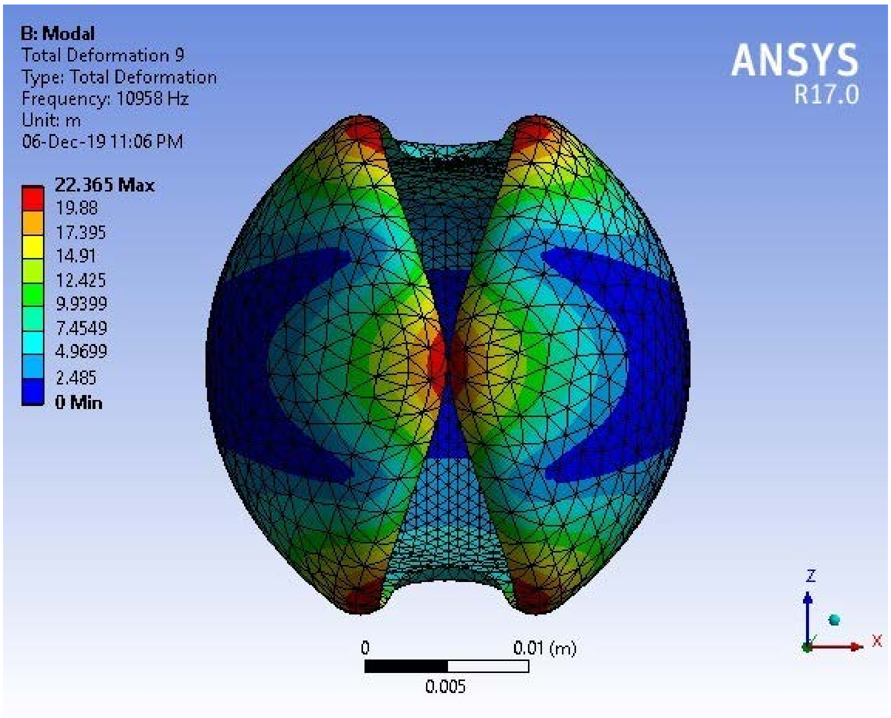


Figure 20: Modal simulation of the bell. Type: Total deformation. Frequency: 10958 Hz

The modal and harmonic simulations show how the shells of the bell vibrate in resonance. In real-life applications, several of these modes of vibration will intersect and emit a sound that results from this combination.

4.2. Harmonic simulation

Multiple damping values were used in order to test its effect on the resonant frequencies. A shift in frequencies was not observed with higher damping values. However, the peaks in the frequency domain started diminishing their amplitude. Moreover, when the damping value was set to zero, the frequency peaks are the same as with the value of 0.2×10^{-3}.

It is important to mention that, while all modes coexist simultaneously while the bell vibrates, higher frequency modes have significantly smaller amplitudes. The fundamental frequency represents the dominating sound produced by the bell’s resonance.

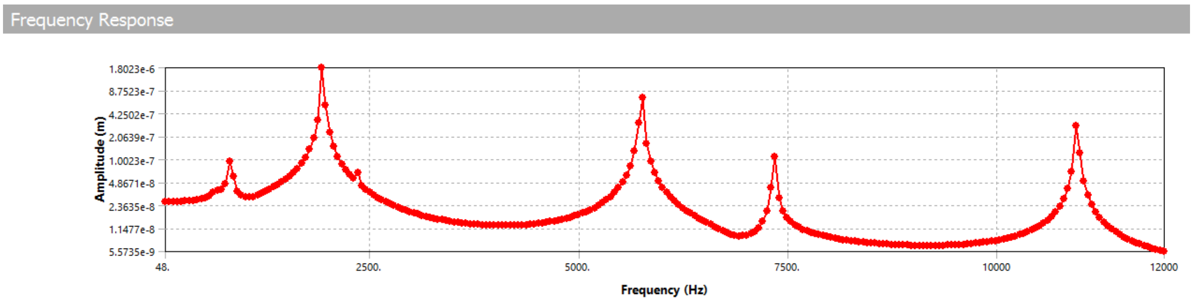


Figure 21: Frequency-domain response

The frequency-domain response shows the peaks along the audible spectrum at which the bells resonate when subjected to the 0.01 N impact force.

4.3. Fundamental frequency

Taking the behavior of the modal simulations into account, the harmonic response simulation's first peak is considered to represent the movement of the whole bell related to the handle, therefore, the fundamental resonant frequency of the bell is represented by the second peak of **Figure 21** (~1900 Hz). In order to obtain this graph from a real sound (time-domain) measurement (amplitude vs. time) an FFT (Fast Fourier Transform) is required in order to identify resonant frequencies in the sound being studied. It is possible to apply the inverse of the FFT (IFFT) to this plot to obtain a time-domain response.

In mathematics, however, it is known that an FFT is irreversible with so little knowledge of the original (time-domain) function. In this case, the frequency-domain response given by ANSYS does not contain enough information to obtain a sound wave function from it. However, the fundamental frequency represents the overall audible frequency produced by the bell.

5. Conclusions and recommendations

5.1. Conclusions

The historic importance of this relies on humankind’s ability and constant pursue of innovation. Dr. Lechtman (1996) proved in her study that such high concentrations of arsenic cannot be accidental or be a product of impurities. Humans, out of their own intellect, were altering the material in order to obtain more desirable mechanical properties and manufacturability.

Arsenic bronze is a very frequently used material throughout the world. It was superior than copper in many aspects, mostly because it was easier to work with, and was more ductile to perform cold work. The acoustic properties of such material have rarely been studied before. This study shows the modes at which the bells resonate and the frequencies that interact with each other to create the bells’ sound. The IFFT shows an amplitude in the time-domain, therefore it can be used to recreate the sound produced by the bells.

Currently, tin bronze is used as opposed to Arsenic Bronze, mainly because arsenic being a known carcinogen, but also because of the remarkable stability of the copper-tin alloy. Pure copper is now available and used because its thermal and electric conductivity. Moreover, the extraction of copper from minerals no longer requires calcinating them because of the existing more modern processes such as electrolysis.

It is crucial to consider that the simulations performed in this study assume that all properties throughout the material stay constant, which was impossible to achieve with the technologies developed by the populations who worked with the metals mentioned in this study. This means that the bells did not all have the same sound, and this study’s aim is not to determine one specific bell’s sound, but to show a sound that encompasses a wide variety of bells with similar characteristics.

The simulation gives an insight to how the bell behaves while vibrating; thus, it shows information about the sound it produced. Small bells like the ones in this study have been used in groups, usually tied together, to amplify the sound they make. As seen in the simulations, the amplitude of the vibrations is small, which means that the bells displace a very small volume of air while vibrating, creating a quiet sound.

The plate approximation gives a clear explanation to why the first three modes are to be ignored from the simulation, given that it is in the fourth mode of the bell where each flap bends in a similar way to the plate first mode.

5.2. Recommendations and future work

As future work, it is recommended to recreate the traditional manufacturing process, creating replicas of both cast and cold worked bells and experimentally measure their acoustic properties. Use standardized beams to test acoustic properties qualities parameters. The author also recommends a comprehensive archaeological study of the soil where the bells were found in order to have a better understanding of the current state of the bells and better preservation techniques for the specific material.

6. References

Chaigne, A., & Kergomard, J. (2016). Acoustics of Musical Instruments. Modern Acoustics and Signal Processing. doi:10.1007/978-1-4939-3679-3

De Ryck, I., Adriaens, A., & Adams, F. (2005). An overview of Mesopotamian bronze metallurgy during the 3rd millennium BC. *Journal of Cultural Heritage*, *6*(3), 261–268. https://doi.org/10.1016/j.culher.2005.04.002

Debut, V., Carvalho, M., Figueiredo, E., Antunes, J., & Silva, R. (2016). The sound of bronze: Virtual resurrection of a broken medieval bell. *Journal of Cultural Heritage*, *19*, 544–554. https://doi.org/10.1016/j.culher.2015.09.007

Díaz, J. A., Serrano, J., & Leiva, E. (2018). Bioleaching of arsenic-bearing copper ores. *Minerals*, *8*(5), 1–19. https://doi.org/10.3390/min8050215

Figueiredo, E., Silva, R. J. C., Braz Fernandes, F. M., & Araújo, M. F. (2010). Some long-term corrosion patterns in archaeological metal artefacts. *Materials Science Forum*, *636–637*, 1030–1035. https://doi.org/10.4028/www.scientific.net/MSF.636-637.1030

Garceau, M. E. (2011). “I call the people.” Church bells in fourteenth-century Catalunya. *Journal of Medieval History*, *37*(2), 197–214. https://doi.org/10.1016/j.jmedhist.2011.02.002

Gary, J. (2006). Linked references are available on JSTOR for this article. *Academy of Management Review*, *31*(2), 386–408. https://doi.org/10.1097/EDE.0b013e3181

Hansapinyo, C., & Poovarodom, N. (2014). Ambient Vibration Tests and Finite Element Analysis for Dynamic Properties of Brick Masonry Inverted Bell-shaped Chedi. *APCBEE Procedia*, *9*, 212–216. https://doi.org/10.1016/j.apcbee.2014.01.038

Hosler, Dorothy, The Sounds and Colors of Power: The Sacred Metallurgical Technology of Ancient West Mexico, Cambridge, MIT Press, 2002.

van Houten, M. H., Schoofs, A. J. G., and van Campen, D. H. (1997) Damping of bells using experimental and numerical methods. *Fifth international congress on sound and vibration december 15-18, 1997.* Adelaide, South Australia

Lechtman, H. (1996). Arsenic bronze: Dirty copper or chosen alloy? A view from the Americas. *Journal of Field Archaeology*, *23*(4), 477–514. https://doi.org/10.1179/009346996791973774

Lechtman, H. (1991). The production of copper-arsenic alloys in the central andes: Highland ores and coastal smelters? *Journal of Field Archaeology*, *18*(1), 43–76. https://doi.org/10.1179/009346991791548780

Mandal, B., & Suzuki, K. (1975). Arsenic round the world: a review. *Advances in Agronomy*, *27*(C), 305–374. https://doi.org/10.1016/S0065-2113(08)70013-0

McLachlan, N., Nigjeh, B. K., & Hasell, A. (2003). The design of bells with harmonic overtones. *The Journal of the Acoustical Society of America*, *114*(1), 505–511. https://doi.org/10.1121/1.1575748

Mechanical Science and Technology, *33*(9), 4345–4352. https://doi.org/10.1007/s12206-019-0830-z

Mott, P. H., Roland, C. M., & Corsaro, R. D. (2002). Acoustic and dynamic mechanical properties of a polyurethane rubber. *The Journal of the Acoustical Society of America*, *111*(4), 1782–1790. https://doi.org/10.1121/1.1459465

Palmer, J., et al (2007). Pre-Columbian metallurgy: Technology, manufacture, and microprobe analyses of copper bells from the Greater Southwest. *Archaeometry*. 40. 361 - 382.

Rao, S. S. (2011). *Mechanical vibrations*. Upper Saddle River: Prentice Hall.

Roebben, G., Bollen, B., Brebels, A., Van Humbeeck, J., & Van der Biest, O. (1997). Impulse excitation apparatus to measure resonant frequencies, elastic moduli, and internal friction at room and high temperature. *Review of Scientific Instruments*, 68(12), 4511–4515. doi:10.1063/1.1148422

Rossing, T. (1984). The Acoustics of Bells: Studying the vibrations of large and small bells helps us understand the sounds of one of the world's oldest musical instruments. American Scientist, 72(5), 440-447. Retrieved from www.jstor.org/stable/27852858

Rossing, T. D., Hampton, D. S., Richardson, B. E., Sathoff, H. J., & Lehr, A. (1988). Vibrational modes of Chinese two-tone bells. *Journal of the Acoustical Society of America*, *83*(1), 369–373. https://doi.org/10.1121/1.396250

Sathoff, H. J., & Rossing, T. D. (1982). Modes of vibration and scaling of handbells. *The Journal of the Acoustical Society of America*, *71*(S1), S63–S64. https://doi.org/10.1121/1.2019494

Sathoff, H. J., & Rossing, T. D. (1980). Modes of vibration and sound radiation from tuned handbells. *Journal of the Acoustical Society of America*, *68*(6), 1600–1607. https://doi.org/10.1121/1.385214

Sproßmann, R., Zauer, M., & Wagenführ, A. (2017). Characterization of acoustic and mechanical properties of common tropical woods used in classical guitars. *Results in Physics*, *7*, 1737–1742. https://doi.org/10.1016/j.rinp.2017.05.006

Strong, W. J. (1985). Acoustics of Bells edited by Thomas D. Rossing . *The Journal of the Acoustical Society of America*, *78*(4), 1447–1448. https://doi.org/10.1121/1.392879

Thornton, C. P., Lamberg-Karlovsky, C. C., Liezers, M., & Young, S. M. M. (2002). On pins and needles: Tracing the evolution of copper-base alloying at Tepe Yahya, Iran, via ICP-MS analysis of common-place items. *Journal of Archaeological Science*, *29*(12), 1451–1460. https://doi.org/10.1006/jasc.2002.0809

Zhang, H. M., Shang, D. G., Lv, S., Liu, X. D., & Zhang, Y. (2019). Finite element simulation of unconstraint vibration treatment for fatigued copper film. *Journal of Mechanical Science and Technology*